VACUUM RENORMALIZATION OF THE CHIRAL-SIGMA MODEL AND THE STRUCTURE OF NEUTRON STARS

Author
Glendenning, N.K.

Publication Date
1987-07-01

LBL-23818
Preprint c.2

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Submitted to Nulcear Physics

Vacuum Renormalization of the Chiral-Sigma Model and the Structure of Neutron Stars

N.K. Glendenning

July 1987

NUCLEAR SCIENCE DIVISION

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

DISCLAIMER

LBL-23818

Vacuum Renormalization of the Chiral-Sigma Model and the Structure of Neutron Stars†

Norman K. Glendenning

Nuclear Science Division

Lawrence Berkeley Laboratory

University of California

Berkeley, California 94720

July 7, 1987

†This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

Vacuum Renormalization of the Chiral-Sigma Model and the Structure of Neutron Stars†

Norman K. Glendenning

Nuclear Science Division

Lawrence Berkeley Laboratory

University of California

Berkeley, California 94720

July 7, 1987

Abstract

Vacuum renormalization corrections are calculated for normal nuclear matter and neutron star matter in the chiral-sigma model. The theory is generalized to include hyperons in equilibrium with nucleons and leptons. The equations of state corresponding to two compression moduli, a 'stiff' and 'soft' one for nuclear matter, are studied. It is shown that fully one half the mass of a neutron star at the limiting mass is composed of matter at less than twice nuclear density. Neutron star masses are therefore moderately sensitive to the properties of matter near saturation and to the domain of the hyperons, but dominated by neither. The predictions for the two equations of state are compared with observed neutron star masses, and only the stiffer is compatible.

PACS 11.10.Gh, 21.10-k, 21.65.+f, 95.30.Cq, 95.70.Np, 97.60.Jd

---

†This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

Contents

Vacuum Renormalization of the Chiral-Sigma Model and the Structure of Neutron Stars

Norman K. Glendenning

July 7, 1987

1 Introduction

In the last few years there has been great interest in relativistic nuclear field theory, both concerning the normal state of nuclear matter and nuclei [1] and states of matter under extreme conditions. Examples of the latter are high density as is found in neutron stars [2] or transiently in high energy nuclear collisions and which may induce a phase transition to a pion-condensed phase [3], or high temperature as may induce a phase transition to abnormal matter, characterized by low effective masses and an abundance of baryon pairs [4]. Presently, the only relativistic field theory that can account for normal nuclei is the scalar-vector-isovector theory involving the coupling of σ, ω and ρ mesons to the baryons [1]. This theory is very successful in describing a large number of nuclear properties but unfortunately is not invariant under the chiral transformation whose symmetry is known to be rather well satisfied at the elementary level. On the other hand the chiral-sigma theory [5] cannot even describe the saturation curve of normal matter [6] unless vacuum corrections are taken into account [7]. Even then, it seems not possible to find solutions for *finite* nuclei that describe normal nuclei [8]. Instead the lowest energy solutions correspond to bubble nuclei, that have the baryon density concentrated on the surface.

Nevertheless, it is of interest to investigate the uniform matter properties of the theory, particularly in the event that a solution to the above dilemma can be found. In this paper we shall take into account vacuum corrections, which are essential to obtain a normal saturation curve [7]. Since we are interested in calculating neutron star properties predicted by this theory, it is essential to generalize it to include the hyperons, because these baryon states become favorable in high density neutron star matter [2]. This raises the question, both for nuclear and for high density matter, as to which baryons are assumed to possess a filled negative-energy sea. We resolve this question by reference to the underlying theory of which the hadronic theory is assumed to be an effective one, valid in the density domain below the quark-gluon phase transition. The stable quarks are the u and d, and these therefore have filled seas. The others are composites on a long time scale. Accordingly renormalization should be carried out on the nucleon only.

Hyperons form an important component in neutron stars, and have been shown to soften the equation of state appreciably in the moderate to high baryon density domain [9]. As previously noted, this could be a critical factor in the first bounce mechanism for supernovae, since the time scales of star collapse are long (seconds) compared to the electroweak processes involved in relaxation of dense nucleon matter into hyperon matter [9]. The situation is opposite in high energy nuclear collisions which are fast compared to these processes so that net strangeness is not developed. Accordingly, for application to neutron stars we generalize the chiral-sigma model to include hyperons.

In the following sections we summarize the chiral-sigma model, derive an expression for the symmetry energy coefficient, fit the parameters of the theory to nuclear matter properties, assuming a 'soft' and 'stiff' compression modulus, and exhibit the corresponding equations of state for nuclear and pure neutron matter, and the various contributions to it. Subsequently, the hyperons are incorporated, and their effect on the equation of state of neutron star matter is computed. The equilibrium composition of such matter is discussed. The structure of neutron stars is calculated, and the effects of beta stability evaluated. Redshifts, masses, radii and binding energies for the two cases of 'stiff' and 'soft' nuclear matter equation of state are computed.

2 The Chiral-Sigma Model

The Lagrangian for the chiral-sigma model is

$$\begin{aligned} \mathcal{L}_\sigma &= \bar{\psi}_N[i\gamma_\mu\partial^\mu - g(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})]\psi_N \\ &+ \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}) - \tfrac{1}{4}\lambda(\sigma^2 + \boldsymbol{\pi}\cdot\boldsymbol{\pi} - \sigma_0^2)^2 \end{aligned} \tag{1}$$

to which we add the Lagrangians for the vector and vector-isovector mesons, ω and $\boldsymbol{\rho}$, which are coupled to the conserved baryon and isovector currents respectively. They give rise to an energy density that is quadratic in the baryon density and isospin density respectively. The first is essential for saturation of nuclear matter and the second is important in accounting for the observed asymmetry energy of nuclear matter, and is of course important to a description of the very isospin asymmetric neutron star matter. Thus,

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_\sigma + [-\tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu] + [-\tfrac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \tfrac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu] \\ &\quad -\bar{\psi}_N[g_\omega\gamma_\mu\omega^\mu + \tfrac{1}{2}g_\rho\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu\cdots]\psi_N \end{aligned} \tag{2}$$

The ellipsis represent other parts of the isospin current contributed by the ρ and π mesons which in principle are present, but which vanish in the normal state of matter [10]. Bold face characters are used to denote isospin vectors, and the notational conventions of ref. [11] are used for the metric and Dirac-gamma matrices.

In this paper we shall be concerned only with the normal non-pion-condensed state of matter, so we take $\pi \equiv 0$, and inconsequentially therefore, also $m_\pi = 0$. The constants appearing in the Lagrangian can be related to the masses through,

$$m_N = g\sigma_0 \tag{3}$$

$$m_\sigma = \sqrt{2\lambda}\sigma_0 \tag{4}$$

The Dirac equation for the baryons is the Euler-Lagrange equation of $\mathcal{L}$, and is readily obtained as,

$$[\gamma_\mu(p^\mu - g_\omega\omega^\mu - \tfrac{1}{2}g_\rho\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu) - g\sigma]\psi_N = 0 \tag{5}$$

This can be easily solved in the mean field approximation. That means that we consider uniform static matter in which the meson fields are replaced by their mean values in the ground state. Without change in notation the meson fields will denote henceforth the mean value, $\sigma = \langle\sigma\rangle$, etc. Then the Dirac equation above can be solved immediately. The corresponding coupling terms in eq.(5) appear also in the Euler-Lagrange equations for the mesons. These source currents are replaced by their expectation value in the ground state defined with respect to the occupation of the above Dirac states up to the Fermi momentum for the desired density. This then poses a self-consistent problem for the determination of the mean meson amplitudes. The mass term in eq.(5) appears in the form $g\sigma$, which is referred to as the *effective* nucleon mass,

$$m_N^\star = g\sigma \tag{6}$$

to be compared with the vacuum mass, eq.(3). The energy eigenvalue is readily obtained as,

$$\epsilon_N(k) = g_\omega\omega_0 + g_\rho\rho_{03}I_{3N} + (k^2 + {m_N^\star}^2)^{1/2} \tag{7}$$

where I_{3N} is the isospin of the baryon (nucleon) N. In writing this we have already anticipated the vanishing of the space-components of ω_μ and ρ_μ (and the isospin 2 and 3 components of the latter). Otherwise the momentum in eq.(7) would be shifted.

The energy density can be found as the diagonal time-component of the stress-energy tensor,

$$\mathcal{T}_{\mu\nu} = -g_{\mu\nu}\mathcal{L} + \sum \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}\partial_\nu\phi \tag{8}$$

where the sum is over the various fields of the Lagrangian, denoted by ϕ. The energy density is then found to be,

$$\begin{aligned}
\epsilon &= -\tfrac{1}{2}m_\omega^2\omega_0^2 - \tfrac{1}{2}m_\rho^2\rho_{03}^2 + \tfrac{1}{4}\lambda(\sigma^2-\sigma_0^2)^2 + \sum_N \frac{2J_N+1}{2\pi^2}\int_0^{k_N}\epsilon_N(k)k^2dk \\
&\qquad -\frac{m_N^4}{8\pi^2}F(\eta) + \frac{m_\sigma^4}{64\pi^2}F(\Delta) \\
&= \tfrac{1}{2}m_\omega^2\omega_0^2 + \tfrac{1}{2}m_\rho^2\rho_{03}^2 + \tfrac{1}{4}\lambda(\sigma^2-\sigma_0^2)^2 \\
&\qquad + \frac{{m_N^\star}^4}{4\pi^2}\sum_N [xu^3 - \tfrac{1}{2}xu - \tfrac{1}{2}\ln(x+u)] \\
&\qquad -\frac{m_N^4}{8\pi^2}F(\eta) + \frac{m_\sigma^4}{64\pi^2}F(\Delta)
\end{aligned} \tag{9}$$

The last two terms have been separately added and represent the one-loop expressions for the renormalization of the energy arising from shifts in the spectrum of the filled Fermi sea of nucleons and the zero point energy of the σ field, in comparison with the vacuum [7].

It has not been found how to calculate the renormalization of the ρ-meson. In the uniform matter case, the only contribution of this meson is a term in the energy density that is quadratic in the isospin density. We may regard this as a phenomenological term, and determine the coupling by the empirical symmetry energy.

Since the baryon eigenvalues, eq.(7), are independent of spin, but depend on isospin projection, the sum over the spin-isospin states of N are expressed in eq.(9) by $(2J_N+1)$ where J_N denotes the spin and the sum on N is over the isospin states of the nucleon. We have used the other definitions,

$$x = k_N/m_N^\star\ , \qquad u = \sqrt{1+x^2} \tag{10}$$

$$\eta = (\sigma/\sigma_0)^2 - 1 \tag{11}$$

$$\begin{aligned}\Delta &= \tfrac{3}{2}\eta + \sum_N \frac{2J_N+1}{2\pi^2}\frac{g^2}{m_\sigma{}^2}\int_0^{k_N}(k^2+m_N^\star{}^2)^{-3/2}k^4dk \\ &= \tfrac{3}{2}\eta + \frac{g^2 m_N{}^2}{\pi^2 m_\sigma{}^2}\sum_N[\tfrac{1}{2}xu + x/u - \tfrac{3}{2}\ln(x+u)]\end{aligned} \tag{12}$$

$$F(y) = (1+y)^2\ln(1+y) - y - \tfrac{3}{2}y^2 \tag{13}$$

The field equation for the scalar field can be obtained as the condition that the energy is stationary at fixed baryon density,

$$\rho = \frac{2}{3\pi^2}k_F{}^3 \tag{14}$$

namely,

$$\lambda\sigma(\sigma^2-\sigma_0^2) + \frac{g m_N^\star{}^3}{2\pi^2}\sum_N[xu - \ln(x+u)] - \frac{m_N^4}{8\pi^2}\frac{\partial F(\eta)}{\partial\sigma} + \frac{m_\sigma^4}{64\pi^2}\frac{\partial F(\Delta)}{\partial\sigma} = 0 \tag{15}$$

The field equations for the ω and ρ are simply,

$$g_\omega\omega_0 = \left(\frac{g_\omega}{m_\omega}\right)^2\rho \tag{16}$$

$$g_\rho\rho_{03} = \left(\frac{g_\rho}{m_\rho}\right)^2\frac{\rho_n-\rho_p}{2} \tag{17}$$

$$\omega_\nu = \rho_\nu = \rho_{\mu,i} = 0, \quad (\nu = 1,2,3),(i=1,2) \tag{18}$$

where the the Lorentz indices are denoted by Greek, and isospin indices by Roman letters. The formal proofs of eq.(18) can be found in ref. [10]. However the physical reason in the case of space-components is that the source term in the Euler-Lagrange equation for these will contain the vector γ, whose expectation over a uniform system (isotropic) vanishes. In the case of the 2 and 3 isospin components of $\boldsymbol{\rho}$, these correspond to the charged mesons, and their diagonal expectation therefore vanishes to conserve charge. The densities and Fermi momenta are related by

$$k_n = k_F(1+t)^{1/3}, \qquad k_p = k_F(1-t)^{1/3} \tag{19}$$

$$t = \frac{\rho_n - \rho_p}{\rho} \tag{20}$$

where k_F is the Fermi momentum defined in eq.(14) for symmetric matter.

For neutron stars it is very important to control the symmetry energy. To compute it in this theory we isolate the parts of the energy that depend on $\rho_n - \rho_p$. The symmetry energy coefficient is the coefficient of t^2 in ϵ/ρ, which we can find as,

$$\begin{aligned} a_{sym} &= \tfrac{1}{2}\Big(\frac{\partial^2(\epsilon/\rho)}{\partial t^2}\Big)_{t=0} \\ &= \Big(\frac{g_\rho}{m_\rho}\Big)^2 \frac{k_F{}^3}{12\pi^2} + \frac{k_F{}^2}{6(k_F{}^2 + m_N^\star{}^2)^{1/2}} \\ &+ \frac{3m_\sigma{}^4}{256k_F{}^3}\Big[\frac{\partial^2 F(\Delta)}{\partial\Delta^2}\Big(\frac{2g^2}{3\pi^2 m_\sigma{}^2}\frac{k_F{}^5}{(k_F{}^2+m_N^\star{}^2)^{3/2}}\Big)^2 \\ &+ \frac{\partial F(\Delta)}{\partial\Delta}\frac{2g^2}{3\pi^2 m_\sigma{}^2}\frac{k_F{}^5}{(k_F{}^2+m_N^\star{}^2)^{3/2}}\Big(\frac{2}{3} - \frac{k_F{}^2}{k_F{}^2+m_N^\star{}^2}\Big)\Big] \end{aligned} \tag{21}$$

The pressure can be found as any one of the diagonal space-components of the stress-energy tensor, and the contribution of the vacuum fluctuation terms can be separately calculated from $p = \rho(d\epsilon/d\rho) - \epsilon$. For systems at constant relative neutron excess, t, (such as nuclear or pure neutron matter) we find,

$$
\begin{aligned}
p &= \tfrac{1}{2}m_\omega^2\omega_0^2 + \tfrac{1}{2}m_\rho^2\rho_{03}^2 - \tfrac{1}{4}\lambda(\sigma^2-\sigma_0^2)^2 \\
&\quad + \frac{{m_N^\star}^4}{12\pi^2}\sum_N [x^3 u - \tfrac{3}{2}xu + \tfrac{3}{2}\ln(x+u)] + \frac{m_N^4}{8\pi^2}F(\eta) \\
&\quad + \frac{m_\sigma^4}{64\pi^2}\Big[-F(\Delta) + \frac{g^2}{6\pi^2 m_\sigma^2}\frac{\partial F(\Delta)}{\partial\Delta}\frac{1}{k_F}\sum_N \frac{k_N^6}{(k_N^2 + {m_N^\star}^2)^{3/2}}\Big]
\end{aligned} \tag{22}
$$

In fitting the theory to the empirical properties of nuclear matter, we shall use the three coupling constants, g, g_ω, g_ρ and the scalar meson mass, m_σ. The vector meson masses are taken from experiment and are $m_\omega = 783\ MeV$ and $m_\rho = 770\ MeV$. Since the value of the nuclear compression modulus is currently debated, we shall use two values for K, one that we refer to as soft, $K = 200\ MeV$, and one that is stiff, $K = 300\ MeV$. The matter properties are shown in Table 1 and the corresponding coupling constants in Table 2. The 'experimental' values quoted for the first three properties in Table 1 are taken from ref. [12]. The effective nucleon mass at saturation is taken from ref. [13].

The equation of state is shown in Fig. 1 and 2 for nuclear matter and pure neutron matter, in the form of binding energy per nucleon as a function of density,

$$B/A = \epsilon/\rho - m_N. \tag{23}$$

Table 1: Nuclear Matter Properties at Saturation

	ρ_0 (fm^{-3})	B/A (MeV)	a_{sym} (MeV)	K (MeV)	$\frac{m_N^\star}{m_N}$
soft	.151	-16.3	32.5	200.	.865
stiff	.152	-16.2	32.5	301.	.838
expt.	.153	-16.3	32.5	?	.83

Also shown are the two contributions V_N and V_σ to the shift of the nucleon and σ vacua, whose sum is V, the two-body contribution E_2 and the sum of the three- and four-body contributions, $E_3 + E_4$. The nucleon renormalization is seen to be strong and repulsive. It almost cancels the two-body attractive contribution in the region of nuclear saturation, but saturates at higher density. Recall that the solution to eq.(15) in the absence of the renormalization terms yields a saturation curve that bears no resemblance to that of normal nuclear matter. Instead the normal state is bifurcated by an abnormal state and terminates.

The 'stiff' and 'soft' equation of state for nuclear matter are compared in Fig. 3.

3 Inclusion of Hyperons

At the high density of neutron stars, where typical interior densities are five times normal nuclear density, the nucleons at the top of the Fermi sea would have more energy than the masses of many of the hyperons. In this case

Table 2: Parameters of the Theory

	$\frac{g_\sigma^2}{4\pi}$	$\frac{g_\omega^2}{4\pi}$	$\frac{g_\rho^2}{4\pi}$	m_σ (MeV)
soft	16.8	2.74	6.3	982
stiff	18.24	3.514	6.08	1071.5

they will be converted through interactions to other baryons which have unfilled eigenstates that lie below the nucleon Fermi momentum. In normal matter, the associated kaons obey the field free Klein-Gordan equation and therefore decay. Strangeness is not conserved on the time-scale of the star. Any neutrinos or gamma rays produced escape from the star and a lower ground state is found consisting of nucleons, hyperons, electrons, muons and perhaps a pion condensate in general equilibrium. We have studied the resulting equilibrium mixture for the scalar-vector-isovector theory previously [2]. The equation of state is softened at the thresholds for the conversion of nucleons to hyperons and we have shown that the limiting mass of neutron stars can be lowered by as much as 1/2-3/4 $M_\odot$ in comparison to the case where ordinary beta equilibrium is established between neutrons, protons and leptons. Therefore we need to generalize the above theory to include the hyperons. We could also include the delta, but the favored charge state, Δ^-, having the same sign for its isospin projection as the neutron and value -3/2, is strongly isospin unfavored by the symmetry energy, and does not appear in the density range of neutron stars [2].

From eq.(3), the obvious generalization of the scalar coupling [14] is

$$\sigma_0 = m_N/g = m_\Lambda/g_\Lambda = m_\Sigma/g_\Sigma \cdots \tag{24}$$

The nucleon terms, N, in eq.(1,2,9,12,15) should be interpreted as sums over the *charge* states of $N, \Lambda, \Sigma, \Xi, \cdots$. The pressure in this general case can be computed from $p = \rho(d\epsilon/d\rho) - \epsilon$. In addition, we add the energy and pressure of the leptons.

Neutron stars are electrically neutral, and they are in chemical equilibrium. The field equations, conditions of chemical equilibrium expressed through the Fermi momenta and the two chemical potentials, μ_e and μ_n, corresponding to the conserved electric charge and baryon number, and condition of charge neutrality, comprise a system of non-linear equations in the quantities,

$$\sigma, \omega_0, \rho_{03}, \mu_n, \mu_e, k_e, k_\mu, k_n, k_p, k_\Lambda, k_{\Sigma^-}, k_{\Sigma^0} \cdots k_{\Xi^-}, \cdots \tag{25}$$

of which there are (7+n) where n is the number of nucleon and hyperon charge states that come into equilibrium at the relevant densities. The field equations and their modifications due to the hyperons were discussed above. The other conditions have been described elsewhere [2].

The equilibrium admixture of nucleons, hyperons and leptons is shown in Fig. 4 for $K = 200\ MeV$ for neutron star matter, which is charge neutral matter in equilibrium. The results are similar for $K = 300\ MeV$. Low density charge neutral uniform matter is of course composed only of neutrons. The Fermi energy of the neutron increases with density and already at $\rho \approx .02 fm^{-3}$ exceeds the threshold for neutron decay to proton and electron. (The neutrinos leak out of the neutron star so there is no Fermi sea of neutrinos.) The next threshold is for muons, which occurs when the electron chemical potential exceeds the muon mass. Thresholds for hyperons occur when

$$\mu_H \equiv \mu_n - q_H \mu_e = \epsilon_H(0) \tag{26}$$

where q_H is the hyperon electric charge and ϵ_H is its eigenvalue given by eq.(7). The first such threshold occurs for the Λ at about three times nuclear density. At not much higher density, the Λ and proton together comprise almost 40 percent of the baryons.

Below the hyperon thresholds, the electron chemical potential is a rapidly increasing function of density, which is reflected in the rapid increase in the lepton populations in the lower density domain. However when the hyperon threshold is reached, charge neutrality can be more economically maintained through the conversion of neutrons to Λ's or protons to Ξ^-'s, etc., rather than neutron beta decay to proton and relativistic electron. The rapid increase in μ_e is therefore arrested, and it never exceeds 280 MeV in the density domain of neutron stars. As observed before [2], this makes kaon condensation very unlikely.

The equation of state for neutron star matter corresponding to the two cases of Table 1 are shown in Fig. 5 and 6. Three comparisons are made in each case, pure neutron matter, matter in which neutrons and protons are in equilibrium with leptons, and the full generalized equilibrium consisting of nucleons, hyperons and leptons. The softening of pure neutron matter by beta decay of some neutrons to protons is evident by the shift to lower pressure, and the additional softening due to hyperons at their thresholds is clearly evident. All three lie below the causal limit, $p = \epsilon$, and reach it only asyptotically far beyond the domain of neutron star densities.

4 Neutron Stars

The structure of a neutron star corresponding to a particular equation of state is found by solving the Oppenheimer-Volkoff equations, which are the special form that Einstein's equations take for spherically symmetric static bodies,

$$4\pi r^2 dp(r) =$$

$$-\frac{GM(r)dM(r)}{r^2}\left(1+\frac{p(r)}{\epsilon(r)}\right)\left(1+\frac{4\pi r^3 p(r)}{M(r)}\right)\left(1-\frac{2GM(r)}{r}\right)^{-1} \quad (27)$$

$$dM(r) = 4\pi r^2 \epsilon(r)\, dr \quad (28)$$

The first equation expresses the condition of hydrostatic equilibrium and is written to display the balance between the force acting on the inner surface of a shell of matter at r due to the net pressure of matter, and the force of gravity acting on the mass, dM, of the shell. The second expresses the mass energy of the shell. For each chosen density of matter at the center of the star, $\epsilon_c = \epsilon(0)$, the equations have a unique solution, yielding a radius, gravitational mass, and chemical composition of the star as well as the distribution of energy density in the star. These quantities are continuous functions of the central density. The mass, as a function of central density, has a maximum value referred to as the limiting mass, for the particular equation of state. Configurations with central densities above that corresponding to the limiting mass are unstable black holes. The limiting mass is especially interesting because an acceptable theory of matter must be able to account for neutron stars whose masses are known. The most accurately measured mass is for PSR1913+16 with $M = 1.451 \pm 0.007 M_\odot$ [15]. The largest measured mass is $1.85^{+0.35}_{-0.30}$ for 4U0900-40. Until recently, measurements of neutron star masses were interpreted as though the stars belonged to a population all having the same mass. In this interpretation the common mass compatible with the existing measurements and their errors is $1.4 \pm 0.2 M_\odot$ [16]. It has been pointed out recently that the theoretical prejudice underlying this interpretation is no longer justified in view of recent developments [17]. In this case the constraint on theory must be taken provisionally as the largest mass that is apparently observed, namely $1.85 M_\odot$.

In Fig. 7 and 8 we show the calculated masses corresponding to the soft and stiff equations of state for pure neutron matter, beta stable neutron-proton matter, and the case of generalized beta equilibrium of nucleons, hyperons and leptons. The last is the one that provides the limiting mass of the corresponding equation of state, the first two being shown to illustrate the magnitude of the effects of beta stability. The effects on the limiting mass are appreciable, neutron-proton stability and nucleon-hyperon stability amounting each to about $1/4 M_\odot$. These effects are not as large as those found for the scalar-vector-isovector theory [9].

We conclude that $K = 200\ MeV$ is marginally compatible with the limit $1.4 \pm 0.2 M_\odot$ in agreement with our earlier analysis of the constraints placed on K by neutron stars [18]. However it cannot account for the mass $1.85 M_\odot$. In fact the "stiff" equation of state can only marginally account for the latter (ie. compatible within the lower error limit).

Now we address the question as to what density regions of the equation of state the mass of neutron stars are sensitive. This is answered in Fig. 9 for the star at the limiting mass. There the fraction of mass of the star, $M(\rho)/M$, that is composed of matter at baryon density greater than ρ is shown. From this figure we learn that about 85 percent of the star's mass resides in matter that is at densities greater than nuclear ($\rho = 0.153 fm^{-3}$) but that half the mass is composed of matter that is at densities less than twice nuclear density! More precisely, 15 % of the mass is contributed by matter that is *below* nuclear density, 35 % between nuclear and twice nuclear, and the remaining 50 % by matter at more than twice nuclear density. Thus the sensitivity to the compression modulus arises in two ways. First although K is a property at saturation density, by continuity of the theory and the causal constraint that the speed of sound in matter cannot exceed the speed of light, the stiffness or softness at saturation is reflected also in the high density equation of state. Secondly, because of the three dimensional geometry, much of the mass of the star is composed of matter at moderate density. The plateau region in Fig. 9 corresponds to the threshold for hyperons that is registered in the equation of state of Fig. 5. The fact that only about a third of the star's mass is contributed by matter at densities above this threshold accounts for the moderate dependance of the limiting mass on the presence of hyperons as was noted above. The central baryon density for the limiting mass star is 6.2 and 5.9 normal nuclear density for the soft and stiff equations of state, respectively.

Gamma-ray bursters may ultimately provide interesting constraints on theories of neutron star structure, *if* the spectral line between 300 and 500 KeV [19] can be unambiguously associated with the gravitational redshift of electron-positron annihilation at the star's surface, *and* the mass of the burster is also determined. At present *corresponding* redshift and mass measurements do not exist. In Fig. 10 the surface gravitational redshift is shown as a function of star mass. It is defined as the fractional shift in the wavelength of light emitted from the surface of the star, and is given by,

$$z = \frac{\Delta\lambda}{\lambda} = e^{\lambda(R)/2} - 1 \tag{29}$$

where the radial metric function is given by,

$$\begin{aligned} e^{-\lambda(r)} &= 1 - \frac{8\pi}{r}\int_0^r \epsilon(r) r^2 dr \\ &= 1 - \frac{2M}{r}, \quad for\ r > R \end{aligned} \tag{30}$$

where M is the star mass, R its radius and $\epsilon(r)$ is the radial distribution of the energy density in the star as obtained from the solution of the Oppenheimer-Volkoff equations.

In Fig. 11 the radius-mass relationship is shown. The radii of the lower mass neutron stars grow very rapidly as the mass decreases, corresponding

Table 3: Properties of Neutron stars at the limiting mass, the central baryon density in units of normal nuclear density, radius, gravitational mass in solar mass units, binding energy in solar mass units, total baryon number, A, and surface gravitational redshift.

	ρ_c/ρ_0	R (km)	$M_{lim}/M_\odot$	$B/M_\odot$	A (10^{57})	z
soft	6.22	12.0	1.30	0.150	1.73	0.215
stiff	5.90	12.3	1.65	0.226	2.23	0.286

to the imminent onset of the instability of bodies whose central density lies between the domain of white dwarfs and neutron stars ($10^9 \lesssim \epsilon_c \lesssim 10^{14}$ g/cm^3). The binding of neutron stars is shown in Fig. 12, and the loss of binding as the lower mass limit is reached from above corresponds to the rapid growth in the radius. The properties of stars at the limiting mass for the two equations of state are summarized in Table 3.

5 Summary

We have studied the vacuum renormalization effects for nuclear and pure neutron matter, and for electrically neutral dense neutron star matter in generalized equilibrium. Two sets of coupling constants were found that yield the empirical saturation density, binding and symmetry energy, but which have compression moduli $K = 200\ MeV$ and $K = 300\ MeV$ for nuclear matter. In both cases, hyperons become appreciable components in the higher density domain of neutron star matter and hence in the cores of neutron stars. They cause a dramatic softening in the equation of state at their threshold at about 3.3 nuclear density. The effect on the limiting neutron star mass of generalized beta equilibrium, as compared to pure neutron matter is a reduction of 1/2 to 3/4 $M_\odot$ depending on whether the equation of state is otherwise stiff or soft. Because the time scale of core collapse is long compared to the electroweak processes involved in the conversion of nucleons to hyperons, it should be taken into account in supernova simulations. About half of this energy is contributed by neutron beta decay to proton and the other half to nucleon conversion to hyperons. The central density of the limiting mass star is 5.9 and 6.2 times the baryon density of normal nuclei, in the two cases, respectively, but fully one half of the mass of the star is contributed by matter at less than twice nuclear density. Consequently neutron star properties depend on the properties of matter in the domain near saturation as well as the domain of the hyperons, and is dominated by neither. Some neutron star properties were calculated for both cases. The softer of the two cases cannot support a star of the mass observed for PSR1913+16 ($M = 1.451 \pm 0.007 M_\odot$). The stiffer has limiting mass of 1.64 $M_\odot$, and can account for the above star, and provisionally for 4U0900-40 ($M = 1.85^{+0.35}_{-0.30} M_\odot$). Although the model is quite different in structure from the scalar-vector-isovector model, the above conclusions are in close agreement with those arrived at previously for the latter model [18,17].

Acknowledgements: I thank Dieter Hartmann for drawing my attention to gamma ray bursters and F. Klinkhamer for discussions on renormalization of the baryons. This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

References

[1] M. Jamion, C. Mahaux and P. Rochus, Phys. Rev. Lett. **43** (1979) 1097; Phys. Rev. **C22** (1980) 2027; B. D. Serot and J. D. Walecka, Phys. Lett. **87B** (1980) 172; C. J. Horowitz and B. D. Serot, Nucl. Phys. **A368** (1981) 413; J. Boguta, Nucl. Phys. **A372** (1981) 386; J. R. Shepard, J. A. McNeil and S. J. Wallace, Phys. Rev. Lett. **50** (1983) 1443; B. C. Clark, S. Hanna, R. L. Mercer and B. D. Serot, Phys. Rev. Lett. **50** (1983) 1644; A. Bouyssy, S. Marcos and Pham van Thieu, Nucl. Phys. **A422** (1984) 541; P.-G. Reinhard, M. Rufa, J. Marhun, W. Greiner and J. Freidrich, Z. Phys. **A323** (1986) 13.

[2] N. K. Glendenning, Astrophys. J. **293** (1985) 470; N. K. Glendenning, Z. Phys. **A 326** (1987) 57; N. K. Glendenning, Phys. Lett. **114B** (1982) 392.

[3] B. Banerjee, N. K. Glendenning and M. Gyulassy, Nucl. Phys. **A361** (1981) 326; N. K. Glendenning, P. Hecking and V. Ruck, Ann. Phys. (N.Y.) **149** (1983) 22.

[4] S. I. A. Garpman, N. K. Glendenning and Y. J. Karant, Nuc. Phys. **A322** (1979) 382; N. K. Glendenning, *Phase Transitions in Nuclear*

Matter in 7th High Energy Heavy Ion Study, 1984, ed. R. Bock, H. H. Gutbrod and R. Stock (GSI report-85-10); N. K. Glendenning, Phys. Lett. **144B** (1984) 158; J. Theis, G. Grabner , G. Buchwald, J. Maruhn, W. Greiner, H. Stocker, J. Polonyi, Phys. Rev. **D28** (1983) 2286; T. Nakai and S. Takagi, Prog. Theor. Phys. **71** (1984) 1118; N. K. Glendenning, Phys. Lett. **185B** (1987) 275; N. K. Glendenning, Nucl. Phys. **A469** (1987) 600.

[5] M. Gell-Mann and M. Levy, Nuovo Ciment. **16** (1960) 53.

[6] A. K. Kerman and L. D. Miller, in "*Second High Energy Heavy Ion Summer Study*" (1974) LBL-3675.

[7] T. D. Lee and G. C. Wick, Phys. Rev. **D9** (1974) 2291; T. D. Lee and M. Margulies, Phys. Rev. **D11** (1975) 1591; M. Margulies, Phys. Rev. **D13** (1976) 1642; E. M. Nyman and M. Rho, Nucl. Phys. **A268** (1976) 408; S. A. Chin, Ann. Phys. (N.Y.) **108** (1977) 301; A. D. Jackson, M. Rho and E. Krotscheck, Nucl. Phys. **407** (1983) 495; B. D. Serot and J. D. Walecka, *The Relativistic Nuclear Many-Body Problem*, in "Advances in Nuclear Physics", eds. J. W. Negele and E. Vogt, 1986; T. L. Ainsworth, E. Baron, G. E. Brown, J. Cooperstein, M. Prakash, Nucl. Phys. **A464** (1987) 740.

[8] J. Kunz, D. Masak and U. Post, Phys Lett. **186B** (1987) 124.

[9] N. K. Glendenning, Z. Phys. A, **327** (1987).

[10] N. K. Glendenning, B. Banerjee and M. Gyulassy, Ann. Phys. (N. Y.) **149** (1983) 1.

[11] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* McGraw Hill, N. Y. (1964).

[12] P Moller, W. D. Myers, W. J. Swiatecki and J. Treiner, Submitted to Atomic Data and Nuclear Data Tables, (1986).

[13] C. H. Johnson, D. J. Horen and C. Mahaux, preprint, *Unified Description of the Neutron -* ^{208}Pb *Mean Field Between -20 and +165 MeV from the Dispersion Relation Constraint*

[14] N. K. Glendenning, Ann. Phys. (N. Y.) **168** (1986) 246.

[15] J. M. Weisenberg and J. H. Taylor, Phys. Rev. Lett. **52** (1984) 1348; J. H. Taylor (private communication, April 1987).

[16] P. C. Joss and S. A. Rappaport, Ann. Rev. of Astr. and Astrophys. **22** (1984) 537.

[17] N. K. Glendenning, *Nuclear Equation of State from Nuclear Masses, Relativistic Collisions, Supernova Explosions and Neutron Star Masses*, LBL-23081.

[18] N. K. Glendenning, Phys. Rev. Lett. **57** (1986) 1120.

[19] E. P. Mazets, S. V. Golenetski, R. L. Aptekar, Y. A. Guran and V. N. Ilyinskii, Nature **290** (1981) 378.

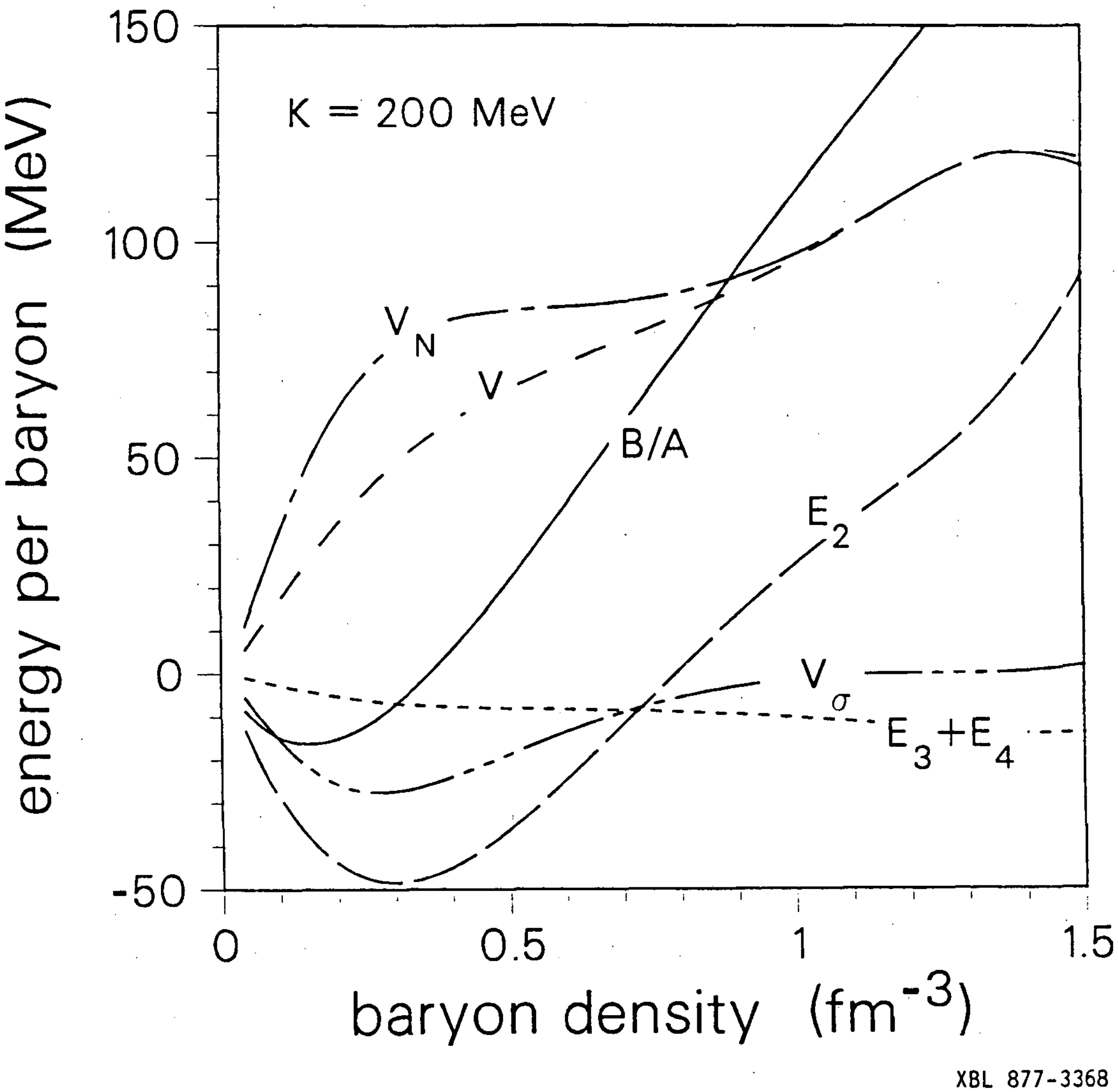


Fig. 1 For the 'soft' equation of state the binding per nucleon is shown for nuclear matter as a function of baryon density. Also shown are the contributions to it, the two, three and four-body parts, and the vacuum polarization energy of the nucleon (V_N) and σ meson (V_σ), and their sum, denoted by V.

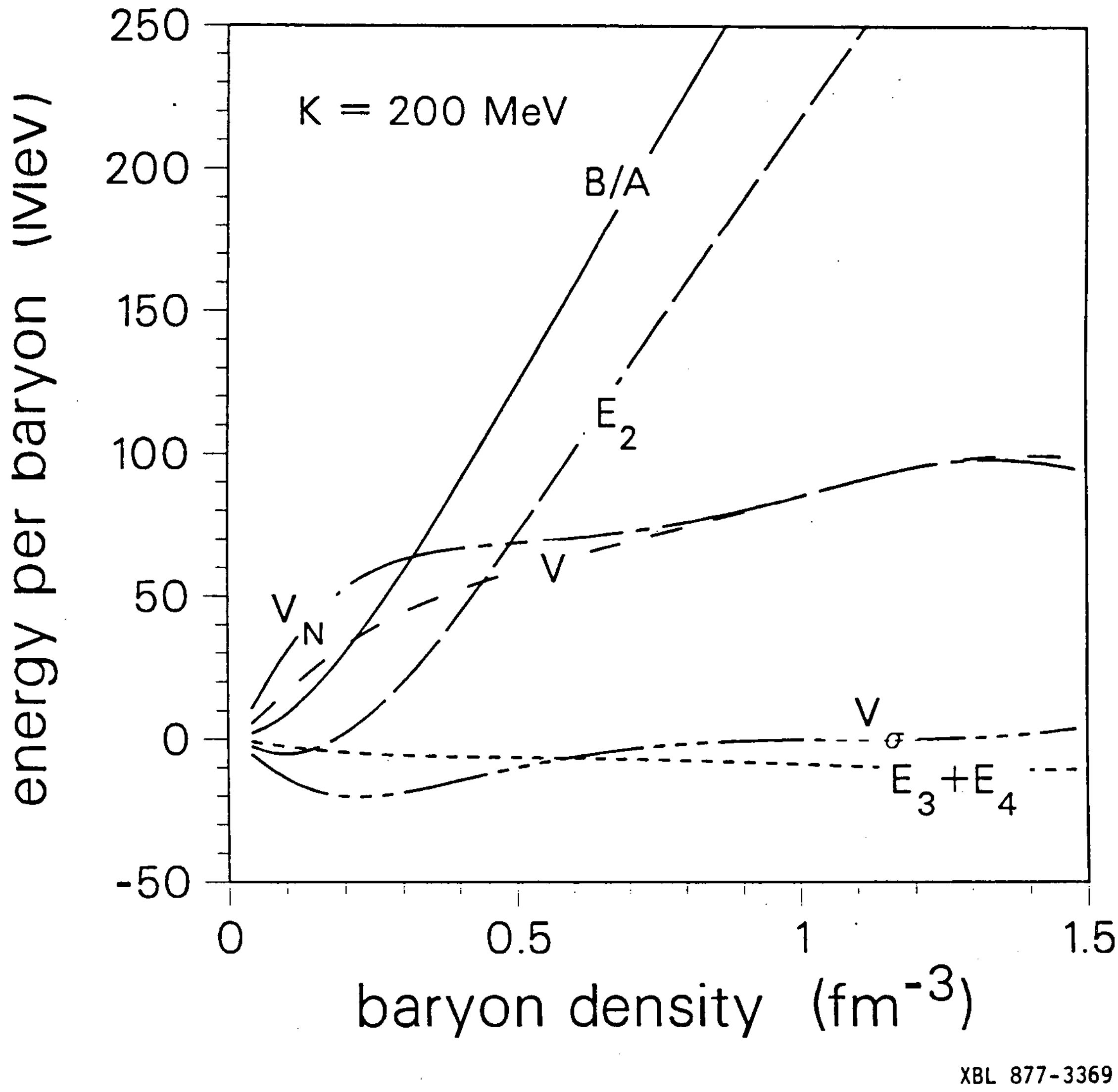


Fig. 2 For the 'soft' equation of state the binding per nucleon is shown for pure neutron matter as a function of baryon density. Also shown are the contributions to it as in Fig. 1. and their sum, denoted by V.

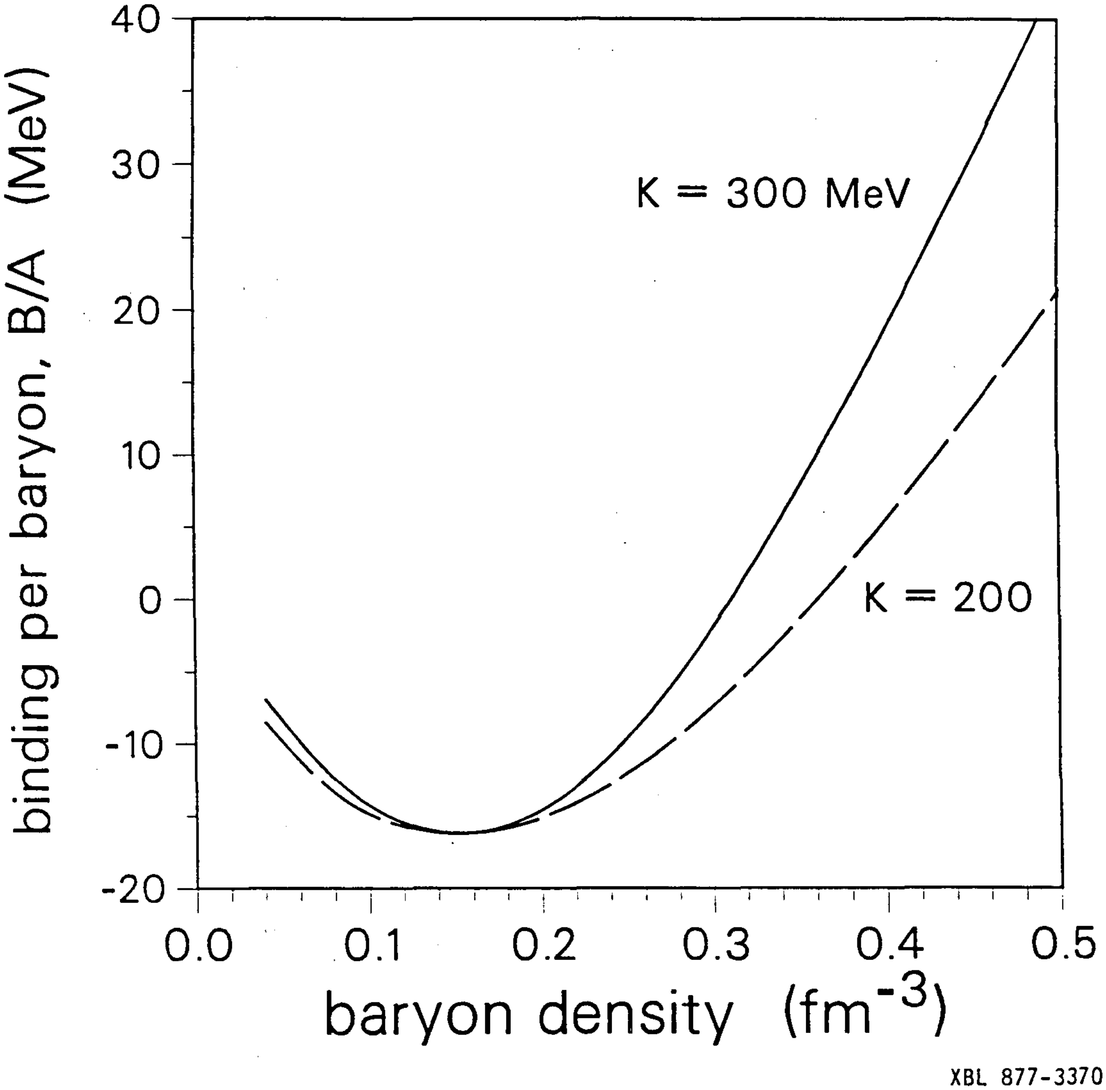


Fig. 3 The nuclear matter equation of statefor the 'soft' and 'stiff' cases.

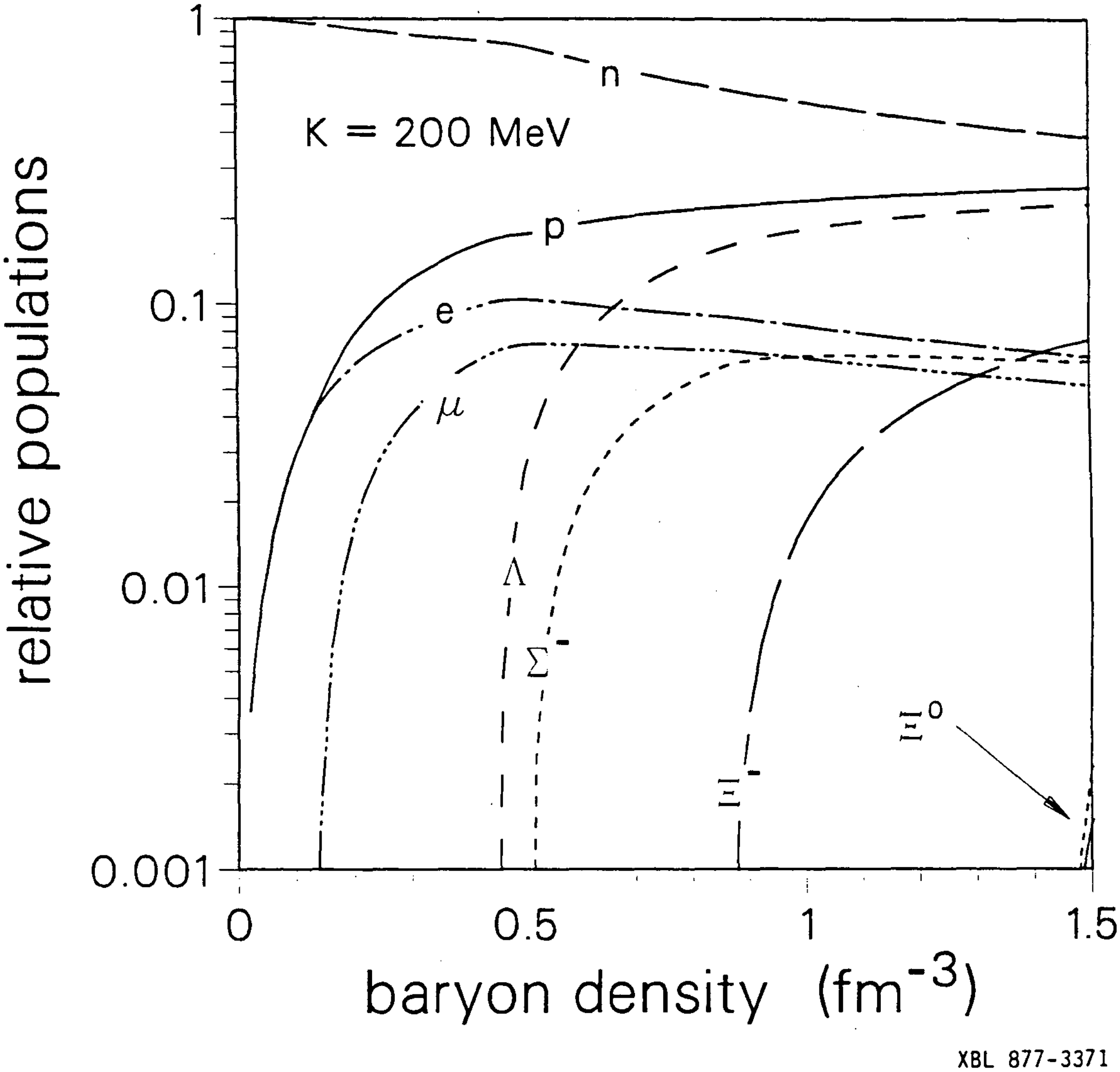


Fig. 4 Populations relative to total baryon density, in charge neutral beta stable neutron star matter as a function of total baryon density.

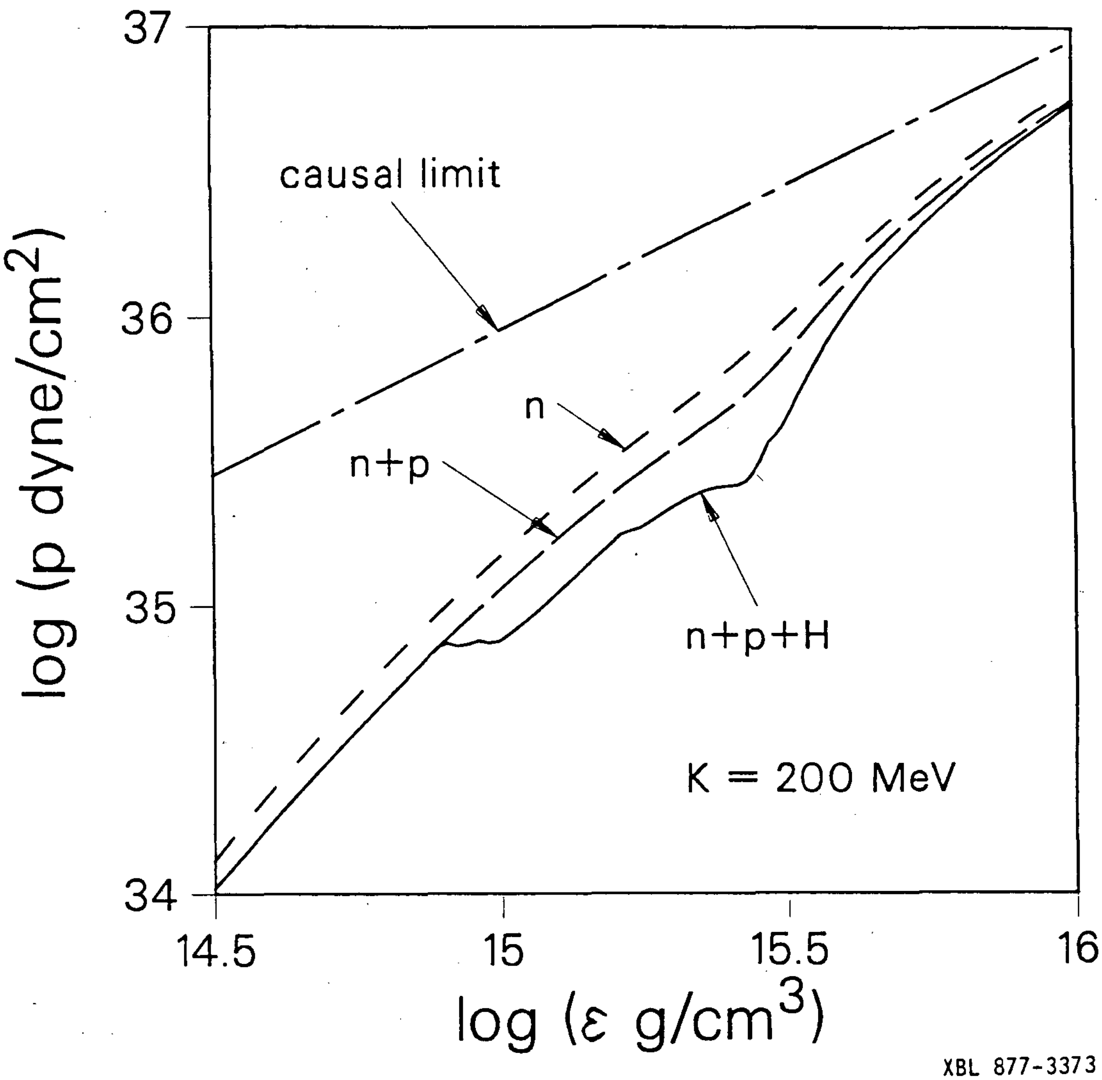


Fig. 5 Equation of state, p vs ϵ, in the case that $K = 200\ MeV$ for nuclear matter. The curve marked 'n' is pure neutron matter, 'n+p' is neutrons and protons in equilibrium with electrons and muons, and 'n+p+H' has hyperons in addition. The causal limit is $p = \epsilon$.

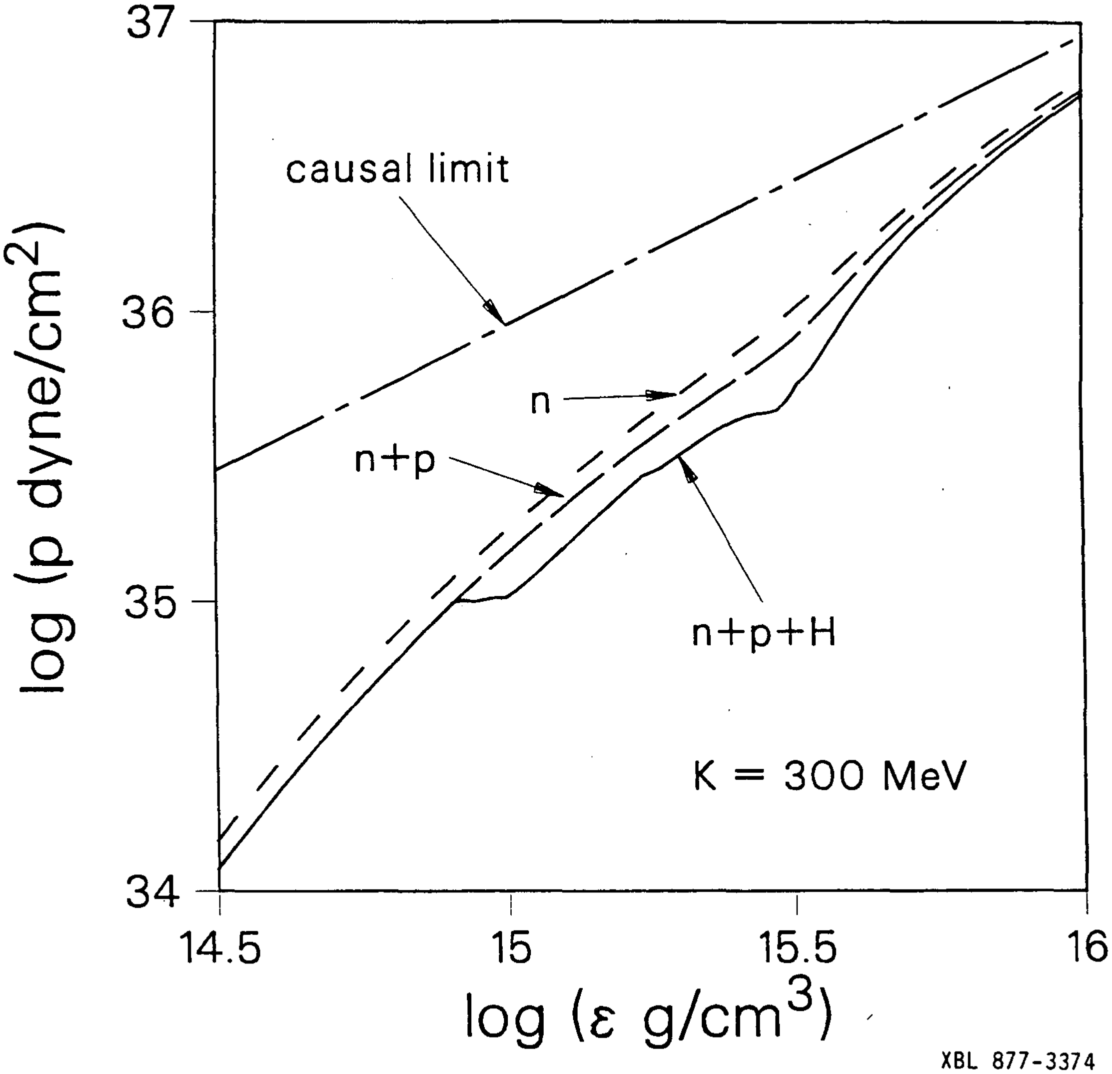


Fig. 6 Equation of state, p vs ϵ, in the case that $K = 300\ MeV$ for nuclear matter. Labeling is as in Fig. 5.

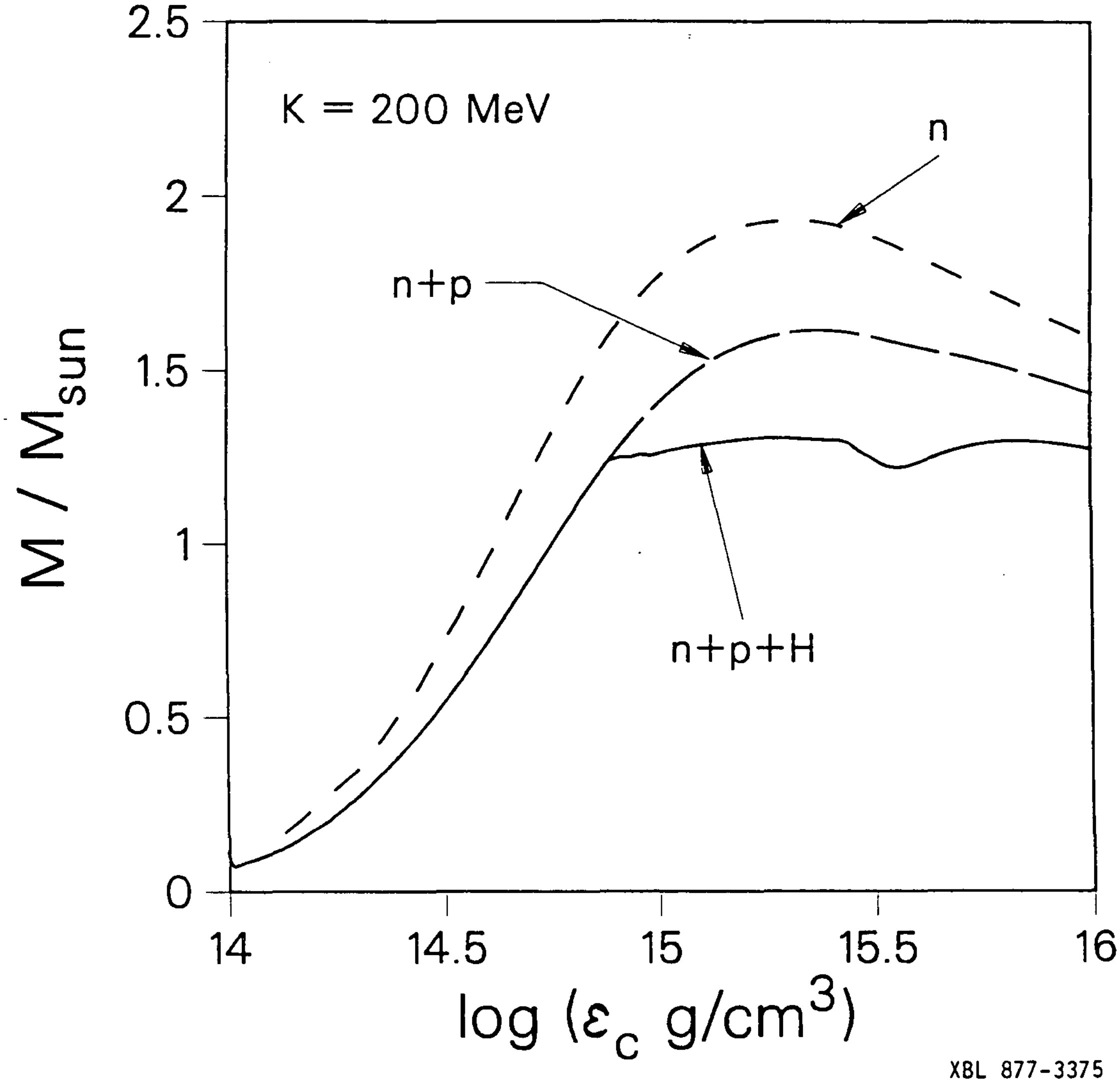


Fig. 7 Neutron star gravitational mass in solar mass units as a function of central energy density for the 'soft' equation of state. The curve marked 'n' is pure neutron matter, 'n+p' is neutrons and protons in equilibrium with electrons and muons, and 'n+p+H' has hyperons in addition.

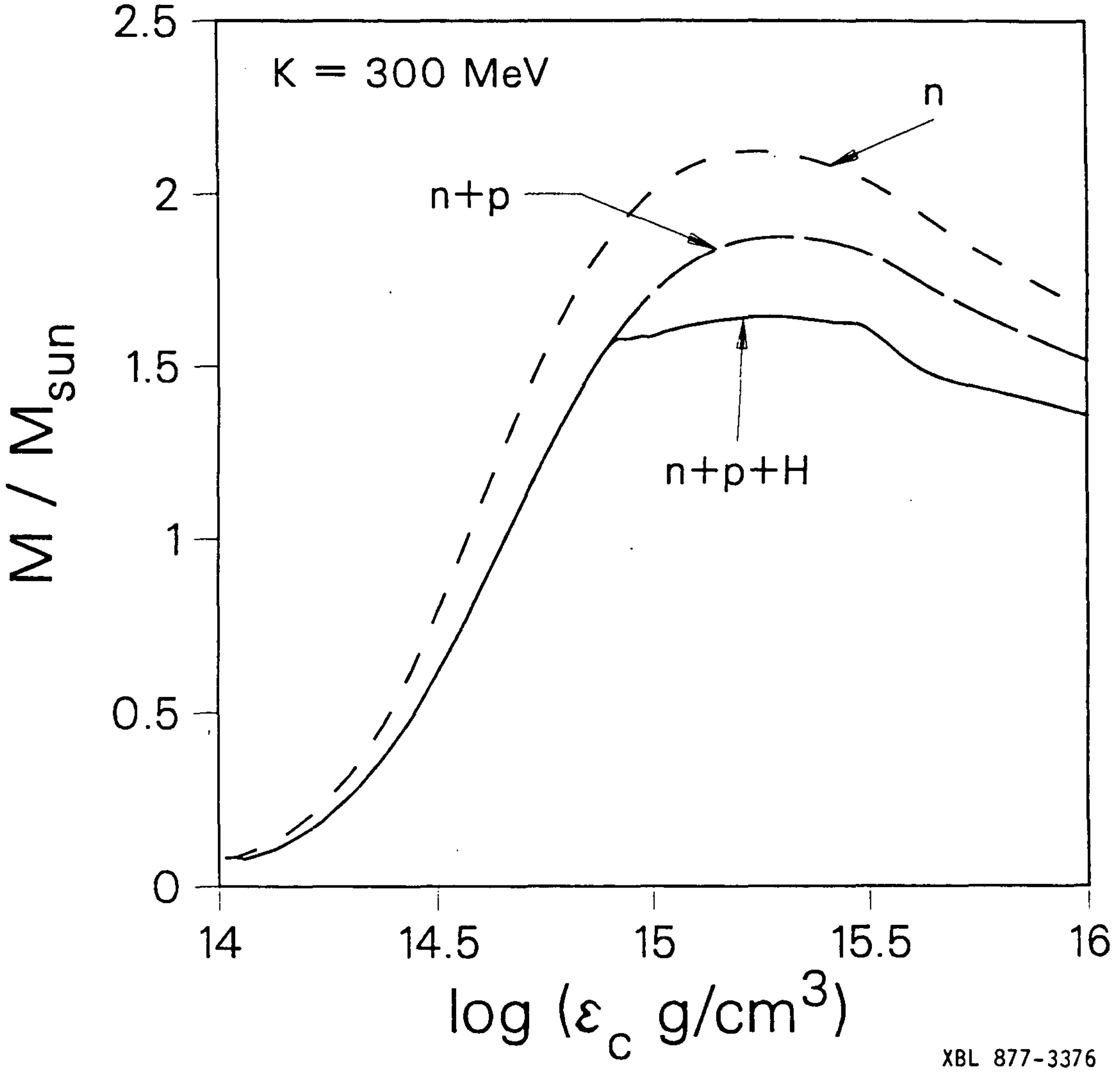
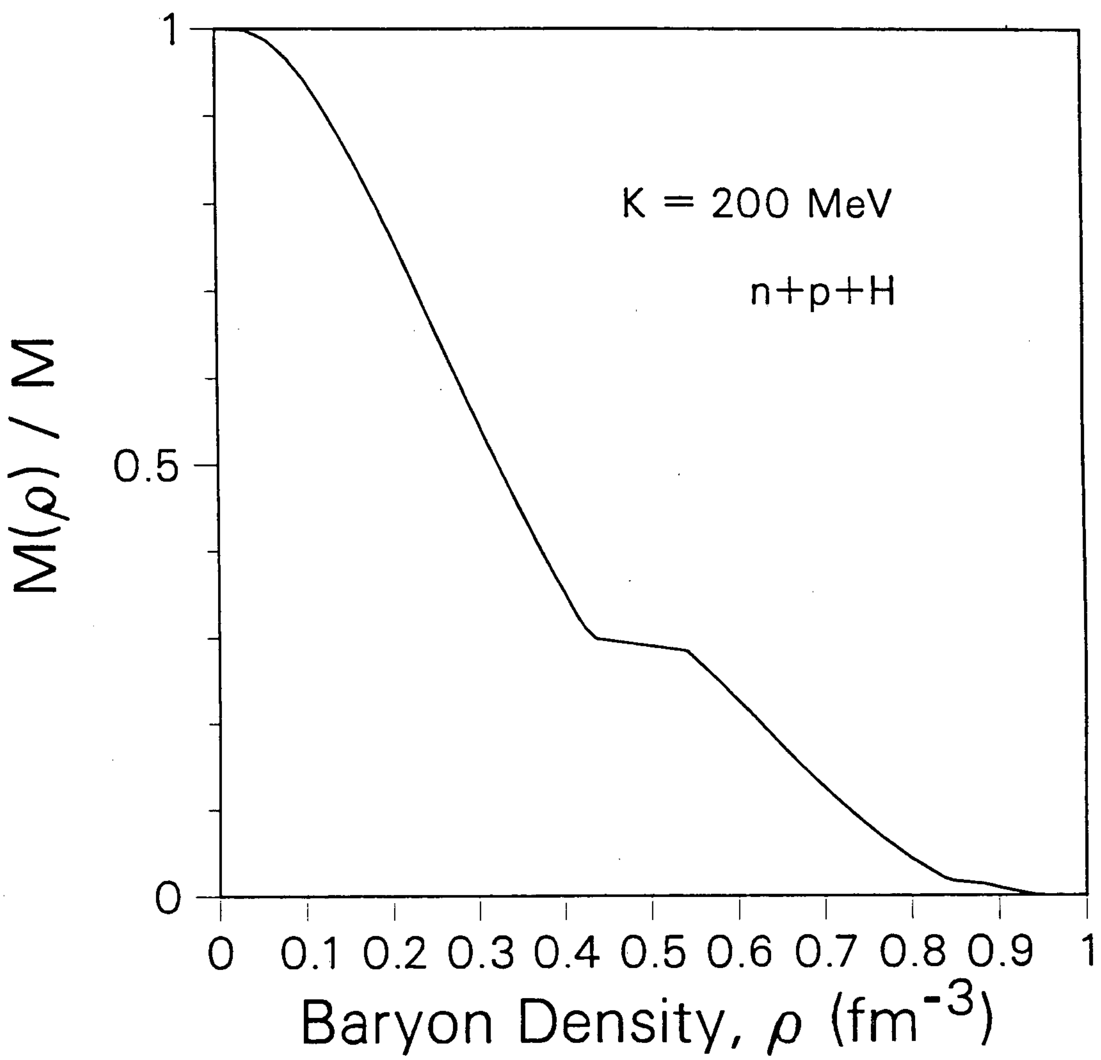


Fig. 8 Neutron star gravitational mass in solar mass units as a function of central energy density for the 'stiff' equation of state. Notation as in Fig. 6.



XBL 877-3377

Fig. 9 For the star at the limiting mass in the case of the 'soft' equation of state, the fraction of the mass of the star $M(\rho)/M$ contained at baryon density greater than ρ is shown as a function of ρ.

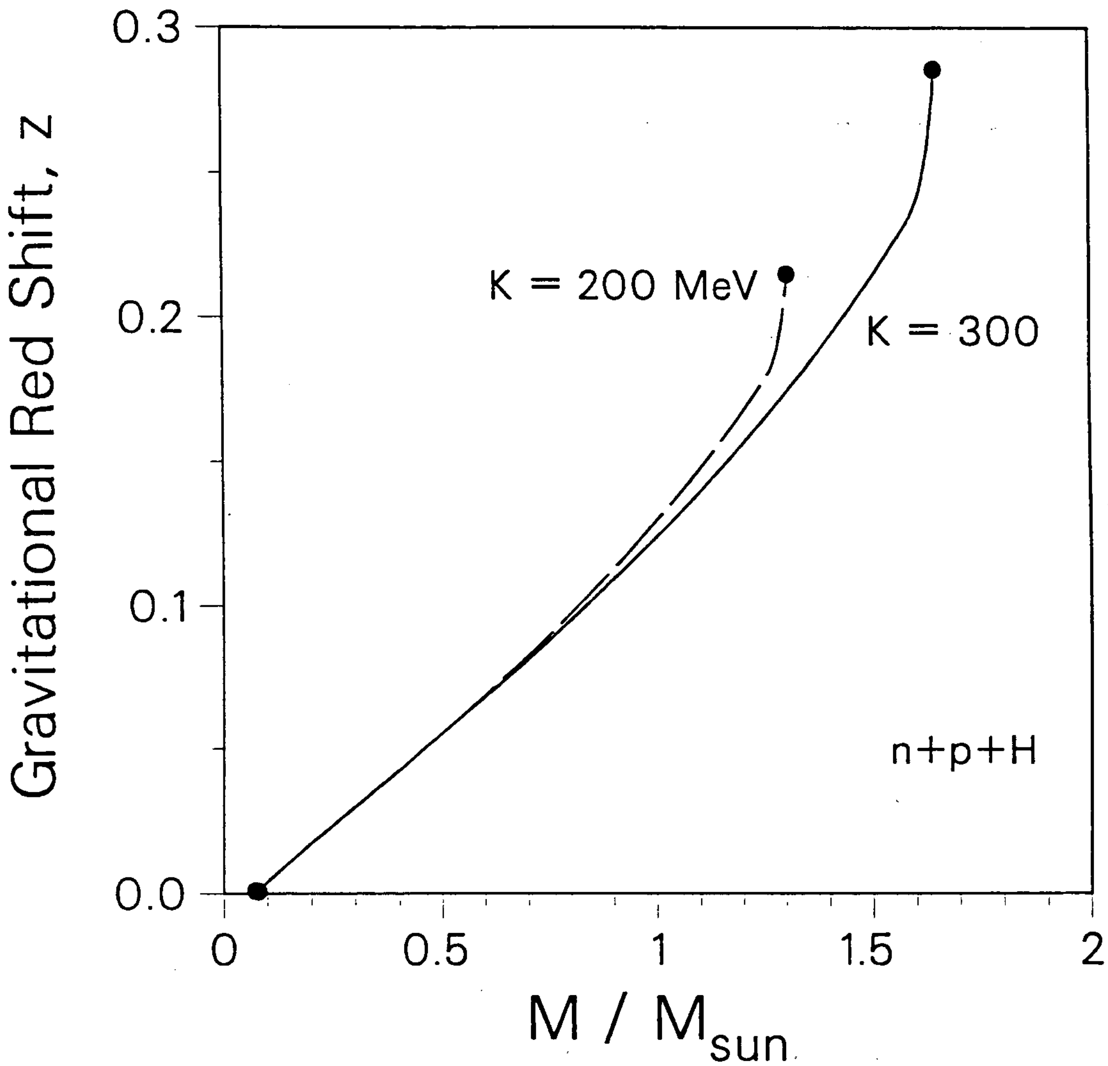


Fig. 10 Gravitational redshift as the fractional shift in the wavelenght of light emited at the surface of a neutron star, as a function of star mass for the 'soft' and 'stiff' cases. The dot at the high end of the curves marks the star at the limiting mass.

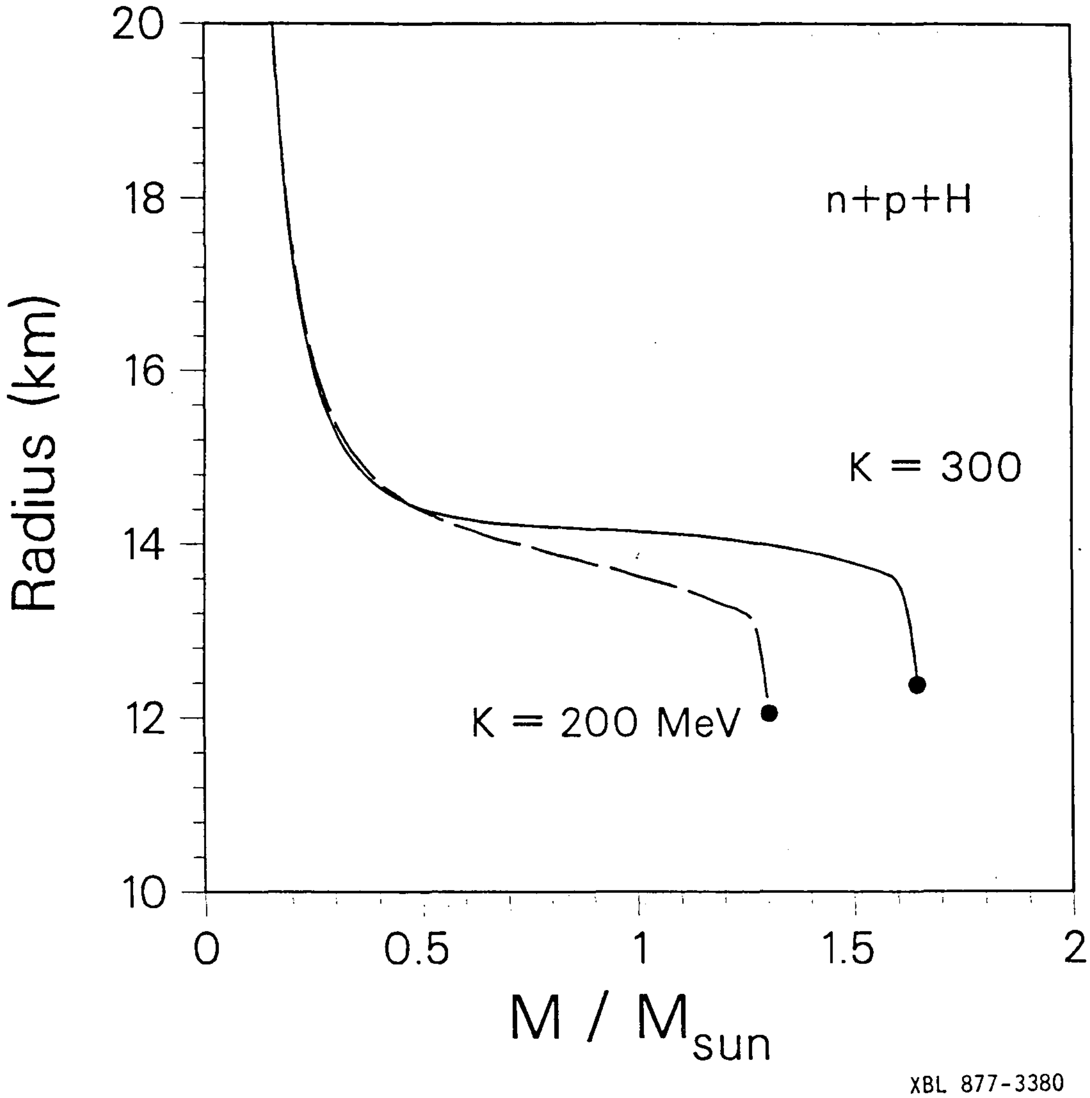
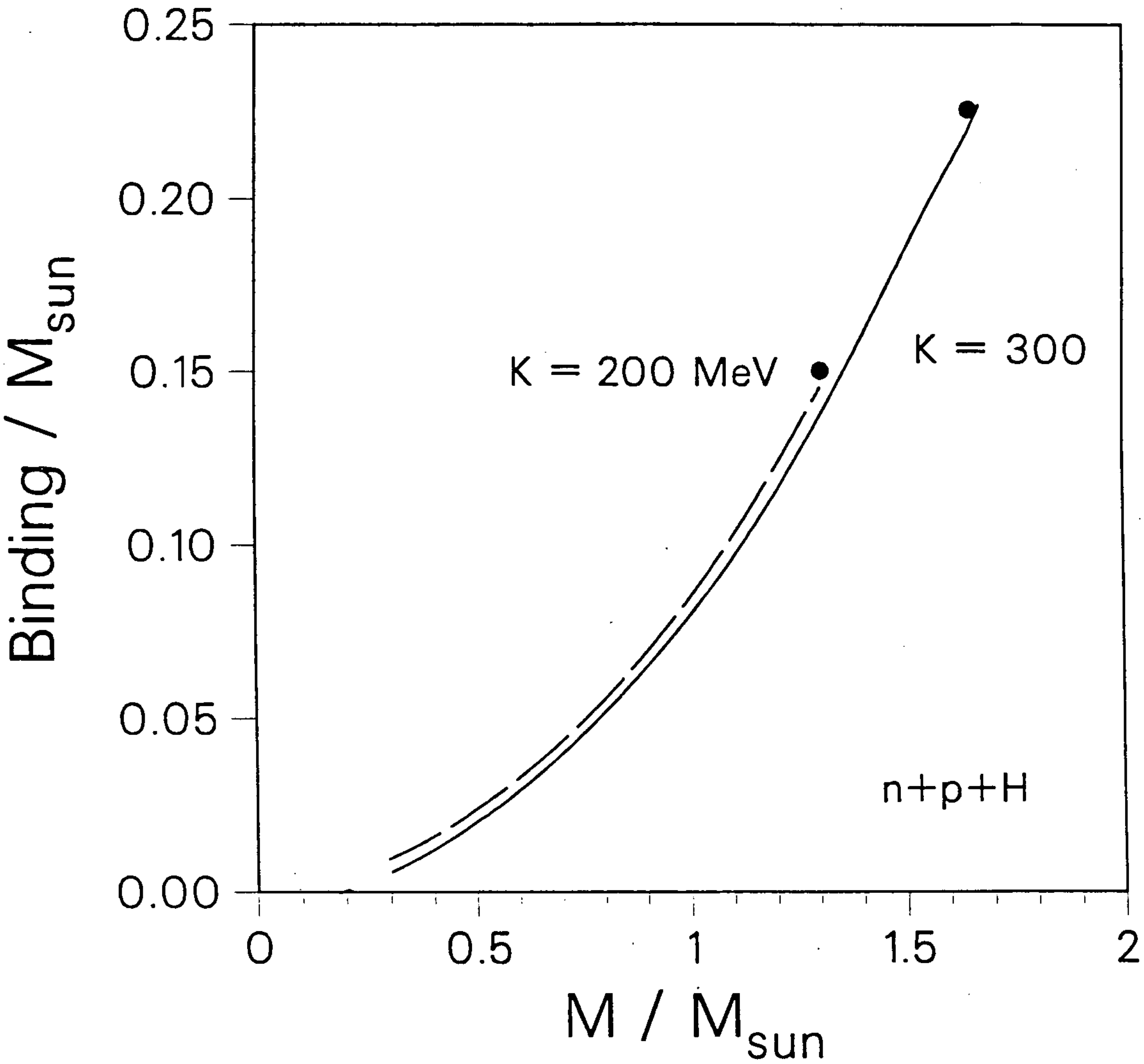


Fig. 11 Neutron star radius as a function of mass for the 'soft' and 'stiff' cases. The dots mark the limiting mass stars in the two cases.



XBL 877-3381

Fig. 12 Neutron star binding energy in solar mass units as a function of mass for the 'soft' and 'stiff' cases.

LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720