UCRL-17329

University of California

Ernest O. Lawrence Radiation Laboratory

MASS PERTURBATIONS IN A
BETHE-SALPETER EQUATION MODEL OF THE NUCLEON

Berkeley, California

DISCLAIMER

Submitted to Physical Review UCRL-17329

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

MASS PERTURBATIONS IN A BETHE-SALPETER EQUATION MODEL OF THE NUCLEON

John Harte

January 18, 1967

MASS PERTURBATIONS IN A BETHE-SALPETER EQUATION MODEL OF THE NUCLEON*

John Harte

Lawrence Radiation Laboratory
University of California
Berkeley, California

January 18, 1967

ABSTRACT

A model of the nucleon based on the Bethe-Salpeter equation is used to investigate the response of the binding energy to small perturbations. Comparisons with the predictions of an N/D model are drawn, and it is shown that the effect of perturbations in the strong-interaction coupling constant may be better described with off-shell dynamics. In this model neither the photon-exchange graph nor the feedback effect can explain the neutron-proton mass difference, but if N^* exchange provides the attractive force to bind the nucleon, then radiative corrections to the $N^*N\pi$ vertex may explain the sign of $m_n - m_p$. Evidence is given for the predominance of pseudovector rather than pseudoscalar πNN coupling.

I. INTRODUCTION

In the past few years, much effort has been made to obtain insight into the behavior of strongly interacting composite systems which are subject to a perturbation. For example, on-shell N/D dynamics has been used to calculate mass differences[1] when a symmetry is broken, and to probe the dynamical origins of octet enhancement.[2] However, evidence[3] of possible inadequacies in the customary N/D approximations has, in part, motivated considerable interest lately in off-shell dynamical models.[4] We discuss here a bound-state model[5] in which the nucleon is assumed to be described by a Bethe-Salpeter wave function, and apply the model to a calculation of the neutron-proton mass difference. We are interested not only in understanding this mass difference, but also in highlighting certain disparities between perturbation calculations in N/D and Bethe-Salpeter dynamics. Lacking any detailed understanding of the nature of the forces that might produce a composite nucleon, we try to emphasize general properties that are independent of the details of the interaction kernel. To simplify the model we consider the nucleon as a pion-nucleon bound state, thus neglecting the other members of the baryon octet and the decuplet in the constituent particle channel.

The important conclusions of this paper are:

1. The model strongly suggests that the pion-nucleon coupling is predominantly pseudovector rather than pseudoscalar.

2. The photon-exchange graph between the π^- and the proton in

the neutron state makes the neutron lighter than the proton in contrast to the apparently replusive effect calculated in an N/D model.[1]

3. The "feedback mechanism",[6] under quite general conditions, cannot yield a sign for the n - p mass difference which is opposite to that obtained from the driving terms. This result does not contain the ambiguity associated with unknown renormalization constants that one finds in feedback models with elementary nucleons.

4. The radiative corrections to the $N^*N\pi$ coupling provide a plausible mechanism to explain the sign of the n - p mass difference if N^* exchange is assumed to provide the predominant contribution to the interaction kernel. This is in contrast to the negligible effect of the coupling-constant renormalizations in the N/D model.

Before proceeding with the details of this model, we mention several ways in which our treatment of the nucleon compares favorably with other models. First, the use of an off-shell Bethe-Salpeter description of the bound state eliminates the infrared divergence difficulty in the photon-exchange correction to the binding energy which apparently plagues the N/D model.[7] A second advantageous feature of our model is that it treats the relativistic effects due to the pion motion and in addition includes some damping provided by nucleon recoil. This latter effect renders our results less sensitive to the unknown high-momentum behavior of the bound-state wave function than would be the case in a static model.

Lastly our model differs significantly from an N/D model in the following way. Let us assume that the exchange of some particle provides the dominant force in a bound-state calculation. Then we can inquire as to the effect on the binding energy if the coupling constant of that exchanged particle is perturbed. N/D dynamics provides a strange answer to this question. In particular, if we consider the nucleon bound state in the $N - \pi$ channel and assume first only the N exchange force with the usual replacement of the left-hand cut by a pole, then the sign of the shift in the binding energy depends crucially on whether that pole is placed to the left or right of the bound-state pole.[8] Indeed, in the static model, the discontinuity across the cut collapses to a δ-function at the bound-state energy, and the shift in the binding energy vanishes identically for arbitrary shifts in the coupling constant. This is because the shift in the binding energy is related to an integral of the product of the discontinuity in the perturbed Born amplitude times the square of the D function, and the latter vanishes identically at the bound-state energy. This aspect of N/D calculations results, we believe, is an underestimation of the magnitude of the effect of coupling-constant perturbations on bound-state energies as well as an ambiguity in the sign of this effect.

We shall see that this spurious, or at best ambiguous, feature is not present in our model, but that the response of the binding energy to changes in the coupling constant of the exchanged particle is determined by off-shell effects. In particular, we shall show that the intuitively expected decrease in the bound-state mass when the attractive force is increased is obtained when the effective coupling of the bound state nucleon into the constituent π - nucleon channel is predominantly pseudovector rather than pseudoscalar. In addition, in contrast to the situation described above in an N/D model, we find that the electromagnetic corrections to the coupling constant of the particle exchanged in the u channel can give a significant contribution to the n - p mass difference.

II. BETHE-SALPETER EQUATION: BOUND-STATE AND PERTURBATION FORMULAE

This section contains a brief review of the Bethe-Salpeter description of a bound state and a derivation of a perturbation formula by means of which we may calculate the electromagnetic shifts in the bound-state energy. We consider a π - nucleon system with incoming momenta p_1, p_2 and outgoing momenta q_1, q_2.

It is convenient to introduce a center-of-momentum (c.m.) variable P and relative momentum variables p and q, defined by

$$P = p_1 + p_2 = q_1 + q_2 \tag{2.1a}$$

and

$$p = \frac{p_1 - p_2}{2} , \quad q = \frac{q_1 - q_2}{2} . \tag{2.1b}$$

We will henceforth remain in the c.m. system and set $\underset{\sim}{P} = 0$. The four-point function, G, satisfies the Bethe-Salpeter equation[9]

$$G(P,p,q) = G_0(P,p)\, \delta(p - q) + G_0(P,p) \int d^4k\; V(P,p,k)\; G(P,k,q), \tag{2.2}$$

where

$$G_0(P,p) = \Delta_F(P,p)\; S_F(P,p) , \tag{2.3}$$

and V is the interaction kernel.

If the Green's function contains a bound-state pole at $P^2 = M_B^2$, then it has been shown that the Green's function may be written in the form[9]

$$G(P,p,q) = i \frac{\chi(P,p) \otimes \bar{\chi}(P,q)}{P^2 - M_B^2} + R(P,p,q) , \tag{2.4}$$

where R may be neglected at $P^2 = M_B^2$. Furthermore, the Bethe-Salpeter wave function, χ, satisfies the homogeneous equation

$$\chi(P,p) = G_0(P,p) \int d^4k \, V(P,p,k) \, \chi(P,k) \tag{2.5}$$

at $P^2 = M_B^2$.

Two additional relations involving the wave function that will be useful later are the normalization condition[10]

$$\begin{aligned} 2iP_\mu = & -\int d^4p \, \bar{\chi}(P,p) \left[\frac{\partial G_0^{-1}(P,p)}{\partial P_\mu} \right] \chi(P,p) \\ & + \iint d^4p \, d^4k \, \bar{\chi}(P,p) \left[\frac{\partial V(P,p,k)}{\partial P_\mu} \right] \chi(P,k) \end{aligned} \tag{2.6}$$

and an equation relating the wave function to the effective coupling constant of the bound state into the constituent particle channel. This coupling constant is defined as the residue of the on-shell T-matrix element at the bound-state pole and can be related to χ by the integral equation

$$T = V + V G V . \tag{2.7}$$

It can be seen from the definition[9] of χ as a time-ordered product of the constituent-particle field operators between the vacuum and the bound state that the nucleon wave function transforms like a spinor. We write

$$\chi_\alpha(P,p) = H_{\alpha\beta}(P,p)\, U_\beta(P,p) , \tag{2.8}$$

and use Eqs. (2.4), (2.5), and (2.7), and the relation between the T-matrix element and the vertex function, $\Gamma_{\alpha\beta}$, to obtain

$$\begin{aligned} T_{\alpha\delta} &= \frac{ig^2}{(2\pi)^4}\, \Gamma_{\alpha\beta}(P,p)\, (\not{P} - M_B)^{-1}_{\beta\gamma}\, \Gamma_{\gamma\delta}(P,p) \\ &= \frac{i[G_0^{-1}(P,p)\, H(P,p)]_{\alpha\beta}}{P^2 - M_B^2}\, U_\beta(P,p) \\ &\quad \otimes \bar{U}_\gamma(P,p)\, [H(P,p)\, \bar{G}_0^{-1}(P,p)]_{\gamma\delta} . \end{aligned} \tag{2.9}$$

If U is now taken to be a free-particle spinor for the bound-state nucleon, then we have

$$U(P) \otimes \bar{U}(P) = (\not{P} + M_B)\, (2M_B)^{-1} , \tag{2.10}$$

and we obtain the result

$$g\, \Gamma(P,p) = (2\pi)^2\, G_0^{-1}(P,p)\, H(P,p) (2M_B)^{-\frac{1}{2}} . \tag{2.11}$$

If we now go on the constituent-particle mass shell, assuming, for definiteness, pseudoscalar πNN coupling, then we have $\Gamma = \Gamma_0 = \gamma_5$ and

$$g\,\gamma_5 = \left| (2\pi)^2\, G_0^{-1}(P,p)\, H(P,p)\, (2M_B)^{-\frac{1}{2}} \right|_{\text{on shell}} \qquad (2.12)$$

is the desired relation between the coupling constant, which is known from experiment, and the Bethe-Salpeter wave function.

We next derive an expression for the change in the bound-state mass in the presence of a small perturbation. To simplify notation we rewrite Eq. (2.5) in the condensed notation

$$G_0^{-1}(M_B)\,\chi(M_B) = V(M_B)\,\chi(M_B)\,. \qquad (2.13)$$

The variable M_B which appears here is to remind us that the equation is only valid at $P^2 = M_B^2$. Now let us write another Bethe-Salpeter equation valid when a small perturbation is applied to the system which results in a bound state with mass M'_B:

$$G_0^{-1'}(M'_B)\,\overline{\chi'}(M'_B) = V'(M'_B)\,\chi'(M'_B)\,. \qquad (2.14)$$

We first translate this equation to $P^2 = M_B^2$ by writing

$$G_0^{-1'}(M'_B) = G_0^{-1'}(M_B) + \frac{\partial G_0^{-1}(M_B)}{\partial M_B} \delta M_B ,$$

$$V'(M'_B) = V'(M_B) + \frac{\partial V\ (M_B)}{\partial M_B} \delta M_B ,$$

and

$$X'(M'_B) = X'(M_B) + \frac{\partial X\ (M_B)}{\partial\ M_B} \delta M_B , \tag{2.15}$$

so that Eq. (2.11) becomes, to lowest order in the perturbation,

$$G_0^{-1'}(M_B)\ X'(M_B) + G_0^{-1'}(M_B) \frac{\partial X\ (M_B)}{\partial M_B} \delta M_B + \frac{\partial G_0^{-1}(M_B)}{\partial M_B} X'(M_B)\ \delta M_B$$

$$= V'(M_B)\ X'(M_B) + \frac{\partial V\ (\overline{M}_B)}{\partial\ M_B} X'(M_B)\ \delta M_B + V'(M_B) \frac{\partial X\ (M_B)}{\partial M_B} \delta M_B . \tag{2.16}$$

Here, $\delta M_B = M'_B - M_B$. If we now define the perturbations in the quantities X, V and G_0^{-1} by the equations

$$G_0^{-1'}(M_B) = G_0^{-1}(M_B) + \delta\ G_0^{-1}(M_B) ,$$

$$V'(M_B) = V(M_B) + \delta\ V(M_B) ,$$

and

$$X'(M_B) = X(M_B) + \delta X(M_B) \tag{2.17}$$

and substitute into Eq. (2.16) we obtain

$$G_0^{-1} X + G_0^{-1} \delta X + G_0^{-1} \frac{\partial X}{\partial M_B} \delta M_B + \delta G_0^{-1} X + \frac{\partial G_0^{-1}}{\partial M_B} \delta M_B X$$
$$= VX + V\delta X + \frac{V\partial X}{\partial M_B} \delta M_B + \delta V X + \frac{\partial V}{\partial M_B} \delta M_B X \ . \tag{2.18}$$

Since all quantities in this equation are to be evaluated at $P^2 = M_B^2$ we have dropped the variable M_B. Now multiply on the left of Eq.(2.18) by $\bar{X}$ and integrate over the relative momentum. Equation (2.13) and its adjoint allow the cancellation of the first three terms on the right and left, leaving, after a rearrangement, the equation

$$\left[\bar{X} \frac{\partial G_0^{-1}}{\partial M_B} X - \bar{X} \frac{\partial V}{\partial M_B} X\right] \delta M_B = \bar{X}\delta V X - \bar{X} \delta G_0^{-1} X \ . \tag{2.19}$$

The combination of terms on the left is exactly that which appears in the normalization equation (Eq. 2.6) and we therefore may simplify Eq.(2.19) to the useful form

$$2i M_B \delta M_B = \bar{X} \delta G_0^{-1} X - \bar{X} \delta V X \ . \tag{2.20}$$

The lowest order contribution to δG_0^{-1} is given by

$$\delta G_0^{-1} = \frac{\partial G_0^{-1}}{\partial m} \delta m + \frac{\partial G_0^{-1}}{\partial \mu} \delta\mu \tag{2.21}$$

where m and μ are the masses of the constituent nucleon and pion.

This term gives rise to the feedback effect; it relates the shift in the bound state mass to the shift in the constituent particle masses. The second term in Eq. (2.20) can be decomposed in similar fashion. If the strong interaction kernel is given by single particle exchange, then we write

$$\delta V = \frac{\partial V}{\partial \lambda} \delta\lambda + \frac{\partial V}{\partial m_{\text{exch.}}} \delta m_{\text{exch.}} + \overline{\delta V} \qquad (2.22)$$

where $\overline{\delta V}$ is a driving term which will be the single photon exchange graph in our model, $m_{\text{exch.}}$ is the mass of the exchanged particle and λ is the product of the coupling constants of the exchanged particle with the constituent particles. A graphical summary of Eqs. (2.21), (2.22), for an interaction kernel consisting of single particle exchange in the u channel, is given in Fig. 1.

III. THE NUCLEON WAVE FUNCTION

The problem remains of finding a suitable Bethe-Salpeter wave function. While there have been some interesting attempts to find approximate solutions to Eq. (2.5), assuming a particular form for the interaction kernel, we will not follow this approach here. Instead, we shall "construct" a wave function in a manner which is independent of the detailed dynamics of the π - N system, taking as our guide the general properties which a Bethe-Salpeter wave function should satisfy.

Let us first consider the analytic structure of the wave function. If the effective coupling constant, g, in Eq. (2.12) is non-zero and finite, then X must have simple poles when the constituent particles are on their mass shells. Thus we can write

$$X(P,p) = G_0(P,p)\, F(P,p)\, U(P) \,. \tag{3.1}$$

The function $F(P,p)$ has the transformation properties of the vertex function as can be seen from Eqs. (2.8), (2.11). For pseudoscalar coupling, we expect

$$F(p) = c\, g(p)\, \gamma_5 \tag{3.2}$$

while for pseudovector coupling we would have

$$F(p) = c\, f(p)\, \gamma_5 \left(\frac{\not{P}}{2} + \not{p}\right) . \qquad (3.3)$$

Here, c is a normalization constant to be determined by Eq. (2.6).

In order to proceed we make the crucial assumption that $f(p)$ and $g(p)$ have no nearby singularities. That is, we assume that the <u>only</u> singularities of the wave function, for small values of the relative momentum, are the poles at the π mass and at the nucleon mass. This is equivalent to saying that $f(p)$ and $g(p)$ are approximately independent of the fourth component of the relative momentum since any integral over the wave function [as in Eqs. (2.6) or (2.20)] will be dominated by the low mass singularities provided for example by the π meson pole. It can be shown[5] that the functions $f(p)$ and $g(p)$ should be insensitive to the relative energy of the two body system if retardation effects in the interaction kernel can be ignored. Retardation may, in fact, be unimportant if the lowest mass singularity in the interaction kernel is at a sufficiently large momentum compared to the binding energy. Therefore, if N and N^* exchange provide the nearest singularity in the cross channel, then the binding energy, which is equal to the π mass, indeed satisfies this condition and the approximation may be expected to be valid. While these arguments are not rigorous, they are plausible and we shall accept their conclusion by setting $f(p)$ and $g(p)$ equal to functions of only the relative three-momentum.

Another condition we can use to further restrict the form of the wave function is the asymptotic behavior of the wave function. It has been shown[11] that a Bethe-Salpeter wave function must decrease faster than a certain power of the relative momentum as $p \to \infty$. For a $\pi - N$ bound state, this requires that

$$\chi(p) < |p|^{-\frac{7}{2}} \tag{3.4}$$

for $|p| \to \infty$. Hence, for large $\underset{\sim}{p}$,

$$g(\underset{\sim}{p}) < |\underset{\sim}{p}|^{-\frac{1}{2}} \tag{3.5}$$

and

$$f(\underset{\sim}{p}) < |\underset{\sim}{p}|^{-\frac{3}{2}} . \tag{3.6}$$

For small $\underset{\sim}{p}$ we expect an asymptotic form for $\int \chi\,(\underset{\sim}{p}, p_0)\, d\, p_0$ which is characteristic of a Schröedinger bound state wave function. This, in fact, is guaranteed by the free propagators appearing in Eq. (3.1).

For $f(p)$ and $g(p)$ we shall use the forms

$$\begin{aligned} g(\underset{\sim}{p}) &= (\underset{\sim}{p}^2 + \Lambda^2)^{-1}, \quad e^{-\underset{\sim}{p}^2/\Lambda^2} \\ f(\underset{\sim}{p}) &= (\underset{\sim}{p}^2 + \Lambda^2)^{-2}, \quad e^{-\underset{\sim}{p}^2/\Lambda^2} \end{aligned} \tag{3.7}$$

containing a single parameter, Λ. This parameter, as well as the overall constant, c, multiplying the wave function, will be determined by Eqs. (2.6), (2.12) and the experimental value of the πNN coupling constant. Clearly if Λ turns out to be too small, say of the order of the π-mass, then the arguments given above to justify the model are inapplicable and the model would be unsupportable.

To conclude this section we apply Eq. (2.6) to the normalization of the wave function and, together with Eq. (2.12), to the determination of the parameters Λ and c. Consider, for definiteness, the wave function corresponding to pure pseudoscalar πNN coupling which is given by

$$\chi(P,p) = \frac{c\, g(\underset{\sim}{p})\, (\frac{\not{P}}{2} - \not{p} + m)\, \gamma_5\, U(P)}{[(\frac{P}{2} - p)^2 - m^2][(\frac{P}{2} + p)^2 - \mu^2]} \qquad (3.8)$$

where $g(\underset{\sim}{p})$ is given by Eq. (3.7). In order to apply Eq. (2.6) we have to know something about the interaction kernel because of the second term which contains a derivative of V with respect to the center of mass energy. If V consists of nucleon exchange then this term vanishes because the nucleon exchange kernel depends only on the relative momentum. However, the N^* exchange kernel does depend on the center of mass energy because of the derivative $N^*N\pi$ interaction. We may expect this term to be small, fortunately, as can be seen from the fact that in the static model, the N^* propagator projects out only the spatial components of the pion momentum. Thus the kernel depends only on the spatial components of the center of mass energy, whereas in applying Eq. (2.6) we take only the fourth component of P_μ. With non-static kinematics we expect this term to be small and an estimate gives a contribution which is approximately μ/m_{N^*} times the contribution of the first term on the right in Eq. (2.6). We will therefore drop the term depending on $\frac{\partial V}{\partial P_\mu}$ although a possible large contribution resulting from more complicated kernels cannot be ruled out.

From the fourth component of the first term in Eq. (2.6) we obtain[12] the normalization condition

$$2i\ m = -c^2 \int d^4p \ \frac{g^2(\underset{\sim}{p})\ (p_0^{\ 2} + \underset{\sim}{p}^2 + m\ p_0 + \frac{m^2}{4})}{[(\frac{P}{2} + p)^2 - \mu^2][(\frac{P}{2} - p)^2 - m^2]^2} \qquad (3.9)$$

after performing some Dirac algebra. We have set $M_B = m$ because the bound state mass is equal to the constituent nucleon mass in lowest order. A second relation between Λ and c is obtained from Eq. 2.12. We write

$$|p\rangle = \frac{-1}{\sqrt{3}}\ |p\pi^o\rangle + \sqrt{\frac{2}{3}}\ |n\ \pi^+\rangle$$
$$|n\rangle = \frac{1}{\sqrt{3}}\ |n\pi^o\rangle - \sqrt{\frac{2}{3}}\ |p\ \pi^-\rangle \qquad (3.10)$$

and hence have

$$g_{\pi^o pp} = \frac{c}{\sqrt{3}}\ \frac{(2\pi)^2}{\sqrt{2m}}\ \ g(\underset{\sim}{p}^2 \approx -\mu^2) \qquad (3.11)$$

where

$$g^2_{\pi^o pp}/4\pi = 14.4. \qquad (3.12)$$

Calculated values for Λ and c appear in Table I for pseudoscalar and pseudovector πNN coupling.

IV. PSEUDOVECTOR OR PSEUDOSCALAR πNN COUPLING?

In this and the following section, we investigate the consequences of our model. We have discussed two forms for the wave function corresponding to pseudoscalar and pseudovector π-nucleon coupling. Strangely enough the model sharply distinguishes between these two possibilities. We make the reasonable postulate that if the magnitude of the coupling constant of an attractive force is increased then the mass of the bound state will decrease. Let us then compute the contribution to δm resulting from the first term in Eq.(2.22):

$$2im\,\delta m = -\delta\lambda \iint d^4p\,d^4q\,\overline{X}(P,p)\,[\frac{\partial V}{\partial \lambda}(P,p,q)]\,X(P,q)\,. \tag{4.1}$$

We need not assume that the kernel consists of the single particle exchange graph as λ could be any overall constant multiplying the kernel. That is, we only assume

$$\frac{\partial V}{\partial \lambda} = \frac{V}{\lambda} \tag{4.2}$$

and therefore we have

$$2i\,m\,\delta m = -\frac{\delta\lambda}{\lambda} \iint d^4p\,d^4q\,\overline{X}(P,q)\,V(P,p,q)\,X(P,q)\,. \tag{4.3}$$

Using the Bethe-Salpeter wave function equation (Eq.(2.5)) we can eliminate the unknown quantity, V, in Eq. (4.3) and obtain

$$2i\, m\, \delta m = -\frac{\delta\lambda}{\lambda} \int d^4p\ \overline{X}(P,p)\, G_0^{-1}(P,p)\, X(P,p)\ . \qquad (4.4)$$

We have written this equation for a one channel problem as this simplification does not affect the validity of the argument. The multichannel generalization will be given later when we compute the effect of the radiative corrections to the $N^*N\pi$ coupling constants on the nucleon masses. For psuedoscalar and pseudovector coupling (Eq. (3.2) and (3.3)) we have

$$2i\, m\, \delta m = -\frac{\delta\lambda}{\lambda}\, \overline{U}(P) \int d^4p\ \frac{\gamma_5(\frac{\not P}{2} - \not p + m)\, \gamma_5\, g^2(\underset{\sim}{p})\, U(P)}{[(\frac{P}{2} - p)^2 - m^2][(\frac{P}{2} + p)^2 - \mu^2]} \qquad (4.5)$$

and

$$2i\, m\, \delta m = -\frac{\delta\lambda}{\lambda}\, \overline{U}(P) \int \frac{d^4p\ (\frac{\not P}{2} + \not p)\, \gamma_5(\frac{\not P}{2} - \not p + m)\, \gamma_5(\frac{\not P}{2} + \not p)\, f^2(\underset{\sim}{p})\, U(P)}{[(\frac{P}{2} - p)^2 - m^2]\ [\frac{P}{2} + p)^2 - \mu^2]} \qquad (4.6)$$

respectively. Now using the identities

$$\overline{U}(P)\, \gamma_5(\frac{\not P}{2} - \not p + m)\, \gamma_5\, U(P) = -(\frac{m}{2} + p_0) \qquad (4.7)$$

and

$$\overline{U}(P)\, (\frac{\not P}{2} + \not p)\, \gamma_5(\frac{\not P}{2} - \not p + m)\, \gamma_5(\frac{\not P}{2} + \not p)\, U(P)$$

$$= 2m\, \underset{\sim}{p}^2 - (\frac{m}{2} + p_0)^3 + (\frac{m}{2} + p_0)\, \underset{\sim}{p}^2 \qquad (4.8)$$

and performing the pole integrals in the p_0 plane we obtain the results

$$2i\ m\ \delta m = +\frac{\delta\lambda}{\lambda}\ \frac{2\pi\ i\ c^2}{4m}\int d^3p\ g^2(\underset{\sim}{p})\ [(\underset{\sim}{p}^2+\mu^2)^{-\frac{1}{2}} - (\underset{\sim}{p}^2+m^2)^{-\frac{1}{2}}] \tag{4.9}$$

and

$$2i\ m\ \delta m = -\frac{\delta\lambda}{\lambda}\ 2\pi\ i\ c^2\int d^3p\ f^2(\underset{\sim}{p})\ [p^2/2\ (\underset{\sim}{p}^2+\mu^2)^{-1} + m(\underset{\sim}{p}^2+m^2)^{-1}] \tag{4.10}$$

For simplicity, we have dropped terms which are of order $\frac{\mu}{2m}$ or smaller for arbitrary $\underset{\sim}{p}$ and which do not affect the argument. In the first of these expressions the right hand side is clearly positive imaginary and the second it is clearly negative imaginary <u>for arbitrary $f(\underset{\sim}{p})$ and $g(\underset{\sim}{p})$</u>. Now, if $\lambda > 0$ corresponds to the attractive case then $\delta\lambda > 0$ corresponds to a perturbation which makes the force more attractive and it is obvious that the pseudovector wavefunction, not the pseudoscalar wave function, has the desired property of yielding a decrease in the mass of the bound state. We conclude that the wave function must be predominantly pseudovector.

V. THE n - p MASS DIFFERENCE

To compute the effect of the photon exchange graph (Fig. 1k) on the binding energy of the neutron we use Eq. (2.20), (2.21) and let[13]

$$\overline{\delta V}(P,p,q) = \frac{ie^2(2\not{P} + \not{q} + \not{p})}{(2\pi)^4 (q - p)^2} . \tag{5.1}$$

We are assuming constant nucleon and pion form factors for simplicity as the convergence of the integral is assured by the high momentum behavior of the wave function. For both pseudovector and pseudoscalar coupling, the result is a negative shift in the neutron mass. The numerical values are shown in Table 2.

Next, we calculate the feedback effect given by Eq. 2.21. Since the quantity $m_n - m_p$ transforms like an isovector, only the mass differences of the constituent nucleons can enter in the calculations. From Eqs. (2.20),(3.10) we have

$$\begin{aligned} 2i\, m\, \delta m_n &= \frac{I}{3}\, \delta m_n + \frac{2I}{3}\, \delta m_p + A_n \\ 2i\, m\, \delta m_p &= \frac{I}{3}\, \delta m_p + \frac{2I}{3}\, \delta m_n + A_p \end{aligned} \tag{5.2}$$

where

$$I = \int d^4p\, \overline{X}(P,p)\, \frac{\partial\, G_0^{-1}(P,p)}{\partial\, m}\, X(P,p) . \tag{5.3}$$

A_n and A_p, are the contributions from the δV term and m is the nucleon mass in the absence of electromagentic effects. Solving for $m_n - m_p = \delta m_n - \delta m_p$, we obtain

$$m_n - m_p = \frac{A_n - A_p}{1 + \frac{I}{6im}} . \tag{5.4}$$

Thus if the feedback effect is to change the sign of the mass difference, we must have

$$I < -6im . \tag{5.5}$$

Now, in the static limit $|I| = 2m$ as a consequence of the normalization condition on X. More generally, with pseudoscalar coupling we obtain

$$I = c^2 \int d^4p \; \frac{g^2(\underset{\sim}{p})\,(p_0{}^2 - \underset{\sim}{p}{}^2 + mp_0 + \frac{m^2}{4})}{[(\frac{P}{2} + p)^2 - \mu^2][(\frac{P}{2} - p)^2 - m^2]^2} . \tag{5.6}$$

Comparing with Eq.(3.9), we see that

$$|I| \leqslant 2m \tag{5.7}$$

with the nucleon recoil correction, and this is true for arbitrary $g(\underset{\sim}{p})$.

A similar result holds for pseudovector coupling[14] and the numerical results are shown in Table 3. Note that the feedback integral has the correct sign to cause a change in the sign of $m_n - m_p$ if the πNN coupling is pseudoscalar, but not if it is pseudovector. Our conclusion, at this point, is that neither the photon exchange graph nor the feedback mechanism can explain the n - p mass difference.

The remaining perturbations are the radiative corrections to the strong binding potential shown in Figs. 1.c, 1.j. We will primarily be interested in the effect of the radiative corrections to the coupling constant of the exchanged particle in a single particle exchange interaction kernel. In order to calculate this effect we first rewrite Eq. (4.4) in a form appropriate to a multichannel problem. We will use Greek subscripts to denote the members of the bound state and constituent nucleon multiplets, Latin subscripts to denote the π mesons and Greek superscripts to denote the members of the multiplet of exchanged particles in the u-channel. Then Eq. (2.5) can be written

$$G_0^{-1} \chi_{\beta\alpha i} = \sum_{\gamma,\delta,j} V^{\gamma}_{\alpha i,\delta j} \chi_{\beta\delta j} . \qquad (5.8)$$

Using Eq. (2.12), we can define the index-free quantities χ and V by

$$\chi_{\beta\alpha i} \equiv g_{\beta\alpha i} \chi \qquad (5.9)$$

and

$$V^{\gamma}_{\alpha i,\delta j} \equiv g^{\gamma}_{\alpha j} g^{\gamma}_{\delta i} V \qquad (5.10)$$

so that Eq. (5.8) becomes

$$G_0^{-1} \chi = g^{-1}_{\beta\alpha i} \sum_{\gamma,\delta,j} g_{\alpha j}{}^{\gamma} g_{\delta i}{}^{\gamma} g_{\beta\delta j} \; V \chi . \tag{5.11}$$

By the same procedure,

$$2i\, m\, \delta m_\beta = - \sum_{\gamma,\alpha,\delta,i,j} \delta(g_{\alpha j}{}^{\gamma} g_{\delta i}{}^{\gamma}) g_{\beta\alpha i} g_{\beta\delta j} \bar{\chi} V \chi \tag{5.12}$$

is the multichannel generalization of Eq. (4.3). Using Eq. (5.11) we obtain

$$2i\, m\, \delta m_\beta = - \sum_{\gamma,\alpha,\delta,j,i} \delta(g_{\alpha j}{}^{\gamma} g_{\delta i}{}^{\gamma}) g_{\beta\alpha i} g_{\beta\delta j}$$

$$\sum_{\lambda,k,\tau} g_{\beta\sigma\ell}(g_{\beta\lambda k} g_{\sigma k}{}^{\tau} g_{\lambda\ell}{}^{\tau})^{-1} \bar{\chi} G_0^{-1} \chi . \tag{5.13}$$

We adopt the convenient normalization for the coupling constants:

$$\sum_{\alpha,j} g_{\beta\alpha j} \; g_{\beta\alpha j} = 1 = \sum_{\alpha,j} g_{\alpha j}{}^{\gamma} \; g_{\alpha j}{}^{\gamma} . \tag{5.14}$$

Then, with a nucleon exchange interaction kernel we have the result

$$2i\, m(\delta m_n - \delta m_p) = - \frac{2}{\sqrt{3}} J(\delta g^{n}_{n\pi^0} + \delta g^{p}_{p\pi^0}) \tag{5.15}$$

where

$$J = \int \bar{\chi}(P,p)\, G_0^{-1}(P,p)\, \chi(P,p)\, d^4p . \tag{5.16}$$

Since the g's are normalized to unity, the quantity X in this expression is given by Eq. (3.8). We note that the mass difference is proportional to deviations from charge symmetry which are known from nuclear physics to be quite small. However, there is reason to believe that N^* exchange should play a more dominant role than N exchange in the interaction kernel since the former is apparently attractive and the latter repulsive in the nucleon state. With N^* exchange, Eq. (5.13) leads to

$$2i\, m(\delta m_n - \delta m_p) = -J\left[\delta g_{n\pi^-}^{N^{*-}} - \delta g_{p\pi^+}^{N^{*++}} + \frac{1}{\sqrt{3}}\,\delta g_{p\pi^-}^{N^{*o}} - \frac{1}{\sqrt{3}}\,\delta g_{n\pi^+}^{N^{*+}}\right] . \qquad (5.17)$$

The numerical values of the integral J, in Eq. (5.16), are given in Table 4. We consider only the case of pseudovector coupling; J has the opposite sign and approximately the same magnitude as that given in the table if we assume pseudoscalar coupling and we have argued above that this implies that the wave function must be predominantly pseudovector.

Some experimental evidence exists for isotopic spin symmetry violations in N^* decays, and therefore we can attempt an estimate of the quantity in brackets in Eq.(5.17) by using the relation

$$\Gamma_{N^*\to N\pi} = \frac{g_{N\pi}^{N^{*2}}|p|^3\,[M_{N^*}^{\;2} + m_N^{\;2} - \mu^2]}{2\;4\pi\;M_{N^*}^{\;2}} \qquad (5.18)$$

and the experimental results[15]

$$\Gamma_{N^{*-}} - \Gamma_{N^{*++}} = 25 \pm 23 \text{ MeV.}$$

$$M_{N^{*-}} - M_{N^{*++}} = 7.9 \pm 6.8 \text{ MeV.} \tag{5.19}$$

$$M_{N^{*o}} \simeq M_{N^{*++}} \qquad \Gamma_{N^{*o}} \simeq \Gamma_{N^{*++}} \quad .$$

N^{*-} exchange can only contribute to the interaction kernel of the proton while N^{*++} exchange can only contribute to the neutron. The larger N^{*-} width implies a larger coupling constant and this results in a more deeply bound proton. We assume the N^{*+} is nearly degenerate with the N^{*o} and N^{*++} as SU_6 gives the relation $M_{N^{*o}} - M_{N^{*+}} = m_n - m_p$ and in any case the $\frac{1}{\sqrt{3}}$ in Eq. (5.17) suppresses, somewhat, the N^{*o} and N^{*+} contributions. The errors in the experimental values of $\delta\Gamma_{N^*} \equiv \Gamma_{N^{*-}} - \Gamma_{N^{*++}}$ and $\delta m_{N^*} \equiv m_{N^{*-}} - m_{N^{*++}}$ are strongly correlated so that, for example, the smaller N^* mass difference should be considered with the smaller width difference. Taking $\delta\Gamma_{N^*} = 2$ MeV, $\delta m_{N^*} = 1.1$ MeV we obtain

$$\delta m_n - \delta m_b = 3.2 \text{ MeV.} \tag{5.20}$$

and this mass difference increases monotonically as $\delta\Gamma_{N^*}$ and δm_{N^*} increase. For example, with $\delta\Gamma_{N^*} = 25$ MeV. and $\delta m_{N^*} = 7.9$ MeV. we obtain

$$\delta m_n - \delta m_p = 8.9 \text{ MeV.} \tag{5.21}$$

Thus all the values of $\delta m_n - \delta m_p$ calculated for values of $\delta\Gamma_{N^*}$ and δm_{N^*} within the large experimental errors have the correct sign but are considerably larger than the experimental value $m_n - m_p = 1.3$ MeV. However, it must be remembered that the nucleon wave function will, in general, represent a combination of pseudovector and pseudoscalar couplings and the pseudoscalar part of the wave function will lead to a contribution to $m_n - m_p$ with the opposite sign, thus diminishing the value calculated above. If the predominant part of the wave function is pseudovector, we are assured of a positive contribution to $m_n - m_p$, but the exact value depends on the relative proportions of the two terms in the wave function. In addition, the photon exchange graph gives a negative contribution to $m_n - m_p$ of approximately 1 MeV. and the feedback effect must be included which multiplies the final result by approximately $\frac{3}{4}$.

It is clear that a reliable calculation of $m_n - m_p$ is impossible at this point. Our conclusion is that the electromagnetic corrections to the $N^* N\pi$ coupling constant can give rise to sizeable shifts in the nucleon masses and this effect is in the direction of yielding a positive value for $m_n - m_p$. This is in contrast to the negligible effect of these coupling constant perturbations calculated in an N/D model.[1]

VI. FURTHER DISCUSSION AND CONCLUSION

We have estimated the effect of the N^* mass differences in the u channel from Eqs. (2.20),(2.22) and find that these give rise to at most a 10% correction to the effects calculated previously. This is because the differention with respect to the N^* mass brings down an extra factor of μ/m_{N^*} in the integrals. This small effect gives a negative contribution to $m_n - m_p$. For the same reason, the effect of the graph in Fig. 1.g should be insignificant.

A source of uncertainty in the calculations arises from our approximation of including only constituent pions and nucleons. It has been suggested by several authors[16] that it may be necessary to include strange particles or the N^* resonances in the constituent channel in order to understand the neutron-proton mass difference. If we continue to assume that the inelastic channels are described by Bethe-Salpeter wave functions of the form discussed in Section 3, even though this assumption becomes less tenable as the masses of the constituent particles increase, then it is possible to partially justify our neglect of these additional channels. For example, the effect of the $K^+ - K^0$ mass difference in the $K\Lambda$ and $K\Sigma$ channels can be estimated from Eqs. (2.20), (2.21) to be at most 0.5 MeV. This estimate involves the $K\Lambda N$ and $K\Sigma N$ coupling constants which are known to be approximately $\frac{1}{3}$ of the πNN couplings.[17] It is more difficult to estimate the importance of the $N^*\pi$ or $N\pi\pi$ channels. In addition, several authors have suggested that the σ mesons may be important in the inelastic channels, but we have not estimated what effect this might have on $m_n - m_p$.

If the experimental data on the charge dependence of the N^* masses and widths inspired sufficient confidence to warrant a precise calculation, then the octet of baryons and the decuplet would have to be included. We have stressed here that the neutron-proton mass difference may be qualitatively understood without this complication.

In conclusion, we wish to reemphasize the fact that the customary approximations used in N/D dynamics and in Bethe-Salpeter equation dynamics give a very different picture of the effect of coupling constant perturbations on bound state masses. We have noted that such perturbations may give a large contribution to off-shell mass difference calculations in spite of the fact that they are often negligible in N/D calculations. This latter phenomenon is, we believe, the consequence of including only the single particle exchange contribution to the force. Frequently the many-particle left hand cuts are distant and may be neglected. In the static model of the nucleon, however, the N/D approximation gives a zero amplitude with a single nucleon exchange Born input and therefore more distant cuts are clearly more important. Therefore in N/D calculations of the properties of the baryon octet or decuplet, where single particle exchange in the u channel gives rise to a short cut lying near or on top of the bound state pole, the force and hence the response to perturbations in the force may be grossly underestimated by the approximation of including only the u channel single particle exchange Born input. This disadvantage is overcome in a Bethe Salpeter equation model which includes an infinite number of left hand cuts.

ACKNOWLEDGMENT

The author expresses appreciation to Drs. D. Beder and R. Van Royen for useful discussions, and to the Lawrence Radiation Laboratory staff for their kind hospitality.

FOOTNOTES AND REFERENCES

* This work was supported by the U. S. Atomic Energy Commission. The author has an AEC Postdoctoral Fellowship under contract No. AT(11 - 1)-34.

1. R. Dashen, Phys. Rev. 135, 1196 (1965); D. Beder, Nuovo Cimento 43, 553 (1966) and 43, 865 (1966); F. Gilman, Phys. Rev. 147, 1094 (1966); H. Chew, Phys. Rev. 150, 1249 (1966).

2. R. Dashen and S. Frautschi, Phys. Rev. 137, 1331 (1965); A. Contogouris, B. Diu, and M. Pusterla, CERN preprint, Theory of Octet Enhancement and the Pseudoscalar-Vector Meson Interaction.

3. R. Sawyer, Phys. Rev. 142, 991 (1966); Y. Kim, Phys. Rev. 142, 1150 (1966); G. Barton, Phys. Rev. 146, 1149 (1966); G. Shaw and D. Wong, Phys. Rev. 147, 1028 (1966).

4. C. Schwartz and C. Zemach, Phys. Rev. 135, B1445 (1965); K. Lin and R. Cutkosky, Phys. Rev. 140, B205 (1965); R. Sawyer, Phys. Rev. 142, 991 (1966); J. Harte, Nuovo Cimento 45, 179 (1966); H. Goldberg, Cornell University preprint, the n-p Mass Difference in an SU_2 Bootstrap Model; S. Biswas and L. Balázs, UCLA report ThR No. 16, Reciprocal Bootstrap of the N and N^* Using the Static Model Bethe-Salpeter Equation.

5. A similar model was used to investigate the decay properties of the Ω^- by J. Harte, Nuovo Cimento 42, 316 (1966).

6. H. Pagels, Phys. Rev. 144, 1261 (1966); H. Fried and T. Truong, Phys. Rev. Letters 16, 559 (1966).

7. G. Barton, Ref. 3.

8. The author thanks D. Beder for a useful discussion on this subject. After the completion of this work a paper by J. Daboul, Physics Letters 23, 592 (1966), containing a treatment of this point was brought to our attention.

9. M. Gell-Mann and F. Low, Phys. Rev. 84, 340 (1951); H. Bethe and E. Salpeter, Phys. Rev. 84, 1232 (1951).

10. S. Mandelstam, Proc. Roy. Soc. A233, 248 (1955); R. Cutkosky and M. Leon, Phys. Rev. 135, B1445 (1964).

11. S. Mandelstam, Ref. 10.

12. The normalization integral obtained from Eq. (2.6) consists of two terms, one of which contains the square of the pion propagator and the other the square of the nucleon propagator. However, these two terms are identically equal which allows us to write Eq. (3.9). Both this equality and the normalization condition follow from charge conservation, or the requirement that the charges of the bound state neutron and proton are the sum of the charges of the constituent particles.

13. A gauge dependent term, $(q - p)_\mu (q - p)_\nu / (q - p)^2$, in the photon propagator gives no contribution to the energy shift due to the symmetric integration over the two internal relative momenta.

14. If Eq. (2.6) receives a large contribution from the term $\frac{\partial V}{\partial P_\mu}$, then these results might be altered. With pseudovector coupling, the result $|I| < 2m$ also depends on the smallness of the ratio μ/m.

15. A. Rosenfeld, et al., Data on Particles and Resonances, Lawrence Radiation Laboratory Report UCRL-8030, January, 1967; G. Gidal, A. Kernan, and S. Kim, Phys. Rev. 141, 1261 (1966).

16. H. Goldberg, Ref. 4; G. Barton, Self-Consistent Bootstrap Model for the Electromagnetic Mass Splitting of Baryons and Mesons, University of Rochester Preprint UR-875-159.

17. M. Lusignole, M. Restignoli, G. Snow, and G. Violini, Physics Letters 21, 229 (1966).

LEGEND

Fig. 1. Electromagnetic effects contributing to the bound-state energy shifts given by Eq. (2.20). a,b give rise to the feed back effect; c,···j are corrections to the strong interaction kernel; k is a driving term. Graphs a,f,i do not contribute to the isovector quantity $m_n - m_p$.

TABLES

Table 1. Values of Λ^2 and c^2 calculated from Eqs. (2.6, 2.12) assuming: a. pseudoscalar πNN coupling,

$$\frac{g^2_{\pi^o pp}}{4\pi} = 14.4 ,$$

b. pseudovector πNN coupling,

$$\frac{f^2_{\pi^o pp}}{4\pi} = 0.08\ \mu^{-2} .$$

$g(\underset{\sim}{p})$	$(\underset{\sim}{p}^2 + \Lambda^2)^{-1}$	$e^{-\underset{\sim}{p}^2/\Lambda^2}$
Λ^2	$49\ \mu^2$	$64\ \mu^2$
c^2	$9.8 \times 10^3\ \mu^5$	$4.7\ \mu$

a.

$f(\underset{\sim}{p})$	$(\underset{\sim}{p}^2 + \Lambda^2)^{-2}$	$e^{-\underset{\sim}{p}^2/\Lambda^2}$
Λ^2	$96\ \mu^2$	$68\ \mu^2$
c^2	$1.7 \times 10^6\ \mu^7$	$2.6 \times 10^{-2}\ \mu^{-1}$

b.

Table II. Contribution of the photon exchange driving term, Fig. (1.j), to δm_n with: a. pseudoscalar πNN coupling, b. pseudovector πNN coupling.

$g(\underset{\sim}{p})$	$(\underset{\sim}{p}^2 + \Lambda^2)^{-1}$	$e^{-\underset{\sim}{p}^2/\Lambda^2}$
δm_n	-1.5 MeV.	-0.9 MeV.

a.

$f(\underset{\sim}{p})$	$(\underset{\sim}{p}^2 + \Lambda^2)^{-2}$	$e^{-\underset{\sim}{p}^2/\Lambda^2}$
δm_n	-1.2 MeV.	-1.0 MeV.

b.

Table III. Value of the feed back integral I, given by Eq. (5.3) assuming: a. pseudoscalar πNN coupling, b. pseudovector πNN coupling.

$g(\underset{\sim}{p})$	$(\underset{\sim}{p}^2 + \Lambda^2)^{-1}$	$e^{-\underset{\sim}{p}^2/\Lambda^2}$
$(2im)^{-1}$ I	-0.78	-0.72

a.

$f(\underset{\sim}{p})$	$(\underset{\sim}{p}^2 + \Lambda^2)^{-2}$	$e^{-\underset{\sim}{p}^2/\Lambda^2}$
$(2im)^{-1}$ I	0.91	0.88

b.

Table IV. Value of the integral J, (Eq. (5.16)) which relates the coupling constant renormalization to the bound state energy shifts for pseudovector πNN coupling. With pseudoscalar coupling, J has approximately the same magnitude, but the opposite sign.

$f(\underset{\sim}{p})$	$(\underset{\sim}{p}^2 + \Lambda^2)^{-2}$	$e^{-\underset{\sim}{p}^2/\Lambda^2}$
J	$-38\ i\ \mu^2$	$-37\ i\ \mu^2$

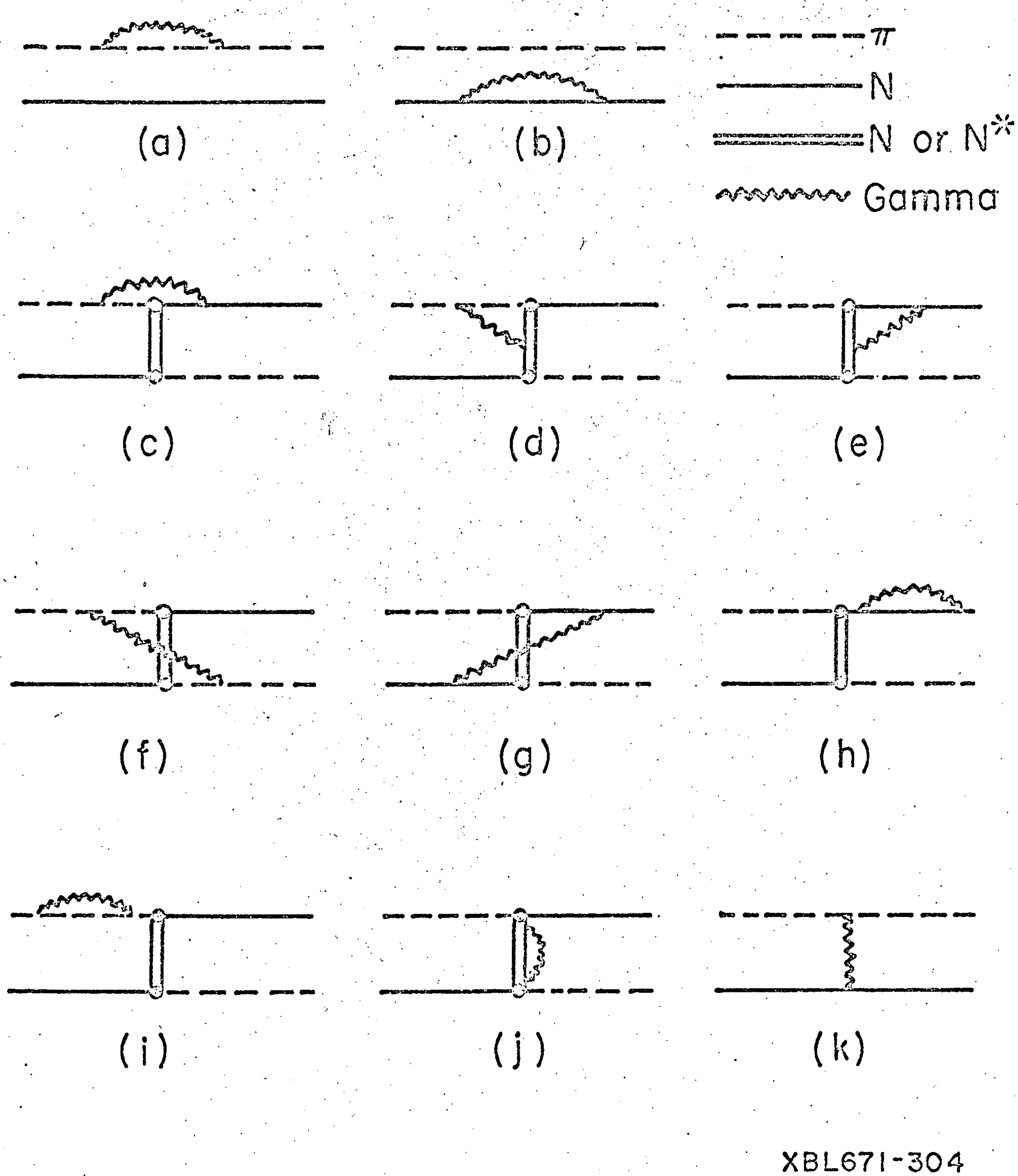

Fig. 1