Mean B-field Effects on Jet Formation in the β-Plane Magnetohydrodynamics Turbulence in the Solar Tachocline

Authors
Chen, Chang-Chun
Diamond, Patrick

Publication Date
2018-11-05

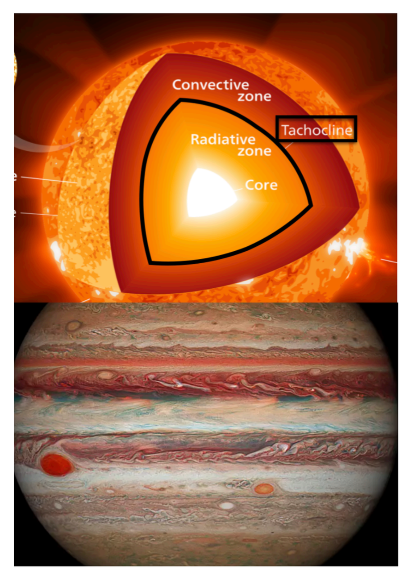
Mean B-field Effects on Jet Formation in β-Plane MHD

60th Annual Meeting of the APS Division of Plasma Physics, 2018
Chang-Chun Chen[1] & Patrick H. Diamond[1]
[1]University of California, San Diego

Abstract

The tachocline is believed to play an important role in solar magnetic activity. Simulations and theories have shown that it plays an important role in generating large-scale magnetic fields. Several studies point out the stratified turbulence inside the tachocline is of significance in redistributing angular momentum on a long timescale. The details of the turbulent transport process, however, remains poorly understood. Tobias et al. found by using a simple β-plane MHD model of the deep tachocline with a weak toroidal magnetic field will suppress the formation of zonal flow. We report an analytical theory of this flow suppression due to the mean B-field, which also enters as a modification of the cross phase in the vorticity flux. A mean field equation for the vorticity and comparisons of real-space and k-space formulations will also be presented by using closure and quasi-linear (QL) approximations.

Introduction



- The tachocline is believed to play an important role in solar magnetic activity
- The lower tachocline is strongly stratified. If we could remove the convection zone of the Sun, we'd see the similar pattern of the tachocline as of the Jupiter atmosphere.
- We consider the β-plane MHD turbulence for this quasi-geostrophic MHD, instead of spherical shell model, for simplicity.
- Spiegel and Zahn (1992) have proposed that the redistribution of angular momentum result in an anisotropic eddy vorticity (friction). Gough and McIntyre (1998), however, argued that the turbulence will act as antifriction and driving mean flow.
- Tobias et al. (2007) asserted this stratified turbulence plays no role of angular momentum transporting due to the cancelation of Maxwell stress and Reynold stress.
- **Physics behind the β-plane turbulence does not just merely depend on Aflvén or Rossby state!**

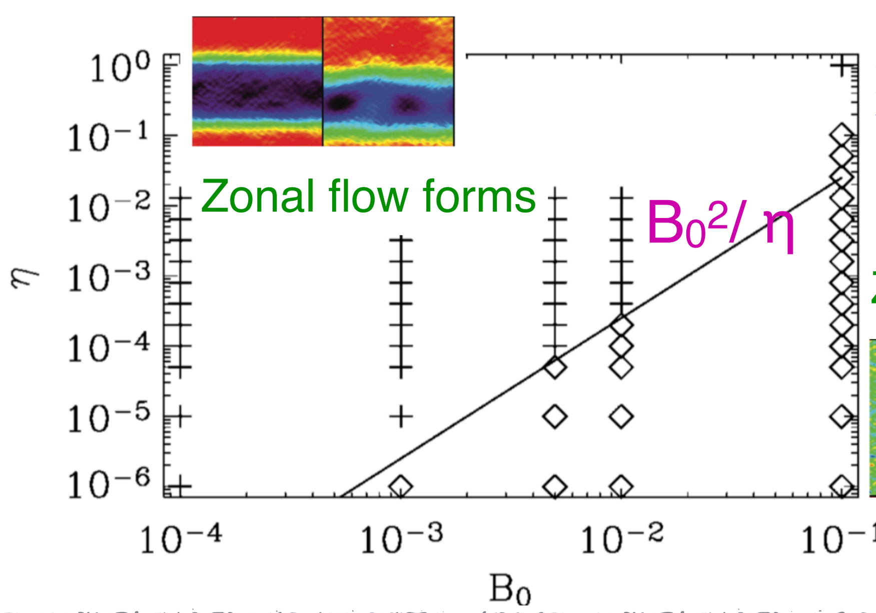


Fig 2. Scaling law for the transition between forward cascades (diamonds) and inverse cascades (plus signs). The line is given by a constant.

Critical Questions

- **What's the dimensionless parameter, which underlines $\frac{B_0^2}{\eta}$?**
- **What's the physics behind this parameter?**

Our work— Forming Zonal Flow in 2D MHD Turbulence (Quasi-Linear Approximation)

From the linear response of vorticity and field potential, we'll have :

$$\begin{cases} \text{Stream Function} & \psi = \psi(x,y,z) \\ \text{Velocity field} & \mathbf{u} = (\frac{\partial\psi}{\partial y}, -\frac{\partial\psi}{\partial x}, 0) \\ \text{Fluid Vorticity} & \zeta = (0,0,\zeta) \\ \text{Potential Field} & \mathbf{A} = (0,0,A) \\ \text{Magnetic Field} & \mathbf{B} = (B_0 + c\frac{\partial A}{\partial y}, -\frac{\partial A}{\partial x}, 0) \end{cases}$$

$$\Gamma = (D_1 - D_2)(-\frac{\partial}{\partial y}\langle\zeta\rangle - \beta) = (D_1 - D_2)\frac{-\partial}{\partial y}(\frac{\langle\zeta\rangle + f_0 + \beta y}{h}) = (D_1 - D_2)\frac{-\partial}{\partial y}\langle PV\rangle,$$

Evolution of Mean Vorticity

$$\boxed{\frac{\partial}{\partial t}\langle\zeta\rangle = -\frac{\partial}{\partial y}\left\{(D_1 - D_2)(-\frac{\partial}{\partial y}PV)\right\}}$$

Dispersion Relation of Rossby-Alfvén wave

$$\boxed{\left(\omega - \mathbf{k}\langle\mathbf{u}\rangle + \frac{\beta k_x}{k^2} + i\nu k^2\right)\left(\omega - \mathbf{k}\langle\mathbf{u}\rangle + i\eta k^2\right) = v_A^2 k_x^2}$$

We calculate $\langle \widetilde{B}_{y,k}\nabla^2\widetilde{A}_k\rangle$ by replacing $\widetilde{B}_{y,k}$, and $\widetilde{A}_k$ with vorticity $\widetilde{\zeta}_k$ (from induction equation):

$$\widetilde{A}_k = \frac{\widetilde{\zeta}}{k^2}(\frac{B_0 k_x}{-\omega - i\eta k^2})$$

Notice that the D_1, D_2 is **transport coefficient** of potential vorticity (PV).

$$D_1 = \sum_k |\widetilde{v}_{y,k}|^2 \frac{\nu k^2 + \omega_A^2\frac{\eta k^2}{\omega^2 + \eta^2 k^4}}{\left(\omega - \omega_A^2\frac{\omega}{\omega^2 + \eta^2 k^4}\right)^2 + \left(\nu k^2 + \omega_A^2\frac{\eta k^2}{\omega^2 + \eta^2 k^4}\right)^2} \quad D_2 = \sum_k |\widetilde{v}_{y,k}|^2 \frac{\omega_A^2 k^2\left[\nu(\omega^2 + \eta^2 k^4) + \omega_A^2\eta\right]}{\omega^2\left(\omega^2 + \eta^2 k^4 - \omega_A^2\right)^2 + k^4\left[\nu(\omega^2 + \eta^2 k^4) + \omega_A^2\eta\right]^2}$$

We have four frequencies: Alfvén frequency $v_A k_x$, Rossby frequency ω_R, νk^2, and ηk^2, where νk^2 and ηk^2 suggest for **the turbulence decorrelation.**

Now, we consider the 3 most relevant limits:

	3 Relavent Limits		Vorticity Flux
1	Pure 2D Fluid (Unmagnetized Wave)	$B_0=0$	$\Gamma = \sum_k \lvert\widetilde{v}_{y,k}\rvert^2 \frac{\nu k^2}{\omega^2 + (\nu k^2)^2}(-\frac{\partial}{\partial y}\langle\zeta\rangle - \beta)$
2	Strong B_0 Field (Alfvénized wave)	$\omega \sim \sqrt{\frac{B_0^2}{\mu_0\rho}}k_x \gg \eta k^2,\ \nu k^2,\ \omega_R$	$\Gamma = 0 + \mathcal{O}(\frac{\eta k^2}{\omega_A})^2$.
3	Weak Field (Electrostatic)	$\eta k^2 \gg \omega \gg \sqrt{\frac{B_0^2}{\mu_0\rho}}k_x \gg \nu k^2$	$\Gamma \sim \sum_k \lvert\widetilde{v}_{y,k}\rvert^2 \frac{v_A^2 k_x^2}{\eta k^2\omega_R^2}(-\frac{\partial}{\partial y}\langle\zeta\rangle - \beta)$

1. Pure 2D Fluid: We retrieve the vorticity flux of unmagnetized fluid! ✔
2. Strong mean B-field: The Reynold stress is **canceled out** by the Maxwell stress, thus vorticity flux is zero with 2^{nd} order correction of $\frac{\eta k^2}{\omega_A} \propto \frac{\eta}{B_0^2}$. ✔
3. **PV transport modification and B_0-field dependence enter through the Reynold stress cross phase $\langle\widetilde{v}_y\widetilde{\zeta}\rangle$, not Reynold-Maxwell csmpetition**:s$\frac{\mathbf{v_A^2 k_x^2}}{\eta\mathbf{k^2}\omega_\mathbf{R}^2} \propto \boxed{\frac{\mathbf{B_0^2}}{\eta}} \ll \mathbf{1}$ we retriese the coefficient! The dimensionless parameter is:

Dimensionless Key Parameter: $\boxed{\frac{k_x^2 v_A^2}{\eta k^2\omega_R} = \frac{k_x^2 v_A^2 l_R^2}{k_x\widetilde{V}\eta}} \ll 1$ ✔

If we stir the system in small scale ($\tilde{V}$) ➔ Rhine scale can be determined!

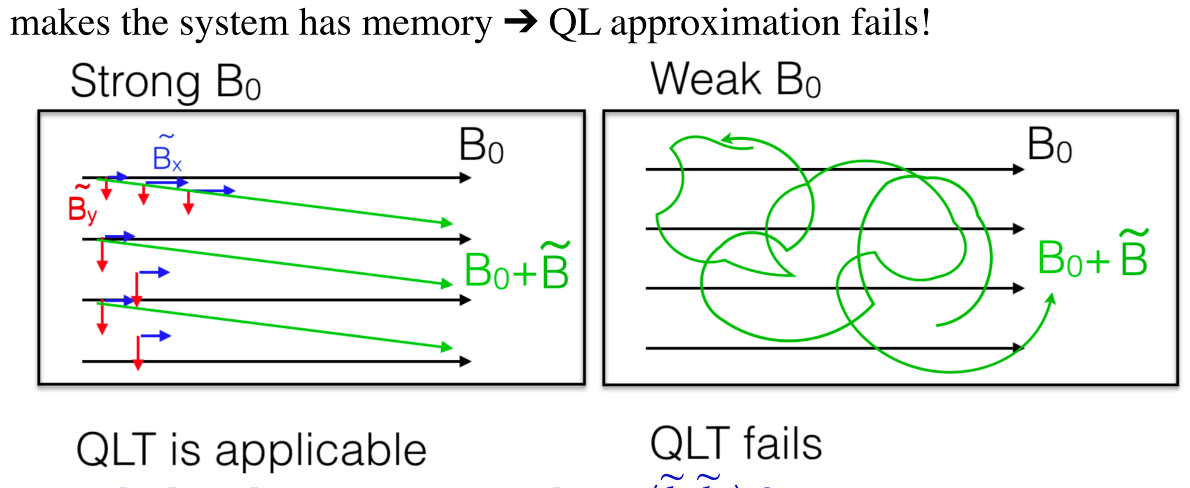
Discussion

Is Quasi-Linear (QL) Theory applicable in weak B-field ? NO

- **Zel'dovich Theorem:** in high magnetic Reynold number R_m : $\langle\widetilde{B}^2\rangle \sim R_m B_0^2$.

Weak mean B-field ➔ **Generate** ➔ **Strong magnetic turbulence (tangled springs)**

➔ makes the system has memory ➔ QL approximation fails!



- **How to calculate the nonzero cross phase $\langle\widetilde{k}_x\widetilde{k}_y\rangle$?**

  We are interested in Maxwell stress ➔ **Symmetry breaking** by zonal shear!

  Starting with $\frac{D}{Dt}k_y = -\frac{\partial}{\partial y}(k_y\langle v_x\rangle)$, and modify cross phase $\langle k_x k_y\rangle$ with $k_y = k_y^{(0)} - k_x\frac{\partial\langle V_x\rangle}{\partial y}\tau_c$.
- Next: Consider weak field where QL approximation fails. We'll recalculate the cross phase in $\langle\widetilde{v}_x\widetilde{v}_y\rangle$ and $\langle\widetilde{B}_x\widetilde{B}_x\rangle$.

For fusion: Many related issue appears in Magnetic fusion energy (MFE) plasmas

- i.e. Effect of weak Resonant Magnetic Perturbation (RMP) or ambient stochastic field on flow evolution?

Previous analyses are using quasilinear Approximation.

Presentation date: 11/05/2018

This work is supported by the U.S. Department of Energy under Award No. DE-FG02-04ER54738

References

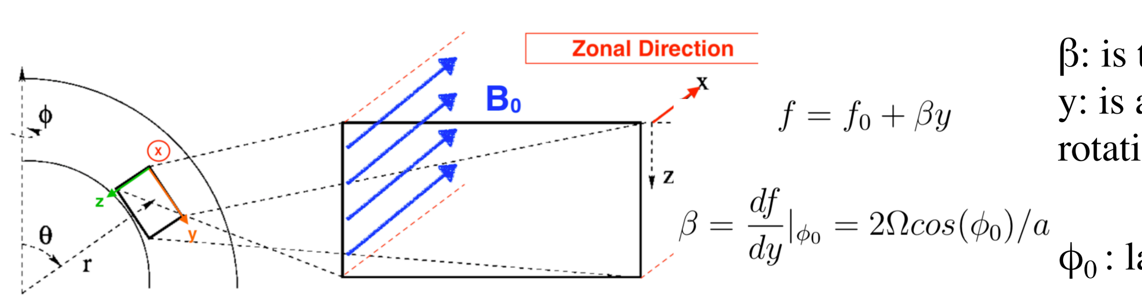
P.H. Diamond, S.-I. Itoh, K. Itoh, L.J. Silvers, β-plane mhd turbulence and dissipation in the solar tachocline. The solar tachocline, 213 (2007)
D.O. Gough, M.E. McIntyre, Inevitability of a magnetic field in the Sun's radiative inte- rior. Nature **394**, 755–757 (1998). doi: 10.1038/29472
E.A. Spiegel, J.-P. Zahn, The solar tachocline. Astronomy and Astrophysics **265**, 106–114 (1992)
S.M. Tobias, P.H. Diamond, D.W. Hughes, β-Plane Magnetohydrodynamic Turbulence in the Solar Tachocline. The Astrophysical Journal Letters **667**, 113–116 (2007). doi:10.1086/521978
G.K. Vallis, *Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-Scale Circulation*, 2nd edn. (Cambridge University Press, 2017). doi:10.1017/9781107588417
Y.B. Zeldovich, The magnetic field in the two-dimensional motion of a conducting turbu- lent fluid. Sov. Phys. JETP **4**, 460–462 (1957)

Basic Physics

1. β-Plane Approximation

Consider a solid sphere– a planet, which is covered by a thin atmosphere. The sphere is rotating in a constant angular velocity. At latitude ϕ_0 the velocity is at the surface is v, and thus the **Coriolis Force** is $2\Omega \times v$. The **Navier-Stokes equations** in a rotating frame can be written as:

$$(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla)\mathbf{v} + 2\Omega \times \mathbf{v} = -\nabla P + \mathbf{F}$$



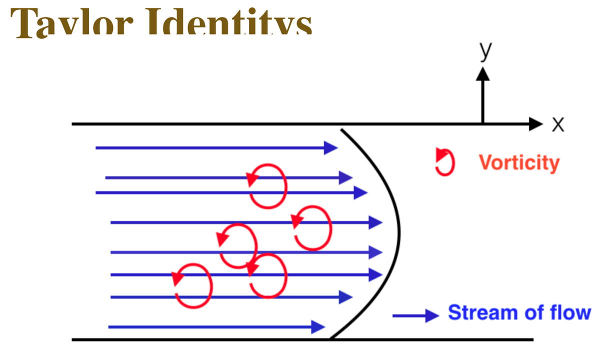
$$f = f_0 + \beta y$$

$$\beta = \frac{df}{dy}|_{\phi_0} = 2\Omega cos(\phi_0)/a$$

β: is the **Rossby parameter**
y: is a meridional distance Ω is angular rotation rate of the planet

ϕ_0 : latitude raising from the equator

2. Taylor Identitys



$$\frac{\partial}{\partial t}\langle v_x\rangle = \langle\widetilde{v}_y\widetilde{\zeta}\rangle_s = -\frac{\partial}{\partial y}\langle\widetilde{v}_y\widetilde{v}_x\rangle_s$$

PV flux transport in y-direction will generate a mean zonal (x-direction) flow.

Shear force in x direction will contributes the formation of zonal flow.

$$\frac{\partial}{\partial t}\langle\rho\rangle = -\nabla\langle\widetilde{v}_y\widetilde{\rho}\rangle - \frac{\partial}{\partial y}\frac{\langle\widetilde{B}_y\widetilde{J}_z\rangle}{\mu_0}$$

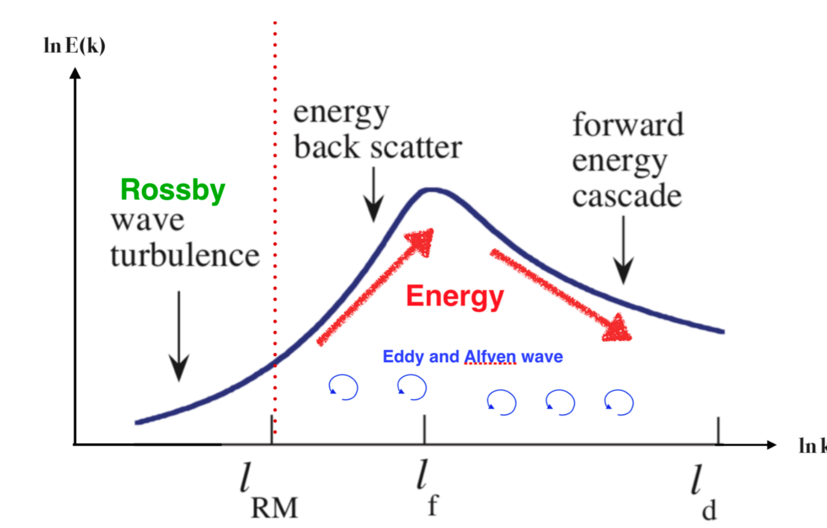
(Evolution of) Vorticity~ (Evolution of) charge density

$$\frac{\partial}{\partial t}\langle v_x\rangle = -\frac{\partial}{\partial y}\left\{\langle\widetilde{v}_{x,k}\widetilde{v}_{y,k}\rangle_s - \frac{\langle\widetilde{B}_{x,k}\widetilde{B}_{y,k}\rangle_s}{\mu_0\rho}\right\}$$

(Evolution of) mean zonal flow

3. Rhines scale in 2D MHD

It's widely accepted that the Zel'dovich Theorem for 2D MHD is applicable to β-plane MHD:



$$\frac{\langle\widetilde{B}^2\rangle}{B_0^2} = \frac{\eta_T}{\eta}, \quad \text{Zel'dovich Theorem}$$

$$l_{RM} = \sqrt{\frac{\widetilde{V}_A}{\beta}}, \quad \text{MHD Rhines scale}$$