Title
Boiling the frog optimally: An experiment on survivor curve shapes and internet revenue

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Intolerable Nuisances: Some Laboratory Evidence on Survivor Curve Shapes*

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Abstract

The fraction of a user population willing to tolerate nuisances of size x is summarized in the survivor curve $S(x)$; its shape is crucial in economic decisions such as pricing and advertising. We report a laboratory experiment that, for the first time, estimates the shape of survivor curves in several different settings. Laboratory subjects engage in a series of six desirable activities, e.g., playing a video game, viewing a chosen video clip, or earning money by answering questions. For each activity and each subject we introduce a chosen level $x \in [x_{\min}, x_{\max}]$ of a particular nuisance, and the subject chooses whether to tolerate the nuisance or to switch to a bland activity for the remaining time. New non-parametric techniques provide bounds on the empirical survivor curves for each activity. Parametric fits of the classic Weibull distribution provide estimates of the survivor curves' shapes. The fitted shape parameter depends on the activity and nuisance, but overall the estimated survivor curves tend to be log-convex. An implication, given the model of [Aperjis and Huberman, 2011], is that introducing nuisances all at once will generally be more profitable than introducing them gradually.

JEL Classification C91 · D40 · L11

Keywords Internet monetization · online advertising · pricing · reference points · adaptation · laboratory experiment

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1 Introduction

A survivor curve summarizes the fraction $S(x)$ of a population that remains after exposure to some scalable nuisance x. Medical researchers use them routinely to describe toxicity, and report the survival rate $S(x)$ as a function of the exposure x to some toxin. Economists' familiar demand curves can be regarded as a special case, where the scalable nuisance x is unit price of a good and $S(x)$ is the chosen quantity normalized by the quantity chosen at zero price.

Our interest is in the typical shape of survivor curves for enjoyable activities that may be abandoned when some nuisance is imposed. A motivating example is an Internet content provider such as the New York Times or YouTube. Users initially may have free access, but eventually the content provider must cover costs by generating revenue via nuisances such as ads and/or subscription fees. Like a monopolistic competitor choosing price, the content provider seeks the optimal nuisance level, trading off fewer remaining users against increased revenue per user. The shape of the survivor curve is clearly crucial.

Section 2 briefly notes connections to strands of literature in economics, psychology and philosophy. Section 3 defines survivor curves more carefully. It emphasizes the distinction between log-concave and log-convex curves, which is essentially the distinction between decreasing and increasing demand elasticity when x is taken to be log price. The distinction is central in the model of Aperjis and Huberman [2011, 2012], who find that a content provider maximizes value by introducing the necessary nuisance all at once if the survivor curve is log-convex, but should introduce it gradually if the curve is log-concave.

Are survivor curves typically log-concave, log-convex, or neither? To the best of our knowledge, previous research provides no clear evidence. Behavior in natural settings is difficult to interpret. For example, website users leave for many reasons unrelated to the chosen nuisance level x, while new users arrive who may have different reactions to x and to the content. Moreover, when x is changed, remaining users may form beliefs about further nuisances that may be introduced later, and such beliefs could vary widely across users. Competitors' nuisance adjustments might also have a major impact.

Laboratory experiments are especially helpful to answer the shape question, because one can control for all these confounding factors, and can systematically vary the nuisance level x. In section 4 we describe an experiment designed to discover the shape of the survivor function over a variety of domains. The experiment confronts 112 human subjects with six different tasks interrupted by nuisances of magnitude $x \in [x_{\min}, x_{\max}]$. We observe 636 decisions of whether to stay with an enjoyable activity or to leave after the nuisance is imposed.

Section 5 collects the results. Novel nonparametric techniques reveal upper and lower bounds on the survivor curve. Preliminary regression analysis shows that the chosen ranges $[x_{\min}, x_{\max}]$ are reasonably well calibrated, that order effects are unimportant, and that behavior is reasonably consistent across tasks. The main finding concerns the shape parameters in Weibull distributions estimated for data from each of the six tasks. Estimation requires extension of established techniques to deal (for the first time that we know of) with doubly censored data. Surprisingly (at least to some of the coauthors), the overall estimate of the shape parameter is well inside the log-convex region.

A concluding discussion notes some caveats, suggests broader applications and implications, and points to future research. Supplementary data analysis and instructions to subjects appear in on-line Appendices.

2 Related Literature

We know of no previous studies estimating the shape of survivor curves for scalable nuisances. There is, of course, a vast literature on the shape of demand curves. Perhaps the most relevant article here is Popescu and Wu [2007], which argues theoretically that firms with risk averse customers maximize profits by gradually increasing or gradually decreasing price. In an adaptation model, Fibich et al. [2005] find that price elasticities increase over time.

A separate strand of literature on adaptation theory considers how users react over time to an introduced inconvenience. A number of papers consider adaptation in the context of repeat-purchase markets and characterize optimal dynamic pricing policies [Kopalle et al., 1996, Fibich et al., 2003, Popescu and Wu, 2007, Nasiry and Popescu, 2010]. In these papers, a firm (usually a monopolist) is facing consumers whose purchase decisions are influenced by past prices through reference price effects. The demand in a given period is assumed to be a function of the current price and the reference price (but does not depend on the number of people that purchased the product in the previous period). In a laboratory experiment, Kahneman et al. [1993] suggest that duration plays a role in the recollection of aversive experiences, with reference points being formed at the peak and end of the negative experience.[1]

There is an active theoretical literature on reference points [Kahneman and Tversky, 1979, Frederick and Loewenstein, 1999, Kőszegi and Rabin, 2006] which has inspired many recent laboratory experiments, including Gneezy [2005] and Baucells et al. [2011]. Abeler et al. [2011] find empirical evidence supporting Kőszegi and Rabin [2006]: payoff expectations seem to anchor reference points, as identified by subjects' effort choices. By contrast Heffetz and List [2011] find no support for the expectations reference point hypothesis. Closely related to this literature we find a number of experimental and empirical studies that focus on the formation of reference points [surveys are provided by Kalyanaram and Winer, 1995, Mazumdar et al., 2005]. In these studies, the inconvenience is the price of a product, and thus the reference point is a reference price. Even though the role of historic prices in forming price expectations is supported in many of these studies, the jury is still out on which specific model best describes how consumers update their reference prices.

In psychology there is a classic strand of literature on "just noticeable differences," which is associated with failures in the transitivity of preferences as in the self-torturer example of Quinn [1990], or in the Sorites ("heap") paradox. The paradox is attributed to Eubulides of Miletus, a disciple of the Megarian school of philosophy, and goes as follows. "No one grain of wheat can be identified as making the difference between being a heap and not being a heap. Given then that one grain of wheat does not make a heap, it would seem to follow that two do not, thus three do not, and so on. In the end it would appear that no amount of wheat can make a heap." [Hyde, 2011] The point is a failure of transitivity: a big difference can be generated by a sequence of unnoticeable differences.

Popular literature has picked up that theme, and suggested that nuisances should be increased imperceptibly. The proverb is that, if only the heat is increased gradually enough, one can keep a frog happy while cooking him. [Goldstein, 2000] says, "Here is how *not* to boil a live frog: boil up a pan of water, pick up the frog and throw it in the pan. The art of frog-boiling is an ancient one."

In the medical literature there is a practical concern over which is more painful, a

[1]The empirical adaptation literature is also related to studies such as Ariely [1998] that examine how remembered pain relates to the time path of pain intensity. It may be worth pointing out that our own concerns are quite different: we shall examine empirically how stay/remain decisions (not recollections) depend on one-shot intensities (not time paths) of nuisances (not pain) in a variety of modalities.

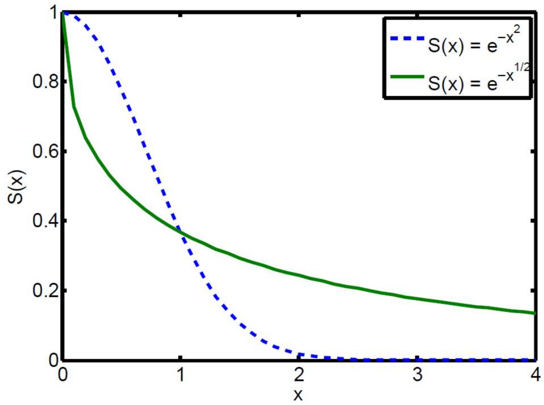


Figure 1: A log-concave (dashed line) and a log-convex (solid line) function.

slow removal of band-aids or a fast one, and results show that a one-shot fast removal is the way to go [Furyk et al., 2009]. Returning to economics, field data suggests that firms generally prefer subdividing price increases but not price decreases [Chen et al., 2008].

3 Theory

A survivor curve $S : [0, \infty) \to [0, 1]$ is a monotone decreasing function that maps the size of a nuisance (or toxic dose or other scalable adversity) $x \geq 0$ into the fraction of "survivors," e.g., those that continue with an enjoyable activity. We seek to estimate the shape of survivor curves in a variety of contexts. In particular, for reasons noted later in this section, we want to distinguish log-concave from log-convex survivor functions.

A function is log-concave if its logarithm is concave. Examples include $S(x) = e^{-x^k}$ with $k > 1$ and $S(x) = (1-x)^k \cdot \mathbf{1}_{[x\in[0,1]]}$ with $k > 1$, where $\mathbf{1}_{[\cdot]}$ is the indicator function. All concave and linear functions are log-concave, but there also exist convex functions that are log-concave.

A function is log-convex if its logarithm is convex. For instance, this is the case if $S(x) = 1/(1+x)^k$ with $k > 0$ or $S(x) = e^{-x^k}$ with $k \in (0, 1)$. To get some intuition for the distinction between log-concave and log-convex survivor curves, consider Figure 1 which shows the log-concave function e^{-x^2} and the log-convex function $e^{-x^{1/2}}$. Note that the dashed line is above the solid line for small nuisances x but for large deviations ($x > 1$ in the Figure) the comparison is reversed. In terms of the website application, the interpretation is that users facing a small nuisance are more likely to stay when their behavior is described by the log-concave function, but users facing a large nuisance are more likely to stay when their survivor function is log-convex.

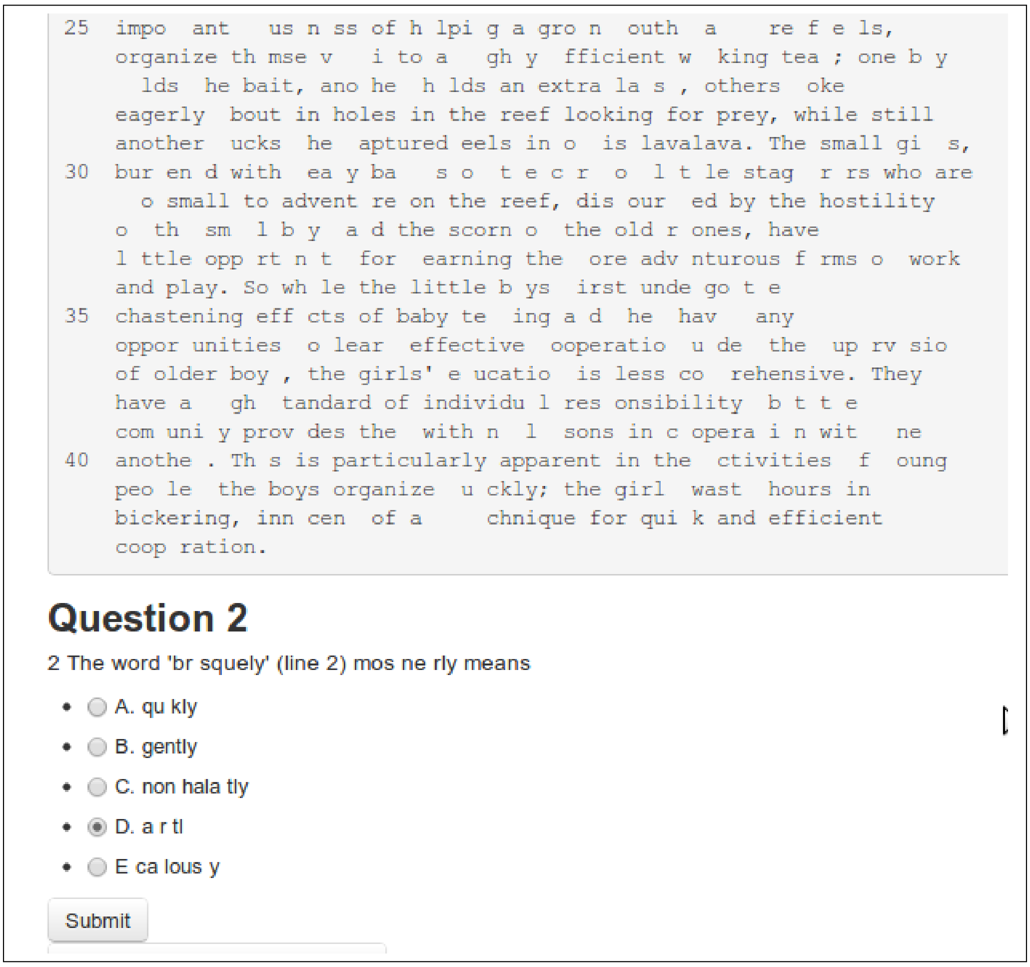
The model of Aperjis and Huberman [2011] takes this reasoning a couple of steps further. Assuming that eventually remaining users completely adapt to any relevant level of the nuisance, the authors show when it is optimal to follow the frog boiling proverb and introduce any necessary level of nuisance gradually, and when it is instead better to introduce that nuisance all at once.

The intuition comes from comparing the survivor fraction $S(x+y)$ to that of $S(x)$ and $S(y)$. One can see from the Figure (or from the definitions of concavity and convexity of $\ln S$) that $S(x+y)S(0) \geq S(x)S(y)$ for any $x, y \geq 0$ if S is log-convex. Thus, in this case the fraction of survivors $S(x+y)S(0)$ after nuisance level $x+y$ is introduced all at once exceeds the fraction $S(x)S(y)$ who survive introduction of nuisance level x at first and then later (after full adaptation) the additional nuisance y. On the other hand, if the survivor curve is log-concave, then we have the reverse inequality $S(x+y)S(0) \leq S(x)S(y)$, which implies that introducing the same level of nuisance in two discrete steps will leave more survivors. Note that this is *not* a result of selection, because the function S is assumed to not change over time.

Aperjis and Huberman [2011] confirm this intuition in a more general setting. Assuming complete adaptation, they show that in the log-concave case it will be optimal for the provider to increase inconvenience gradually according to a particular schedule that trades off the cost of delaying revenue collected via the nuisance against the benefits of keeping more users. Conversely, in the log-convex case, it is optimal to introduce the necessary nuisance all at once.

4 Methods

The laboratory experiment presented subjects with tasks of the following sort. First, they engaged in a pleasurable activity, such as putting on earphones and watching an 8 minute video clip — their choice either of an interview of John Stewart at The O'Reilly Factor, or else a selection of the 10 most popular ads shown to viewers of the 2010 Super Bowl. (Pilot experiments included a longer list of videos, but these two were the most popular.) Then, after 100 seconds, an annoying computer-generated voice at $z \in [30, 80]$ decibels began reading the decimal expansion of $\pi = 3.14159...$. Subjects knew that the only way to escape the auditory nuisance was to click a button that immediately switched them to a bland activity, in this case watching a video of gentle waves breaking at La Jolla beach, for the remaining 6 minutes or so. Of course, a higher fraction of subjects switched when $z = 80$ decibels than when $z = 30$, and intermediate fractions switched at intermediate values z of the nuisance.


impo ant us n ss of h lpi g a gro n outh a re f e ls, organize th mse v i to a gh y fficient w king tea ; one b y lds he bait, ano he h lds an extra la s , others oke eagerly bout in holes in the reef looking for prey, while still another ucks he aptured eels in o is lavalava. The small gi s, bur en d with ea y ba s o t e c r o l t le stag r rs who are o small to advent re on the reef, dis our ed by the hostility o th sm l b y a d the scorn o the old r ones, have l ttle opp rt n t for earning the ore adv nturous f rms o work and play. So wh le the little b ys irst unde go t e chastening eff cts of baby te ing a d he hav any oppor unities o lear effective ooperatio u de the up rv sio of older boy , the girls' e ucatio is less co rehensive. They have a gh tandard of individu l res onsibility b t t e com uni y prov des the with n l sons in c opera i n wit ne anothe . Th s is particularly apparent in the ctivities f oung peo le the boys organize u ckly; the girl wast hours in bickering, inn cen of a chnique for qui k and efficient coop ration.

Question 2

2 The word 'br squely' (line 2) mos ne rly means

- A. qu kly
- B. gently
- C. non hala tly
- D. a r tl
- E ca lous y

Submit

Figure 2: SAT task with nuisance level $x = 4$, corresponding to probability $z = 0.15$ of dropping each letter.

We also presented subjects with visual nuisances, like large flashing pop-up ads that interrupted a video clip for 15 seconds, leaving only z nuisance-free seconds between interruptions, with z ranging from 5 to 30 seconds. Figure 2 shows a text-based nuisance for the task of answering SAT questions, with a \$0.40/\$0.10 payment for each correct/incorrect answer. The nuisance is the random omission of each letter with probability $z \in [0.06, 0.21]$; e.g., $z = 0.15$ in Figure 2. Subjects could escape the nuisance entirely by clicking a button, but then would be paid for the remainder of the 8 minute

Task	Activity A	Activity B	Inconvenience:	Range of z
1. Movie/Pi	Watch Movie	Watch Waves	π digits	[30, 80] decibels
2. Movie/Pop	Watch Movie	Watch Waves	15 sec Pop-up	every [5, 30] sec
3. Slug	Slug ($$$$)	Slug ($)	Jitter	[0.10, 0.25] rate
4. Read	Read Article	Count Bits	Drop Letters	[0.15, 0.30] rate
5. SAT	SAT Questions ($$$$)	SAT Questions ($)	Drop Letters	[0.06, 0.21] rate
6. Pay	Watch Movie	Watch Waves	Pay to Stay	[1, 23] cent fee

Table 1: Task specification.

period at the much lower rate of \$0.10/\$0.02.

We presented each subject with six distinct tasks that shared the common structure depicted in Figure 3. The subject starts with an engaging activity (A activity), which after 100 seconds is interrupted by a scalable nuisance of size z (or level x) that remains attached to the A activity thereafter. She can escape the nuisance at any time by clicking a button to switch to a "bland" activity (B activity) where she will remain for the rest of the 6-8 minute period. Her choice of whether or not to switch is a data point that helps us estimate the shape of the survivor curve.

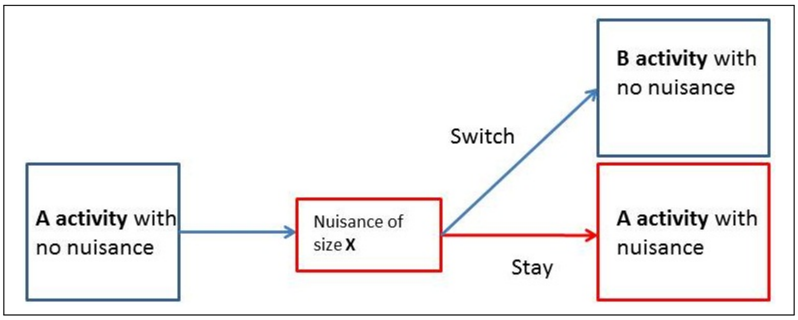


Figure 3: Task structure. The nuisance is introduced to activity A after 100 seconds and remains for the rest of the 6-8 minute period.

Table 1 summarizes the six combinations of A activity, scalable nuisance, and B activity presented to each subject. Of the entries not yet mentioned, Slug is a simple video game similar to Snake (see Appendix B for a detailed description of the activity), and the jitter nuisance involves a random turn (left or right) each pixel with probability $z \in [.10, .25]$. The Pay to Stay nuisance is a one time fee of z cents deducted from a 500 cent endowment, which can be avoided only by switching to the B activity. The B activity Count Bits is illustrated in Figure 4 below. Paid activities are indicated by ($$$$), and B activities paid at 1/4 the rate are indicated by ($).

Nuisance values z were chosen to span each task-specific range $[z_{\min}, z_{\max}]$ by six evenly spaced values. These values were coded as nuisance levels $x = 1, 2, ..., 6$ comparable across tasks, as detailed in Appendix A. The task-specific nuisance ranges were chosen to avoid the inefficient sampling that occurs when $S(x(z))$ is very close to 0 or 1. Based on a few pilot sessions, we aimed to have $S(x(z_{\text{min}}))$ in the vicinity of 0.8 and $S(x(z_{\text{max}}))$ in the vicinity of 0.20.

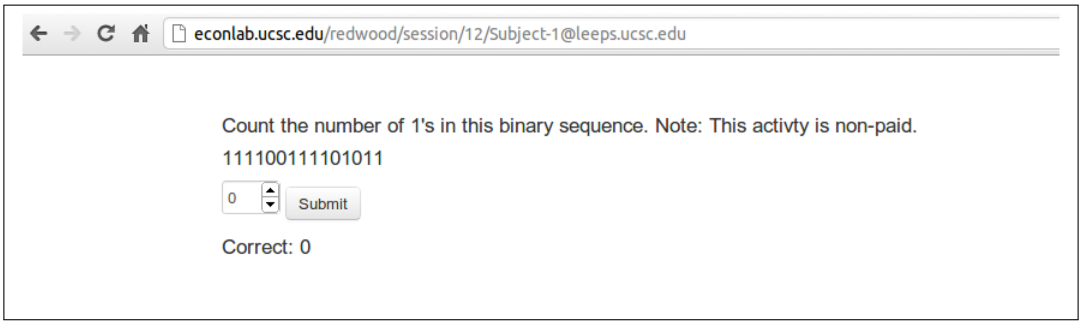


Figure 4: Counting Bits. The subject is asked to count the number of ones in a random binary string of 15 digits. If incorrect, she is asked to try again. If correct, she goes on to a new string. The task repeats until the end of the 6 minute period.

4.1 Procedure

We recruited 112 human subjects, most of them undergraduates majoring in Economics, Biology or Engineering. Each subject participated in only one of the 16 sessions we ran. Sessions lasted 70 to 90 minutes, including the time used to read instructions and to pay subjects.

Upon arrival, each subject was assigned to an isolated computer terminal, and general instructions for the experiment were read aloud; a copy appears in On-line Appendix C. Next, subjects practiced all B activities, in order to ensure that they knew exactly what they would do if they decided to switch to a bland activity. Subjects were then given specific instructions for the first of the six tasks, after completion they received instructions for the second task, completed it and were given instructions for the third task, etc. The order of the six tasks was varied in a balanced manner across sessions. In each session we randomly assigned each subject's nuisance level x, but limited the choice to one of the two nuisance bins that we created; either $x = 1, 3, 5$ or $x = 2, 4, 6$ in each session. These bins allowed us to have for each activity in each session a sizeable number of observations with the same treatment level.

Before each task it was announced whether A and B would be paid activities. If they were, then a detailed description of the payment schedule was given. If they weren't paid, then we emphasized it in the instructions. Subjects would know how much money they had made at the end of each paid task, and once the experiment was over, they were paid individually. Payments included a \$5 show-up fee and ranged from \$12 to \$27 (some subjects proved very proficient at Slug); they averaged around \$16.

5 Results

The experiment yielded 636 observations of whether ($Y(i, j, x) = 1$) or not ($Y(i, j, x) = 0$) subject i decided to switch after experiencing inconvenience level x in task j. Due to implementation glitches, we lost one Slug data point and the SAT data in two sessions (35 data points); hence the slight shortfall from the intended $6 \times 112 = 672$ observations.

We begin with a non-parametric summary of the data. Recall that for each subject and each activity, we see only the choice of whether or not to switch for a single nuisance level x. It seems safe to assume that if a subject switched (did not switch) at x then she would still switch (not switch) at any greater nuisance level $y > x$ (lesser level $y < x$), but there is no obvious way to guess how switchers (nonswitchers) would respond to lesser (greater) nuisances. It follows that a conservative upper bound $S_j^U(x)$ for the survivor curve in task j is defined by assuming that everyone survives (i.e., doesn't switch) except those who are seen to have died (switched) at that or lesser nuisance levels. Using the indicator function $I_{[y \leq x]}$ to pick out those levels, we have

$$S_j^U(x) = \frac{n_j - \sum_{i=1}^{n_j} Y(i, j, y) I_{[y \leq x]}}{n_j}, \tag{1}$$

where n_j is the number of observations of task j, usually 112. Likewise, noting that $1 - Y(i, j, x)$ picks out the non-switchers,

$$S_j^L(x) = \frac{\sum_{i=1}^{n_j} (1 - Y(i, j, y)) I_{[y \geq x]}}{n_j} \tag{2}$$

is a conservative lower bound for the survivor curve $S(x)$, and is defined by assuming that nobody survives except those seen to survive level x or higher levels.

Figure 5 shows these bounds on the empirical survivor curves for each task, as well as the midrange between the bounds. The Figure also includes bars showing the fraction of observations at each nuisance level $x = 1, ..., 6$ that were switches ($Y = 1$), and also includes dots showing the fraction of non-switchers observed at adjacent nuisance levels.

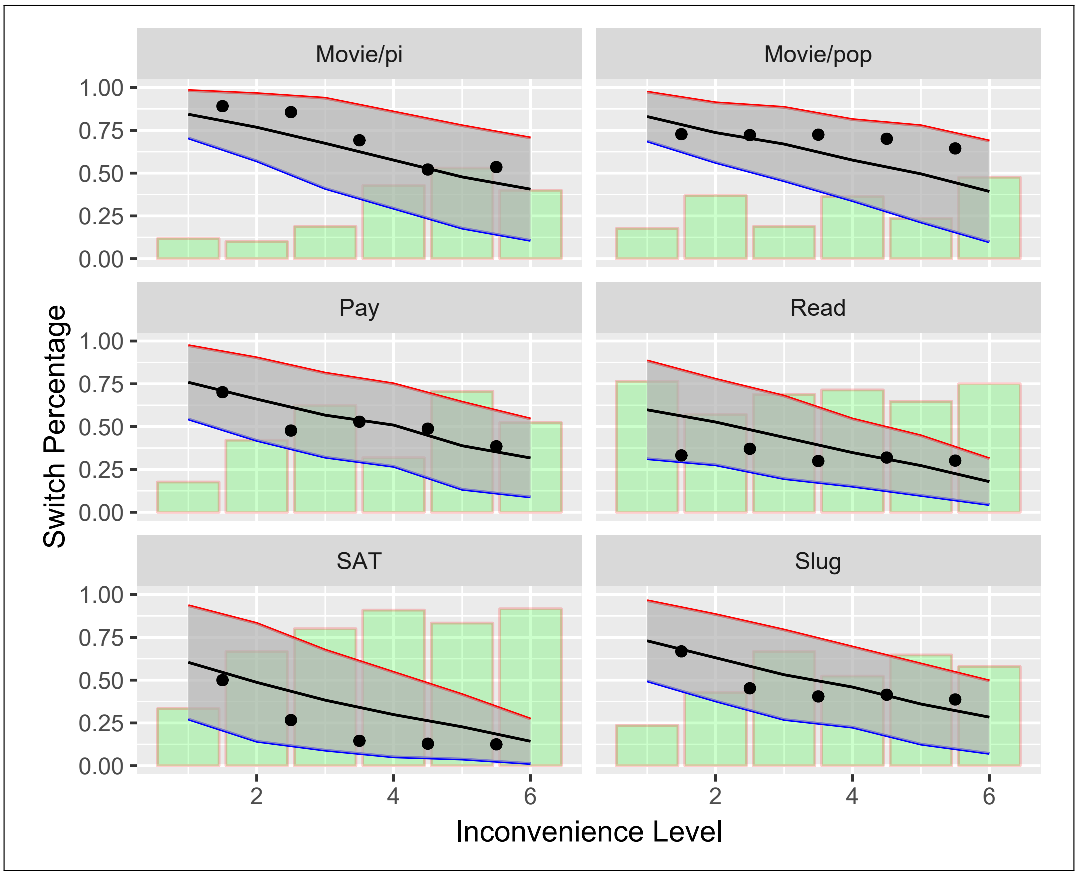


Figure 5: Overview of raw data. The red (blue) lines show the empirical upper bound $S_j^U(x)$ (lower bound $S_j^L(x)$) of the survivor curve for each task j, and the black line is their average $(S_j^U(x)+S_j^U(x))/2$. The percentage of subjects that switched ($Y = 1$) for each inconvenience level (x) are represented by the green bars. The dots are the average percentage of not switching ($Y = 0$) between every two consecutive inconvenience levels (x).

Do the different nuisance levels make a difference? Do the different tasks have different overall switch rates? To answer these preliminary questions, we run a Probit regression of the binary outcome $Y(i, j, x)$ on dummies for nuisance levels and the different tasks. Columns 1 to 4 of Table 2 report coefficient estimates (and standard errors) obtained from the entire data set, while the remaining columns report estimates obtained from the separate task data.

The composite estimates show (see especially Column 3) highly significant and essentially monotonic impact of nuisances, and the task effects are also all highly significant, except Movie/Pop, whose impact is not significantly different than that of the baseline

	Composite Estimates				Individual Task Estimates					
					Pi	Pop	Slug	Read	SAT	Pay
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
N-level=2.	0.299*		0.359*	0.735***	-0.275	0.490	0.966	-0.610	0.938*	0.168
	(0.180)		(0.190)	(0.280)	(0.921)	(0.862)	(0.847)	(0.820)	(0.523)	(0.818)
N-level=3.	0.578***		0.633***	0.649***	0.375	0.109	1.202**	-0.296	0.736	1.206**
	(0.171)		(0.187)	(0.177)	(0.545)	(0.511)	(0.486)	(0.473)	(0.646)	(0.501)
N-level=4.	0.524***		0.642***	0.968***	0.797	0.652	1.349	0.0489	11.15***	-0.473
	(0.165)		(0.175)	(0.257)	(0.951)	(0.872)	(0.837)	(0.815)	(1.002)	(0.817)
N-level=5.	0.746***		0.845***	0.913***	1.128*	0.220	1.157**	-0.507	16.01***	1.517***
	(0.164)		(0.178)	(0.199)	(0.585)	(0.521)	(0.464)	(0.544)	(1.196)	(0.503)
N-level=6.	0.737***		0.859***	1.265***	0.762	0.742	1.349	-0.227	15.91***	0.479
	(0.179)		(0.196)	(0.281)	(0.917)	(0.893)	(0.830)	(0.835)	(1.195)	(0.832)
Task=2.		0.0511	0.0591	0.180						
		(0.174)	(0.176)	(0.194)						
Task=3.		0.551***	0.583***	0.708***						
		(0.174)	(0.176)	(0.185)						
Task=4.		1.029***	1.072***	1.322***						
		(0.172)	(0.174)	(0.243)						
Task=5.		1.144***	1.238***	1.339***						
		(0.199)	(0.191)	(0.238)						
Task=6.		0.428**	0.444**	0.554***						
		(0.170)	(0.177)	(0.185)						
Constant	-0.524***	-0.540***	-1.133***	-1.282***	-0.948**	-1.072**	-0.763*	0.735*	-0.0847	-0.554
	(0.125)	(0.125)	(0.186)	(0.221)	(0.478)	(0.417)	(0.401)	(0.378)	(0.439)	(0.383)
N	636	636	636	636	112	112	111	112	74	112
Order Dummies	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Probit Estimates (and standard errors). Dependent variable is Switch (Y). Independent dummy variables are included for Nuisance levels 2-6 (level 1 is excluded) and for Tasks 2-6 (Task 1, Movie/Pi, is excluded). Order dummies are explained in Appendix A, which includes additional robustness checks. Errors are clustered at the subject level. The symbols * , ** and *** respectively indicate p-values less thant 0.10, 0.05 and 0.01.

task, Movie/Pi. Not surprisingly, given the smaller sample sizes and heterogeneity, the impacts estimated from individual task data (Columns 6-10) are mostly not statistically not significant.

Appendix A reports additional robustness checks at the individual and aggregate level, and confirms that there were no important session or sequence effects. It also reports a Fisher Exact test comparing switch rates for each value of x across tasks. The switch rates are not statistically significant, suggesting that all tasks share a similar underlying distribution of subject tolerance for nuisances. We shall now examine that question more directly.

5.1 Estimation Strategy

A major objective of our experiment is to detect log-concavity or log-convexity of the survivor curve $S(x)$ separately in each of our six tasks. Recall that each subject i contributes an independent observation (switch or not) for each separate curve j. Recall also that each observation is informative in one direction but not the other; more formally expressed, each observation is either left censored or right censored. If subject j switches ($Y = 1$) to the bland activity B when facing nuisance level x, then we infer that her switching threshold y is somewhere in the interval $(0, x]$, and so this observation is left censored (LC). Therefore the likelihood of the observation is given not by the density of the distribution of thresholds at x but rather by the cumulative distribution function F evaluated at that point:

$$F(x) \equiv P(y \leq x).$$

On the other hand, if subject j stays in activity A ($Y = 0$), then we infer that her threshold is in the interval (x, ∞), and the observation is right censored (RC). The likelihood of such an observation is

$$S(x) = P(y > x),$$

where $S(x) \equiv 1 - F(x)$ is the probability that the subject "survives" the introduction of the inconvenience.

This likelihood function applies to any parametric family of survivor curves. We use the family most often used for survivor curve analysis, the standard two-parameter Weibull family (e.g., Johnson et al. [1994], Rinne [2008]). Recall that the Weibull distribution has density

$$f(x;\gamma,\kappa)=\begin{cases}\frac{\kappa}{\gamma}(\frac{x}{\gamma})^{\kappa-1}e^{-(\frac{x}{\gamma})^{\kappa}} & if\, x\geq 0\\ 0 & if\, x<0,\end{cases}$$

where $\kappa > 0$ is the shape parameter and $\gamma > 0$ is a scale parameter for the distribution. The corresponding cdf is $F(x;\kappa,\gamma) = 1 - e^{-\left(\frac{x}{\gamma}\right)^{\kappa}}$, and thus the survivor function is $S(x;\kappa,\gamma) = e^{-\left(\frac{x}{\gamma}\right)^{\kappa}}$.

Besides being standard, the Weibull family has the very convenient property that the shape parameter κ alone determines the hazard function and log-convexity versus log-concavity. To see this, recall that the hazard rate $h(x) = f(x)/S(x) = -\frac{d\ln S}{dx}$ for any survivor function S is the density for switching at nuisance level x conditional on not switching at a lower level. For the Weibull distribution this function is easily seen to be proportional to $x^{\kappa-1}$. Thus $h(x)$ is an increasing function if $\kappa > 1$ and is decreasing if $\kappa < 1$. [Bagnoli and Bergstrom, 2005] derives the implications regarding log-convexity and log concavity. To summarize,

- $S(x)$ is log-convex (and the hazard rate is strictly decreasing) if $0 < \kappa < 1$, and
- $S(x)$ is log-concave (and the hazard rate is increasing) if $\kappa \geq 1$.

Econometric packages usually include the Weibull distribution, and sometimes can deal with singly censored data, but we must build our own likelihood function to deal with doubly censored data. It follows from the preceding discussion that the likelihood function for data $Y = (Y_{ij})$ is:

$$\begin{aligned} L(\gamma,\kappa|Y) &= \prod_{Y_{ij}\in LC} P(X<x_i|\gamma,\kappa) \prod_{Y_{ij}\in RC} P(X>x_i|\gamma,\kappa)\\ &= \prod_{Y_{ij}\in LC}\left(1-e^{-\left(\frac{x_i}{\gamma}\right)^{\kappa}}\right)\prod_{Y_{ij}\in RC} e^{-\left(\frac{x_i}{\gamma}\right)^{\kappa}}. \end{aligned} \tag{3}$$

We maximize the expression in (3) over the parameter space using standard non-linear minimization techniques (a Newton-type algorithm) in the statistical package R to obtain point estimates of the shape parameter κ. The results are reported in Table 3 along with the centered 90% confidence interval obtained through bootstrap procedures (clustered at the individual level) appropriate for finite samples.

5.2 Fitted Parameters

Task	Shape Parameter	90% Confidence Interval
Movie/π	1.17	[0.59, 1.99]
Movie/Pop	0.43	[0.11, 0.89]
Slug	0.60	[0.23, 1.09]
Read	-	-
SAT	0.92	[0.48, 1.48]
Pay	0.62	[0.24, 1.10]
All Six Tasks	**0.50**	**[0.33, 0.68]**
Five Estimable Tasks	**0.652**	**[0.45, 0.86]**

Table 3: Weibull estimation results. Estimates in the next to last line pool all data, and those in the last line pool data for all tasks except Read.

Several things stand out in Table 3. First, four of the six point estimates are for a shape parameter below 1. One exception is for the task Movie/π, which has an estimate close to 1, but with a confidence interval that includes a considerable interval below 1. The other exception is Read, where MLE does not converge. Looking back at Figure 5, one gets the impression that there is insufficient variation across the chosen range $[0.15, 0.30]$ of the nuisance (letter drop probability). Perhaps a contributing factor is that some of the subjects apparently enjoyed the B activity, bit counting, more than the A activity. Of the four $\kappa < 1$ estimates, three of them (Movie/Pop, Slug and Pay) have confidence intervals mainly or entirely in the log-convex region.

The pooled data, whether or not we include the problematic Read data, yield a shape parameter clearly below 1. Indeed, the bootstrap histograms shown in Figure 6 have negligible probability mass for $\kappa > 1$. We conclude that the overall survivor curve of our subjects is log-convex.

Supplementary graphs in Appendix A plot the fitted Weibull survivor curves against the empirical upper and lower bounds; the results generally seem reasonable (see Figure 8).

6 Discussion

Our pooled data yield a Weibull shape parameter estimate clearly inside the log-convex region $\kappa < 1$. Different tasks (combinations of pleasurable activities, nuisances, and bland activities) generate somewhat different behavior, but none of the six task-specific

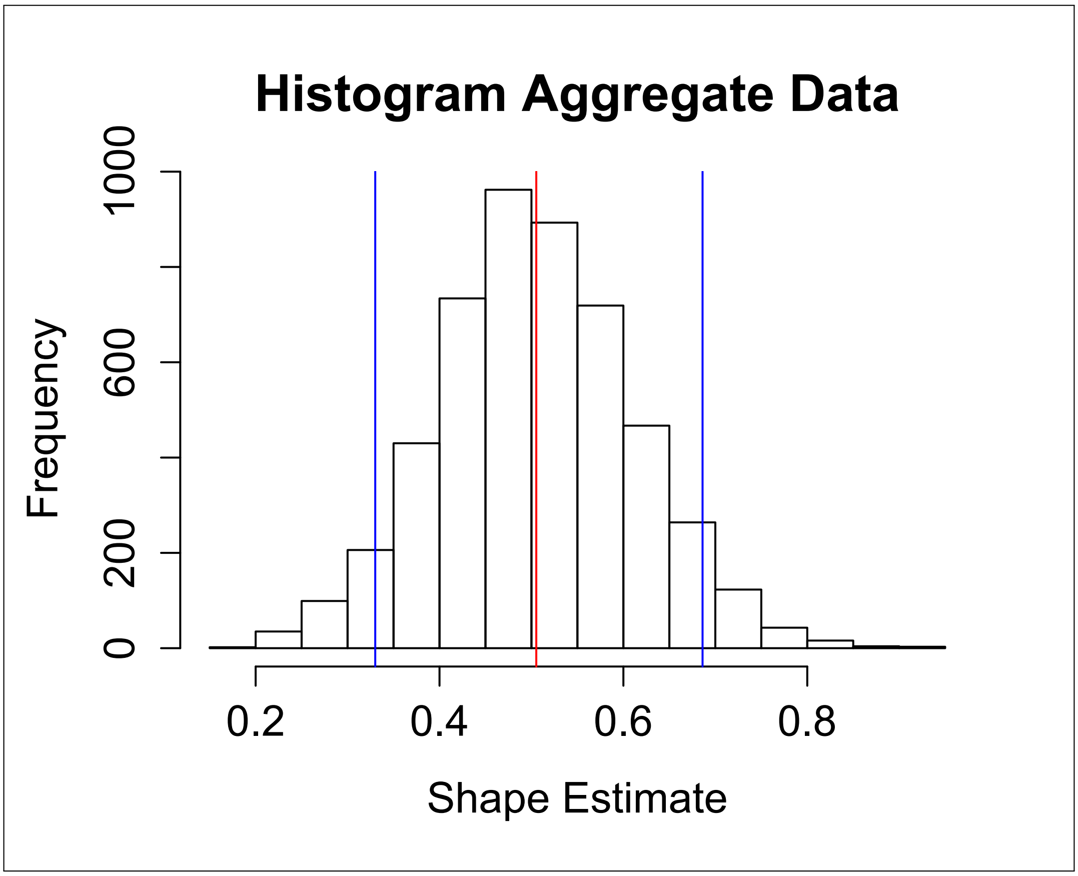


Figure 6: Histogram of bootstrap estimations of the shape parameter κ for the composite data across all tasks. The average estimate is shown as a red vertical line, and the 90% confidence interval is bounded by the two blue vertical lines.

estimates is clearly inconsistent with a log-convex shape. Three of them definitely point to log-convexity, two are near enough to $\kappa = 1$ to leave the question open, and one estimate fails to converge. Overall, then, our study — the first to estimate the shape of survivor curves in response to avoidable nuisances — suggests that log-convexity is typical.

As with any empirical finding, several caveats are in order. Our results are based on the decisions of more than 100 human subjects recruited from a subject pool consisting mostly of undergraduate students in a US university. It is entirely possible that other populations would be more or less tolerant of nuisances than ours, and thus have survivor curves with different scale or location. However, it seems to us rather implausible that they would yield survivor curves with much different *shape* than ours, but of course that can only be confirmed through further research. Also, we only looked at six different tasks. Although our tasks spanned a considerable range, there still are innumerable other activities and nuisances that might be considered, and each might produce a different shape parameter. Again, only further research can determine how many of these tasks have survivor curves that are log-convex.

A direct implication of a Weibull shape parameter $\kappa < 1$ is that the hazard rate (in other contexts sometimes called the failure rate or inverse Mills ratio) is decreasing. This means that, proportionately speaking, we lose more participants at low intensity; the few who remain at high intensity are less apt to switch when we ratchet up intensity a bit more.

A practical implication arises in the theoretical framework of Aperjis and Huberman [2011]: web content providers should introduce their necessary nuisances all at once. Contrary to the Goldstein proverb, it seems that the best way to boil a frog may actually be to drop it into a pan of boiling water.[2] Of course, that theoretical framework includes a strong assumption that we do not test — that people eventually fully adjust to any particular nuisance level. Like many assumptions, it is probably a good approximation within some range but might fail badly far enough outside that range. It seems reasonable to conjecture that violations of the full-adjustment assumption would tend even more to favor introducing a necessary nuisance all at once.[3]

In any case, we hope that our study encourages other empirical investigations of survivor curves and of all-at-once versus gradual changes in nuisances or useful features, and new theoretical investigations of generalized demand curves.

[2]Anecdotally, our result is in line with those of actual frog boiling attempts, as reported in online interviews by Dr. Victor Hutchinson Emeritus Professor of Biology at the University of Oklahoma http://srel.uga.edu/outreach/ecoviews/ecoview071223.htm

[3] We thank an anonymous referee for this conjecture.

7 Appendix A: Details

Details on Inconveniences

Table 4 summarizes the inconvenience levels and the number of observations at each level. Along it, Table 5 reports the p-values of a Fisher exact test comparing switch rates across tasks for each value of x. As mentioned, the results suggest that all tasks share a similar underlying distribution of subject tolerance for nuisances.

Inconvenience	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Pi volume	30[11]	40[20]	50[12]	60[22]	70[13]	80[20]
Pop-up	30[11]	25[19]	20[12]	15[22]	10[13]	5[21]
Jitter	.10[11]	.13[21]	.16[12]	.19[22]	.22[13]	.25[19]
Reading	.15[11]	.18[21]	.21[12]	.24[21]	.27[13]	.30[20]
SAT	.6[9]	.9[8]	.12[11]	.15[11]	.18[12]	.21[12]
Movie Pay	1[11]	5[19]	9[12]	13[22]	17[13]	23[21]

Table 4: Nuisance Values [and Numbers of Observations]. The units are respectively: decibels, seconds between pop-up, probability of jitter per pixel, probability of dropping each letter (both for Reading and SAT), and cents.

Task	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6
Task 1	-	0.405	0.559	0.225	0.310	0.423
Task 2	-	-	0.666	0.694	0.511	0.403
Task 3	-	-	-	0.666	0.984	0.974
Task 4	-	-	-	-	0.599	0.345
Task 5	-	-	-	-	-	0.911
Task 6	-	-	-	-	-	-

Table 5: P-values of the Fisher exact test comparing switching decisions for all treatment levels.

Robustness Checks

Table 6 presents robustness checks to supplement Table 4. Additional variables include ordering ($i.order$), or sequence of tasks in the session, and several dummies $pibigpop_{i,j}$, $popbigpi_{i,j}$, $readbigsat_{i,j}$, $satbigread_{i,j}$ that test for similar activities with different levels of nuisance. For example, the dummy $pibigpop_{i,j}$ ($popbigpi_{i,j}$) indicates trials in which the nuisance level for Movie/Pi (Movie/Pop) is higher than the nuisance level for Movie/Pop (Movie/Pi); similarly $readbigsat_{i,j}$ ($satbigread_{i,j}$) is a dummy for the case where Reading (SAT) has a higher nuisance level than does SAT (Reading).

	Composite Estimate	Pi	Pop	Slug	Read	SAT	Pay
	(4)	(5)	(6)	(7)	(8)	(9)	(10)
2.order	-0.315	-9.724***	-0.253	-0.384	0.101	-5.601***	-1.179**
	(0.223)	(1.104)	(0.694)	(0.509)	(0.550)	(0.894)	(0.569)
3.order	-0.552	0.687	0.361	-0.779	-0.265		-0.773
	(0.382)	(1.043)	(0.886)	(0.812)	(0.856)		(0.883)
4.order	-0.516	-0.494	-0.0734	-0.508	-0.270	-15.26***	-0.240
	(0.334)	(0.861)	(0.775)	(0.740)	(0.748)	(1.214)	(0.736)
5.order	-0.0737	0.290	-0.0111	-0.726	-0.619	0.570	1.167
	(0.287)	(0.924)	(0.883)	(0.846)	(1.009)	(0.796)	(0.835)
6.order	-0.248	-0.00678	0.512	-0.203	0.215	-0.611	0.786
	(0.356)	(0.870)	(0.792)	(0.760)	(0.886)	(0.489)	(0.762)
7.order	0.443	0.551	0.413	0.237	-0.0855		-0.635
	(0.408)	(0.840)	(0.452)	(0.467)	(0.495)		(0.480)
1.readbigsat	-0.249				0.266		
	(0.261)				(0.455)		
1.satbigread	0.0678					-10.22***	
	(0.515)					(1.107)	
1.popbigpi	-0.302		0.172				
	(0.408)		(0.658)				
1.pibigpop	1.155**	9.975***					
	(0.536)	(0.638)					
Cons	-1.282***	-0.948**	-1.072**	-0.763*	0.735*	-0.0847	-0.554
	(0.221)	(0.478)	(0.417)	(0.401)	(0.378)	(0.439)	(0.383)
N	636	112	112	111	112	74	112
Order Dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 6: Switching Probit Model, Continued.

The composite results show that none of the 6 task orderings employed in the experiment has a statistical effect on the decisions of subjects, and only one of the nuisance level difference dummies (the indicator that Movie/Pi has a higher nuisance level than Movie/Po, which there are only 8 observations) seems to have an effect. Of the 16 session dummies, only one is significant at the 5% level.

Individual task data gives very similar results. Only one ordering (2.order, which is Pi, Pop, Slug, Read, SAT, Pay) seems to have any impact relative to baseline in any of the tasks (in this case, on Pi, SAT and Pay but not on Pop, Slug or Read).

We conclude that our results are robust, and that the few dummies with significant estimates probably arise from small sample bias.

Fit of the Estimation for all Activities

In Figure 7 we present a summary the pooled data across all treatments along with the survivor curve resulting from a Weibull distribution with the estimated parameters from Table 3. The fit of the curve to our empirical data points is remarkably good. Additionally we present the survivor curves resulting from our estimated parameters for each activity in Figure 8.

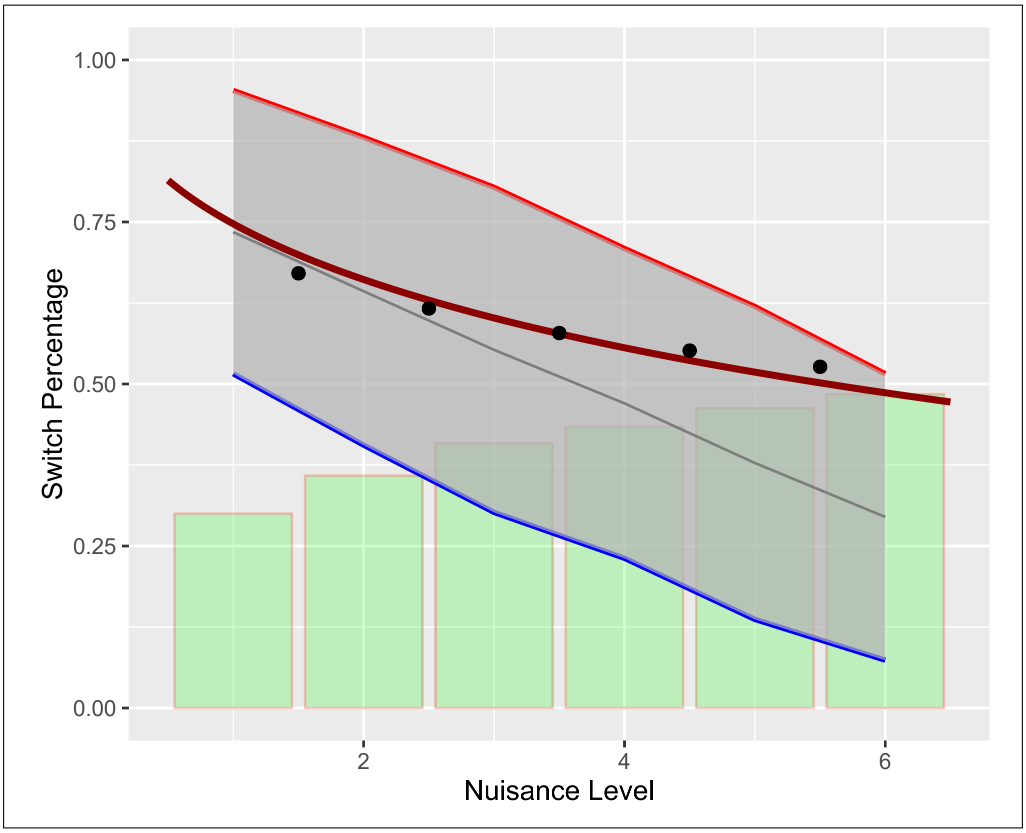


Figure 7: Overview the pooled data across all activities. The red (blue) line shows the upper (lower) bound of the $S(x)$. The black line is the average between the upper and lower bound. The percentage of subjects that switched ($Y = 1$) for each inconvenience level (x) are represented by the green bars. The dots are the average percentage of not switching ($Y = 0$) between every two consecutive inconvenience levels (x). The thicker dark red line is the survival curve resulting from a Weibull distribution with the estimated parameters.

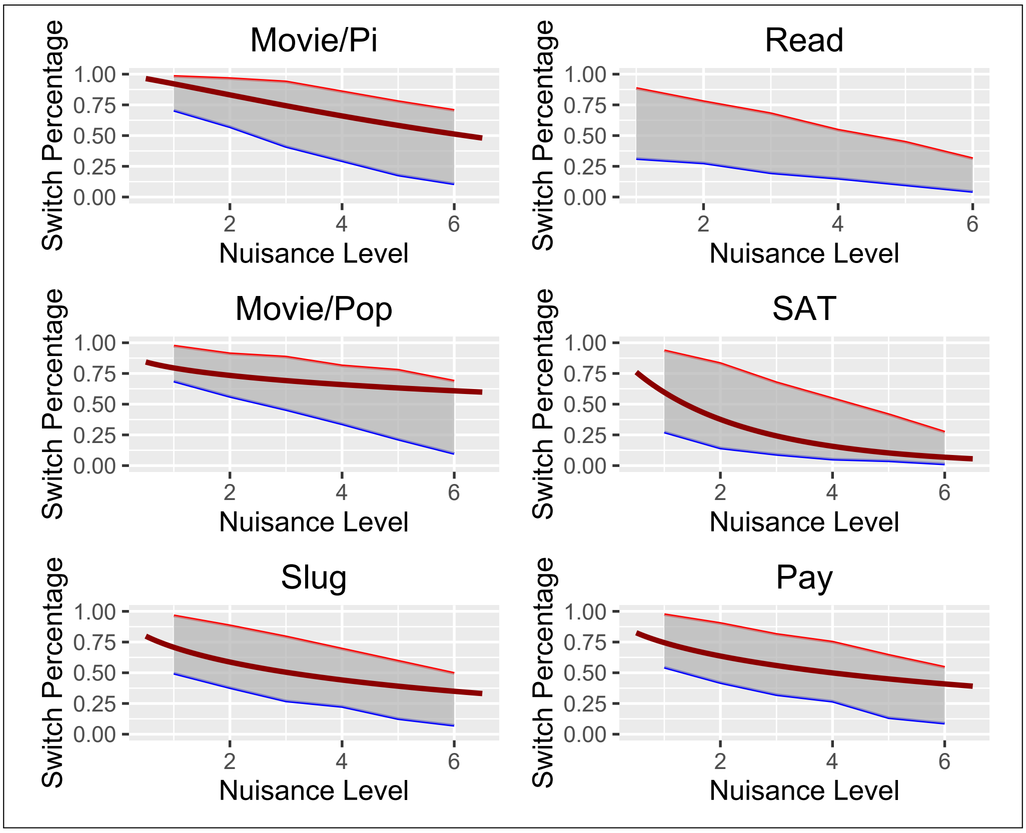


Figure 8: Resulting survival curves from estimated parameters for each activity. The red (blue) line shows the upper (lower) bound of the $S(x)$. The thicker dark red line is the survival curve resulting from a Weibull distribution with the estimated parameters. Note that for activity Read there was no convergence of the MLE.

8 Appendix B: Description of activities

In this appendix we list all the activities that were not described in detail in the methods section.

Movie/Pop: Subjects were presented with a menu of two video clips (an interview of Zack Galifianakis by Letterman, and a clip on how to do crossover moves in basketball). After 100 seconds of visualization, a 15 second long pop-up would appear on the screen. This pop-up would partially cover the video clip, and have flashing colors; moreover, while the pop-up was on the screen, the movie clip would continue playing in the background but with no sound. The unit of the nuisance $x \in [5, 30]$ is the number of seconds between consecutive pop-ups, e.g., if a subject was assigned a nuisance level of $x = 5$, then she would have a 15 second pop-up every 5 seconds. If the subject decided that the nuisance was too big, then she could switch to the bland activity which, as in all movie activities, was a video of gentle waves breaking at La Jolla beach. Once a subject switched to the bland activity she would remain there until the end of the round. Rounds lasted 8 minutes.

Note on wave watching: The bland activity for all movie activities is "watching waves." We decided to use this video because as it has no plot, that is, its "replay value" is very high, allowing us to reuse it with almost no loss in its (relative) attractiveness.

Slug: Slug is a version of the classic video game Snake. Snake was a popular arcade game in the 1970's but gained world-wide acceptance in 1998 as it became the standard pre-loaded game in Nokia phones. The game has been used as "Easter egg" by both Youtube and Gmail. In this game the objective is to get "food," which corresponds to colored pixels that appear at random points of the enclosed "playing space." Each time the player gets to food she earns points, but the slug increases in size, making it harder to maneuver. To get to the food subjects control the slug with the keyboard arrows. If the slug bumps into the walls of the enclosed playing space, or if it hits itself, the player loses. Losing has no cost in points, the subject just need to restart the game by pressing the refresh button (F5 on the keyboard), and the game starts over with the same amount of accumulated points. As mentioned, points are awarded by getting to food; 10 points for regular food and 40 points for bonus food. The difference between these two types of food is that bonus food only stays on screen during 10 seconds, while normal food is there until eaten. Food is color coded, with bonus food being yellow, and regular food blue. Each point was worth \$0.01. The jitter nuisance would start 50 seconds into the round, and involves a random turn (left or right) each pixel with probability $x \in [.10, .25]$. The bland activity towards which subjects could switch was the same exact game without the jittering nuisance, but paying only one fourth of the amounts in the original activity (i.e., 10 points per bonus food, and 5 points for each piece of regular food "eaten"). Each round lasted 7 minutes.

Read: Subjects are given a menu with a series of articles from the New York Times (an article on the Proposition B for LA county, an article on veterans of the Iraq war coming back to the US, and an article on fee increase at the UC system). The nuisance $x \in [.15, .30]$ is the (independent) probability for each letter of being dropped. The first 15% of the text would be nuisance free. On the other hand, the text was presented broken into paragraphs. To ensure that subjects actually read the text, they could only move to the next paragraph by clicking a "next" button that would appear 10 seconds after the start of every new paragraph. The bland activity was counting bits, which presented subjects with a binary string of 15 digits, and asked them to count how many 1's were in the string. If the answer was correct, then a new string was generated. If the answer was wrong the subject would be given a new opportunity until he answered correctly. This would last until the end of the round, which was 6 minutes long.

SAT: Subjects could pick between two different texts taken from an SAT practice web-page. The text would be presented to subjects along with only one of the 8 multiple choice questions they had in this round. All answers were final, and once a choice was made the next question would appear, with no way of going back. This was a paid activity and each correct answer would pay \$0.40, while each incorrect answer would penalize \$0.10. The nuisance for this activity was letter dropping, and worked exactly as in the Read activity. In this case each letter was dropped with probability $x \in [0.06, 0.21]$. The bland activity was the same task with all the letters, but paying one fourth (i.e., \$0.10 for each correct answer and -\$0.02 per incorrect answer). If a subject decided to switch, she would not start over all the questions, but would start the bland activity at the same question where she switched to activity B.

9 Appendix C: Instructions

Upon entering the lab subjects were read an initial set of instructions that described the structure of the experiment but did not give any details on the activities or inconveniences they would encounter; subjects were told that detailed instructions would be given before each round. These instructions appeared on separate pages for each separate task. However, to save space below, we omit the page breaks and put the detailed task instructions together in a single document.

9.1 General instructions

Welcome! This is an economics experiment. You will be a player in many periods of an interactive decision-making game. If you pay close attention to these instructions, you can earn a significant sum of money. It will be paid to you in cash at the end of the last period.

It is important that you remain silent and do not look at other people's work. If you have any questions, or need assistance of any kind, please raise your hand and we will come to you. We expect and appreciate your cooperation today.

The Experiment:

This experiment will have six different rounds. In each round you will begin with an enjoyable activity that we refer to as Activity A. At any time during the round you can switch to another activity, Activity B. The experimenter will announce the A and B activities for that round before it starts.

At the same time, the experimenter will also announce an "annoyance" that will accompany Activity A at some point during that round. If, after experiencing the annoyance, you think you would prefer Activity B, then simply click the button on your screen. It will immediately switch you to B, where you will remain for the rest of the round. You will never be interrupted by any annoyance in Activity B. Key points:

- You will start each round participating in an A activity.
- A activities will be interrupted by specific annoyances (announced before the round).

- At any point during the round you can switch from activity A to activity B (announced before the round)
- You can switch from A to B, but never from B to A.
- B activities do not have any interruptions.

Also note:

- Some rounds include a paid Activity and some do not.
- You automatically get to experience an A activity each round. To make sure that you are familiar with all with B activities, you will practice with all of them before the experiment starts.
- For some of the activities the audio output is needed. Please check if you have headphones attached to your computer. If you have your own, feel free to use them. You will be able to adjust the volume through the "speaker icon" on the upper right corner of your screen.
- Do not start Activity A until the experimenter announces that it is time to do so.

9.2 Specific activity instructions

Round Movie/Pi (8 minutes):

Activity A: Watching a video. You will choose it from a menu that will appear on screen.

Annoyance: While watching the video, at some point you will start to hear a computerized voice reading the first few thousand digits of the decimal expansion of $\pi = 3.14159\ldots$ This will continue at the same volume until the end of the round, or until you switch to activity B.

Activity B: Watching a video of waves breaking at La Jolla beach. This is not a paid round.

Round Movie/pop (8 minutes):

Activity A: Watching a video. You will choose it from a menu that will appear on screen.

Annoyance: While watching the video, at some point a pop-up will appear on your screen and mute the audio. These pop-ups are 15 second long, and will appear at regular intervals on your screen. The time remaining is shown on the pop-up.

Activity B: Watching a video of waves breaking at La Jolla beach. This is not a paid round.

Round Slug (7 minutes):

Activity A: Playing a game called “Slug”, very similar to the popular game “Snake.” Use your arrow keys to control a hungry slug. The slug gets longer as it eats food, and you earn points:

- Regular food (Blue Pixel): will stay on screen until you eat it, each piece that you eat which gives you 20 points.
- Bonus food (Yellow Pixel): gives you 40 points, will appear randomly and only lasts for 10 seconds on screen, if you don’t eat it during this time it disappears.

Your slug will “die” whenever it collides either with an edge of its rectangle or with its own body. But the points you earned are stored and accumulated, and you can begin again with a new slug. Just hit the refresh page key (F5) and the game will restart with a new short slug.

Annoyance: At some point the slug starts to “jitter.” That is, with some probability, it will change direction randomly each time it reaches a new pixel. The jitter rate (probability) will remain the same for Activity A the rest of the round.

Activity B: Playing the same game, “Slug,” but with two differences:

- The slug will not jitter
- You will earn points at 1/4 the previous rate: 5 points per blue pixel, 10 per yellow.

Round Read (6 minutes):

Activity A: Reading newspaper articles. You will choose one from a menu, and the text will appear on your screen. The text will be broken up into different pages. After 10 seconds "next page" button will appear. Just click the button to move to the next page. On the last page, please press the button to indicate when you are done reading the article.

Annoyance: In this activity the annoyance will be that some letters of the text will be missing. With a certain probability letters will be dropped from the article. This will apply to all the text, except the very beginning. As usual, press the button if you would rather go to the B activity than continue trying to read the article with missing letters.

Activity B: Counting the number of 1's in a string of 0's and 1's. If enter the correct number, then you will get 1 point and a new array of numbers will be randomly generated for you to count. If your answer is incorrect, then you will not get any points and will still have the same array of binary numbers for you to count. There is no limit to the number of attempts for each array. This is a activity — you get no money for the points!

Round SAT (8 minutes):

Activity A: Answering SAT questions. You will pick one of two sets of multiple choice questions. You will get paid 40 points per correct answer and will lose 10 points for incorrect answers. Your points are accumulated as you go and are shown on the screen. You will get to see 1 question at a time which you will be able to answer. Once you have answered a question you will NOT be able to change it, so you choice is always final.

Annoyance:: Except for the first question, some letters of the text will be missing. With a certain probability each letter will be dropped from each SAT question. As usual, you can press the button that takes you to activity B at any moment of the round.

Activity B: In this case the B activity will be the same SAT text, except it will have all the letters in the text, and it will pay you 10 points per correct answer and subtract 2 points if the answer is incorrect. If you switch to activity B you will start at the same point where you decided to change from A to B. So, for example, if you decided to switch at question 3, you will start activity B at question 3. Note that you can come out with negative earnings from this activity.

Movie/Pay (8 minutes):

Activity A: In this round you will be offered to pick from a series of clips to watch. On top of this you will be endowed with 500 points for you to keep.

Annoyance: Some seconds into the video you will be asked to pay a fee (in experimental points) if you want to continue watching the video.

Activity B: If you don't pay, the video will switch to waves breaking at La Jolla beach.

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