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1 Generalities

This project investigates the role of radiation in Rayleigh-Taylor instabilities by performing linear stability analyses of a plane parallel background equilibrium, with a semi-infinite medium 1 overlying a semi-infinite medium 2, in a gravitational field $\mathbf{g} = -g\mathbf{e_z}$ and a radiation flux $\mathbf{F}$ normal to the discontinuity. The width of the interface, which prohibited any mixing, is ignored throughout, imposing to the tangential wavenumber k of the eulerian perturbations considered here an upper bound equal to the reciprocal of the photon mean free path and allowing under this conditions the use of Hugoniot-Rankine-like jump equations. Vanishing of the perturbation away from the interface was also a required boundary condition. Analytical complexity limits the hitherto obtained results to essentially two opposite regimes corresponding to different approximations of the energy equations, namely the optically thin isothermal regime and the gas+radiation adiabatic optically thick regime which we now describe in turn.

2 The (very) optically thin limit

In the optically thin regime, absorption of the incoming UV radiation dominates the momentum transfer while radiative equilibrium with reemitted IR photons, which readily escape because of the lower associated opacity, sets the gas temperature. However, in the limit of zero optical depth, provided the density dependence of the monochromatic opacity can be factored out of the wavelength's, the gas temperature converges toward a finite value and the background density profile becomes exponential, leaving our system with no clearly defined discontinuity. Our framework can be restored if we focus our attention on the case of a *chemical* discontinuity, which allow a discontinuous sound speed, and hence density despite a continuous gas pressure. We assume that the specific opacity is independent of density in each region and that we can neglect attenuation of the UV flux, such that the radiation force can be lumped in an effective gravity field $\mathbf{g_{eff}} = \mathbf{g} + \frac{\kappa_F}{c}\mathbf{F}$, with the peculiarity that it may be different on either side of the interface. Under the aforementioned assumptions, the problem can be solved exactly, with the complex frequency ω obeying the following dispersion relation:

$$\omega^8 - 2(ka)^2\omega^6 + \left((ka)^4 + g_1 g_2 k^2\right)\omega^4 + k^4(g_1 - g_2)h\omega^2 - k^6h^2 = 0, \qquad (1)$$

with $a \equiv \sqrt{a_1^2 + a_2^2}$ and $h \equiv g_1 a_2^2 - g_2 a_1^2$. The instability criterion, $\rho_1 g_1 > \rho_2 g_2$, amounts, when expressed in a coordinate-free manner, to the statement that the effective gravity per unit volume *toward* the interface is greater than that *away* from it. The asymptotic growth rate

$$\omega^2 = -k\frac{\rho_1 g_1 - \rho_2 g_2}{\rho_1 + \rho_2} + O(1) \qquad (2)$$

agrees with the classical Rayleigh-Taylor result for a uniform effective gravity field.
A possible application of this could consist in the interface between ionized and neutral hydrogen in HII regions expanding via radiation pressure. The radiation flux would be absorbed at the interface and thus the effective gravity in the (hopefully acceptably isothermal) neutral zone would have no radiative contribution. A random number: 1 pc away from a star with $L = 10^5 \mathrm{L}_\odot$, with $\kappa = 1$ kg/m^2 and 1 particle per mm^3 in the neutral zone, the fastest growth timescale would be $\frac{r}{\kappa}\sqrt{\frac{c}{\rho L}} \sim 10^7$ a.

3 The (very) optically thick limit

In the optically thick limit, assuming Local Thermodynamic Equilibrium, the radiation field is a Planckian locked at the temperature T of the gas and the radiation force may be approximated as $\mathbf{G_0} = \frac{\kappa_F \rho}{c}\mathbf{F_0} \approx -\nabla P_r$ and lumped with the gas pressure gradient. If we can neglect the divergence of the flux perturbation in the gas+radiation energy equation (the 'adiabatic approximation'), the total specific entropy of the gas+radiation field is conserved for a given lagrangian fluid element, which somewhat simplifies the analytics. Focussing on perturbations that decay rapidly compared to the equilibrium scale height, and in the limit where the density of the lower medium 2 is very small compared to that of medium 1 (we henceforth drop '1' subscripts), we obtain the following dispersion relation:

$$-\frac{\mathcal{C}}{a^2}\omega^6 + \left(\frac{\mathcal{C}(\mathcal{C}-\mathcal{B})g^2}{a^4} + k^2\right)\omega^4 + \mathcal{B}\left(\frac{kg\omega}{a}\right)^2 - g^2k^4 = 0, \qquad (3)$$

with $\mathcal{B} = \frac{1}{D}\left(16(E-1)x^2+(24E-8)x+E(5+\frac{\gamma}{\gamma-1})-1+\frac{\gamma E}{4(\gamma-1)x}\right)$, $\mathcal{C} = \frac{1}{D}\left(12x+\frac{1}{\gamma-1}\right)$ and $D = 16x^2+20x+\frac{\gamma}{\gamma-1}$, $E \equiv \frac{\kappa_F F_0}{gc}$ and $x \equiv \frac{P_r}{P_g}$.
The relevant root, in the domain of validity of the approximations used, quite generically adopts its asymptotic behaviour:

$$\omega = -gk, \tag{4}$$

which really is the standard Rayleigh-Taylor instability, all radiative effects having been cast in a mere modification of the equation of state. We note the qualitative difference with the local RHD overstabilities studied by Blaes and Socrates (2003), in optically thick media but without our adiabatic approximation, where radiative flux sips in rarefied zones created by acoustic disturbances, giving rise to buoyant 'photon bubbles', and where the instability criterion involves the opacity law, which is simply absent in the adiabatic approximation except as a constraint for its validity. For large wavelengths, there may be a hint for a convective instability near the Eddington limit but the perturbation locality requirement is violated. For large wavenumbers, the adiabatic approximation breaks down since the divergence of the flux can no longer be neglected in the energy equation. Inasmuch it is a diffusion term that becomes important, it may be suspected that the instability will be damped for this wavelength range.
One application of this work would be massive star formation by accretion beyond 20-30 $M_\odot$ in spite of gravity being overcome by radiation pressure, as studied by Krumholz *et al.* (2009). The latter authors witnessed in their simulation that although enhanced luminosity first blew gas away and formed rarefied bubbles aligned along the polar axis, their walls subsequently lost axisymmetry and fingers protruded around which rather than through which radiation flowed, enabling their accretion onto the central stars via their disks.
However, it seems that the flux is somewhat too high to even allow a finite window of validity of the adiabatic calculation but that same fact coincidentally appears to allow the instability criterion of Blaes and Socrates (2003) to be satisfied, and it may be conjectured that this local instability has an interface-related counterpart, which would be the primary driver of the clumping seen in Krumholz *et al.* (2009). Then, in the zones complementary to the 'photon bubbles', where the flux-to-mass ratio is lowered, the 'adiabatic Rayleigh-Taylor instability' would take over. All these instabilities would grow on timescales comparable to the free-fall (i.e. scale height sound crossing) time, of order $1-10\,\mathrm{ka}$, as needed to account for the simulation.

4 Conclusion

The role of radiation in Rayleigh-Taylor instabilities dramatically depends on the regime considered. In the optically thin isothermal regime, the radiation force acts as an ('equivalence principle-breaking') effective gravity, whereas in the optically thick adiabatic regime, radiation merely causes a modified equation of state, and therefore is more on the side of pressure than gravity. The intermediate regime seem to harbour yet another qualitatively different effects: one would be the interface-problem version of the local 'photon bubbles' instabilities studied by Blaes and Socrates (2003) for optically thick media; another an optical depth effect in optically thin media as in Krolik (1977), a global stability analysis of a constant-density finite optically thin slab subject to the Boussinesq approximation. Analytical complexity prevented these regimes from being explored during this project.

References

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