Title

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Abstract

We consider a two-tier cellular system where there is a macrocell base station equipped with multiple antennas transmitting to multiple microcells, each equipped with a microcell base station (relay) equipped with one antenna. Each microcell has one user. We explore the degrees of freedom (DoF) of several configurations of this network with the assumption that no channel state information (CSI) of the users is available at the macrocell base station and different assumptions on the CSI between other transmitter-receiver pairs. With very limited CSI and the relays, the achievable DoF can be greatly increased compared to the case when there is no relays.

I. INTRODUCTION

While the capacity of Gaussian MIMO broadcast channel is known with perfect channel state information at the transmitter (CSIT) [1], the capacity is unknown in general with no CSIT [2], [3], [4], [5]. With no CSIT, the capacity may be much less than the case with CSIT. For example, consider a vector broadcast channel where the transmitter is equipped with multiple antennas and each receiver is equipped with a single antenna. If each receiver experiences statistically identical fading, then TDMA is optimal and the maximum degrees of freedom is one [2], [3]. While this is a pessimistic result, there are more recent optimistic results [5] where feasibility of interference alignment is demonstrated to achieve more DoF even with no CSIT. The premise of interference alignment is the relativity of alignment, i.e., receivers should be statistically distinguishable at the transmitters. If all channels are statistically equivalent, interference alignment cannot be exploited. In this paper, we show that even in this case, relays can be exploited to achieve more DoF even with no CSIT or limited CSIT.

We consider a two-tier cellular system where a macrocell base station has multiple transmit antennas. It transmits to multiple microcells, each equipped with a microcell base station transmitter (relay) equipped with one antenna. Each microcell has one user. Two scenarios are considered. The first one is the case when the microcells are separated far enough so that the transmissions from microcell base stations do not interfere with other microcells. In this case we show that if the macrocell base station is equipped with 2 antennas, then with no CSIT each user can achieve $\frac{2}{K+1}$ DoF where K is the number of microcells. Note that in [6], [7], an outer bound for the K user MISO broadcast channel with 2 antennas at the transmitter and with limited CSIT is shown to be $\frac{2K}{K+1}$, which is also applicable to no CSIT case. Therefore, with relays and *no CSIT*, we can achieve the maximum DoF that can be achieved for the case without relays. In addition, we consider the case where the macrocell base station is equipped with 3 antennas. With no CSIT, it is difficult to achieve more DoF than the 2 antennas case. However, we show that with limited CSI, $\frac{3}{2}$ DoF can be achieved. Note that this is the outer bound for the case when there is no relays and CSIT. In particular, the instantaneous CSIT and even *delayed* CSIT from the macrocell to microcell base stations as well as partial CSI at microcell base stations are exploited. Delayed CSIT refers to the case when the current channel state is not available to the transmitter but is available with some delay. Recent works [8], [9] have shown that delayed CSIT can be exploited to achieve interference alignment in wireless networks. The key to our result is an interference management technique called interference neutralization [10]. The second scenario we consider is the case when microcells are interfering with each other. Similar to previous case, with delayed CSIT at the macrocell station and partial CSI at the microcell base stations, $\frac{3}{2}$ DoF can be still achieved with the 2 microcells and 3 antennas at the macrocell base station. We also show that with less partial CSI, $\frac{6}{5}$ DoF can be achieved.

II. NO CSIT

A macrocell base station has 2 transmit antennas. It transmits to K microcells, each equipped with a microcell base station transmitter (relay) equipped with one antenna. Each microcell has one user. The microcells are separated far enough so the transmissions from microcell base stations do not interfere with other microcells. Fig. 1 illustrates the two-microcell case. We assume *no channel state information at the transmitters* (CSIT), i.e., neither the macrocell nor microcell base stations are aware of the channels of the users and the macrocell base station does not know the channel to macrocell base stations. The receivers are assumed to have perfect channel knowledge. In addition, each user knows not only the channel associated with itself but also the channel from the macrocell base station to its microcell base station.

Our goal is to achieve $\frac{2}{K+1}$ DoF per user for a total of $\frac{2K}{K+1}$ DoF. More importantly, this is done with no CSIT. The achievable scheme is described as follows:

- Use the channel $K+1$ times to deliver two symbols to each user.
- The ith, $i \in \{1, \cdots, K\}$, channel use is dedicated to user i only. Specifically, in the ith time slot, two symbols for user i are sent simultaneously from two transmit antennas of the macrocell base station, providing user i one linear combination

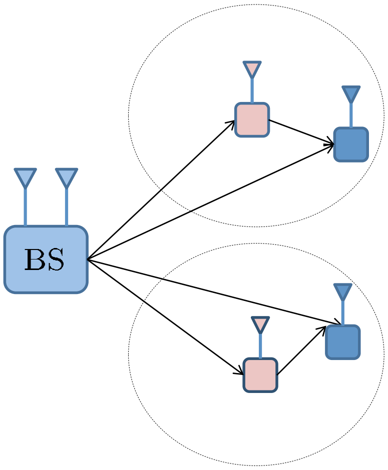

Fig. 1. A macrocell base station with 2 antennas transmitting to 2 microcells

of two desired symbols. In the same time, it also provides one linear combination of the two desired symbols to the ith microcell base station. Due to generic channel coefficients, these two linear combinations are linearly independent almost surely.

- In the final time slot, the ith microcell base station sends the received signal in the ith time slot to its respective user simultaneously.

At the end of the $(K+1)$th time slot, each user receives two linearly independent combinations of two desired symbols, one from the macocell base station and another from the microcell base station and thus he can resolve them to achieve 2 DoF.

Remark: The achievable scheme can be also applied to the multicast setting where there are M users in each microcell demanding the same message from the macrocell base station. In this case, each user can still get $\frac{2}{K+1}$ DoF even if there is no CSIT.

III. Partial CSIT

In previous section, we explore some examples where even with no CSIT, the achievable DoF can be greatly increased if we use relays. However, the same scheme cannot be generalized if we increase the number of antennas at the macrocell base station. For example, consider the case where the macrocell base station is equipped with 3 antennas and it transmits to two microcells, which is shown in Fig. 2. With no CSIT, it is very difficult to achieve more than $\frac{4}{3}$ DoF. Next, we will show that under the assumption of partial CSIT, it is able to achieve more DoF.

A. *Perfect CSIT from macrocell base station to microcell base station*

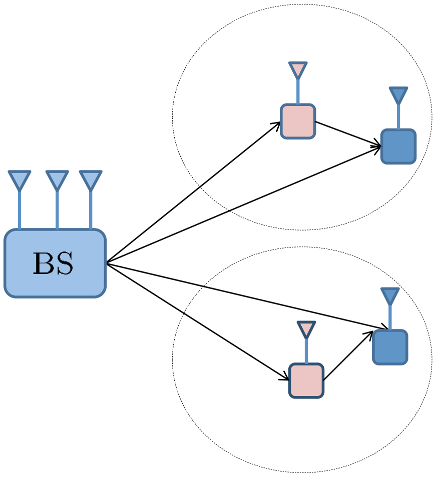

Fig. 2. A macrocell base station with 3 antennas transmitting to 2 microcells

Let us consider the case where the macrocell base station is equipped with 3 antennas and it transmits to two microcells as shown in Fig. 2. Similar to the previous case, we assume the macrocell base station is unaware of the channels of the users. However, we assume it has instantaneous channel state information to the microcell base stations. The validity of this assumption lies in the fact that most microcell base stations are stationary and thus the CSI is easier to obtain. In addition, each microcell base station is assumed to know the ratio of the channel of its respective receiver to the channel coefficient from the first antenna of the macrocell base station to that user.

The goal is to achieve $\frac{3}{2}$ DoF. This is done by sending 3 symbols to each user in 4 time slots, and the achievable scheme is described as follows:

- In the first time slot, the macrocell base station sends three symbols to user 1, i.e., $x_1^{[1]}, x_2^{[1]}, x_3^{[1]}$, along beamforming vectors $\mathbf{v}_1^{[1]}, \mathbf{v}_2^{[1]}, \mathbf{v}_3^{[1]}$, respectively. The vector $\mathbf{v}_1^{[1]}$ can be chosen randomly. However, $\mathbf{v}_2^{[1]}, \mathbf{v}_3^{[1]}$ are chosen as two linearly independent vectors in the null space of the channel from the macrocell base station to the second microcell base station.
- In the second time slot, the macrocell base station sends three symbols to user 2, i.e., $x_1^{[2]}, x_2^{[2]}, x_3^{[2]}$, along beamforming vectors $\mathbf{v}_1^{[2]}, \mathbf{v}_2^{[2]}, \mathbf{v}_3^{[2]}$, respectively. The vector $\mathbf{v}_1^{[2]}$ can be chosen randomly. However, $\mathbf{v}_2^{[2]}, \mathbf{v}_3^{[2]}$ are chosen as two linearly independent vectors in the null space of the channel from the macrocell base station to the first microcell base station.
- In the third time slot, each microcell base station amplifies and forwards its received desired signal to its respective user.
- In the final time slot, the macrocell base station sends $x_1^{[1]} + x_1^{[2]}$ from its first antenna. Simultaneously, the first microcell base station sends $-\frac{h_1^{[1]}}{h^{[1r_1]}} x_1^{[2]}$ and the second microcell base station sends $-\frac{h_1^{[2]}}{h^{[2r_2]}} x_1^{[1]}$ where $h_1^{[i]}$ and $h^{[ir_i]}$, $i \in \{1, 2\}$, are the channel coefficients from the first antenna of the macrocell base station and the ith microcell base station to user i, respectively. As a result, the received signal ignoring the noise at user 1 is $h_1^{[1]}(x_1^{[1]} + x_1^{[2]}) - h^{[1r_1]} \frac{h_1^{[1]}}{h^{[1r_1]}} x_1^{[2]} = h_1^{[1]} x_1^{[1]}$. Similarly, the received signal at user 2 is $h_1^{[2]} x_1^{[2]}$. Note that the ith microcell base station only needs to know the ratio of $h_1^{[i]}$ to $h^{[ir_i]}$, not the individual one.

Therefore, after 4 time slots, each user gets 3 linearly independent combinations of three desired symbols and thus can resolve them. The key to this scheme is the final time slot. In this time slot, the same interference symbol is received at the undesired receiver through two paths and more importantly with complementary scaling factors. Thus, they can be cancelled over the air. This interference management scheme is called interference neutralization [10].

B. Delayed CSIT from macrocell base station to microcell base station

In previous section, we assume instantaneous CSIT from the macrocell base station to the microcell base station (relay). However, if the relays are also mobile, instantaneous channel information to the relays is difficult to obtain and CSIT may only be available with a delay. We will show even with delayed CSIT, it is able to achieve the same DoF as the instantaneous CSIT case in the setting discussed in previous section. Except for the assumption of delayed CSIT, other assumptions on the CSI are the same as the previous section. The achievable scheme is as follows:

- Use the channel 4 times to deliver three symbols to each user.
- In the first time slot, the macrocell base station sends three symbols, $x_1^{[1]}, x_2^{[1]}, x_3^{[1]}$, for user 1, one from each transmit antenna. Then the received signal ignoring the noise at the second microcell base station is $L^{[r_2]}(1) = h_1^{[r_2]}(1) x_1^{[1]} + h_2^{[r_2]}(1) x_2^{[1]} + h_3^{[r_2]}(1) x_3^{[1]}$ where $h_i^{[r_2]}(1)$, $i \in \{1, 2, 3\}$, is the channel coefficient from the ith antenna of the macrocell base station to the second microcell base station in time slot 1.
- In the second time slot, the macrocell base station sends three symbols, $x_1^{[2]}, x_2^{[2]}, x_3^{[2]}$, for user 2, one from each transmit antenna. Then the received signal ignoring the noise at the first microcell base station is $L^{[r_1]}(2) = h_1^{[r_1]}(2) x_1^{[2]} + h_2^{[r_1]}(2) x_2^{[2]} + h_3^{[r_1]}(2) x_3^{[2]}$ where $h_i^{[r_1]}(2)$, $i \in \{1, 2, 3\}$, is the channel coefficient from the ith antenna of the macrocell base station to the first microcell base station in time slot 2.
- In the third time slot, each microcell base station amplifies and forwards its received desired signal to its respective user.
- In the final time slot, the macrocell base station sends $L^{[r_2]}(1) + L^{[r_1]}(2)$ from its first antenna. Simultaneously, the first microcell base station sends $-\frac{h_1^{[1]}}{h^{[1r_1]}} L^{[r_1]}(2)$ and the second microcell base station sends $-\frac{h_1^{[2]}}{h^{[2r_2]}} L^{[r_2]}(1)$ where $h_1^{[i]}$ and $h^{[ir_i]}$, $i \in \{1, 2\}$, are the channel coefficients from the first antenna of the macrocell base station and the ith microcell base station to user i, respectively. It can be easily seen that the received signals ignoring the noise at user 1 and 2 are $h_1^{[1]} L^{[r_2]}(1)$ and $h_1^{[2]} L^{[r_1]}(2)$, respectively. Note that in order for the macrocell base station sends $L^{[r_2]}(1) + L^{[r_1]}(2)$, it needs to know the channel $h_i^{[r_2]}(1), h_i^{[r_1]}(2)$, i.e., the channel in previous time slots.

Therefore, after 4 time slots, each user receives three linearly independent combinations of three desired symbols and thus is able to resolve them.

C. Interfering Relays

In previous sections, we assume microcells are located far enough and do not interfere with each other. In this section, we consider the case when they interfere with each other. For example, as shown in Fig. 3, two microcells have overlap and one

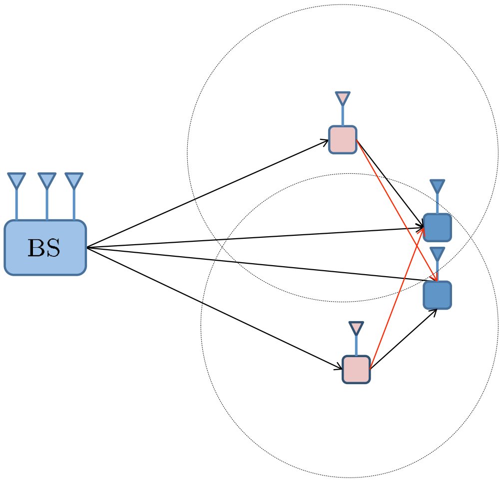

Fig. 3. A macrocell base station with 3 antennas transmitting to 2 interfering microcells

will interfere with the users in the other microcell. In this case, it is not difficult to see with delayed CSIT and partial CSI at the relays, using similar scheme described in previous section, each user is still able to achieve $\frac{3}{4}$ DoF for a total of $\frac{3}{2}$ DoF. Specifically, the achievable scheme is the same as the one described in Section III-B except for the third time slot. In this case, the macrocell base station sends $L^{[r_1]}(1)+L^{[r_2]}(2)$ from its first antenna. Note that $L^{[r_1]}(1)$ and $L^{[r_2]}(2)$ are linear combinations of desired symbols for user 1 and 2, respectively. To cancel interference of $L^{[r_1]}(1)$ at user 2, the first relay simultaneously sends $-\frac{h_1^{[2]}}{h^{[2r_1]}}L^{[r_1]}(1)$ where $h^{[2r_1]}$ is the channel from the first relay to user 2. Similarly, the second relay sends $-\frac{h_1^{[1]}}{h^{[1r_2]}}L^{[r_2]}(2)$ where $h^{[1r_2]}$ is the channel coefficient from the second relay to user 1. Note that in this scheme four ratios of channels are needed at two relays to cancel interference and these ratios contain the channel from the macrocell base station to users. Next, we will show that with two ratios of channels available to relays which do not involve the channels from the macrocell base station to users, $\frac{6}{5}$ DoF can be achieved.

Again, we assume the macrocell base station only has delayed CSIT to the microcell base stations and no CSI of users. The achievable scheme is as follows:

- Use 5 time slots to deliver 3 symbols to each user.
- In the first time slot, the macrocell base station sends three desired symbols , $x_1^{[1]}, x_2^{[1]}, x_3^{[1]}$ for user 1, one from each transmit antenna, providing one linear combination of desired symbols at user 1. Then the received signal ignoring noise at the ith microcell base statoins is $L^{[r_i]}(1) = h_1^{[r_i]}(1)x_1^{[1]} + h_2^{[r_i]}(1)x_2^{[1]} + h_3^{[r_i]}(1)x_3^{[1]}$ where $h_k^{[r_i]}$, $\forall i \in \{1,2\}, k \in \{1,2,3\}$, is the channel coefficient from kth antenna of the macrocell base station to the ith microcell base station.
- In the second time slot, the macrocell base station sends three desired symbols $x_1^{[2]}, x_2^{[2]}, x_3^{[2]}$, for user 2, one from each transmit antenna, providing one linear combination of desired symbols at user 2. Then the received signal ignoring noise at the ith microcell base statoins is $L^{[r_i]}(2) = h_1^{[r_i]}(2)x_1^{[2]} + h_2^{[r_i]}(2)x_2^{[2]} + h_3^{[r_i]}(2)x_3^{[2]}$.
- In the third time slot, the macrocell base station sends $L^{[r_1]}(1) - L^{[r_2]}(1)$ from its first transmit antenna, providing another linear combination of desired symbols at user 1. Note that to achieve this, it needs to know the channel coefficients in the first time slot. Once the second microcell base station receives this signal, it can normalize the signal and then add it to $L^{[r_2]}(1)$ which was received in the first time slot, producing $L^{[r_1]}(1)$.
- In the fourth time slot, the macrocell base station sends $L^{[r_1]}(2) - L^{[r_2]}(2)$ from its first transmit antenna, providing another linear combination of desired symbols at user 2. Note that to achieve this, it needs to know the channel coefficients in the second time slot. Once the second microcell base station receives this signal, it can normalize the signal and then add it to $L^{[r_2]}(2)$ which was received in the second time slot, producing $L^{[r_1]}(2)$.
- In the final time slot, the first microcell base station sends $-\frac{h^{[2r_2]}}{h^{[2r_1]}}L^{[r_1]}(1) - \frac{h^{[1r_2]}}{h^{[1r_1]}}L^{[r_1]}(2)$. The second microcell base station sends $L^{[r_1]}(1) + L^{[r_1]}(2)$. It can be easily seen that the received signals ignoring noise at user 1 and 2 are $(h^{[1r_2]} - h^{[1r_1]}\frac{h^{[2r_2]}}{h^{[2r_1]}})L^{[r_1]}(1)$ and $(h^{[2r_2]} - h^{[2r_1]}\frac{h^{[1r_2]}}{h^{[1r_1]}})L^{[r_1]}(2)$, respectively. In this time slot, the third linear combination is provided to each user simultaneously.

After 5 time slots, it can be easily seen that each user receives 3 linearly independent combinations of its desired symbols and thus is able to resolve them.

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