Title
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Abstract

Tables of Quarter-Squares, as discussed recently in this *Journal*, were once an alternative to logarithms for multiplication, but their use declined about the time that reasonably priced four-function calculators came into the market after 1900. Here I offer additional references, including both earlier and more recent examples of those tables, and suggest some additional sources of their decline, notably their limitations on certain kinds of computations such as the Galton-Watson and other sociological calculations of the sort that George W. Jones, compiler of one key set of tables, taught at Cornell.

Introduction

More than a century ago George William Jones (born 1837, died 1911), a Cornell University professor, offered a course on "Probabilities and Least Squares with Sociologic Applications, Including Some Recent Work of Galton" [44, page 301]. Jones was also a purveyor of mathematics books, among them a book of mathematical tables. The latter included a table of Quarter-Squares [23].

Printed mathematical tables played an important role as computational aids at least from the time of Napier and Briggs, and their earlier relatives dated back at least to about 2600 BC and a Sumerian multiplication table for finding areas from lengths of sides [41, page 27]. By 1960, however, there were hints that their importance was waning. That was the year the name of the periodical *Mathematical Tables and Other Aids to Computation* was changed to *Mathematics of Computation* [29], [30]. Subsequently, they fell into disuse about the same time slide rules did, with the advent of small electronic calculators.

Lately, however, printed mathematical tables have been receiving some overdue attention, including a book by Martin Campbell-Kelly and others, as well as shorter items in this *Journal* [1], [9], [7], [25], [36]. In one of those that focused specifically on tables of Quarter-Squares [1], it was noted that they had been an alternative to logarithms for multiplication, but fell into disuse about the time affordable four-function mechanical calculators became available after 1900, possibly suggesting that those calculators may have been major contributors to the demise of Quarter-Squares.

Here I offer more examples of those tables, both earlier and more recent, and suggest some additional sources of their decline. In particular, they were less well suited than logarithms for computations such as those for the Galton-Watson process, as Jones would have learned in teaching sociological applications in the aforementioned course. Furthermore, the death of Jones, who had been a main purveyor of Quarter-Squares tables, contributed to their demise, as his book was put out of print.

Theory of Quarter-Squares

Like logarithms, Quarter-Squares permit substitution of one set of operations for another set of operations. Such substitution has the potential of being preferable in some sense, such as obtaining the result more readily, or more precisely.

Logarithms are commonly introduced with the assertion that addition of logarithms is easier than multiplication of the original numbers. However a valid comparison is actually more complicated than that, and ease of computation is not the only consideration.

Instead of one multiplication, logarithms permit substitution of one addition, along with two lookups and one reverse lookup in a table. Similarly, instead of one multiplication, Quarter-Squares permit substitution of one addition, two subtractions, and two lookups in a different table. (Other instances of such substitute operations arise in Laplace transforms and in the method of prosthaphaeresis. For the latter see [8] or [48, page 23].)

Whether any particular computation will actually be made easier depends on several matters, including how many operations of each type are required, the level of accuracy required in the result, and what particular numbers are to be used in the computation, as well as any inherently different levels of difficulty of different operations. It also depends on what other computational aids are available; for example, someone using only pen and paper may have found addition easier than multiplication, while someone also using a slide rule or Crelle's Tables may have found the opposite.

Let q denote the Quarter-Square function, with $q(w) = w^2/4$. Then it is readily shown that

$$xy = q(x + y) - q(x - y)$$

This identity replaces multiplication, on the left side, with addition and subtraction, on the right side. The right side also requires two separate instances of evaluation of the Quarter-Square function, ordinarily by table lookup.

A noteworthy special case of the product of two numbers arises when they are equal. In this case the identity

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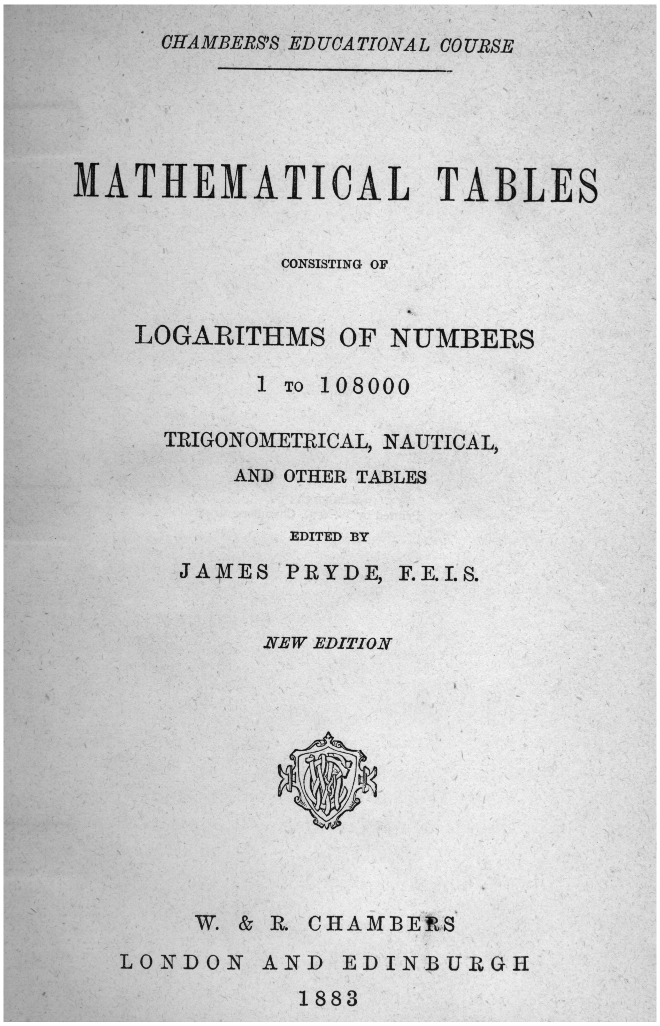
becomes $x^2 = q(2x)$, which replaces squaring with doubling. Also, in reverse lookup it replaces extraction of a square root with halving. However, squares and square roots were called for sufficiently often that their own specialized tables were published; for example [23, Tables X and XII].



CHAMBERS'S EDUCATIONAL COURSE

MATHEMATICAL TABLES

CONSISTING OF

LOGARITHMS OF NUMBERS

1 TO 108000

TRIGONOMETRICAL, NAUTICAL,

AND OTHER TABLES

EDITED BY

JAMES PRYDE, F.E.I.S.

NEW EDITION

W. & R. CHAMBERS

LONDON AND EDINBURGH

1883

Figure 1. Title page from 1883 Chambers book which includes Quarter-Squares.

In his *History of Mathematics* [43, vol I, page 283; vol II, page 123] David E. Smith traced the Quarter-Square identity back to Al-Karkhi, who had written in Baghdad around the year 1010 AD. Smith also speculated that it may have come from earlier Hindu sources. The Quarter-Square was described not as a numerical 'table', however, but as a 'rule' (i.e., mathematical formula).

Howard Eves [13, page 59] traced it back even further. He stated the identity in the form:

$$\left(\frac{x-y}{2}\right)^2 = \left(\frac{x+y}{2}\right)^2 - xy$$

and attributed it to Euclid, in *Elements*, Book II, Proposition 5.

This would indicate that the *principle* of multiplication by means of Quarter-Squares was known more than two millennia ago, and long before the principle of multiplication by means of logarithms was known.

Tables of Quarter-Squares would be another matter, however; Euclid was not to my knowledge in the business of publishing numerical tables.

Quarter-Square Tables

The earliest Quarter-Square table of which I am aware was published in 1876 by William St George Kent [26]. Other Quarter-Square tables include an 1888 work by Joseph Blater [5], a 1909 work by Joseph Bojko [6], and a 1933 work by Joseph Plassmann [37].

None of these is common, and I have not actually inspected them, only seen citations. Searches on WorldCat find Blater in only thirteen libraries worldwide, Bojko in six, Plassmann in four, and Kent in only two.

The Blater Quarter-Square table was cited in one of the Napier Tercentenary volumes, in the review of tables by Herbert Bell and J. R. Milne [3, page 48]. None of the others cited herein was listed for Quarter-Squares, although both Jones and Chambers books (see below) were cited under other headings, such as Gaussian logarithms in the Jones book.

Quarter-Square tables by Blater, by Bojko, and by Plassmann were cited (but not reprinted) in the volume by Jahnke and Emde [20, page 296].

The oldest table of Quarter-Squares in my possession is in an 1883 printing of *Chambers Mathematical Tables* [38]. See Figure 1. That volume is a stated New Edition, but itself bears no reference to dates of earlier editions. A more recent Chambers publication (dating from 1973 but without Quarter-Squares) traces the Chambers Tables back to 1844 and gives 1878 as publication date of the new edition [40, Preface].

The second-earliest Quarter-Squares table in my collection dates to 1894. This is in a book of tables by George W. Jones [23, pages 154-157]. Apparently Jones' book of tables was a success; it went through 14 editions between 1889 and (posthumously) 1918.

An even more recent Quarter-Squares table in my possession, which will be discussed below, is in a 1951 reprint of the 1930 edition of *Chambers Seven-Figure Mathematical Tables,* edited by James Pryde [39, pages lx-lxi, 439-452].

These tables give Quarter-Squares of integers beginning at 1. (Actually they omit the fractional part of the Quarter-Square of each odd number. This causes no difficulty, since (x-y) is odd whenever (x+y) is, so the two omitted fractions cancel; see [1] or [23, page 11].)

The size of a Quarter-Square table might best be characterized in terms of the largest number whose Quarter-Square is given. Note however that this is not the maximum value of x, nor the maximum value of y, nor the maximum value of their product xy, but the maximum value of the *sum* of the two. The computation of the product xy requires the lookup of the Quarter-Square of the argument (x+y).

In the Jones table, the values of the argument go up to 2,000, whose Quarter-Square is 1,000,000. In the Chambers table, the values of the argument go up to 5,100, whose Quarter-Square is 6,502,500. Based on its title, I understand the Blater table to have values of the argument up to 200,000, whose Quarter-Square is 10^{10}, and likewise for the Plassmann table. My information on the Kent table does not indicate how far it goes.

George W. Jones and his Tables

A professor at Cornell University, George W. Jones was among the early mathematicians to take note of sociological applications of mathematics. In 1887, his course which had previously borne the description “Probabilities and Insurance” was changed to “Probabilities and Least Squares with Sociologic Applications, Including Some Recent Work of Galton” [44, page 301].

My collection includes two editions of his book of tables and a booklet excerpted therefrom [21], [23], [24], as well as a copy of his Algebra book [22].

The 1889 (apparently first) edition of his *Tables* does *not* include the table of Quarter-Squares. Its content was as follows: logarithms of numbers; constants; sines and tangents of small angles; trigonometric functions; Naperian logarithms; and meridional parts for latitudes 0-75 degrees in nautical miles.

Somewhere between 1889 and 1894 (the fifth edition) the publisher was changed (from Dudley Finch to Macmillan), and the nautical table was dropped, but otherwise the collection was greatly expanded, and the additions included a table of Quarter-Squares. (See Figure 2.) These were, however, placed near the end of the book among others Jones classified as ‘Minor Tables’ [23, page 11]. The other ‘minor tables’ in the Jones collection included natural logarithms; prime and composite numbers; squares, cubes, square roots, cube roots, reciprocals; Bessel coefficients and binomial coefficients for interpolation; and the normal or Gaussian probability distribution then called ‘Errors of Observations’. (See Figure 3.)

The 1905 booklet [24] is a mixture of 4-place logarithms and advertisements for Jones’ books. Of note here is the price of his *Logarithmic Tables* book, then in its tenth edition, which could be had for the grand sum of $1.00 including postage, or $.75 cash with carriage at buyer’s expense. This price level is unchanged from 1894, which is shown in Figure 4. The price will be relevant in consideration below of why Quarter-Square tables fell into disuse.

In addition to these tables, Jones published a series of mathematics textbooks, including works on algebra, geometry, and trigonometry.

Jones’ algebra book in my possession, although by no means a mathematical sociology text, does contain exercises with a sociological flavor.

For example, the page that gives exercises about capacities of reservoirs and pipes also gives exercises about worker productivity [22, page 99]. Similarly, his exercises on combinatorics include some dealing with committee formation [22, page 257]. His exercises on mortality would seem familiar territory to any sociology student taking a demographic methods course today [22, pages 259, 263].

154 XV. QUARTER-SQUARES.

0	0	1	2	3	4	5	6	7	8	9
0	0	0	1	2	4	6	9	12	16	20
1	25	30	36	42	49	56	64	72	81	90
2	100	110	121	132	144	156	169	182	196	210
3	225	240	256	272	289	306	324	342	361	380
4	400	420	441	462	484	506	529	552	576	600
5	625	650	676	702	729	756	784	812	841	870
6	900	930	961	992	1024	1056	1089	1122	1156	1190
7	1225	1260	1296	1332	1369	1406	1444	1482	1521	1560
8	1600	1640	1681	1722	1764	1806	1849	1892	1936	1980
9	2025	2070	2116	2162	2209	2256	2304	2352	2401	2450

Figure 2. Excerpt from Jones 1894 table of Quarter-Squares

Attractions of Quarter-Square Tables

As discussed recently in *JOS*, the Quarter-Squares method has the advantage of giving *exact* results, something that would appeal to those who compute financial results to the last cent [1].

Exact results would presumably have appealed as well to a sociologist such as Franklin Henry Giddings (born 1855, died 1931) who would apparently rather deliver exhortations about ‘exact science’ than soil his hands with inexact results from a slide rule [34].

In a 1909 article on the then-current situation in sociology, compiled by L. L. Bernard, Giddings opined as follows: “The present tendency [in sociology] is to loaf and to generalize. I speak of the subject, not of any one institution. We need men not afraid to work; who will get busy with the adding machine and the logarithms, and give us *exact studies*, such as we get from the psychological laboratories, not to speak of the biological and physical laboratories. *Sociology can be made an exact, quantitative science,* if we can get *industrious* men interested in it” [16, emphasis in original].

Logarithms of choice would have been of the ten-place variety [46], [45], as used by astronomers and others in what Giddings called the physical laboratories; Jones’ logarithms were ‘merely’ six-place. (On the other computational aid cited by Giddings, the adding machine, see [32], [33], [34].)

But I have found no indication that Giddings used Quarter-Squares tables. He may have been unaware that Quarter-Squares tables produce exact products, or may even have been unaware of their existence, although he and Jones used the same publisher, Macmillan, for their books.

Another attraction of Quarter-Square tables, shared with logarithm tables, is that one can perform multiplication of factors outside the scope of the table by shifting the decimals appropriately. Note, however, that this will not always yield the aforementioned exact results.

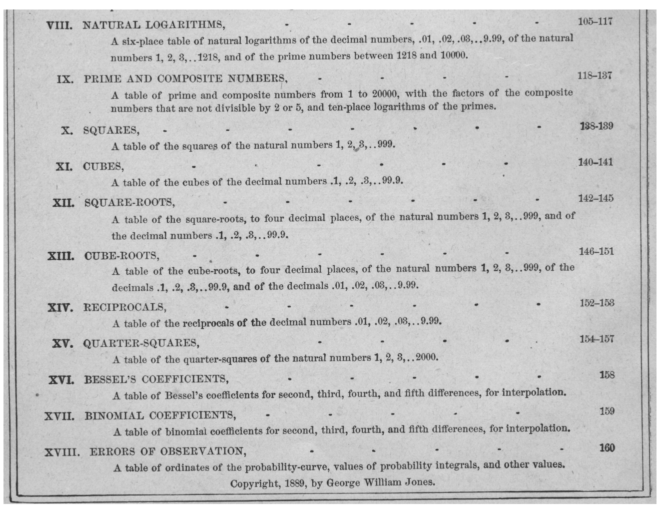
Limitations of Quarter-Square Tables

Most collections of mathematical tables did *not* include Quarter-Squares. Furthermore, the examples in my possession are included near the end of collections of assorted tables, almost as if an afterthought (cf. Figure 3). Thus I am not entirely sure that Quarter-Square tables ever were widely used as an alternative to logarithms.

Along with their attractions, Quarter-Square tables also have limitations. While approximate rather than exact, logarithms are applicable to problems that cannot be handled by Quarter-Squares.

Logarithms, whether in the form of numerical tables or in the form of scales as on a slide rule, work equally well for division as for multiplication. Not so with Quarter-Squares, whose straightforward means of multiplication has no counterpart for division.

Thus, as Jones would have learned very early in developing his course on sociological applications of probability, even one of the most basic probability computations, estimation of an unknown probability by calculation of an observed proportion, highlights limitations of Quarter-Squares. This may help explain why Jones relegated the Quarter-Square table to the back of his book among 'minor tables'.



VIII. NATURAL LOGARITHMS, - - - - - 105–117
A six-place table of natural logarithms of the decimal numbers, .01, .02, .03,..9.99, of the natural numbers 1, 2, 3,..1218, and of the prime numbers between 1218 and 10000.

IX. PRIME AND COMPOSITE NUMBERS, - - - - 118–137
A table of prime and composite numbers from 1 to 20000, with the factors of the composite numbers that are not divisible by 2 or 5, and ten-place logarithms of the primes.

X. SQUARES, - - - - - - - 138-139
A table of the squares of the natural numbers 1, 2, 3,..999.

XI. CUBES, - - - - - 140–141
A table of the cubes of the decimal numbers .1, .2, .3,..99.9.

XII. SQUARE-ROOTS, - - - - - - 142–145
A table of the square-roots, to four decimal places, of the natural numbers 1, 2, 3,..999, and of the decimal numbers .1, .2, .3,..99.9.

XIII. CUBE-ROOTS, - - - - - - 146–151
A table of the cube-roots, to four decimal places, of the natural numbers 1, 2, 3,..999, of the decimals .1, .2, .3,..99.9, and of the decimals .01, .02, .03,..9.99.

XIV. RECIPROCALS, - - - - - - 152–153
A table of the reciprocals of the decimal numbers .01, .02, .03,..9.99.

XV. QUARTER-SQUARES, - - - - 154–157
A table of the quarter-squares of the natural numbers 1, 2, 3,..2000.

XVI. BESSEL'S COEFFICIENTS, - - - - - 158
A table of Bessel's coefficients for second, third, fourth, and fifth differences, for interpolation.

XVII. BINOMIAL COEFFICIENTS, - - - - - 159
A table of binomial coefficients for second, third, fourth, and fifth differences, for interpolation.

XVIII. ERRORS OF OBSERVATION, - - - - - 160
A table of ordinates of the probability-curve, values of probability integrals, and other values.

Copyright, 1889, by George William Jones.

Figure 3. Excerpt from Contents of Jones 1894 showing Quarter-Squares and other 'minor tables'

Consider a simple division, x/z, as for example an event occurring in x cases out of z possible cases. True, the Quarter-Square identity can be rewritten in terms of division. Substituting $y = 1/z$ yields:

$$x/z = q(x + 1/z) - q(x - 1/z)$$

However, that does not fit into the Quarter-Square tables, which have integer arguments only. If x and z are integers, as presumed in writing x/z on the left, then neither $(x + 1/z)$ nor $(x - 1/z)$ will be an integer, so the table will be inapplicable.

Interpolation, as often used to evaluate other mathematical functions between tabulated values, requires some means of multiplication, the lack of which is the whole motivation for the Quarter Square table, and some means of division, the lack of which is a shortcoming of the Quarter-Square tables. (While some other types of tables provide 'proportional parts' supplements to facilitate interpolation to one extra place, none of the Quarter-Square tables I have inspected does so.)

Division could be performed in two stages, using two different tables, first looking up the reciprocal of the denominator in a table of reciprocals (e.g., [23, Table XIV]), then multiplying it by the numerator, using the Quarter-Squares table. Feasible, but more cumbersome than using logarithms.

Sociological Work of Galton

Jones' course on probability and least squares, according to its catalog description, provided sociological applications, including some then-recent work of Francis Galton.

Lacking a more detailed syllabus, one can speculate about precisely what that involved. The prime candidate in my assessment was from Galton's study of what he called Hereditary Genius. An offshoot of that study involved modeling the process of family lines growing or dying out [15], [47] to assess whether family lines of prominent people were more prone to die out than family lines of less noteworthy people. This contribution by Galton and Watson was published in 1874, just a few years before Jones offered the aforementioned course.

This model, known as the 'branching process' or the 'Galton-Watson process', was later applied more generally rather than limited to the sociological context in which it had been developed. These included studying whether some genetic variation would persist or die out, and studying whether a nuclear reaction would result in a fizzle or an explosion [17], [27], [28].

The 'generating function', a mathematical device created by Watson for use on this sociological problem, subsequently acquired an important place in probability theory [14, chapter XI].

Here let $p_0, p_1, p_2, p_3, ...$ denote the probabilities that a man will have 0, 1, 2, 3, ... sons who survive to maturity. Then the generating function for the number of sons is the polynomial

$$f(x) = p_0 + p_1x + p_2x^2 + p_3x^3 + ...$$

and the probability that the family line will eventually die out is a solution of the equation $f(x) = x$, namely the smallest solution in the [0, 1] interval.

Quarter-Squares would be of little assistance here: those tables can handle the multiplications, but the generating function also involves squares, cubes, etc., which Quarter-Squares can handle only by repeated multiplication. A slide rule, particularly one with log-log scales of Roget, would have been much more convenient [42].

Logarithm tables are more versatile than Quarter-Square tables. While multiplication can readily be done on either, logarithm tables can also be used in various other problems. For example, a sociologist studying population growth (or a financial person doing interest computations) has use for the formula for exponential growth, $P_t = P_0 e^{rt}$. To determine the growth rate, r, from known population sizes at two time points, one needs $r = ln(P_2/P_1)/(\Delta t)$. Either of these (depending on which quantities are known and which is to be computed) can be performed readily using a log-log slide rule, or less readily using a simpler slide rule or logarithm tables, but not using Quarter-Square tables.

Complementary and Competing Tools

A famous 1950s advertisement depicted a room full of engineers, each busily using his slide rule, with a caption stating that one of IBM's new computers was almost like 150 extra engineers. In the economically expansive 1950s it might indeed have been an *additional* 150 engineers rather than a replacement for 150 engineers, although in leaner times some administrators would surely have given the ad the latter interpretation.

The replacement of slide rules by a mainframe *did not happen* – neither then nor subsequently. The two coexisted, and the era of the slide rule as everyday tool was ended not by later mainframes that were even more powerful, but by small calculators. (See [35] for references.)

The reason is that the mainframe was not really in the same market as the slide rule. The computer was more powerful, to be sure, but it was so expensive that it would have been a shared resource unlike the slide rule, where every scientist or engineer had his or her own. The person who relied on the computer for calculations would spend a lot of time in a queue waiting for a turn to use it, while the person who relied on a slide rule would have the calculation completed and checked and be on to subsequent tasks. Someone who used both, carefully choosing which calculations would really benefit from the computer, was better off than either who used one tool exclusively.

Something similar may have been involved in the relationship between tables and four-function calculators in the early 20th century. As mentioned above, in both 1894 and 1905 Jones' book of tables was offered at $1.00 per copy, postage included [23], [24]. I do not have 1894 nor 1905 prices for four-function calculators, but they must have been at least as high as prices for such machines in the 1920s which were reported by McCarthy [31, page 50].

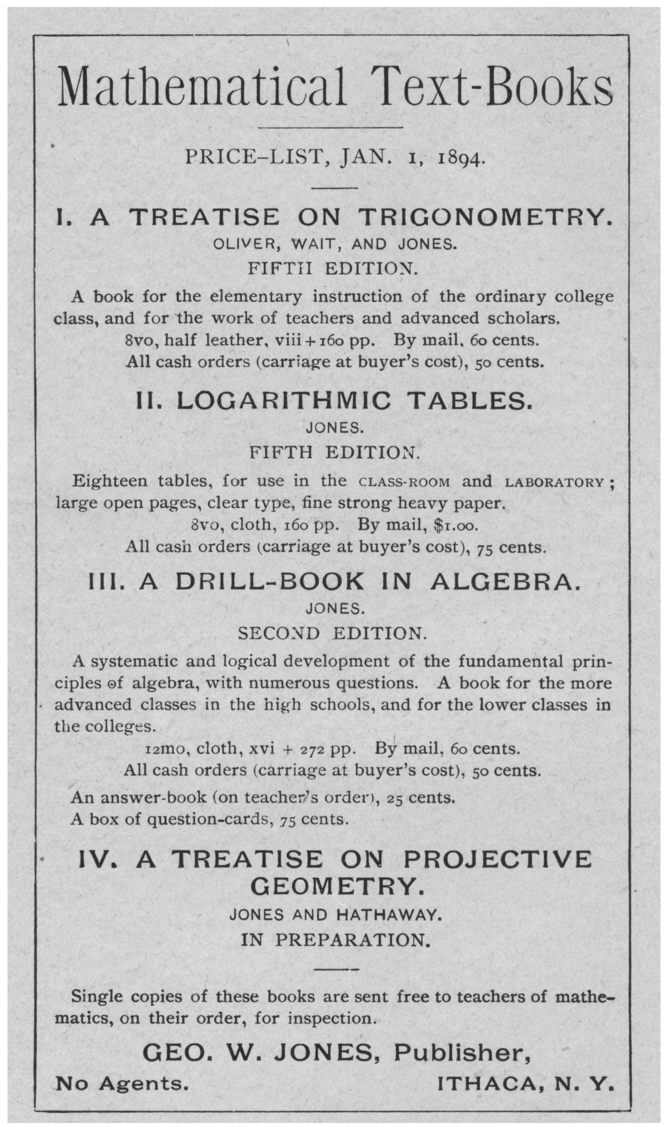
The $1.00 price for Jones' book of tables may be compared with 1924 prices as follows: Brunsviga four-function calculators starting at $275; and Monroe four-function calculators starting at $200 for crank-operated and $500 for electrically-operated models [31, pages 35, 72, 70, 82].

Machines costing hundreds of dollars may have had difficulty displacing the $1.00 book of tables for the same reason the computer was unable to displace the slide rule.

The increasingly widespread use of slide rules doubtless contributed to the demise of Quarter-Square tables. But the slide rule would not have ended use of Quarter-Square tables any more than it ended use of logarithm tables.

Users of logarithms had their choice of table lookup and a mechanical counterpart in the form of a slide rule. In the era of Jones and Chambers tables there was not, to my knowledge, any mechanical counterpart of Quarter-Square tables.

The slide rule's three-place accuracy was sufficient for many problems. And for those where greater accuracy was required the real competition to Quarter-Square tables may have been the previously mentioned ten-place logarithms, or the multiplication table itself, particularly in its extended form by Crelle.



Mathematical Text-Books

PRICE–LIST, JAN. 1, 1894.

I. A TREATISE ON TRIGONOMETRY.
OLIVER, WAIT, AND JONES.
FIFTH EDITION.
A book for the elementary instruction of the ordinary college class, and for the work of teachers and advanced scholars.
8vo, half leather, viii + 160 pp. By mail, 60 cents.
All cash orders (carriage at buyer's cost), 50 cents.

II. LOGARITHMIC TABLES.
JONES.
FIFTH EDITION.
Eighteen tables, for use in the CLASS-ROOM and LABORATORY; large open pages, clear type, fine strong heavy paper.
8vo, cloth, 160 pp. By mail, $1.00.
All cash orders (carriage at buyer's cost), 75 cents.

III. A DRILL-BOOK IN ALGEBRA.
JONES.
SECOND EDITION.
A systematic and logical development of the fundamental principles of algebra, with numerous questions. A book for the more advanced classes in the high schools, and for the lower classes in the colleges.
12mo, cloth, xvi + 272 pp. By mail, 60 cents.
All cash orders (carriage at buyer's cost), 50 cents.
An answer-book (on teacher's order), 25 cents.
A box of question-cards, 75 cents.

IV. A TREATISE ON PROJECTIVE GEOMETRY.
JONES AND HATHAWAY.
IN PREPARATION.

Single copies of these books are sent free to teachers of mathematics, on their order, for inspection.

GEO. W. JONES, Publisher,
No Agents. ITHACA, N. Y.

Figure 4. 1894 Pricelist for Jones books

Crelle's Extended Multiplication Table

August Leopold Crelle (born 1780, died 1855) published a book that went through several editions, remained in print for well over a century, and was translated into several languages. The content of this successful tome was simply the multiplication table. Published originally in 1820, Crelle's *Rechentafeln* consisted of a book-length multiplication table, which listed the product of every two integers from 1 to 1,000 [12].

In 1857, after Crelle's death, his tables were republished, under the editorship of and with a foreword by Karl Bremiker.

In the 1857 edition there are two facing title pages, in French and German; the foreword and instructions are in parallel columns in the two languages. (I do not know whether the 1820 edition had front material in any language but German.)

English was added later. Still, the language of the front material may not be that crucial for use of the table itself. Someone who knows how to use a multiplication table could bypass the instructions, and likewise may not greatly care whether the title is 'Rechentafeln' or 'Tables de calcul' or 'Calculating Tables'. The 1907 edition, edited by Oskar Seeliger, remained in print until at least 1954 [12].

The 1960s, with their mainframe computers, brought not the end of Crelle-type tables, but publication of a computer-generated equivalent by Kirk Carlsten and Karsten Hellebust [10]. According to its preface, this volume was "computed and tabulated on IBM electronic computing equipment and then reproduced photographically, to eliminate the possible human error factor in typesetting" – Charles Babbage's dream finally achieved.

Carlsten and Hellebust, although making no explicit mention of Crelle or other predecessors, were clearly following the tradition of Crelle.

On the criterion of portability a Quarter-Squares table had a considerable advantage over the volume by Crelle (or the more recent one by Carlsten and Hellebust). But these multiplication tables gave a product directly, requiring no addition nor subtraction, nor more than a single table lookup.

QUARTER SQUARES.

No.	Quar. Sq.	No.	Quar. Sq.	No.	Quar. Sq.	No.	Quar. Sq.	No.	Quar. Sq.	No.	Quar. Sq.	No.	Quar. Sq.
1	0	61	930	121	3660	181	8190	241	1 4520	301	2 2650	361	3 2580
2	1	62	961	122	3721	182	8281	242	1 4641	302	2 2801	362	3 2761
3	2	63	992	123	3782	183	8372	243	1 4762	303	2 2952	363	3 2942
4	4	64	1024	124	3844	184	8464	244	1 4884	304	2 3104	364	3 3124
5	6	65	1056	125	3906	185	8556	245	1 5006	305	2 3256	365	3 3306
6	9	66	1089	126	3969	186	8649	246	1 5129	306	2 3409	366	3 3489
7	12	67	1122	127	4032	187	8742	247	1 5252	307	2 3562	367	3 3672
8	16	68	1156	128	4096	188	8836	248	1 5376	308	2 3716	368	3 3856
9	20	69	1190	129	4160	189	8930	249	1 5500	309	2 3870	369	3 4040
10	25	70	1225	130	4225	190	9025	250	1 5625	310	2 4025	370	3 4225
11	30	71	1260	131	4290	191	9120	251	1 5750	311	2 4180	371	3 4410
12	36	72	1296	132	4356	192	9216	252	1 5876	312	2 4336	372	3 4596
13	42	73	1332	133	4422	193	9312	253	1 6002	313	2 4492	373	3 4782
14	49	74	1369	134	4489	194	9409	254	1 6129	314	2 4649	374	3 4969
15	56	75	1406	135	4556	195	9506	255	1 6256	315	2 4806	375	3 5156

Figure 5. Excerpt from Chambers 1951 Quarter-Squares table.

The Demise of Jones and his Tables

George W. Jones died in 1911. This may have had more impact than most authors' deaths have on a particular type of computation, for two reasons.

First, judging from how many copies are still around today, Jones' may have been one of only two widely distributed Quarter-Squares tables, the other being Chambers'. The Jones book of tables was apparently put out of print after a posthumous edition published in 1918.

By that time Macmillan had developed a series of mathematics textbooks under the editorship of Earle Raymond Hedrick. The latter had his own book of tables, first published in 1913 and revised in 1920 [18]. The Hedrick book, which might reasonably be considered a replacement for Macmillan's earlier book of tables by Jones, did *not* include Quarter-Squares. (The Hedrick volume had an interesting additional feature: the very last page was a do-it-yourself slide rule, intended to be cut out and pasted onto cardboard backing [18].)

Second, Jones himself had during his lifetime failed to promote the features of his book that set it apart from dozens of other books of tables. He called it simply *Logarithmic Tables* even though it included quite a few other tables, Quarter-Squares among them. Furthermore, he placed them in the back of the book and labeled them 'minor tables' rather than emphasizing that they helped distinguish his book from others; see Figure 3.

Chambers, in contrast, was positioned as *Mathematical Tables*, with two features that distinguished it from most: (a) the variety of different tables included (recognized in its title, as well as in its actual content), and (b) its accuracy (seven figures, distinguishing it from many others with five figure accuracy, recognized in the title of later editions). Jones had six figure logarithms, but they were buried as Table 3, not highlighted as Table 1 and promoted in the book title.

Still, the Chambers book, like the Jones book, gave little prominence to its Quarter-Squares table, placing it as the next-to-last table in a book of more than 500 pages.

The Chambers book, including its Quarter-Squares table, did continue publication for several more decades. The most recent Quarter-Squares table in my possession is in a 1951 reprinting of the 1930 edition of *Chambers Seven-Figure Mathematical Tables,* edited by James Pryde [39, pages lx-lxi, 439-452]. An excerpt is shown in Figure 5.

Conclusion

In 1954 Chambers published a College Edition that included only the logarithmic and trigonometrical tables. However, they received "very numerous requests" and in response restored some of the omitted tables in what they then called the 'Full Edition' [40, Preface].

Not restored, however, was the table of Quarter-Squares.

It is hazardous to make end-of-era pronouncements (cf. [35]), but it is safe to say that I am not presently aware of Quarter-Squares tables published since 1954.

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