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Kinship networks and discrete structure theory: applications and implications

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Abstract

Confusions between substantive and relational concepts of kinship as a social network have led to a number of problems that are clarified by a temporally ordered relational theory of network structure. The ordered-network approach gives rise to a novel means of graphing the social field of kinship relations, while allowing kinship to be locally defined in culturally relative terms. Its utility is exemplified in applications to kinships among US Presidents, Old Testament Canaanites, and native Australians of Groote Eylandt. The formal concepts treated in the mapping of kinship networks are: kinship axioms, parental graph structure, core, circuits of consanguineally and affinally linked kin, sides and divides, homeomorphic mappings, homomorphisms as potentially simplifying mappings of kinship, elementary structure, and order-structure. Representational theorems are proven about homeomorphisms, cores and circuits, and the ambiguity of elementary structures. The last set of theorems leads to clarifying and redefining some of the basic concepts of elementary, semi-complex and complex structures of kinship in terms of properties of generationally ordered networks. The conclusions of the formal argument are 'post-structural' in the narrow sense of demonstrating the need for specifying contingent historical processes in the structural analysis of kinship as a social field. The open-ended approach to change, one that is implied by the study of ordered structures that unfold in a temporal succession, connects to issues of population variability, selection, and evolutionary processes. The kinship structures that are mapped in this approach are not intended as any sort of complete representations of kinship 'systems', but merely as scaffoldings that help to bring into view kinship as a social field, providing a baseline for other mappings (which may be superimposed) of social processes such as communicative fields, exchange processes, transmission of learned behaviors, social rights and inheritance, political and religious succession, and the like.

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1. Introduction

Cultural constructs mediate the biological grid of kinship. Kinship is a communicative, group-reinforced set of concepts, implying some form of ramifying network, building off culturally recognized forms of parentage. In spite of the centrality of social networks to sociocultural studies, theoretical interest in kinship is, with certain exceptions (e.g. Kelly, 1977, 1985), virtually moribund, except in French anthropology, today. Following a debate initiated by Leach (1961) over the supposed 'substance' of kinship, Schneider (1984) defined kinship as a non-comparable subject by emphasizing the fact that its cultural constituents are only locally valid. For him, the very language of kinship comparisons was suspect, containing implicit Western assumptions, e.g. that 'individuals', 'blood' relations, or 'nuclear' families are invariably the basic kinship constituents. Lévi-Strauss (1969) had attempted to remove these conceptual barriers by defining an elementary unit of kinship that included at least minimal components of a larger 'system of exchange'. However, Schneider (1965) saw a structural approach as overreliant on statistical and mechanical models themselves open to critique. He did not see beyond the problems he had raised to any creative solution. He did not consider network concepts at all.

Network ideas of kinship might have contributed more foundationally to re-thinking the study of kinship, but have remained underdeveloped. British social anthropological fieldwork, in areas where kinship was often a salient principle in social organization, did make major contributions to the concepts of roles and networks of relations, but network concepts tended to be used only as metaphors for the concrete basis of social structure (Radcliffe-Brown, 1940, 1952). In the 1950s the British developed methodologies to analyze social networks (Barnes, 1954, 1971, 1972; Nadel, 1957; Mitchell 1969), but they rarely used them to explore networks much beyond the small group.

In the face of the issues that the Leach and Schneider critiques raised in anthropological consciousness, what is required is a foundational reconstitution of the field of study of kinship. The present paper is more modestly concerned with the representation of genealogical graphs, but we argue that many of the insights and strengths of the French and British approaches, and also the American concepts of kinship as a cultural construct, [1] can be combined in a formal

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[1] It is obvious, for example, that kinship phenomena can be investigated at the level of biology, of the sociology of culturally constituted kin relations, the linguistics of kinship terms and usages, of behavioral observations, of individual cognition and subjectivity, and so forth. Much of Schneider's critique boils down to the problems that occur when these levels are conflated. Clearly, one may also attempt to keep them separate. The formal approach developed here is simply a skeleton for recording parental relations (which may be defined according to any of these criteria) which give shape to a kinship network: other data may be analyzed separately or viewed in terms of a mapping onto this skeleton if this framework proves to have some theoretical utility. Like the genealogical diagram, this framework is primarily a device for recording (and ordering) data, not a 'theory' of kinship. 'Theory' in this study concerns the formal statements about kinship structures and the logico-deductive relations among them: formal concepts which can nonetheless help to elucidate substantive problems of kinship theory.

approach that is responsive to Schneider's (1965, 1984) critiques. We provide a canonical representation of kinship networks that allows formulation of more rigorous concepts and methods for comparative analysis, and can serve as a basis for comprehensive general theories to be tested by patterns of observed behavior, as well as language use and cognition.

2. Issues of representation: the map is not the territory

It is of fundamental importance to a field of study constituted by the study of social relations to have a canonical representation: a proper series of visualizations for the graphs of kinship. There are at present no canonical graphs for kinship networks. [2] The anthropological standard is the genealogical diagram in which males and females are given separate identifiers (e.g. triangles and circles), and are linked by double lines to show marriages or unions. Gendered offspring of unions are identified by a second type of line. One can try to express the genealogical diagram as a graph where the vertices are individuals labeled by gender, and there are two fundamental relations, one for couples and one for their offspring. This is not a true graph, since the edges for offspring are not between vertices but between vertices (children) and pairs of vertices (couples). To draw offspring lines between vertices, we would have to decide which is more primary, the mother or the father, or whether both are necessary (the genetic graph). Whether the father is a primary or derivative relation (a child's mother's husband) takes us back to Schneider's conundrum of 'substance' vs. 'cultural construction' of kinship.

To find a canonical representation of kinship, we must concern ourselves with kinship not as 'substance' (blood, affinity, or other qualities) but as order relations, with networks of relations ramifying out of those between parents and children. (These networks, of course, allow myriad substantive interpretations, but to keep a primary focus on the idea of kinship, we can exclude metaphorical extensions such as 'religious brotherhood' which lack a defining element in parental relations.) However culturally defined and variable, some concept of parent in relation to

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[2] Social network researchers may be surprised to learn that their canonical graph, with individuals as vertices and kin-ties as edges, is inadequate. Imagine even the simplest nuclear family: do we draw a complete graph and label each of the edges with a particular type of kinship relation? Or do we draw and label only certain relations as elementary (e.g. parent/child, husband/wife) and omit the others as derivative or secondary (e.g. brother/sister)? If the latter, which are the elementary relations? Expanding this dilemma to larger networks, concatenation of kinship links leads to either the complete graph with multiple kin-type labels, or to the choice of elementary relations as generators. In either case we are caught in cultural definitions of kinship which turn out not to be of universal applicability. The answer to the question 'What is the simplest graph to represent kinship networks?' is neither trivial nor obvious, and a simple graphic representation of kinship has eluded social anthropologists for a century.

child is found in all societies. Every human child learns the possibility of having one or more parents, and the concept of adoptive parents or 'parental' caretakers is easily understood. Parentage, [3] however variable culturally, is a sine-qua-non of kinship. That the 'biological' mating resulting in offspring may be unknown, or that parentage only implies a (not always certain) mating or marriage, is an example of the cultural or situational variability in parental relations. The universal axioms of kinship, then, may be as weak as the relational statement that parents precede their children in time, but need not include any statements about the content of kinship, such as marriage as a human universal. Axioms of the latter kind are no longer relational: they enter the domain of cultural definitions, and as such, lack universality as they encounter cross-cultural variety.

In representing kinship relationally, as Weil noted (Lévi-Strauss and Enbon, 1988, p. 52), there is no need from a graphical or mathematical standpoint to characterize the substance or cultural definition of marriage or reproduction. While the empirical cultural content remains to be filled in, the problem of formal representation of kinship is concerned with the *relations* or *orderings* between culturally given unions such as reproductive or other types of matings, not the cultural characteristics of the unions themselves. In graph theoretic terms, vertices and their labels, even while associated, are distinct. *The map (graph) is not the territory (kinship)*. Kinship algebras have failed over such confusions: by raising axiomatic questions as to whether a mother's husband is a father, or a woman's child's father is a husband, they have confused formal representation with a 'substance' of kinship, whose variously cultural labelings will necessarily confound the generality of the axioms. Not that mathematicians care: but scientists and anthropologists rightly do care about questions of generality in their use of axioms or definitions, and as about the uniqueness properties of analytic concepts.

3. The uniqueness property

Guilbaud (1970, see Lévi-Strauss and Eribon, 1988; p. 83) used Weil's insight, about separating out the 'orderings' of kinship from questions of 'content', to construct graphs of marriage systems where individuals ('ideally') are obligated to

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[3] Malinowski (1930) argued for parenthood as the fundamental kinship relation, but overlaid this view with his theory of the importance of the nuclear family as a universal institution. Parenthood, however, also implies a system of relations for finding mates outside a kin group with its culturally constituted rules against incest, as Lévi-Strauss argues. Our use of parenthood as the fundamental relation of kinship does not entail Malinowski's functionalist or extensionist views of kinship. Nor need one insist that parentage is exclusively biological, as the concept of parentage can represent sociological or adoptive parenting as well. Indeed, in many societies it is the sociological parents who are regarded as the primary kin when they are not identical to the biological parents.

marry within a particular category. [4] It remained for Jorion (1984) to build more directly on Weil's (1949) insight to derive a more canonical kinship graph. He noted that from an order perspective, quite generally, *individuals link their parents' reproductive union to their own*. Irrespective of how mating or marriages are conceived or culturally embellished, the fact that individuals are the links between reproductive unions is a sufficient basis for a completely general graphic representation of kinship networks. While Guilbaud stuck to narrow algebras of marriage-class systems, Jorion (1984) expanded on these axioms. White and Jorion (1992) went further to generalize the representation of kinship networks.

All people are born, whatever the cultural variants of the 'substantive' definition of 'mating', 'marrying' or 'parenting'. In something so simple, does there lie a basis for a conceptual underpinning that may lead, after needed scientific and ethnographic work, to rethinking foundational issues as in the 'descent versus alliance' debates in social anthropology? Rather than trying to separate marriage versus descent as aspects of kinship (with endless argument ensuing about their relative priorities), reexamining our choice of scientific representation of the problem may lead to a new relational conception of the issue. Are not parentage and couplings, of which 'marriage' and 'descent' are merely one pair of instantiations, *relationally entwined*?

The starting point of this paper, then, is that *mapping of kinship networks in a graph where vertices are couplings (culturally interpreted) and edges are individuals* (following White and Jorion, 1992) *brings the social 'field' of kinship into view*. This relational construction of kinship, submerging the normal Western 'ontological' priority given to individuals, has considerable advantage over the usual genealogical diagrams where vertices are individuals. It also has utility in the comparative study of kinship, since definitions and axioms may be employed about common

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[4] The type of kinship graph that was initially stimulated by the work of Lévi-Strauss and Weil in 1949 (and given form by Guilbaud in 1962 at l'Ecole des Hautes Etudes en Sciences Sociales) was only one of the varieties of kinship graphs developed by Guilbaud in the early 1960s (see Cuisinier, 1962; Bertin, 1967; Guilbaud, 1970). Guilbaud's graphs were mainly applied to permutation group models of kinship (Weil, 1949). The group-theory approach to kinship, stressing models of a timeless repetition of the transformational structures of kinship, fit Lévi-Strauss's concept of kinship as a 'cognized' domain in which the universals of thought could make sense out of the 'impossible complexity' of actual kinship (1965:125). In contrast to Andre Weil's group-theoretic approach to kinship, the mathematical idea utilized here of the p-graph as a network derives from theories of ordered sets (partial orders, Galois lattices). Vincent Duquenne oriented DRW to the foundations of lattice and ordering theories, on which the current approach is based, and gave helpful stimulus and advice to programming. The p-graph is an ordering of kinship relations between couples by individuals who link their parents to their own parental coupling. It is dual to ordinary kinship graphs since individuals are the edges and couplings are vertices. It is also dual as an ordering by generations, where upward chains generate ever more inclusive sets of ancestors while descent lines include only a single member of each couple or singleton. The p-graph, in contrast to assumptions that structural analyses are ahistorical, allows us to include the flow of historical time. It is only as a second step that we impose upon them or reduce them to 'cognized' transformational structures repeating themselves in time, the concern of Lévi-Strauss, that may represent natives or observers 'thinking about kinship' as social rules and conventions.

ordering properties of the kinship field, with wide tolerance for cultural variations. Human culture is not so relativistic that we may expect to find a belief that it is children who have given life to their parents by a reversal of time's arrow! Even beliefs in cycles of rebirth imply a temporal sequence. Our mapping, simply of the order relations in kinship, provides a more foundational representation of the implicit universal in social fields of parent–child relations: events taken to constitute the relation are ordered in time. In terms of the multiple cultural definitions of kinship, *our mapping also has a uniqueness property*: generator relations of parentage (applying both to males and females) are given precedence over derived kinship relations such as between siblings or blood kin, yet relations of affinity through 'coupling' are not treated as derived relations but are *intrinsic* to the mapping. Variations both in culturally defined parentage and coupling relations may be accommodated without sacrificing the uniqueness of the representation. Given this uniqueness, the many different cultural features of kinship can be mapped onto the canonical graph, without loss of generality.

In contrast, *with individuals as nodes in a kinship graph, the cultural decomposition of kinship lacks a uniqueness property* (see White, 1974, for further discussion of algebraic decomposibility and the relevance of uniqueness properties for formal tools used in sociocultural analysis). Anthropologists dispute the universality of marriage or a husband/wife relation, for example, and disagree about the primacy of marriage versus descent in kinship networks. These become issues for communities that are stable over many generations, where primary kin relations, however defined (among them mother, father, brother, sister, son, daughter, husband, wife – often denoted by letters M, F, B, Z, S, D, H, W), may interconnect to form a network that relates nearly everyone by multiple compound ties. This leads to the question of which relations are 'primary'. In some cases, anthropologists view father (F) as culturally 'secondary', the mother's husband at the birth of a child. In other cases husband (H) is culturally viewed as a woman's child's father rather than a primary relation. Nor is there a basis for agreement on how kinship relations are composed from elementary relations to form a kinship 'system'. In the semantics of the cultural labels that we might attach to the edges of a complete kinship graph with individuals as nodes, kinship definitions become refractory. The refractory quality of studies like those of Schneider (1980) on the subjective semantics of kinship labels, and the type of critique we see in Schneider (1984), are understandable in these terms.

4. Relational concepts and orderings

Can mathematical insights help in the study of kinship and kinship networks? Having framed here an account of the evolution of a canonical graph for kinship from mathematicians' insight about orderings, we are now in a position to ask the question, what does the kinship graph have to do with the mathematics of ordering, or discrete structure analysis? Consider the orderings that are implicit in the genealogical relations of descent and marriage, or reproductive union. Mating

pairs in a population are ordered by rank of descent of one or both mates, as defined by:

1. Male/male ancestral trees.
2. Female/female ancestral trees.
3. Composite forests of trees, intersecting at common ancestors.

We can represent these three partial orders by a common diagram containing two distinct order relations (and their composite), where:

$a <_m b$ iff there is an upward path from a to b that starts with a male link (a contains some male descendant of b).

$a <_f b$ iff there is an upward path from a to b that starts with a female link (a contains some female descendant of b).

$a < b = \{a <_m b$ or $a <_f b\}$ iff there is any upward path from a to b (a contains some descendant of b).

This diagram is called a *p-graph* (White and Jorion, 1992), where p- is a mnemonic for parenté (kinship), deriving from the parental relation that composes the graph. The French word parenté is also a reminder that kinship is also potentially constituted (again: variably by culture) by marriage relations (Dumont, 1971). Vertices are sets of people (including couples or singletons), some of them parental, connected by two types of descent lines for gendered individuals who connect their parents' couplings to their own (or to self as child). If a, b, c exist such that $a <_m b$ and $a <_f c$, a represents a *coupling* (e.g. mating; parents may also be single, mate unknown). If a and c exist such that $c <_m a$ and c is *adjacent* to a, then c contains the *son* of a. If a and b exist such that $b <_f a$ and are adjacent, then b contains the *daughter* of a. Because the p-graph diagram represents individuals as (e.g. descent) lines and couplings (e.g. marriage or matings) as time-ordered intersections between (descent) lines, it is well suited to representation and analysis of kinship networks. The graph allows easy identification, for example, of marriages between those who are already kin: if a and b exist in P such that $a <_m b$ and $a <_f b$ (both members of couple a have pair b as common ancestors) then marriage a is between relatives.

The *dual* of an ordered graph is constructed by assigning points for each of the edges, and adding a directed connection from point A to point B if their corresponding edges in the original graph share this directed orientation as a path through a given vertex (in the p-graph no extra undirected links between siblings are introduced). The resulting dual of the p-graph is the *genetic graph* known to graph theorists (Ore, 1963; pp. 62–67), in which individuals are vertices, and directed edges (i.e. arcs) run from children to parents. Genetic graphs, however, are a more cumbersome representation in that they may have up to two times as many vertices and twice the number of edges than the equivalent p-graph anti-dual, and extra links must be added for purposes of sociological analysis in order to specify the types of relations between parents. While the p-graph has its *dual* in the genetic graph, it is much simpler in structure. The dual of the dual of a

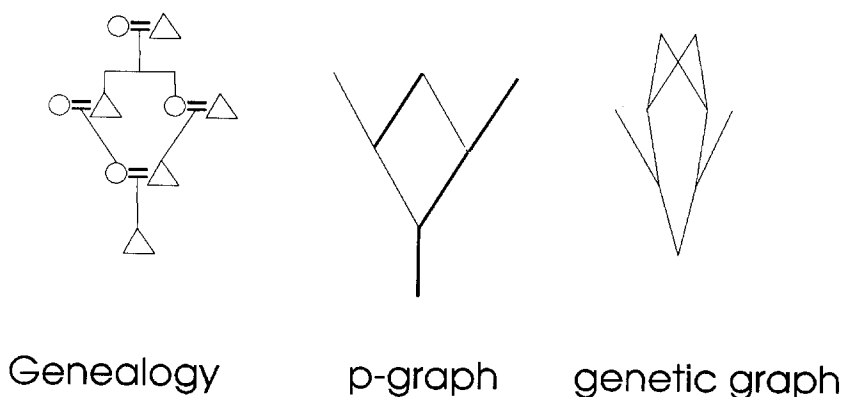Genealogy   p-graph   genetic graph

Fig. 1. Equivalent graphs of the same genealogical data.

p-graph, incidentally, must be simplified by equivalence of parental nodes before the simplicity of the original p-graph is recovered.

Fig. 1 provides a comparison of the conventional genealogy, the p-graph, and the genetic graph in representing the same body of information. The p-graph in the center codes gender by the darkness of the lines: darker lines for males, and lighter lines for females. Its dual, the genetic graph to the right, adds points and lines to restore individuals as points, but at the cost of redundancy, loss of information about gender, and ease of interpreting the types of 'closed circuits' of marriages between blood kin. The closed parallelogram in the center of the p-graph, on the other hand, expresses the exact relationship of married relatives, which in the present case is a MBD marriage.

5. Biases in the doctrine of 'genealogical unity of humankind'

Schneider (1984; pp 187–195) argues that the study of kinship has been constituted by three axioms that establish a doctrine of the 'genealogical unity of mankind':

(1) 'kinship is universal'.

(2) 'kinship has to do with human reproduction and the relations concomitant to that process. Hence a system of relative products based on the primitives of mother, father, (parent), husband, wife, (spouse), son, daughter, (child), is simply developed and extended from that nucleus.'

(3) the special corollary, that 'blood is thicker than water' deduces that "How far out the genealogy is extended... varies from culture to culture and... diminishes beyond the relations of primary kin."

Schneider rightly finds these axioms objectionable: they single out biological reproduction as the defining feature of kinship, and take an element of European folk culture ('blood is thicker than water') as the source for postulating how

variation is organized in the way kinship bonds are constituted (p. 193). He advocates abandoning any sort of assumption about the university of kinship in line with his view of the mission of anthropology:

Anthropology, then, is the study of particular cultures. *The first task of anthropology, prerequisite to all others, is to understand and formulate the symbols and meanings and their configurations that a particular culture consists of.* (p. 196, emphasis in the original).

If the culture contains the assumption that sexual intercourse is necessary to human reproduction one cannot just stop there.... To merely establish that the culture postulates that one person engenders another is insufficient: is the relationship held to be significant for that very reason or is that just one of the facts of life that are not really important, in terms of which social action is regulated? ...if [my] solution were accepted ... kinship might then become a special custom distinctive of European culture, an interesting oddity at worst, like the Toda bow ceremony. (p. 201).

We agree with Schneider's critique in that Axioms (2) and (3) must be rejected as universals in the study of kinship. Nor does Axiom (1) have our support in terms of universal salience of husband/wife bonds, marriage as a cultural universal, or substantive universality of beliefs that sexual intercourse is important to biological reproduction.

When one asks questions in the context of particular cultures about topics such as sexual intercourse, reproduction, or parentage, however, one assumes that there is some 'translation' of certain concepts which make it possible to ask the questions. Our ability to ask comparable questions is the basis for a discipline such as anthropology, whether we conceive it like Schneider or in other terms.

What we are asserting in the present exposition of how to represent certain empirical but culturally defined facts pertaining to 'kinship' is that one can ask questions about concepts relating to parentage, for example, in any number of human cultures, and find a great variety of responses. We do not require that the answers be consistent with the European bias of Schneider's Axioms (2) and (3), or with any presumed substantive universals held to be implicit in his Axiom (1). In this respect we agree with Schneider.

It is not so much that we differ with Schneider in his critique as that we address here a series of questions that he omits to undertake. What are some of possible and useful comparisons that one can make when assembling culturally diverse answers to a range of questions that explore the various domains of kinship? Schneider might deny that there is any domain of kinship in all cultures, but by that he would mean that the axioms or meanings defined as important by Europeans (Schneider's Axioms (1)–(3)) may not be important in a given culture. With this we would agree. In our view, however, there are various domains of kinship questions, one of which has to do with culturally defined relations of parentage. In asking questions about parentage, we need not privilege European

ideas such as 'blood is thicker than water', but we do need to recognize that beliefs about parentage may or may not entail concepts about coupling, mating, marriage, etc., just as they may or may not entail concepts about common 'blood', descent, etc.

Our concern, then, is with establishing a comparative framework for the study of genealogical networks: specifically, with how we represent our data in response to questions about parentage and genealogical networks, broadly defined. We think it useful to map out the network of what people in different cultures assert to the interpersonal relations between parents and children, however these relations are defined to form a 'genealogical' network. Unlike conventional genealogical diagrams, with their egocentric bias, the graph theoretic (p-graph) form that we choose here to represent culturally defined facts about genealogical connections does not privilege ideas of ancestry over other ideas about how connections are organized in genealogical networks or about what kinds of bonds are important.

Thus, consistent with Schneider's critique, we make no prior predictions about what kinds of kinship bonds, if any, will be important, or about the importance of 'genealogical connections' generally relative to other kinship phenomena. There are a host of other meanings and cultural facts in particular cultures that may relate in various ways and degrees to genealogy. For this reason, the use of p-graphs in the representation of genealogical networks may be of very general interest: first because they are capable of accommodating a wide variety of culturally constituted kinship relations; and second because they provide a comparable framework for mapping out a variety of other kinship phenomena, posing a much broader and open ended series of questions about the cross-cultural diversity of so-called kinship systems. Indeed, this approach may raise questions about whether and how kinship phenomena are 'systematic' in the first place.

The formal concept of kinship networks introduced in this paper is not intended as a theory of substantive universals of kinship, but as one means of representing certain comparable kinship phenomena and of mapping other phenomena that may or may not be related. Our presentation of a comparable concept of a kinship network is followed by definitions, axioms, theorems and methods of reduced representation that make such graphs workable as tools for representing and modeling kinship. The examples show the use of the kinship network p-graph representation and use topological and homomorphic reductions to analyze properties of kinship graphs and to represent some of these properties in more simplified models. [5] The examples also show diverse ways in which kinship is culturally constituted, ranging from kinships among American Presidents, in which common ancestry is of salient interest, to an Australian case where kinship roles

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[5] Two IBM PC programs, PGRAPH and PAR-CALC, were written by Douglas R. White to produce graphic representations of kinship networks as parental orders and to perform calculations on consanguinal relations between spouses. PGRAPH also does the homomorphic reductions of kinship networks illustrated here. PAR-CALC owes its basic algorithm to Jorion and Lally (1983). PAR-PLOT, extensively rewritten by White from an initial version provided by Linton C. Freeman, converts the PGRAPH output into commands for HPGL laser printing of high resolution graphics.

and vocabularies are formulated quite independently of the relative genealogical positions of social actors.

Where we disagree with Schneider's is his axiom that "symbols and meanings and their configurations" are what "a particular culture consists of". He privileges an ideational understanding of culture over the possibility that an understanding is required of both material processes and behaviors in addition to 'symbols and meanings'. In the study of any cultural phenomena such as kinship, we regard the tasks of mapping out behaviors and material processes as well as ideational symbols, rules and meanings to be important.

6. Illustration: kinship networks of US Presidents

Our representation of kinship networks provides a universal grid onto which culturally constituted features of kinship can be mapped and analyzed separately for each individual case, or in relation to other cases. An example from Roberts' (1989) *Ancestors of American Presidents* will illustrate how multiple lines of descent may be graphically depicted for complex kinship networks. Fig. 2 shows the common descent of Presidents Ulysses S. Grant, S. Grover Cleveland and Gerald R. Ford Jr. from Anna White of Windsor, Conn. (daughter of Robert
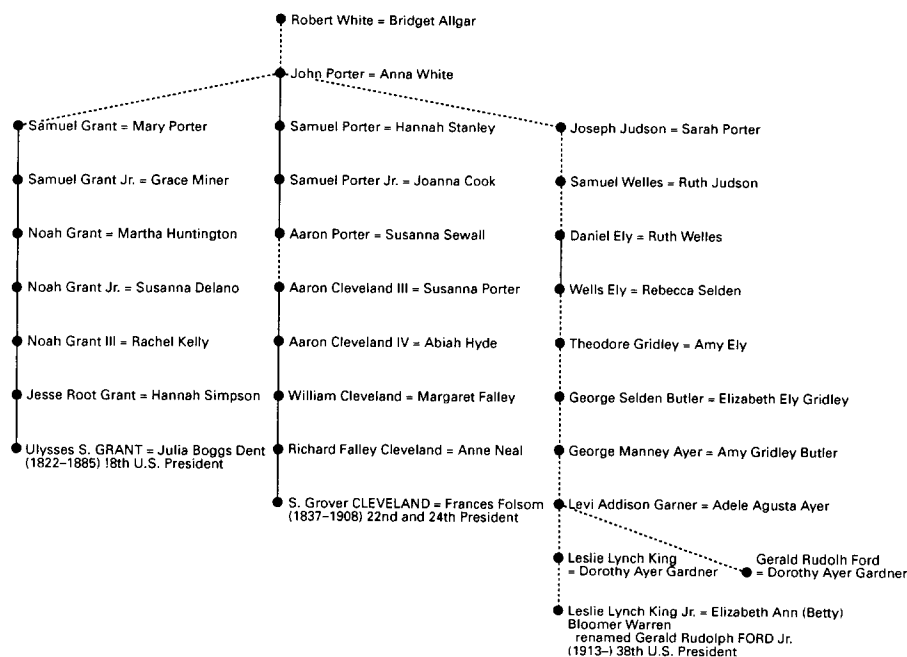
Fig. 2. Families linking Presidents Grant, Cleveland, Ford.

White of England). Roberts' genealogical notation is translated into our graph of the corresponding kinship net.

In our graph of a kinship network (of which the ego-centered genetic graph is a dual), solid and dotted edges are individual males and females, respectively (solid/dotted and other conventions may be reversed for emphasis), and each edge is directed from an individual's own to his or her parents' marriage. Vertices where the bottoms of two edges meet (as a woman and her mate) are typically marriages (wife and husband), and edges below the vertex are the resultant children. (Note: The term "arc" is here synomyous with a directed edge, and "edge" is an arc followed indifferently with respect to direction.)

Fig. 3 expands this example just a bit to show common descent from two ancestral couples (White and Wheeler) for four Presidents, adding James A. Garfield. Here we see one case of intermarriage between lines (George Ayer = Amy Butler) in the ancestry of Gerald Ford, Jr. This marriage link makes Ford a descendant of both White and Wheeler.

Fig. 4 shows kinship links for 15 US Presidents (the six in Fig. 3 plus Washington, Adams Jr., Adams, Hayes, Taft, Coolidge, Roosevelt, Nixon, and Bush) taken from the first six of Roberts' 81-page compilation of kinships among American Presidents. [6] The linking ancestors here are the two mentioned above (White and Wheeler) and four additional ancestors from England (Thomas Morse, Henry Squire, Henry Spenser, Robert Foote).

It is not surprising that many of the US Presidents, descended predominantly from English stock, are recruited from a connected kinship network. Fig. 5 shows something about the general social context by which American Presidents are linked. This is a reduced graph of what we call the *core* of a kinship network: only those vertices remain that connect two or more vertices in the core. Here, many of the Presidents themselves drop out, since they are at the ends of chains of descendants, connecting only themselves to the graph. They are 'outliers', as it were, connected by a single link. The core structure, however, begins to show something about the kinds of connectivities that we find in the kinship network. How often does intermarriage repeat between the vertical lines that represent bilateral descent? That is, do we find blood marriages (however distant) in this network? Answer: very rarely (they are necessarily contained in the core), and in later generations only between very distant blood relatives (e.g. marriage 17). Is there what French social anthropologists such as Segalen (1991) call "relinking" between families? Yes, there are quite a few marriages that relink families already connected by marriage. Marriage 142, for example, relinks two families already connected by marriage 150. Marriage 17 relinks three families already intermarried through couples 150 and 136, and so forth. The various closed paths or circuits in the core network, however, can be described here as relative endogamy within a larger group ('social circles'), not by specific intermarriage rules.

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[6] Data on the US Presidents were input courtesy of Christina Klein, GSM, UC Irvine (a member of our 1992 seminar) from the first four pages of charts in Roberts (1989).

Robert White = Bridget Allgar

John Wheeler = Agnes Yeomans

John Porter = Anna White

Aguila Chase = Ann Wheeler

Henry Wheeler = Abigail Allen

Samuel Grant = Mary Porter

Samuel Porter = Hannah Stanley

Joseph Judson = Sarah Porter

John Chase = LydiaChallis

James Wheeler = Grizzel Squire

Samuel Grant Jr. = Grace Miner

Samuel Porter Jr. = Joanna Cook

Samuel Welles = Ruth Judson

Jacob Chase = Joanna Davis

James WheelerJr. = Elizabeth West

Noah Grant = Martha Huntington

Aaron Porter = Susanna Sewall

Daniel Ely = Ruth Welles

Ezra Chase = Judith Davis

Ebenezer Ingells = Elizabeth Wheeler

Noah Grant Jr. = Susanna Delano

Aaron Cleveland III = Susanna Porter

Wells Ely = Rebecca Selden

William Chase = Abigail Gove

Henry Ingalls = Sybil Carpenter

Noah Grant III = Rachel Kelley

Aaron Cleveland IV = Abiah Hyde

Theodore Gridley = Amy Ely

Samuel Ayer = Polly Chase

James Ballou IV = Mehitable Ingalls

Jesse R. Grant = Hannah Simpson

William Cleveland = Margaret Falley

George S. Butler = Eliz. E. Gridley

John V. Ayer = Edna V. Manney

Abram Garfield = Eliza Ballou

Ulysses S. Grant = Julia B. Dent
(1822–1885) 18th U.S. President

Richard F.Cleveland = Anne Neal

George M. Ayer = Amy G. Butler

James A. Garfield = Lucretia Rudolph
(1831–1881) 20th U.S. President

S. Grover Cleveland = Frances Folsom
(1837–1908) 22nd and 24th President

Levi A. Gardner = Adele A. Ayer

Leslie L. King = Dorothy A. Gardner

Gerald R. Ford = Dorthy A. Gardner

Gerald R. Ford Jr. = Elizabeth A. (Betty) Bloomer Warren
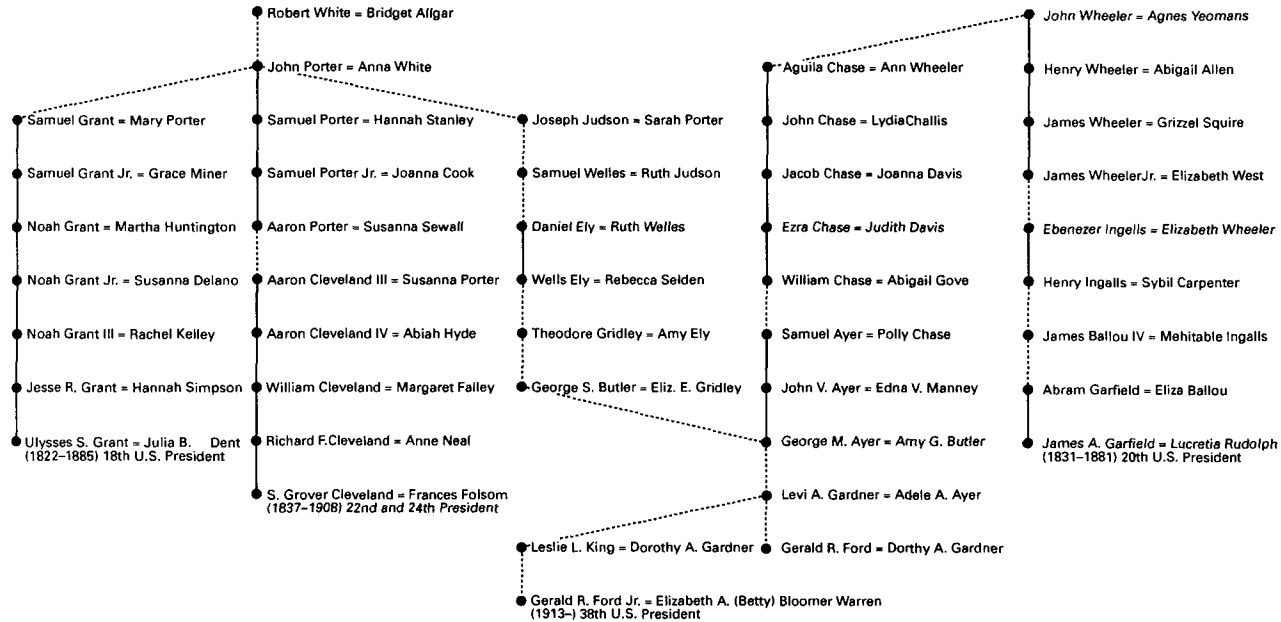(1913–) 38th U.S. President

Fig. 3. Descent lines linking Presidents Grant, Cleveland, Ford, Garfield (source: Roberts 1989).
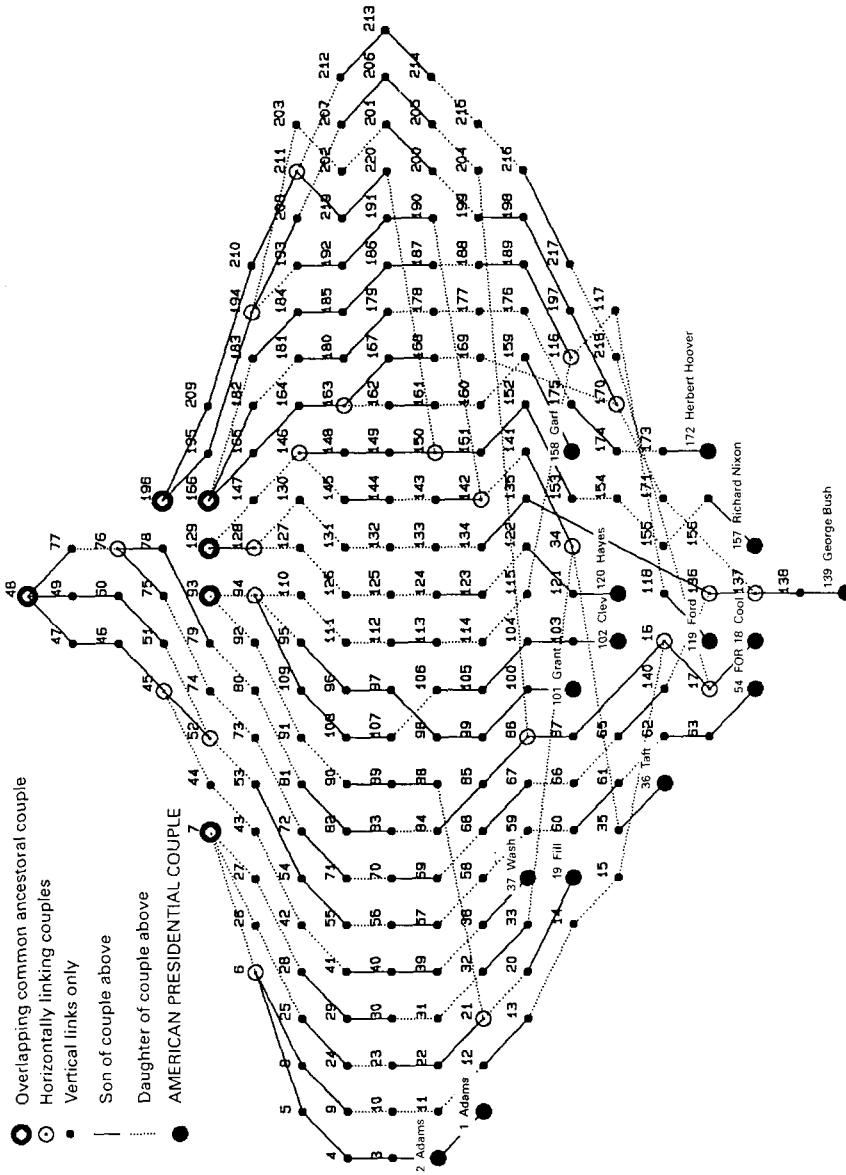
Fig. 4. A few of the related ancestries of US Presidents given by Roberts (1989).

If we delete marriage 77 in Fig. 4, a 'cutpoint' in the graph, we see that there are two *blocks* in which every vertex in the graph lies on a connected path from any other pair of vertices in the same block. This begins to tell us something about the overall structure of these 'social circles' and raise further problems about how to characterize them.

To see additional features of a kinship network, it will be useful to introduce formal definitions. Two kinship graphs are *homeomorphic* if both can be obtained from the same graph by a sequence of subdivisions of lines (Hage and Harary 1991 pp. 219–220). For example, where branching Fa's, Mo's and Mo's and Mo's Fa's Fa's ancestries are unknown, instead of Mo's Fa's Mo's parents, it may be useful to create a single link, such as Mo's ancestors twice removed (MFM parents). By condensed labeling of the reduced link, one retains information on the structure of the kinship net, but simplifies the graph.

Fig. 6 shows a homeomorphic reduction of line subdivision for the Ancestries of US Presidents of Fig. 4. Vertical lines containing no branches are simplified as links from a male or female descendant to an ancestor but the bilateral links to that ancestor are unspecified. Lines are no longer to parents but links to ancestors. *This is the minimum of ancestral links needed to capture the structure of overlapping ancestries, marriages between blood relatives, and marriage circles*, as among our 15 Presidents in Fig. 4.

The structural information needed to assess circuits in the Presidential network is visible in Fig. 6 as well as Fig. 5. Here again we see dense cross-linkage and complex 'marriage circles' as between couples 136 (ancestors 48 and 129), 17 (ancestors 48 and 196) and 150 (completing the circle with ancestors 129 and 196), as well as two cases of marriage between blood relatives. The network in Figs. 4–6 has two distant cousin marriages (couples 52 and 17, ancestors of Roosevelt and Coolidge, respectively) and a number of 'marriage circles' where each of two pairs of individuals in two couples have kin connections, as where A1–B1 are related by blood, B1–B2 by marriage, B2–A2 by blood, and A2–A1 by marriage, forming a closed circle of kin connections. Both marriage circles and blood marriages evidence a tendency toward endogamy or intermarriage in bilateral descent lines. There are three instances of affinal relinking of the descendants of pairs of ancestors to the right of Fig. 4–6, and numerous examples (five or more) in which pairs of families that are affinally linked are relinked in succeeding generations to form marriage circles. The high ratio of affinal relinking to blood marriages is consistent with endogamy through marriage links rather than a preference for marriage with blood kin. [7]

Fig. 7 simplifies the Presidential kinship network further to show simply the top six common ancestors and the 15 Presidents. All are connected via overlapping ancestries, and six of the 15 Presidents are related to two or more of the six common ancestors. Moreover, seven of the eight Presidents up to 1900 have only

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[7] Appropriate statistics need to be worked out to compute expected baseline rates of different types of marriage circuits given different population parameters.
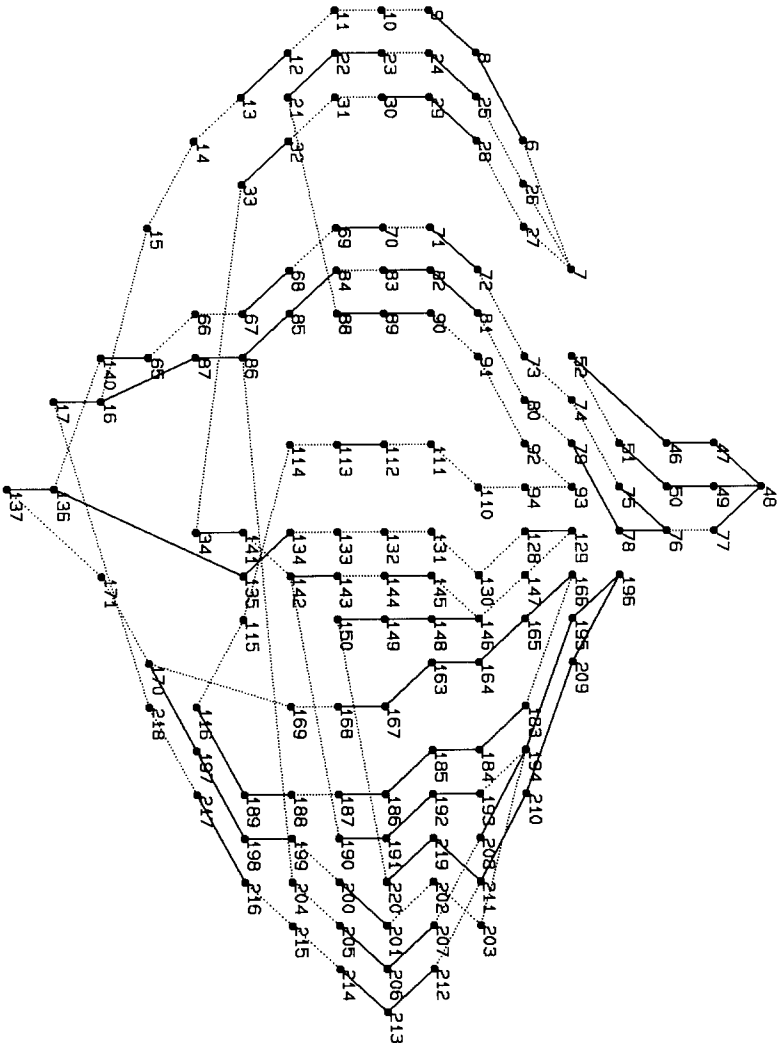
Fig. 5. Core of related ancestries of american presidents.

Daughter of a bilateral descendent of couple above
Son of a bilateral descendent of couple above

Fig. 6. Related ancestries of US Presidents with vertical links simplified.

one such ancestor (Fillmore has two), while five of the seven Presidents after 1900 have two or more such ancestors (Taft and Coolidge are related to three; Bush to four).

Presidential kinships as defined by Roberts are a function of ancestry not affinal ties, and their ancestral ties are often quite remote. In this respect, "Probably anyone...with between twenty and fifty 1620–1650 New England ancestors will

Fig. 7. Bipartite graph of some American Presidents and six common ancestors.

have five to ten presidents among his distant kin" (Roberts, 1989, xii), as do nearly all of the 18 Presidents with considerable New England ancestry. But socialization in elite social circles is clearly important in recruitment to Presidential office, and it would seem that fewer of the Presidential wives come from these circles than the Presidents themselves. [8]

If it can be inferred from the visible links in the network that kinship serves as a means of socialization for and recruitment to the highest executive office in the US, then what is the explanation for this structure? The endogamy of 'old family' Yankee (or Jamestown founder) elite social circles is a feature of American social structure independent of the Presidency. Endogamy is relative not absolute: some offspring, for example, may marry out or be lost to self-reproducing elite social circles. Given the 'old families' as the early political elites, and the transmission of political ties through the generations, network members who keep their ties in later generations are likely to be advantaged in political life. The network itself carries a political memory that those socialized within it can draw upon. One must be careful not to attribute unwarranted teleological properties to network members, such as intention to marry within the group to produce Presidential candidates. In marrying someone of the right social circles, however, there is commonly an intention to produce offspring who will be well socialized for membership in the group, and group membership may dispose to the Presidency, for reasons that the network itself may explain.

7. Formal concepts and theory of kinship networks

We now explore the structure of kinship networks through a formal exposition. The formal concept of a kinship network K is an ordered 5-tuple (X, Y, P, F, G) consisting of sets X and Y (each including one or more null elements), respectively designated *females* and *males*, or *feminine* and *masculine*, and a set P containing *ordered parental ensembles*, where each ensemble p in P consists of m (possibly null) elements in X and n (possibly null) elements in Y. One can also formulate as many types of null-codings as one wants to fit particular cultural logics or sociological problems. For example, single parents, or spouse unknown, or unmar-

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[8] No study comparable to Roberts' has been done of the distant kinships of First Ladies and other American Presidents. From Roberts' data it would appear that Presidents are apparently less likely to acquire kinship links to past Presidents through their wives than through their parents, which would be consistent with a bias for those who become President to have grown up in a somewhat more elite social stratum that includes bilateral ties with past Presidents. Of the First Ladies, Frances Cleveland, Nancy Reagan and Edith Roosevelt are related to one another and to British Royalty (Roberts, p. 285); Grace Coolidge is related to her husband and to J.Q. Adams (p. 277), Ellen Wilson and Bess Truman are related to other Presidents and British Royalty, and Eleanor Roosevelt is the niece of Theodore Roosevelt (p. 223).

ried children, may be accommodated as singleton individuals (with different types of null-counterparts to indicate their differences). They may be represented as $(x, 0)$ or $(0, y)$ in P for single parents, $(x, -1)$ or $(-1, y)$ for unmarried children, $(x, {}^*)$ or $({}^*, y)$ for adult bachelors or spinstresses, $(x, ?)$ or $(?, y)$ for spouse-unknown. For single x in X and y in Y, a pair (x, y) in P is a 'couple' (such as a mating, marriage or other potentially 'parental' pairing) if neither x or y is null, and a singleton individual if one member of (x, y) is a null element.

The definition of P sets is formulated to allow for maximum cultural diversity in the operating definitions of kinship, coupling, and parenting. Definitions of sex or gender are purposely left under-specified, as are the potentially parental ensembles. There are innumerable societies which distinguish between various types of parents, e.g. a 'natural' parent, 'adoptive' parent, 'foster' parent, or other 'sociological' parent. There are also cases where more than one 'biological' father is thought to exist (e.g. Yanomamö or Trio kinship). The cultural constructions used to define kinship relations may follow biological conceptions of kinship or allow latitude for membership in the sets X and Y in terms of concepts of culturally constructed gender, latitude in defining the pairings in P, or in cultural definitions of 'parents' and 'offspring'.

While there may be sociological diversity in P sets, for each conceptual type of parentage (biological, adoptive, foster, sociological, ritual, etc.), we designate a parental structure such that there are at most one set of parents of a given type. This structure of parenthood is stated as a formal theory, following the tradition of (Suppes, 1968, 1977), in the following set-theoretic definitions.

Definition 1. A structure $S = (P, (F, G)^i)$, where P is a set (of 'couples') and F and G are relations on this set, is a *parent structure* if and only if the following axioms hold for every i'th parental type, with its nominal parental relations F and G, and for every a, b in P:

Axiom 1. For each a in P there are at most one b and one c in P such that aFb and aGc. (*Nominal parents are gender-paired and unique; every 'couple' of nominal parents itself has at most two such sets, one for the male, one for the female.*)

Corollary 1. G and F may be redefined as functions in P.

If elements in the set P are composed at most of two ('coupled') nominal individuals, then an individual has at most two nominal parents, and nominal parents are of opposite sex, and unique (see the axioms given for genetic graphs by Ore, 1963, p. 60).

Axiom 2. Let $<_A$ be an *ancestry relation* such that for all a, b in P where $a <_A b$, there exists a path from a to b through a concatenation of the functions F and G. Then, there exists no a in P such that $a <_A a$. (*No one is their own ancestor.*)

Corollary 2. *The ordered ancestry relation $<_A$ is irreflexive and acyclic.* [9] *(Parentage is asymmetric and is partially ordered in a time dimension; generational succession is implied: one's parents generation and their mating precedes one's own).*

Proof. If not, then $a <_A a$, contradicting Axiom 2.

One may wish to impose additional restrictions on a kinship network, such as avoidance of incest between siblings: for each a, b, c in P such that aFb and aGc, b and c are distinct and non-overlapping. This condition does not represent a cross-cultural universal, however. For some purposes, one might want to impose a *parental invariance* condition whereby a person cannot maintain that their parents in the context of one spouse are different than their parents in the context of another spouse, as might be common among fugitives. The following would suffice to define this condition, that *one's parents are the same regardless of one's marriages*: For all pairs $x = (a, b)$, $y = (a, c)$, $z = (d, b)$ in P where a, d are in Y and b, c are in X, then $xF = yF$ and $xG = zG$.

The functions F and G refer to only one pair of nominal parents at a time. For more than one type of parents we may superscript successive pairs of unique functions, e.g. F^1 and G^1, F^2 and G^2, etc. Axioms 1 and 2 must hold for each separate type of nominal parents, that is, for sociocultural extensions of the parental concept [10] as well as biological definitions of parentage.

The *parental graph* (*p-graph*) K_P of a kinship network consists of the ordered triple *(P, F, G)* of the set of vertices P representing potential parents or matings, a set of arcs in relation F from (female elements of) pairs in P to their parents in P, and the set of arcs in relation G from (male elements of) pairs in P to their parents in P. The union P_{FG} of these two relations is the relation between one member of a pair in P and both members of another that we may call *parentage*. *The transitive closure of the relation P_{FG}* forms the directed *ancestry* relation $<_A$ indicating that one member of a pair in P has as ancestors both members of another pair or element p in P. It follows from Corollary 2 that generational rank can be assigned to every vertex (i.e. to matings or marriages, and to individuals by

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[9] While we do not insist on the relevance of Axiom 2 in every case of sociological or adoptive parenting, this biological assumption is usually taken as the 'model' for culturally defined parenting.

[10] The *ancestral graph K_A* consists of the ordered pair $(P, <_A)$ of couples (or null-paired singletons) and their ancestral order relations. $K_A = (P, <_A)$ is a two-component or *binary partial order of ancestral / descendant relations among the vertices in P*, each of which has at most two immediate ancestors, depending on whether the pair in P contains a male with parents, a female with parents, or both.

A kinship graph K_P may be represented as a *three valued graph* where any ordered pair of vertices (x, y) in $P \times P$ elements in U, V, or U and V. With the following axiom added only *two values* are required, so that (x,y) may be in U, V but not both:

Axiom A. If aFb then not aGb.

This is the principle of *reproductive avoidance by brother and sister* – a typical law of culture, but not necessarily universal, as witnessed by the Ancient Egyptian royalty.

rank of their parents' marriage) in terms of discrete levels (but not uniquely determined) in a p-graph kinship net.

Kinship relations are calculated on the parental graph of a kinship network (White and Jorion, 1992) by the relations F and G (also defined as functions by Corollary 1), that record parentage for males and females, and the inverse relations that index the daughters and sons (*filles* and *garçons* in the french mnemonic) as offspring of each set of parents. Thus, f and g are defined for a,b in P by:

aFb iff bfa and aGb iff bga. (*Parentage is inverse to having children.*)

For some purposes, we may want to expand the formal definition of a kinship net. For example, an 8-tuple (X, Y, P, F, G, A, B, C) might also include a set A that pairs any number of attributes with individuals in X and Y (such as Presidential Office as in Fig. 4), a set B that pairs observed or normative behaviors with ordered pairs of individuals, and a set C of culturally defined relations on P that correspond to normative expectations, etc.

Two types of reductions of kinship are useful in moving from kinship links in networks of concrete individuals to models of more abstract properties of kinship systems. These are the topological reductions of homeomorphisms and cores, and the algebraic reductions of homomorphisms.

8. Topological reductions of p-graph networks

Definition 2. A *homeomorphic reduction* h: $(P, F, G) \dashrightarrow (P^*, F^*, G^*)$ maps vertices in P and arcs in $P \times P$ to a subset P^* of vertices in P with relabeled arcs F^*, G^* in $P^* \times P^*$ where for all distinct a,b in P^* $c P$, *the original graph can be recovered by a sequence of subdivisions and relabelings*, described by each aR^*b, as to how a is connected to b by a succession of pairs of vertices-and-edges, $aR_0 x_1 \dots R_k b$, and where R_i for $i = 0,k$ are the arcs of F or G in the original graph, with their appropriate orientations for R_0 and each successive $x_i R_i$, and $h(x_i) = b$.

Theorem 1. If h: $(P, F, G) \dashrightarrow (P^*, F^*, G^*)$ is a homeomorphic reduction, then Axioms 1 and 2 still apply to $(P^*, F^*, G^*) = h(P, F, G)$.

Proof. A homeomorphic reduction still defines a p-graph in which individuals or couples are ordered by ancestries. The reduction applies to a line of ancestry from B to A ($B < A$) only if it is *linear*, that is, traced from descendant B to ancestral couple A without knowledge of bifurcating relatives. In such a case, while the homeomorphism merges the line of successive ancestors into a single line, uniqueness of the collapsed ancestry relations (Axiom 1) is retained, and the ordering of ancestry (Axiom 2) is not violated.

Definition 3. The *cores* of a parent structure $S = (P, G, F)$ are the maximal sets $C_i (i = 1$ to $l)$ of vertices in P, together with the arcs defined by G and F within each core set $(C_i \times C_i)$, where each vertex in a maximal set C_i has edges with at least two other vertices in C_i (2-cores: see Seidman, 1983). Distinct cores are disconnected, since two cores cannot be connected without constituting a single maximal core. Since the total set of vertices in the cores of a kinship network is calculated recursively by eliminating vertices in P that are not doubly connected to others in the total set (the remainder of which converge to the core sets in successive iterations), the maximal sets of the cores are unique.

Definition 4. A *circuit* in $S = (P, G, F)$ is a closed path, composed of G, F or their inverse (child) relations, in which no edge is traversed more than once, and no vertex is crossed more than once. Every consanguinal marriage necessarily defines a *vertical circuit*, closed by common ancestry of the couple. *Lateral circuits* ('relinking' or marriage circles) are those circuits that are not vertical (not closed by 'consanguinal' or 'blood' marriages between those with common ancestry).

Definition 5. The (regular) *blocks* of a parent structure $S = (P, G, F)$ are the maximal sets $D_j (j = 1$ to $J, J > 2)$ of vertices in P where removal of at least two vertices is required to disconnect the graph of a block (Hage and Harary, 1991; p. 139). It follows that every block D_j is a subset of some core C_i. There may be two or more blocks in a single core, in which case they share a point or are connected by *bridges* (a single path of connecting edges). Every vertex in a block is connected by two or more independent paths (if not, removal of one vertex would suffice to disconnect the block), and lies on a connected path connecting any other pair of vertices within its block. The regular blocks, bridge, or $(J = 2)$ blocks of a graph are a partition of its edge set (see Chartrand and Lesniak, 1986, p. 47–51).

Theorem 2 (*blocks*). Two vertices in P of a parent structure $S = (P, G, F)$ are in the same *block* of S if and only if they lie on a common circuit in S (for proof: see Hage and Harary, 1991, p. 80).

The *blocks* of a network (Definition 5) provide evidence of endogamy in the form of marriage circles (lateral circuits) or common ancestry marriages (vertical circuits), all of which are necessarily contained in the blocks (Theorem 2). Connections are multiple or 'reroutable' in the blocks of a network.

Definition 6. The common ancestry or *vertical blocks* of a parent structure $S = (P, G, F)$ are the maximal sets $B_k (k = 1$ to $K)$ of vertices in P contained in vertical circuits that share one or more edges. Each distinct vertical block B_k is a subset of some block D_j.

The following definitions derive from the work of Houseman and White (1994).

Definition 7. The *divides* of a parent structure $S = (P, G, F)$ are a bi-partition of the vertices in P having the property that for any a,b,c in P where aFb and aGc, the parent vertices b and c are members of opposing sets in the bi-partition.

Definition 8. The *sides* of a parent structure $S = (P, G, F)$ are a pair of divides such that for all a,b in P, a and b always belong to the same divide if aGb (*male sides*) or to the same divide if aFb (*female sides*).

Theorem 3*a*. To evaluate the existence of sides or divides, it is necessary and sufficient to examine the blocks of a kinship network: if each meets the criteria of sides or divides, then the entire kinship network meets these criteria.

Proof. Elements outside the core are connected only by single links, so their bipartite assignments can also always be made consistent with those in the core. Only a vertex in a block may generate inconsistencies, and only with respect to other vertices within the block. Since interconnections between blocks are single-link chains, bipartite coding within blocks can always be adjusted relative to one another, but cannot create other inconsistencies beyond those in the blocks.

Theorem 3*b*. If sides exist for a connected set V of vertices in P, they are unique.

Proof. Given vertices a, b in V and male sides, if aGb or bGa then a and b belong to the same side; if aFb or bFa then they belong to the opposite side, hence relative sidedness is determinate for all connected vertices in V; similarly for female sides.

Divides are not necessarily unique, even for a connected set of vertices in P, since relative assignment of divides in adjacent generations is indeterminate.

9. Tracing succession and 'common ancestry' marriage: refiguring kinship transmission in the genealogy of Canaan

Marriages among those with common ancestors are the stuff of many kinship alliance models. They are also widely seen as a primary means by which not only alliances are made but succession, inheritance, and transmission of cultural knowledge are governed or manipulated. Definition 6 allows us to focus on the part of the core of a kinship network containing the vertices that define common ancestry marriages.

Some of kinship strategies of the Genesis story of the line of Patriarchs may be illustrated with reference to the kinship network of the patriarchs of Canaan shown in Fig. 8. As described in the Old Testament, Abram in Ur, son of Terah, grandson of Nahor, and descendant of Shem, gathered a group of his nomadic tribesmen to go to the land of Canaan (near Bethel in the Levant), then to Egypt. There, as a means of guarding against jealousies, he makes public the fact that his wife Sarai is his close kinswoman (according to Genesis, a half-sister, but identified by the Hebrew source Josephus as Iscah, a daughter of his brother Haran). He returns safely to Canaan to rescue brother Haran's son Lot from capture and is obliged to battle foreign kings. Following disputes between their shepherds, Abram and Lot separate, and Lot settles near the Jordan valley cities of Sodom and
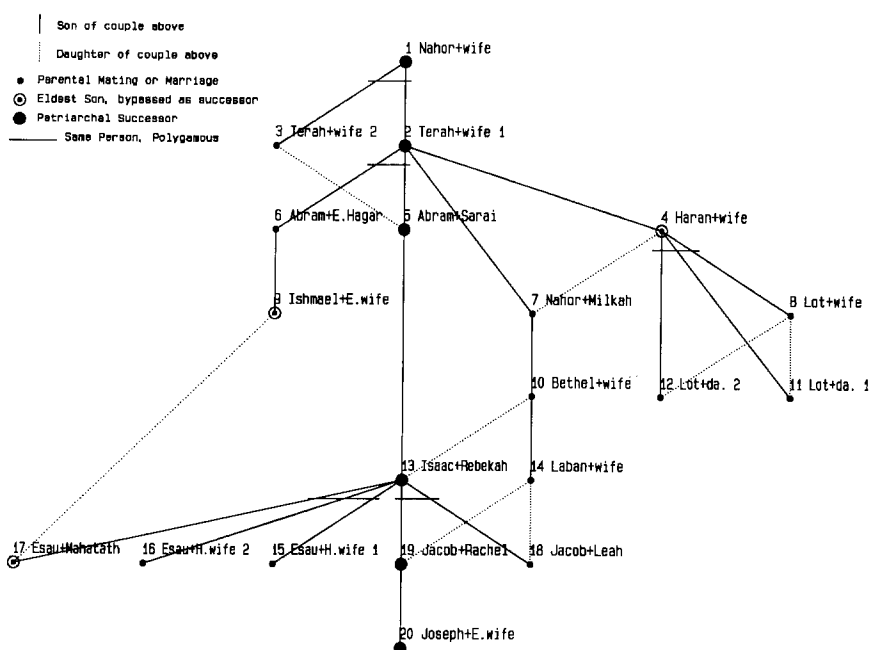
Fig. 8. Marriage and succession in the Old Testament Genealogy of Canaan – patriarchical successors marry co-descendants.

Gomorrah. Sodom and Gomorrah are destroyed and Lot and his daughters saved, but Lot's wife perishes. Lot is then seduced by his daughters who seek to have children to preserve his patriline. In Fig. 8, the edges joining Lot to matings with his daughters are not marriages but illustrate the importance attached to paternal succession in the Canaanite mythos, outweighing, in the case of Lot's daughters, the injunction against incest.

By a covenant with Yahweh, Abram and his infertile wife Sarai became Abraham and Sarah with their son Isaac the resultant 'seed of Israel'. Sarah's objections to succession by Ishmael, the elder son by the Egyptian servant Hagar, lead to their banishment and to succession by Isaac. Isaac wanders beyond Jordan, but marries Abram's brother's son's daughter (and brother's daughter's son's daughter) Rebekah. Their elder son Esau (representative of the hunter–raider in the tribal division of labor) takes Hittite wives and is displaced from succession by the machinations of Rebekah and a double artifice (Gen. xxv, 29–34 and xxvii) by the younger brother Jacob (representative of the shepherds). Jacob obtains the rights and privileges of seniority and returns to the Aramean homeland of Abraham, receiving a revelation from God at Bethel. Two strategic marriages with his mother's brother's daughters (also great granddaughters of Abram's brother) cement his ties, and he leads a reunification of his tribe, whence his legendary fame as the founder of the tribe of Israel.

In Fig. 8, Terah, Abram, or Jacob, each with two wives, and Esau with his three wives, are represented by multiple edges (scored by an extra horizontal line), indicative of polygynous marriage. Successors to the priestly line are shown in the central vertical axis. The rightful heirs by primogeniture (Haran over Abram, Ishmael over Isaac, Esau over Jacob), bypassed by the succession of their younger brothers, are shown with open circles.

In this context of intense conflict and psychic drama, a variety of themes emerge about conflicting kinship principles. Three counter-motifs are played out against a general cultural backdrop of primogeniture and patriarchal authority. One concerns the claims of women belonging to the patrilineage to have their sons take precedence over the sons of foreign women. The second, given that marriage patterns oscillate between alliances with women of Egyptian, Hittite, and Elamite origin and marriages with lineage members, is the intermittent prohibition of outside marriage, strengthening the solidarity of the family and tribe, and keeping its knowledge and heritage intact. The third is the struggle of the younger sons (especially the favorites of a mother who is a lineage member) for rights to succession superseding those of the eldest brother. The younger sibling theme is beautifully documented in an account by Forsyth (1991), and is consistent with an earlier interpretation by Niditch (1987). What the present kinship analysis adds to an understanding of these themes is the role of intra-lineage marriages in strengthening the mother's identification with her son as a member of her own charismatic patrilineage.

The patriarchal line of priesthood of the Old Testament begins with Abraham, Isaac and Jacob, and is faced with the problem of transmitting a sacred body of knowledge (historical experience that contains sacred lessons given a special convenant with God) down the generations as well as uniting one or more nomadic tribes of shepherds, hunters, and raiders. The younger sons of the Canaanites patriarchs cement their alliances and succeed to leadership over the claims of their elder brothers through strategic marriages with women of their own lineage who also trace their descent back to the earlier charismatic patriarchs of the lineage. These women, as opposed to the foreign born wives of the elder sons, assist their sons in turn in the succession.

We will examine two different interpretations of the common ancestry cores of this segment of Canaanite mytho-history, one from a male viewpoint with Sarah as Abraham's half-sister, the other from a female viewpoint with Sarah as Abraham's niece, following the Josephus interpre·ation. Fig. 8 shows the male-oriented view, and Fig. 9 the female-oriented view. The male view, with Sarai interpreted as Abram's half-sister, helps to explicate the story of conflicts over succession unfolding within a single patriline (White and Jorion, 1992), with wives who are members of this line intervening on behalf of their son's succession. Their sons tend uniformly to be the younger ones, since the elder sons are uniformly the offspring of other marriages to women outside the line.

A more female-oriented view depicted in Fig. 9, in contrast, adds the Hebrew interpretation of Sarai as a niece of Abram, and is drawn to show a startingly different structure. Here the women (shown as dark lines) are grouped into two

females sides (Definition 8), with males of the Canaanite patriline shuttling back and forth between them in the marriages of successive generations. The only marriage inconsistent with the interpretation of female sides is that of Esau, who 'wrongly' marries a daughter of Ishmael (of his own side) rather than a daughter of the opposite female 'side' of the family. If this marriage were a violation of an alternating female-sided marriage rule, it would help to explain the choice of Jacob over Esau in the succession (see White and Jorion, 1992). While knowledge of further genealogical links of the female sides are unknown, the semblance of a classificatory female principle emerges from this representation of matrimonial sidedness, with 'patriarchal' succession alternating between the two sides. Jacob's long brideservice to Laban, accusations by his sons that the family wealth had gone with their sisters' marriages to Jacob, and the stealing of Laban's household gods by his daughter Rachel would be consistent with emphasis on female-sidedness. The 'male' and 'female' views, however, may illustrate kinship principles that were co-existent, as many of the conflicts that are narrated involve competing female and male principles of rights and privileges. As with the theme of succession by younger sons, questions of exegesis are raised: were the 'Patriarchs' patriarchal? Was 'patriarchy' an interpretation of later texts?

10. A binomial test for marriage sides

Fig. 9, showing a tenuous pattern approximating female sides in network data, is precisely the type of case where we would want to test a null hypothesis that such a result might easily occur at random. The evidence to confirm or disconfirm sidedness exists only in independent circuits. Circuits with female sides, for example, must have an even number of male links in order to consistently connect the two female sides. Given a particular model of sides among descent lines connected by intermarriages, such as Fig. 9, let c be the number of two-sided circuits (i.e. with this feature), and d the number of discordant circuits without sidedness. Here, $c = 5$, if we ignore duplicate links to sisters (or $c = 7$ if they are included), and $d = 1$. There are a total of $n = c + d = 6$ circuits, and the null hypothesis may be tested by the binomial theorem, just as we would assess the likelihood of six heads and only one tail in tossing a fair coin seven times:

$$p = \sum_{k=0}^{d(=1)} (6 \text{ choose } k)(1/2)^{6-k}(1/2)^k = (1 + 6)(1/2)^6 = .109.$$

In this case, the network pattern of female sides is not strong enough to reject the null hypothesis at $p < 0.05$, and only barely significant ($p = 0.03$) if we include sororal couplings with the same man. Note that this computation depends on the number of independent circuits, not of the total circuits in the graph, and that for a given block with $|E|$ edges and n points, the number of independent circuits is $|E| - n + 1$ (Gibbons, 1985, p. 56).
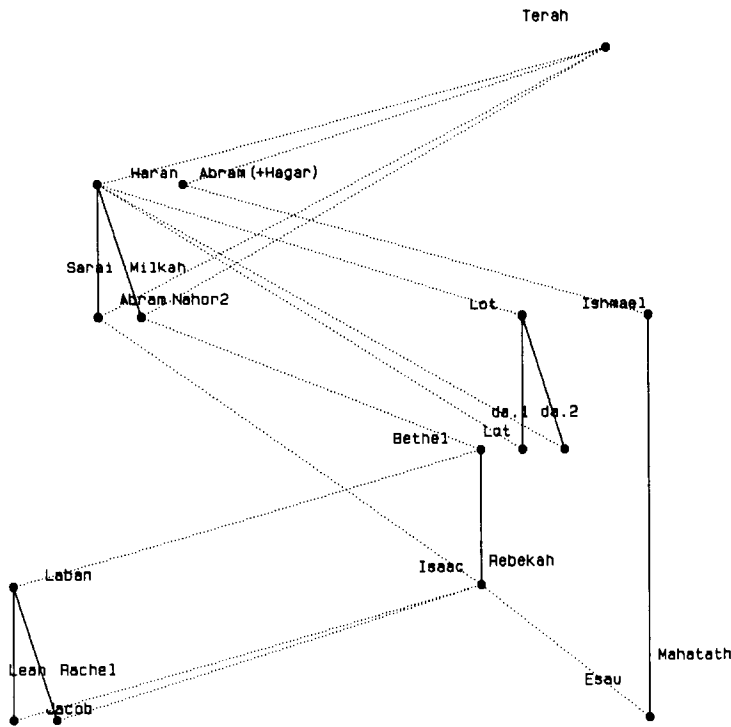
Fig. 9. Female view of the Canaan genealogy (females, dark lines).

11. Homomorphic reductions of p-graph networks

Definition 9. A parental graph *homomorphism* is a mapping h: $(P, U, V) \dashrightarrow (P^*, U^*, V^*)$ of vertices in P to a reduced set P^*, where for all a,b in P, $h(aUb) = haU^*hb$ and $h(aVb) = haV^*hb$. *Kinship relations are preserved in a homomorphic mapping*, but not necessarily uniquely: Axiom 1 no longer automatically applies. Without further restrictions on the homomorphism, the images U^*, V^* of parentage are not necessarily functions, since the mapping does not prevent the relations of a gendered child-set to parent-sets from becoming one to many. This homomorphism is thus defined in terms of the *relations U, V* rather than the functions *F, G* that define them.

Definition 10. A parental graph homomorphism h: $(P, F, G) \dashrightarrow (P^*, F^*, G^*)$ is *order preserving* if and only if Axioms 1 and 2 still apply to the new marriage classes. An order homomorphism requires equivalence classes of couples with respect to the uniqueness of their gendered parental classes, and retains the axiom of generational succession, thus *defining a reduced p-graph for classes of couples*.

Jacob's double marriage with matrilateral cross-cousins, Lot's two daughters lying with their father, and Nahor and Abram's marriages to daughters of their brother Haran (in one interpretation) are 'equivalent' marriages under the principle of an order homomorphism: these marriages of one polygamous individual or two same sex siblings form a single class with equivalent parent classes of the spouse. In general, edges for multiple marriages by the same person, siblings of the same sex, or successive generational sets in the same descent line may be reduced to the same equivalence classes if their spouses, for each set, come from a single equivalence class in the reduction.

12. Illustration: a Groote Eylandt 'gerontocratic' marriage network

Marriages and detailed genealogical data [11] were recorded by Rose (1960) in 1941 and 1948 for 221 of the 300–350 living individuals (0.33 per square mile) of the Groote Eylandt Winindiljangwa, and his data collection was extended by Peter Worsley [12] in 1952/53. While the kinship network among the living was commonly three generations in depth, we produced a database with five generations and 400-plus marriages from further compilation of kin relations using names of deceased parents, comments about prior marriages and marginal notes about relatedness. The marriage network forms a single connected graph, and the core of the graph has 290 marriages, shown in Fig. 10. Here, males are the darker and more vertical lines, forming about 32 groups related through males. These groups are divided into two moieties. As Rose noted, all but one or two marriages were between opposite moieties, so that the moieties also satisfy the definition for *sides* (our Definition 8). The sides have been separated, one on the left, the other to the right, so that nearly all the marriages are between a son from one side and a daughter on the other (here $p < 0.0000000000001$). Since males are segregated on the respective sides of their moieties, all the female lines (of the same moiety as their brothers) cross between the two sides. Fig. 10 is remarkably well-structured in
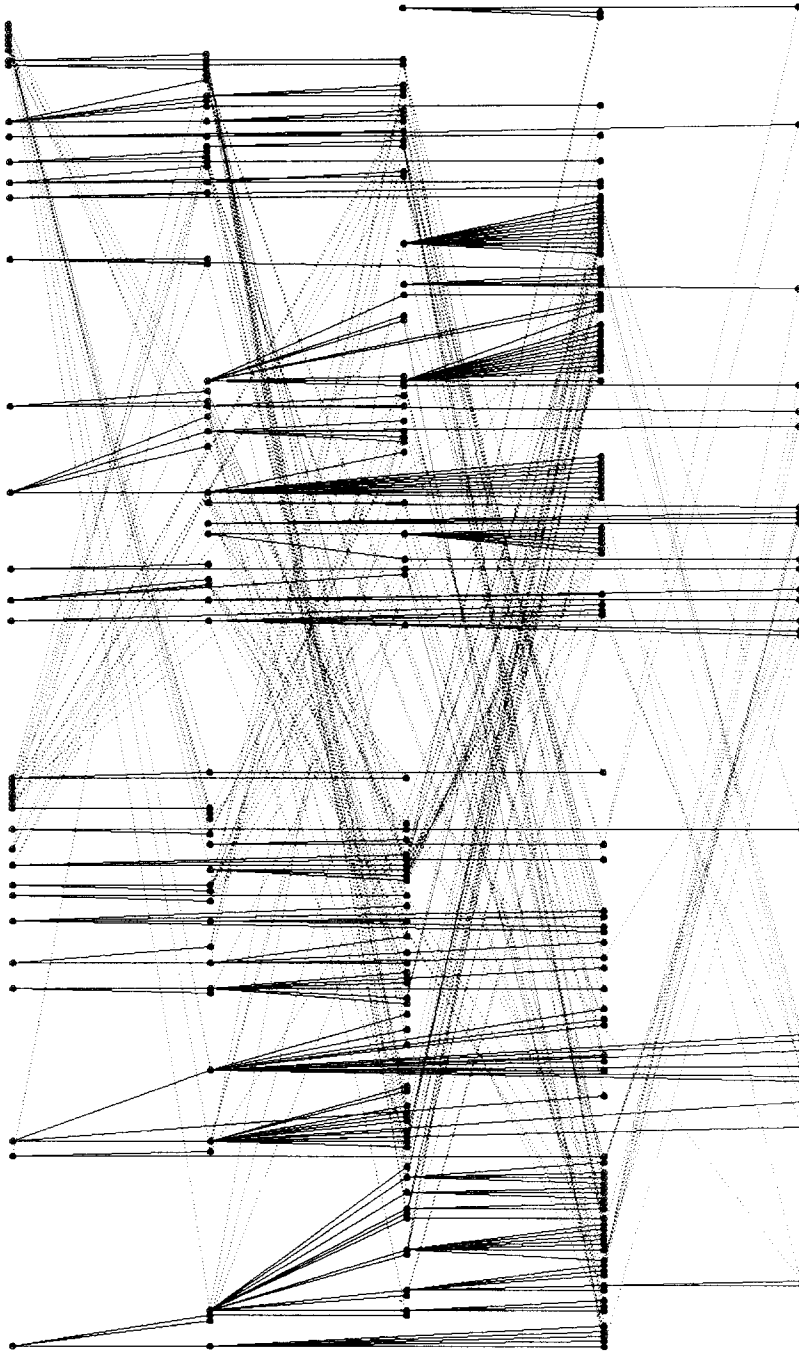

Fig. 10. Groote Eylandt moieties for core network.

terms of a dual organization of marriages. Of the two marriages found to violate moiety exogamy, neither produced children (one man was speared for marrying his half-sister; the other banished himself from the group).

There are many types of short- as well long-lived arrangements involved in the couplings of Fig. 10. Of the 170 marriages on which notes are available, 81, for example, resulted from girls being 'promised' to boys or older men at a time when the girls were infants or yet unborn. In most of these cases a girl's preadolescent brother, if one was available, was also given as an initiate to the prospective husband. Another 48 marriages resulted from the stealing of a wife already married or of a girl who had been promised. Stealing or death of the spouse prevented 17 of 81 girls promised (21%) from being taken. Twenty or more wives were inherited after the husband died, five 'given' to a relative, and in some way four 'exchanged', two 'bought' and one 'given' in compensation.

Betrothal of a daughter's unborn girl to another couple's son is common in Australian gerontocratic societies. The son may be two decades older when the girl comes of age. In such promissory marriages, if the 'exchanging' couples are also of the same generation, the son will be of a higher generation than his bride. There roughly 20 cases consistent with generational skewing where two or more brothers (or a man or his brother) married two or more sisters. Conversely, there were only two generationally unskewed sister exchanges (usually disallowed since a promised wife's young brother was usually given to the prospective husband as an initiate: in one case the husband later gave his sister to the initiate, 45). Generational differences between spouses are given in Table 1, and are mildly skewed (1 : 1.15) towards a quicker generational time for females. It might be considered remarkable that there is not more generational skewing, given the gerontocratic structure of marriage and the promising of infant brides. Wife inheritance, however, works in the direction of adding extra male generations while female generation is held constant. While wife-stealing is often by older men, when done by younger men against the wives of older men, it may also contain a counter-tendency to equalize the generational differences of spouses.

With so many types of marriage, it is not uncommon for them to happen as a quick succession of events: females promised very young or before birth, girls often stolen, sometimes stolen again, sometimes returned or exchanged or given once more, at some point settling down to a stable marriage, only to be inherited by the husband's kin when the husband dies, etc. If some of these marriage fail to produce children, they may leave no permanent mark on the kinship structure. To

Table 1

	Males higher	Females higher
No generation difference	103 (57%)	162 (85%)
One generation difference	59	25
Two generations difference	27	3
Three generations difference	1	0

answer the question of whether generational skewing might be something that is transient, occurring mostly with initial or 'temporary' marriages that do not leave issue, we examine Fig. 11, in which marriages that produced no children are eliminated. The marriages in this figure, where the types are known, are coded P = promised, S = stolen, T = taken or given by other means, and I = inherited. Table 2 shows the distribution of marriage types classified by generational differences. Males tend to be of higher generation in arranged or promissory marriages, but both types of generational differences occur with wife stealing and in inherited marriages. Tables 1 and 2 are consistent in showing that about 33% of the marriages are generationally skewed, 11% by two generations or more, with much higher skewing in women's first marriages.

The Winindiljangwa have two sets of classificatory kinship terms for generations within their own moiety, one for people glossed with ego's patriline (e.g. fa's fa's totem), and one for relatives glossed with their mother's mother's and sister's daughter's patriline. Moiety exogamy encompasses totemic exogamy within these lines, but analysis of the actual patterns of marriage between relatives within the network in Fig. 10 supports an idea not recognized by Rose: cognatic exogamy is also present. There are virtually no marriages between descendants of the same grandparents, each of which is known and identified by totem. Totems have to do with myths that can be told about localities in the mythic 'dreaming', and rights to sing and teach associated sets of songs.

In the opposite moiety, classificatory sister and fa's mo's lines are distinguished in each generation from mo's fa's and the da's lines. Descendants of the 'MoBr' class in the mo's fa's line are called, if male, by the same term as 'MoBr' class 'I' terms include: 'M' = 'MBD' in which more distant relatives fall outside the cognatic marriage proscriptions. Also marriageable *except* for close relatives are patriline descendants of the 'FZ' class in ego's moiety whose female patriline offspring are also called '*FZ*', who are again marriageable (class '*F*' includes 'FZ' = 'FZD', 'Omaha' terms). Descendants of the 'MMBDD' class in ego's moiety are also marriageable (in the '*O*' or 'MMBDD' class) except for close relatives, but do not transmit their kin-class to the next generation.

Groote Islandt males' first-preference in marriage is with the '*O*' class ('MMBDD'). In the five-generation network in Fig. 10, however, analysis of actual blood kin marriages of 39 married men who do have a MMBDD suitable for marriage shows a striking result: none married an actual MMBDD. In general, those sought in marriage in the preferred classes are not close consanguineal relatives but more distant, affinal, or even unrelated 'classificatory' kin. The moiety structure is also open to the assimilation of outsiders by simply assigning a moiety opposite their spouse. What is surprising from an analysis of network marriages is that of the total number of couples, fewer than 1% are traceable blood relatives, although there are plenty of female kin in the opposite moiety available to marry, given even the cognatic avoidance of marriage between close kin.

Groote Eylandt kinship allows surprising variability in choice of spouse. Except for cognatic exogamy of co-descendants of grandparents, marriage is permitted with any clan or totem in the opposing moiety. Still, principal choices for over 90%

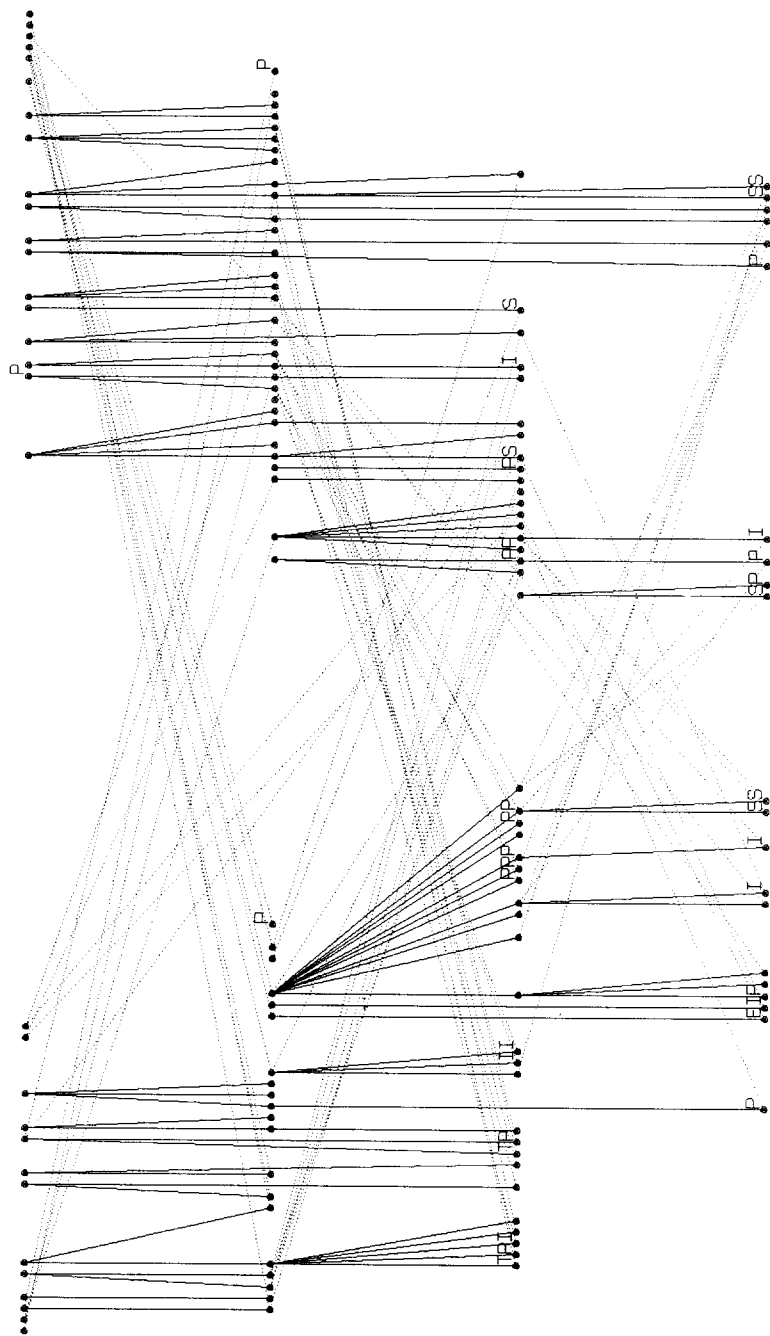marriage types P=promised, I=inherited, S=stolen, T=taken

Fig. 11. Groote Eylandt reduced core.

Table 2

	Males higher Promised		Same / \ Stolen		Females higher Inherited		Totals (%)	
No generation difference	/ 9 \		/ 7 \		/ 7 \		23	66
One generation difference	2	0	2	3	2	0	9	22
Two generations difference	2	0	0	0	0	1	3	11

of the marriages are between three preference-ordered kin-term classes: 'O', 'I' and 'F'. Moreover, Rose's analysis of marriages with respect to kin terms (his Table 14, p. 58) shows a matrilateral bias for 75% of the male marriages: an older brother marries 'O' ('MMBDD') in 33% of the marriages and 'I' (classificatory 'M = MBD') in 42%; while on the sister's or patrilateral side 17% are with 'F' (classificatory 'ZD = FZD').

Marriage arrangements serve also to tie preadolescent brothers of a prospective bride as an initiate to her prospective husband. These boy-initiates, in preadolescence, formerly went to reside with and serve the new husband after he took his child-bride. Thus, marriage preferences also expressed preferences for 'initiators' of young boys. The heterogeneous categories of 'MMBDD' and 'MBD', then, interpreted as a distant matrilateral 'category' marriage preference, also allocate sons (brides' brothers) as initiates to descendants of classificatory (mother's) mother's brothers. Rose's Table 14 shows, from the male initiates' perspective, an opposing matrilateral category bias in 71% of the girl's (rather than boy's) marriages: a person in the older sister category to ego marries 'O' ('MMBDS') in 32% of the cases, and 'I' in 39%; on the sister's or patrilateral side, 18% are with 'F'. Thus, in 71% of the cases, a boy is initiated to someone who is linked to or 'like' a descendant of the 'MMB' or 'MB' categories. From the husband's point of view, the reciprocal of 'O' is 'MFZDD' (likely to be termed reciprocally 'O') and that of 'F' is 'I' – both matrilateral categories (thus; 50% of the marriages). The reciprocal of 'I' is 'F,' a patrilateral category (39% of the marriages).

Nearly all these marriage and initiatory arrangements, and even stealing, giving or exchanging of spouses and/or initiates, exclude traceable consanguineal relationships (six exceptions fall inside the cognatic prohibitions). There are no FZD daughter marriages (out of 159 possible), and only two MBD marriages (out of several hundred possible: the two exceptional MBD marriages are by one man with two sisters). There are two generationally skewed blood marriages, with ZD and a half-ZD. The only other blood marriages are a F half-ZD, M half-BD, and FF half-BDD.

13. Static and dynamic models of Groote Eylandt kinship

Readers familiar with algebraic 'models' of Australian kinship might have anticipated that a marriage network such as for Groote Eylandt would lend itself easily to the kinds of homomorphisms discussed above. Rose, on the other hand,
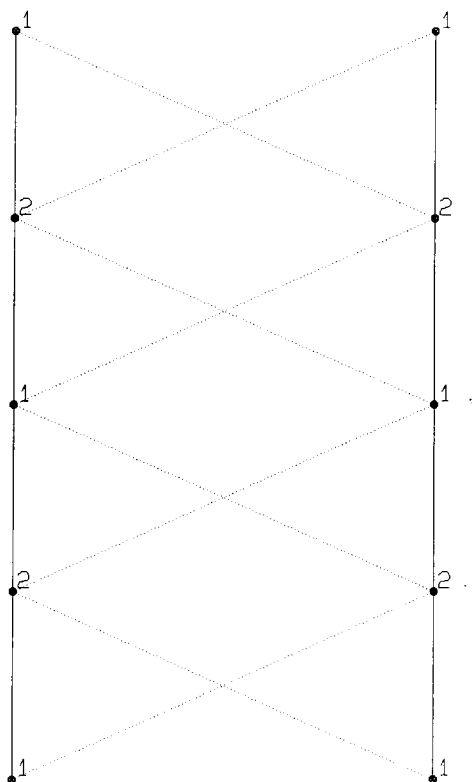
Fig. 12. Kariera-type system.

shows how at every level in the kinship terminology, the term applied to a spouse of someone in a consanguineal marriage class is variable among three or more categories. Those familiar with the actual ethnographies and the variability in Australian marriage practices might have anticipated that an algebraic reduction of Groote Eylandt kinship networks would yield relatively little by way of a structural model of marriage rules, beyond the obvious fact of matrimonial moieties. There are certain limited homomorphic reductions of this network, but nothing that suggests a general structural formula, beyond the dual organization of marriages.

If there were no generational skewing, Fig. 10 minus the skewed marriages could generate Fig. 12 as an order homomorphism that: (1) collapses 'sides' as a merging of individual patrilines into two major lines that preserve uniqueness of G^* and (2) collapses female links between sides at the same generational level, preserving the uniqueness of F^*.

The model in Fig. 12 resembles that of the 'Kariera' system as defined by Radcliffe-Brown (Lévi-Strauss 1969, p. 159), but in fact it conveys only a few of the constraints on marriages, and is not a model of marriage rules such as 'Kariera' at all. Kariera rules are the simplest of a set of four-group marriage class systems, based on classificatory sister exchange and bilateral cross-cousin marriage. The Groote Eylandt, if they had no skewed marriages, could fit this model even though they prohibit first cousin marriages and repeated sister exchanges in adjacent generations. Even so defined, non-prescriptively, the Groote Eylandt show no evidence of fitting such a model. Thirty-three percent of their marriages are generationally skewed, which is quite high by comparative standards, and the same levels of skewness hold up for marriages with or without children, and for various types of marriage. Yet there is no alternative model that fits much better than Fig. 12 (like Denham et al.'s, 1979, Alyawara case, for example): the skewing does not fit a regular pattern. Generational skewing is of a sliding nature, with daughters sometimes given to two adjacent generations in another male line. Furthermore, there is no regular fit between the relative generational standing of dyads in Fig. 11 and the classificatory kinship terms for the dyads as studied exhaustively by Rose.

Instead of looking for a structural pattern, we may look in this case for a dynamic pattern. We have seen that there is a succession of marriage types, from promissory arrangement before a girl is born to a series of other transfers (giving, exchanging, stealing) that may end with intergenerational inheritance by male kin. In a woman's lifespan she is likely to be married to a senior man at the beginning, then men her age, and finally men who are younger. A woman's 'position' as wife is not fixed generationally relative to the husband by a structural rule, such as a classificatory type of kinsman. To be sure, marriages within the 'proper' classificatory kinship category (depending on how defined) range between 33–75%, but only in 1% are there traceable connections of consanguineal kinship.

In one example of marriage 'arrangement' discussed by Rose (1960; p. 71), the couples arranging the exchange were classificatory cross-cousins (a man and woman plus her brother in different moieties). Their relative generational status was ambiguous. Fig. 13, organized by the principle of exchange relations, which gives two female generations for every male generation, shows the principal actors. A brother/sister pair (154 and 155, ages 23 and 30) discuss with his 'O' cross-cousin (77, age 39 'MMBDS', who later becomes a 'MBS/MB' both to him and his sister; wife 133, age 39, their classificatory sibling) the marriage of his son (51, age 2) to the as yet unborn daughter of 155's daughter (13, age 8). The prospective mother (13) of the promised girl has a prior link to the prospective groom, who was her prospective husband's other prospective wife's FBSS (she called his father a classificatory sibling and his mother 'F' or 'FZD = ZD'). These families, then, are already linked by prior exchange relationships. While 13 and 51 are at one level of the same 'generation' and 13 is only slightly older, 51 had called 13 his 'FZ' and she becomes his 'mother-in-law' through the betrothal. Sometimes the promising of daughters is more direct, as in the case where Tamaradaracka gave his daughter 101 to 191, who called Tamaradaracka 'I' ('MBS', but with no traceable relation).
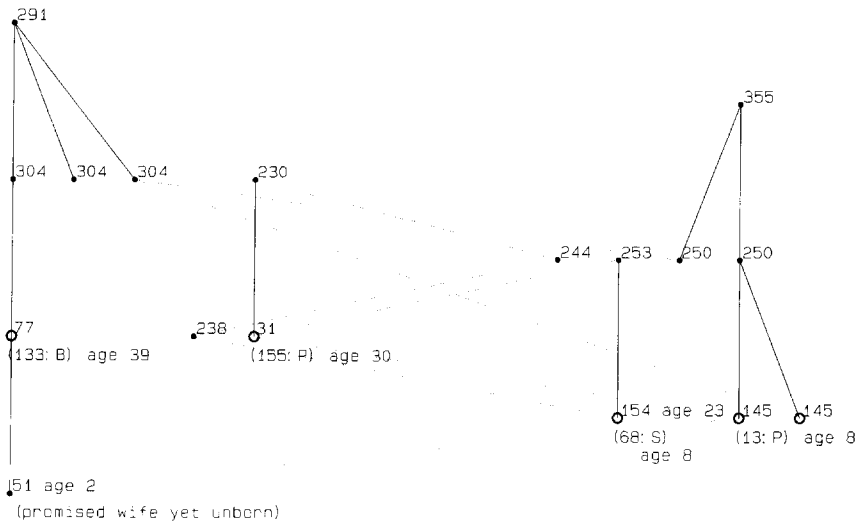
Fig. 13. Groote Eylandt exchange partners.

If we view generation as dynamically sliding, recognizing an asymmetry that makes the wife generationally younger at the start but older with respect to her successive spouses at the end of her lifespan, we begin to glimpse a logic to the Winindiljangwa kinship system. They employ their '*I*' and '*F*' kin terms so as to *equate adjacent generations* ('Omaha' terms), treating as equivalent certain sets of children and grandchildren of senior relatives. This type of rule is often associated with cognatic rules of exogamy whereby one may marry into any number of other clans, but not with co-descendants of certain of ego's senior relatives. The effect of 'Omaha' type skewing, as shown in Table 3, is to raise the generational term for matrilateral cross-cousin (rows 1,2), and reciprocally lower the term for the patrilateral cross-cousin (rows 1',2'):

For the Winindiljangwa, the net effect of these equations is to create additional 'cross-cousins' as potential exchange partners in arranging marriages while simultaneously enlarging the cross-cousin categories (and generational skewing) of those whose marriages in the lower generations are arranged. These arrangements serve the gerontocratic bias of the marriage exchanges in a double capacity. The

Table 3

	Kin	Term	Gloss	Equation
1.	'I'	neba	MBS, MB	[MMSS = MMS]
1'	'F'	nabura	FZS, S (woman spkg.), ZS (man spkg.)	[FFDS = FDS]
2.	'I'	dengda	MBD, M	[MMSD = MMD]
2'	'F'	dabura	FZD, D (woman spkg.), ZD (man spkg.)	[FFDD = FDD]

male-initiatory aspect of marriage arrangements gives not only a bride but her young brother to the initiator. The logic of preferences is to seek both a prospective bride and a prospective initiator in the 'matrilateral' category where the preference value of alliances is strongest.

How can the preferred relation between both the bride-taker/boy-initiator and the bride/boy-initiate (who are siblings) be *reciprocally matrilateral* when matrilateral and patrilateral are opposing categories for genealogical cross-cousins? That is, simple matrilateral cross-cousin marriage cannot satisfy this dual requirement, since one class of the 'MBD/FZS' category is matrilateral, but its reciprocal is patrilateral. Dual matrilateral preference is possible, however, within the 'MMBDD'/'MFZDS' categories. A displacement from classificatory first-to second-cousin allows both marriage preferences and preferences or initiators of boys to be matrilateral. The reciprocally matrilateral preference for classificatory cross-cousins is compatible with a gerontocratic bias where a boy's family prefers an initiator for their son who is a matrilateral class descendant, while the prospective groom and initiator of the bride's young brother prefers to take a bride who is also a reciprocally matrilateral class descendant.

Variability in the prescribed kinship behaviors that differentiate classes of kinsmen is also consistent with a reciprocal matrilateral bias in classificatory cousin marriage and initiation of brides' younger brothers. For the three preferred kinship categories that correspond in practice to 90% of actual spouses or 'initiators' O, F, and I, for both sexes (f,m), Table 4 gives the relevant behavioral distinctions given by Rose (p. 219). The behaviors range along two dimensions: elaboration of avoidance and of joking behaviors. These form distinct Guttman scales that apply across the set of all 15 kin terms, differentiated by sex. From a

Table 4

Kin term	Sex	Avoidance behaviors, CANNOT			
		TOUCH	NAME	SIT	TALK/GIVE
O	f	Y	Y	Y	Y
O	m	Y	Y	Y	N
F	f	Y	Y	?	N
I	f	Y	N	N	N classificatory
I	f	N	N	N	N genealogical relative
F	m	N	N	N	N
I	m	N	N	N	N

Kin term	Sex	Joking Behaviors, CAN			
		JOKE	LOOK	TALK	
O	f	N	N	N	
O	m	N	Y	Y	
I	f	N	Y	Y	classificatory, single
F	f	Y	Y	Y	
I	f	Y	Y	Y	classificatory, married
I	f	Y	Y	Y	genealogical relative
F	m	Y	Y	Y	
I	m	Y	Y	Y	

perspective of the prospective husband, prospective spouses are all touch-avoidance relatives (CANNOT TOUCH/TALK/SIT in Table 4), but only classificatory matrilateral second cousin preferences (O f) are talk-avoidant, while the less sought-after classificatory patrilateral F f cross cousin are joking relations (CAN LOOK/JOKE/NAME). Further, the preferred category of boy-initiates (O m) is a talk-but-not-joke relative, while the less preferred (F m and I m) are joking relatives.

Our analysis, then, suggests that the sliding and flexibility in the relative generations of spouses in the marriage network is well matched by the sliding and flexibility in the 'Omaha' kinship terminology. Generational uniformity in marriage (e.g. as rigidly defined in Fig. 12) is incompatible with the Groote Eylandt gerontocratic bias in the marriage and initiatory arrangements. Generational time is on average a bit faster for females than for males (averaging about seven female generations for every six male generations), but much faster (2 : 1) if only the first marriages of females are considered (Fig. 13). More detailed analysis of the dynamics of actual marriage transactions within the network, their relations to kin term usage, as well as documented examples of how the same dyad will address one another changes over time, might be fruitful for a better understanding of kinship dynamics. About 20% of the pairs of persons whose kin term usages were studied by Rose, for example, actually changed their use of dyadic kin terms over the 12 year period between Rose's fieldwork and that of Peter Worsley. This may reflect changing relationships as people go through successive marriages in their lifetime.

Thus, while we cannot recover a structural formula for marriage 'type' of the Groote Eylandt in terms of genealogical relationships (unless perhaps in dynamic form, with selective kinship erasures following remarriage), our network analysis has demonstrated that a more dynamic analysis is needed, and that static structural formulas are likely to be misleading. [13] As for kinship terms (see Bearman, 1984), it has yet to be determined whether the application of the terminology is dependent

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[13] Bearman's (1984) exemplary blockmodel analysis of a random sample of 76 aborigines used nine of the 16 abstract kin term relationship that accounted for 77% of all kinship ties. His main finding was that age and sex do not induce equivalent patterns of kinship, and equivalent positions in the network of relationships defined by kinship usage were not defined in terms of equivalent genealogical position. We mapped his eight kin-term positions into our network of the genealogical relations, and verified that genealogical relationships were not related in any regular way to the 'classificatory' social positions. His results suggest the possibility of a set of exchange roles that are not based on common descent but cross-cut the lineages, and are transmitted in alternating generations. This remains an intriguing possibility, but beyond the scope of the present paper. Without further analysis of kin-term matrices, we have not been able to determine whether the structural model that he proposes for classificatory social positions do correspond to an Aranda-type kinship structure. What his results clearly do support, however, is the general idea that 'effective' kinship relations among in Groote Eylandt involve a cultural construction that goes far beyond the network structure of ties laid down by strictly genealogical relationships. If his model is correct, it implies a p-graph of shifting 'effective' kinship ties that generate the current pattern of marriage alliances, 'classificatory' merging of individuals as if they belonged to the same 'effective' descent lines, and equivalence of alternating 'effective' generations.

on more remote genealogical connections than we have been able to establish by going back five generations (which seems unlikely), or on more dynamic and shifting class membership criteria that change with successive marriages, as demonstrated by Worsley's data.

The Groote Eylandt have moieties and a 'classificatory' kinship terminology, but lack the marriage classes defining 'elementary structure', where marriage is organized by alliance categories, as a reflex of social structure. Yet 'Omaha' type kinship systems are typically 'semi-complex': a hybrid of prohibitions that are egocentrically defined (like cognatic exogamy) and/or categorically defined, with a statistical distribution of 'positive' choices among multiple categories. This case demonstrates that moieties are complex forms, precipitates of complex historically situated forms that we still know little about. [14]

14. Elementary structures

Elementary structures allow collapse of the order dimension of time implied in Axiom 2.

Definition 11. A homomorphism $h: (P, F, G) \dashrightarrow (P^*, F^*, G^*)$ is *class consistent* if and only if Axiom 1 (unique and gendered parents) still applies to the new marriage classes.

Fig. 10, for example, cannot be reduced to Fig. 12 by a class homomorphism: the Groote Eylandt parental classes are not unique by generation.

In class-reduced (c-)graphs, ancestries may cycle so that points appear ancestral to themselves, violating Axiom 2. This is no longer a p-graph but a *cyclical marriage graph* (Guilbaud, 1970) with certain 'classificatory' marriage-type relations between pairs of marriage classes (Weil, 1949). This is necessarily the case with an elementary kinship structure defined below where we explore the problems of 'classificatory' relations.

Definition 12. A structure $S^* = (P^*, F^*, G^*)$ is an *elementary* (*permutation group*) *kinship structure* if and only if Axiom 1 still applies (unique and gendered parental class), F^*, G^* are permutations (not excluding identity relations) of P^* (group closure), and for all a^*, b^*, c^* in P^*, if $a^*F^*b^*$ and $a^*G^*c^*$ then b^* and c^* are distinct and non-overlapping.

The last condition requires elementary structures to describe rules of exogamy rather than endogamy, since class a^* is endogamous if and only if $a^*F^* = a^*G^*$. The fundamental insight contained in the definition of elementary structure is that for a system of exogamous kinship groups to reproduce itself unchanged over

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[14] See Houseman and White (1996): this observation courtesy of Houseman.

A

Moiety

B

Karlera classes:
two patrilines,
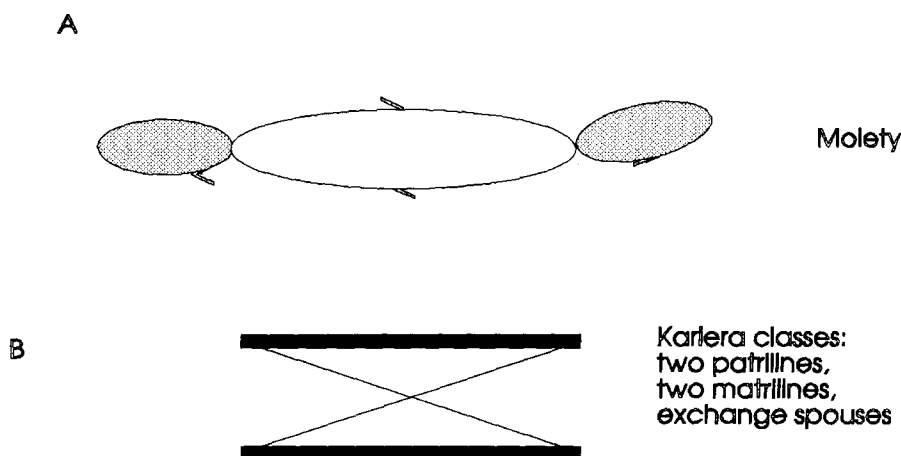two matrilines,
exchange spouses

Fig. 14. Two elementary structure graphs.

generations, every marriage group must be reproduced by couples whose male and female sources from these same groups are unique and distinct. If there are N groups, and parental classes are unique, then each must contribute its females (likewise for males) to a single group, and to reach all N groups these contributions cannot be overlapping, hence the permutation operator. The circulation of males and females in successive generations between the N groups defines two sets of permutation cycles among them.

Permutation cycles in an elementary structure graph correspond to repetition of a sequence of marriage classes in m generations in the G^* or male descent line, and of n generations in the F^* or female descent line. Elementary structures as cyclical marriage graphs represent group-theoretic algebras of marriage-class systems. Fig. 14(a) and (b) shows two elementary structure graphs, and a series of others are found in Tjon Sie Fat (1983, 1990)

For an empirical example, the marriages in Fig. 10 can be reduced to an elementary structure graph, as in Fig. 14(a). Here there are only two points, one for each moiety, both of which are exogamous. The relations of parentage are a permutation in F^* and an identity in G^*, which is equivalent to saying that the moiety is patrilineal.

However, if we deleted the generationally skewed marriages in Fig. 10, this graph could be reduced to a c-graph (Fig. 12) by Definition 11, and further reduced to another elementary structure graph in Fig. 14(b) with four points and two intersecting sets of exogamous sides. Here, each male side is connected bidirectionally by heavy lines, each female side by lighter lines. The relations of parentage are a permutation in F^* and a permutation in G^*.

Definition 13. A homomorphism h: $(P, F, G) \dashrightarrow (P^*, F^*, G^*)$ is *elementary* if and only if Axiom 1 (unique and gendered parents) still applies to the new marriage classes and $S^* = (P^*, F^*, G^*)$ is an *elementary structure*.

15. Elementary homomorphisms: the importance of circuits

The following theorem makes an important point about the interpretation of elementary homomorphisms.

Theorem 4 (*elementary ambiguity*). If a p-graph contains no circuits, it has elementary homomorphisms to any and every elementary structure.

Proof. Every node in an elementary structure is a 'universal' node in having connections to precisely two parents and two offspring, a son and a daughter. A p-graph with no circuits is a tree. One may lay down any point in the tree on any point on the elementary graph, and wrap the rest of the tree onto the elementary graph link by link, according to whether the next link is mother, father, son, or daughter.

Corollary 3. *Only the circuits in a p-graph (hence: its blocks in the core) can be structured so as to contradict an elementary homomorphism.*

Order, class, and elementary homomorphisms are ambiguous with respect to what they tell us about marriage rules. What do Figs. 12 and 14(a), for example, imply about marriage classes? Does a network which has Fig. 12 as its homomorphic model imply sister exchange and bilateral cross-cousin marriage? Do Figs. 14(a) and 14(b) also have these implications? Theorem 4 and its corollary state that while circuits are the important elements of kinship structure, no implications can be read from elementary structure models unless we define our homomorphisms more stringently.

Furthermore, in an empirical kinship network, indefinite repetition of circuits can occur over successive generations without necessarily producing blood marriages:

Theorem 5 (*exchange circuits*). A system of marriage (exchange) circuits between consanguinal kinship segments (however defined) over successive generations need not entail consanguinal marriages.

Proof. An infinite series of marriages between lineages (or other consanguinal kin groups) can be accommodated in successive generations so long as marriages in later generations involve collateral lines from those involved in prior marriages.

Groote Islandt has provided a dramatic demonstration of this principle, whereby consanguineal marriages can be avoided in an empirical kinship network. As a cautionary note about interpreting 'elementary structures', then:

Corollary 4. '*Marriage rules' in an elementary (permutation group) structure appear to 'prescribe' blood marriage in terms of 'classificatory' rules for marriage partners, but none of these partners are necessarily related by consanguinity.*

Proof. follows directly from Theorem 5.

In sum, we need more stringent restrictions on the conditions for homomorphic representations of a marriage network if we want to be sure that the model represents actual marriage practices. To do so we want first to redefine the *circuits* that exist in the empirical graph as types of compound relations.

Definition 14. For a parental structure $S = (P, F, G)$, let T be a circuit-set of non-empty relations generated by composition of **F,G** and their inverses **f,g** such that each element R in T contains at least one of the circuits in S, that is, for each such R there exists a vertex b such that *bRb*.

Definition 15. The homomorphism of a parental structure $S = (P, F, G)$ is said to be *extended* by an induced mapping $h:T \dashrightarrow T^*$ on its circuit set T such that for all b in P and R in T, *bRb* implies bR^*b, where $R^* = hR$. Note that by the definition of a circuit, the composite path of a loop *bRb* cannot repeat any arc or vertex twice.

Definition 16. An extended homomorphism of a parental structure $S = (P, F, G)$ and generated set T is *full* if and only if, for all b^* in P^* and R^* in T^*, $b^*R^*b^*$ implies there exists b in P such that $hb = b^*$ and *bRb*.

The full homomorphism is too strong a criterion for models of marriage rules, since it requires every *loop* in the homomorphic image graph to have a homomorphic mapping from a *circuit* in the original graph. Moreover, attempts to provide workable definitions of homomorphisms that read back from an elementary structure image to the original graph (such as the regular homomorphisms defined by White and Reitz, 1983) are unworkable as criteria for fit.

Definition 17. A minimal circuit in $S = (P, G, F)$ is one that has no proper subset of vertices that form a circuit via a closed path composed of *G,F* or their inverse *g,f* relations.

Definition 18. An (extended) homomorphism of a parent structure $S = (P, G, F)$ is *fit* to an elementary structure $S^* = (P^*, G^*, F^*)$ if and only if S^* *contains an image of all paths and minimal circuits in S as paths and circuits in* S^*.

Theorem 6 ('fit'). A parent structure $S = (P, G, F)$ is an (extended) *fit* homomorphism to an elementary structure $S^* = (P^*, G^*, F^*)$ if and only if S^* *contains an image of all minimal circuits in the blocks of S as circuits in* S^*.

Proof. any path that is not a circuit in S can be mapped onto a corresponding path in S^*; only circuits in S that are not in S^* can violate the conditions of the homomorphism. See theorem 4 for the proof that any path not contained in a

circuit is irrelevant for fit to an elementary structure, and theorem 2 for the containment of all circuits in blocks.

In a fit elementary homomorphism, minimal circuits (composed of distinct vertices and edges) in the original graph imply the existence of identical circuits (again: with distinct vertices and edges) in the image graph. The reverse is not the case, however. For example, the p-graph in Fig. 12 has a fit homomorphism in Fig. 14(b): minimal circuits in 12 (cross-cousin marriages, brother-sister exchange) are found in 14(b). But there are minimal circuits in 14(b) such as the classificatory marriage to ZSD that are not found empirically in Fig. 12. Further, Figs. 12 and 14(b) are homomorphically fit images of Fig. 10 with its generationally skewed marriages removed even though there are FZD marriage circuits in the image graphs that are not found in the original, and the MBD marriage circuits in the image are not empirically symmetric between the moieties in the original graph.

We cannot, in general, read back from elementary structure models to the empirical properties of the kinship networks they are intended to characterize. In losing the order dimension of time, elementary structure models become so over-simplified (e.g. symmetrized) as to lose their utility as empirical models.

16. Homomorphisms to elementary order-structures

Definition 19. An elementary *order-structure* is a p-graph that has an elementary homomorphism, with three classes of elements: unique starters each with a single son and daughter class; unique terminators each with two unique parental classes, and intermediates with a single son and daughter and two unique parental classes.

Fig. 12 is such a structure.

Definition 20. An extended homomorphism $h: S = (P, F, G, T) \dashrightarrow S^*$ is *order-fit* to an elementary order-structure $S^* = (P^*, F^*, G^*, T^*)$ if and only if for all b^* in P^* and R^* in T^*, every *minimal circuit* $b^* R^* b^*$ implies there exists b in P such that $hb = b^*$ and bRb is a minimal circuit in S.

Order-fit elementary homomorphisms are not a 'mechanical model' in which every circuit in a the image implies 100% compliance to a 'marriage rule' in the original graph. They define a loose fit between empirical properties of a kinship network that are carried over to a model, and properties of a model which are realized at least minimally in the empirical network. Neither are they a 'statistical model' in the way they define criteria for minimal fitness. Surely, however, statistical criteria, as well as a host of other evidence, can also be used to evaluate the relevance of a particular model of a marriage network. Perhaps order-fit homomorphisms are best used when they are thought of and further defined as 'smart bombs' which require targeting on what particular properties of the circuits in a marriage network are the important ones that a model ought to capture.

Now, in spite of the relative looseness of 'order-fit' homomorphisms, Fig. 12 is

still not an order-fit image of Fig. 10, even with generationally skewed marriages deleted: the FZD marriages (a minimal circuit on the image graph) are not found at all in the original network, and MBD marriages are not found there symmetrically.

One of the consequences of this more formal line of thinking, then, as in the case of Groote Eylandt, is that *matrimonial moieties that have no implications for marriage types, other than the implied existence of divides plus a descent rule (sexually aligned sides), have no elementary order-structures.* The sides in Fig. 9 (deleting one exceptional marriage), for example, also lack an order-fit homomorphism to an elementary order-structure. Moreover, the pure case of bipartitite 'divides', without a descent rule, imply no necessary fit to an elementary order-structure. Reduction to the divide-sets themselves, for example, does not even provide a class homomorphism since parental classes are not unique. The pure forms of moieties, sidedness and dividedness, then, lack elementary order-structures and may be, in Lévi-Strauss' terms, complex or semi-complex.

17. Post-structural implications

Our attempt to disambiguate Lévi-Strauss' concept of 'elementary structure' leads to his conclusion that elementary (order-) structures are characterized by 'positive' marriage rules, but replaces his concept of 'timeless' algebraic structures of marriage rules with temporally driven, historically contingent 'post-structural' process models. By 'post-structural' here, we simply mean the need for bringing back historical processes into structural analysis, a need that is expressed in various other 'post-structural' approaches.

The graph-theoretic approach to genealogical networks exemplified here is intended to help bring the formal analysis of kinship into closer agreement with some of the current research practices, especially in France, more typical of post-structural approaches (for a current summary, see Godelier and Trautmann, 1996).

First, ambiguities of elementary structural models, and the difficulties of reading back empirical properties of kinship networks from them, suggest that they do not provide adequate criteria for evaluating the fitness of structural models of marriage rules. The ambiguities arise because temporal orderings are lost in these models. Hence, we return to defining elementary structures in terms of ordered relations. The present approach to homomorphic models is more akin to post-structural approaches that reincorporate the temporal dimension to disambiguate the relation between empirical orders and idealized models of concrete historical processes.

Second, the properties of a post-structural type of model, whether in the present instance through the p-graph itself or reduced homomorphic representations of some of its structural properties, are not immanent in a timeless 'structure' but are historically conditioned and may change at any point in time.

18. Conclusion

Conceptually, it is no small achievement to represent kinship networks as graphs, and to replace cumbersome 'genetic graphs' (Ore, 1963) with more succinct parental graphs that have genetic graphs as their dual. Koestler (1964), Tufte (1983, 1990) and others have shown the importance of visualization in the development of scientific specialties; Klovdahl (1981) has argued in particular that social network analysis and its concepts (such as centrality, reachability, role position, clique, flow, etc.) would not have developed as they did without graph theoretic images and measurements.

The concern here is not only with the global structure of kinship networks, but mapping onto these networks real-time processes that are constituted in differentiated fields of social relations. Internal differentiation is of critical relevance to how fundamental processes in the constitution of societies are self-organized and perpetually reorganized. Just as selection operates on variability at the level of frequencies in a gene pool, selective adaptation operates on variability in populations of behaviors.

Because kinship networks refer to the means of reproduction, they are necessarily implicated, along with other principles of recruitment, in constituting the basic social, political, religious and economic units of any society. Consequently, the principles that underlie kinship networks will affect the basic elements of social and economic life in every society. These units and their functions, however, are multiplex and overlapping. It is not sufficient to make a 'statistical model' of one principle of grouping or aggregation, then of another, and expect a mechanical matching between them to emerge as a mapping of social organization. Rather than statistical models, we are interested in representing the raw underlying reproductive network, its general constraints or organizing principles, and the variant ways in which different kinds of social action and grouping are overlaid on this network. Out of this kind of recombinatory social variety there emerge the internal potentials for gradients of learning about different kinds of social adaptations, with actors having differentiated local fields of action that nonetheless fit together into a total pattern. The heterogeneity in this total pattern defines its complexity, its ability to absorb inputs at one time, store or 'memorize' them, and use them to modify outputs at some future time.

is elsewhere developed by Houseman and White (1996). Initial ideas for the graphic representation were provided by Jorion (White and Jorion, 1992). It was his contribution to show how p-graph data are given as vectors for an ordered array of couples. Parental couples are given in one array for males, in another for females. Computations on kinship relations use vector operations wherever possible, which greatly enhances the speed at which they are carried out. For suggesting the possibility of our collaboration we are indebted to Françoise Héritier-Augé. Work by DRW on the p-graph computer program in 1991–92 was supported by the Maison des Sciences de l'Homme, the Maison Suger, and Ministère de la Recherche et de la Technologie, within the framework of an international and interdisciplinary working group on discrete structures in the social sciences created around the support and research facilities of the Maison Suger. Support for programming developments during 1992 was also provided by Alain Degenne's LASMAS research group at IRESCO and the Ministère de la Recherche. The later phases of this research were funded by a 1993–95 NSF award #SBR-9310033 for 'Network Analysis of Kinship, Social Transmission and Exchange'. We are also endebted to Jeff Stern for questions that provoked some of the theoretical formalizations, to Jim Hess, Christina Klein and Ben Jester (members of the 1992 network theory seminar at UCI), to Linton Freeman for help with the laser graphic representation, and to Bill Batchelder, Phil Bonacich, Patricia Skyhorse and James Ferguson for editorial suggestions.

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