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# Modeling the Impact of QoS Pricing on ISP Integrated Services and OTT Services

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**Abstract**— We are concerned with whether a vertically integrated broadband and content provider can unreasonably advantage itself over competing content providers, either by selling quality-of-service (QoS) to content providers at unreasonably high prices, or by refusing to provide access to QoS to competing content. We address this question by modeling the competition between one such vertically integrated provider and one over-the-top (OTT) content provider. We analytically determine when the broadband provider will sell QoS and when the OTT content provider or users will purchase QoS. We characterize the optimal QoS and video service prices. The ISP's market share increases with the difference in the value of the two video services and decreases with the difference in the corresponding costs. Numerical results illustrate the effect of QoS price on content price, and the variation of market share and profit with QoS price. The ISP may sell QoS to users at a lower price than when QoS is sold to the OTT provider.

**Keywords**— QoS, charging, pricing, business models

## I. INTRODUCTION (HEADING 1)

Historically, many broadband providers have offered both packet-switched broadband Internet service and circuit-switched video service to end users. However, most broadband providers are now moving their circuit-switched video service to video-over-IP [1], and are multiplexing this video-over-IP traffic with their broadband Internet traffic. These vertically integrated video services are thus competing with over-the-top (OTT) video services, both for customers and for network capacity.

Broadband providers are deploying quality-of-service (QoS) technologies to improve the performance of video-over-IP, either by reserving network resources for their video traffic (e.g. using IntServ) or by prioritizing their video traffic over other traffic (e.g. using DiffServ) [2]<sup>1</sup>. Broadband providers may have an incentive to sell QoS to OTT video service providers or directly to end users. Alternatively, vertically integrated broadband and video service providers may have an incentive to refuse to provide access to QoS to competing video service providers, if that decision will result in a higher total profit for its broadband and video services.

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<sup>1</sup> In the latter case, we presume that video packets are marked as high priority by the content provider and that the broadband provider respects this marking if either the content provider or the end user has purchased QoS.

One of the questions driving the net neutrality debate has been whether a vertically integrated broadband and content provider can unreasonably advantage itself over competing content providers either by selling QoS to content providers at unreasonably high prices or by refusing to provide access to QoS to competing content.

There is a small academic literature on the impact of QoS pricing on the competition between a vertically integrated broadband provider's content service and OTT content services. One set of papers explores competition from an economics viewpoint. Kocsis and de Bijl [3] apply economic concepts to explore whether bargaining power allows broadband providers to extract surplus from OTT service providers through QoS. However, no mathematical model is proposed. Weisman and Kulick [4] similarly apply economic concepts to explore price discrimination by a broadband provider. They claim that the weight of the economic evidence suggests that both differential pricing and price discrimination by broadband providers toward content providers increases both static and dynamic efficiency, and are thus likely welfare enhancing. However, again no mathematical model is proposed.

Another set of papers focuses on OTT service providers. Baldry et al. [5] argue that the OTT providers are taking advantage of flat rate end user broadband Internet access to provide services that compete with the broadband provider's content services. They believe that that addressing this issue will require regulation, and they propose some regulatory approaches. However, no mathematical analysis is given to validate and solve the competition issues. Nooren et al. [6] use a systematic value chain analysis to investigate how net neutrality interacts with video distribution. However, the focus is not on competition between OTT providers and a broadband provider's integrated services, and no mathematical model is proposed to evaluate the video distribution value chain.

Another set of papers analyzes vertical integration or traffic discrimination from the Internet policy perspective. Jordan [7] provides an analysis of potential discrimination using QoS. He proposes a prohibition on unreasonable discrimination, which would allow charging users for QoS but place limits on charging content providers for QoS. Grunwald [8] analyzes the possibility that ISPs may use QoS or traffic prioritization to discriminate certain services against others, and lead to unfair

competition. Waterman and Choi [9] argue that vertically integrated broadband providers have plausible incentives to favor their affiliated content and to restrict entry of OTT content providers. As with the literature discussed above, none of these papers propose a mathematical model of the competition.

As a complement to these approaches, here we postulate that a simple mathematical model may lend further insight into whether a vertically integrated broadband and content provider can unreasonably advantage itself over competing content providers either by selling QoS to content providers at unreasonably high prices or by refusing to provide access to QoS to competing content. We are interested in when a broadband provider will find it profitable to deploy QoS, and when it does, whether it will choose to sell QoS. If a broadband provider charges content providers for QoS, we are interested in when a content provider will choose to purchase QoS. If a broadband provider charges end users directly for QoS, we are interested in which users will choose to purchase QoS. We are particularly interested in a comparison of these two approaches to charging for QoS.

The mathematical model presented in section II considers competition between one vertically integrated broadband and video service provider (henceforth referred to as the ISP) and one OTT video service provider. The two video service providers offer services that differ both by the amount of content they provide and the type of content (e.g. the amount of movies versus television programs). End users are similarly differentiated by their preference for the type of content. End users decide which service to subscribe to (if any), so as to maximize their surplus. The ISP decides whether to deploy QoS, and sets its video service price and the QoS price (if it deploys QoS) to maximize its profit, defined as the sum of its video service revenue and its QoS revenue (if any) minus the corresponding incremental costs<sup>2</sup>. The OTT content provider sets its video service price to maximize its profit, define as its video service revenue minus the corresponding incremental content cost and the QoS cost (if any).

We both derive analytical results and provide numerical results. For the analytical results, we must make a set of assumptions to simplify the model and the range of parameters considered. In section III, we analyze the duopoly competition game under different QoS prices. The price, market share and profit of the ISP and of the OTT content provider at the Nash equilibrium are derived. In section IV, we present numerical results based on current Internet statistics. In addition to verifying the analytical results, we illustrate when the broadband provider does not sell QoS; the effect of QoS price on content price; the variation of each content provider's market share and profit with QoS price.

## II. MARKET MODELS FOR QoS

In this section, we formulate a mathematical model that considers competition between one vertically integrated broadband and video service provider (denoted ISP/CP1) and

one OTT video service provider (denoted CP2). The two video service providers offer services that differ both by the amount of content they provide and the type of content. A common economic model for horizontal product differentiation is the Hotelling model [10], which we use here. Content provider  $j$  is characterized by a pair of parameters  $(H_j, V_j)$ , where  $H_j \in [0,1]$  characterizes the type of content and  $V_j$  is the maximum amount that any user would pay for the service. User  $i$  is similarly characterized by her preference, denoted  $\theta_i \in [0,1]$ , for the type of content. The distance  $|\theta_i - H_j|$  between the preference of user  $i$  and the type of content provided by CP  $j$  is the basis for the valuation of user  $i$  for the video service provided by CP  $j$ . The utility of user  $i$ , when subscribing to CP  $j$  without QoS, is given by:

$$U_{i,j}^b \triangleq \max(V_j - t_j |\theta_i - H_j|, 0)$$

where  $t_j > 0$  is the marginal penalty on user utility caused by the deviation of CP  $j$ 's content type from user  $i$ 's preference. If user  $i$  is served with QoS, it is common to assume that user utility is increased proportionally, due to better service quality. Thus, the utility of user  $i$ , when subscribing to CP  $j$  with QoS, is given by:

$$U_{i,j}^q = \max((1+r)(V_j - t_j |\theta_i - H_j|), 0)$$

where  $r$  is the increased proportion of user utility due to QoS.

### A. ISP Chooses not to Deploy QoS

We first consider the competition between ISP/CP1 and CP2 when the ISP chooses not to deploy QoS. Denote the price charged by CP  $j$  by  $P_j$ .<sup>3</sup> The surplus of user  $i$ , when subscribing to CP  $j$  without QoS, is thus defined as utility minus the payment to CP  $j$ :

$$S_{i,j} = U_{i,j}^b - P_j = \max(V_j - t |\theta_i - H_j|, 0) - P_j$$

Denote the content provider choice of user  $i$  by  $T_i \in \{0,1,2\}$ , where  $T_i=0$  indicates user  $i$  chooses not to subscribe to any CP.<sup>4</sup> End users decide which service to subscribe to (if any), so as to maximize their surplus:

$$T_i = \arg \max_j S_{i,j} \quad (1)$$

Denote the set of users who subscribe to ISP  $j$  by  $\mathbb{N}_j = \{\text{user } i \mid T_i = j\}$ . Thus the demands for both content providers are functions of the set of prices:  $N_j(P_1, P_2) = |\mathbb{N}_j|$ .

Denote the incremental cost to serve each user who subscribes to CP  $j$  without QoS by  $C_j$ .<sup>5</sup> Denote the profit earned by CP  $j$  by  $\pi_j = N_j(P_j - C_j)$ .<sup>6</sup> Both content providers maximize their profits by controlling their prices:  $\max_{P_j} \pi_j$ .

<sup>2</sup> If an ISP deploys QoS and attempts to sell it to the OTT content provider, then the ISP is presumed to use it for its own video service.

<sup>3</sup> We ignore the user payment to the ISP for Internet access, since it affects neither the competition for video service nor the sale of QoS.

<sup>4</sup> For convenience, we denote  $V_0=0$  and  $P_0=0$ .

<sup>5</sup> The incremental cost includes costs for servers, content, and any network capacity above that already purchased by the user.

<sup>6</sup> The profit does not reflect fixed costs.

The competition between CP1 and CP2 through setting  $P_1$  and  $P_2$  is a continuous game, because  $P_1$  and  $P_2$  can be any positive values. Thus, we focus on the local Nash equilibriums [11] in these continuous games. The local Nash equilibrium derived in the following model analysis may also be a global Nash equilibrium. We will illustrate these equilibriums in more detail in the next section.

### B. QoS Sold to Content Provider

We next consider a market in which the ISP deploys QoS and uses it for its own integrated video service CP1. The ISP may decide to offer QoS to CP2 at a price  $P_q$  per user, or it may decide not to offer QoS to CP2 and to use QoS solely for CP1 (denoted by  $P_q=\infty$ ). If the ISP offers QoS to CP2, then CP2 decides whether or not to purchase QoS for all of its users;  $Q=0$  denotes that CP2 does not purchase QoS and  $Q=1$  denotes CP2 purchases QoS. We will consider the ISPs and CP2's choices in sections III and IV.

User  $i$ 's surplus when subscribing to CP  $j$  is thus given by:

$$S_{i,1} = U_{i,1}^q - P_1, \quad S_{i,2} = \begin{cases} U_{i,2}^b - P_2, & \text{if } Q=0 \\ U_{i,2}^q - P_2, & \text{if } Q=1 \end{cases}$$

As before, end users decide which service to subscribe to (if any), so as to maximize their surplus according to (1). Denote the incremental network cost per user to deploy QoS by  $d$ . In the short term (i.e. given  $P_q$  and  $Q$ ), ISP/CP1 and CP2 compete by setting  $P_1$  and  $P_2$  to maximize their profits  $\pi_1 = N_1(P_1 - C_1 - d) + QN_2(P_q - d)$  and  $\pi_2 = N_2(P_2 - C_2 - QP_q)$ , respectively. The first term in  $\pi_1$  is the profit earned from CP1's users; the second term in  $\pi_1$  is the profit, if any, earned from selling QoS to CP2.

Denote the profit of ISP/CP1 and the profit of CP2 at the local Nash equilibrium  $(P_1^*, P_2^*)$  by  $\pi_1^*$  and  $\pi_2^*$ , respectively.

$\pi_1^*$  and  $\pi_2^*$  are functions of  $P_q$  and  $Q$ . Thus, in the long term (considered in sections III and IV), the ISP sets the QoS price  $P_q$  to maximize profit, and CP2 decides whether or not to buy QoS:  $\max_{P_q} \pi_1^*, \max_Q \pi_2^*$ .

### C. QoS Sold to Users

We finally consider a market in which the ISP deploys QoS and offers it to end users at price  $P_q$ . Subscribers to either content provider can experience the benefit from QoS only if they directly buy QoS in addition to the video service. The surplus of user  $i$ , when subscribing to CP  $j$ , is thus the maxima of the surplus when buying QoS and the surplus when not buying QoS:

$$S_{i,j} = \max(U_{i,j}^b - P_j, U_{i,j}^q - P_j - P_q)$$

In the short term (given  $P_q$ ), as before end users decide which service to subscribe to (if any), so as to maximize their surplus according to (1). Denote the number of users who buy QoS and subscribe to CP  $j$  by  $N_j^q$ . ISP/CP1 and CP2 compete

by setting  $P_1$  and  $P_2$  to maximize profits  $\pi_1 = N_1(P_1 - C_1) + (N_1^q + N_2^q)(P_q - d)$  and  $\pi_2 = N_2(P_2 - C_2)$ , respectively. In the long term, ISP maximizes the profit of ISP/CP1 by controlling  $P_q$ :  $\max_{P_q} \pi_1^*$ .

## III. MARKET SHARES UNDER DIFFERENT QOS PRICES

In this section, we analyze the duopoly competition game under a fixed QoS price  $P_q$ , i.e. competition through the video service prices  $P_1$  and  $P_2$ . The price, market share and profit of the ISP and of the OTT content provider at the Nash equilibrium are derived. For these analytical results, we must make a set of assumptions to simplify the model and the range of parameters considered. Some of these assumptions will be relaxed in the numerical results section below.

**Assumption A:**  $H_1=0, H_2=1, t_1=t_2=t$ . This places CP1 and CP2's content types at the extreme points, and user utility decreases with the same marginal penalty under both content providers, when the content type deviates from a user's preference.

**Assumption B:** User preferences  $\theta$  are uniformly distributed between 0 and 1, i.e.  $f(\theta)=1$ , for  $0 \leq \theta \leq 1$ . It is common in economic models to assume a uniform distribution of user preferences. Since the range  $0 \leq \theta \leq 1$  is arbitrary, the distribution does not by itself matter.

**Assumption C:**  $V_1 + V_2 > 3t + C_1 + C_2$  and  $|(V_1 - V_2) - (C_1 - C_2)| < 3t$ . When QoS is not deployed, this assumption assures that all users subscribe to one content provider and that no content provider can monopolize the market.

**Assumption D:**  $-(3+r)t < [(1+r)V_1 - V_2] - [(C_1 + d) - C_2] < (3+2r)t$

$$(r^2 + 4r + 3)V_1 + (2r + 3)V_2 >$$

and

$$(2r^2 + 9r + 9)t + (3+r)(C_1 + d) + (3+2r)C_2$$

When the ISP uses QoS solely for CP1's subscribers, this assumption assures that all users will subscribe to one content provider and that no content provider can monopolize the market.

**Assumption E:**  $(1+r)(V_1 + V_2) > 3(1+r)t + (C_1 + C_2)$  and

$$|(1+r)(V_1 - V_2) - (C_1 - C_2)| < 3(1+r)t$$

. When QoS is provided by the ISP for free, this assumption assures that all users will subscribe to one content provider and that no content provider can monopolize the market.

**Assumption F:**  $V_1 \geq V_2, C_1 \geq C_2$ . This assumption restricts the range of the parameters considered in the analysis to the case in which the ISP provides a video service that is of higher value to its customers than the OTT's service is to its customers ( $V_1 \geq V_2$ ), and in which its cost per subscriber is correspondingly higher.

**Assumption G:**  $rV_1 \geq d, rV_2 \geq d$ . This assumption assures that if QoS is offered at cost directly to users, there are some subscribers of each content provider that would purchase it.

### A. ISP Chooses not to Deploy QoS

With these assumptions, we return to consideration of competition between ISP/CP1 and CP2 when the ISP chooses not to deploy QoS. In this market, we can show that there is a partition of users between the two content providers, and that CP1's market share increases with the difference between the maximum value of the two services ( $V_1 - V_2$ ) and decreases with the difference between their two costs ( $C_1 - C_2$ ). Denote the user preference threshold  $\theta^{(b)} = 0.5 + [(V_1 - V_2) - (C_1 - C_2)]/6t$ .

**Theorem 1:** Suppose assumptions A-C hold, and QoS is not deployed by the ISP. There exists a global Nash equilibrium such that users with preferences  $0 \leq \theta < \theta^{(b)}$  subscribe to CP1 and users with preferences  $\theta^{(b)} < \theta \leq 1$  subscribe to CP2. ■

The threshold  $\theta^{(b)}$  that determines the market shares of ISP/CP1 and CP2 is determined by  $(V_1 - V_2)/t$  and  $(C_1 - C_2)/t$ . In addition to the aforementioned dependence on the difference between the maximum value of the two services and the difference between their two costs, we also observe that the market share depend on  $t$ , the marginal penalty on user utility caused by the deviation from user  $i$ 's preference. Smaller values of  $t$  result in video subscribers who are more homogeneous in their preferences. When the difference between CP1's and CP's maximum value exceeds the corresponding cost difference (i.e.  $V_1 - V_2 > C_1 - C_2$ ), CP1's market share is greater than 50% and increasing with the heterogeneity of user preferences. Similarly, when  $V_1 - V_2 < C_1 - C_2$ , CP2's market share is greater than 50% and increasing with the heterogeneity of user preferences.

### B. QoS Sold to Content Provider

We next return to the market in which the ISP deploys QoS and uses it for its own integrated video service CP1. We first consider the situation when CP2 does not purchase QoS from the ISP, either because the ISP decides not to sell QoS to CP2 or because the QoS price is high enough that CP2 chooses not to purchase it. We can show that there is again a partition of users between the two content providers, and that CP1's market share increases with the difference between the maximum value of the two services, which is now  $(1+r)V_1 - V_2$ , and decreases with the difference between their two costs, which is now  $C_1 + d - C_2$ . Denote the user preference threshold

$$\theta^{(c0)} = \frac{3+r}{6+3r} + \frac{[(1+r)V_1 - V_2] - [(C_1 + d) - C_2]}{(6+3r)t}.$$

**Theorem 2:** Suppose assumptions A, B, D and G hold, and CP2 does not purchase QoS from the ISP (i.e.  $Q=0$ ). There exists a global Nash equilibrium such that users with preferences  $0 \leq \theta < \theta^{(c0)}$  subscribe to CP1 and users with preferences  $\theta^{(c0)} < \theta \leq 1$  subscribe to CP2. The price of each CP

$$\text{is: } \begin{cases} P_1^{(c0)} = ((3+r)t + [(1+r)V_1 - V_2] + 2(C_1 + d) + C_2)/3 \\ P_2^{(c0)} = ((3+2r)t - [(1+r)V_1 - V_2] + (C_1 + d) + 2C_2)/3 \end{cases} \blacksquare$$

We then consider the situation when CP2 does purchase QoS from the ISP at a price  $P_q$  per user. We can show that there is again a partition of users. However, in some cases not

all users subscribe to a video service. The cases depend on the QoS price. Denote the following QoS price thresholds:

$$P_q^{(c1)} \triangleq ((1+r)(V_1 + V_2) - 3(1+r)t - (C_1 + C_2))/2$$

$$P_q^{(c2)} \triangleq ((1+r)(2V_1 + 4V_2) - 6(1+r)t - 2C_1 - 4C_2)/6$$

$$P_q^{(c3)} \triangleq (1+r)(V_1 + V_2) - (C_1 + d + C_2) - 2(1+r)t$$

$$P_q^{(c4)} \triangleq (1+r)V_2 - C_2$$

Denote the following user preference thresholds:

$$\theta^{(c1)} = 0.5 + ((1+r)(V_1 - V_2) - (C_1 - C_2))/6(1+r)t$$

$$\theta^{(c2)} = 1 - ((1+r)V_2 - (C_2 + P_q))/2(1+r)t$$

$$\theta^{(c3)} = ((1+r)V_1 - (C_1 + d))/2(1+r)t$$

**Theorem 3:** Suppose assumptions A-B and E-G hold, the ISP may sell QoS to content providers, and CP2 purchases QoS from the ISP (i.e.  $Q=1$ ). There exists a global Nash equilibrium as follows:

Case 1: When  $P_q < P_q^{(c1)}$ , users with preferences  $0 \leq \theta < \theta^{(c1)}$  subscribe to CP1 and users with preferences  $\theta^{(c1)} < \theta \leq 1$  subscribe to CP2.

Case 2: When  $P_q^{(c1)} \leq P_q < P_q^{(c2)}$ , users with preferences  $0 \leq \theta < \theta^{(c1)}$  subscribe to CP1 and users with preferences  $\theta^{(c1)} < \theta \leq 1$  subscribe to CP2.

Case 3: When  $P_q^{(c2)} \leq P_q < P_q^{(c3)}$ , users with preferences  $0 \leq \theta < \theta^{(c2)}$  subscribe to CP1 and users with preferences  $\theta^{(c2)} < \theta \leq 1$  subscribe to CP2.

Case 4: When  $P_q^{(c3)} \leq P_q < P_q^{(c4)}$ , users with preferences  $0 \leq \theta < \theta^{(c3)}$  subscribe to CP1 and users with preferences  $\theta^{(c2)} < \theta \leq 1$  subscribe to CP2. ■

Theorem 3 gives the price and market share of ISP/CP1 and of CP2 under different QoS prices. When the QoS price is small (case 1), both CP1 and CP2 pass the entire QoS charge to the end users, and thus their market shares do not depend on the QoS price. Note, however, that in addition to passing on the QoS charge, CP1 extracts an additional portion of its subscribers' surplus due to QoS. When the QoS price is a bit higher (case 2), neither content provider passes the entire QoS charge to the end users, because doing so would reduce their market share and profit. As  $P_q$  increases, although the market shares remain constant, CP2's profit decreases due to the lack of pass-through of the entire QoS charge, and ISP/CP1's profit increases due to the sale of QoS. When the QoS price is moderate (case 3), CP2 can no longer afford to keep its price constant, and it will resume increasing its price with further increases in the QoS price. In response, CP1 can afford to decrease its price, resulting in an increasing CP1 market share and increasing ISP/CP1 profit. When the QoS price is large (case 4), there is now a set of users with moderate preferences that no longer subscribe to either content provider. CP2's profit continues to decrease with increases in the QoS price, but CP1's profit is a unimodal function of the QoS price.

### C. QoS Sold to Users

Finally, we return to the market in which the ISP deploys QoS and offers it to end users at price  $P_q$ . We can show that if the QoS price is low enough, then subscribers with preferences close to the content mix of one of the two content providers, i.e. those who place a high value on the service, will choose to purchase QoS. Denote the following user preference thresholds:

$$\underline{\theta} \triangleq \max\left(\min\left(\frac{(rV_1 - P_q)}{rt}, 1\right), 0\right)$$

$$\bar{\theta} \triangleq \min\left(\max\left(1 - \frac{(rV_2 - P_q)}{rt}, 0\right), 1\right)$$

**Lemma 1:** Suppose assumption A holds. If  $P_q \leq rV_1$ , then only those subscribers to CP1 with preferences  $0 \leq \theta_i \leq \underline{\theta}$  purchase QoS. If  $P_q > rV_1$ , then no CP1 subscribers purchase QoS. If  $P_q \leq rV_2$ , then only those subscribers to CP2 with preferences  $\bar{\theta} \leq \theta_i \leq 1$  purchase QoS. If  $P_q > rV_2$ , then no CP2 subscribers purchase QoS. ■

Given these decisions, we can show that if there exists a Nash equilibrium, then there is again a partition of users between the two content providers:

**Lemma 2:** Suppose assumptions A-E hold, and the ISP may sell QoS to users. If there exists a local Nash equilibrium, then there exists a threshold  $\hat{\theta}$  such that users with preferences  $0 \leq \theta < \hat{\theta}$  subscribe to CP1 and users with preferences  $\hat{\theta} < \theta \leq 1$  subscribe to CP2. ■

However, the existence of a local Nash equilibrium is not guaranteed. Denote the following QoS price thresholds:

$$P_q^{(u1)} \triangleq \max\left(r(V_1 + 5V_2)/6 - rt/2 - r(C_1 - C_2)/6(1+r), 0\right)$$

$$P_q^{(u2)} = r\left((1+r)V_1 + (5+3r)V_2 - (3+2r)t - [(C_1 + d) - C_2]\right)/(6+3r)$$

$$P_q^{(u3)} = r\left((5+2r)V_1 + V_2 - (3+r)t + [(C_1 + d) - C_2]\right)/(6+3r)$$

$$P_q^{(u4)} \triangleq r\left((5V_1 + V_2) + (C_1 - C_2) - 3t\right)/6$$

Denote the following user preference thresholds:

$$\theta^{(u1)} = \left((1+r)(V_1 - V_2) + 3(1+r)t - (C_1 - C_2)\right)/6(1+r)t$$

$$\theta^{(u2)} = \left((1+r)V_1 - V_2 + (3+r)t - [(C_1 + d) - C_2]\right)/(6+3r)t$$

$$\theta^{(u3)} = \left(3t + (V_1 - V_2) - (C_1 - C_2)\right)/6t$$

**Theorem 4:** Suppose assumptions A-G hold and the ISP may sell QoS to users. When  $P_q^{(u1)} \leq P_q \leq P_q^{(u2)}$  or  $P_q^{(u3)} \leq P_q \leq P_q^{(u4)}$ , there is no local Nash equilibrium. When  $0 \leq P_q < P_q^{(u1)}$ ,  $P_q^{(u2)} < P_q < P_q^{(u3)}$ , or  $P_q^{(u4)} < P_q$ , there exists a local Nash equilibrium as follows:

Case 1: When  $0 \leq P_q < P_q^{(u1)}$ , users with preferences  $0 \leq \theta \leq \theta^{(u1)}$  subscribe to CP1 and users with preferences  $\theta^{(u1)} \leq \theta \leq 1$  subscribe to CP2. All users purchase QoS.

Case 2: When  $P_q^{(u2)} < P_q < P_q^{(u3)}$ , users with preferences  $0 \leq \theta < \theta^{(u2)}$  subscribe to CP1 and users with preferences  $\theta^{(u2)} \leq \theta \leq 1$  subscribe to CP2. All CP1 subscribers purchase QoS; only CP2 subscribers with preferences  $\bar{\theta} \leq \theta_i \leq 1$  purchase QoS.

Case 3: When  $P_q^{(u4)} < P_q$ , users with preferences  $0 \leq \theta \leq \theta^{(u3)}$  subscribe to CP1 and users with preferences  $\theta^{(u3)} \leq \theta \leq 1$  subscribe to CP2. Only CP1 subscribers with preferences  $0 \leq \theta_i \leq \underline{\theta}$  and CP2 subscribers with preferences  $\bar{\theta} \leq \theta_i \leq 1$  purchase QoS. ■

Theorem 4 gives the market share of ISP/CP1 and of CP2 under different QoS prices, when QoS is sold directly to users. It also shows how many users purchase QoS. When the QoS price is small (case 1), all users buy QoS. Thus, we see similar behaviors as when QoS is sold to the content providers (Theorem 3, case 1). When the QoS price is moderate (case 2), while all CP1 subscribers continue to purchase QoS, some CP2 subscribers (those with lower valuations on CP2's video service) choose to no longer purchase QoS. The marginal users on which the content providers compete are thus indifferent between CP1 with QoS and CP2 without QoS. We see similar behaviors as when QoS is used solely for CP1 (Theorem 2), except that ISP/CP1 earns an additional profit from the sale of QoS to CP2's users. When the QoS price is high (case 3), there are some subscribers to both CP1 and CP2 who do not value the service enough to purchase QoS at this price. The marginal users on which the content providers compete are thus indifferent between CP1 without QoS and CP2 without QoS (Theorem 1). We see similar behavior as when ISP chooses not to deploy QoS, except that ISP/CP1 earns an additional profit from the sale of QoS to CP1's users and CP2's users.

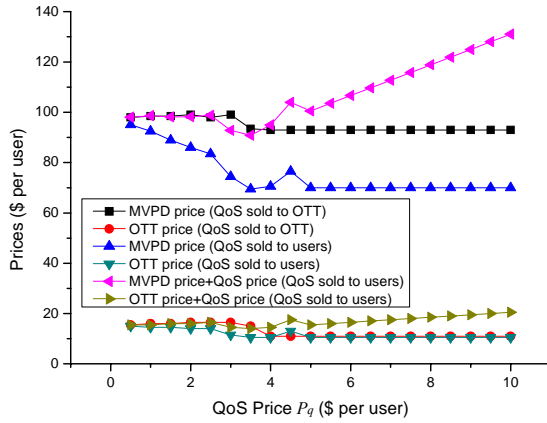
## IV. NUMERICAL RESULTS

In this section, we numerically evaluate the competition between a vertically integrated cable broadband provider (ISP) that also offers a Multichannel Video Programming Distributor (MVPD) service (CP1) and an OTT provider (CP2) that offers a competing video service. The average prices charged by MVPD providers are approximately  $P_1 = \$64/\text{month}$  and by OTT providers approximately  $P_2 = \$10/\text{month}$  [12]. The average profit margins over the past five years for DirecTV (a MVPD provider) and Netflix (an OTT provider) are 9.41% and 4.22%, respectively [13][14], which gives  $C_1 = P_1(1 - 0.0941) \approx \$58/\text{month}$  and  $C_2 = P_2(1 - 0.0422) \approx \$9.5/\text{month}$ . We set QoS cost per user for the MVPD provider  $d_1 = \gamma C_1$  and for the OTT provider  $d_2 = \gamma C_2$ , where  $\gamma = 0.3$ . Among 118 million United States households, 60.6 million subscribe only to MVPD service, 6.4 million subscribe only to OTT service, and 40.3 million subscribe to both [12][15], i.e. 51.4% of households have  $(S_{i,1} > 0, S_{i,2} \leq 0)$ , 5.4% have  $(S_{i,1} \leq 0, S_{i,2} > 0)$  and 34.2% have  $(S_{i,1} > 0, S_{i,2} > 0)$ . We set  $H_1, H_2, t_1, t_2, V_1$  and  $V_2$  to fit these user subscription choices at the competition equilibrium without QoS; this gives  $V_1 = 92.4, V_2 = 16.5, t_1 = 55, t_2 = 34$  (\$ per month) and  $H_1 = 0.44, H_2 = 0.71$ . The increased proportion of user utility is  $r = 0.3$ . Each user can choose either to subscribe to a single content provider or can choose not to subscribe to either, so as to maximize their

surplus  $S_{ij}$ . When QoS is sold to the OTT provider, the OTT provider can choose to buy QoS for all its users at price  $P_q$  (in \$ per user), or can choose not to buy QoS. When QoS is sold to the users, subscribers to the MVPD service have the option to buy QoS at price  $P_q d_1/d_2$ , and subscribers to the OTT service provider have the option to buy QoS at price  $P_q$ .

Fig. 1 shows the variation with the QoS price of the MVPD price, the OTT price and the total user payment for video service and QoS (if purchased). When QoS is sold to the OTT provider at a relatively low price ( $P_q < \$3.5$ ), the OTT price increases slightly with the QoS price, passing on a part of the QoS charge to the end users. In contrast, the cable provider does not have much incentive to change its MVPD price, which is already optimized to maximize the joint profit of its broadband and video businesses. When QoS is sold to the OTT provider at a moderate to high price ( $P_q \geq \$4$ ), the OTT provider chooses not to buy QoS from the ISP, and some users drop it due to its lower service quality. The MVPD continues to deploy and use QoS, and it decreases its MVPD price so as to gain a larger market share. The cable provider thus earns 71% market share at a slightly lower MVPD price, while the OTT provider is only left with 10% market share.

Fig. 1. MVPD price, OTT price, and total user payment with QoS subscription vs. QoS price.

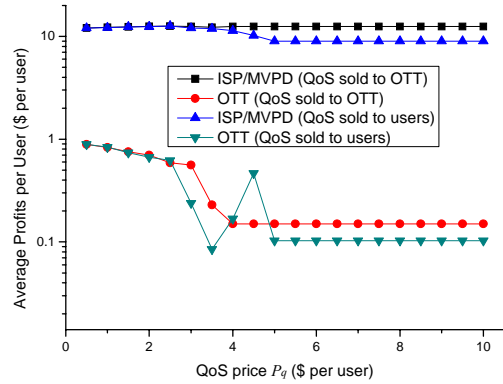


When QoS is sold to end users at a relatively low price ( $P_q < \$2.5$ ), all video service subscribers buy QoS. The cable provider decreases its MVPD price proportionally with the QoS price, so that the total price  $P_1 + P_q d_1/d_2$  to its MVPD subscribers remains constant. In response, the OTT decreases its price slightly, but not be enough to hold the total price  $P_2 + P_q$  to its subscribers constant. As a result, a few users switch from the OTT to the MVPD. When QoS is sold to end users at a moderate price ( $\$2.5 \leq P_q < \$3.5$ ), the cable provider seeks to further dominate the market by reducing its MVPD price, so that the total price paid by its MVPD subscribers who purchase QoS falls. Although the OTT provider also reduces its price in response, it still loses market share from 14% to 9% while the MVPD market share increases from 54% to 78%. When QoS is sold to end users at a moderate to high price ( $\$3.5 \leq P_q < \$4$ ), some video subscribers stop buying QoS. In this environment, the cable provider has less advantage from selling QoS, and it ends the price war, which leads to both a

higher MVPD price and a higher OTT price. If QoS is sold to end users at a high price ( $P_q \geq \$4$ ), no users buy QoS, which is the same as the case when the ISP does not deploy QoS.

Fig. 2 shows the variation with the QoS price of the profits of the cable provider and the OTT provider. When the QoS price is relatively small ( $P_q \leq \$2.5$ ), regardless of whom it sells QoS to, the cable provider can always earn more profit by increasing the QoS price. In contrast, the OTT provider earns less profit because it is losing market share and may slightly reduce its OTT price. When QoS is sold to the OTT provider, the cable provider earns the maximum profit by pricing QoS high enough ( $P_q \geq \$4$ ) so that the OTT provider does not buy it. However, this maximum profit is very close to profit earned when  $P_q = \$3.5$  (the maximum QoS price that the OTT provider can accept). Thus, the cable provider may either use QoS solely for its MVPD service or sell QoS to the OTT provider at a high price. In contrast, the OTT provider always earns the minimum profit when  $P_q$  is large. When QoS is sold to users, the cable provider starts a price war when  $\$2.5 \leq P_q < \$3.5$ , which leads to lower profits for both providers. However, when  $P_q \geq \$3.5$ , some light users do not buy QoS. The profit of the cable provider decreases due to less advantage from selling QoS. When  $P_q \geq \$5$ , the QoS price is too expensive for all users. Both providers earn less profit. The cable provider's profit is maximized when  $P_q = \$2.5$ , which is smaller than the optimal QoS price ( $P_q \geq \$4$ ) if QoS is sold to the OTT provider.

Fig. 2. The profit of cable ISP with integrated MVPD and the profit of OTT provider vs. QoS price.



## V. CONCLUSION

We modeled the impact of different QoS pricing strategies on the competition between an ISPs' integrated services and an OTT content provider. We analytically determined when the ISP will sell QoS and when the OTT provider or users will purchase QoS. When QoS is sold to users, only those users who place a high value on the video service purchase QoS. We characterized the optimal QoS and video service prices. We proved that the ISP's market share increases with the difference in the value of the two video services and decreases with the difference in the corresponding costs. Numerical results show the ISP may sell QoS to users at a lower price than when QoS is sold to the OTT provider.

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