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Phase diagram of the integer quantum Hall effect in p-type Germanium

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We experimentally study the phase diagram of the integer quantized Hall effect, as a function of density and magnetic field. We used a two dimensional hole system confined in a Ge/SiGe quantum well, where all energy levels are resolved, because the Zeeman splitting is comparable to the cyclotron energy. At low fields and close to the quantum Hall liquid-to-insulator transition, we observe the floating up of the lowest energy level, but *no floating* of any higher levels, rather a merging of these levels into the insulating state. For a given filling factor, only direct transitions between the insulating phase and higher quantum Hall liquids are observed as a function of density. Finally, we observe a peak in the critical resistivity around filling factor one.

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Two dimensional electronic systems, subject to a perpendicular magnetic field exhibit a large variety of phenomena. The most interesting one, is the quantization of the Hall resistivity, ρ_{xy}, in terms of the quantum unit of resistance h/e^2. In a pioneering work, Kivelson, Lee and Zhang (KLZ) [1], proposed a theoretical global phase diagram (GPD) of the quantum Hall effect expressed in terms of "laws of corresponding states". Their phase diagram is composed of two different types of stable zero temperature (T) phases, the quantum Hall liquid phases and the insulating phase. In a quantum Hall liquid phase, the diagonal resistivity, $\rho_{xx} = 0$, and the Hall resistivity $\rho_{xy} = h/\nu e^2$. (We will only consider the case in which the filling factor ν is an integer.) In the insulating phase ρ_{xx} diverges at zero T. The different phases in KLZ's GPD are a function of disorder strength and magnetic field (B). At high enough disorder only the insulating phase is present at finite B. At lower disorder, when different quantum Hall liquids exist, only transitions between the $\nu = 1$ quantum Hall liquid and the insulating phase are allowed. This means, for example, that the $\nu = 3$ state has to first undergo a transition to $\nu = 2$ and $\nu = 1$ before reaching the insulating phase. This rule relies on the "floating up" of the extended states at vanishing B, first discussed by Khmel'nitzkii and Laughlin [2]. The "floating" argument was later questioned by recent experiments, in which direct transitions between $\nu \geq 3$ states and the insulating phase have been observed [3,4,5].

During the past years, there has been a number of theoretical and experimental papers focusing on the existence of the "floating up" of extended states. Some experiments in n-type GaAs [6] and in Si-MOSFET [7,8] are consistent with the "floating up" scenario and other experiments in SI-MOSFET [3] and in p-type GaAs are

inconsistent with it. Theoretically, this issue is also under debate [9,10]. To analyze this aspect further, we present new results for a different system, i.e., a two dimensional hole system (2DHS) confined in a Ge/SiGe quantum well [11], in which the lowest energy level "floats up" but **no** higher energy level.

Addressing this issue experimentally, involves extracting the zero T physics. A phase is insulating, when ρ_{xx} diverges as $T \to 0$ as opposed to the quantum Hall liquid phases, where $\rho_{xy} \to h/\nu e^2$ and $\rho_{xx} \to 0$ with vanishing T. Therefore, the transition can be defined as the B-field, where the T-dependence of ρ_{xx} changes direction [12,13]. This method was later successfully used by several groups to determine parts of the phase diagram [4,5,11,12,13,14]. We will follow the same method and discuss the differences compared to other methods in connection with our data.

In order to illustrate our procedure, we have plotted in fig. 1a, ρ_{xx} and ρ_{xy} as a function of B for different T's, ranging from 0.4 K to 4.2 K. The insulating-to-quantum Hall liquid transition at B_C^L can easily be identified from the crossing point of the ρ_{xx} curves at different T's. With increasing B, ρ_{xy} first develops a plateau at $\nu = 3$, then at $\nu = 2$ and finally at $\nu = 1$. This is an example of a direct transition between the $\nu = 3$ state and the insulating phase at $B < B_C^L$. At higher fields, the quantum Hall liquid $\nu = 1$ state-to-insulator is shown in fig. 1b. Here again the transition at B_C is obtained from the crossing point of the ρ_{xx} traces. In both cases (fig. 1a and fig. 1b) ρ_{xy} shows no features across the transition. In the case of B_C, ρ_{xy} even remains quantized as was noted previously [15,16]. In the inset of fig. 1b, we observe that the transition does not occur at the maximum of the diagonal conductivity, σ_{xx}. On the other hand, at B_C and at B_C^L, the Hall conductivity, σ_{xy}, is T-independent. One

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could, therefore, alternatively use the T-dependence of σ_{xy} to define the transition point [17], but this method has the disadvantage, that σ_{xy} is not measured directly, because the conductivities are obtained by inverting the resistivity tensor.
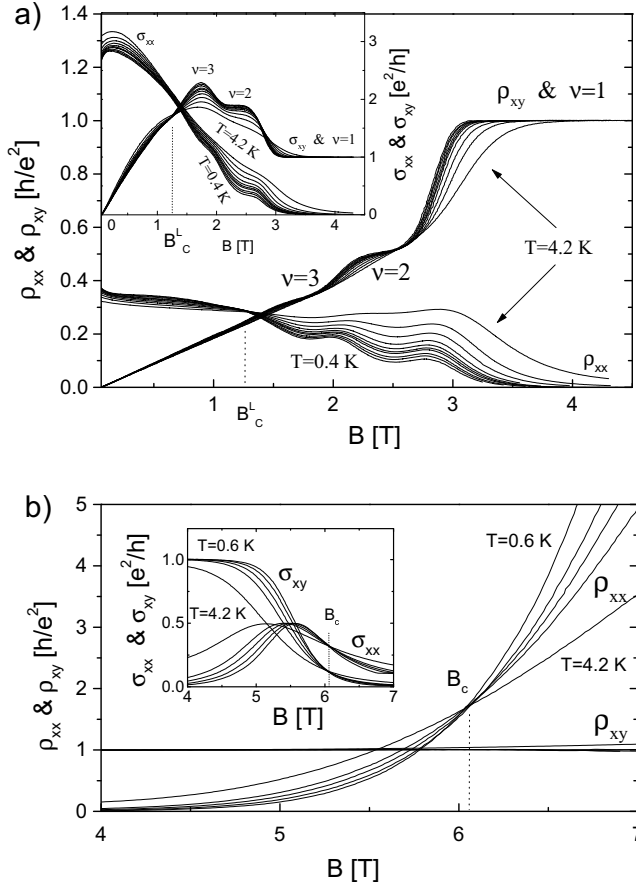

FIG. 1. Magnetic field dependence of ρ_{xx} and ρ_{xy} for temperatures ranging between 0.6 K and 4.2 K. In the insets, the conductivities obtained from ρ_{xx} and ρ_{xy} are plotted as a function of the magnetic field. B_C and B_C^L are the critical magnetic fields for which ρ_{xx} is temperature independent ($B_C^L = 1.3T$ and $B_C = 6.1T$). Fig. a) and b) were obtained from different cool-downs and densities ($n = 1.2 \times 10^{11} cm^{-2}$ and $n = 0.8 \times 10^{11} cm^{-2}$).

When extracting the transition from the crossing point we imply that only two types of states exist in our system: a quantum Hall and an insulating state. The crossing point, which defines a T-independent point, can be reliably taken as the transition point. However, in recent experiments a zero-field metallic-like state in a 2D system was observed [18]. In these systems, for high enough densities and at $B = 0$ the resistance decreases and saturates with decreasing T, which invalidates the use of the crossing point to extract the boundary. Therefore,

alternatively the peak in the conductivity was used to determine the transition [3,19]. The question of the existence of only an insulating state at $B = 0$ is even more fundamental. In our Ge/SiGe system the insulating behavior is well defined in the low density limit with an increase of ρ_{xx} by more than an order of magnitude when T is lowered from 1K to 0.3 K. This is in contrast to our highest density, where the increase in ρ_{xx} is only a few percent for the same T span. Are these two behaviors characteristics of the same insulator? If yes, our procedure is well defined. On the contrary, if these two regions belong to different zero-T phases one could expect signatures at our experimental T. However, we do not observe any qualitative change of the T-dependence in our entire density range, which indicates a transition or cross-over to a metallic-like state at $B = 0$. This is correlated with the existence of a well defined crossing point as a function of B. On the quantum Hall side we have the reverse T-dependence with a similar magnitude close to the transition.

The next step is to obtain the entire phase diagram, by varying the density and B. In most previous experimental phase diagrams the degeneracy of the Landau levels could not be resolved down to the insulating phase. Typically the Zeeman splitting was much smaller than the cyclotron gap [20]. In holes there is an additional degeneracy of the valence band, which leads to a two band system, which affects the phase diagram [3,19]. We avoid these difficulties because our 2DHS is confined in a strained Ge layer, where the strain removes the degeneracy of the valence band. The 150 Å thick Ge layer is sandwiched in between $Si_{0.4}Ge_{0.6}$ layers, where Boron modulation doping is placed. The crucial property of this system is that the Zeeman activation energy is only 30 % smaller than the cyclotron activation energy. As a result, the Zeeman energy is resolved down to our lowest density so that we observe all even and odd filling factors. To vary the density we use a top gate with an insulating silicon oxynitride layer grown between the cap layer and the gate. By applying a gate voltage between 0 V and 6 V we could vary the density between $n = 0.5 - 5 \times 10^{11} cm^{-2}$ with no leakage. The highest mobility we obtained was 20×10^3 cm$^2/Vs$ at zero gate voltage. The measurements were performed in a dilution refrigerator and a He$_3$ system at T's ranging between 40 mK and 4.2 K, using standard AC lock-in technique with an excitation current of 0.1 nA. DC measurements were performed in order to check for consistency.

We determine the transition between two quantum Hall liquids from the local Max$\{d\rho_{xy}/dB\}$ [21] at 40 mK. This point is T-independent at low T and the size of the dots are representative of the differences obtained from the 200mK and 400mK curves. However, other choices for determining the transition are possible, such as Max$\{\rho_{xx}\}$ or Max$\{d\sigma_{xy}/dB\}$ [22]. When using these other methods we found that they give similar results.

The insulator-to-quantum Hall transitions (solid dots) were extracted from the crossing points of ρ_{xx} at three different T's (40mK, 200mK and 400mK), which also gives an estimate of the experimental inaccuracy represented by the size of the dots. The results are summarized in fig. 2.
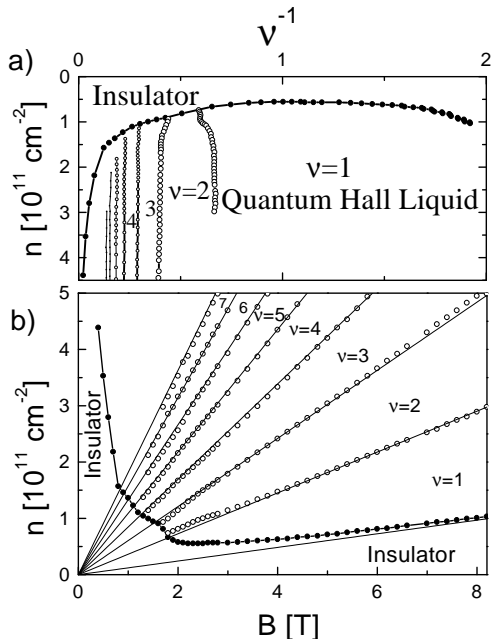

FIG. 2. a) the phase diagram in terms of the inverse of the filling factor. Solid dots are insulator-to-quantum Hall transitions and open dots quantum Hall-to-quantum Hall transitions. b) the Landau level fan.

At a fixed ν and with decreasing density, there are direct transitions between the quantum Hall phases $\nu = 1, 2, 3, ...$, and the insulating phase. This is the main result of fig. 2a and is in strong contrast to KLZ's GPD, where the insulating phase is only bordered by the $\nu = 1$ quantum Hall liquid. These results are very similar to those obtained in Si MOSFET [3], where a merging of the transition states into the insulating phase and an absence of floating was observed. However, in Si MOSFET each Landau level is doubly degenerate due to the additional valley band degeneracy. Therefore, the $\nu = 5$ state corresponds to the second Landau level. Other results with Si MOSFET (using an arbitrary cut off value of the conductivity to determine the transition) were interpreted as consistent with the floating up scenario [7,8] like in n-type GaAs/AlGaAs [6]. p-type GaAs/AlGaAs has a similar valley band degeneracy, but no floating was observed and mainly even filling factors were resolved [19]. An interesting result was obtained by Lee *et al.* [5], who used a multiple GaAs quantum well and obtained similar transitions for high even filling factors. As an alternative method to determine the transition, the disappearance of the activated behavior was used in ref. [23]. Transitions up to $\nu = 3$ were observed in a similar Ge/SiGe system by Song *et al.* [4].

To analyze the termination of the Landau levels as a function of B, we plotted in fig. 2b the same data as in fig. 2a, but as a function of B. At high B, the experimental points (open and solid dots) follow very precisely the $n = (i + \frac{1}{2})\frac{eB}{h}$ lines, where $i = 0, 1, 2, ...$ The insulating phase below the lowest energy level ($i = 0$) can be understood in terms of simple energy level physics. The energy levels are broadened by disorder, which leads to a smooth density of states around them and to the formation of mini-bands. The states at the center of the mini-bands are extended, but the states with energies away from the band center are localized [24]. In this way, at $T = 0$ and when the Fermi energy (F_F) is below the lowest energy level, E_F is pinned to the localized states and the system becomes insulating. The deviation from the linear dependence ($i = 0$), when B is decreased, is due to the disorder broadening. Lowering B decreases the gaps and leads to an overlap of the energy levels. Therefore, some states from higher energy levels increase the density of states below the lowest energy level ($i = 0$), which leads to the apparent "floating up" of the lowest energy level, when E_F is pinned to the $i = 0$ level. When assuming a symmetric broadening, the transition occurs at $\nu = 1/2 + \nu_h$. In this case, ν_h is simply the density of states of the higher energy levels (normalized to one energy level) integrated between zero and the $i = 0$ level. Hence, ν_h is a measure of the overlap between the density of states of higher energy levels and the lowest energy level. Applying the same argument to higher energy levels does not hold, because the contribution of states from even higher levels is compensated by the states lost from lower levels. Therefore, broadening cannot affect the higher energy levels and, indeed, we observe **no floating** of higher energy levels. These higher energy levels simply merge straight into the insulating phase.

Before discussing our results in relation to existing theoretical ones, we need to make an important point. Most theories, starting with KLZ's GPD, are a function of the B-field strength and the disorder strength. At first sight our system is very different, as we replaced the disorder axis by the density. One could argue that, when the density is reduced, screening becomes smaller. However, in two dimensions and at $B = 0$ the screening is largely density independent. The situation is slightly different with a quantizing B-field because the screening becomes nonlinear [25]. In addition, screening is also T-dependent. On the other hand, it is the ratio of the disorder potential fluctuation over the quantizing energy, which can be effectively tuned by the density. Therefore, our results suggest that at strong level mixing (which can be estimated by ν_h), the extended states at the center of each energy level disappear and localization takes over. This observation is in agreement with recent theoretical and numerical calculations. Fogler, for instance, showed that

the levitation of extended states remains very weak even at low B [9]. Using tight binding models, Liu *et al.* [26] and more recently Sheng and Weng [27,28] showed evidence for the disappearance of extended states without floating. Inspired by recent experimental results, a numerical phase diagram based on the tight binding model has been obtained [29]. The similarity between their GPD and ours (fig. 2) is very striking.
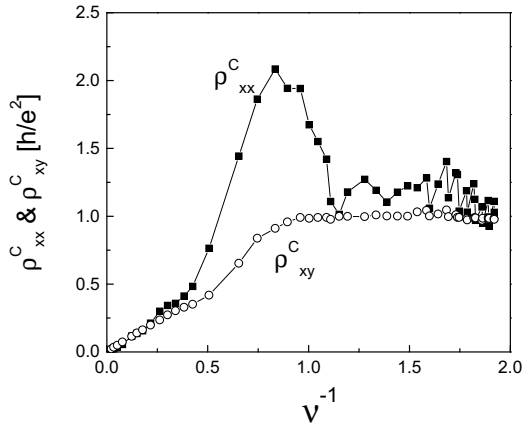

FIG. 3. Inverse filling factor dependence of ρ_{xx}^C and ρ_{xy}^C.

We now turn to the analysis of the resistivities at the transition. In fig. 3, we have plotted the critical resistivities, ρ_{xx}^C and ρ_{xy}^C defined as the value of the resistivity at the quantum Hall liquid-to-insulator transition. This work was inspired by a recent letter by Song *et al.* [4], who obtained a similar curve for $\nu > 1$. Our striking new result is the peak of ρ_{xx}^C close to $\nu = 1$ before ρ_{xx}^C saturates around the quantized value h/e^2. In addition, ρ_{xy}^C follows the classical expression for $\nu > 1$ indicative of a Hall insulator, which becomes closely quantized for $\nu < 1$, bordering the quantized Hall insulator. A similar peak in ρ_{xx}^C was obtained very recently by Hanein *et al.* [30] in systems exhibiting a zero B metal-insulator transition. But in their case ρ_{xx}^C tends to h/e^2, with vanishing B, as opposed to zero in our case. The physical origin of this peak is not understood.

Summarizing, we have experimentally mapped out the phase diagram of the integer quantum Hall effect, as a function of density and magnetic field. At low fields and close to the quantum Hall liquid-to-insulator transition, we have observed the floating up of the lowest energy level, but no floating of any higher levels, rather a merging of these levels into the insulating state. These results are consistent with the disappearance of extended states without levitation due to the broadening of the energy levels. Along the transition, we observe a peak in the critical resistivity around filling factor one.

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