Hiding the cosmological constant

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Abstract

Perhaps standard effective field theory arguments are right, and vacuum fluctuations really *do* generate a huge cosmological constant. I show that if one does not assume homogeneity and an arrow of time at the Planck scale, a very large class of general relativistic initial data exhibit expansions, shears, and curvatures that are enormous at small scales, but quickly average to zero macroscopically. Subsequent evolution is more complex, but I argue that quantum fluctuations may preserve these properties. The resulting picture is a version of Wheeler's "spacetime foam," in which the cosmological constant produces high curvature at the Planck scale but is nearly invisible at observable scales.

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1. The cosmological constant problem

Quantum fluctuations of the vacuum are expected to generate a very high energy density, which should manifest itself as an enormous cosmological constant. We don't know how to calculate this quantity exactly, and it remains possible that it is suppressed, exponentially [1] or otherwise [2], but standard effective field theory arguments predict a value $\Lambda \sim \pm 1/\ell^2$, where the cut-off length ℓ is usually taken to be the Planck length ℓ_P [3,4]. The sign of Λ depends on the exact particle content of the Universe—bosons and fermions contribute with opposite signs—but unless a remarkable cancellation occurs, the predicted value is huge.

We do, in fact, observe an accelerated expansion of the Universe that could be due to a cosmological constant. But a Planck-scale cosmological constant is some 120 orders of magnitude too large, making it what has been called "the worst theoretical prediction in the history of physics" [5]. It is generally assumed that Λ must either be canceled by incredibly precise fine tuning or eliminated by some other form of special pleading—anthropic selection [6], nonlocal modifications of the gravitational action [7], or the like. The problem is made especially intractable by the mixing of scales: Λ is generated near the Planck scale, but observed at cosmological scales.

Here I propose a simple but radical alternative. Perhaps our Universe really *does* have a cosmological constant of order $1/\ell_P^2$. If one assumes homogeneity, this is, of course, immediately ruled out by observation. But if Λ is generated by Planck scale quantum fluctuations, there is no reason to expect homogeneity at that scale. This notion was anticipated by Wheeler [8], who called the resulting picture "spacetime foam."

For a Riemannian space—one with a positive definite metric—it is easy to imagine high curvature at small scales that averages to zero macroscopically. For a spacetime, though, one might worry that a cosmological constant entails exponential expansion (at least if the anisotropy is not too big [9]), and it is not obvious how such behavior can be averaged away. But general relativity is time-reversal invariant; for every expanding solution there is a corresponding contracting solution, and these may indeed compensate.

In what follows, I make this idea more concrete. Starting with the initial value formulation of general relativity with a large cosmological constant, I show that a very large class of initial data has a local Hubble constant that is huge at the Planck scale but tiny macroscopically. For an infinite subset of data, the macroscopic spatial curvature is also very small, and has a vanishing first time derivative. A "macroscopic" region here need not be very large: a cubic centimeter contains some 10^{100} Planck-size regions.

An initial value formulation is not enough, of course; one must also show that these features are preserved dynamically. Higher order time derivatives depend on finer details, and are harder to analyze. If the initial inhomogeneities are generated by quantum fluctuations, though, I argue that these fluctuations should also preserve the crucial properties that camouflage the cosmological constant.

These arguments do not provide a complete answer to the cosmological constant problem. They do not, for example, explain the apparent existence of a very small Λ at macroscopic scales. More generally, one would have to show that long wavelength excitations on this foamlike background obey a macroscopic version of the Einstein field equations, a form of the much-studied but unresolved "averaging problem" [10]. But the results here suggest, at least, that we may have been looking for answers at the wrong scales.

2. The initial value formulation

Let Σ be a compact three-dimensional manifold, interpreted as a Cauchy surface of a space-time. The initial data for general relativity on Σ consist of a spatial metric g_{ij} and an extrinsic curvature $K^i{}_j$. These are not arbitrary, but must satisfy a set of constraints. If the contribution of matter is negligible compared to the cosmological constant, these are

$$R + K^2 - K^i{}_j K^j{}_i - 2\Lambda = 0, \tag{2.1a}$$

$$D_i(K^i{}_j - \delta^i_j K) = 0, \tag{2.1b}$$

where R is the scalar curvature of the metric g_{ij}, D_i is the covariant derivative compatible with that metric, and $K = K^i{}_i$. This is the classical formalism that translates most natural into canonical quantum theory; the constraint (2.1a) becomes the Wheeler-DeWitt equation, while (2.1b) imposes spatial diffeomorphism invariance.

The dynamical evolution of this data is described by the equations

$$\mathcal{L}_n g_{ij} = 2 g_{ik} K^k{}_j, \tag{2.2a}$$

$$\mathcal{L}_n K^i{}_j = -R^i{}_j - K K^i{}_j + \Lambda \delta^i{}_j + \frac{D^i D_j N}{N}, \tag{2.2b}$$

where $\mathcal{L}_n$ is the Lie derivative along the unit normal to Σ, essentially a covariant time derivative, and N is the lapse function, which determines the arbitrary position-dependent separation of successive time slices. (For simplicity, I have taken the shift vector to be zero.) It is sometimes useful to split off the trace of the extrinsic curvature, writing

$$K^i{}_j = \sigma^i{}_j + \frac{1}{3} K \delta^i_j. \tag{2.3}$$

K is the expansion—by (2.2a), it is the local Hubble constant, the logarithmic derivative of the volume element—while $\sigma^i{}_j$ is the shear tensor. The shear scalar is defined as $\sigma^2 = \frac{1}{2}\sigma^i{}_j \sigma^j{}_i$.

The evolution equations (2.2a)–(2.2b) and the constraints (2.1a)–(2.1b) have rather different status in quantum gravity. Assuming a gravitational version of Ehrenfest's theorem, the evolution equations should hold for averages, but observed values of geometric quantities will be subject to quantum fluctuations, presumably of order one at the Planck scale. The constraints are different: while their precise form may be modified by quantum effects, some version of the constraints is likely to hold *exactly*. In an operator formalism, for instance, the Wheeler-DeWitt equation is the statement that the constraints exactly annihilate physical states. In most path integral approaches, only configurations that satisfy the constraints appear in the sum over histories (though with some ambiguity [11]). The constraints thus capture the quantum structure at the Planck scale in a way the evolution equations do not.

We will now need two properties of the initial value formulation:

1. The equations are time reversal invariant: if (g, K) is allowed initial data for a manifold Σ, so is $(g, -K)$.

2. Two manifolds Σ_1 and Σ_2 with initial data (g_1, K_1) and (g_2, K_2) can be "glued" to form a manifold $\Sigma_1 \# \Sigma_2$ for which the initial data is unchanged outside arbitrarily small neighborhoods of the points where the gluing is performed [12, 13].

More precisely, $\Sigma_1 \# \Sigma_2$ is topologically the connected sum of Σ_1 and Σ_2, formed by cutting balls out of each manifold and identifying the boundaries. Geometrically, pick open sets $U_1 \subset \Sigma_1$ and $U_2 \subset \Sigma_2$, restricted only by the generic condition that the initial data is "not too symmetric," in the sense that the domains of dependence of U_1 and U_2 contain no Killing vectors. Pick points $p_1 \in U_1$ and $p_2 \in U_2$, cut geodesic balls B_1 and B_2 of arbitrarily small radius ϵ around each, and join the boundaries. Then $\Sigma_1 \# \Sigma_2$ admits initial data that exactly coincides with the original data outside $U_1 \cup U_2$ and is close to the original data, in a suitable norm, inside $U_1 \cup U_2$ but outside $B_1 \cup B_2$.

Now, as a preliminary construction, pick a three-manifold Σ with a fixed open set U and a point $p \in U$, and specify initial data (g, K). Let $\bar{\Sigma}$ be an identical copy of Σ, but with initial data $(g, -K)$. Glue the two manifolds symmetrically at p to form a connected sum $\widetilde{\Sigma} = \Sigma \# \bar{\Sigma}$. By symmetry, $\widetilde{\Sigma}$ will have an isometry $(g, K) \to (g, -K)$. While the definition of an averaged tensor is ambiguous [10], any average that respects this symmetry will clearly give $\langle K^i{}_j \rangle = 0$.

Next, much more generally, consider a large collection of manifolds $\Sigma_1, \Sigma_2, \ldots, \Sigma_N$, each with its own random initial data (g_α, K_α). Form the glued manifold

$$\widetilde{\Sigma} = \Sigma_1 \# \Sigma_2 \# \ldots \# \Sigma_N \,. \tag{2.4}$$

As long as we do not assume an arrow of time at this scale, the data (g, K) and $(g, -K)$ for any particular Σ_α will be equally likely. Thus, again, any sensible average over a large enough number of components should give $\langle K^i{}_j \rangle \sim 0$. Exactly how fast the average will go to zero depends on the number and distribution of manifolds and initial data sets, but as noted above, a cubic centimeter already contains some 10^{100} Planck-size regions.

These results imply that $\langle \mathcal{L}_n \sqrt{g} \rangle = 0$ and $\langle \sigma^i{}_j \rangle = 0$. It is also easy to check that $\langle \mathcal{L}_n R \rangle = 0$. To first order, the averaged spatial geometry is thus stationary. To match our Universe, we would also like the average spatial curvature to be small. For the initial values, the only restriction comes from averaging the constraint (2.1a):

$$\langle R \rangle = 2 \langle \sigma^2 \rangle + 2\Lambda - \frac{2}{3} \langle K^2 \rangle \,. \tag{2.5}$$

It is thus evident that if $\langle K^2 \rangle$ is large (for positive Λ) or $\langle \sigma^2 \rangle$ is large (for negative Λ), the cosmological constant can be "absorbed" in fluctuations of extrinsic curvature.

Let me stress that I am not starting with a spacetime and searching for a special hypersurface on which $\langle K^i{}_j \rangle = 0$. That would be an artificial procedure, and there would be no reason to expect such a hypersurface to be physically interesting. Rather, I am taking an arbitrary hypersurface and giving it initial data chosen randomly from a large collection. Further requirements may be added to make this data "nice," but as long as these allow Planck-scale inhomogeneity and do not pick out a microscopic arrow of time, the conclusions should not change.

This construction allows $\widetilde{\Sigma}$ to have an arbitrarily complicated topology. Indeed, any orientable compact three-manifold has a unique decomposition as a connected sum of "prime" manifolds [14, 15]. But $\widetilde{\Sigma}$ may also be topologically trivia: if each Σ_α is a three-sphere, the connected sum is also a three-sphere. We can thus reach a large set of initial data by starting with any data, cutting out a collection of Planck-size balls, changing the data on the balls, and gluing them back. The geometry of the "necks" between components is rather special, though, and it is an open question how much of the space of initial data can be reached this way.

For the special case of local spherical symmetry, one can perform a similar construction much more explicitly [16]. This case is "too symmetric" to meet the genericity condition for the gluing theorem. Nevertheless, it is possible to explicitly construct a "ring" of three-spheres with arbitrary signs of K, though more general gluings typically require breaking the symmetry.

3. Evolution

We have established that on an initial hypersurface, a large class of initial data can hide the macroscopic effects of a cosmological constant. But is this feature preserved in time? This is a hard question, whose answer almost certainly requires a better understanding of quantum gravity. In particular, the evolution equations (2.2a)–(2.2b) are classical approximations, which do not include the quantum fluctuations that presumably create the complex microscopic structure we are interested in.

Naively, we might have two expectations:
– Expanding regions grow in time, while contracting regions shrink, so eventually the expanding regions should dominate in a volume average (though this may take an arbitrarily long time [17]).
– But nothing in this construction picks out a "preferred" initial time, so if the "foamy" structure is generated by quantum fluctuations, it should replicate itself: expanding regions should themselves fill up with new curvature fluctuations.

Without a better understanding of how (or whether) quantum fluctuations generate spacetime foam, it is unlikely that we can fully resolve this question. Still, it is worth exploring what we *do* know of the evolution to look for hints.

3.1. Classical evolution

Let us first ask whether the classical evolution (2.2a)–(2.2b) can preserve the averaged structure. This is similar to Buchert's question, in a somewhat different context [18], of whether a non-equilibrium "cosmic equation of state" can lead to a stationary averaged configuration.

This is at least a well posed question, although one that is more difficult than it might seem. First, we can only hope to learn about short-time evolution. The initial data described here typically evolves to form singularities, with minimal spheres in the the connecting "necks" behaving like black hole horizons [17]. It is generally assumed that quantum gravity will resolve such singularities, and we have independent evidence that Planck-scale fluctuations can disrupt the causal structure of spacetime [19], requiring a quantum treatment. Classically, though, these singularities signal a breakdown of evolution.

Second, there are notorious ambiguities in defining time derivatives of averages. To determine the derivative of an average $\langle \bullet \rangle$ over a region $\mathcal{U}$, we must specify how $\mathcal{U}$ changes in time. If $\mathcal{U}$ is fixed in terms of some set of coordinates, the result will not be invariant; if it is defined in terms of geometric quantities, it will typically be time-dependent. Further, averages are often (although not always [20]) defined in term of integrals with a dynamical integration measure, providing an added source of time dependence [21].

Third, even if we know what we mean by "average," it's not always clear what we mean by "time." The choice of a time-slicing in general relativity is not unique; in the initial value formalism, it depends on an arbitrarily chosen, position-dependent lapse function N. The local

derivative with respect to proper time, for instance, is $\frac{1}{N}\partial_t$, and a variable lapse function cannot be simply passed through an integral to define an average derivative.

We might first ask whether *any* classical evolution—any choice of lapse function N—preserves the condition $\langle K \rangle = 0$. Let us follow [21] and define a spatial average

$$\langle X \rangle_{\mathcal{U}} = \frac{1}{V_{\mathcal{U}}} \int_{\mathcal{U}} X \sqrt{g} \, d^3x \quad \text{with} \quad V_{\mathcal{U}} = \int_{\mathcal{U}} \sqrt{g} \, d^3x \,, \tag{3.1}$$

and suppose the region $\mathcal{U}$ is defined in some time-independent way. Then, from (2.2a)–(2.2b),

$$\frac{d}{dt}\langle K \rangle = \frac{1}{V_{\mathcal{U}}} \int_{\mathcal{U}} N \mathcal{L}_n (K \sqrt{g}) \, d^3x$$

$$= \frac{1}{V_{\mathcal{U}}} \int_{\mathcal{U}} N(-R + 3\Lambda) \sqrt{g} \, d^3x = \frac{1}{V_{\mathcal{U}}} \int_{\mathcal{U}} N \left(\Lambda + \frac{2}{3} K^2 - 2\sigma^2 \right) \sqrt{g} \, d^3x \,. \tag{3.2}$$

If we choose a uniform time-slicing $N = 1$, this becomes

$$\frac{d}{dt}\langle K \rangle = -\langle R \rangle + 3\Lambda \,, \tag{3.3}$$

which is essentially the second Friedmann equation for averaged quantities. But given the "foamy" nature of the initial geometry, there is no reason to choose a constant lapse function at the Planck scale. As long as the integrand in (3.2) doesn't have a definite sign—that is, as long as the shear (for positive Λ) or expansion (for negative Λ) is large in some regions—there will be an infinite number of choices of N for which the right-hand side of (3.2) vanishes. We can further choose N to be invariant under $(g, K) \rightarrow (g, -K)$, which will guarantee that any integrand of the form $N^m K^{2n+1}$ will also average to zero.[*]

The second derivative is also simple:

$$\frac{d^2}{dt^2}\langle K \rangle = \frac{1}{V_{\mathcal{U}}} \int_{\mathcal{U}} \left[(\dot{N} + NK) \left(\Lambda + \frac{2}{3} K^2 - 2\sigma^2 \right) + 2N^2 K^{ij} R_{ij} \right] \sqrt{g} \, d^3x \,. \tag{3.4}$$

The last term contains an odd power of K, and goes to zero for large $\mathcal{U}$. The first term has exactly the same form as (3.2), and $\dot{N}$ can be specified independently, so if the first derivative can be made to vanish, the second derivative can as well.

Higher derivatives are more complicated. $\mathcal{L}_n^3 K$, for instance, contains derivative terms like $K\Delta K$ and higher order correlations like $\langle K^4 \rangle - \langle K^2 \rangle^2$, which probe structure at shorter distances. But each new derivative of $\langle K \rangle$ also comes with a new time derivative of N, which can be specified independently. Hence there is thus no obvious obstruction to choosing a time-slicing for which all of the time derivatives of $\langle K \rangle$ vanish.

This is a strong claim. If correct, it implies the local existence of a foliation of spacetime by slices of vanishing average expansion, despite the presence of very high curvature at the Planck scale. Of course, the foliation will itself vary rapidly at the Planck scale, but given the "foamy" structure of the three-geometry, that should come as no surprise. Whether one can simultaneously choose a lapse function for which $\langle R \rangle$ remains small is a more difficult question, requiring future work.

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[*]One might worry that the requirement $\langle N(R - 3\Lambda) \rangle \sim 0$ could force average curvature $\langle R \rangle$ to be large. For $N = 1$, this is the case. But for a lapse function with Planck scale structure, $\langle NR \rangle$ can be very different from $\langle R \rangle$.

3.2. Quantum evolution

In the absence of a quantum theory of gravity, much less can be said about the full quantum evolution. Note, though, that in many approaches to the quantum theory, the constraints—which are under better control here than the evolution equations—are the primary objects. The Wheeler-DeWitt equation, for instance, is simply the constraint (2.1a), written as an operator equation [22], and it (plus spatial diffeomorphism invariance) is supposed to capture the entire quantum theory. Of course, in some sense this merely hides the nature of time evolution, as part of the notorious "problem of time" in quantum gravity [23]. But it also suggests a different way of posing the question: does a typical solution of the Wheeler-DeWitt equation on a typical time slice have a foamlike structure at the Planck scale?

It would be interesting to connect this approach to Hawking's four-dimensional Euclidean spacetime foam [24]. This task is difficult without a better understanding of the full four-dimensional evolution of our initial data. But the "necks" in the connected sum (2.4) are similar to the throats of a Schwarzschild black hole [17], for which the Euclidean continuation is understood, so some progress might be possible. It would also be worthwhile to look further at the $\Lambda < 0$ case in light of the AdS/CFT correspondence. Here, there has been interesting work on the question of which topologies contribute, although mainly in the context of black holes and three-dimensional gravity [25].

4. What this proposal does, and does not, do

As early as 1957, Wheeler argued that

> ... it is essential to allow for fluctuations in the metric and gravitational interactions
> in any proper treatment of the compensation problem—the problem of compensation
> of "infinite" energies that is so central to the physics of fields and particles [26].

What I am proposing is a concrete realization of this vision. Several previous attempts have been made to model spacetime foam—see, for instance, [24, 27, 28]—but only a few have addressed the cosmological constant problem [29–32]. The new ingredients here are the ability to construct explicit initial data and the crucial realization that time reversal invariance allows, and perhaps even requires, the expansion and shear to average to zero.

This proposal addresses the "old" cosmological constant problem, the problem of large vacuum energy. It does not tell us whether the observed accelerated expansion of the Universe is caused by a small residual cosmological constant. It was recently proposed that higher order correlations of vacuum fluctuations might produce a small cosmological constant [33,34]; these would presumably show up here in higher correlations of the metric and extrinsic curvature, which appear in higher derivatives of averaged expansion and curvature.

While this proposal offers a natural explanation for small macroscopic expansion and shear, the requirement of small spatial curvature is less obvious. It is certainly possible to choose data for which $\langle R \rangle$ is small, and there are hints that this may be preferred by the gravitational partition function, but a better understanding is needed. Of course, the answer may be dynamical. In a standard closed FLRW cosmology, after all, the spatial curvature is initially very high and decreases in time. There is some evidence that the same is true here: the second time derivative

of the averaged curvature $\langle R \rangle$ can be calculated in the manner of the preceding section, and while the result depends on the lapse function, most of the terms are negative definite.

The proposal also does not attempt to explain the emergence of a macroscopic arrow of time, an important and complicated question but one that is probably logically independent. Nor have I shown that long wavelength disturbances sitting on top of Planck-scale spacetime foam will be described by classical general relativity. This is the notorious "averaging problem" [10,20,21], the problem of how the nonlinearities of general relativity interact with the process of taking averages. Here, though, effective field theory arguments may help [3]. Nothing in this construction has broken spatial diffeomorphism invariance, so at a minimum the effective action should involve only terms invariant under that symmetry. This implies a Hořava-Lifshitz action [35], of which the action of general relativity is a special case. If, as I have argued, there is also nothing "preferred" about the initial time slice, then time reparametrization invariance should also be a symmetry, and the large scale effective action should take the usual Einstein-Hilbert form.

So far, I have treated a quantum gravitational problem semiclassically, appealing to quantum mechanics to generate Planck-scale structure but relying on classical general relativity to describe constraints and evolution. We might next consider coherent states centered on the configurations described here, and construct more general wave functions as superpositions. But this would force us to confront some of the standard problems of quantum gravity: the metric and extrinsic curvature are not true observables, and to define an average we would have to figure out what "at the same point" means in different components of the wave function.

Interesting technical questions remain as well. The gluing construction I have used provides a large set of initial data, but it is not known just how much of the total space of initial data is covered. More generally, gluing is certainly not the *only* way to produce data with no arrow of time at the Planck scale, and a full understanding of the measure such data is still lacking. It would also be useful to further investigate higher order correlations, or, conversely, to see to what extent further restrictions (e.g., $\langle \mathcal{L}_n^3 K \rangle = 0$) limit the possible initial data.

For all these limitations, though, this proposal suggests a simple and radical solution to a deep problem. If a large cosmological constant is generated by vacuum fluctuations at the Planck scale, then perhaps that is also the place to look for answers. I have shown that at least in principle, hiding a Planck scale cosmological constant in Planck scale curvature fluctuations is not only possible, but can be quite natural. We may have simply been looking in the wrong place.

Acknowledgments

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