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Implementation of 1D Ground Response Analysis in Probabilistic Assessments of Ground Shaking Potential

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Abstract

Results of 1D ground response analyses are typically not incorporated into probabilistic seismic hazard analyses (PSHA) in a statistically robust way. Often ground response is incorporated into PSHA using deterministic amplification factors. This simplistic method generates results that are intrinsically arbitrary and often unconservative. The main problem in probabilistically linking PSHA and ground response lies in quantifying the dispersion that is appropriate for use with ground response analysis results. We review two alternative procedures for quantifying this uncertainty and illustrate their differences with respect to dispersion values of spectral acceleration at the surface for various site conditions.

Introduction

The problem of how to link probabilistic assessments of ground motion hazard with the results of geotechnical ground response analysis has long been vexing to engineers and seismologists. The term "ground response analysis" is used here to refer to one-dimensional (1D) modeling of shear wave propagation through a soil column, the response of which will generally be nonlinear for the ground motion levels of interest in engineering applications. Such analyses can be performed using equivalent-linear or fully nonlinear modeling of the material response. We do not discuss the relative merits of these two methods of analysis. Our remarks in this article are predicated on the assumption that the method of 1D analysis being utilized is appropriate for the strain conditions in the soil.

The question of how to incorporate the results of 1D modeling into PSHA has recently been considered by two independent teams of investigators (Bazzurro and Cornell, 2004a; Baturay and Stewart, 2003). In each case, specific recommendations were provided regarding how the median and standard deviation of spectral acceleration at the soil surface should be estimated. However, distinct approaches were utilized to arrive at those recommendations. The purpose of this article is to briefly review the manner in which the recommendations were developed, the content of the recommendations, and to compare the standard deviation terms that would be obtained by the two approaches for a variety of site conditions. The implementation of both procedures into the open-source hazard code OpenSHA (http://www.opensha.org/) is currently in progress.

Ground Response Implementation Procedures

Bazzurro and Cornell Procedure

Bazzurro and Cornell (2004a) present two procedures for implementation of ground response analysis results into PSHA. The first one involves convolution of amplification functions (*AF*) calculated from ground response analysis with a site hazard curve developed for reference site conditions (usually rock). Such amplification *functions* account for the frequency dependence of amplification *factors*. The second procedure involves modifying the median and standard deviation of a ground motion intensity measure estimated from a rock attenuation relationship. The two procedures produce nearly identical hazard estimates (Bazzurro and Cornell, 2004a). Although the first procedure is more general, for the sake of convenience and brevity our remarks here will focus on the second one.

In Bazzurro and Cornell's second procedure, it is assumed that the median and standard deviation of the spectral acceleration on rock are available from an attenuation relationship. These are denoted $\hat{S}_a^r(f)$ for the median and $\sigma_{\ln S_a^r(f)}$ for the standard deviation. The procedure's objective is to modify these statistical moments to account for ground response effects in a PSHA environment.

Ground response analyses are performed using an appropriate equivalent-linear or nonlinear code using multiple rock input motions (possibly scaled to different levels). The procedure allows the analyst to account for randomness in the site soil properties as well. Although some 1D ground response analysis computer codes allow the use of more than one component of the same recording, this methodology was originally developed using one randomly selected horizontal component. The results consist of a suite of calculated ground surface motions. The ratio of the spectral accelerations of a ground surface motion to its corresponding rock motion is taken as an amplification factor (*AF*), which is evaluated for frequency *f* as,

$$AF(f) = \frac{S_a^s(f)}{S_a^r(f)} \tag{1}$$

where $S_a^r(f)$ is the rock spectral acceleration at frequency *f* and $S_a^s(f)$ is the soil spectral acceleration. An example of the relationship between $S_a^r(f)$ and *AF*(*f*) is shown in Figure 1, where each circle represents the results of a ground response analysis for a rock record. The decrease of *AF*(*f*) with $S_a^r(f)$ is due to sediment nonlinearity. This procedure states that in order to estimate the expected *AF*(*f*) at a given frequency *f* one needs to know only the value of $S_a^r(f)$. No other information about the spectrum of the input motion and about the characteristics of the causative event (e.g., the magnitude) is strictly necessary.

An alternative procedure for input motion selection that is often used in practice involves scaling a suite of input (real or synthetic) rock motions to have the same PGA^r. These motions are often from earthquakes with a narrow range of magnitudes that is compatible with disaggregation of the site hazard. The analysis is often repeated with suites of records of different severity to evaluate how *AF*(*f*) varies with increasingly nonlinear soil behavior. Hence, in the traditional approach the amplitude of the reference motion is described by PGA^r (and, perhaps, implicitly by magnitude) in lieu of $S_a^r(f)$.

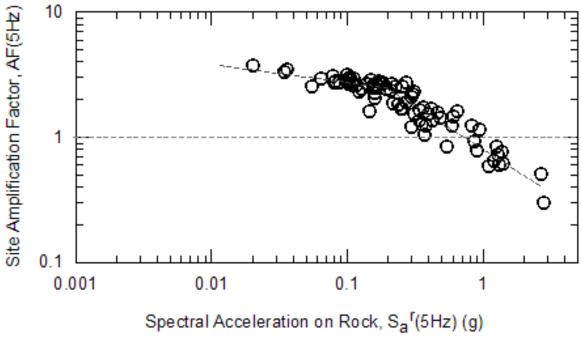

Figure 1. Example of a site amplification function calculated from ground response analyses plotted against the reference motion amplitude (here $S_a^r(5Hz)$). Adapted from (Bazzurro and Cornell, 2004b).

Bazzurro and Cornell (2004a) assume that a statistically appropriate linear fit line (or a set of linear fit lines) can be established through the data, as illustrated in Figure 1. The equation for the line (or a segment of the line) is written as follows:

$$\ln AF(f) \approx c_0 + c_1 \ln S_a^r(f) + \varepsilon_{\ln AF^{Sa}(f)} \sigma_{\ln AF^{Sa}(f)} \tag{2a}$$

where *AF*(*f*) is the site amplification at frequency *f*; c_0 and c_1 are linear regression coefficients; $\sigma_{\ln AF^{Sa}(f)}$ is the standard deviation of the *AF*(*f*) - $S_a^r(f)$ relationship (i.e., the standard error of estimation from the statistical regression); and $\varepsilon_{\ln AF^{Sa}(f)}$ is a standard normal variate. Note that, for brevity, we dropped *r* as a reference to rock conditions from PGA and $S_a(f)$ when they appear as superscript above in the $\sigma_{\ln AF^{Sa}(f)}$ and $\varepsilon_{\ln AF^{Sa}(f)}$ terms.

If PGA^r is used as the ground motion parameter for the reference site condition, Eq. 2a becomes:

$$\ln AF(f) \approx b_0 + b_1 \ln PGA^r + \varepsilon_{\ln AF^{PGA}(f)} \sigma_{\ln AF^{PGA}(f)} \tag{2b}$$

where b_0 and b_1 are regression coefficients analogous to c_0 and c_1 and $\sigma_{\ln AF^{PGA}(f)}$ and $\varepsilon_{\ln AF^{PGA}(f)}$ are analogous to the quantities above but here the conditioning is done on PGA^r rather than on $S_a^r(f)$. Note that the *r* superscript is again dropped from these ε and σ terms

Once the regression represented by Eq. 2a has been completed, Bazzurro and Cornell (2004a) showed that the median and standard deviation of the soil motion can be evaluated as:

$$\ln \hat{S}_a^s(f) \approx c_0 + (c_1+1)\ln \hat{S}_a^r(f) \quad (3) \qquad \text{and} \qquad \sigma_{\ln S_a^s(f)} \approx \sqrt{(c_1+1)^2 \sigma^2_{\ln s_a^r(f)} + \sigma^2_{\ln AF^{Sa}(f)}} \quad (4)$$

where $\hat{S}_a^s(f)$ is the median spectral acceleration at frequency f on the soil surface and $\sigma_{\ln S_a^s(f)}$ is the resulting standard deviation of spectral acceleration on soil. The approximations in Eqs. 3-4 are due to the assumption of linearity in Eq. 2a. It is interesting to note that c_1 is often negative (as is the case in Fig. 1) and this, under certain circumstances, can cause $\sigma_{\ln S_a^s(f)}$ to be less than or equal to $\sigma_{\ln S_a^r(f)}$.

We have derived similar expressions to Eq. 3-4 when PGA^r is the reference site ground motion amplitude parameter utilized in the estimation of $AF(f)$ (see Eq. 2b). Assuming that $\ln S_a^r(f)$, $\ln PGA^r$ and $\ln AF(f)$ are normally distributed, the soil motion is related to reference (rock) motion as:

$$\ln S_a^s(f) \approx \ln \hat{S}_a^r(f) + \varepsilon_{\ln S_a^r(f)}\sigma_{\ln S_a^r(f)} + \left[b_0 + b_1\left(\ln P\hat{G}A^r + \varepsilon_{\ln PGA}\sigma_{\ln PGA^r}\right)\right] + \varepsilon_{\ln AF^{PGA}(f)}\sigma_{\ln AF^{PGA}(f)} \tag{5}$$

where $\ln P\hat{G}A^r$ represents the median PGA for rock in natural log units and $\sigma_{\ln PGA^r}$ represents the corresponding standard deviation, both taken from an attenuation relationship. The term in brackets on the right side of Eq. 5 represents the median of $AF(f)$, and is taken from Eq. 2b. Based on Eq. 5, the median of $S_a^s(f)$ is given by

$$\ln \hat{S}_a^s(f) \approx b_0 + \ln \hat{S}_a^r(f) + b_1 \ln P\hat{G}A^r \tag{6}$$

Eq. 5 contains three standard normal ε variates. Hence, the total standard deviation of the spectral acceleration at the soil surface evaluated by means of a SRSS operation contains six terms (Ang and Tang, 1975, p. 193), three of which are cross terms that are different from zero only if the correlation coefficient between the corresponding variables is not zero. To evaluate the total standard deviation, we account for the correlation between ε terms for $\ln S_a^r(f)$ and $\ln PGA^r$ of the same horizontal component using the model of Baker and Cornell (2005a). For shallow crustal earthquakes and NEHRP Class C-D soil this correlation coefficient for $\ln PGA$ and, for example, $\ln S_a$ at 0.3s period is approximately 0.7. We assume that this value holds for the reference soil conditions used here as well. The correlation between $\varepsilon_{\ln S_a^r(f)}$ and $\varepsilon_{\ln AF^{PGA}(f)}$ is expected to be zero. The weak negative correlation between $\varepsilon_{\ln PGA^r}$ and $\varepsilon_{\ln AF^{PGA}(f)}$ is conservatively neglected (see Bazzurro and Cornell, 2004a for discussion). With these assumptions, the total standard deviation of $\ln S_a^s(f)$ can be derived as

$$\sigma_{\ln S_a^s(f)} \approx \sqrt{\sigma^2_{\ln s_a^r(f)} + b_1^2\sigma^2_{\ln PGA^r} + \sigma^2_{\ln AF^{PGA}(f)|} + 2b_1\rho\sigma_{\ln PGA^r}\ \sigma_{\ln s_a^r(f)}} \tag{7}$$

where ρ is the correlation coefficient between ε terms for $\ln S_a^r(f)$ and $\ln PGA^r$ mentioned above. As noted before for c_1, the value of the coefficient b_1 is usually negative and, therefore, the cross term tends to reduce the variability in the spectral acceleration at the soil surface.

Under certain conditions, the modified median and standard deviation obtained from Eqs. 3-4 or Eqs. 6-7 can be used directly within the hazard integral in lieu of those from a conventional prediction equation for the appropriate soil category. The use is legitimate provided that the definition of $S_a^r(f)$ and PGA^r used for the development of the rock ground motion attenuation relationship and for the prediction of $AF(f)$ is the same in both cases. A mismatch can occur, for example, if the attenuation relationship is developed for the *geometric mean* of $S_a^r(f)$ or PGA^r for the two horizontal components and the parameter used as the predictor of $AF(f)$ is instead the $S_a^r(f)$ or PGA^r of an *arbitrary component* used for ground response analysis. The mismatch occurs because the dispersion of a ground motion parameter's geometric mean is smaller than the dispersion of the same parameter from an arbitrary component. It is therefore important to be consistent with the type of ground motion (geometric mean or arbitrary component) utilized for PSHA and for ground response analysis. Baker and Cornell (2005b) present three different approaches to avoid such a mismatch and, hence, allow one to use Eqs. 3-4 and Eqs. 6-7:

A critical assumption implicit to the above procedures is that site effects are correctly simulated by 1D ground response analysis. More specifically, this assumption implies that: (1) the median value of *AF* is correct, which in turn implies that the median prediction of soil site intensity measures is unbiased and (2) the standard deviation of the soil site intensity measures is correct.

Baturay and Stewart Procedure

Empirical ground motion studies estimate standard deviation using a consistent framework – a model is developed, residuals between data and the model are calculated, and the standard deviation is calculated from the residuals. This process is fundamentally different from the procedure used to develop Eqs. 4 and 7.

Baturay and Stewart (2003) evaluated standard deviation within site categories using the empirical approach. The "model" that they used was applied to 134 recordings from 68 sites in California. Sites selected for use in the analyses had well-characterized ground conditions, including in situ measurements of shear wave velocity and detailed descriptions of soil types, as well as available recordings. Applying the model involved the following steps:

(1) A target response spectrum for rock site conditions was estimated from ground motion prediction equations for rock modified to account for rupture directivity, weathered rock effects, and event-specific bias (see Baturay and Stewart (2003) for details).

(2) Suites of time histories with appropriate magnitude, distance, and rupture directivity characteristics were scaled to match the target spectrum in an average sense over the period range 0 – 1.0 s and then spectrum-matched such that the median spectrum of the suite matched the target spectrum while retaining natural record-to-record variability.

(3) Equivalent-linear ground response analyses were performed for the sites using the input motion suite from (2); the median of the calculated soil motions was taken as the estimate of *Sa* at the soil surface.

Residuals were calculated between the geometric mean of the data (i.e., recordings) and the geometric mean of two horizontal components of estimated spectral acceleration from (3). An exception is made for near-fault sites, where residuals are calculated separately for the fault-normal and fault parallel components of motion. The standard deviation of the residuals is termed an "intra-event" standard deviation because event-to-event variability in the data was eliminated by use of the event term. These standard deviation terms were compiled for data within various site categories as shown in Figure 2.

The results in Figure 2 underlie Baturay and Stewart's (2003) procedure for implementing ground response results into hazard calculations. Conceptually, this procedure is similar to what was described in the previous section in which the median and standard deviation are modified from those obtained using a ground motion prediction equation for reference (rock) site conditions. However, the following differences are noted:

1. The median is only assumed to be unbiased for mid- to high-frequencies ($f > 1$ Hz). This assumption is made because ground response predictions were found to have an under-prediction bias at low frequencies for deep stiff soil sites. That bias likely results from basin effects, which are not captured by one-dimensional ground response analysis. A significant correlation between the residuals of ground response analyses and basin depth provides evidence that basin geometry affected the ground motions at low frequencies (Figure 6 of Baturay and Stewart, 2003).
2. The intra-event standard deviation at high frequencies ($f > 1$ Hz) is taken as the empirical value for the appropriate site category (i.e., the values indicated in Figure 2).
3. At lower frequencies, the median and standard deviation are evaluated using empirical approaches in lieu of ground response analyses. This is recommended because empirical procedures capture (at least in an average sense) the relatively complex site response physics that can significantly contribute to observed intensity measures at these low frequencies (e.g., Choi et al., 2005).
4. No adjustments to the standard deviation are needed to account for the effect of conditioning on geometric mean or an arbitrary component of motion. However, the results are only applicable with ground motion prediction equations for the geometric mean of $S_a^r(f)$ or PGA^r.

A complete description of this implementation procedure is given in Baturay and Stewart (2003), and includes details such as accounting for inter-event variability and site-specific variability of the median site amplification due to the randomness of input motion phasing and the dispersion of fundamental soil properties.

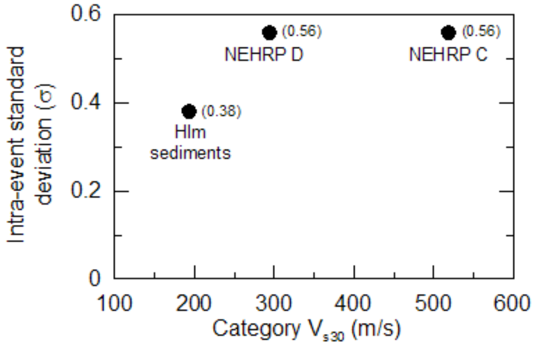

Figure 2. Standard deviation of spectral acceleration at the soil surface at high frequency (3.33 Hz) associated with ground motion prediction model that includes ground response analysis. The standard deviation terms are relatively insensitive to frequency for $f > 1$ Hz. Hlm indicates sediments of Holocene age and lacustrine or marine origin (often similar to NEHRP E).

Comparison of the Two Procedures

In Figure 3, the intra-event empirical standard deviation terms from Baturay and Stewart (2003) are compared to those from the "theoretical" approach represented by Eq. 4 or 7. The Baturay and Stewart (2003) standard deviation terms are applicable across a range of high frequencies ($f > 1$ Hz). We first apply the theoretical approach using ground response analysis results derived for a broad array of site profiles within geologic categories by Silva et al. (1999). Similar results are then shown for an array of site profiles considered representative of the central and eastern U.S. for which ground response analysis results are presented in EPRI (1993). In both the Silva et al. (1999) and EPRI (1993) studies, site factors are described using PGA^r as the reference site amplitude parameter (hence, standard deviations are calculated using Eq. 7). We show results for a spectral period of 0.3 s (i.e., $1/f = 0.3$ sec) and plot them in Figure 3 at the median value of V_{s30} for the assemblage of site profiles that was analyzed.

A second application uses site-specific ground response analysis results from Bazzurro and Cornell (2004b). To be consistent with Silva et al. and EPRI, PGA^r is used with Eq. 7 as the reference ground motion parameter.

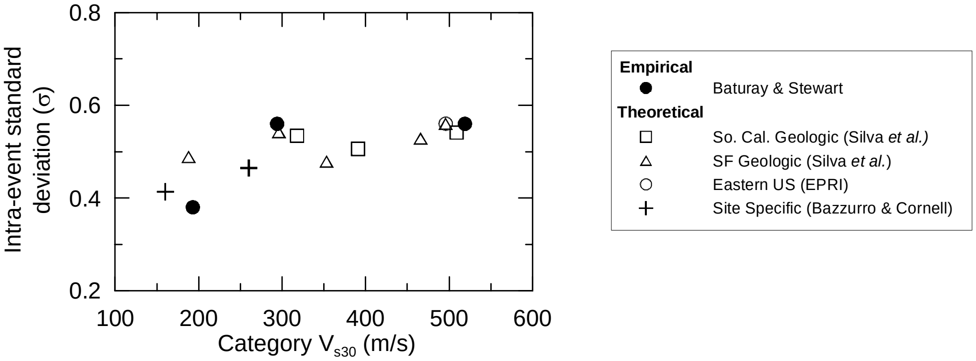

Figure 3. Standard deviation of response spectral acceleration at high frequency for empirical and theoretical procedures for combining rock attenuation relations (Abrahamson and Silva (1997)) with ground response analysis results for M > 7.

As shown in Figure 3, standard deviation terms evaluated by both methods generally indicate lower standard deviation at high frequencies for soft soils (e.g., Holocene-age lacustrine and marine and NEHRP Category E sediments) than for relatively stiff sites. We expect ground response analyses to capture the essential physics of site response for soft soil sites, and this relatively good "knowledge" of site response translates into lower standard deviations by both approaches.

For stiffer sites, empirical standard deviation terms generally exceed those from the theoretical approach, although the difference is small. Unfortunately, for these stiff site conditions, the only available comparison is between empirical results (based on site-specific analyses) and results for site categories (not site-specific) by Silva et al. (1999) and EPRI (1993). It would be expected that the standard deviation of $AF(f)$ should be lower for site-specific analyses than for site category analyses. For example, values of $\sigma_{\ln AF^{PGA}(f)}$ provided by Silva et al. (1999) for southern California Quaternary alluvium vary from approximately 0.3 to 0.45 depending on the level of shaking. Bazzurro and Cornell (2004b) found this standard deviation term to be about 0.25-0.4 for site-specific studies. If the category standard deviation terms shown in Figure 3 are too large for site-specific application, it is likely by a factor of 0.05 or less; the overall difference from the empirical results likely remains on the order of 0.1.

Conclusions

Local site conditions can significantly affect design ground motions, and hence procedures are needed to implement the results of site-specific ground response calculations into probabilistic hazard analysis. Two distinct approaches that have been presented previously in the literature are compared here. Our preliminary findings suggest that a higher standard deviation may result from the empirical approach. The preliminary comparisons shown here suggest these differences may be negligible for soft soils but as high as approximately 0.1 in natural log units for stiff soils. At a 10% exceedance probability in 50 year hazard level in seismically active areas such as southern California, a variation of 0.1 in standard deviation could result in 10% differences in ground motions calculated from PSHA (Field and Petersen, 2000). This effect on ground motions is large enough to be significant in practice. Additional comparisons (similar to those in Figure 3) for relatively stiff site conditions are needed to confirm our preliminary findings regarding the differences between standard deviations from the theoretical and empirical approaches. Nonetheless, our opinion is that both approaches have merit, and both should be considered in PSHA for critical facilities. The alternative approaches can be readily implemented using logic trees, which are an ordinary component of hazard analyses when alternative viable models exist for a given phenomenon (e.g., Baecher and Christian, 2003, Chapter 20).

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