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Opportunistic Interference Management: A New Approach for Multi-Antenna Downlink Cellular Networks

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Abstract

A new approach for multi-antenna broadcast channels in cellular networks based on multiuser diversity concept is introduced. The technique called Opportunistic Interference Management (OIM), achieves Dirty Paper Coding (DPC) capacity asymptotically with minimum feedback required reported to date in literature. When there are K antennas at the base station with M mobile users in the cell, the proposed technique only requires K integer numbers related to channel state information (CSI) between mobile users and base station. The encoding and decoding complexity of this scheme is the same as that of point-to-point communications which makes the implementation of this technique easy. An antenna selection scheme is proposed at the base station to reduce the minimum required mobile users significantly at the expense of reasonable increase in feedback. In order to guarantee fairness, a new algorithm is presented which incorporates OIM into existing GSM standard.

I. Introduction

Multiuser diversity scheme [1] is an alternative approach to more traditional techniques like time division multiple access (TDMA) to increase the capacity of wireless cellular networks. The main idea behind this approach is that the base station selects a mobile user that has the best channel condition by taking advantage of the time varying nature of fading channels, thus maximizing the signal-to-noise ratio (SNR).

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Traditionally, fading and interference have been viewed as the two major impeding factors in increasing the capacity of wireless cellular networks. Opportunistic Interference Management (OIM) scheme is an approach that takes advantage of the fading in the channel to reduce the negative effects of interference.

Several schemes have been developed that achieve optimal dirty paper coding capacity by utilizing beamforming [2], [3]. Most recent studies [4]–[7] have investigated the effect of partial finite-rate feedback on the capacity of MIMO broadcast channels in networks with limited number of users M.

We present OIM technique for the downlink of wireless cellular networks in which d ($d \leq K$) independent data streams can be broadcasted to d ($d \ll M$) mobile stations with single antenna such that these data streams do not interfere with each other[1]. Furthermore, the mean value of d, i.e. $D = \mathbf{E}[d]$, can be any number up to the maximum value of K as long as M is large enough. Therefore, OIM is capable of achieving the maximum multiplexing gain as long as there is a minimum number of mobile stations in the network. The feedback requirement to transmit K independent data streams is proportional to K. The original multiuser diversity concept was based on searching for the best channel to communicate, while our approach shows that searching simultaneously for the best and worst channels can lead to significant capacity gains. This technique can asymptotically achieve the capacity of DPC when M is very large. OIM scheme does not require mobile stations to cooperate for synchronization during transmission. It achieves optimal K maximum multiplexing gain in the downlink of cellular systems as long as $K = \Theta(\log M)$. However, in most practical cellular networks, there may not be too many mobiles users in a cell. Therefore, it is important to reduce the minimum required number of mobile users. This paper also introduces an antenna selection technique at the base station such that it reduces the minimum required number of mobile users significantly. This improvement is achieved at the expense of modest increase in the feedback requirement and additional computational complexity at the base station receiver.

The remaining of this paper is organized as follows. Section II, presents an overview of related work. Section III introduces the OIM scheme and the model used in our analysis. Section IV presents the theoretical analysis and corresponding numerical results. Section V focuses on the antenna selection scheme and the lower and upper bounds computation of multiplexing gain as a function of M. Fairness issues and practical considerations are discussed in Section VI and the paper is concluded in Section VII.

[1]Note that d is a random variable.

II. Related Work

Knopp and Humblet [1] derived the optimum capacity for the uplink of a wireless cellular network taking advantage of multiuser diversity. They proved that if the “best” channel (i.e. the channel with the highest SNR in the network) is selected, then all of the power should be allocated to the user with the ”best channel” instead of using a water-filling power control technique. Tse extended this result into the downlink (broadcast) case of a wireless cellular network [8]. Furthermore, Viswanath et al. [9] used a similar idea for the downlink channel and employed the so-called “dumb antennas” by taking advantage of opportunistic beamforming. Grossglauser et al. [10] extended the multiuser diversity concept into mobile ad hoc networks and took advantage of the mobility of nodes to scale the network capacity. All above schemes have taken advantage of multiuser diversity concept to combat the two major obstacles in wireless networks, namely, fading and interference.

Interference alignment [11] is another technique to manage interference. The main idea in this approach is to use part of the degrees of freedom available at a node to transmit the information signal and the remaining part to transmit the interference. For example, they consider $K \times M$ MIMO interference channel and demonstrate that the number of achievable degrees of freedom is $\frac{KM}{K+M-1}$. The drawback of interference alignment is that the system requires full knowledge of the CSI. This condition is very difficult to implement in practice, and feedback of CSI is MK complex numbers in a $K \times M$ interference channel. The advantage of interference alignment is that there is no minimum number of users required to implement this technique.

Sharif and Hassibi introduced a technique [2], [3] based on random beamforming concept to search for the best SINR in the network. Their approach requires M complex numbers for feedback instead of complete CSI information, and achieves the same capacity of $K \log\log M$ similar to DPC when M goes to infinity. There are major differences between our approach and the design in [2], [3]. First, our approach does not require beamforming, while the techniques proposed in [2], [3] take advantage of random beamforming. Second, the feedback requirement in our scheme is proportional to K integers while this value is proportional to M complex numbers in [2], [3]. When M grows, the feedback information in [2], [3] grows linearly, while this complexity is constant with the number of antennas at the base station in our scheme. Our approach achieves DPC asymptotic capacity of $K \log\log M$ with minimum feedback requirement.

DPC provides the optimal $K \log\log M$ sum-rate capacity which is the maximum multiplexing and multiuser diversity gains. These gains are achieved at the expense of full CSI requirement and infinite-rate feedback M when M tends to infinity. In this paper, we present a new scheduling scheme which requires only minimum finite-rate feedback K and yet retains the optimal multiplexing and multiuser diversity gains achievable by dirty paper coding.

To the best of our knowledge, [12] and [4] are the only two publications with some similarities to our approach. Diaz et al. [12] proposed "1-bit" feedback from the mobile users instead of CSI information to the base station with the total feedback still proportional to M. While Tajer et al. [4] scheduling scheme is asymptotically optimal, it also exhibits a good performance for practical network sizes. They also showed [4] that by appropriate design of the feedback mechanism, they can refrain the aggregate feedback from increasing with the number of mobile users and for asymptotically large networks, the total number of feedback is bounded by $K \log K$ bits.

In this paper, we present new approaches to reduce the minimum required number of mobile users M to achieve DPC capacity while maintaining the same feedback requirement of K (or equivalently $K \log K$ bits). Our approach is fundamentally different from random beamforming approach [3] while they both achieve the same asymptotic capacity. It is noteworthy to mention that our approach can be easily extended to distributed systems such as ad hoc networks [13] while random beamforming approaches cannot be extended to distributed systems. Finally, we propose a practical technique to incorporate this scheme to existing cellular networks. There are other schemes in literature [5]–[7], [14] that achieve DPC capacity or close to that capacity with feedback requirement that is proportional to M.

III. OPPORTUNISTIC INTERFERENCE MANAGEMENT

A. Network Model

We investigate the problem of optimal transmission in the downlink of a cellular network when the base station has independent messages for the mobile stations in the network. Clearly if the base station has only K antennas, it can transmit at most K independent data streams at any given time. We assume that all mobile stations have a single antenna for communication. The channel between the base station and mobile stations $\mathbf{H}$ is an $M \times K$ matrix with elements h_{ji}, where $i \in [1, 2, \ldots, K]$ is the antenna index of the base station and $j \in [1, 2, \ldots, M]$ is the mobile user index. We consider block fading model where the channel coefficients are constant during coherence interval of T. Then the received signal $\mathbf{Y}^{M \times 1}$ is expressed as

$$\mathbf{Y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{x}$ is the transmit $K \times 1$ signal vector and $\mathbf{n}$ is the $M \times 1$ noise vector. The noise at each of the receive antennas is i.i.d. with $\mathcal{CN}(0, \sigma_n^2)$ distribution.

B. The scheduling protocol

During the first phase of communication, the base-station antennas sequentially transmit K pilot signals. In this period, all the mobile stations listen to these known messages. After the last pilot signal is transmitted, mobile stations evaluate the SNR for each antenna. If the SNR satisfies certain conditions for a mobile node, that particular mobile station will be selected by the base station. The mobile station is selected when the SNR for one transmit antenna is greater than a pre-determined threshold SNR_{tr} and below another pre-determined threshold of INR_{tr} for the remaining $K-1$ antennas.

In the second phase of communication, the mobile stations that satisfy SNR criteria will notify the base station that they have the required condition to receive packets during the remaining time period of T. We will not discuss the channel access protocol required for these mobile stations to contact the base station or the case when two mobile stations satisfy OIM condition for the same base station antenna. We assume that this will be resolved by some handshake between the mobile stations and the base station. Note that, if we choose appropriate values for SNR_{tr} and INR_{tr} such that $\text{SNR}_{\text{tr}} \gg \text{INR}_{\text{tr}}$, then the base station can simultaneously transmit different packets from its antennas to different mobile stations. The mobile stations only receive their respective packets with a strong signal and can treat the rest of the packets as noise. The value of SNR_{tr} (or INR_{tr}) can be selected as high (or low) as required for a given system, as long as M is large enough.

In general, there is a relationship between average number of antennas with OIM condition, $D = \mathbf{E}(d)$, and number of mobile stations, M. Clearly, OIM decreases the encoding and decoding complexity of MIMO broadcasting channel significantly[2] at the expense of the presence of large number of mobile stations. Fig. 1 demonstrates the system that is used here. Without loss of generality, we assume that the user i for $i \in [1, 2, \ldots, d]$ is assigned to antenna i in the base station. In this figure, solid and dotted lines represent strong and weak channels between an antenna at the base station and a mobile station respectively. Note that if there is no line between the base station and mobile stations, then it means the

[2]For OIM technique, the encoding and decoding of multiple antennas reduces to simple point-to-point communication because the channels are decoupled from each other and no longer interfere significantly with each other.

channel is a random parameter based on the channel probability distribution function. For simplicity, Fig. 1 only illustrates the strong channel case.

IV. Theoretical Analysis

Let's define SNR_{ji} as the signal-to-noise ratio when antenna j is transmitting packet to mobile station i in the downlink. Further denote INR_{ji} as the interference-to-noise ratio between transmit antenna j and receiver i. The objective of OIM is to identify d mobile stations out of M choices to satisfy the following criteria

$$\begin{aligned}
&\text{SNR}_{ii} \geq \text{SNR}_{\text{tr}}, \quad 1 \leq i \leq d, \\
&\text{INR}_{ji} \leq \text{INR}_{\text{tr}}, \quad 1 \leq j \leq K, 1 \leq i \leq d, j \neq i
\end{aligned} \tag{2}$$

The above condition in (2) states that each one of the d mobile stations has a very good channel to a single antenna of the base station and strong fading to the other $K-1$ antennas of base station as shown in Fig. 1. After all the mobile users with OIM condition return their feedback to the base station, then the base station will select those mobile stations to participate in the communication phase such that the maximum multiplexing gain is achieved. Note that it is possible that two mobile users satisfy OIM condition for the same base station antenna.

The sum rate in the downlink of wireless cellular channel can be written as

$$\begin{aligned}
R_{\text{proposed}} &= \sum_{i=1}^{d} \log\left(1 + \text{SINR}_{ii}\right) \\
&= \sum_{i=1}^{d} \log\left(1 + \frac{\text{SNR}_{ii}}{\sum_{j=1, j\neq i}^{d-1} \text{INR}_{ji} + 1}\right) \\
&\geq d \log\left(1 + \frac{\text{SNR}_{\text{tr}}}{(K-1)\text{INR}_{\text{tr}} + 1}\right) \\
&= d \log(1 + \text{SINR}_{\text{tr}})
\end{aligned} \tag{3}$$

where SINR_{ii} and SINR_{tr} are defined as

$$\text{SINR}_{ii} = \frac{\text{SNR}_{ii}}{\sum_{j=1, j\neq i}^{d-1} \text{INR}_{ji} + 1}, \forall i = 1, 2, \cdots, d \tag{4}$$

and

$$\text{SINR}_{\text{tr}} = \frac{\text{SNR}_{\text{tr}}}{(K-1)\text{INR}_{\text{tr}} + 1}, \tag{5}$$

respectively.

First, the mean value of multiplexing gain d is derived. Then, we will prove that for any value of SINR_{tr}, there exists a minimum value of M that satisfies Eq. (3). Finally, we prove that our approach achieves the optimum capacity of DPC asymptotically.

For the rest of paper, the channel distribution is considered to be Rayleigh fading but OIM can be implemented for other time-varying channel distributions. Note that for an i.i.d. Rayleigh fading channel $\mathbf{H}$, the probability distribution function (pdf) of SNR (or INR) is given by [15]

$$p(\text{z}) = \begin{cases} \frac{1}{\sigma}\exp\left(-\frac{z}{\sigma}\right), & z > 0 \\ 0, & z \leq 0 \end{cases} \tag{6}$$

where z is the SNR (or INR) value and $\mathbf{E_H}(z) = \sigma$, $\mathbf{Var_H}(z) = \sigma^2$. Equivalently, $\sqrt{\sigma/2}$ is the parameter for Rayleigh fading distribution which shows the strength of the fading channel.

A. Exact Analysis

Let's define event A for any mobile station that satisfies the condition in Eq. (2). Since the channels between the base station and the mobile stations are i.i.d., then the probability of this event can be derived as

$$\text{Pr}(A) = \binom{K}{1}\int_{\text{SNR}_{\text{tr}}}^{\infty} p(z)dz \left(\int_0^{\text{INR}_{\text{tr}}} p(z)dz\right)^{K-1} = \binom{K}{1} e^{-\frac{\text{SNR}_{\text{tr}}}{\sigma}}\left(1 - e^{-\frac{\text{INR}_{\text{tr}}}{\sigma}}\right)^{K-1} \tag{7}$$

Our objective is to maximize this probability based on network parameters. Maximizing $\text{Pr}(A)$ will minimize the number of required mobile stations M as will be proved later. Note that among all network parameters $K, \text{SNR}_{\text{tr}}, \text{INR}_{\text{tr}}$, and σ, the values of K and σ are really related to the physical properties of the network and are not design parameters. Further, the parameters SNR_{tr} and INR_{tr} can be replaced with a single parameter SINR_{tr} using Eq. (5).

Let X be the random variable related to the number of mobile stations satisfying the OIM condition for Eq. (2). Note that it is possible that two mobile stations satisfy OIM condition for the same base-station antenna. The probability of $X = x$ is computed as

$$\text{Pr}_A(X = x) = \binom{M}{x}(\text{Pr}(A))^x(1 - \text{Pr}(A))^{M-x}. \tag{8}$$

We plan to solve this problem by formulating it as "bins and balls" problem. Note that there are x balls that satisfy the OIM condition. The probability distribution of x is given in Eq. (8). Let's define event B as the event of associating y specific base-station antennas (or bins) to x given mobile stations (or balls) which satisfy the OIM condition and denote it by $\Pr_B(d = y|X = x)$. Note that this probability includes the possibility that some of y antennas are not associated to any of x mobile stations and some correspond to more than one mobile station, i.e., some bins are empty and some bins have more than one ball in them. This conditional probability is equal to

$$\Pr_B(d = y|X = x) = \left(\frac{y}{K}\right)^x, \qquad y \leq K \tag{9}$$

Let's again define another event C as the event of associating y specific base-station antennas to x given mobile stations satisfying the OIM condition so that there is no antenna in this set that is not associated to at least one of the x mobile stations and denote it by $\Pr_C(d = y|X = x)$. This conditional probability can be derived as

$$\Pr_C(d = y|X = x) = \begin{cases} \Pr_B(d = 1|X = x), & y = 1 \\ \Pr_B(d = y|X = x) - \sum_{j=1}^{y-1} \binom{y}{j} \cdot (\Pr_C(d = j|X = x)), & 1 < y \leq \min(x, K) \\ 0. & y > \min(x, K) \end{cases} \tag{10}$$

This equation is derived iteratively and in order to initialize it for $y = 1$, we utilize $\Pr_B(d = 1|X = x)$. Since $\Pr_C(d = y|X = x)$ represents the probability of selecting a specific combination of y antennas, the total possible choices should be considered as another event D. Event D is the event of associating y base-station antennas chosen from all the base-station antennas to the given x mobile stations satisfying the OIM condition so that there is no antenna in this set that is not associated to at least one of the x mobile stations. $\Pr_D(d = y|X = x)$ is computed as

$$\Pr_D(d = y|X = x) = \binom{K}{y} \Pr_C(d = y|X = x). \tag{11}$$

Finally, we derive the expected value of d using this probability.

Theorem 1. *The average multiplexing gain is*

$$D = \mathbf{E}(d) = K\left(1 - \left(1 - \frac{\Pr(A)}{K}\right)^M\right) \tag{12}$$

Proof: See appendix. ■

Hence, for specific values of $\Pr(A)$ and K, the minimum required value of M when D varies would be

$$M = \frac{\log\left(1 - \dfrac{D}{K}\right)}{\log\left(1 - \dfrac{\Pr(A)}{K}\right)} \tag{13}$$

This equation simply shows that when the minimum required number of mobile stations, M, goes to infinity we can even have the multiplexing gain of K.

B. *Numerical Results*

Our simulation results are based on exact analysis of interference management technique. In Fig. 2 we have proved that the simulation data are actually consistent with theoretical results. we have run simulations for different K's and fixed value of $Pr(A)$. Fig. 3 illustrates the minimum required value for M when D varies and for $K = 3$ or 5, $\sigma = 10$ and $\mathrm{INR_{th}} = 1$. As we can see from this result, when the SINR$_{tr}$ requirement increases, the number of mobile stations required to implement this technique increases significantly. Therefore, using capacity approaching techniques such as Turbo code or LDPC that requires very low SINR$_{tr}$ will help to implement this technique with modest number of MS users. Besides, from this figure we notice that there is a tradeoff between the total number of the mobile stations M and the number of the nodes $K - D$ needed to do cooperative communication utilizing technique such as distributed MIMO. For example when $K = 3$, the capacity of the network increases twofold with only 100 mobile stations in the network.

Fig. 4 demonstrates the relationship between the minimum number of mobile stations required for different channel fading conditions. The result clearly shows that as the fading of the channel increases up to a certain value, the minimum required number for M decreases and after that the minimum required number for M increases. As we mentioned it earlier, the new multi-user diversity scheme performs better when the fading strength in channel is so that we can take advantage of both strong and weak channels. When the fading strength is increased from zero up to a specific value, while having strong channels, we can take advantage of the weak channels in the system to reduce the required number of mobile stations. However, after passing a specific value of the channel fading strength, most of the channels in system become weak channels and so we can no longer take advantage of the strong channels and therefore the required number of mobile stations increases. Note that the original multi-user diversity concept performs better by only taking advantage of strong channels.

In order to reduce the minimum required number of mobile users further, we can allow each mobile user to utilize two antennas and try to select one of the antennas that satisfies OIM condition. However, such increase in the number of antennas does not require space-time encoding or decoding. From base station point of view, the additional antenna for each mobile user is equivalent of increasing the number of mobile users twofold or equivalently, the actual minimum number of mobile users required to achieve a multiplexing gain is reduced by a factor of 2.

C. Scaling Law Analysis

In this subsection, we will prove that the sum-rate of the proposed scheme under OIM condition achieves the optimum asymptotic DPC capacity, i.e. $K \log\log M$. We can use equation (13) to minimize the required number of mobile users M in terms of $\Pr(A)$ when the average multiplexing gain is fixed to a constant D. Since the numerator in this equation is negative, in order to minimize M we need just to minimize $(\Pr(A))^{-1}$ such that the SINR_{tr} condition in Eq. (5) is satisfied.

$$\begin{aligned} &\text{minimize} \qquad (\Pr(A))^{-1} &(14)\\ &\text{subject to} \qquad \text{SINR}_{\text{tr}} = \frac{\text{SNR}_{\text{tr}}}{(K-1)\text{INR}_{\text{tr}}+1} &(15)\end{aligned}$$

This optimization problem can be rewritten as

$$\min_{Eq.(15)} \left((\Pr(A))^{-1}\right) = \frac{1}{K} \min_{Eq.(15)} \left(\frac{e^{\frac{\text{SNR}_{\text{tr}}}{\sigma}}}{\left(1-e^{-\frac{\text{INR}_{\text{tr}}}{\sigma}}\right)^{K-1}} \right) \overset{(a)}{=} \frac{1}{K} e^{\frac{\text{SINR}_{\text{tr}}}{\sigma}} \min_{\text{INR}_{\text{tr}}} \left(\frac{e^{(K-1)\frac{\text{SINR}_{\text{tr}}\text{INR}_{\text{tr}}}{\sigma}}}{\left(1-e^{-\frac{\text{INR}_{\text{tr}}}{\sigma}}\right)^{K-1}} \right) \tag{16}$$

We derive the equality (a) by replacing SNR_{tr} with INR_{tr} and SINR_{tr} using Eq. (15). Since in practice a successful communication occurs when we have a predetermined minimum value for SINR_{tr}, therefore we fix the value of SINR_{tr} and attempt to optimize the above equation based on INR_{tr}. The minimization can be done by taking the first derivative with respect to INR_{tr} and making it equal to zero. The solution for INR^*_{tr} is

$$\text{INR}^*_{\text{tr}} = \sigma \log\left(1 + \frac{1}{\text{SINR}_{\text{tr}}}\right) \tag{17}$$

We can check that for this optimal value of INR^*_{tr} the second derivative has in fact a positive value and so this is in the minimizing point. The maximum value of $\Pr^*(A)$ is found to be

$$\Pr^*(A) = K \exp\left(-\frac{\text{SINR}_{\text{tr}}}{\sigma}\right) \frac{\text{SINR}_{\text{tr}}{}^{(K-1)\text{SINR}_{\text{tr}}}}{(1+\text{SINR}_{\text{tr}})^{(K-1)(1+\text{SINR}_{\text{tr}})}} \tag{18}$$

Minimum value of M also can be found by plugging $\Pr^*(A)$ into the equation (13)

$$M^* = \frac{\log\left(1-\dfrac{D}{K}\right)}{\log\left(1-\dfrac{\Pr^*(A)}{K}\right)} < -K\log\left(1-\frac{D}{K}\right)\frac{1}{\Pr^*(A)} \tag{19}$$

Now we investigate the asymptotic behavior of the network (i.e. $M \to \infty$) and try to compute the maximum achievable capacity and scaling laws for this scheme. When M tends to infinity, $\Pr^*(A) \to 0$ and from equation (18), SINR_{tr} goes to infinity too. Therefore

$$\Omega\left(\frac{M}{-K\log\left(1-\frac{D}{K}\right)}\right) = \lim_{M\to\infty}\frac{1}{\Pr^*(A)} = \lim_{\text{SINR}_{\text{tr}}\to\infty}\frac{1}{K}\left(1+\text{SINR}_{\text{tr}}\right)^{K-1} e^{\frac{\text{SINR}_{\text{tr}}}{\sigma}}\left(\frac{1+\text{SINR}_{\text{tr}}}{\text{SINR}_{\text{tr}}}\right)^{\text{SINR}_{\text{tr}}(K-1)}$$

$$= \frac{1}{K}e^{K-1}\lim_{\text{SINR}_{\text{tr}}\to\infty} e^{\frac{\text{SINR}_{\text{tr}}}{\sigma}}\left(1+\text{SINR}_{\text{tr}}\right)^{K-1} = O\left(\frac{1}{K}e^{K-1}e^{2\frac{\text{SINR}_{\text{tr}}}{\sigma}}\right)^{3} \tag{20}$$

The lower bound of SINR_{tr} is asymptotically computed as

$$\lim_{M\to\infty}\text{SINR}_{\text{tr}}^{\max} = \Omega\left(\frac{\sigma}{2}\log\left(\frac{-1}{e^{K-1}\log\left(1-\frac{D}{K}\right)}M\right)\right) = \Omega(\log M). \tag{21}$$

Thus, $\text{SINR}_{\text{tr}}^{\max}$ scales at least with $\Omega(\log M)$. If we assume $\text{SINR}_{\text{tr}} = \Theta\left(\frac{\sigma}{4}\log M\right) = \Theta\left(\frac{\sigma}{2}\log M^{1/2}\right)$. and with Eq. (20), we have $M^{1/2} = O\left(-e^{K-1}\log\left(1-\frac{D}{K}\right)\right) = O\left(Ke^{K}\right) = O\left(e^{2K}\right)$. This result implies that $K = \Omega\left(\log M\right)$ is achievable. Then the scaling laws of OIM scheme is

$$R_{\text{proposed}} = \Omega(K\log\log M). \tag{22}$$

We have proved that OIM achieves DPC asymptotic capacity.

$$\lim_{M\to\infty} R_{\text{OIM}} = R_{\text{DPC}} = \Theta(K\log\log M) \tag{23}$$

This result implies that

$$\lim_{M\to\infty}\text{SINR}_{\text{tr}} = \Theta(\log M). \tag{24}$$

[3]Given two functions $f(n)$ and $g(n)$. We say that $f = O(g(n))$ if $\sup_n(f(n)/g(n)) < \infty$. We say that $f(n) = \Omega(g(n))$ if $g(n) = O(f(n))$. If both $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$, then we say $f(n) = \Theta(g(n))$.

Our objective is to show, via simulation, that when SINR_{tr} grows proportional to $\Theta(\log M)$, the maximum multiplexing gain of k can be achieved when M tends to infinity. Let's define SINR_{tr} as

$$\text{SINR}_{\text{tr}} = \frac{\sigma}{c_0} \log \left(\left(\frac{1}{e} \right)^{K-1} M \right). \tag{25}$$

where c_0 is a constant value. In practical cellular systems, it is possible that the minimum number of mobile users may not be available in a cell. Note that it is easy to show that for any value of K, M and σ, the designer can select the appropriate value for SINR_{tr} such that the maximum multiplexing gain is achieved at the expense of reduced rate for each individual mobile user, i.e., $D = K$.

Fig. 5 confirms that when SINR_{tr} grows logarithmically with M, this approach achieves the maximum multiplexing gain for different values of c_o based on Eq. (25).

It is noteworthy to point out that when the value of σ is small or equivalently, if the channel fading is not strong, then OIM cannot converge to the maximum multiplexing gain of K rapidly. In the new multiuser diversity scheme that is introduced in this paper, both strong and weak channels are important. When the fading coefficient σ becomes stronger up to a specific value, this technique performs better. After passing this specific value of channel fading strength, the system is less probable to have strong channels and so the OIM can not take advantage of the strong channels. Thus, for a constant number of mobile stations, the average multiplexing gain after passing that specific value of σ decreases. Fig. 6 illustrates this important point.

When $K = 1$, then our approach is similar to that of [1]. Moreover if $M \to \infty$ and $D = K$, then our scheme has the same asymptotic scaling laws capacity result as that of [3]. The cost of the proposed scheme is the need for a minimum number of mobile stations, M. In most practical cellular systems, in any given frequency and time inside a cell, there is only one assigned mobile station while this technique suggests that we can have up to the number of base-station antennas utilizing the same spectrum at the same time with no bandwidth expansion. Clearly, this approach can increase the capacity of wireless cellular networks significantly. This gain is achieved with modest feedback requirement which is proportional to the number of antennas at the base station.

D. Feedback requirements

A natural question regarding our OIM scheme is what the number of MS users that satisfy the interference management criterion is? Clearly, this number is a random variable, which we denote by

X. We will prove that this value is at most K with probability arbitrarily close to one if the network parameters are appropriately selected. More specifically, the probability that $X \leq K$ MS users satisfy the interference management criteria denoted as η can be arbitrarily close to 1 if we select proper SINR_{tr} based on network parameters such as fading parameter σ and M.

For any mobile station, the probability that it satisfies the interference management condition is $K \times$ $\Pr(A)$, i.e., the mobile station has a very strong channel with a single base station antenna and a very weak channel (deep fade) with all other base station antennas. The number of the mobile stations satisfying the interference management criteria is a random variable X satisfying binomial distribution whose probability density function (pdf) is given by Eq. (8). Therefore, the cumulative distribution function can be expressed as

$$\Pr(X \leq K) = \sum_{i=0}^{K} \binom{M}{i} (P(A))^i (1 - P(A))^{M-i} \geq \eta, \tag{26}$$

where $0 < \eta < 1$ can be arbitrarily close to 1 , i.e., $\eta = 99\%$.

It will be shown that the number of mobile users X (which is a random variable) with OIM constraint is always smaller than K with probability arbitrarily close to 1 with the correct choice of network parameters. Note that, for any value of K, M and σ, the designer can select the appropriate value for SINR_{tr} such that with probability close to 1 the value of random variable X is less than K as numerically shown in Fig. 7. Given that the number of active MSs in a cell is known to the BS, the BS can adjust the SINR_{tr} value such that the number of MS users qualifying the OIM condition does not increase significantly. This is a significant improvement compared to the dirty paper coding or techniques introduced in [2], [3], which require $K \times M$ and M CSI feedback information respectively. When M increases, the feedback information also increases accordingly. However, OIM requires $\Theta(K)$ CSI feedback regardless of the number of mobile stations with probability arbitrarily close to 1 as long as the SINR_{tr} is adjusted appropriately. For any values of K, M and σ, the designer can select the appropriate value for SINR_{tr} such that with probability close to 1 the value of random variable X is less than K as shown in Fig. 7.

V. Antenna Selection Algorithm

Antenna selection diversity [16] is a low-cost low-complexity alternative to capture many of the advantages of MIMO systems by choosing the path with the highest SNR among all channels between the base station antennas and a mobile user. In our approach, we assume that an average multiplexing gain of D is desired while there are actually K antennas at the base station such that $K \gg D$. Further, we define a new parameter L such that it is the minimum number of channels between a single base station antenna and a mobile user that their SNR is below INR_{tr}. Since the number of antennas at the base station is now assumed to be a large number, therefore the number of pilot signals that should be transmitted by the base station antennas increases linearly with K.

Unlike the original OIM technique described in Section III, we no longer require the mobile users to send their information when they have one strong channel and $K-1$ weak channels. Under the new scheme, each mobile user that has at least one strong channel and at least L weak channels, sends its information to the base station. In our system model (Fig. 1) we also capture the case where $D < L$, as the dotted / weak channels between the users and the base station antennas may expand the derived multiplexing gain. Under the new scheme, the mobile users should notify which channels are strong, which ones are weak and perhaps some channels are neither strong nor weak channel. Hence, each mobile user responds with more additional information than the original OIM technique. Notice that it is possible for a mobile station to have more than one strong channel and it is the task of the base station to choose one strong channel based on its total information and sends its data over that channel.

Having large number of antennas at the base station is a reasonable assumption and we need to select a subset of these antennas such that an equal number of mobile users have OIM capability with respect to these antennas. Each mobile user that has at least L weak channels and at least one strong channel sends its information to the base station. Our objective is to search amongst all the mobile users that send their feedback information to the base station and select the set of mobile users with maximum multiplexing gain. In this set, every mobile user should have weak channels with all of the other antennas that have strong channels associated to other mobile stations.

There are mainly two ways to carry out this search. The optimum search is based on the exhaustive search among all possible combinations of mobile users such that it has the maximum multiplexing gain. This exhaustive search can be carried in practice using backtracking algorithm [17]. Backtracking is a general algorithm for finding all (or some) solutions to some computational problem, that incrementally builds candidates to the solutions, and abandons each partial candidate as soon as it determines that it cannot possibly lead to a valid or the best solution. This algorithm actually searches among all the different combinations of channels and selects the ones that combined result provides the maximum parallel transmissions. The disadvantage of the optimal solution is significant computational complexity at the base station and the time required to complete the search such that it is not practical to implement the optimal search.

Our proposed sub-optimal approach is inspired based on antenna selection techniques. In this approach, at first we search among the mobile stations that have reported their OIM information to the base station and we select the mobile station with maximum number of weak antennas. We create a table with the number of antennas related to this mobile user as values of the first row[4]. Then based on this set, we choose the next mobile user that has the largest subset of this set. Note that the second mobile user satisfies the OIM constraint for this subset. We continue this algorithm until we find a group of mobile users that satisfies this scheme and have the largest multiplexing gain. In the original OIM approach, we have proved analytically that the number of mobile users sending feedback to the base station is less than K with probability going to 1. The proposed antenna selection scheme will increase the feedback and there is clearly a tradeoff between minimum required mobile users for a given multiplexing gain and feedback requirement.

A. Description of the problem

We first review the method developed in Section IV and discuss its main drawback which is the logarithmic relationship between K and M. The main contribution of this section is to reduce the minimum required number of mobile users M to achieve the maximum multiplexing gain.

The communication in OIM technique takes place in two phases. During the first phase of communication, the base-station antennas sequentially transmit K pilot signals. In this period, all the mobile stations listen to these known messages. After the last pilot signal is transmitted, mobile stations evaluate the SNR for each antenna. If the SNR for only one transmit antenna is greater than a pre-determined threshold SNR_{tr} and below another pre-determined threshold of INR_{tr} for the remaining $K-1$ antennas, that particular mobile station will select that particular antenna at the base station.

Given that more than one mobile station may be found with this property, in the second phase of communication, the mobile stations notify the base station that they have the required criterion to receive packets during the remaining time period of T. We assume there is a channel access protocol for these

[4]If there are more than one mobile user with the largest number of antennas satisfying the OIM constraint, then we choose all of them and do the parallel search for all of them to find the best solution.

mobile stations to contact the base station and also the base station will resolve the case when two mobile stations have similar property for the same antenna by some protocol.

Note that, if we choose appropriate values for SNR_{tr} and INR_{tr} such that $\text{SNR}_{tr} \gg \text{INR}_{tr}$, then the base station can simultaneously transmit different packets from its antennas to different mobile stations. The mobile stations only receive their respective packets with a strong signal and can treat the rest of the packets as noise. The value of SNR_{tr} (or INR_{tr}) can be selected as high (or low) as required for a given system, as long as M is large enough.

Suppose that there are on average D antennas that can be matched to corresponding mobile stations with the above property. Further, we select another $K-D$ mobile stations such that they do not have the above property and require cooperation among themselves to decode the $K-D$ data streams. Note that these $K-D$ nodes can potentially operate similar to a distributed MIMO system.

It has been proved in Section IV that $K = \Theta(\log M)$ in order to achieve DPC asymptotic capacity. Similar relationship was also reported for random beamforming technique in [3]. However, from practical point of view, usually there exists far less number of mobile users in a base station. It is important to achieve average multiplexing gain of D for some small values of M. This paper introduces an antenna selection technique that achieves this goal at the expense of modest increase in the number of feedback requirement and some additional complexity at the base station. We will demonstrate later with simulation that the minimum required number of mobile users decreases significantly by using this new antenna selection technique.

B. Theoretical Bounds of Antenna Selection

In this section, we investigate the theoretical aspects of antenna selection for both of the optimal and sub-optimal search techniques. In the antenna selection scenario we define another event A' which is the event that a specific mobile user satisfies the OIM with antenna selection which means that it has at least one strong channel and at least L weak channels. The probability of this event is

$$\begin{aligned}\Pr(A') &= \Pr\left(\text{Strong} \geq 1, \text{Weak} \geq L\right) = \sum_{i=L}^{K-1} \sum_{j=1}^{K-i} \Pr\left(\text{Strong} = j, \text{Weak} = i\right) \\ &= \sum_{i=L}^{K-1} \sum_{j=1}^{K-i} \binom{K}{j} \binom{K-j}{i} \left(\int_0^{\text{INR}_{\text{tr}}} p(z)dz\right)^i \left(\int_{\text{SNR}_{\text{tr}}}^{\infty} p(z)dz\right)^j \left(\int_{\text{INR}_{\text{tr}}}^{\text{SNR}_{\text{tr}}} p(z)dz\right)^{K-i-j} \end{aligned} \tag{27}$$

We also define the event B', as the event of associating y specific antennas to the mobile stations who are satisfying in the OIM condition for antenna selection. This event is defined like the event B. We can also define the events C' and D' similar to the events ,C and D. The corresponding probabilities can therefore be found from equations like (9), (10) and (11) if we use event A' instead of event A. The main difference with the original OIM that we discussed earlier is that unlike the original OIM, in the antenna selection scenario not all of the OIM reported mobile stations can be used for multiplexing. In the antenna selection scenario, just a number of mobile stations that have reported their OIM information are selected and can start transmission. In order to incorporate this difference into our model we define a new event E for the antenna selection scenario which is the event of having multiplexing gain of z when x mobile stations are satisfying in the OIM conditions. If we define the random variable Y as the number of associated antennas to these x OIM reported mobile stations so that there is no antenna that is not associated to at least one of the mobile-stations then the average multiplexing gain can be found as

$$D_{AS} = \sum_{x=1}^{M} \sum_{y=1}^{K} \sum_{z=0}^{y} z \mathrm{Pr}_E(d = z | X = x, Y = y) \mathrm{Pr}_{D'}(Y = y | X = x) \mathrm{Pr}_{A'}(X = x) \tag{28}$$

Note that the only difference between the antenna selection and the original OIM in terms of the average multiplexing gain is the event E. Therefore, if we assume that the probability of this event is one or in other words if we assume that all of the OIM reporting mobile stations are participating in the multiplexing without any search, since events A',B',C' and D' are similar to the events A,B,C and D the equations (7), (9), (10) and (11) still hold for events A',B',C' and D' and therefore in this case also we can use theorem 1 to find the expected value of multiplexing gain.

$$D_{UB} = K \left(1 - \left(1 - \frac{\mathrm{Pr}(A')}{K} \right)^M \right) \tag{29}$$

Hence the multiplexing gain from the above equation acts as an upper bound for both of the optimal and sub-optimal search techniques. Note that the only difference between the optimal and sub-optimal search techniques from the analytical point of view is the probability of event E, $\mathrm{Pr}_E(d = z | X = x, Y = y)$, which is actually a very complicated probability to find in case of optimal search and even in sub-optimal scenario.

In order to find a lower bound for the average multiplexing gain suppose that we have only $L + 1$ antennas at the base station. The antenna selection is simply reduced to the original OIM in this case and according to theorem 1, average multiplexing gain in this case is

$$D_{LB} = (L+1)\left(1 - \left(1 - \frac{\Pr(A')}{L+1}\right)^M\right) \tag{30}$$

Regardless of the search technique that we use, if now we increase the number of antennas in the base station, the average multiplexing gain would not decrease since we are increasing the probability of finding mobile stations which satisfy in the OIM conditions for antenna selection. Therefore D_{LB} acts as a lower bound for the antenna selection scenario. So we have

$$D_{LB} \leq D_{SubOptimal} \leq D_{Optimal} \leq D_{UB} \tag{31}$$

C. Numerical Results

Fig. 8 compares our analytical lower and upper bounds of M with simulation results as a function of D when $\text{SNR}_{tr} = 40$, $K = 10$, $\text{INR}_{tr} = 2$ and $\sigma = 10$. The simulation results are plotted for two different choices of L. We have also plotted our theoretical upper and lower bounds to compare them with simulation results. This figure shows that our upper bound is good bound while the lower bound is a very loose bound and needs to be improved upon. When the number of mobile stations is increased to very large numbers the upper bound also becomes a loose upper bound according to simulation results. Proposing better lower and upper bounds remains as a future work to this contribution. From this figure also we can verify that the simulation results of sub-optimal search are close to the optimal exhaustive search. However, as the difference between K and L increases, the difference between the optimal and sub-optimal search is also increased. This point can also be verified from Fig. 9.

Fig. 9 compares the performance of our sub-optimal search to that of the optimal search for different values of L and when $\text{SNR}_{tr} = 40$, $K = 10$, $\text{INR}_{tr} = 2$. If L is equal to 6 or 7 the difference between the optimal and sub-optimal search techniques is so small that can not be distinguished from the figure. However, when L is reduced this difference becomes noticeable. Especially for $L = 4$, this difference becomes so high that we can say that the sub-optimal search is saturated to the value of 1.3.

It is clear from these results that our proposed antenna selection technique reduces the minimum required number of mobile users significantly. For example for $D = 1$ and $K = 10$ when $\text{SNR}_{tr} = 40$, the optimal search only requires 35 mobile users while the suboptimal search requires 120 or 600 mobile users when $L = 2$ or $L = 3$ respectively. When $K = 10$, $D = 1$ and for the same set of parameters as before, the optimal search requires 17 users while the suboptimal search requires 19, 26, and 53 users for $L = 1$,

$L = 2$, and $L = 3$ respectively.

One important question is that why a multiplexing of one, i.e. $D = 1$, is important while one may think that we only need a single mobile user to achieve this gain. The answer relies on the following fact. In OIM scheme a multiplexing gain of one means that there are at least one antenna at the base station that has a deep fade with that particular user. Since the selection of mobile users that have OIM capability is completely random and depends on the time-variant fading nature of the channel, then the problem of fairness becomes a major issue. However, this technique can be incorporated into the current wireless standards. For example, in a TDMA system we can use the L antennas at the base station for regular TDMA communication since their signal is extremely weak at the receiver of mobile users that are participating in OIM, i.e., they are not interfering with those transmissions. On the other hand, nodes utilizing OIM can affect TDMA receiver but we will show in Section VI that they can be orthogonalized at the TDMA receiver side using a technique that does not require any channel knowledge. The technique is fundamentally different from beamforming concept.

Table V-C demonstrate the simulation results for the antenna selection technique when $k = 10$ and 5 respectively. The number in the second, third and forth columns represent the number of mobile users required to achieve multiplexing gain of 1, 2, and 3 respectively. The values inside parenthesis are the number of nodes that feedback their information to the base station. From these tables, it can be concluded that in general few nodes usually send their feedback information which makes this technique practical. Further, as K increases the minimum required number of mobile users to achieve certain multiplexing gain decreases. For example, for a multiplexing gain of 2, i.e., $D = 2$, we need 336 and 550 mobile users when K is equal to 10 and 5 respectively when optimal search is conducted. For a multiplexing gain of 1, we only need 15 and 30 mobile users when K is equal to 10 and 5 respectively. These small numbers of mobile users are quite practical in wireless cellular networks. These results are obtained based on the network parameters of $\text{SNR}_{tr} = 40, \text{INR}_{tr} = 2,$ and $\sigma = 10$.

VI. Practical Related Issues

There are still two important issues with OIM scheme. One is the fact that in current cellular systems, the assignment of users is based on pre-determined schemes such as time-division. The other issue is the fairness problem which is important so that all users have minimum access to the channel. For example, some mobile users may be close to the base station for a long period of time with line of sight. In the following section, we provide an approach to incorporate OIM scheme into existing TDMA systems to

L	(Optimal)	(Sub-Optimal)
$L = 1$	15	20
$L = 2$	23	29
$L = 3$	48	61
$L = 4$	151	181
$L = 5$	651	780
$L = 6$	4400	4700

TABLE I
MINIMUM REQUIRED NUMBER OF MOBILE STATIONS IN ORDER TO HAVE $(D = 1)$ WHEN $(K = 10)$

assure fairness in terms of accessing the channel for all users. The extension of this approach to other standards such as CDMA is straightforward.

A. Fairness under TDMA Scheme

In this section, we propose one practical approach for existing GSM cellular systems to guarantee the fairness and Quality of Service (QoS) for TDMA users while allowing other users to take advantage of OIM scheme without interrupting the main user. For any TDMA user, the received signal vector can be written as

$$\mathbf{R}_{\text{TDMA}}^{\text{T}} = \mathbf{S}_{\text{TDMA}}^{\text{T}} h_{\text{TDMA}} + \sum_{i=1}^{d} S_i h_i \mathbf{V}^{\text{T}} + \mathbf{n}^{\text{T}}, \tag{32}$$

where $\mathbf{R}_{\text{TDMA}}$ and $\mathbf{S}_{\text{TDMA}}$ are the TDMA signal vectors received by a mobile user and transmitted by an antenna in the base station respectively, provided that this antenna does not participate in OIM scheme, i.e., $d < K$. The superscript T represents transpose of a vector, S_i and V^T are the signal transmitted by the antenna that is utilizing OIM scheme and a vector with unit weight that will be multiplied by each signal S_i respectively. $\mathbf{n}$ is the additive Gaussian noise vector with zero mean i.i.d. elements and variance of σ_n. h_{TDMA} and h_i are the CSI between base station and mobile users that are participating in TDMA and OIM scheme respectively.

At the receiver, we multiply the received vector by a vector $\mathbf{U}$. This vector is orthonormal to $\mathbf{V}$, i.e., $\mathbf{U}\mathbf{V}^T = 0$. Thus, the received signal will be equal to

$$\begin{aligned} \mathbf{U}\mathbf{R}_{\text{TDMA}}^{\text{T}} &= \mathbf{U}\mathbf{S}_{\text{TDMA}}^{\text{T}} h_{\text{TDMA}} + \sum_{i=1}^{d} s_i h_i \mathbf{U}\mathbf{V}^T + \mathbf{U}\mathbf{n}^{\text{T}} \\ &= \mathbf{U}\mathbf{S}_{\text{TDMA}}^{\text{T}} h_{\text{TDMA}} + \mathbf{n}' \end{aligned} \tag{33}$$

Note that the signals transmitted utilizing OIM scheme are now multiplied by this new vector $\mathbf{V}$. Even though the TDMA user does not have the OIM capability and therefore other users are interfering with this user, but when we multiply the orthogonal vector $\mathbf{U}$ by the received vector, we can get rid of these interfering signals. Further, the vector $\mathbf{V}$ does not have any relationship with CSI and we are not really using any beamforming scheme. We will later describe the criterion for selecting this vector. For block fading channel, this vector only requires to be of length 2. We notice that by the new transmission policy, we have reduced the actual rate of signals participating in OIM scheme by a factor proportional to the length of vector $\mathbf{V}$. However, the rate of TDMA signal is still one symbol per channel use.

If the wireless channel is block fading, then $\mathbf{U} = [u_1, u_2]$ and $\mathbf{V} = [v_1, v_2]$ are enough for implementation. However, for fast fading the implementation of this technique is more complicated and we omit that here. For the rest of the paper, we assume that the QPSK signals are used for transmission. Since the TDMA vector signal is multiplied by $\mathbf{U}$ as shown in Eq. (33), then our criterion for designing this signal is based on the condition that the combination of multiple QPSK signals results in optimum separation of points in the two-dimensional space. This condition will help in decoding performance of the received signal. Note that again this vector is not really a function of channel matrix as it is common in beamforming techniques.

For a combination of two QPSK signals, an appropriate choice would be a 16-QAM signal. It has been shown in [18] that any combination of QPSK signals can be mapped into M-QAM signals. For the specific case of 16-QAM, we have

$$\text{16-QAM} = \sum_{i=0}^{1} 2^i \left(\frac{\sqrt{2}}{2}\right) (j^{x_i}) \exp\left(\frac{\pi j}{4}\right) \tag{34}$$

where $x_i \in Z_4 = \{0, 1, 2, 3\}$. The QPSK constellation can be realized as QPSK $= j^{x_i}$. Thus, one can use shift and rotation operation to create M-QAM constellations from QPSK symbols. It is easy from Eq. (34) to show that the normalized values of vectors $\mathbf{U}$ and $\mathbf{V}$ are

$$\mathbf{U} = \sqrt{\frac{2}{5}} \exp\left(\frac{\pi j}{4}\right) \left[\frac{\sqrt{2}}{2}, \sqrt{2}\right] \tag{35}$$

and

$$\mathbf{V} = \sqrt{\frac{2}{5}} \left[\sqrt{2}, -\frac{\sqrt{2}}{2}\right] \tag{36}$$

respectively. Since the vector $\mathbf{U}$ is normalized, then the variance of Gaussian noise remains the same.

Note that with this signalling at the base station, the Quality of Service (QoS) and fairness for all users are guaranteed in a time-division approach while other users can utilize the spectrum taking advantage of OIM scheme.

B. Signaling requirement

One of the main advantages of this technique is the fact that, by taking advantage of multiuser diversity, we reduce a distributed MIMO system in the downlink of wireless cellular networks into a group of parallel single-input single output (SISO) systems. For this reason, all challenges and complexities related to space-time signal processing design can be replaced by simple point-to-point communications while achieving maximum capacity as long as the number of mobile stations is adequate. This significant simplification of the signalling in the cellular systems is an additional advantage of our OIM scheme.

VII. Conclusion

In this paper, we proposed an Opportunistic Interference Management (OIM) technique that asymptotically achieves DPC capacity with minimum feedback by taking advantage of the multiuser diversity and fading channel in the network to minimize the negative effects of interference in wireless cellular networks. Besides, this technique requires simple encoding and decoding for the downlink of wireless cellular networks similar to that of point-to-point communications. Furthermore, we investigated the effect of using antenna selection techniques for OIM scheme to reduce the minimum required mobile users for a given multiplexing gain in a multi-user environment. It has been shown through simulation that with as few as 19 mobile users one can achieve some multiplexing gain in the downlink of wireless cellular systems Finally, a practical way to guarantee the fairness in existing TDMA cellular systems is proposed.

Our future work will concentrate on improving the minimum required mobile users for higher multiplexing gains than what we have derived with antenna selection technique. Extension of this technique to ad hoc networks will be also investigated in the future.

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Appendix

A. Proof of theorem 1

Lemma 1. *Suppose a_n is a recursive sequence with $a_1 = 1$ and*

$$a_n = n^x - \sum_{j=1}^{n-1} \binom{n}{j} a_j \quad for \quad n > 1. \tag{37}$$

Where x is an arbitrary real number. Then

$$a_n = \sum_{j=1}^{n} \binom{n}{j} (-1)^{n-j} j^x \tag{38}$$

Proof: We use induction to prove the lemma. For $n = 1$, the correctness of the lemma can be easily verified. Suppose for $i = 1, 2, ..., n-1$ the lemma is established. Our goal is to show that it is also established for $i = n$. If we plug the corresponding values of a_i from (38) for $i = 1, 2, ..., n-1$ and use (37) we have

$$\begin{aligned}
a_n &= n^x - \sum_{i=1}^{n-1} \binom{n}{i} a_i \\
&= n^x - \sum_{i=1}^{n-1} \sum_{j=1}^{i} \binom{n}{i} \binom{i}{j} (-1)^{i-j} j^x \\
&= n^x - \sum_{j=1}^{n-1} \sum_{i=j}^{n-1} \binom{n}{i} \binom{i}{j} (-1)^{i-j} j^x \\
&= n^x - \sum_{j=1}^{n-1} \sum_{i=j}^{n-1} \binom{n}{j} \binom{n-j}{i-j} (-1)^{i-j} j^x \\
&= n^x - \sum_{j=1}^{n-1} \binom{n}{j} j^x \sum_{i=j}^{n-1} \binom{n-j}{i-j} (-1)^{i-j} \\
&= n^x - \sum_{j=1}^{n-1} \binom{n}{j} j^x \sum_{m=0}^{n-j-1} \binom{n-j}{m} (-1)^{m} \\
&= n^x + \sum_{j=1}^{n-1} \binom{n}{j} (-1)^{n-j} j^x \\
&= \sum_{j=1}^{n} \binom{n}{j} (-1)^{n-j} j^x
\end{aligned}$$

■

We will use this lemma to prove theorem 1. Let $a_y = K^x \mathrm{Pr}_C(d = y|X = x)$ and use (10) to write

$$
\begin{aligned}
a_1 &= 1 \\
a_y &= y^x - \sum_{j=1}^{y-1} \binom{y}{j} a_j \quad \text{for} \quad y > 1
\end{aligned}
\tag{39}
$$

Using lemma 1 the recursive sequence a_y can be rewritten as

$$
a_y = \sum_{j=1}^{y} \binom{y}{j} (-1)^{y-j} j^x \quad \text{for} \qquad 1 \le y \le \min(x, K) \tag{40}
$$

Therefore, we have

$$
\mathrm{Pr}_C(d = y|X = x) = \sum_{j=1}^{y} \binom{y}{j} (-1)^{y-j} \left(\frac{j}{K}\right)^x \quad \text{for} \qquad 1 \le y \le \min(x, K) \tag{41}
$$

$$
\mathrm{Pr}_D(d = y|X = x) = \sum_{j=1}^{y} \binom{K}{y} \binom{y}{j} (-1)^{y-j} \left(\frac{j}{K}\right)^x \quad \text{for} \qquad 1 \le y \le \min(x, K) \tag{42}
$$

Equation (10) implies that $\mathrm{Pr}_C(d = y|X = x)$ and $\mathrm{Pr}_D(d = y|X = x)$ are zero for $y > \min(x, K)$. Now that we have closed form formulas for these probabilities we can compute the expected value of d.

$$
\begin{aligned}
\mathbf{E}(d|X = x) &= \sum_{y=1}^{K} y \mathrm{Pr}_D(d = y|X = x) \\
&= \sum_{y=1}^{\min(x,K)} \sum_{j=1}^{y} y \binom{K}{y} \binom{y}{j} (-1)^{y-j} \left(\frac{j}{K}\right)^x \\
&= \sum_{j=1}^{\min(x,K)} \sum_{y=j}^{\min(x,K)} y \binom{K}{y} \binom{y}{j} (-1)^{y-j} \left(\frac{j}{K}\right)^x \\
&= \sum_{j=1}^{\min(x,K)} \sum_{y=j}^{\min(x,K)} y \binom{K}{j} \binom{K-j}{y-j} (-1)^{y-j} \left(\frac{j}{K}\right)^x \\
&= \sum_{j=1}^{\min(x,K)} \binom{K}{j} \left(\frac{j}{K}\right)^x \sum_{y=j}^{\min(x,K)} y \binom{K-j}{y-j} (-1)^{y-j} \\
&= \sum_{j=1}^{\min(x,K)} \binom{K}{j} \left(\frac{j}{K}\right)^x \sum_{m=0}^{\min(x,K)-j} (m+j) \binom{K-j}{m} (-1)^m
\end{aligned}
\tag{43}
$$

If $x \geq K$ then $\min(x,K) = K$ and thus in the above derivation we can use the identity

$$\sum_{m=0}^{K-j}(m+j)\binom{K-j}{m}(-1)^m = j\delta[K-j] - \delta[K-j-1] \tag{44}$$

to write

$$\begin{aligned}\mathbf{E}(d|X=x) &= \sum_{j=1}^{K}\binom{K}{j}\left(\frac{j}{K}\right)^x (j\delta[K-j] - \delta[K-j-1]) \\ &= K - K\left(\frac{K-1}{K}\right)^x \end{aligned}\tag{45}$$

However, if $x < K$ Then $\min(x,K) = x$ and we can use the identity

$$\sum_{m=0}^{x-j}(m+j)\binom{K-j}{m}(-1)^m = \frac{(-1)^{x-j}(xK-xj-j)}{(K-j-1)}\binom{K-j-1}{x-j} \tag{46}$$

to write:

$$\begin{aligned}\mathbf{E}(d|X=x) &= \sum_{j=1}^{x}\binom{K}{j}\left(\frac{j}{K}\right)^x \left(\frac{(-1)^{x-j}(xK-xj-j)}{(K-j-1)}\binom{K-j-1}{x-j}\right) \\ &= K - K\left(\frac{K-1}{K}\right)^x \end{aligned}\tag{47}$$

All of the identities (44), (46), (47) can be verified by a mathematical software like Mathematica. Till now we have proved that for every x

$$\mathbf{E}(d|X=x) = K - K\left(\frac{K-1}{K}\right)^x \tag{48}$$

Now we proceed to find D.

$$\begin{aligned}D = \mathbf{E}(d) &= \sum_{x=1}^{M}\sum_{y=1}^{K} y\mathbf{Pr}_D(d=y|X=x)\mathrm{Pr}_A(X=x) \\ &= \sum_{x=1}^{M}\mathbf{E}(d|X=x)\mathrm{Pr}_A(X=x) \\ &= \sum_{x=1}^{M}\left(K - K\left(\frac{K-1}{K}\right)^x\right)\binom{M}{x}(\mathrm{Pr}(A))^x(1-\mathrm{Pr}(A))^{M-x} \\ &= K\left(1-\left(1-\frac{\mathrm{Pr}(A)}{K}\right)^M\right) \end{aligned}\tag{49}$$

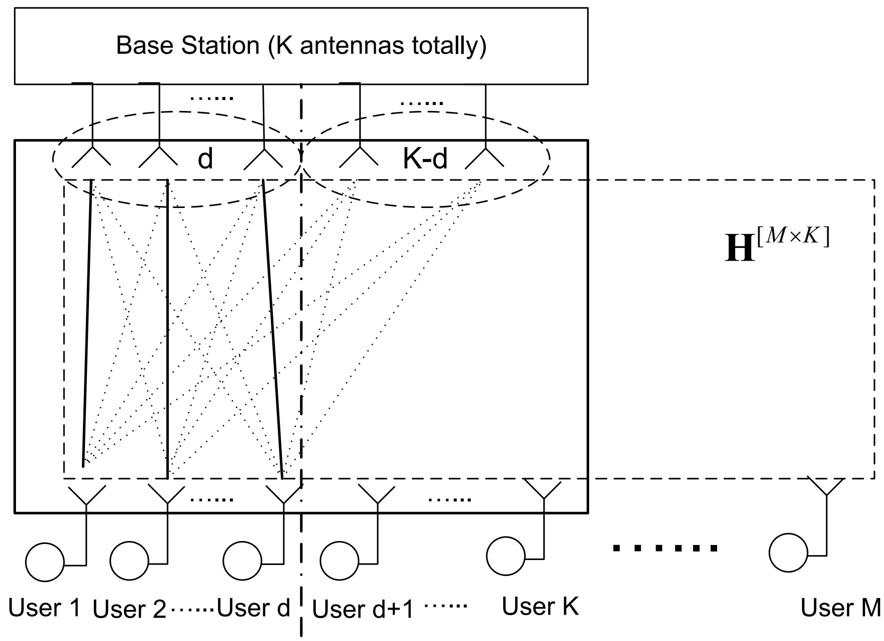

Fig. 1. Wireless cellular network model

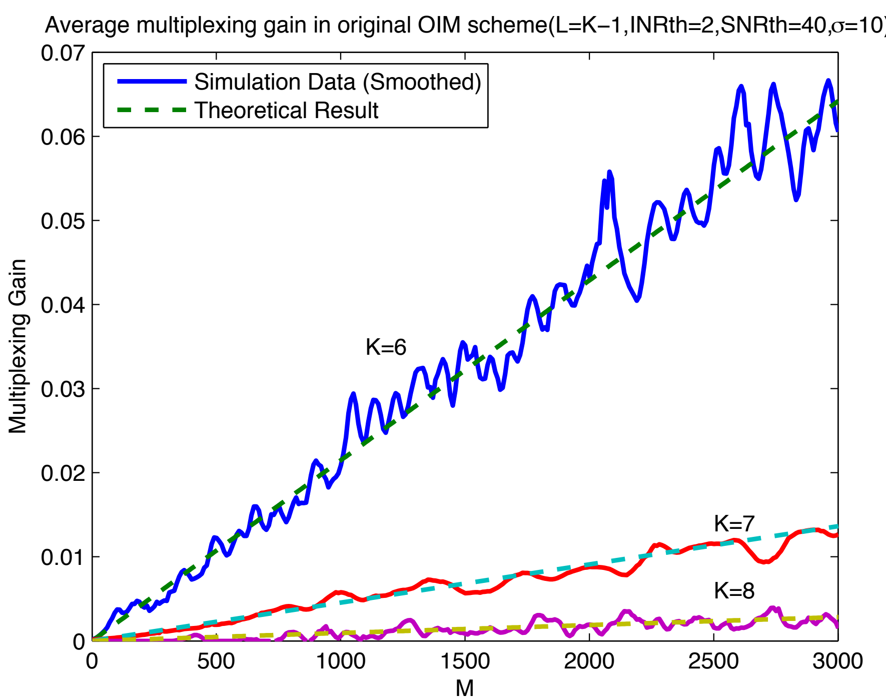

Fig. 2. Original OIM simulations are consistent with analytical results.

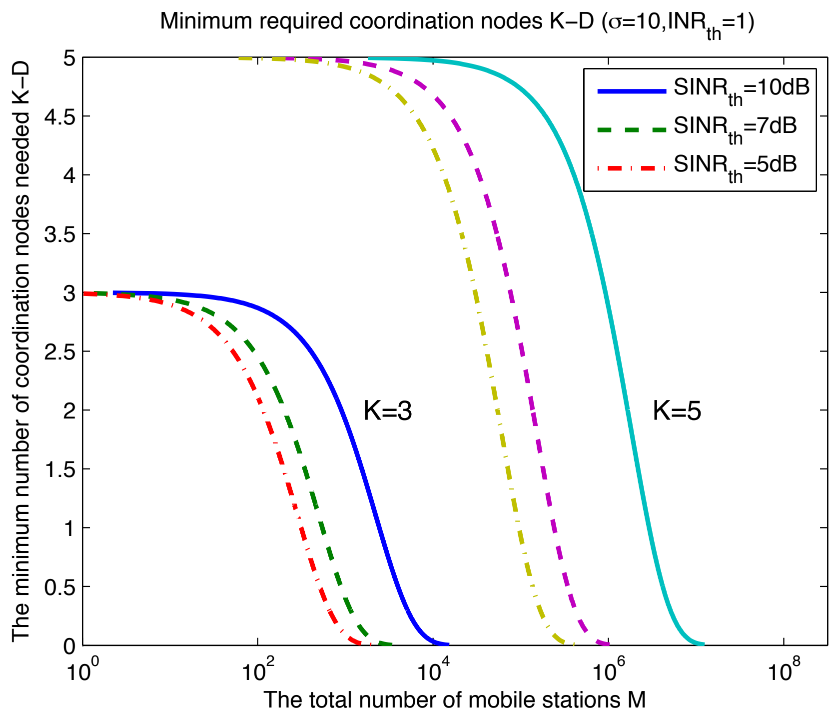

Fig. 3. Simulation results for different values of SINR

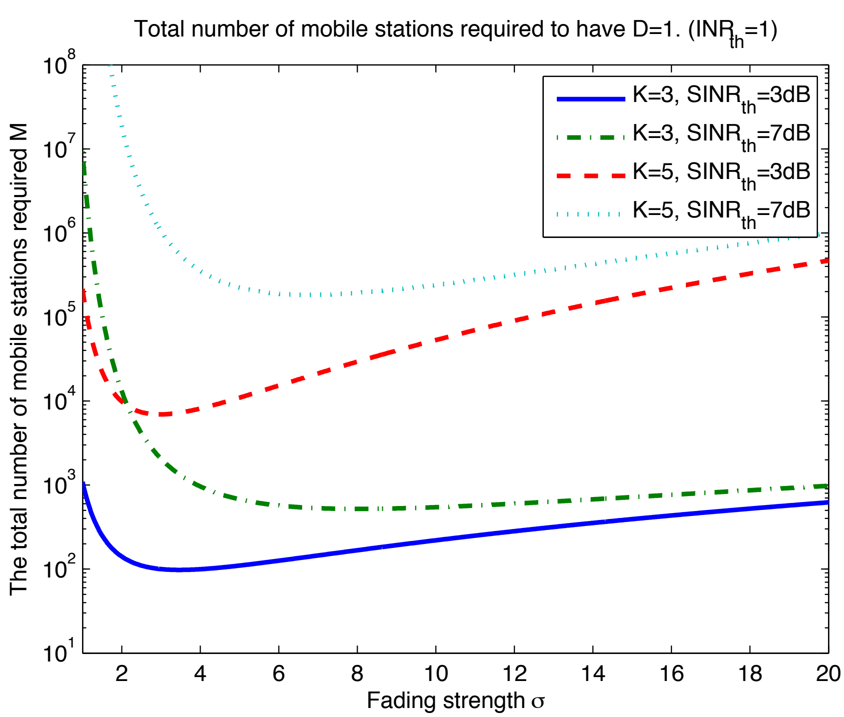

Fig. 4. Simulation results for different fading channel environments

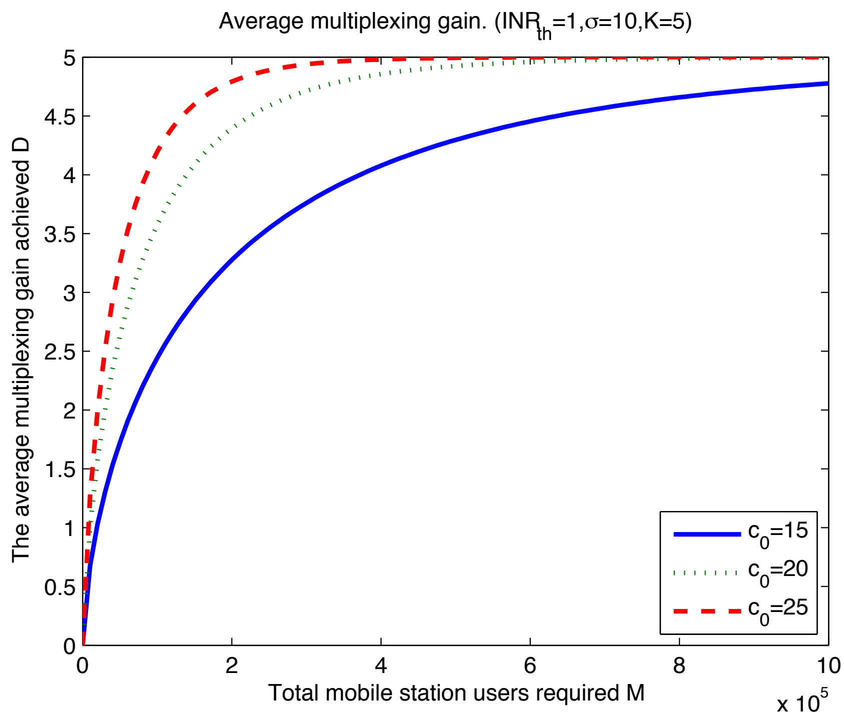

Fig. 5. Simulation results demonstrate DPC capacity and maximum multiplexing gain are achieved simultaneously.

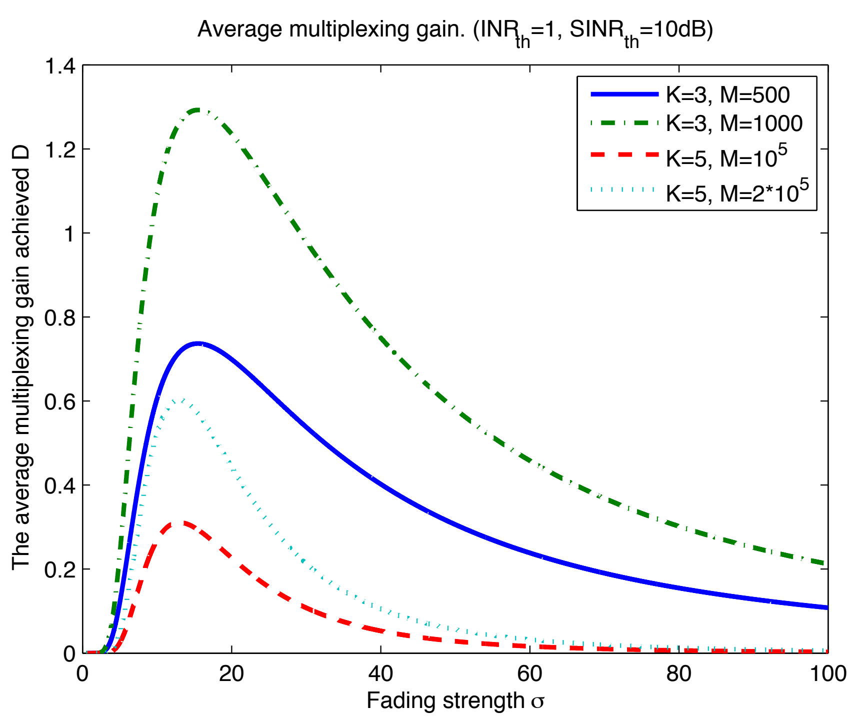

Fig. 6. Simulation results demonstrate relationship between fading strength and multiplexing gain.

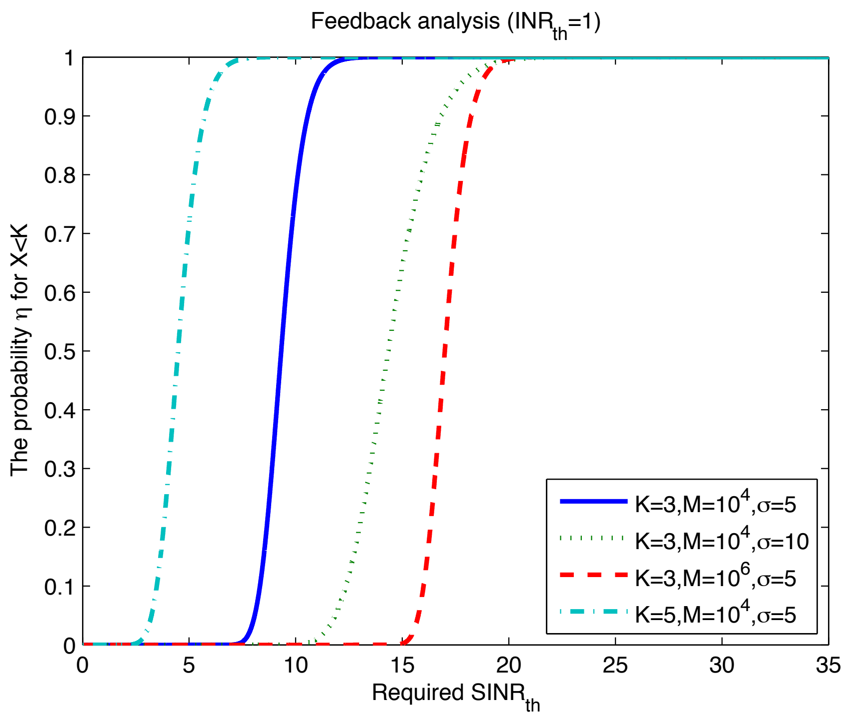

Fig. 7. The feedback is at most K with almost sure

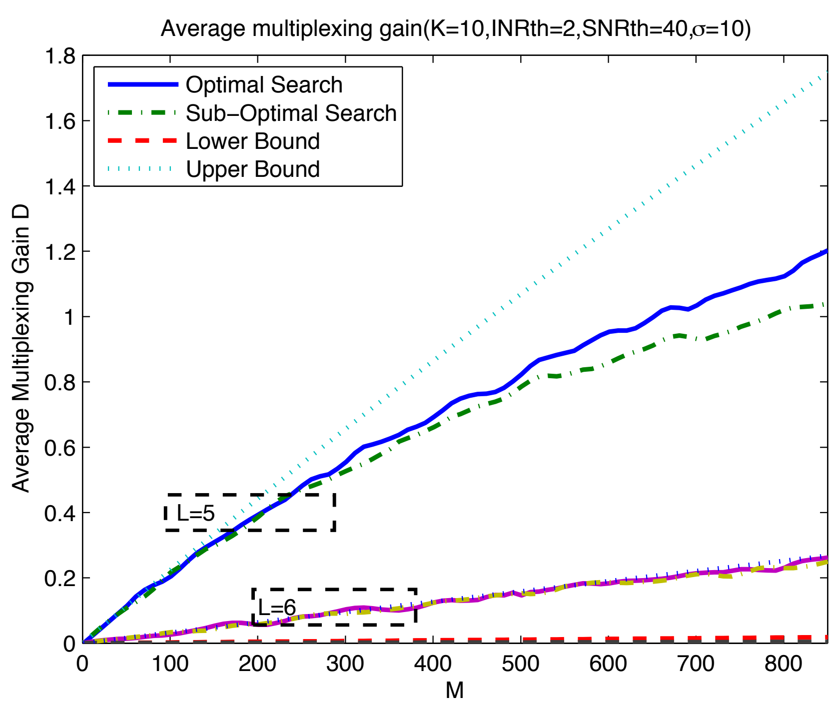

Fig. 8. Upper and lower bounds vs. optimal and sub-optimal search algorithm for K=10

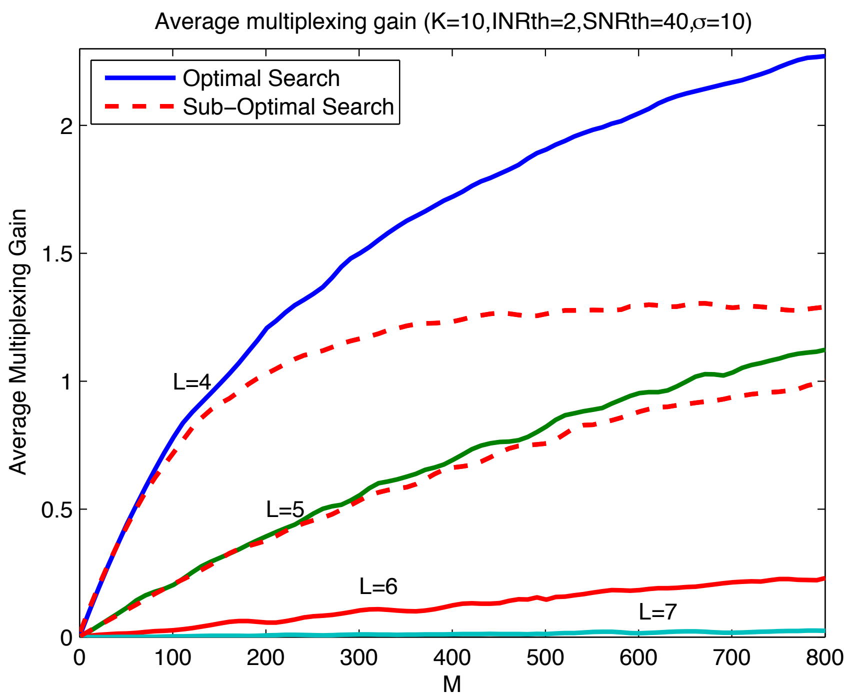

Fig. 9. Trade off between complexity and minimum number of users required for K=10