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# Neutrino Mass Hierarchy and Stepwise Spectral Swapping of Supernova Neutrino Flavors

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We examine a phenomenon recently predicted by numerical simulations of supernova neutrino flavor evolution: the swapping of supernova  $\nu_e$  and  $\nu_{\mu,\tau}$  energy spectra below (above) energy  $E_C$  for the normal (inverted) neutrino mass hierarchy. We present the results of large-scale numerical calculations which show that in the normal neutrino mass hierarchy case,  $E_C$  decreases as the assumed effective  $2 \times 2$  vacuum  $\nu_e \rightleftharpoons \nu_{\mu,\tau}$  mixing angle ( $\approx \theta_{13}$ ) is decreased. In contrast, these calculations indicate that  $E_C$  is essentially independent of the vacuum mixing angle in the inverted neutrino mass hierarchy case. With a good neutrino signal from a future galactic supernova, the above results could be used to determine the neutrino mass hierarchy even if  $\theta_{13}$  is too small to be measured by terrestrial neutrino oscillation experiments.

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In this Letter we point out how two grand themes in contemporary science, physics beyond the standard model of elementary particles and the physics of stars undergoing gravitational collapse, overlap in a way that could allow unique insight into the nature of neutrinos. Recent experiments have established that neutrinos have nonvanishing rest masses and that the flavor states  $\nu_e$ ,  $\nu_\mu$ , and  $\nu_\tau$  for these particles are mixtures of the vacuum mass eigenstates  $\nu_1$ ,  $\nu_2$ , and  $\nu_3$  (see, e.g., Refs. [1,2] for recent reviews). However, key issues remain unresolved. Among these is the nature of the neutrino mass hierarchy: the sign of the mass-squared difference  $\delta m_{31}^2 = m_3^2 - m_1^2 \approx \pm \delta m_{\text{atm}}^2$  remains unknown. Here  $\delta m_{\text{atm}}^2$  is the neutrino mass-squared difference associated with atmospheric neutrino oscillations, and the plus (minus) sign corresponds to the normal (inverted) neutrino mass hierarchy. Conventional laboratory experimental resolution of the mass hierarchy issue is problematic, in part because neutrino rest masses are tiny and because  $\theta_{13}$ , the mixing angle relating  $\nu_e$  to  $\nu_3$ , is small. One possible way to probe the neutrino mass hierarchy is to analyze neutrino signals from galactic supernovae. Supernova neutrinos can experience significant flavor transformation through the Mikheyev-Smirnov-Wolfenstein (MSW) effect [3,4] as they stream out from the surface of a proto-neutron star (with very high matter density) into the vacuum. In addition, it has been pointed out that neutrino-neutrino forward scattering can provide an additional source for neutrino refractive indices [5–8]. This neutrino self-coupling is especially important in the supernova environment because neutrino fluxes are large. Therefore, previous studies of supernova neutrino oscillations based on the pure MSW effect [9–15] may not apply in some scenarios. Following suggestions in Refs. [16–18], in this Letter we present and analyze new large-scale numerical calculations of supernova neutrino flavor evolution that suggest a novel method to determine the neutrino

mass hierarchy. This method is independent of absolute neutrino masses and can work even for tiny  $\theta_{13}$ .

Our method is based on a stunning feature revealed by recent numerical simulations of supernova neutrino flavor evolution [16,19]: (i) For the normal neutrino mass hierarchy case,  $\nu_e$  and  $\nu_{\mu,\tau}$  swap their energy spectra at energies below a transition energy  $E_C$ , but retain their original spectra at higher energies; (ii) for the inverted neutrino mass hierarchy case, the situation is exactly the opposite. This phenomenon is known as “stepwise spectral swapping” [16] or “spectral split” [18].

The stepwise swapping of the  $\nu_e$  and  $\nu_{\mu,\tau}$  energy spectra has its origin in nonlinear neutrino self-coupling. Assuming coherent neutrino propagation and the efficacy of the mean field approach [20,21], for  $2 \times 2$  flavor evolution, it is possible to define the neutrino flavor isospin (NFIS) as [22]

$$\mathbf{s}_\nu \equiv \psi_\nu^\dagger \frac{\boldsymbol{\sigma}}{2} \psi_\nu \quad \text{and} \quad \mathbf{s}_{\bar{\nu}} \equiv (\sigma_y \psi_{\bar{\nu}})^\dagger \frac{\boldsymbol{\sigma}}{2} (\sigma_y \psi_{\bar{\nu}}) \quad (1)$$

for a neutrino (with flavor wave function  $\psi_\nu$ ) and an antineutrino ( $\psi_{\bar{\nu}}$ ), respectively. Flavor evolution for neutrino or antineutrino mode  $i$  is described by precession of a corresponding NFIS  $\mathbf{s}_i$  around an effective field:

$$\frac{d}{dt} \mathbf{s}_i = \mathbf{s}_i \times \left[ \omega_i \mathbf{H}_V - \sqrt{2} G_F n_e \hat{\mathbf{e}}_z^f - 2\sqrt{2} G_F \sum_j (1 - \cos \vartheta_{ij}) n_j \mathbf{s}_j \right]. \quad (2)$$

Here  $\hat{\mathbf{e}}_{x,y,z}^f$  are the flavor-basis unit vectors in flavor space,  $\mathbf{H}_V \equiv -\sin 2\theta_\nu \hat{\mathbf{e}}_x^f + \cos 2\theta_\nu \hat{\mathbf{e}}_z^f$  generates vacuum mixing for a nonvanishing effective mixing angle  $\theta_\nu$ ,  $\omega_i = \pm \delta m^2 / 2E_i$  is the vacuum precession angular velocity around  $\mathbf{H}_V$  for a NFIS corresponding to a neutrino (plus sign) or antineutrino (minus sign) with energy  $E_i$ ,  $G_F$  is the

Fermi constant,  $n_e$  is the net electron number density,  $\vartheta_{ij}$  is the angle between the propagation directions of neutrinos in modes  $i$  and  $j$ , and  $n_j$  is the number density of neutrinos in mode  $j$ . Because flavor transformation in the  $\nu_e \rightleftharpoons \nu_{\mu,\tau}$  and  $\bar{\nu}_e \rightleftharpoons \bar{\nu}_{\mu,\tau}$  channels is the most important in supernovae (e.g., for shock reheating [23,24] and nucleosynthesis [24–27]), and because  $\delta m_{\text{atm}}^2$  will give flavor transformation deeper in the supernova envelope than will  $\delta m_{\odot}^2$ , the mass-squared difference associated with solar neutrino oscillations, we take  $\delta m^2 = \pm 3 \times 10^{-3} \text{ eV}^2 \simeq \pm \delta m_{\text{atm}}^2$  and  $\theta_\nu \simeq \theta_{13} \ll 1$ . For this  $2 \times 2$  mixing we use  $\nu_{\tau^*}$  to designate the relevant linear combination of  $\nu_\mu$  and  $\nu_\tau$  [28].

In a stepwise-swapping scenario, the probability  $P_{\nu\nu}$  for neutrinos to remain in their initial flavor states is

$$P_{\nu\nu}(\omega) \simeq \frac{1}{2} \left[ 1 - \text{sgn}(\omega - \omega_{\text{pr}}^0) \right], \quad (3)$$

where  $\text{sgn}(\xi) = \xi/|\xi|$  is the sign of  $\xi$ , and  $\omega_{\text{pr}}^0 = \delta m^2/2E_C$  specifies the transition energy  $E_C$ . Equation (3) is slightly different from Eq. (57b) in Ref. [29], with different conventions for  $\theta_\nu$  and  $\delta m^2$ . This stepwise spectral swapping feature has been demonstrated in two different approaches [16,19,30,31]: “multiangle” simulations, where flavor evolution on independently followed neutrino trajectories is self-consistently coupled, and “single-angle” simulations, where the evolution history of radially propagating neutrinos is assumed to apply to all trajectories. Single-angle calculations capture the qualitative features of stepwise spectral swapping, and they suggest the following generic explanation for this phenomenon.

Because neutrinos and antineutrinos are in flavor eigenstates when they leave the neutrino sphere, they naturally form a “bipolar system” in which the corresponding NFIS’s form two oppositely oriented groups [22]. (Note that  $\nu_e/\bar{\nu}_{\tau^*}$  and  $\bar{\nu}_e/\nu_{\tau^*}$  correspond to NFIS’s in the directions of  $+\hat{\mathbf{e}}_z^f$  and  $-\hat{\mathbf{e}}_z^f$ , respectively.) This bipolar system behaves like a gyroscopic pendulum in flavor space [32], which can have both nutation and precession modes. These neutrinos and antineutrinos initially follow a quasistatic, MSW-like solution near the neutrino sphere before being driven away from this solution by the collective nutation of the gyroscopic pendulum [29]. Subsequently, the gyroscopic pendulum can execute regular precession around  $\mathbf{H}_V$ , corresponding to the collective precession of NFIS’s in flavor space [17]. This precession, although not perfectly regular, is indeed found in both single-angle and multi-angle simulations [16,17]. If NFIS’s stay in the regular collective precession mode, a stepwise spectral swapping given by Eq. (3) will occur when the neutrino fluxes decrease toward 0 [16–18]. In this case,  $\omega_{\text{pr}}^0$  in Eq. (3) is just the precession angular velocity for vanishing neutrino fluxes.

Strictly speaking, the regular precession mode obtains for  $n_e = 0$  where the “lepton number”

$$\mathcal{L} \equiv \int_0^\infty [f_{\nu_1}(E) - f_{\nu_3}(E) - f_{\bar{\nu}_1}(E) + f_{\bar{\nu}_3}(E)] dE \quad (4)$$

is conserved [32]. Here  $f_{\nu_1(\bar{\nu}_1)}(E)$  and  $f_{\nu_3(\bar{\nu}_3)}(E)$  are the distribution functions specifying the populations of the corresponding neutrino (antineutrino) vacuum mass eigenstates within energy interval  $dE$ . These are normalized to the total (summing over all states) neutrino number density  $n_\nu^{\text{tot}}$  and in general evolve with time. The conservation of  $\mathcal{L}$  is exact for  $n_e = 0$  and holds even when neutrino number densities change [17]. Because the presence of the matter field does not change the collective precession qualitatively [17,29], the conservation of  $\mathcal{L}$  can be used to compute  $E_C$  [18].

In the inverted neutrino mass hierarchy case, flavor transformation is suppressed when  $n_e$  and neutrino fluxes are high. As  $\nu_e$  are dominant in supernovae, the bipolar system of neutrinos and antineutrinos resembles a gyroscopic pendulum near its highest point (displacement angle equal to  $\pi$ ) in flavor space. For a simple bipolar system initially consisting of monoenergetic  $\nu_e$  and  $\bar{\nu}_e$ , the analogy is exact, with the initial displacement angle being  $\pi - 2\theta_\nu$ . When the total neutrino flux decreases below some critical value, the flavor pendulum evolves away from its maximum displacement [17,32]. Its nutation then pushes neutrinos and antineutrinos into the collective precession mode. The presence of a matter field does not change this nutation qualitatively, but effectively reduces the mixing angle [22,32]. Therefore,  $\mathcal{L}$  is essentially unchanged before the precession mode begins. As a result, when  $n_e$  and neutrino fluxes become small,  $f_{\nu_1(\bar{\nu}_1)}(E)$  and  $f_{\nu_3(\bar{\nu}_3)}(E)$  are related simply through Eq. (3) to the initial neutrino energy spectra at the neutrino sphere. Specifically, for  $\theta_\nu \ll 1$  we have

$$\begin{aligned} \mathcal{L} &\simeq \int_0^{E_C} [f_{\nu_e}(E) - f_{\nu_{\tau^*}}(E)] dE \\ &\quad + \int_{E_C}^\infty [f_{\nu_{\tau^*}}(E) - f_{\nu_e}(E)] dE + \frac{n_{\bar{\nu}_e} - n_{\bar{\nu}_{\tau^*}}}{n_\nu^{\text{tot}}} \quad (5) \\ &\simeq \frac{n_{\nu_e} - n_{\nu_{\tau^*}} - n_{\bar{\nu}_e} + n_{\bar{\nu}_{\tau^*}}}{n_\nu^{\text{tot}}}, \end{aligned}$$

where, e.g.,  $f_{\nu_e}(E)$  and  $n_{\nu_e}(E)$  are the initial spectrum (normalized to  $n_\nu^{\text{tot}}$ ) and number density, respectively, of  $\nu_e$  at the neutrino sphere. The transition energy  $E_C$  can then be found from Eq. (5).

The conservation of  $\mathcal{L}$  can also be used to find  $E_C$  for the normal mass hierarchy case. However, in this case,  $\mathcal{L}$  cannot be related simply to the initial neutrino spectra at the neutrino sphere through Eq. (3). This is because there is a resonance in the quasistatic MSW-like solution initially followed by neutrinos and antineutrinos. For example, in the large neutrino luminosity limit, all neutrinos and antineutrinos are synchronized [33] and experience simultaneously an MSW-like resonance near the radius where a single  $\nu_e$  with a representative energy  $E_{\text{sync}}$  would encoun-

ter a conventional MSW resonance [26]. During this MSW-like evolution,  $\mathcal{L}$  is decreased. Using the initial spectra for supernova neutrinos and Eqs. (3) and (4), it can be shown that the less  $\mathcal{L}$  is reduced, the smaller  $E_C$  becomes. If  $\theta_\nu$  is tiny and/or the neutrino luminosities are not large enough, the MSW-like conversion will be non-adiabatic, and  $\mathcal{L}$  will change very little. In this case,  $E_C \rightarrow 0$  and the stepwise nature of the swapping of  $\nu_e$  and  $\nu_{\mu,\tau}$  spectra becomes unobservable. On the other hand, given sufficiently large values of  $\theta_\nu$  and/or neutrino luminosities, MSW-like flavor conversion can be adiabatic and efficient for a large range of neutrino energies. This can engender nearly complete swapping of the entire neutrino and anti-neutrino spectra and, consequently,  $\mathcal{L}$  can retain its magnitude but reverse its sign. In this case,  $E_C$  for the subsequent stepwise spectral swapping is roughly the same as in the inverted neutrino mass hierarchy case.

Though the transition energy  $E_C$  is sensitive to  $\theta_\nu$  in the normal mass hierarchy case, it appears to be essentially independent of  $\theta_\nu$  in the inverted mass hierarchy case. We have carried out multiangle simulations under the same conditions as those discussed in Refs. [16,19] except with smaller  $\theta_\nu$ . The probability  $P_{\nu_e\nu_e}(E, \vartheta_0)$  at radius  $r = 250$  km is plotted in Fig. 1 for both a normal mass hierarchy case ( $\theta_\nu = 0.01$ , left panel) and an inverted mass hierarchy case ( $\theta_\nu = 10^{-9}$ , right panel). Here  $\vartheta_0$  is the angle between the propagation direction of the neutrino and the normal at its emission position on the neutrino sphere (see Fig. 1 of Ref. [19]). A comparison of the results shown in Fig. 1 with those for  $\theta_\nu = 0.1$  shown in Fig. 3 of Ref. [19] is revealing. In the normal neutrino mass hierarchy case,  $E_C$  decreases from  $\sim 10$  MeV to  $\sim 3$  MeV as  $\theta_\nu$  is reduced from 0.1 to 0.01. However, for the inverted

neutrino mass hierarchy case,  $E_C$  is essentially unchanged as  $\theta_\nu$  is decreased by 8 orders of magnitude. The value  $E_C \simeq 8.4$  MeV calculated from Eq. (5) for this case agrees very well with the numerical results. We note that  $E_C$  has a slight dependence on neutrino trajectory ( $\cos\vartheta_0$ ) in the multiangle simulations for  $\theta_\nu \ll 0.1$ .

The apparent insensitivity of  $E_C$  to  $\theta_\nu$  in the inverted neutrino mass hierarchy case requires discussion. We note that the neutrino system transitions from the MSW-like evolution to the collective precession mode through nutation. If the system does not develop significant nutation while it is in the collective flavor transformation regime, it will not enter the collective precession mode, and therefore, stepwise spectral swapping will not occur. For a uniform and isotropic gas of monoenergetic neutrinos initially in pure  $\nu_e$  and  $\bar{\nu}_e$  states, the nutation time scale is  $T_{\text{nut}} \sim -\ln\theta_\nu$  in the inverted mass hierarchy case [32]. Estimating the nutation time scale for realistic supernova neutrino systems is problematic, partly because this quantity depends on time-varying electron and neutrino number densities [22,32,34]. In Fig. 2 we plot as a function of  $\theta_\nu$  the radius  $r_X$  (as defined in Ref. [16]) where the energy-averaged value of  $P_{\nu_e\nu_e}$  drops below 0.9 and significant nutation develops in our single-angle simulations for the inverted mass hierarchy. Our single-angle calculations suggest that the onset of significant nutation is nearly independent of  $\theta_\nu$  as it is decreased from 0.1 to  $\simeq 10^{-20}$ . As  $\theta_\nu$  is decreased further,  $r_X$  begins to increase. We expect that for sufficiently small  $\theta_\nu$ , the onset of significant nutation is pushed to so large a radius that the corresponding neutrino number density becomes too low to generate any collective flavor evolution or stepwise spectral swapping.

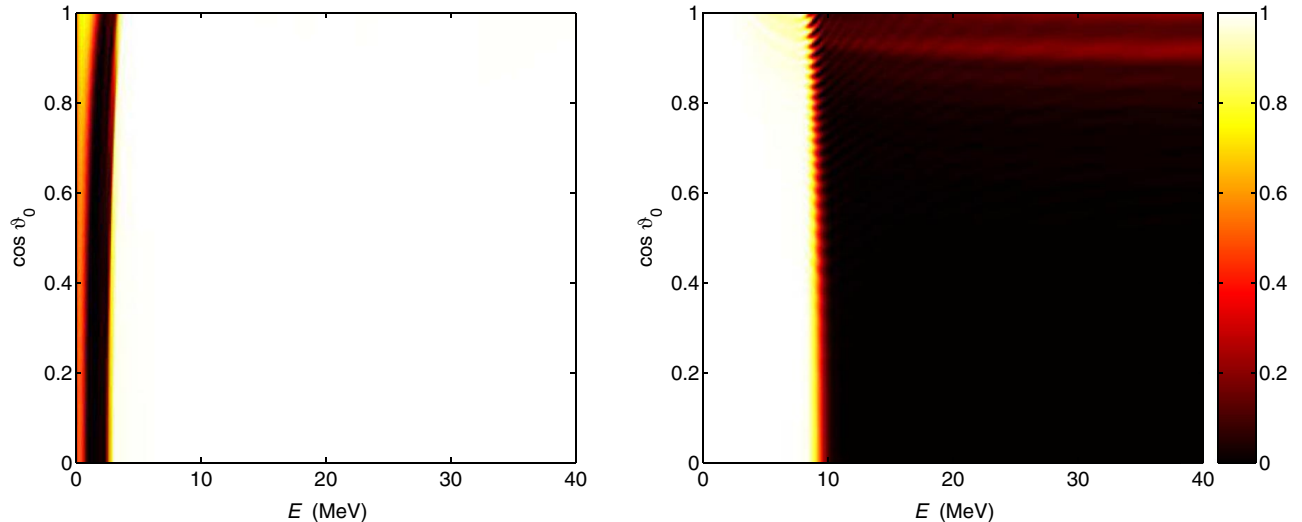


FIG. 1 (color online). The neutrino survival probability  $P_{\nu\nu}$  as a function of neutrino emission angle  $\vartheta_0$  (relative to the normal at the emission point on the neutrino sphere) and energy  $E$ . The left panel is calculated for a normal neutrino mass hierarchy with  $\theta_\nu = 0.01$ , and the right panel is for an inverted neutrino mass hierarchy with  $\theta_\nu = 10^{-9}$ . These results are taken at radius  $r = 250$  km. Except for  $\theta_\nu$ , all parameters are the same as those for Fig. 3 of Ref. [19].

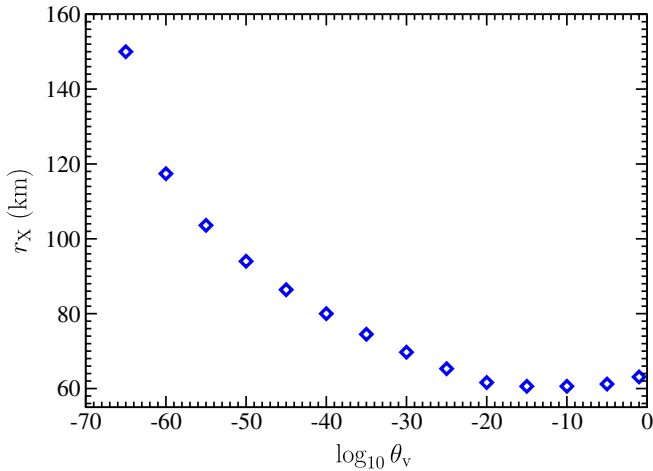


FIG. 2 (color online). Single-angle simulation results for the dependence of  $r_X$  on  $\theta_v$  in the inverted neutrino mass hierarchy case. Here  $\theta_v$  is the effective  $2 \times 2$  vacuum mixing angle ( $\simeq \theta_{13}$ ), and  $r_X$  is the radius where the energy-averaged value of  $P_{\nu_e \nu_e}$  drops below 0.9. Except for  $\theta_v$ , all parameters are the same as those for Fig. 2 of Ref. [19].

Just after the bounce of the supernova core, when the supernova shock breaks through the neutrino sphere, there is a brief intense burst of neutrinos which are emitted predominantly as  $\nu_e$ 's. Lacking in  $\bar{\nu}_e$ 's [35], this burst is not likely to be affected by stepwise spectral swapping at the  $\delta m_{\text{atm}}^2$  scale (but see Ref. [36]). Later, both neutrinos and antineutrinos are emitted and they form a bipolar system. If  $\theta_{13}$  is not too small, for the normal mass hierarchy case, stepwise spectral swapping at the  $\delta m_{\text{atm}}^2$  scale can be observable at late times when matter has been sufficiently condensed toward the proto-neutron star. For the inverted mass hierarchy case, however, stepwise spectral swapping occurs even at early times [22,31] for essentially any nonvanishing  $\theta_{13}$ . In light of the insensitivity of  $E_C$  to  $\theta_{13}$  in the latter case, supernova neutrino signals can offer a unique probe of the neutrino mass hierarchy even for  $\theta_{13}$  too small to be measured by conventional neutrino oscillation experiments.

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