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Since their introduction by Barenblatt et al. (1960), double-porosity models have been widely used for simulating flow in fractured reservoirs. In a dual-porosity system, the matrix blocks provide most of the storage of the reservoir, whereas the fractures provide the global transmissivity. Initially, most work on dual-porosity models emphasized the development of analytical solutions to idealized reservoir problems. Increasingly, the dual-porosity approach is being implemented by numerical reservoir simulators. Accurate numerical simulation of a dual-porosity problem often requires a prohibitively large number of computational cells in order to resolve the transient pressure gradients in the matrix blocks. In this paper we discuss a new dual-porosity model that utilizes a nonlinear differential equation to approximate the fracture/matrix interactions. When implemented into a numerical simulator, it eliminates the need to discretize the matrix blocks, and thereby allows more efficient simulation of reservoir problems.

When a single-phase, slightly compressible fluid flows through a macroscopically-homogeneous fractured medium, the fluid pressure P_f in the fractures is governed by

$$\phi_f c_f \frac{\partial P_f(x,t)}{\partial t} = \frac{k_f}{\mu} \nabla^2 P_f(x,t) + Q(x,t), \tag{1}$$

where t is time, x is the position vector of a point in the fracture continuum, k_f is the effective permeability of the fracture continuum, ϕ_f is the total fracture porosity, and c_f is the total compressibility of the fractures and the fluid within them. Q is a volumetric source/sink term representing fluid flow from the matrix blocks to the fracture system, per unit of total volume.

One commonly-used type of dual-porosity model allows global flow only through the fracture network, with the matrix blocks serving as continuously-distributed sources/sinks of fluid for the fractures. The matrix blocks at each location in the fracture continuum are represented by a single average pressure, $P_m(x,t)$. Conservation of mass for the matrix block leads to the following equation for P_m:

$$\phi_m c_m \frac{\partial P_m(x,t)}{\partial t} = -Q(x,t). \tag{2}$$

In order to close the system given by (1) and (2), an equation is needed to relate Q to P_f and P_m. Warren and Root (1963) assumed that Q is proportional to $P_f - P_m$:

$$Q(x,t) = \frac{-\alpha k_m}{\mu}(P_f - P_m), \tag{3}$$

where α has dimensions of 1/area. Eq. (3) is often referred to as the ''quasi-steady-state'' approximation (Chen, 1989). This terminology follows from consideration of the problem in which there is an instantaneous change in the fracture pressure P_f, which serves as the boundary condition for the matrix block, which will be assumed to be a sphere of radius a_m. Differentiation of the most-slowly-decaying Fourier component in the expression for the average pressure, which is the dominant component at large times, leads to an equation of the form (2,3), with $\alpha = \pi^2/a_m^2$. Other matrix block shapes, such as slabs or cubes, lead to long-time behavior governed by (2,3), but with different expressions for α.

An exact coupling term could be developed in terms of the step-function response of a single matrix matrix by using the convolution principle. This would lead to an integro-differential system of equations, which most reservoir simulators are not suited to solve. Pruess and Wu (1989) and Dykhuizen (1990) improved upon the quasi-steady-state model by approximating flow in the matrix blocks with trial functions that satisfy the boundary conditions and global mass conservation. We have taken the approach of utilizing a nonlinear ordinary differential equation which, in some sense, approximates the linear partial differential equation that actually governs P_m. This equation, first proposed by Vermeulen (1953) in the context of ion-exchange chromatography, is:

$$\phi_m c_m \frac{\partial P_m}{\partial t} = \frac{\pi^2 k_m}{2\mu a_m^2} \frac{(P_f - P_i)^2 - (P_m - P_i)^2}{(P_m - P_i)}, \tag{4}$$

where P_i is the initial pressure. When P_m is close to P_f, (4) reduces to (3), and is therefore accurate in the long-time regime. We have found that (4) is also very accurate in the small-time limit, as will be briefly demonstrated.

We have tested (4) under situations in which the fracture pressure, which serves as the boundary condition for the matrix block, is a known function of time, thereby isolating the matrix pressure response from that of the overall reservoir. Fig. 1 shows the mean matrix block pressure, when the fracture pressure increases abruptly from P_i to P_o at $t=0$. The solution to (4) very closely approximates the exact solution, whereas the prediction of (2,3) is not accurate until the process is nearly complete.

As another example, consider the case where $P_f = P_i + Bt^m$. The leading term in the solution to (4) for small times is

$$P_m = P_i + \left[\frac{k_m}{\phi_m \mu c_m a_m^2}\right]^2 \frac{\pi B}{\sqrt{2m+1}} t^{m+1/2} . \tag{5}$$

The exact small-time approximation can be found from the solution (Crank, 1975) for diffusion into a semi-infinite media, which applies to all geometries as $t \rightarrow 0$:

$$P_m = P_i + \left[\frac{k_m}{\phi_m \mu c_m a_m^2}\right]^2 \frac{3B\,\Gamma(m+1)}{\Gamma(m+3/2)} t^{m+1/2} . \tag{6}$$

The solution to (2,3) in this case is

$$P_m = P_i + \left[\frac{k_m}{\phi_m \mu c_m a_m^2}\right] \frac{\pi^2 B}{m+1} t^{m+1} . \tag{7}$$

The quasi-steady-state equation (2) incorrectly predicts that $P_m \approx t^{m+1}$, whereas (4) correctly predicts $P_m \approx t^{m+1/2}$. Although the numerical constant in (7) is too small, the error varies from only $7\% \rightarrow 26\%$ as m varies from $0 \rightarrow \infty$.

Having shown that (4) accurately predicts the mean matrix pressure, we have incorporated it as a fracture/matrix coupling term in the simulator TOUGH (Pruess, 1987). In solving problems with this modified simulator, we assume that each computational cell represents an element of the fracture continuum, with the fracture/matrix interaction computed from (4). As a test of this approach, consider flow from a

boundary that is maintained at pressure P_o, into a semi-infinite formation that is initially at P_i. The permeabilities are taken as $k_f = 10^{-15}\,\mathrm{m}^2$ and $k_m = 10^{-18}\,\mathrm{m}^2$, the porosities as $\phi_f = 0.001$ and $\phi_m = 0.1$, and the matrix block radii as $a_m = 1\,\mathrm{m}$. The temperature is 20 °C, and the boundary and initial pressures are $P_i = 10\,\mathrm{MPa}$ and $P_o = 11\,\mathrm{MPa}$.

The flowrate into the formation is shown in Fig. 2. In the case "MINC - 1 shell", the matrix blocks were represented by a single computational cell; this is a numerical implementation of the Warren-Root equation (3). In the case "MINC - 10 shells" each matrix block was discretized into 10 concentric shells. All three computations predict the correct pressure response in the short and long-time limits, when $P_f \simeq t^{-1/2}$. In the intermediate-time regime, when the matrix blocks near the inlet are being filled, the Warren-Root method incorrectly predicts $P_f \simeq constant$, whereas the new method correctly leads to the known (Nitao and Buscheck, 1991) $t^{-1/4}$ pressure dependence.

The computational time required for simulating a given problem with a code such as TOUGH grows linearly with the number of computational cells, since most of the computing effort consists in inverting a sparse matrix by Gaussian elimination. Since the nonlinear coupling equation removes the need for discretizing the matrix blocks, the savings in computational time compared to the fully-discretized simulation is about 90%.

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REFERENCES

Barenblatt, G. I., Zheltov, Y. P., and Kochina, I. N., Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks, Journal of Applied Mathematics and Mechanics, 24, 1286-1303, 1960.

Chen, Z.-X., Transient flow of slightly compressible fluids through double-porosity, double-permeability systems - A state-of-the-art review, Transport in Porous Media, 4, 147-184, 1989.

Crank, J., The Mathematics of Diffusion, Clarendon Press, Oxford, 1975.

Dykhuizen, R. C., A new coupling term for dual-porosity models, Water Resources Research, 26, 351-356, 1990.

Nitao, J. J., and Buscheck, T. A., Infiltration of a liquid front in an unsaturated, fractured porous medium, Water Resources Research, 27, 2099-2112, 1991.

Pruess, K., TOUGH User's Guide, Report LBL-20700, Lawrence Berkeley Laboratory, Berkeley, Calif., 1987.

Pruess, K., and Wu, Y.-S., A new semi-analytical method for numerical simulation of fluid and heat flow in fractured reservoirs, Paper SPE-18426, Society of Petroleum Engineers, Dallas, 1989.

Vermeulen, T., Theory of irreversible and constant-pattern solid diffusion, Industrial and Engineering Chemistry, 45, 1664-1670, 1953.

Warren, J. E., and Root, P. J., The behavior of naturally fractured reservoirs, Society of Petroleum Engineers Journal, 3, 245-255, 1963.

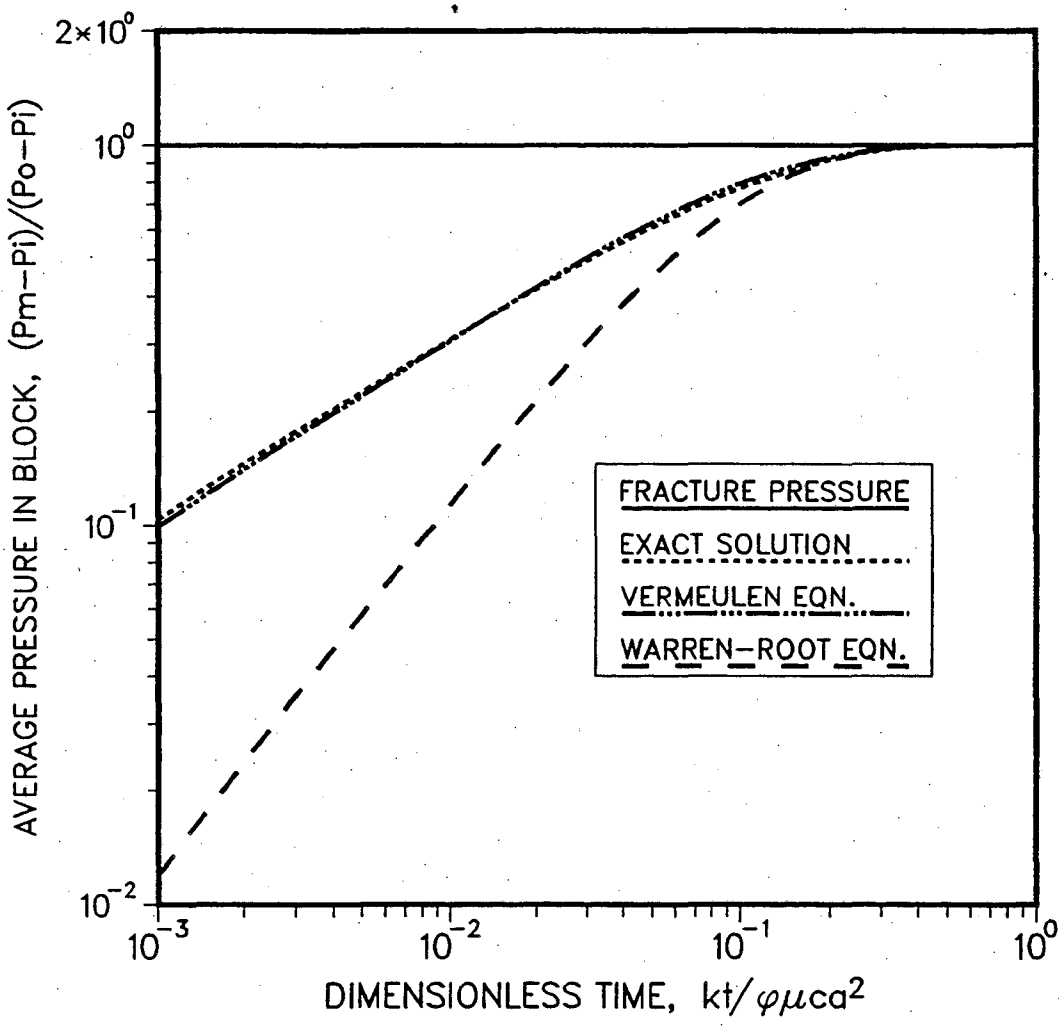

Fig. 1. Average matrix pressure in a spherical block that is subjected to a step-function increase in pressure at its outer boundary.

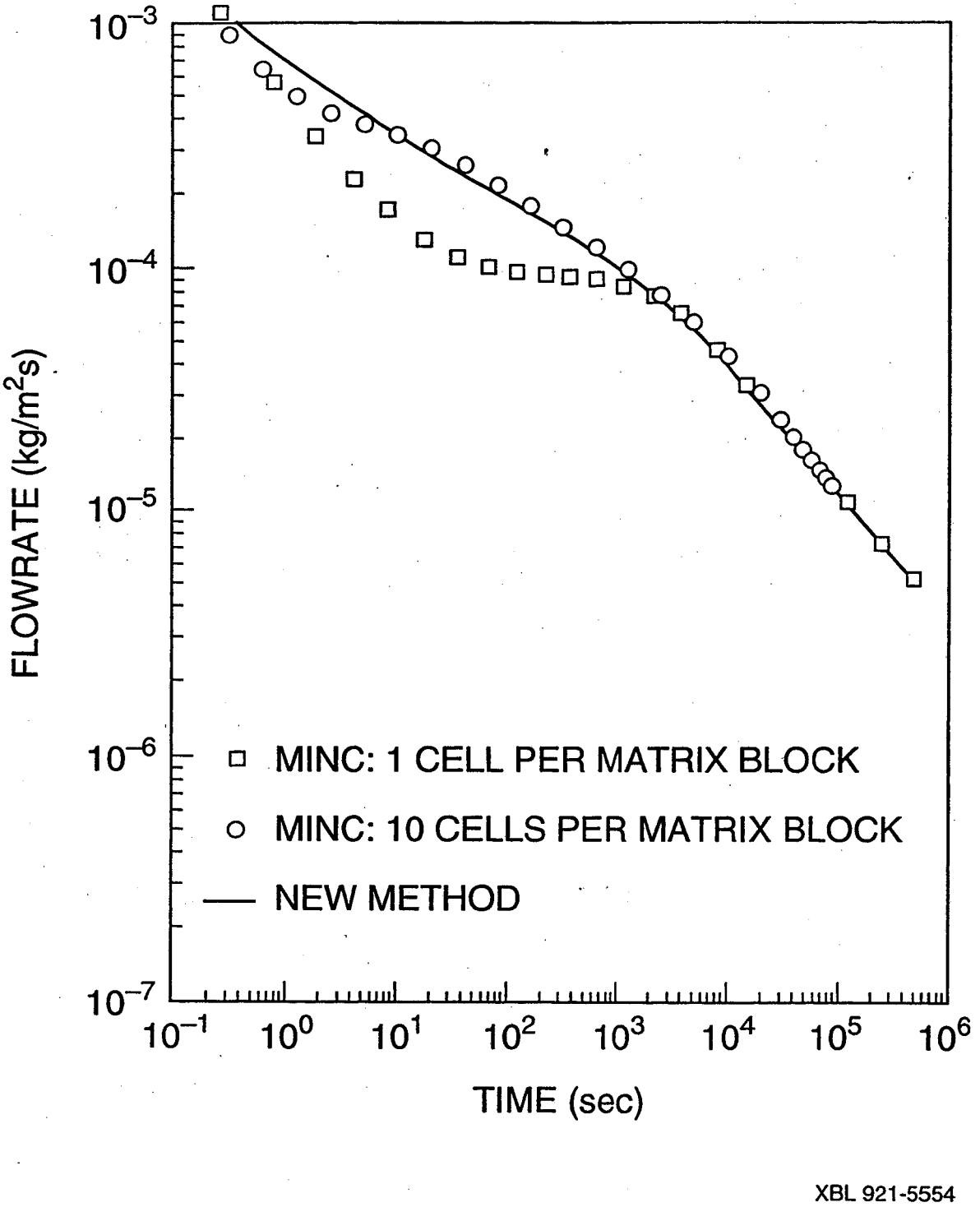

Fig. 2. Flowrate into a one-dimensional fractured formation, as described in the text.

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