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Essays in Risk Management and Financial Econometrics

by

Haoyang Liu

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

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in the

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of the

University of California, Berkeley

Committee in charge:

Professor Nancy Wallace, Co-chair
Professor Christopher Palmer, Co-chair
Professor Amir Mohsenzadeh Kerman
Professor Noureddine El Karoui

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Essays in Risk Management and Financial Econometrics

Abstract

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Haoyang Liu

Doctor of Philosophy in Business Administration

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Professor Nancy Wallace, Co-chair

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This dissertation consists of three chapters that concern risk management and financial econometrics. Fannie Mae and Freddie Mac's implicit government guarantee is widely argued to cause irresponsible risk taking. Despite moral-hazard concerns, this paper presents evidence that Fannie Mae and Freddie Mac (the GSEs) more effectively managed home price risks during the 2000-2006 housing boom than private insurers. Mortgage origination data reveal that the GSEs were selecting loans with increasingly higher percentage of down payments, or lower loan to value ratios (LTVs), in boom areas than in other areas. Furthermore, the decline of LTVs in boom areas stems entirely from the segment insured by the GSEs only, and none of the decline stems from the segment co-insured by private mortgage insurers. Private mortgage insurers also did not lower their exposure to home price risks along other dimensions, including the percentage of high LTV GSE loans they insured. To quantify how the GSEs' portfolios would have performed under alternative home price scenarios, I build an insurance valuation model based on competing-risk hazard regressions, calibrated Hull and White term-structure model, and forecasting prepayment and default speeds. I find that the GSEs' risk management would have been sufficient for the historically average 32% mean reversion but insufficient for the realized 95% mean reversion between 2006 and 2011. My results highlight that post-crisis reform of the mortgage insurance industry should carefully consider additional factors besides moral hazard, such as mortgage insurers' future home price assumptions.

The second chapter studies high dimensional time series, with application to estimating the mean variance frontier. One persistent challenge in macroeconmics and finance is how to draw inference from data with a large cross section but short time series. Financial econometric techniques all are designed for large time series and small cross-sections. However, financial data typically has a large cross section and short time series (large-N small-T). One particular large-N small-T impact is the underestimation of risk in the mean variance frontier. We propose a correction for the finite sample bias when the underlying returns are high dimensional linear time series. Our algorithm first corrects for the bias in eigenvalues of the asset return covariance matrix, and then estimate the contribution of each leading factor to the mean variance frontier. A cross validation

method is employed to select the optimal number of leading factors. Performance of the proposed methods is examined through extensive simulation studies.

The third chapter studies how expected home prices affect borrowers' default behavior. One of the penalties mortgage defaulters face is being locked out of the mortgage market and missing the home price appreciation. I find that this penalty deters some borrowers from defaulting. A higher future home price growth implies a lower ex-ante default probability. Furthermore, high credit score borrowers react more to past home price declines and future home price appreciation than low credit score borrowers. This suggests that high credit score borrowers are more likely to be strategic defaulters. A model is built to study the effect of changing the cooling off period. In high expected home price appreciation areas, a longer cooling-off period amplifies the impact of each foreclosure. In low expected home price appreciation areas, a longer cooling-off period reduces the number of foreclosures.

To Dwight

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Chapter 1

Do Government Guarantees Inhibit Risk Management? Evidence from Fannie Mae and Freddie Mac

1.1   Introduction

From flood insurance to deposit insurance, from Medicare to mortgage default insurance, the risk management of publicly sponsored insurance draws frequent scrutiny both from academia and the popular press (Bin et al., 2008; Michel-Kerjan, 2010; McMillan, 2007; King, 1994; Oberlander, 1997; Pear, 1996). Economists have long reasoned that because taxpayers, not private investors, bear the downside risk of public insurance, the insurance managers have limited incentives to manage risk. Perhaps one of the most criticized public insurers is the "public/private partnership" of Fannie Mae and Freddie Mac (Bernanke, 2015; Acharya et al., 2011). The two government-sponsored enterprises (GSEs) had profit maximizing shareholders, but they also carried an implicit government guarantee, essentially eliminating any downside risk. Both academics and policy makers, including the Obama administration, have argued that this flawed structure caused the GSEs to "take on irresponsible risks" (Treasury, 2011; Acharya et al., 2011; Quigley, 2006; Jaffee et al., 2007; Hermalin and Jaffee, 1996). Based on this view, many post-crisis proposals suggest that the GSEs should be privatized, gradually replaced by private mortgage insurance companies, or only allowed to passively follow prices set by private insurers. (Treasury, 2011; Acharya et al., 2011; Jaffee and Quigley, 2012; Elenev et al., 2016).

   A crucial assumption underlying these proposals is that by solving the incentive problem, private insurers will more effectively manage risk and set fairer prices than the GSEs did. Inconsistent with this assumption, I present evidence that the GSEs more effectively managed home price risk than private mortgage insurers did during the 2000s housing bubble. By increasing the percentage of down payments in boom areas, the GSEs reduced their exposure to a housing downturn. In contrast, little evidence suggests that private insurers were aware of the housing bubble or took precautionary measures for the looming home price crash. To study how the GSEs' risk manage-

ment would have performed under alternative home price environments, I construct a mortgage insurance valuation model based on competing-risk hazard regressions, calibrated Hull and White term-structure model and forecasting prepayment and default speeds.

My analysis is based on the segmentation of GSE loans along LTVs. GSE loans with LTVs at or below 80% are insured by the GSEs only. For GSE loans with LTVs above 80%, private insurers take first loss position before the GSEs cover additional losses (Frame et al., 2015). I document that in the LTVs at or below 80% segment, boom areas had a greater relative decline of LTVs than other areas. By lowering the LTVs in these boom areas with greater risk of home price mean reversion (Cutler et al., 1991), the GSEs reduced their exposure to home price risk through lower default rates and lower losses given defaults. In contrast, in the LTVs above 80% segment, I find no evidence that private insurers lowered their home price risk along dimensions where they could have adjusted. For instance, in the LTVs above 80% segment, LTVs did not decline more in boom areas than elsewhere. Also, the share of mortgages with LTVs above 80% did not decline more in boom areas. Another example is that private mortgage insurers slightly increased their percentage of covered losses more in boom areas than in other areas. These results suggest that the GSEs more effectively managed their home price risk compared with private insurers.

The first set of empirical challenges of my paper is interpretation of the central result of declining LTVs in boom areas among GSE loans with LTVs at or below 80%. I conduct a series of tests to confirm that my results are most likely driven by the GSEs' risk management. Two plausible alternative explanations for this result are mortgagor demand-side story and reverse causality. For the demand-side story, perhaps declining LTVs in boom areas stem from borrowers' reluctance to take out high LTV loans rather than the GSEs' risk management. This is plausible because home buyers who traded up in 2005 might have carried the equity from their previous homes to their next mortgages. Trade-up buyers from boom areas had more equity in their previous homes than buyers from other areas, and thus voluntarily took low LTV loans. To address this, I run separate regressions for first-time home buyers and other buyers. First-time home buyers are particularly unlikely to be affected by the wealth effect from past home price appreciation. In fact, it is far more likely that first-time home buyers would prefer higher LTVs instead of lower LTVs after rapid home price growth. I show that the relationship between home price appreciation and decline in LTVs among first-time home buyers and other buyers are both strongly significant and have similar magnitudes.

Another concern is reverse causality. Changes in LTV requirements, or collateral constraints, have causal impacts on home prices (Kiyotaki et al., 1997; Caballero and Krishnamurthy, 2001; Corbae and Quintin, 2015a; Sommer et al., 2013; Iacoviello, 2005). I show that the reverse causal story predicts the opposite sign of my coefficients. Lowering LTVs, or equivalently increasing percentages of down payments, would bring down home prices, not trigger a housing boom. My coefficients are likely biased towards zero by the reverse causal story. Another alternative story in this context is the mechanical effect of conforming loan limits. GSE-insured mortgages are required to be below a year-specific dollar amount. In high home price CBSAs with a large recent home price boom, their LTVs would have to be lowered to continue to satisfy the conforming loan limits. To address this, I conduct tests in two subsamples, one with only low-medium home price CBSAs, one restricting to loans under 95% of conforming loan limits. I find that the relationship between decline of LTVs and home price appreciation persists in these two subsamples.

Other alternative stories that I address include dynamic selection into GSEs' Fixed Rate Mortgages (FRMs), competition from private label loans, second liens and effect of binding debt-to-income ratios (DTI) constraints.

LTVs are only one of the loan and borrower characteristics through which the GSEs manage default risk. To understand how risk management using LTVs fits in the overall risk structure of GSE loans, I study how other loan and borrower characteristics changed during the housing boom, including FICO scores, DTIs, original loan amount, interest rates and owner occupation. The two questions I aim to answer are: were LTVs the GSEs' main tool managing home price risk? Can risk taking along other dimensions undo the risk reduction along LTVs? Perhaps the most important among other loan and borrower characteristics is FICO scores. I show that average FICO scores of GSE loans declined more in boom areas than in other areas. In other words, the GSEs took an increasing amount of FICO risk in boom areas than elsewhere. A natural question is whether the risk taking along FICO scores is larger than risk management along LTVs. Using elasticities of default with respect to FICO scores and LTVs estimated from a proportional hazard model, I show that by a very conservative estimate, the risk reduction through LTVs is at least 3.6 times as large as risk taking through FICO scores.

Between 2008 and 2011, the GSEs lost $215 billion from their insurance business and received $187 billion capital injection from the federal government (Frame et al., 2015). This clearly indicates that their risk management was insufficient for the 2006-2011 housing bust. However this could be driven by the 95% home price mean reversion between 2006 and 2011, which was much larger than the historically average 32% mean reversion (Glaeser, 2013; Glaeser and Nathanson, 2016; Cutler et al., 1991). To quantify how the GSEs would have performed under alternative home price scenarios, I build a valuation model projecting the total discounted guarantee fees collected by the GSEs and costs paid by the GSEs to investors under four home price mean reversion scenarios. To do this, I first estimate how loan and borrower characteristics, the coupon gap, unemployment rate and home price appreciation affect borrowers' default and prepayment decision using competing-risk hazard regressions. These hazard parameters, together with four different assumed home price paths and projected future interest rates from a Hull and White term-structure model, are used to forecast prepayment and default speeds. The final step is to transform the future prepayment and default speeds to cash flows and discount them. Using this framework, I answer two questions: taking into account risk adjustment along all mortgage and borrower characteristics, did the GSEs' indeed lower their risk in boom areas than other areas? Would the GSEs' risk management have been sufficient for a typical housing downturn?

Figures 1.2 to 1.5 illustrate the results from the structural valuation model. From Figure 1.2, we see that risk management by the GSEs results in a net lower risk in boom areas compared to other areas if home prices stay constant. However, the magnitude of risk management is small. For example, Figure 1.3 shows that under a 10% home price mean reversion, boom areas would already have larger normalized cost than elsewhere. Figure 1.4 shows that under the historically average 32% mean reversion, all CBSAs would collect sufficient revenue to cover losses. This suggests that the GSEs' risk management would have been sufficient for an average downturn. Figure 1.5 shows that under the realized 95% mean reversion, the GSEs' cost would be much higher than revenue in many boom areas.

Besides the papers mentioned above, my findings contribute to the following literature: 1) understanding home price expectations, especially during the 2000s housing bubble; 2) GSEs' loan selection and risk management; 3) LTVs' role in housing policy; 4) The interaction between collateral constraints and home prices.

For the home price expectation literature, my findings complement Cheng et al. (2014) and Cortés (2015). Using investment bankers' personal home transactions, Cheng et al. (2014) showed that private label securitization agents did not show an awareness of the housing bubble. In fact, some groups of private label securitization bankers were particularly aggressive in expanding their housing portfolios. In this paper, I show that in contrast to the private label securitization chains, the GSEs were aware of the housing bubble, highlighting the different beliefs of public and private mortgage insurers. In particular, the lack of relationship between the change in private label LTVs and home price appreciation detailed in Section 1.4 is also consistent with the findings of Cheng et al. (2014). Cortés (2015) show that local lenders forecasted the housing bust and reduced their market shares in bubble areas.

Understanding beliefs about home prices is crucial because the magnitudes of the last housing cycle far exceed what can be explained by credit expansion alone (Glaeser, 2013). Between 2001 and 2005, home prices rose by 103% in Phoenix, 110% in Las Vegas and 154% in Los Angeles, much larger than the causal effects of credit expansion (Di Maggio and Kermani, 2015; Glaeser, 2013) [1]. Exuberant expectation of home prices is argued to be a major cause for the boom-bust cycle (Glaeser, 2013). Recent studies also show that credit expansion itself was more likely to be driven by home price beliefs instead of changes in lending technologies (Adelino et al., 2016). In other words, the massive credit expansion was more from inflated optimism about home prices making lenders insensitive to borrower and loan characteristics, rather than a change in financial technology, for example the securitization of subprime mortgages fueling credit to low income borrowers. On one hand, we have the strong fact that housing markets mean revert (Cutler et al., 1991). On the other hand it seems that during each boom, people tend to think that "this time it's different". These two opposite effects make it difficult to infer whether people were really aware of the housing bubbles. On top of that, during the housing boom, there was a lively debate among prominent economists on whether home prices were reasonable (Himmelberg et al., 2005; McCarthy and Peach, 2004; Gallin, 2006, 2008; Davis et al., 2008).[2] The results from this paper show that the GSEs, the dominant insurers of the mortgage market, did take precautionary measures for the looming housing crash.

On the GSEs' loan selection, my paper is built on the premise of Kulkarni (2016) and Hurst et al. (2016), namely, the GSEs charge uniform prices across different areas, but adjust along the extensive margin. Kulkarni (2016) shows that the GSEs select more loans from lender-friendly states than neighboring borrower-friendly states. My contribution is strong evidence that the GSEs reacted to housing boom by lowering the share of high LTV loans in boom areas.

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[1]To put those numbers in context, even with the very strong wage growth in the Bay Area between 2011 and 2015, home prices in San Jose rose by 60% , less than half of the home price appreciation Los Angeles experienced between 2000 and 2005.

[2]Among the optimists are Himmelberg et al. (2005) and McCarthy and Peach (2004). Among the pessimists are Gallin (2006, 2008) and Davis et al. (2008).

The LTV ratio is a macroprudential policy tool widely used to intervene in home prices in other countries, including Hong Kong, China, the Netherlands, Sweden, Singapore and New Zealand (Wong et al., 2011; Shin et al., 2011; Lim et al., 2011; Borio and Shim, 2007). Academics also suggest that during a boom period, banks should use the long run home prices, instead of the current market prices, for mortgage underwriting (Glaeser, 2013). In the U.S., the government is reluctant to directly express views on asset prices. Unless asset prices have a large effect on inflation, the Federal Reserve Board tends not to adjust monetary policy for them. In this paper, I show that although the GSEs do not explicitly claim LTVs as a policy tool to express views on home prices, they do use LTVs in managing their home price risk.

I also contribute to the vast literature understanding how collateral constraints affect home prices and asset prices in general (Kiyotaki et al., 1997; Caballero and Krishnamurthy, 2001; Corbae and Quintin, 2015a; Sommer et al., 2013; Iacoviello, 2005). My contribution is that home prices can also affect collateral constraints, even if this channel is not explicitly stated by the GSEs.

The paper proceeds as follows. Section 1.2 gives a brief introduction of the institutional background. Section 1.3 describes data used in my analysis. Section 1.4 presents evidence that the GSEs reacted to the housing bubble by lowering LTVs in boom areas. Section 1.5 presents the insurance valuation framework. Section 1.6 concludes.

1.2   Institutional Background

This section gives a brief overview of the institutional settings studied in this paper. For more details, I refer the reader to Jaffee and Quigley (2012); Frame et al. (2015); Weiss et al. (2012). We first briefly discuss the history and business models of Fannie Mae and Freddie Mac. Then we discuss private mortgage insurance companies. We also present a numerical example of how private mortgage insurers take first loss positions for high LTV GSE loans.

Fannie Mae and Freddie Mac

Fannie Mae and Freddie Mac were established as government-sponsored enterprises by 1968 and 1970 legislation (Jaffee and Quigley, 2012). They are private entities in that they have profit-maximizing shareholders with stocks traded on the New York Stock Exchange. They are also public entities in the sense that they were chartered by Congress, with some board members selected by the president. Their structure as government-sponsored enterprises is to remove their activity and debt from the federal budget, while still achieving some public policy goals.

Fannie Mae and Freddie Mac's activities take two broad forms. First, their credit guarantee business involves providing mortgage insurance. They purchase a pool of mortgages from originators—typically banks or mortgage companies and then issue a security that receives cash flows from the mortgage payments, also called a mortgage backed security. They promise mortgage backed security investors timely payments of principal and interest, even if there are defaults and losses on the underlying loans. In return, the firms receive a monthly "guarantee fee" (Frame et al.

(2015)). The second form of Fannie Mae and Freddie Mac's business is to invest in assets including whole mortgages, their own agency mortgage-backed securities, nonagency mortgage-backed securities, and other types of fixed income securities. (Frame et al. (2015)).

Notice that during the 2008-2011 crisis period, the credit guarantee business lost $215 billion. The investment business generated $85 billion profit during 2009-2011, despite the large initial loss of $83 billion in 2008 (Frame et al., 2015).

Private Mortgage Insurers

Private mortgage insurers are companies that provide mortgage insurance similar to the ones provided by the GSEs. They primarily provide credit enhancement for GSE loans with LTVs above 80%.

Because the default rate strongly co-moves with housing cycles, private mortgage insurers have had concentrated failures. In the 1930s, all 50 or so private mortgage insurance companies became insolvent (Weiss et al., 2012). From the mid-1930s until the 1950s no private mortgage insurers existed and Federal Housing Administration (FHA) was the only provider of mortgage insurance. In the 1980s crash, about half of the private mortgage insurance companies stopped underwriting insurance. Only about a dozen companies survived. Due to the 2006-2011 housing crash, three out of the eight major mortgage insurers failed, and one was placed into receivership. They also changed their behavior handling claims, for example, rejecting a unprecedented high fraction of claims and delaying settlements, so that they would suffer less loss at the expense of their clients.

Numerical Example of Insurance for High LTV GSE Loans

This section presents a numerical example of how private insurers take first loss positions for high LTV GSE loans, with the GSEs covering any additional losses.

Consider a mortgage with an initial balance of $270,000 for a house valued $300,000 at origination. Since the initial LTV is 90%, higher than the 80% threshold, the mortgage requires private mortgage insurance to be eligible for the GSEs' purchase. The median percentage of loan balance covered by private insurers, also called *coverage ratio*, is 25% for high LTV GSE loans. Assume that two years latter after origination, the borrower defaulted. At the time of default, the remaining balance was $260,000 and the house value was $150,000. The total loss for the lender is $110,000, the difference between remaining balance and house value. Assuming a 25% coverage ratio, the private insurer would cover $25\% \times \$270,000 = \$67,500$. The GSEs would cover $\$110,000 - \$67,500 = \$42,500$.

Notice that in this example, since a private insurer covers 25% of the initial balance, the net LTV for the GSEs is $75\% \times 270,000/300,000 = 67.5\%$, much lower than the 80% threshold. This is typical for high LTV GSE loans. In other words, just along the LTV dimension, high LTV GSE loans are less risky than an 80% LTV GSE loan on the GSEs' balance sheet.

1.3 Data Description and Summary Statistics

The analysis in this paper is based on three different types of data: loan level mortgage origination and performance data, home price indexes and interest rate data. In the following, I go through how each of the data sets is constructed. Loan level mortgage origination data are used to study how loan characteristics, for example LTVs and FICO scores, evolved during the housing boom. House price indexes are used to differentiate between boom areas and other areas. Loan performance data, house price indexes and interest rates data are used to build a mortgage insurance valuation framework, studying how the GSEs' risk management would have performed under different house price mean reversion scenarios.

Loan Level Mortgage Origination and Performance Data

In this paper, I study both GSE loans and private label loans. For GSE loans, I use the public data collected from Fannie Mae and Freddie Mac's websites. Private label loans in my sample come from ABSNet. Both data have rich mortgage characteristics, including original LTV, original CLTV, FICO score, loan amount, loan purpose (purchase or refinance) and detailed monthly loan performance. The GSE data also have a variable indicating if a loan is taken by a first-time home buyer. As I will argue, this variable helps me address an important alternative story from the wealth effect of past home price appreciation. I keep all first lien purchase mortgages.

Home Price Index

CBSA level home price indexes are collected from FHFA. I choose CBSA level home price indexes over zip code level home price indexes because the finest geographic code in the public GSE data are at the CBSA level. FHFA home price indexes are typically used for mortgage modeling and stress testing.

Interest Rate Data

To value mortgage insurance, I collect interest rate data from Yield Book. Yield Book is a fixed income valuation service provided by Citi group widely used on Wall Street. They also provide historical data related to fixed income trading, including interest rate data. Interest rates affect the valuation of mortgage insurance through two ways. First, forecast interest rates are the discount rates for both the fixed leg and the floating leg of mortgage insurances. Second, a larger coupon gap, defined as the original ten year rate minus the current ten year rate, gives borrowers stronger incentives to refinance. This determines how long the insurance provider expects to collect premium and how long the insurance provider is exposed to house price risk.

Summary Statistics

Table 1.1 reports the summary statistics. There are 321 CBSAs in my sample and 1926 CBSA-year level observations between 2000 and 2006. In most of my specifications, separate regressions are run for GSE loans for first-time home buyers and other buyers, with LTVs below 80.5% or above 80.5%[3]. Thus Table 1.1 reports changes in loan and borrower characteristics separately for these four subsamples.

We can see that 2000-2006 was a period with strong home price and wage growth. The rising debt-to-income ratios indicate that mortgage debt growth out paced wage growth. Interest rates significantly declined during this boom period. In both the above and below 80.5% segments, LTVs changed little on average. For first-time home buyers, the standard deviation of changes in log LTVs for the below 80.5% segment is three times the standard deviation of changes in log LTVs for the above 80.5% segment.

1.4 Home Price Risk Management through LTVs

This section presents evidence that during the 2000-2006 housing boom, the GSEs actively lowered their home price risk exposure through LTVs while private insurers did not. The central piece of evidence is the strong relationship between home price appreciation and the decline of average LTVs among mortgages insured by the GSEs only, *i*.e., mortgages with LTVs at or below 80%. In contrast, little evidence suggests that private insurers lowered their home price exposure for mortgages with LTVs above 80%, where they would take first loss positions.

I first present the empirical model and my estimation samples. To isolate two important alternative explanations, the mechanical effect of conforming loan limits and wealth effect from past home price appreciation, my preferred sample is purchase mortgages taken by first-time home buyers from low to middle home price CBSAs. I then present the main results.

I also conduct robustness tests for private insurers and the GSEs respectively. I show that private insurers did not lower their home price exposure through two other channels along which they could have adjusted: share of high LTV loans and coverage percentage. I first focus on the results for the GSEs. I conduct a series of robustness tests to verify that the most likely explanation for my results is the GSEs' dynamic home price risk management. Alternative explanations that I address include reverse causality, upper bound on DTIs, risk adjustment along other dimensions, challenges in saving for 20% down payments, borrowers voluntarily switching to second liens, and private label ARMs or FRMs.

The main goal of this paper is to compare the GSEs' home risk management with private insurers'. Section 1.4 complements the main goal by presenting results for private label loans, showing that they are consistent with existing results in Cheng et al. (2014), which finds that

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[3]I use 80.5% instead of 80% as the threshold because loans with LTVs just above 80% within round errors are treated by the GSEs as at 80%. They are exempt from the requirement for private mortgage insurance. By using 80.5% as the threshold, loans with LTVs just above 80% are classified as below or at 80%, consistent with the GSEs' definition.

investment bankers working for the private label securitization chain were unaware of the housing bubble.

Empirical Model

The main specification is

$$\Delta \log LTV_{ct,LTV \leq 80.5\%} = \beta_{LTV}^{LTV \leq 80.5\%} \Delta \log HPI_{ct} + \beta_X \Delta X_{ct} + \alpha_c + \gamma_t + \epsilon_{ct}, \quad (1.1)$$

$$\Delta \log LTV_{ct,LTV > 80.5\%} = \beta_{LTV}^{LTV > 80.5\%} \Delta \log HPI_{ct} + \beta_X \Delta X_{ct} + \alpha_c + \gamma_t + \epsilon_{ct}, \quad (1.2)$$

where α_c and γ_t are geographic and year fixed effects respectively; $\Delta \log LTV_{ct,LTV \leq 80.5\%}$ and $\Delta \log LTV_{ct,LTV > 80.5\%}$ are changes in logged average LTVs from year t to year $t+1$ in CBSA c for the LTVs below 80% and LTVs above 80% segments respectively. Separate regressions are run for GSEs loans for first-time home buyers and GSEs loans for other buyers, explained in Section 1.4. $\Delta \log HPI_{ct}$ are changes in log home prices. ΔX_{ct} are CBSA-year level control variables, including changes in macroeconomic conditions measured by unemployment and average wage, changes in loan and borrower characteristics including average FICO scores, debt-to-income ratios, interest rates and percentage of owner occupied mortgages.

LTVs are among the many loan and borrower characteristics the GSEs and private insurers adjust. To complement the findings for LTVs, I also estimate the following regression for FICO scores

$$\Delta \log FICO_{ct} = \beta_{FICO}^{LTV \leq 80.5\%} \Delta \log HPI_{ct} + \beta_X \Delta X_{ct} + \alpha_c + \gamma_t + \epsilon_{ct}. \quad (1.3)$$

Naturally, in the FICO score regressions, the set of control variables include changes in LTVs and exclude changes in FICO scores. Separate regressions are run for GSE loans by first-time home buyers (the full sample or the segment with LTVs below 80%) and private label ARMs. Regression (1.3) is studied for two reasons. First, they are to study how the GSEs and private insurers adjusted FICO scores to understand how LTV adjustment fits in the overall home risk management, detailed in the following. Second, the FICO score results are used to eliminate some alternative stories. For example, one concern is that rising house prices in boom areas drove up mortgage payments, forcing some low income borrowers to switch to ARMs with lower current interest rates than FRMs. This leaves the FRM borrower pool with good borrowers having enough savings for large down payments, explaining why LTVs declined in boom areas. If my results were driven by this story, we would expect a relative increase of FICO scores among GSE loans and a relative decline of FICO scores among ARMs from boom areas. However, Table 1.7 finds the exact opposite. Among GSE loans, FICO scores relatively declined in boom areas, while among ARMs, FICO scores relatively increased in boom areas.

Below I argue that my preferred sample is GSE first-time buyers from low-to-middle home price areas to address two important alternative explanations. I also discuss the mechanical effect of conforming loan limits and how it is addressed by excluding high home price CBSAs or restricting the sample to loans under 95% of the conforming loan limits. I also describe how the wealth effect from past home price appreciation is addressed by restricting to first-time home buyers.

Mechanical Effect of Conforming Loan Limit

One important alternative explanation for declining LTVs in boom areas is the mechanical effect of the conforming loan limits (CLLs). The loan amount of GSE insured mortgages is required to be under the CLLs. Since LTVs are loan amount divided by house prices, in areas where house prices are high, LTVs of GSE loans have to be low to satisfy the CLL requirement. Thus past home price appreciation could mechanically bring down average LTVs among GSE loans.

To address this, I construct two samples where the CLLs are not binding. In the first sample, only CBSAs with low to middle home prices by 2006 are kept. Low to middle home price CBSAs are defined as CBSAs with more than 80% of private label loans in 2006 under the CLLs. I use the distribution of private label loans because their loan amount is not required to be under the CLLs. Notice that the 80% selection criteria is very strict. All CBSAs in the remaining sample had home prices at least one hundred thousand dollars below the CLLs throughout the housing boom. Figure 1.1 plots the excluded and selected CBSAs. Intuitively, many of the CBSAs on the two coasts have high home prices and are thus excluded. Some boom CBSAs with initial low home prices, including part of inland California and Florida, are kept in the remaining sample because their prices started low. This provides us with enough variation in HPA to study the relationship between HPA and changes in LTV. I report results from both the full sample and the low to middle home price CBSAs sample. In the second sample, I keep loans below 95% of the CLLs. The gap between their loan amount and the CLLs ensure that the CLLs are not binding for this universe of loans. Results under this robustness test is presented in Table 1.12. I can see that my results persist in this subsample.

Ialth Effect from Past Home Price Appreciation

One could also argue that the decline of LTV for GSE loans was driven by borrowers' demand for low LTV loans, not driven by the GSEs' supply of low LTV loans. This is especially plausible, since buyers who traded up in 2005 might have carried the equity in their previous home, mainly accumulated from the rapid home price appreciation in the last two years, to their new home. Thus, trade-up buyers from boom areas in 2005 might naturally ask for a low LTV, unrelated to GSEs' risk management. To address this, I run separate regressions for first-time home buyers and other buyers. First time home buyers are particularly unlikely to be affected by the wealth effect from past home price appreciation. In fact, because of their limited savings, it is far more likely that first-time buyers on average would prefer lower LTVs than higher LTVs after large recent home price growth. I show that the relationship between home price appreciation and decline of LTVs persists in the first-time home buyer sample. This is more consistent with the supply side story from GSEs' risk management than the demand side story of boom area borrowers voluntarily taking low LTV loans.

Main Result

Table 1.2 and Table 1.3 report results from regressions (1.1) and (1.2), estimating the effect of home price appreciation on LTVs. Table 1.2 reports results from all CBSAs while Table 1.3 excludes high home price CBSAs from the sample.

Panel A of Table 1.2 focuses on first-time home buyers. Columns 1-3 are for the LTVs below 80% segment. Columns 4-6 are for the LTVs above 80% segment. Columns 1 and 4 control for year fixed effects only. Columns 2 and 5 add CBSA-year level controls, including changes in macroeconomic conditions, changes in loan and borrower characteristics. Columns 3 and 6 further add CBSA fixed effects. The most striking contrast in Panel A is the strongly negative coefficient in column 3 and the statistically insignificant coefficient in column 6. In the LTVs below 80% segment, LTVs significantly declined in boom areas than other areas, while there is no such pattern in the above 80% segment.

Panel B of Table 1.2 reports the results for other buyers. Comparing the coefficients for other buyers with the corresponding coefficients for first-time buyers in Panel A, we see that the LTVs always declined more among loans taken by other buyers than first-time buyers in boom areas. This is consistent with our hypothesis that second time home buyers voluntarily lowered their LTVs in response to home price booms from a wealth effect.

Table 1.3 excludes high home price areas to rule out the mechanical effect of conforming limits discussed in Section 1.4. As discussed in Section 1.4 and Section 1.4, my preferred sample is GSE first-time home buyers from low to middle home price CBSAs, corresponding to Panel A of Table 1.3. Columns 1-3 show that the relationship between decline of LTVs and home price appreciation is always strong across different specifications in the LTVs below 80% segment. Columns 4-6 show that in the LTVs above 80% segment, boom areas did not have a disproportionately larger declines of LTVs. The estimate -0.053 in column 3 of panel A implies that a 10% home price appreciation leads to a 0.51% decline of LTVs.

Comparing Panel A and Panel B in Table 1.3, we see that in the low to middle home price sample, the coefficients for GSE first-time home buyers and GSE other buyers are similar to each other. The strong home price appreciation in boom areas should have led to large wealth differences between first-time home buyers and second time home buyers, and potentially different preferences for LTVs. With large equity in their previous homes, it is natural to expect second time home buyers to carry some equity to their next purchase loans. The similar coefficients for the two buyer groups are more consistent with the supply side story of the GSEs' risk management than the demand side story of boom area borrowers asking for lower LTVs.

Robustness Tests for Private Insurers

One challenge in interpreting Table 1.3 is that the share of loans with LTVs above 80.5% might have changed over time. For instance, private insurers might have been concerned about the housing bubble and insured a decreasing number of LTVs above 80.5% loans in boom areas than the other areas. To rule out this alternative story, I run the following regression

$$\Delta \log P_{ct,LTV>80.5\%} = \beta_{LTV>80.5\%}\Delta \log HPI_{ct} + \beta_X \Delta X_{ct} + \alpha_c + \gamma_t + \epsilon_{ct}, \qquad (1.4)$$

where $P_{ct,LTV>80.5\%}$ is the percentage of loans with LTV above 80.5% in CBSA c and year t. Panel A of Table 1.4 reports the results. I see that in my preferred sample, boom areas did not have a larger relative decline of share of high LTV GSE loans.

Another potential channel for private mortgage insurance companies' home price risk management is coverage percentage, the maximum percentage of loan amount they cover in case of defaults. They could have relatively lowered their covered losses in boom areas to reduce home price risk. Panel B of Table 1.4 reports results from the following regression

$$\Delta \log CP_{ct} \quad = \quad \beta_{LTV>80.5\%}\Delta \log HPI_{ct} + \beta_X \Delta X_{ct} + \alpha_c + \gamma_t + \epsilon_{ct}, \qquad (1.5)$$

where CP_{ct} is coverage percentage in CBSA c year t. I can see that private mortgage insurance companies did not lower insurance percentages more in boom areas than elsewhere. The positive and significant coefficient in column (3) panel B means that they relatively increased percentages of covered losses in boom areas, the opposite of reducing home price risk exposure.

Robustness Tests for the GSEs

This section conducts a series of robustness tests to verify that the most likely explanation for the relative decline of LTVs in boom areas among the LTVs below 80% segment is the GSEs' home price risk management.

I show that the reverse causal story predicts the opposite sign of my findings. In other words, my estimates are biased towards zeros by the reverse causal story of stricter LTV requirements lowering house prices. I address three alternative stories related to borrowers' mortgage choice. All three of them are based on the idea that high LTV borrowers from boom areas were not credit rationed by the GSEs, but voluntarily switched to other mortgage products, including private label FRMs, ARMs, or second liens. I present evidence inconsistent with each of these three stories. For example, one could argue that borrowers might have switched to private label FRMs because they offered better interest rates. However, Table 1.5 shows that the GSEs offered better interest rates than private label FRMs in almost all market segments in each CBSA. Thus my results are more likely to be driven by the GSEs' voluntary risk management rather than being forced by competition from the private label segment.

Another alternative story is upper bounds on DTIs. If borrowers are already taking out the maximum loan amount allowed by DTI upper bounds, any more home price appreciation would lower LTVs. To address this, I drop loans with high DTIs under a number of DTI thresholds. My results persist in these low DTI subsamples.

I compare risk adjustment along FICO scores with risk management along LTVs and verify that LTVs is the main channel for the GSEs' home price risk management. The GSEs relatively lowered FICO scores in boom areas, or took more risk along FICO scores in boom areas than elsewhere. I show that the risk reduction along LTVs is much larger than risk taking along FICO scores.

I also address a more subtle alternative story. It could be that borrowers always try to lower their LTVs to 80% to avoid a private mortgage insurance premium. However, it was relatively

easy to do with low home prices before the housing boom than the much higher home prices after the housing boom. This manifests as non-boom areas having an increasing concentration of LTVs at 80% than boom areas, explaining why boom area LTVs relatively declined in the LTVs under 80% segment. If my results are driven by this story, we would expect that after dropping loans with LTVs at 80%, the coefficients will become insignificant. However, I show that the results are stronger after dropping loans with LTVs between 79.5% and 80.5%.

Reverse Causality

One could argue that regressions (1.1) and (1.2) are subject to reserve causality. After all, the literature studying credit constraints and home prices largely focus on how credit conditions, including LTV requirements, would affect house prices (Corbae and Quintin (2015a); Sommer et al. (2013)). This reverse causal story predicts the opposite signs of my findings. The credit condition affecting house price channel predicts that declining LTVs, or equivalently requiring higher percentage of down payments, would lower home prices. In contrast, I find that LTVs for GSE loans declined in boom CBSAs, the opposite of the causal effect of LTV requirement on house prices.

Borrowers' Mortgage Choice

The risk management story essentially means that high LTV borrowers from boom areas were credit rationed by the GSEs for home price risk management. One could argue that rather than being credit rationed, these borrowers switched to other products by choice. This section addresses three alternative stories along this line, that high LTV borrowers from boom areas voluntarily switched to private label FRMs, ARMs, or second liens.

   The first example is that borrowers switched to private label FRMs because they offered better terms. To test this theory, I use interest rate data for both GSE FRMs and private label FRMs to test if private label FRMs offered better interest rates relative to GSE loans towards the end of the housing boom than in the beginning of the housing boom. I first divide mortgages into sixteen segments along two dimensions, LTV and FICO score. Along the LTV dimension, I divide LTV to four ranges: below 79.5%, between 79.5% and 80.5%, between 80.5% and 90%, and above 90%.[4] Along the FICO score dimension, I divide the spectrum into four ranges: below 660, between 660 and 720, between 720 and 760, and above 760. There are sixteen combinations of LTV ranges and FICO score ranges. For each CBSA and each combination of LTV and FICO score, I collapse the median interest rate for both GSE FRMs and private label FRMs. In each year, there are more than 4000 CBSA-segment combinations. Table 1.5 reports the percentage of CBSA-segment combinations in which private label FRMs had a lower median interest rate than GSE FRMs. I can see that in every year between 2000 and 2006, GSEs had an interest rate advantage in more than 94% of the CBSA-segment combinations. More importantly, the percentage of CBSA-segments in which private label loans had an edge was declining through the housing boom. Also, all of the CBSA-segments in which private loans had an interest rate edge in 2005 and 2006 were small

---

[4]I define between 79.5% and 80.5% as a separate segment because many mortgages have LTVs very close to 80%.

CBSA-segments, with 83 out of 133 having 3 or fewer private label FRMs. These results show that competition from private label loans is unlikely to drive my results.

The second alternative story is about borrowers switching to ARMs. It could be that because of the rising home prices, many borrowers in boom areas found FRM payments unaffordable and switched to ARMs for temporarily lower interest rates. With these low income borrowers leaving the FRM pool in boom areas, the remaining FRM pool improved and LTVs for GSE FRMs declined. There are two pieces of evidence inconsistent with this argument. First, an assumption in this argument is that relatively low income borrowers switched to ARMs in boom areas. However, as illustrated in columns 4-6 of Table 1.7, average FICO scores for ARMs relatively increased in boom areas across different specifications. In boom areas, good borrowers, rather than low income borrowers were more likely to leave the FRM pool than the other areas. The second piece of evidence is that, illustrated in columns 1-3 of Table 1.7, average FICO scores for GSE first-time home buyers relatively declined in boom areas throughout the housing boom. It is unclear that the relative improvement of GSE loans' LTVs in boom areas is driven by improving borrower quality.

The third mortgage choice alternative explanation is that boom area borrowers could have switched to low LTV GSE loans but took out second liens instead. To address this, I use changes in combined-loan-to-value ratios (CLTVs) as the left-hand-side variable in the following regressions

$$\Delta \log CLTV_{ct,LTV \leq 80.5\%} = \beta_{CLTV}^{LTV \leq 80.5\%} \Delta \log HPI_{ct} + \beta_X \Delta X_{ct} + \alpha_c + \gamma_t + \epsilon_{ct}, \quad (1.6)$$

$$\Delta \log CLTV_{ct,LTV > 80.5\%} = \beta_{CLTV}^{LTV > 80.5\%} \Delta \log HPI_{ct} + \beta_X \Delta X_{ct} + \alpha_c + \gamma_t + \epsilon_{ct}. \quad (1.7)$$

Table 1.6 reports the results. Comparing Table 1.6 with Table 1.3, we see that changing the left-hand-side variable to changes in CLTVs has almost no impact on the coefficients.

Upper Bound on Debt-to-Income Ratio

Another alternative story is that the debt-to-income ratios in boom areas might have got to the upper bounds allowed by the GSEs. Under a rising home price environment, these upper bounds on loan amounts make larger down payments, or lower LTVs necessary. To rule out this story, I drop loans with high debt-to-income ratios by a number of different thresholds. By dropping these loans with possibly binding debt-to-income ratios, I test if the relationship between home price appreciation and decline of LTVs still holds. Table 1.8 report the results. I can see that in the low DTI subsamples, my results continue to hold.

Comparing Magnitudes of Different Dimensions

LTVs are one of the dimensions through which the GSEs adjust risk. Other dimensions include FICO scores, debt-to-income ratios (DTIs), original loan amount, interest rates, and owner occupancy. One could argue that LTVs were not the main dimension of the GSEs' risk management. For example, the GSEs might adjust FICO scores between boom areas and non-boom areas much more than their LTV adjustment. For a complete analysis of the net risk management by the GSEs, I build a structural valuation framework in Section 1.5 based on calibration of term structure model, hazard regressions and forecasting future prepayment and default speeds. In this section, I conduct

back of envelope analysis for the GSEs' risk taking or risk management along perhaps the most important dimension besides LTVs, FICO scores.

Table 1.9 compares how the GSEs adjusted LTVs and FICO scores side by side. Column 2 shows that average FICO scores of GSE loans declined more in boom areas than other areas. In other words, the GSEs took an increasing amount of risk in boom areas than other areas along the FICO score dimension. A natural question is whether the risk taking along FICO scores is larger than risk reduction along LTVs or the reverse.

I present evidence that risk reduction along LTVs dominates risk taking along FICO scores. Conceptually, lower LTVs reduce the GSEs' losses through two channels while lower FICO scores increase the GSEs' losses through only one channel. Lower LTVs reduce both default probabilities and loss-given-default. Lower FICO scores only increase default probabilities, and have ambiguous impact on loss-given-default. The loss-given-default channel could be much larger than the default probability channel under moderate home price declines.

If we focus on the default probability channel, to compare the effects on default probabilities, I use the elasticities of default with respect to LTVs and FICO scores estimated from hazard regressions detailed in Section 1.5. From Table 1.14, we see that their default elasticities have similar magnitudes, while Table 1.9 shows that the GSEs' LTV response is 3.6 times as large as the GSE's FICO score response to rising home prices. Thus the GSEs' LTV home risk reduction dominates their FICO risk taking.

Difficulty of Obtaining 20% Down Payments

To address the story that rising home prices changed the difficulty of obtaining 20% down payments, I drop loans with binding LTVs-LTVs at 80%. Table 1.10 reports the results. I see that relationship between home price appreciation and decline of LTVs persists.

Results for Private Label FRMs

While the main goal of the paper is to study the GSEs and private insurers' home price risk management, in this section, I study if my results are consistent with previous results for the private label segment studied in Cheng et al. (2014). Using securitization investment bankers' personal home transactions, Cheng et al. (2014) find no evidence that investment bankers foresaw the housing crash. My results are consistent with their findings. Table 1.11 reports the result for private label FRMs. I see that after controlling for changes in macroeconomic conditions, loan and borrower characteristics, CBSA and year fixed effects, private label FRMs had relatively increasing LTVs in boom areas. This is consistent with the idea that the private label segment was unaware of the housing bubble.

1.5 Valuation of Insurance

In this section, I construct an insurance valuation framework quantifying the risk management of the GSEs. The goal of the framework is to answer two questions: 1) Combining risk adjustment using all loan and borrower characteristics, did the GSEs indeed lower their risk exposure more in boom areas than other areas?; 2) Was the GSEs' risk management sufficient? For the second question, the GSEs' $215 losses from the insurance business between 2008 and 2011 suggest that their risk management was insufficient. However, one contributing factor to the large losses is the larger than historical average house price mean reversion between 2006 and 2011. For every $1 home price increase between 2001 and 2006 in a CBSA, it on average gave back 95¢between 2006 and 2011. This 95% mean reversion was much larger than the 32% historically average mean reversion. Thus the GSEs' failure alone does not imply that the GSEs' risk management was insufficient based on reasonable assumptions ex-ante. To quantify how the GSEs' portfolios would have performed under alternative scenarios, I consider four hypothetical housing markets: 0% mean reversion, or prices staying constant, 10% mean reversion, 32% mean reversion and 95% mean reversion.

My valuation framework calculates the discounted cash flows for the mortgage insurance underwritten by the GSEs in 2005, the peak year of the housing boom. Section 1.5 describes the cash flows. Section 1.5 presents the competing-risk hazard regressions, from which the estimated hazard parameters are used to forecast default and prepayment speeds, and the cash flows. Section 1.5 introduces the term structure model, used both as the discount rates valuing the cash flows and to calculate the coupon gap, an important predictor for default and prepayment risks. Section 1.5 summarizes the data-generating process. Section 1.5 presents the results.

To make the computation manageable. I restrict my loan sample to purchase mortgages from the 100 largest CBSAs in the U.S. They represent about 75% of all mortgages in the full sample.

Insurance Cash Flows

Mortgage default insurance provided by the GSEs have two legs of cash flows. The first leg is the premium collected by the GSEs from investors over the life of mortgages. The second leg is paid by the GSEs to investors to cover losses when borrowers default and the collateral value is lower than the remaining balance. To be consistent with the credit default swap terminologies, I refer to the insurance premium leg as the fixed leg, and the leg of loss covered by the GSEs as the floating leg.

Both legs of the cash flows are random. The fixed leg is collected by the GSEs until mortgage termination, which are random events. Reasons for termination include default, being paid in full till maturity, and prepayment caused by, for example, moving or refinance. The floating leg, paid when borrowers default, is random as well. Table 1.13 summarizes how the cash flows evolve each month by different mortgage outcomes.

Hazard Model

Hazard Model Specification

Many factors affect borrowers' default and prepayment decisions. For example, we have the intuition that borrowers with lower initial FICO scores are more likely to default. Another example is that when interest rates decline, borrowers have larger incentives to refinance and are more likely to prepay. The goal of the competing-risk model presented in this section is to study the relative importance of how different factors affect default and prepayment risks. I estimate the following proportional competing-risk hazard model specified in equations (1.8) and (1.9)

$$
\begin{aligned}
\lambda_{ic}^{\text{Default}}(t) &\equiv \lim_{\xi \to 0} \frac{1}{\xi} \Pr_{ic}^{\text{Default}}(t - \xi < \tau \leq t | \tau > t - \xi, X) && (1.8) \\
&= \exp\left(X'_{ict}\beta^{\text{Default}}\right)\lambda_0^{\text{Default}}(t), \\
\lambda_{ic}^{\text{Prepay}}(t) &\equiv \lim_{\xi \to 0} \frac{1}{\xi} \Pr_{ic}^{\text{Prepay}}(t - \xi < \tau \leq t | \tau > t - \xi, X) && (1.9) \\
&= \exp\left(X'_{ict}\beta^{\text{Prepay}}\right)\lambda_0^{\text{Prepay}}(t),
\end{aligned}
$$

where

$$
\begin{aligned}
X'_{ict}\beta &= \theta_{\text{HPA}}\log(\text{HPA}_{ct}) + \theta_{\text{Unemp}}\text{Unemployment}_{ct} + \theta_C(\text{Coupon Gap}) \\
&\quad + W'_{Bi}\theta_B + W'_{Li}\theta_L + \alpha_c, \\
\text{Coupon Gap} &= r_{10,\text{origination}} - r_{10,t}, \\
\text{HPA}_{ct} &= \text{HP}_{ct}/\text{HP}_{c0}
\end{aligned}
$$

$\lambda_{ic}^{\text{Default}}(t)$ and $\lambda_{ic}^{\text{Prepay}}(t)$ are the latent instantaneous default and prepayment probabilities for individual i from CBSA g with loan age t months respectively. $\lambda_0^{\text{Default}}(t)$ and $\lambda_0^{\text{Prepay}}(t)$ are the baseline default and prepayment hazard functions, estimated nonparametrically, following Han and Hausman (1990). In specifications (1.8) and (1.9), $\exp\left(X'_{ict}\beta^{\text{Default}}\right)$ and $\exp\left(X'_{ict}\beta^{\text{Prepay}}\right)$ proportionally scale up or down the hazards, depending on the signs of X_{ict} and coefficients β^{Default} and β^{Prepay}. β^{Default} and β^{Prepay} are the main parameters of interests, measuring how different factors affect default and prepayment risks. Covariates X_{ict} cover static and dynamic variables. Static variables include initial loan and borrower characteristics, denoted as W_{Li} and W_{Bi} respectively, including log FICO score, first-time home buyer indicator, owner occupancy, log original loan amount, log original LTV, the difference between the original interest rate and the original ten year rate. Dynamic covariates include log cumulative home price changes since origination HPA_{it}, coupon gap, defined as the difference between the ten year rate at origination and the current ten year rate, and unemployment rate. The estimation sample is mortgage performance data between 2000 and 2005. I truncate the performance data at the end of 2005 to make my estimation sample comparable to the data available for pricing mortgage insurance in 2005.

Estimation

The competing-risk hazard model specified in equations (1.8) and (1.9) is a continuous time model. However, loan performance is observed at the end of each month in discrete time. Assuming that the time varying covariates X_{ict} are constants in each discrete time interval $[t-1, t]$, the continuous time model in (1.8) and (1.9) can be transformed to a discrete time model. In greater details, let $S(t) = \Pr(\tau > t)$ denote the survivor function and let $\Lambda(t) = -\log(S(t))$. $\Lambda(t)$ is also called the integrated hazard function because it satisfies the familiar identity

$$\Lambda(t) = \int_0^t \lambda(\tau)d\tau. \tag{1.10}$$

Using $\Lambda(t) = -\log(S(t))$ and identity (3.-3), the probability of survival between $t-1$ and t conditional on that one survived the first $t-1$ periods is

$$
\begin{aligned}
\Pr\left(\tau > t \mid \tau > t-1\right) &= \frac{S(t)}{S(t-1)} \\
&= \exp\left(\Lambda(t-1) - \Lambda(t)\right) \\
&= \exp\left(-\int_{t-1}^t \lambda(\tau)d\tau\right)
\end{aligned}
$$

In general $\lambda(\tau)$ depends on $X_{ic\tau}$, which is time varying between $t-1$ and t. Assuming that $X_{ic\tau}$ are constants when τ is between $t-1$ and t. We have

$$
\begin{aligned}
\Pr\left(\tau > t \mid \tau > t-1\right) &= \exp\left(-\int_{t-1}^t \lambda(\tau)d\tau\right) \\
&= \exp\left(-\exp\left(X'_{ict}\beta\right)\lambda_0(t)\right),
\end{aligned}
$$

or equivalently

$$
\begin{aligned}
\log\left(\log\left(\frac{S(t-1)}{S(t)}\right)\right) &= X'_{ict}\beta + \log(\lambda_0(t)) \\
\log\left(-\log\left(1 - \Pr\left(\tau \in (t-1, t] \mid \tau > t-1\right)\right)\right) &= X'_{ict}\beta + \log(\lambda_0(t)),
\end{aligned}
$$

which is the complementary log-log model I estimate in discrete time.

Results

Table 1.14 reports the hazard regression results. All coefficients have the expected signs. For example, a high LTV loan is much more likely to default than a low LTV loan. The coefficient on log(Original LTV) in column (1) implies that a 5% higher LTV increases default probability by 34.5%. A high FICO score borrower is much less likely to default and more likely to prepay than a low FICO score borrower. A positive and large coupon gap gives the borrower strong incentive to refinance, and leads to a larger prepayment risk. A 10% larger home price appreciation leads to a 36.7% lower default hazard and a 14.2% higher prepayment hazard.

Interest Rate Model

I calibrate the following Hull-White term-structure model

$$dr = (\theta(t) - \alpha r)dt + \sigma dw. \tag{1.4}$$

The calibration process is a three-step procedure:

1) From Yield Book, I collect interest rates, or equivalently discount factors, for ten maturities including 1-month, 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, and 30-year. I first construct a continuous yield curve for all possible maturities by fitting the discount function expressed as

$$Z(t) = e^{at + bt^2 + ct^3 + dt^4 + et^5} \tag{1.5}$$

   using observed discount factors for the ten available maturities. The estimated $\hat{Z}(t)$ is then used to calculate the forward rate function $\hat{f}(t, t')$.

2) Calibrate parameters α and σ in equation (1.4).

3) Calculate $\hat{\theta}(t)$ using the estimated forward rate $\hat{f}(0, t)$ from step 1), and estimated $\hat{\alpha}$ and $\hat{\sigma}$ from step 2).

Note that loans in my sample were originated in different months in 2005. To forecast how interest rates would evolve after loan originations, I calibrate the term-structure model for each origination month independently. For step 1), parameters a through e are estimated from first taking the log of both sides of 1.5 and then running a linear regression.

For step 2), I estimate α and σ using caplet prices and discount rates. Caplets are essentially European call options where the underlying is the interest rate with cash flow at time T_i proportional to $\max(r(T_{i-1}, T_i) - r_K, 0)$, where $r(T_{i-1}, T_i)$ is the floating rate at time T_{i-1} and maturity T_i, r_K is the strike interest rate. Intuitively, caplet prices, as prices for interest rate options, contribute to estimating the volatility parameter σ and the mean reversion parameter α in the term-structure model (1.4). α and σ are chosen to best fit all caplet prices by minimizing the following function

$$\min_{\alpha, \sigma} \sqrt{\sum_{i=1}^{I} \left(\frac{model_i(\alpha, \sigma) - market_i}{market_i} \right)^2}$$

where $model_i(\alpha, \sigma)$ and $market_i$ are, correspondingly, model and market caplet i cash prices. Model prices, $model_i(\alpha, \sigma)$-s, are based on the modified Black-Sholes formula.

For step 3), $\hat{\theta}(t)$ is calculated as

$$\hat{\theta}(t) = \frac{\partial \hat{f}(0, t)}{\partial t} + \hat{\alpha}\hat{f}(0, t) + \frac{\hat{\sigma}^2}{2\hat{\alpha}} \left(1 - e^{-2\hat{\alpha}t} \right),$$

where $\hat{f}(0, t)$ is the estimated interest rate between time 0 and time t

$$\hat{f}(0, t) = -\hat{a}t - \hat{b}t^2 - \hat{c}t^3 - \hat{d}t^4 - \hat{e}t^5.$$

Data-Generating Process

Data in my simulations are generated from the following sources. Loan and borrower characteristics are from the GSE public data. I keep purchase mortgages originated in 2005 from the largest 100 CBSAs in the U.S. There are 596,911 loans in my sample. I calculate the discounted cash flows for insurance underwritten on each loan and then collapse to the CBSA level and report the results in Section 1.5. Hazard parameters—measuring how covariates, loan and borrower characteristics affect default and prepayment speeds—are reported in Table 1.14. Interest rates are simulated using estimated parameters from Section 1.5. For each month, I simulate 200 antithetics interest rate paths. Future home prices follow assumed mean reversion. For example, for a 10% mean reversion, between 2005 and 2010, each CBSA would give back 10% of the increase in home prices between 2000 and 2005. Unemployment rates are simulated from AR(1) processes, with persistence parameters estimated for each CBSA using historical unemployment rates.

Results

Figure 1.2 through Figure 1.5 illustrate the results. Figure 1.2 focuses on the 0% mean reversion, or prices staying constant scenario. The assumption to test under this scenario is whether or not the GSEs took more precautionary measures in boom areas compared with other areas. As argued in Section 1.4, risk management for mortgages is inherently a multidimensional problem because mortgages have many risk characteristics including FICO scores, debt-to-income ratios (DTIs), original loan amount, interest rates, and owner occupancy. To study whether the GSEs indeed lowered their risk exposure in boom areas more than other areas, a valuation model taking into account all characteristics, like the one presented in this section, is necessary. From Figure 1.2, we see that if home prices stay constant, boom areas indeed would have smaller losses than other areas. This is consistent with the hypothesis that the GSEs were aware of the housing bubble and lowered their home price risk exposure by originating safer loans in boom areas than elsewhere.

A natural question to ask is how large the GSEs' risk management was. To study this, I consider a 10% mean reversion scenario. Under a 10% mean reversion assumed in Figure 1.3, boom areas would have larger losses than other areas. This suggests that the implied GSEs' model mean reversion was small, between 0% and 10%, assuming that the GSEs adjust their insurance to be actuarially fair between CBSAs. However, there are confounding factors that might have prevented the GSEs from implementing their believed house price mean reversion. One example is political pressure for uniform access to mortgage credit between CBSAs. Another example is that if the GSEs had asked for even higher percentages of down payments from boom areas, very few borrowers from boom areas could provide the down payments.

Two additional natural scenarios to test are the historically average 32%, and the realized 95% mean reversion. Somewhat surprisingly, under a 32% mean reversion assumed in Figure 1.4, all CBSAs would have costs lower than even 40% of their projected revenue. This implies that under a historically average mean reversion, the GSEs' expected revenue would be more than twice of the expected costs in every CBSA. Figure 1.5 shows the results under the realized 95% mean reversion scenario. In this case, guarantee fees collected in boom CBSAs are clearly insufficient to cover

the costs, with 10 CBSAs having projected costs over fives times of the projected revenue. This explains the unprecedented losses and government bailouts for the GSEs during the 2008 crash.

1.6   Conclusion

A common critique of the "public-private" partnership of Fannie Mae and Freddie Mac is that their implicit government guarantee reduces incentives for risk management and fosters irresponsible risk taking. Evidence from this paper suggests that Fannie Mae and Freddie Mac more effectively managed home price risk during the 2001-2006 housing boom than private mortgage insurance companies did.

These somewhat surprising results are nevertheless consistent with the history of private mortgage insurance industry, including its repeated and concentrated failures. Most recently in the 2008 crash, three out of the eight largest private mortgage insurers failed. However, perhaps overshadowed by the highly publicized and controversial bailout of the GSEs, private mortgage insurers' failures have received relatively little attention from academics and the popular press. Many post-crisis proposals also assume that replacing the GSEs by private insurers would be a Panacea. My results suggest that privatizing the GSEs alone is unlikely to ensure sufficient risk management in the mortgage insurance industry. Additional factors besides incentives, such as assumptions about future house prices, are important in shaping risk management practices. One way to establish reasonable house price assumptions is to stress test mortgage insurers, forcing the industry to consider their exposure to the housing downturn scenarios proposed by regulators.

The mortgage insurance industry plays a crucial role in financing Americans' mortgages. Their insurance reduce or remove mortgage default risks, thereby enhancing the liquidity of mortgage backed securities and lowering homebuyers' borrowing costs. The risks they face and the optimal regulatory structure for them deserve more study to prevent them from being a source of systemic risk in the financial system.

1.7   Figures and Tables

Figure 1.1: CBSAs with High or Low-Middle Home Prices

Low-Middel HPI
High HPI
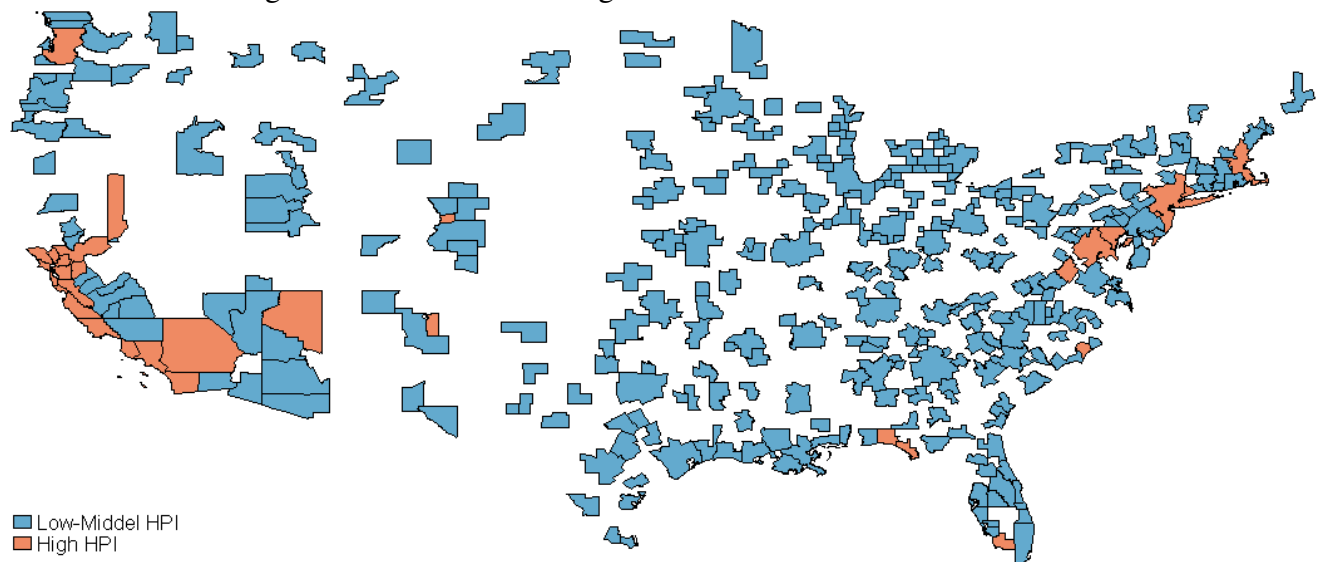
Source: ABSNet, author's own calculations. This figures plots CBSAs classified as high and low-middle home-price CBSAs. Low-middle home-price CBSAs are defined as CBSAs with more than 80% of private label loans in 2006 under the conforming loan limits. As explained in Section 1.4, the low-middle home price CBSA subsample is used to address the potential effect of the conforming loan limits. To isolate this effect, I report estimation results from both the full sample and the subsample of low-middle home-price CBSAs.

Figure 1.2: Projected Cost/Projected Guarantee Fees (0% Mean Reversion)

Source: Fannie Mae, Freddie Mac, FHFA, Yield Book and author's calculation. This figure plots the result from the default insurance valuation model, presented in Section 1.5. The simulation setting assumes a 0% home price mean reversion, or home prices staying constant. Each dot represents a CBSA among the largest 100 CBSAs in the U.S. The vertical axis is the total discounted cost for the GSEs from the insurance contracts normalized by the total discounted revenue. The magenta line is the fitted line. The valuation model builds on the competing-risk hazard estimates reported in Table 1.14 and a calibrated Hull-White term-structure model. Details of the competing-risk hazard regressions are described in Section 1.5. The Hull-White term-structure model and its calibration procedure is described in Section 1.5.

Figure 1.3: Projected Cost/Projected Guarantee Fees (10% Mean Reversion)

Source: Fannie Mae, Freddie Mac, FHFA, Yield Book and author's calculation. This figure plots the result from the default insurance valuation model, presented in Section 1.5. The simulation setting assumes a 10% home price mean reversion. Each dot represents a CBSA among the largest 100 CBSAs in the U.S. The vertical axis is the total discounted cost for the GSEs from the insurance contracts normalized by the total discounted revenue. The magenta line is the fitted line. The valuation model builds on the competing-risk hazard estimates reported in Table 1.14 and a calibrated Hull-White term-structure model. Details of the competing-risk hazard regressions are described in Section 1.5. The Hull-White term-structure model and its calibration procedure is described in Section 1.5.

Figure 1.4: Projected Cost/Projected Guarantee Fees (32% Mean Reversion)
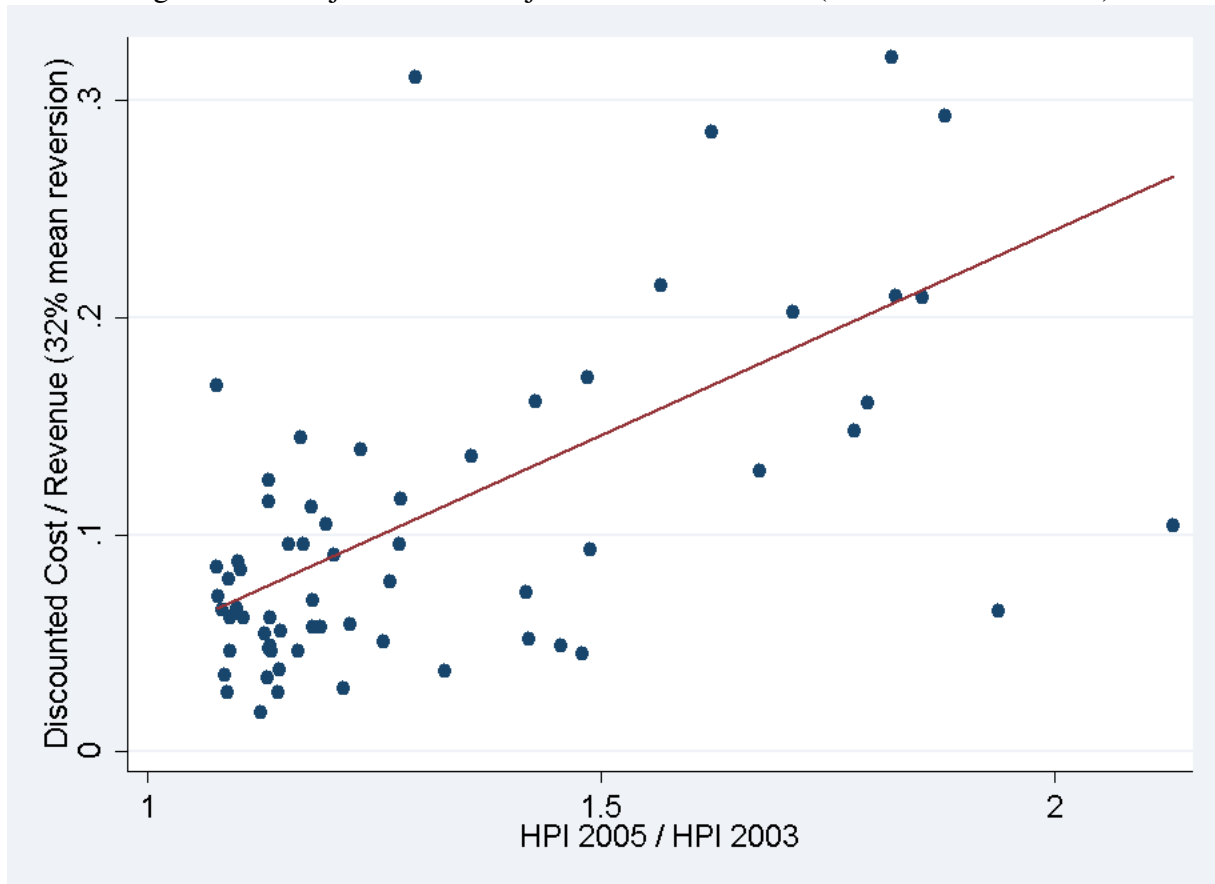
Source: Fannie Mae, Freddie Mac, FHFA, Yield Book and author's calculation. This figure plots the result from the default insurance valuation model, presented in Section 1.5. The simulation setting assumes a 32% home price mean reversion. Each dot represents a CBSA among the largest 100 CBSAs in the U.S. The vertical axis is the total discounted cost for the GSEs from the insurance contracts normalized by the total discounted revenue. The magenta line is the fitted line. The valuation model builds on the competing-risk hazard estimates reported in Table 1.14 and a calibrated Hull-White term-structure model. Details of the competing-risk hazard regressions are described in Section 1.5. The Hull-White term-structure model and its calibration procedure is described in Section 1.5.

Figure 1.5: Projected Cost/Projected Guarantee Fees (95% Mean Reversion)
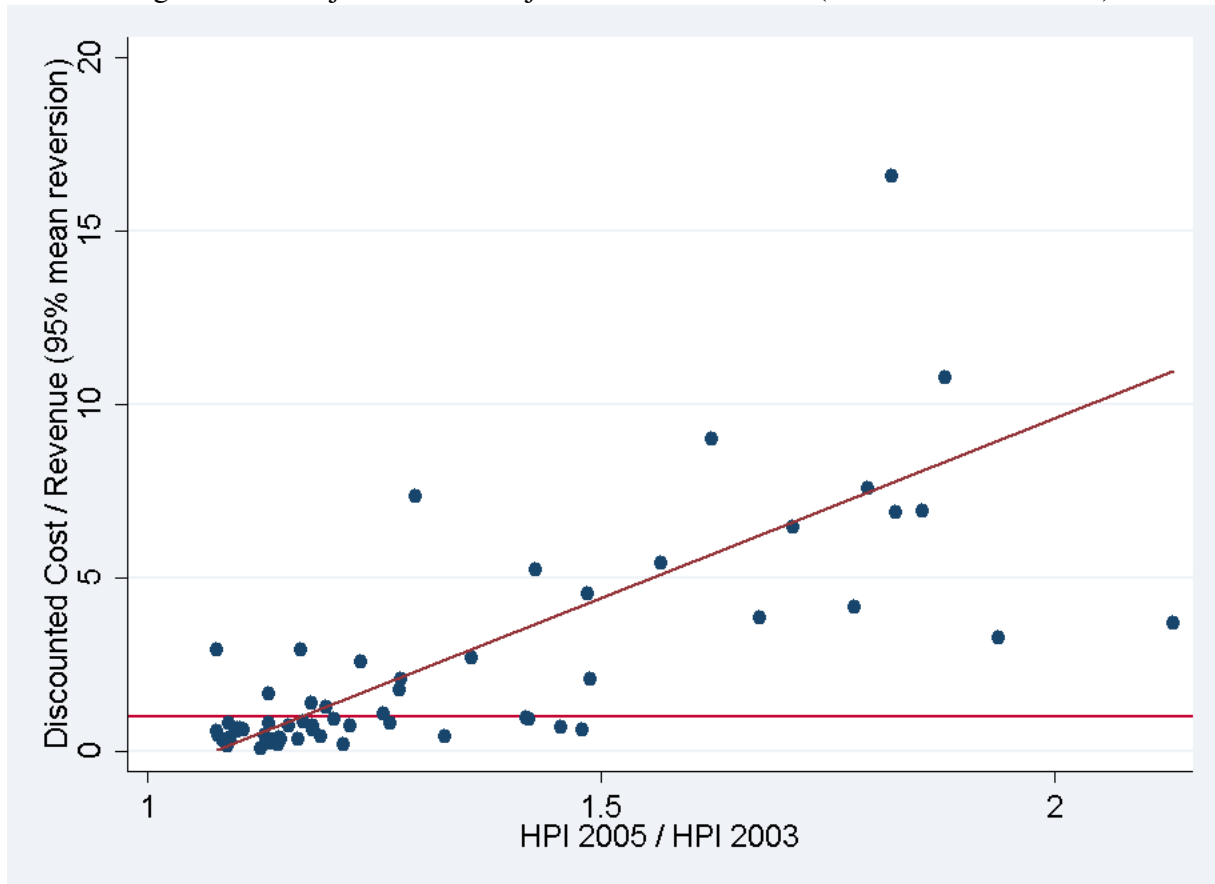
Source: Fannie Mae, Freddie Mac, FHFA, Yield Book and author's calculation. This figure plots the result from the default insurance valuation model, presented in Section 1.5. The simulation setting assumes a 95% home price mean reversion. Each dot represents a CBSA among the largest 100 CBSAs in the U.S. The vertical axis is the total discounted cost for the GSEs from the insurance contracts normalized by the total discounted revenue. The magenta line is the fitted line. The valuation model builds on the competing-risk hazard estimates reported in Table 1.14 and a calibrated Hull-White term-structure model. Details of the competing-risk hazard regressions are described in Section 1.5. The Hull-White term-structure model and its calibration procedure is described in Section 1.5.

Table 1.1: Summary Statistics

	Mean	Std	Min	Max
$\Delta \log(HPI)$	0.080	0.058	-0.072	0.364
Δ(Unemployment)	0.138	0.830	-2.300	3.500
Δ(Wage)	749	1001	-13151	4509
$\Delta \log(LTV)$				
First-Time Buyers with LTVs $\leq 80.5\%$	0.001	0.015	-0.270	0.219
First-Time Buyers with LTVs $> 80.5\%$	0.001	0.004	-0.040	0.050
Other Buyers with LTVs $\leq 80.5\%$	-0.001	0.016	-0.134	0.081
Other Buyers with LTVs $> 80.5\%$	-0.000	0.003	-0.025	0.024
$\Delta(FICO)$				
First-Time Buyers with LTVs $\leq 80.5\%$	1.741	5.166	-51.333	73.167
First-Time Buyers with LTVs $> 80.5\%$	-0.019	5.864	-104.000	70.500
Other Buyers with LTVs $\leq 80.5\%$	1.942	2.967	-27.333	28.896
Other Buyers with LTVs $> 80.5\%$	0.459	4.138	-50.333	33.985
$\Delta(DTI)$				
First-Time Buyers with LTVs $\leq 80.5\%$	0.528	1.804	-16.450	17.136
First-Time Buyers with LTVs $> 80.5\%$	0.530	1.892	-16.348	22.500
Other Buyers with LTVs $\leq 80.5\%$	0.610	1.390	-9.320	11.700
Other Buyers with LTVs $> 80.5\%$	0.601	1.530	-23.333	35.500
Δ(Owner-Occupation)				
First-Time Buyers with LTVs $\leq 80.5\%$	0.000	0.005	-0.087	0.077
First-Time Buyers with LTVs $> 80.5\%$	-0.000	0.005	-0.077	0.077
Other Buyers with LTVs $\leq 80.5\%$	-0.010	0.021	-0.161	0.194
Other Buyers with LTVs $> 80.5\%$	-0.024	0.027	-0.500	0.208
Δ (Interest Rate)				
First-Time Buyers with LTVs $\leq 80.5\%$	-0.280	0.568	-1.342	0.881
First-Time Buyers with LTVs $> 80.5\%$	-0.458	0.536	-1.377	0.855
Other Buyers with LTVs $\leq 80.5\%$	-0.316	0.560	-1.194	0.762
Other Buyers with LTVs $> 80.5\%$	-0.464	0.519	-1.238	1.542
Observations (CBSA-years)	1926			

Source: Fannie Mae, Freddie Mac, FHFA, IRS, BLS, author's own calculations. This table displays summary statistics for changes in loan and borrower characteristics in my sample of GSE loans at the CBSA-year level.

Table 1.2: Effect of Home-Price Growth on Loan-to-Value Ratios

A: GSE First-Time Home Buyers

	LTVs $\leq 80.5\%$ Segement			LTVs $> 80.5\%$ Segement		
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \log(HPI)$	-0.082***	-0.087***	-0.086***	-0.006***	-0.008***	-0.003
	(0.006)	(0.006)	(0.010)	(0.002)	(0.002)	(0.003)
Year FEs	y	y	y	y	y	y
Controls	n	y	y	n	y	y
CBSA FEs	n	n	y	n	n	y
Observations	1926	1926	1926	1926	1926	1926
R-squared	0.19	0.20	0.28	0.04	0.09	0.14

B: GSE Other Buyers

	LTVs $\leq 80.5\%$ Segement			LTVs $> 80.5\%$ Segement		
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \log(HPI)$	-0.125***	-0.133***	-0.129***	-0.012***	-0.013***	-0.012***
	(0.005)	(0.005)	(0.007)	(0.001)	(0.001)	(0.002)
Year FEs	y	y	y	y	y	y
Controls	n	y	y	n	y	y
CBSA FEs	n	n	y	n	n	y
Observations	1926	1926	1926	1926	1926	1926
R-squared	0.48	0.52	0.61	0.19	0.23	0.29

Source: Fannie Mae, Freddie Mac, FHFA, IRS, BLS, author's own calculations. This table reports estimates of Equations (1.1) and (1.2) in the text. The dependent variable is the annual change in log average LTV at the CBSA-year level. Panel A and Panel B are for GSE first-time home buyers and other buyers respectively. CBSA-year level controls include changes in macroeconomic conditions measured by unemployment rates and average wage, and changes in loan and borrower characteristics including average FICO scores, debt-to-income ratios, interest rates and percentage of owner occupied mortgages.

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 1.3: Effect of Home Price Growth on Loan-to-Value Ratios (Excluding High HP Areas)

A: GSE First Time Home Buyers, Excluding High Home Price CBSAs

	LTVs $\leq 80.5\%$ Segement			LTVs $> 80.5\%$ Segement		
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \log(HPI)$	-0.042***	-0.047***	-0.054***	-0.003	-0.006**	-0.001
	(0.007)	(0.007)	(0.011)	(0.002)	(0.002)	(0.004)
Year FEs	y	y	y	y	y	y
Controls	n	y	y	n	y	y
CBSA FEs	n	n	y	n	n	y
Observations	1794	1794	1794	1794	1794	1794
R-squared	0.09	0.10	0.15	0.04	0.09	0.14

B: GSE Other Buyers, Excluding High Home Price CBSAs

	LTVs $\leq 80.5\%$ Segement			LTVs $> 80.5\%$ Segement		
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \log(HPI)$	-0.064***	-0.067***	-0.064***	-0.011***	-0.011***	-0.012***
	(0.004)	(0.004)	(0.006)	(0.001)	(0.001)	(0.002)
Year FEs	y	y	y	y	y	y
Controls	n	y	y	n	y	y
CBSA FEs	n	n	y	n	n	y
Observations	1794	1794	1794	1794	1794	1794
R-squared	0.48	0.50	0.54	0.18	0.22	0.28

Source: Fannie Mae, Freddie Mac, FHFA, IRS, BLS, author's own calculations. This table reports estimates of Equations (1.1) and (1.2) in the text. The dependent variable is the annual change in log average LTV at the CBSA-year level. The difference of this table from Table 1.2 is that high home price CBSAs, colored orange in Figure 1.1, are excluded from the sample to address the potential effect of conforming loan limits. Panel A and Panel B are for GSE first-time home buyers and other buyers respectively. CBSA-year level controls include changes in macroeconomic conditions measured by unemployment rates and average wage, and changes in loan and borrower characteristics including average FICO scores, debt-to-income ratios, interest rates and percentage of owner occupied mortgages.

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 1.4: Effect of Home Price Growth on Private Insurers' Risk Management

A: $\Delta \log$(Share of Loans with LTVs $> 80.5\%$)

	(1)	(2)	(3)
$\Delta \log(HPI)$	0.055	-0.036	0.080
	(0.053)	(0.055)	(0.077)
Year FEs	y	y	y
Controls	n	y	y
CBSA FEs	n	n	y
Observations	1794	1794	1794
R-squared	0.31	0.36	0.46

B: $\Delta \log$(Private Insurers' Coverage Percentage)

	(1)	(2)	(3)
$\Delta \log(HPI)$	0.039**	-0.014	0.069***
	(0.020)	(0.016)	(0.025)
Year FEs	y	y	y
Controls	n	y	y
CBSA FEs	n	n	y
Observations	1794	1794	1794
R-squared	0.25	0.48	0.55

Source: Fannie Mae, Freddie Mac, FHFA, IRS, BLS, author's own calculations. Panel A and B report estimates of Equations (1.4) and (1.5) in the text respectively. In Panel A, the dependent variable is the annual change in log percentage of loans with LTVs above 80.5%. In Panel B, the dependent variable is the annual change in insurance percentage, percentage of initial loan balance covered by private insurers. The estimation sample is loans by GSE first-time home buyers from low-to-middel CBSAs, colored blue in Figure 1.1. CBSA-year level controls include changes in macroeconomic conditions measured by unemployment rates and average wage, and changes in loan and borrower characteristics including average FICO scores, debt-to-income ratios, interest rates and percentage of owner occupied mortgages.

Table 1.5: Percentage of CBSA-mortgage segments where Private Labeled FRMs had Lower Interest Rates than GSE FRMs

	(1) Full Sample	(2) Low-to-Middle Home Price Sample
2000	5.2%	5.2%
2001	2.9%	2.9%
2002	5.2%	4.5%
2003	3.2%	3.6%
2004	2.1%	2.3%
2005	0.9%	1.0%
2006	2.1%	2.2%

Source: Fannie Mae, Freddie Mac, ABSNet, author's own calculations. This table reports the percentage of CBSA-mortgage segments where private label originators offered better interest rates than the GSEs during the housing boom. Mortgages from each CBSA-year are divided into sixteen segments along two dimensions, LTVs and FICO scores. Along the LTV dimension, the cutoffs are 79.5%, 80.5% and 90%. Along the FICO score dimension, the cutoffs are 660, 720 and 760. For each CBSA-year-LTV-FICO segment, I collapse the median interest rates for GSE FRMs and private label FRMs. This table reports the percentage of CBSA-mortgage segments where private label loans had lower median interest rates than GSE loans.

Table 1.6: Effect of Home Price Growth on Combined-Loan-to-Value Ratios

A: GSE First Time Home Buyers, Excluding High Home Price CBSAs

	LTVs $\leq 80.5\%$ Segement			LTVs $> 80.5\%$ Segement		
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \log(HPI)$	-0.049***	-0.059***	-0.054***	-0.003	-0.006**	-0.001
	(0.009)	(0.010)	(0.015)	(0.002)	(0.002)	(0.004)
Year FEs	y	y	y	y	y	y
Controls	n	y	y	n	y	y
CBSA FEs	n	n	y	n	n	y
Observations	1794	1794	1794	1794	1794	1794
R-squared	0.05	0.06	0.10	0.04	0.09	0.15

Panel B: GSE Other Buyers, Excluding High Home Price CBSAs

	LTVs $\leq 80.5\%$ Segement			LTVs $> 80.5\%$ Segement		
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \log(HPI)$	-0.087***	-0.090***	-0.077***	-0.011***	-0.011***	-0.012***
	(0.005)	(0.005)	(0.007)	(0.001)	(0.001)	(0.002)
Year FEs	y	y	y	y	y	y
Controls	n	y	y	n	y	y
CBSA FEs	n	n	y	n	n	y
Observations	1794	1794	1794	1794	1794	1794
R-squared	0.39	0.40	0.44	0.19	0.22	0.28

Source: Fannie Mae, Freddie Mac, FHFA, IRS, BLS, author's own calculations. This table reports estimates of Equations (1.6) and (1.7) in the text. The dependent variable is the annual change in log average CLTV, as opposed to LTV in Table 1.3, at the CBSA-year level. High home price CBSAs, colored orange in Figure 1.1, are excluded from the sample to address the potential effect of conforming loan limits. Panel A and Panel B are for GSE first-time home buyers and other buyers respectively. CBSA-year level controls include changes in macroeconomic conditions measured by unemployment rates and average wage, and changes in loan and borrower characteristics including average FICO scores, debt-to-income ratios, interest rates and percentage of owner occupied mortgages.

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 1.7: Effect of Home Price Growth on FICO

	Excluding High Home Price CBSAs					
	GSE First Time Buyers			Private Label ARMs		
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \log(HPI)$	-0.009***	-0.010***	-0.020***	0.059***	0.009***	0.019***
	(0.003)	(0.003)	(0.005)	(0.003)	(0.003)	(0.004)
Year FEs	y	y	y	y	y	y
Controls	n	y	y	n	y	y
CBSA FEs	n	n	y	n	n	y
Observations	1794	1794	1794	1794	1794	1794
R-squared	0.21	0.26	0.31	0.70	0.86	0.87

Source: Fannie Mae, Freddie Mac, FHFA, IRS, BLS, author's own calculations. This table reports estimates of Equation (1.3) in the text. The dependent variable is the annual change in log average FICO score. High home price CBSAs, colored orange in Figure 1.1, are excluded from the sample to address the potential effect of conforming loan limits. CBSA-year level controls include changes in macroeconomic conditions measured by unemployment rates and average wage, and changes in loan and borrower characteristics including average FICO scores, debt-to-income ratios, interest rates and percentage of owner occupied mortgages.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 1.8: Effect of Home Price Growth on LTVs for Various Low Debt-to-Income Ratio Subsamples

	GSE First Time Home Buyers, Excluding High Home Price CBSAs					
	Full Sample (1)	DTI≤60% (2)	DTI≤55% (3)	DTI≤50% (4)	DTI≤45% (5)	DTI≤40% (6)
$\Delta \log(HPI)$	-0.051***	-0.050***	-0.050***	-0.053***	-0.057***	-0.060***
	(0.016)	(0.016)	(0.017)	(0.017)	(0.018)	(0.020)
Obs	1794	1794	1794	1794	1794	1794
R squ	0.12	0.12	0.12	0.11	0.11	0.11

Source: Fannie Mae, Freddie Mac, FHFA, IRS, BLS, author's own calculations. This table reports estimates of Equation (1.1) in the text for low debt-to-income ratio subsamples. The dependent variable is the annual change in log average LTV at the CBSA-year level. CBSA-year level controls include changes in macroeconomic conditions measured by unemployment rates and average wage, and changes in loan and borrower characteristics including average FICO scores, debt-to-income ratios, interest rates and percentage of owner occupied mortgages.

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 1.9: The Relative Importance of Risk Taking in FICO and LTVs

	(1) Changes in log LTVs	(2) Changes in log FICO
GSE First Time Home Buyers with LTVs $\leq 80.5\%$ Excluding High Home Price CBSAs		
$\Delta \log(HPI)$	-0.054***	-0.015***
	(0.011)	(0.006)
Year FEs	y	y
Controls	y	y
CBSA FEs	y	y
Observations	1794	1794
R-squared	0.15	0.19

Source: Fannie Mae, Freddie Mac, FHFA, IRS, BLS, author's own calculations. This table reports estimates of Equations (1.1) and (1.3) in the text for GSE loans by first time home buyers with LTVs below 80.5%. The dependent variable is the annual change in log average LTV in column 1, and annual change in log FICO score in column 2. High home price CBSAs, colored orange in Figure 1.1, are excluded from the sample to address the potential effect of conforming loan limits. CBSA-year level controls include changes in macroeconomic conditions measured by unemployment rates and average wage, and changes in loan and borrower characteristics including average FICO scores, debt-to-income ratios, interest rates and percentage of owner occupied mortgages. In the LTV regression reported in column 1, changes in LTVs are excluded from the controls. In the FICO score regression reported in column 2, changes in FICO scores are excluded from the controls.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 1.10: Robustness of LTV Results to Bunching at 80%

	GSE First Time Home Buyers with LTVs $\leq 79.5\%$ Excluding High Home Price CBSAs		
	(1)	(2)	(3)
$\Delta \log(HPI)$	-0.069***	-0.068***	-0.063**
	(0.018)	(0.019)	(0.028)
Year FEs	y	y	y
Controls	n	y	y
CBSA FEs	n	n	y
Observations	1794	1771	1771
R-squared	0.05	0.06	0.10

Source: Fannie Mae, Freddie Mac, FHFA, IRS, BLS, author's own calculations. This table reports estimates of Equations (1.1) in the text for GSE loans by first time home buyers with LTVs below 79.5%. The dependent variable is the annual change in log average LTV. High home price CBSAs, colored orange in Figure 1.1, are excluded from the sample to address the potential effect of conforming loan limits. CBSA-year level controls include changes in macroeconomic conditions measured by unemployment rates and average wage, and changes in loan and borrower characteristics including average FICO scores, debt-to-income ratios, interest rates and percentage of owner occupied mortgages.

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 1.11: Results for Private Label FRMs

	Private Label FRMs *Excluding High Home Price CBSAs*		
	(1)	(2)	(3)
$\Delta \log(HPI)$	-0.026***	-0.020***	0.017*
	(0.005)	(0.006)	(0.009)
Year FEs	y	y	y
Controls	n	y	y
CBSA FEs	n	n	y
Observations	1792	1792	1792
R-squared	0.69	0.71	0.74

Source: ABSNet, FHFA, IRS, BLS, author's own calculations. This table reports estimates of Equations (1.1) in the text for private label FRMs. The dependent variable is the annual change in log average LTV. High home price CBSAs, colored orange in Figure 1.1, are excluded from the sample to address the potential effect of conforming loan limits. CBSA-year level controls include changes in macroeconomic conditions measured by unemployment rates and average wage, and changes in loan and borrower characteristics including average FICO scores, debt-to-income ratios, interest rates and percentage of owner occupied mortgages.

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 1.12: Robustness of LTV Results to Conforming Loan Limits

	GSE Loans by First Time Home Buyers with Loan Amount under 95% *of CLLs*		
	(1)	(2)	(3)
$\Delta \log(HPI)$	-0.042***	-0.047***	-0.055***
	(0.007)	(0.007)	(0.011)
Year FEs	y	y	y
Controls	n	y	y
CBSA FEs	n	n	y
Observations	1794	1794	1794
R-squared	0.09	0.10	0.15

Source: Fannie Mae, Freddie Mac, FHFA, IRS, BLS, author's own calculations. This table reports estimates of Equations (1.1) in the text for GSE loans by first time home buyers with with loan amount under 95% of conforming loan limits. The dependent variable is the annual change in log average LTV. CBSA-year level controls include changes in macroeconomic conditions measured by unemployment rates and average wage, and changes in loan and borrower characteristics including average FICO scores, debt-to-income ratios, interest rates and percentage of owner occupied mortgages.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 1.13: Cash Flows of Default Insurance

Mortgage Monthly Outcome	Cash Flows	
	Fixed Leg (Revenue)	Floating Leg (Cost)
Defaulted	0	Loan balance $-$ Value of the house
Prepaid or Matured	0	0
Paid Down	$\approx \frac{0.2\%}{12}$ of remaining balance	0

This table summaries in each month how the cash flows evolve according to the loan outcomes.

Table 1.14: Default and Prepayment Hazard Estimates

	(1)	(2)
	Default Risk	Prepayment Risk
	A: Static Covariates	
log(FICO)	-6.095***	1.233***
	(0.156)	(0.119)
First-Time Home Buyer	-0.071***	-0.053***
	(0.024)	(0.004)
Owner Occupied	-0.396***	0.274***
	(0.112)	(0.026)
Original r - Original 10 Year Rate	0.898***	0.771***
	(0.023)	(0.021)
log(Original Amount)	-0.045	0.773***
	(0.100)	(0.021)
log(Original LTV)	6.077***	-0.160***
	(0.272)	(0.026)
	B: Dynamic Covariates	
log(Cumulative HPA)	-4.790***	1.392***
	(0.629)	(0.226)
Coupon Gap	0.245***	0.912***
	(0.016)	(0.025)
Unemployment	0.099***	0.138***
	(0.035)	(0.017)
CBSA FEs	y	y
Observations	106,965,734	119,834,487

Source: Fannie Mae, Freddie Mac, FHFA, Yield Book, BLS, author's own calculations. This table shows estimates using maximum likelihood estimator of the hazard functions in (1.8) and (1.9) in the text, estimated using a continuous-time nonparametric baseline hazard function. Estimated coefficients are the effect of a given covariate on the log hazard rate of a mortgage. Details of the estimation procedure are described in Section 1.5. Panel A and Panel B report the coefficients for static and dynamic covariates respectively.

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Chapter 2

Estimation of Linear Process Spectra with an Application to Determining the Mean-Variance Frontier for Time Series

Co-authored with Alexander Aue and Debashis Paul

2.1   Introduction

One persistent challenge in macroeconmics and finance consists of devising inference procedures for short time series data of length T with a large number N of cross sections. As Cochrane (2005) argues:

"Our econometric techniques all are designed for large time series and small cross-sections. Our data has a large cross section and short time series. A large unsolved problem in finance is the development of appropriate large-N small-T tools for evaluating asset pricing models."

One particular large-N small-T impact mentioned in Cochrane (2005) is the underestimation of risk in the mean-variance frontier. This Mean-Variance Frontier (henceforth, MVF) measures the minimum risk to be taken in order to achieve a set of expected returns.

As Cochrane (2005) articulated, apart from statistical uncertainty due to sampling, the large dimensionality of the data leads to peculiar discrepancies between the sample MVF compared to its population counterpart. From Figure 3.1, we can see that the ex-post frontier, calculated from finite samples, significantly underestimates risk compared to the true, ex-ante frontier. Existing works solve this problem for time independent data (Kan and Zhou, 2007; El Karoui, 2013). In this paper, we propose a novel algorithm correcting the bias in MVF for time series returns.

Figure 2.2 illustrates the main result via Monte Carlo simulation. Dashed lines around each solid line constitute the one standard error bands around the respective mean estimates. The mean estimate of the proposed LinShrink algorithm closely matches the truth, outperforming the other two algorithms when the data is sampled from a high-dimensional, second-order moving average process. The magenta curve displays the result of the IndShrink method proposed by El Karoui
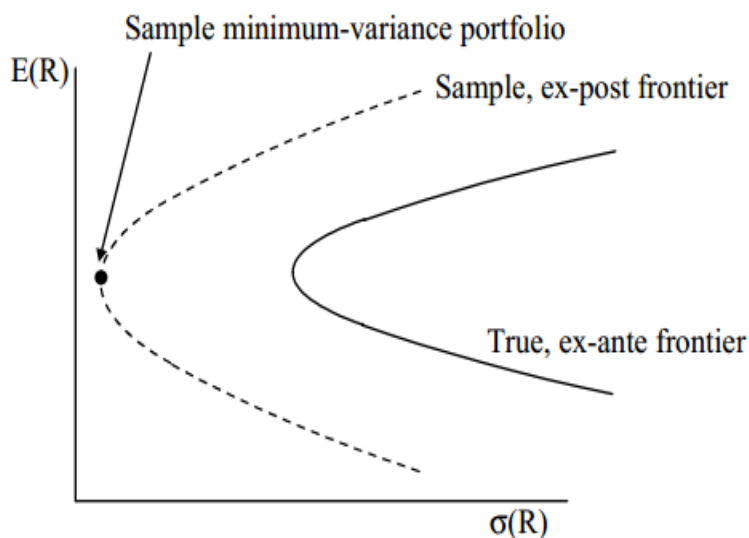
Figure 2.1: Ex-ante and Ex-Post frontiers. Source: Cochrane (2005).

(2013), stressing that his algorithm yields an unbiased estimator when the underlying data generating process is time independent. The difference between the magenta curve and the green curve is due to the model misspecification between a time independent model and a time series model. Given that most time series in finance and econometrics have important inter-temporal correlations, it is important to design an algorithm that accounts for the time dependence.

A simple justification of risk underestimation by the naive estimate of MVF, that is, the sample version of the minimum portfolio risk formula, follows from Jensen's inequality. The difference between the population and sample versions only gets larger with increasing N. To understand this intuitively, consider the extreme case of mutually uncorrelated assets. Here, the covariance matrix Σ is a diagonal matrix and the population correlation of returns from any pair of assets is 0. However, the sample correlations could still be significantly positive or negative. These positive or negative sample correlations make assets look like good hedges for each other, despite their independence in the population. These pseudo hedges are the fundamental reason why risk is underestimated in finite samples. This is particularly an issue when the number of assets N is large. There are $N(N-1)/2$ number of distinct correlations, not to mention correlations between linear combinations, or portfolios, of assets. Thus, when N is large, some pairs of assets should appear to be correlated, or good hedges, with a very high probability despite the cross sectional independence assumption.

The issue of bias in the empirical estimate of MVF gets more intricate when, in addition, there is temporal dependence across observations. The aim of this paper, then, is to explore this aspect in detail by making use of a widely used class of time series – linear processes – as a benchmark for explaining the phenomena. In particular, RMT is utilized using the results developed in **?**
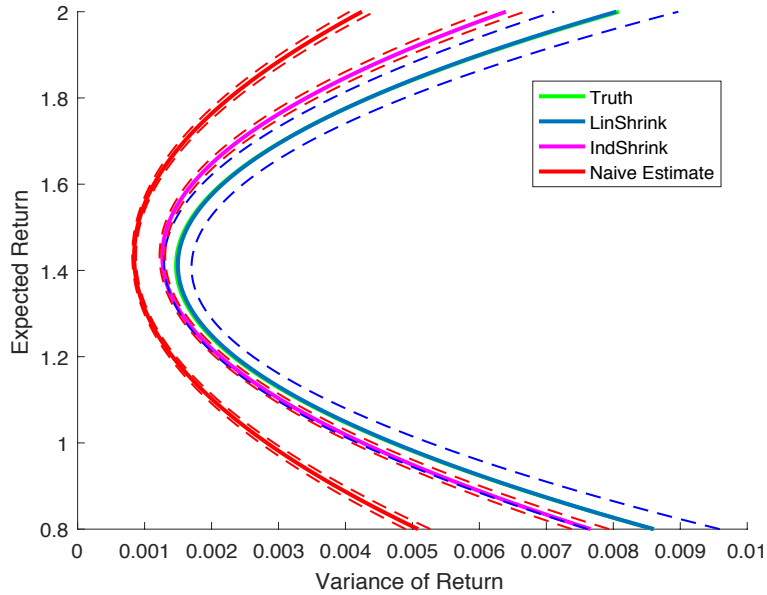
Figure 2.2: MVF estimation for an MA(2) process with $N = 1000$, $T = 3000$. LinShrink is the proposed algorithm, IndShrink stands for shrinkage assuming independence, and the naive estimate uses the standard sample covariance matrix estimate.

as a starting point. Throughout the exposition, significant differences and intricacies associated with the temporal dependence in the returns are emphasized, in particular when related to issues regarding MVF estimation.

To articulate the main ideas, note that the basic problem of MVF can be formulated as that of estimating the quadratic form $\mathbf{B}^T \mathbf{\Sigma}^{-1} \mathbf{B}$, where $\mathbf{\Sigma} = \mathrm{Cov}(X_t)$ denotes the $N \times N$ covariance matrix for asset return vector X_t, $t = 1, \ldots, T$ and $\mathbf{B}$ is an $N \times K$ matrix. In MVF, $\mathbf{B}$ is an $N \times 2$ matrix with expected returns of the N assets as the first column, and every entry in the second column being equal to 1. As mentioned above, the "natural estimate" $\mathbf{B}^T \mathbf{S}^{-1} \mathbf{B}$ is biased, even if the population mean return is assumed to be known. In this paper, a correction for the finite sample bias is proposed for $(X_t \colon t \in \mathbb{Z})$ in the class of high-dimensional linear time series of the form $X_t = \sum_{\ell=0}^{\infty} \mathbf{A}_\ell Z_{t-\ell}$, where the coefficient matrices $(\mathbf{A}_\ell \colon \ell \in \mathbb{N}_0)$ are symmetric and simultaneously diagonalizable, and $(Z_t \colon t \in \mathbb{Z})$ an independent, identically distributed (i.i.d.) process of innovations with independent coordinates.[1] Under the stated assumptions, **?** established the existence of a limiting spectral distribution for any finite order symmetrized sample autocovariance matrix. The latter result forms the cornerstone of the estimation procedures proposed in this paper. Note that the assumed process on one hand generalizes the i.i.d. observations that have mostly been assumed in the literature on (high-dimensional) portfolio optimization and encompasses the stationary Autoregressive Moving Average (ARMA) processes widely used in time series model-

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[1] In this paper, $\mathbb{Z}$, $\mathbb{N}$ and $\mathbb{N}_0$ denote the integers, the positive integers and the nonnegative integers, respectively.

ing, with the restriction of simultaneously diagonalizable coefficients. On the other hand, it also constitutes a nontrivial extension to existing results in RMT literature on the asymptotic behavior of spectra of sample covariance matrices in large dimensions, such as the works of El Karoui (2008) and Ledoit and Wolf (2012, 2015). As is demonstrated later, estimation of the eigenvalue distribution of these coefficient matrices is a challenge due to the highly nonlinear nature of the relationship between the sample spectra and the population parameters. The first contribution of this paper is to develop an algorithm for the estimation of these eigenvalue distributions. It is also described how solving this problem allows for designing an effective algorithm, termed LinShrink, that includes a model selection step for estimating the quantity of primary interest in this paper, namely the MVF, in a time series context. The second contribution of this paper is the extension of the algorithm to the scenario where asset returns follow a factor structure with the idiosyncratic term possessing the linear process structure specified above. Related work for high-dimensional factor models may be found in Bai (2003), Bai and Ng (2002), and Onatski (2009, 2012).

Although the results are illustrated in the classical MVF context, the proposed methodology has the potential for further applications. Quadratic forms of the type $\mathbf{B}^T \Sigma^{-1} \mathbf{B}$ arise in many fields of time series econometrics; for example, in computing standard errors in the generalized methods of moments and in linear discriminant analysis. Moreover, estimating the eigenvalue distribution of coefficients of the linear time series provides avenues for designing appropriate shrinkage rules for objects such as the spectral density matrix associated with it. The latter object and functionals derived from it are especially important in prediction problems. Extensions of the proposed method to these and other specific contexts constitute promising directions for future research.

Besides the papers mentioned above, our work contributes to the vast literature on asset pricing and optimal portfolio choice. The pathbreaking work of Markowitz Markowitz (2009) utilized optimization theory to determine the optimal MVF under the assumption of known mean and covariances of the returns. Since then, decades of work in econometrics and finance have addressed various facets of MVF estimation under statistical and parameter uncertainties. The importance of the latter was revealed by Barberis (2000), who emphasized that even after incorporating parameter uncertainty, there is sufficient predictability in returns, thus making it important to take estimation risk into account when allocating assets to stocks.

? were among the earliest to study the phenomena associated with high dimensionality on the portfolio risk estimation. Kan and Zhou (2007) considered the effects of model uncertainty and studied the implications of combinations of portfolio choices. The bias in the empirical MVF can be significant even when T and N are of comparable sizes. The bias in this setting was quantified by Bai et al. (2009) and **?** through the use of the Random Matrix Theorey (henceforth, RMT) framework, assuming that the returns across time are independent. The latter works, as well as El Karoui (2013), also suggested methods for correcting for this bias under the same assumptions. It should be mentioned here that the use of RMT in explaining behavior of high-dimensional financial data was pioneered by **??**. A broad overview of this growing literature can be found in Bouchaud and Potters (2009), and Paul and Aue (2014).

The approaches put forth in Bai et al. (2009); **?**); El Karoui (2013) rely on various ways of reducing the bias in the estimation of quadratic forms of the type $\mathbf{a}^T \Sigma^{-1} \mathbf{b}$, where Σ is the population return covariance matrix. The vectors $\mathbf{a}$ and $\mathbf{b}$ are typically either the population mean returns μ or

some fixed vector such as the vector of ones. One redeeming feature of their work are the minimal structural assumptions on the population parameters. Recently, another line of study within the MVF estimation framework has come into the focus that seeks to modify prior work through imposing various constraints on the portfolio weights. Empirical research, for example in **??**Brodie et al. (2009), has shown that such constraints can enhance portfolio performance. Some of these empirical results were validated through theoretical justifications in **?**. Very recently, Ao et al. (2016) proposed a new sparsity constraint-based estimation of MVF and showed its consistency.

The remainder of the paper is organized as the follows. Section 2.2 reviews existing results, sets the model and states the main assumptions. Section 2.3 describes in detail the proposed estimation algorithm, including a thresholding and model selection algorithm. Section 2.4 discusses the application of the proposed methodology to the Markowitz portfolio problem and an extension to factor models with known factors. Section 2.5 reports empirical results from extensive Monte Carlo simulations.

2.2 Overview of Results in ?

The methodology developed in this paper is built upon the theoretical analysis of the behavior of empirical spectral distribution of symmetrized sample autocovariance matrices carried out in Liu et al. **?**, whose results are linked to the present work in the following way. Here, interest is in estimating a generalized quadratic form of the type $\mathbf{B}^T \mathbf{\Sigma}^{-1} \mathbf{B}$ with a strategy based on the spectral decomposition of the covariance matrix $\mathbf{\Sigma}$. Denoting the distinct (ordered) eigenvalues of $\mathbf{\Sigma}$ by $(\sigma_j \colon j = 1, \ldots, J)$ and corresponding eigen-projection matrices $(\mathbf{P}_j \colon j = 1, \ldots, J)$, where $J \in \{1, \ldots, p\}$, the quadratic form can be expressed as

$$\mathbf{B}^T \mathbf{\Sigma}^{-1} \mathbf{B} = \sum_{j=1}^{J} \frac{1}{\sigma_j} \mathbf{B}^T \mathbf{P}_j \mathbf{B}. \tag{2.1}$$

In a nutshell, the strategy is based upon splitting the estimation problem into two steps. The first step involves the estimation of the eigenvalues $(\sigma_j \colon j = 1, \ldots, J)$ together with their multiplicities. This can be equivalently expressed in the form of the *empirical spectral distribution (ESD)* of $\mathbf{\Sigma}$. The second step involves estimation of the parameters $\mathbf{\Theta}_j = \mathbf{B}^T \mathbf{P}_j \mathbf{B}$, $j = 1, \ldots, J$. The main results of **?** provide the foundation for estimating the ESD of $\mathbf{\Sigma}$. Given the estimate of the ESD of $\mathbf{\Sigma}$, one can make use of a carefully constructed regression formulation, again based on the derivations in **?**, to estimate the parameters $(\mathbf{\Theta}_j \colon j = 1, \ldots, J)$. The final estimate of $\mathbf{B}^T \mathbf{\Sigma}^{-1} \mathbf{B}$ is obtained by combining the two component estimates.

To elaborate further, **?** established the existence of nonrandom limits of the ESD of the sample covariance matrix $\mathbf{S}$ and symmetrized autocovariance matrices for linear process of the kind described in Section 2.1. These results are expressed in terms of the *Stieltjes transform* of the ESDs of the sample autocovariance matrices and the coefficients of the linear process. The proposed estimation strategy for the ESD of $\mathbf{\Sigma}$ makes explicit use of the relationships between the Stieltjes transforms of the ESD of $\mathbf{S}$ and that of the coefficient matrices to formulate an optimization

problem where the ESDs of the coefficient matrices are used as unknown parameters. Section 2.2 contains details on the model and the main notions, including the definition of the high-dimensional setting, symmetrized autocovariance matrices, Stieltjes transform and the description of the linear time series model. For convenience, Section 2.2 summarizes the main results of **?**.

Setting

The limiting scenario considered considered here is the classical high dimensional setting when the number of dimensions N grows with the number of observations T, so that $N = N(T)$ is assumed to be a function of the sample size satisfying

$$\lim_{T \to \infty} \frac{N}{T} = c \in (0, \infty). \tag{2.1}$$

A sequence of random vectors $(X_t \colon t \in \mathbb{Z})$ with values in $\mathbb{C}^N$ is called a linear process or moving average process of order infinity, abbreviated by the acronym MA(∞), if it has the representation

$$X_t = \sum_{\ell=0}^{\infty} \mathbf{A}_\ell Z_{t-\ell}, \qquad t \in \mathbb{Z}, \tag{2.1}$$

where $(Z_t \colon t \in \mathbb{Z})$ denotes a sequence of independent, identically distributed N-dimensional random vectors whose entries are independent and satisfy $\mathbb{E}[Z_{nt}] = 0$, $\mathbb{E}[|Z_{nt}|^2] = 1$, and $\mathbb{E}[|Z_{nt}|^4] < \infty$, where Z_{nt} denotes the nth coordinate of Z_t. In the complex-valued case this is meant as $\mathbb{E}[\mathrm{Re}(Z_{nt})^2] = \mathbb{E}[\mathrm{Im}(Z_{nt})^2] = 1/2$. It is also assumed that real and imaginary parts are independent.

The symmetrized lag-τ sample autocovariance associated with the process $(X_t \colon t \in \mathbb{Z})$ is defined as

$$\mathbf{C}_\tau = \frac{1}{2T} \sum_{t=1}^{T-\tau} \left(X_t X_{t+\tau}^* + X_{t+\tau} X_t^* \right), \qquad \tau \in \mathbb{N}_0,$$

assuming $X_1, \ldots, X_T$ have been observed. For $\tau = 0$, this definition gives the covariance matrix $\mathbf{S} = \mathbf{C}_0$. Let

$$\hat{F}_\tau(\sigma) = \frac{1}{p} \sum_{n=1}^{N} \mathbb{I}_{\{\sigma_{n,\tau} \leq \sigma\}},$$

denote the empirical spectral distribution (ESD) of $\mathbf{C}_\tau$, where $\sigma_{1,\tau}, \ldots, \sigma_{N,\tau}$ are the eigenvalues of $\mathbf{C}_\tau$.

Assumption 2.2.1 below lists several additional assumptions on the coefficient matrices $\mathbf{A}_\ell$ of the linear process in (2.2). The essence of this assumption is that, up to an unknown rotation matrix $\mathbf{U}$, the coordinates of the observation vector X_t, form uncorrelated stationary time series with the coefficients in the linear process representation being functionally related in a suitably smooth manner, as indicated by the behavior of a set of continuous functions f_ℓ.

Assumption 2.2.1. *(a) Set $\mathbf{A}_0 = \mathbf{I}$, the $N \times N$ identity matrix.*

(b) The matrices $(\mathbf{A}_\ell\colon \ell \in \mathbb{N}_0)$ are simultaneously diagonalizable Hermitian matrices satisfying $\|\mathbf{A}_\ell\| \leq \bar{\lambda}_{\mathbf{A}_\ell}$ for all $\ell \in \mathbb{N}_0$ and large N with

$$\sum_{\ell=0}^{\infty} \bar{\lambda}_{\mathbf{A}_\ell} \leq \bar{\lambda}_{\mathbf{A}} < \infty \qquad and \qquad \sum_{\ell=0}^{\infty} \ell \bar{\lambda}_{\mathbf{A}_\ell} \leq \bar{\lambda}'_{\mathbf{A}} < \infty.$$

Note that one can set $\bar{\lambda}_{\mathbf{A}_0} = 1$.

(c) There are continuous functions $f_\ell\colon \mathbb{R}^m \to \mathbb{R}$, $\ell \in \mathbb{N}_0$, such that, for every N, there is a set of points $\lambda_1, \ldots, \lambda_N \in \mathbb{R}^m$, not necessarily distinct, and a unitary $N \times N$ matrix $\mathbf{U}$ such that

$$\mathbf{U}^* \mathbf{A}_\ell \mathbf{U} = \mathrm{diag}(f_\ell(\lambda_1), \ldots, f_\ell(\lambda_N)), \qquad \ell \in \mathbb{N},$$

and $f_0(\lambda) = 1$. Note that the functions f_ℓ could also be allowed to depend on $N = N(T)$ as long as they converge uniformly to continuous functions as $T \to \infty$.

(d) With probability one, $F_p^{\mathbf{A}}$, the ESD of $\{\lambda_1, \ldots, \lambda_N\}$, converges weakly to a nonrandom probability distribution function $F^{\mathbf{A}}$ on $\mathbb{R}^m$ as $N \to \infty$.

One classical route to formulate results in the high-dimensional setting prescribed in (2.2) is through the use of the Stieltjes transform, which transforms a distribution to a function defined on $\mathbb{C}^+$, where $\mathbb{C}^+ = \{x + iy\colon x \in \mathbb{R}, y > 0\}$ denotes the upper complex half plane. The Stieltjes transform of a distribution function F on the real line is the function

$$s_F\colon \mathbb{C}^+ \to \mathbb{C}^+, \ z \mapsto s_F(z) = \int \frac{1}{\sigma - z} dF(\sigma).$$

It can be shown that s_F is analytic on $\mathbb{C}^+$ and that the distribution function F can be reconstructed from s_F using an inversion formula. See Bai and Silverstein (2010) or Paul and Aue (2014) for further descriptions on Stieltjes transforms and their usage in random matrix theory.

Large-Sample Spectral Behavior of $\mathbf{C}_\tau$

Denote by $\mathbf{A} = [\mathbf{A}_0 : \mathbf{A}_1 : \cdots]$ the matrix collecting the coefficient matrices of the linear process $(X_t\colon t \in \mathbb{Z})$. Define the transfer functions

$$\psi(\lambda, \nu) = \sum_{\ell=0}^{\infty} e^{i\ell\nu} f_\ell(\lambda) \qquad and \qquad \psi(\mathbf{A}, \nu) = \sum_{\ell=0}^{\infty} e^{i\ell\nu} \mathbf{A}_\ell, \tag{2.1}$$

and the power transfer functions

$$h(\lambda, \nu) = |\psi(\lambda, \nu)|^2 \qquad and \qquad H(\mathbf{A}, \nu) = \psi(\mathbf{A}, \nu)\psi(\mathbf{A}, \nu)^*.$$

Note that the contribution of the temporal dependence of the underlying time series on the asymptotic behavior of $\hat{F}_\tau$ is quantified through $h(\lambda, \nu)$. Specifically, $h(\lambda_n, \nu)$ with λ_n as in part (c) of Assumption 2.2.1 is (up to normalization) the spectral density of the nth coordinate of the process rotated with the help of the unitary matrix $\mathbf{U}$. With these definitions, the main results of **?** can be stated as follows.

Theorem 1. *If a complex-valued linear process $(X_t \colon t \in \mathbb{Z})$ with independent, identically distributed Z_{nt}, $\mathbb{E}[Z_{nt}] = 0$, $\mathbb{E}[\mathrm{Re}(Z_{nt})^2] = \mathbb{E}[\mathrm{Im}(Z_{nt})^2] = 1/2$, $\mathrm{Re}(Z_{nt})$ and $\mathrm{Im}(Z_{nt})$ independent, and $\mathbb{E}[|Z_{nt}|^4] < \infty$, satisfies Assumption 2.2.1, then, with probability one and in the high-dimensional setting (2.2), $\hat{F}_\tau$ converges to a nonrandom probability distribution F_τ with Stieltjes transform s_τ determined by the equation*

$$s_\tau(z) = \int \left[\frac{1}{2\pi} \int_0^{2\pi} \frac{\cos(\tau\nu) h(\lambda, \nu)}{1 + c\cos(\tau\nu) K_\tau(z, \nu)} d\nu - z \right]^{-1} dF^{\mathbf{A}}(\lambda), \tag{2.1}$$

where $K_\tau \colon \mathbb{C}^+ \times [0, 2\pi] \to \mathbb{C}^+$ is a Stieltjes kernel, that is, $K_\tau(\cdot, \nu)$ is the Stieltjes transform of a measure with total mass $m_\nu = \int h(\lambda, \nu) dF^{\mathbf{A}}(\lambda)$ for every fixed $\nu \in [0, 2\pi]$, whenever $m_\nu > 0$. Moreover, K_τ is the unique solution of

$$K_\tau(z, \nu) = \int \left[\frac{1}{2\pi} \int_0^{2\pi} \frac{\cos(\tau\nu') h(\lambda, \nu')}{1 + c\cos(\tau\nu') K_\tau(z, \nu')} d\nu' - z \right]^{-1} h(\lambda, \nu) \, dF^{\mathbf{A}}(\lambda) \tag{2.1}$$

subject to the restriction that K_τ is a Stieltjes kernel. Otherwise, if $m_\nu = 0$, then $K_\tau(z, \nu)$ is identically zero on $\mathbb{C}^+$ and so still satisfies (1).

Theorem 2. *If a real-valued linear process $(X_t \colon t \in \mathbb{Z})$ with independent, identically distributed real-valued Z_{nt}, $\mathbb{E}[Z_{nt}] = 0$, $\mathbb{E}[Z_{nt}^2] = 1$ and $\mathbb{E}[Z_{nt}^4] < \infty$, satisfies Assumption 2.2.1 with real symmetric coefficient matrices $(\mathbf{A}_\ell \colon \ell \in \mathbb{N}_0)$, then the result of Theorem 1 is retained.*

2.3 Estimation Strategy

The estimation strategy for the quadratic form $\mathbf{B}^T \mathbf{\Sigma}^{-1} \mathbf{B}$ is based on two steps that are summarized in the following. All details are given in subsequent sections.

(1) *Estimate the ESD of $\mathbf{\Sigma}$:* Since under the assumed model, the ESD of $\mathbf{\Sigma}$ is determined by $F^{\mathbf{A}}$ and the (known) functions $(f_\ell \colon \ell \in \mathbb{N})$, a strategy is formulated for the estimation of $F^{\mathbf{A}}$ within the linear process framework. For computational tractability, the assumed model is chosen to be a finite-order MA process that can serve as an approximation to the true, and typically unknown, infinite-order linear process. Once the process is specified, the system of equations (1) and (1) describing the Stieltjes transform of the limiting spectral distribution can be utilized for a collection of symmetrized sample autocovariance matrices, including the sample covariance matrix $\mathbf{S}$, to formulate an optimization problem involving a discrepancy measure between the empirical and limiting Stieltjes transforms, with $F^{\mathbf{A}}$ serving as the unknown parameter of interest. To enable this optimization, $F^{\mathbf{A}}$ is parametrized by treating it as a mixture of point masses, though more general formulations are feasible. The proposed approach to estimation of $F^{\mathbf{A}}$, and consequently the ESD of $\mathbf{\Sigma}$, is connected to, but significantly more involved than, the approach for estimating the distribution of eigenvalues of the population covariance matrix adopted by El Karoui (2008). However, this approach

is quite distinct from other recent estimation procedures for the spectrum of the population covariance matrix, such as those by Bai et al. (2010) and Ledoit and Wolf (2015). Details of the first step are given in Section 2.3.

(2) *Estimate the parameters* $\mathbf{\Theta}_1, \ldots, \mathbf{\Theta}_J$: Recall that $\mathbf{\Theta}_j = \mathbf{B}^T \mathbf{P}_j \mathbf{B}$, where $\mathbf{P}_j$ is the eigen-projection matrix of $\mathbf{\Sigma}$ corresponding to its jth distinct eigenvalue σ_j, $j = 1, \ldots, J$. For the estimation of these parameters, make use of the *deterministic equivalent* of the resolvent $\mathbf{R}(z) := (\mathbf{S} - z\mathbf{I}_p)^{-1}$, $z \in \mathbb{C}^+$. This result is a key ingredient in **?** for establishing the limiting spectral distribution (LSD) of $\mathbf{S}$ and allows for the use of a regression problem formulation, minimizing the sum of squared distances between the matrix-valued quantities $\mathbf{B}^T \mathbf{R}(z) \mathbf{B}$ and their limiting values, which can be expressed in terms of functionals of $F^{\mathbf{A}}$ (dependent on z) and the parameters $(\mathbf{\Theta}_j : j = 1, \ldots, J)$, with the discrepancy measure between the empirical and limiting quantities then summed over a suitably dense set of values of $z \in \mathbb{C}^+$. Substituting the estimates of the ESDs of the coefficient matrices, this objective function is minimized with respect to the parameters $(\mathbf{\Theta}_j : j = 1, \ldots, J)$ to find their estimators. Details of the second estimation step are given in Section 2.3.

A few comments regarding the estimation procedure are in order. First, the MA representation of the linear time series formulation allows us to make meaningful approximations of the process by finite order MA processes. This kind of approximation is important for a stable implementation of the proposed algorithms and turns out to be quite effective, as is demonstrated through numerical studies. Second, the estimate of the ESD of $\mathbf{\Sigma}$ is derived from the estimate $F^{\mathbf{A}}$, even though the latter object could be a higher dimensional distribution if the assumed MA process is of order larger than one. Moreover, the quadratic form of interest, $\mathbf{B}^T \mathbf{\Sigma}^{-1} \mathbf{B}$ is a lower dimensional estimation object. This suggests, and is supported by our numerical studies, that the estimation of the latter is simpler in the sense that even when the estimation of $F^{\mathbf{A}}$ is not very accurate, the estimate of the quadratic form could still remain quite accurate. Moreover, when it comes to estimation of the quadratic form, there is some redundancy in that not all of the eigen-subspaces of $\mathbf{\Sigma}$ may contribute significantly to the object $\mathbf{B}^T \mathbf{\Sigma}^{-1} \mathbf{B}$. This means that a model selection procedure choosing a set of significant $\mathbf{\Theta}_j$ can be more efficient. Indeed, such a model selection strategy is developed using the principle of cross validation. Finally, even though the description here focuses on the linear process formulation for the observed return, to deal with more realistic scenarios, the estimation strategy is extended to factor models whose idiosyncratic term follows the linear process structure described above. The corresponding description is given in Section 2.4 below.

Estimating the ESD of $\mathbf{\Sigma}$

The essence of step (1) is to invert equations (1) and (1). By this we mean that we observe $s_\tau(z)$ on the left hand side of (1) and want to estimate $F^{\mathbf{A}}$ on the right hand side. The following steps are involved in our estimation procedure.

(a) Discretization of $F^{\mathbf{A}}$. To estimate $F^{\mathbf{A}}$, use the discrete distribution

$$\hat{F}^{\mathbf{A}} = \sum_{j=1}^{J} \hat{\eta}_j \delta_{\alpha_j},$$

as an approximation, where $\hat{\eta}_j$ are weights, δ_{α_j} Dirac measures and J the number of grid points $\alpha_j \in \mathbb{R}^q$. For example, for an MA(2) process with $q = 2$, the number of grid points is the product of the grid size for each coordinate corresponding to the MA coefficients. In this formulation, $\hat{\eta}_j$ are the objects to estimate.

(b) Picking a finite collection of $z \in \mathbb{C}^+$. Observe first that, for every given z, (1) and (1) constitute a separate system of equations. For fixed z and τ, compute as discrepancy measure the squared error loss between the Stieltjes transform of the empirical and limiting ESDs, that is, the two sides of (1). Pick then a finite collection $\mathcal{D}$ of $z \in \mathbb{C}^+$ and a finite collection $\mathcal{T}$ of lag orders $\tau \geq 0$. The foregoing gives as optimization criterion the sum over $\mathcal{D}$ and $\mathcal{T}$ and the optimal $\hat{F}^{\mathbf{A}}$ is chosen as the one that minimizes this criterion.

The collection $\mathcal{D}$ is chosen to satisfy two criteria. The first criterion is that the real parts need to cover the range of empirical eigenvalues. The second criterion is that the imaginary parts need to be reasonably close to zero. The closer to zero the imaginary parts of z are, the more sensitive is the Stieltjes transform to the changes to the model parameters. However, when the imaginary parts are too close to zero, the Stieltjes transforms also become noisy. Thus, in the numerical studies, the real parts are chosen to be equally spaced over the range of the empirical distributions of the eigenvalues, while the imaginary parts are chosen to follow the geometric sequence of $1/4$, $1/2$, 1, 2 and 4.

(c) Reformulating (1) and (1) as a system of equations. To solve equations (1) and (1) for $F^{\mathbf{A}}$, solving for the intermediate object $K_\tau(z, \nu)$ defined in (1) is needed. For any given z, $K_\tau(z, \nu)$ is a function defined on $[0, 2\pi]$. Rewrite $K_\tau(z, \nu)$ so that it is a combination of a finite number of trigonometric functions. Thus estimating function $K_\tau(z, \nu)$ becomes equivalent estimating a finite number of parameters. For example, for an MA(q) process, (1) can be written as,

$$
\begin{aligned}
K_\tau(z, \nu) &= \int \frac{h(\lambda, \nu)}{\mathcal{M}_\tau(z, \lambda) - z} dF^{\mathbf{A}}(\lambda) \\
&= \sum_{\ell=0}^{q} \sum_{k=\ell+1}^{q} 2\cos((k - \ell)\nu) \int \frac{f_\ell(\lambda) f_k(\lambda)}{\mathcal{M}_\tau(z, \lambda) - z} dF^{\mathbf{A}}(\lambda) + \sum_{\ell=0}^{q} \int \frac{|f_\ell(\lambda)|^2}{\mathcal{M}_\tau(z, \lambda) - z} dF^{\mathbf{A}}(\lambda),
\end{aligned}
$$

where

$$\mathcal{M}_\tau(z, \lambda) = \frac{1}{2\pi} \int_0^{2\pi} \frac{\cos(\tau\theta) h(\lambda, \theta)}{1 + c\cos(\tau\theta) K_\tau(z, \theta)} d\theta, \tag{2.1}$$

and $z \in \mathbb{C}^+$. For $0 \leq \ell \leq q$ define

$$s_{\ell, \tau}(z) = \int \frac{|f_\ell(\lambda)|^2}{\mathcal{M}_\tau(z, \lambda) - z} dF^{\mathbf{A}}(\lambda),$$

and, for $0 \leq \ell < \ell' \leq q$, let

$$s_{\ell,\ell',\tau}(z) = s_{\ell',\ell,\tau}(z) = \int \frac{f_\ell(\lambda) f_{\ell'}(\lambda)}{\mathcal{M}_\tau(z,\lambda) - z} \, dF^{\mathbf{A}}(\lambda).$$

Then, (1) and (1) can be formulated as a system of equations involving $s_{\ell,\tau}(z)$ and $s_{\ell,\ell',\tau}(z)$, which are solved using Newton's method for any given value of $z \in \mathbb{C}^+$. The existence and uniqueness of the latter solution is guaranteed by the results in Liu et al. **?**.

Estimation of Quadratic Form

To accommodate step (2), use the fact that, for any $z \in \mathbb{C}^+$,

$$\mathbf{B}^T \mathbf{R}(z) \mathbf{B} \approx \sum_{j=1}^{J} \frac{1}{\mathcal{M}(z,\alpha_j) - z} \mathbf{\Theta}_j, \tag{2.0}$$

where J is the number of grid points defined in Section 2.3, $\mathbf{R}(z) = (\mathbf{S} - zI)^{-1}$, $\mathbf{\Theta}_j = \mathbf{B}^T \mathbf{P}_j \mathbf{B}$, and $M(z,\alpha)$ is the kernel defined in (2.3), determined by the Stieltjes kernel describing the LSD of $\mathbf{S}$. It should be noted here that the approximation in (2.3) holds in a limiting sense, as articulated through the notion of *deterministic equivalent of the resolvent* $\mathbf{R}(z)$, the details of which can be found in **?**. Notice that $\mathbf{\Theta}_1, \ldots, \mathbf{\Theta}_J$ are symmetric (indeed, non-negative definite) and $\sum_{j=1}^{J} \mathbf{\Theta}_j = \mathbf{B}^T \mathbf{B}$. Equation (2.3) leads to the following estimation strategy for $\mathbf{B}^T \mathbf{\Sigma}^{-1} \mathbf{B}$.

- *Step 1:* Let $\mathcal{D} \subset \mathbb{C}^+$ be the finite grid from Section 2.3.Then, estimate $\mathbf{\Theta} = (\mathbf{\Theta}_1 : \cdots : \mathbf{\Theta}_J)$ by

$$\hat{\mathbf{\Theta}} = \arg \min_{\mathbf{\Theta}_1,\ldots,\mathbf{\Theta}_J \in \mathcal{S}^{K \times K}} \sum_{z \in \mathcal{D}} \left\| \mathbf{B}^T \mathbf{R}(z) \mathbf{B} - \sum_{j=1}^{J} \frac{1}{\mathcal{M}(z,\alpha_j) - z} \mathbf{\Theta}_j \right\|_F^2, \tag{2.0}$$

  where $\mathcal{S}^{K \times K}$ denotes the class of $K \times K$ symmetric matrices.

- *Step 2:* Estimate $\mathbf{B}^T \mathbf{\Sigma}^{-1} \mathbf{B}$ by

$$\sum_{j=1}^{J} \frac{1}{\psi(\alpha_j, 0)} \hat{\mathbf{\Theta}}_j. \tag{2.0}$$

Thresholding and Model Selection

One challenge in the proposed algorithm is choosing how many grid points $\alpha_j \in \mathbb{R}^q$ to keep from step (1) of Section 2.3. On one hand, there should be sufficiently many grid points to ensure an accurate approximation of $F^{\mathbf{A}}$. On the other hand, there should sufficiently few grid points so that their corresponding weights can be estimated with high precision. One natural solution is to threshold eigenvalues with weights below some pre-specified tuning parameter ξ. Below, an algorithm for a given value of ξ is described first and a discussion is then added on how to choose the tuning parameter based on a model selection approach.

Algorithm 2.3.1 (Thresholding). *Perform the following three steps.*

- *Thresholding. Let $\xi \geq 0$ be a threshold. For each $j \in \{1, \ldots, J\}$, threshold the estimated weights $\hat{\eta}_j$ at ξ. Define $\mathcal{J}(\xi) = \{j \in \{1, \ldots, J\} \colon \hat{\eta}_j > \xi\}$ and drop the grid point α_j if $j \notin \mathcal{J}(\xi)$.*

- *Reweighting. For all $j \in \mathcal{J}(\xi)$, assign the updated weight*

$$\hat{\eta}_j(\xi) = \frac{\hat{\eta}_j}{\sum_{j' \in \mathcal{J}(\xi)} \hat{\eta}_{j'}}.$$

  Then the thresholded estimate of $F^{\mathbf{A}}$ is

$$\hat{F}_\xi^{\mathbf{A}} = \sum_{j \in \mathcal{J}(\xi)} \hat{\eta}_j(\xi) \delta_{\alpha_j}. \tag{2.0}$$

- *Estimation of $\mathbf{\Theta}$: Estimate the redefined $\mathbf{\Theta} = (\mathbf{\Theta}_j \colon j \in \mathcal{J}(\xi))$ by minimizing*

$$\sum_{z \in \mathcal{D}} \left\| \mathbf{B}^T \mathbf{R}(z) \mathbf{B} - \sum_{j \in \mathcal{J}(\xi)} \frac{1}{\hat{\mathcal{M}}_\xi(z, \alpha_j) - z} \mathbf{\Theta}_j \right\|_F^2,$$

  where $\hat{\mathcal{M}}_\xi(z, \alpha)$ is the kernel $\mathcal{M}_0(z, \alpha)$ in (2.3) determined by the distribution $\hat{F}_\xi^{\mathbf{A}}$. Here, the restriction that $\mathbf{\Theta}_j$, $j \in \mathcal{J}(\xi)$, are symmetric is imposed again. Finally, estimate $\mathbf{B}^T \mathbf{\Sigma}^{-1} \mathbf{B}$ by

$$\hat{\mathbf{\Phi}}(\xi) = \sum_{j \in \mathcal{J}(\xi)} \frac{1}{\psi(\alpha_j, 0)} \hat{\mathbf{\Theta}}_j(\xi). \tag{2.0}$$

The thresholding strategy outlined in Algorithm 2.3.1 could be performed for a sequence of nonnegative ξ (the maximal ξ being the point beyond which $\mathcal{J}(\xi)$ is the empty set). In practice, however, a particular value of the thresholding parameter ξ needs to be chosen. This is a model selection problem, and a simple cross-validation strategy is proposed to solve it. Accordingly, split the data $\{X_1, \ldots, X_T\}$ into two parts, a *training set* consisting of the first half of the observations $\{X_1, \ldots, X_{T/2}\}$ and a *test set* of the second half observations $\{X_{T/2+1}, \ldots, X_T\}$.

Algorithm 2.3.2 (Model selection). *Perform the following five steps.*

- *Given a $\xi \geq 0$, obtain the estimate $\hat{F}_\xi^{\mathbf{A},Train}$ based on the training data. Note that the dimension-to-sample size ratio needs to be adjusted to $2N/T$ rather than N/T while carrying out this procedure.*

- *Estimate $\mathbf{\Theta}_j$, $j \in \mathcal{J}^{Train}(\xi)$, from the training data by the procedure described in Algorithm 2.3.1. Let the corresponding estimates be denoted by $(\hat{\mathbf{\Theta}}_j^{Train}(\xi) \colon j \in \mathcal{J}^{Train}(\xi))$.*

- *Let $\mathbf{S}_{0,Test}$ denote the sample covariance matrix for the test data, and let $\mathbf{R}_{Test}(z)$ be the corresponding resolvent. Let $\hat{\mathcal{M}}_\xi^{Test}(z, \alpha)$ be the analog of $\mathcal{M}_0(z, \alpha)$ when using the ratio*

$2N/T$ in the computation for the LSD, while using $\hat{F}_\xi^{\mathbf{A},Train}$ for the distribution. Compute then the forward cross validation score

$$\mathrm{CV}_f(\xi) = \sum_{z \in \mathcal{D}} \left\| \mathbf{B}^T \mathbf{R}_{Test}(z) \mathbf{B} - \sum_{j \in \mathcal{J}^{Train}(\xi)} \frac{1}{\hat{\mathcal{M}}_\xi^{Test}(z, \alpha_j) - z} \hat{\mathbf{\Theta}}_j^{Train}(\xi) \right\|_F^2.$$

- *Flip the training data set and the test data set to calculate the backward cross validation score $\mathrm{CV}_b(\xi)$. Take the sum $\mathrm{CV}(\xi) = \mathrm{CV}_f(\xi) + \mathrm{CV}_b(\xi)$ as the score for model selection.*
- *Define ξ_{opt} to be the value of ξ for which $\mathrm{CV}_f(\xi)$ is minimized subject to the restriction that $\hat{\Phi}(\xi)$ defined in (2.3.1) is nonnegative definite. The latter restriction helps to further narrow down the scope of plausible models and thereby improves statistical efficiency.*

Note that, although in the simulation study below $F^{\mathbf{A}}$ is set up as a discrete distribution, the proposed algorithm can be extended to continuous $F^{\mathbf{A}}$; for example through an approximation with a set of spline functions. Hence, estimating a continuous $F^{\mathbf{A}}$ becomes equivalent to estimating the coefficients in the spline representation. This, however, can be implemented using numerical methods similar to the ones utilized for the discrete $F^{\mathbf{A}}$ under consideration here.

2.4 Extensions

Application to the Markowitz Portfolio Problem

As documented in El Karoui **?**, the Markowitz portfolio problem can be formulated as a special case of the quadratic program

$$\min_{w \in \mathbb{R}^p} \frac{1}{2} w^T \Sigma w \qquad \text{subject to } w^T v_k = u_k, \quad k = 1, \dots, K.$$

Let $\mathbf{V} = [v_1 : \cdots : v_K]$ be the $p \times K$ matrix whose kth column is v_k and $U = (u_1, \dots, u_K)^T$ the K-dimensional vector whose kth entry is u_k. Define the $K \times K$ matrix

$$\mathbf{Q} = \mathbf{V}^T \Sigma^{-1} \mathbf{V}, \tag{2.0}$$

assuming that the columns of $\mathbf{V}$ are such that $\mathbf{Q}$ is invertible. The solution of the quadratic program with linear equality constraints (2.4) is

$$w_{\text{optimal}} = \Sigma^{-1} \mathbf{V}^T \mathbf{Q}^{-1} \mathbf{V} \tag{2.0}$$

and

$$w_{\text{optimal}}^T \Sigma w_{\text{optimal}} = U^T \mathbf{Q}^{-1} U. \tag{2.0}$$

The Markowitz portfolio problem fits into the above framework of quadratic programs. Its aim is to find the minimum risk that one has to absorb in order to achieve an expected portfolio

return. For each expected portfolio return, there is one minimum risk determined by the smallest variance of portfolio returns, so it is standard to plot the variance of portfolio returns against the expected portfolio returns, as in Figures 3.1 and 2.2. The Markowitz portfolio problem has two linear constraints, so that $K = 2$. The first constraint is that the total weight is 1, while the second constraint is that the expected portfolio return is equal to μ_P, that is,

$$w^T e = 1,$$
$$w^T \mu = \mu_P,$$

where e is a p-dimensional vector whose entries are all equal to 1, μ is a vector of expected asset returns in the portfolio and μ_P is a particular expected portfolio return. Consequently, the respective quantities in (2.4) and (2.4) become

$$U_P = \begin{bmatrix} 1 \\ \mu_P \end{bmatrix} \quad \text{and} \quad \mathbf{V} = [\, e : \mu \,],$$

noting that the dependence of U on μ_P will be expressed by the subscript P. Expected returns μ are typically unknown and commonly estimated by the sample mean $\hat{\mu}$. For each fixed μ_P, the goal is then to estimate the minimum risk in (2.4). The algorithm for this is listed in the following.

(1) Since the matrix $\mathbf{V}$ involves unknown expected asset returns, the first step is to calculate $\hat{\mathbf{V}} = [\, e : \hat{\mu} \,]$, where $\hat{\mu}$ is the sample mean.

(2) The proposed algorithm provides the estimate $\hat{\mathbf{Q}} = \hat{\mathbf{V}}^T \hat{\mathbf{\Sigma}}^{-1} \hat{\mathbf{V}}$ for $\mathbf{Q}$ in (2.4).

(3) The estimate for the minimum risk in (2.4) is then $U_P^T \hat{\mathbf{Q}}^{-1} U_P$.

Note that, for each expected portfolio return, a minimum risk needs to be estimated. However, all estimated minimum risks share the same $\hat{\mathbf{Q}}$, which hence has to be estimated only once.

Extension to Factor Models

In this section, an extension of the proposed algorithm to incorporate factor structures is discussed. In the framework discussed below, it is assumed that there are M known factors, the context being situations for which the leading factors in asset returns (for example the market return factor, "small minus big" and "high minus low" factors in the Fama-French three-factor model, see Fama and French (1993)) can reasonably be considered as known. When the leading factors are unknown, they can be estimated from the leading eigenvectors of the sample covariance matrix with well established procedures described in the literature; for example, in Onatski Onatski (2009, 2010). Focusing here on the known factors framework, asset returns may be written as

$$Y_t = \sum_{m=1}^M L_m f_{m,t} + X_t, \qquad X_t = \sum_{\ell=0}^\infty \mathbf{A}_\ell Z_{t-\ell}, \qquad t \in \mathbb{Z},$$

where $f_{m,t}$ is the observable return of factor m at time t, and L_m the corresponding unknown $N \times 1$ factor loading, $m = 1, \ldots, M$. Assume that $\mathbf{L} = [L_1 : \cdots : L_M]$ is orthogonal to the eigenvectors

of the linear process coefficient matrices $(\mathbf{A}_\ell \colon \ell \in \mathbb{N})$. The estimation procedure for the quadratic form $\mathbf{B}^T \mathbf{\Sigma}^{-1} \mathbf{B}$ may be adjusted to this factor model setting in the following way.

(1) Estimate the matrix of factor loadings by $\mathbf{L}$ by regressing Y_t on the factor vector $\mathbf{f}_t = (f_{1,t}, \ldots, f_{M,t})^T$. This yields the least squares estimate $\hat{\mathbf{L}} = [\hat{L}_1 : \cdots : \hat{L}_M]$.

(2) Estimate the covariance matrix of factor returns by $\sum_{m=1}^{M} \hat{\sigma}^2(f_m) \hat{L}_m \hat{L}_m^T$, where $\hat{L}_m$ is the estimated loading vector on factor m and $\hat{\sigma}(f_m)$ its estimated standard deviation. In matrix notation, the estimated covariance matrix for the factor structure is therefore

$$\hat{\mathbf{L}} \hat{\mathbf{\Delta}} \hat{\mathbf{L}}^T,$$

where $\hat{\mathbf{\Delta}} = \mathrm{diag}(\hat{\sigma}(f_1), \ldots, \hat{\sigma}(f_M))$ is an $M \times M$ diagonal matrix.

(3) Note that

$$\mathbf{B}^T \mathbf{\Sigma}^{-1} \mathbf{B} = \mathbf{B}^T \left(\mathbf{L}(\mathbf{I}_M + \mathbf{\Delta})^{-1} \mathbf{L}^T + \left(\mathbf{P}_L^\perp + \sum_{\ell=0}^{\infty} \mathbf{A}_\ell \mathbf{A}_\ell' \right)^- \right)^{-1} \mathbf{B}$$

$$= \mathbf{B}^T \mathbf{L}(\mathbf{I}_M + \mathbf{\Delta})^{-1} \mathbf{L}^T \mathbf{B} + \mathbf{B}_L^T \left(\mathbf{I}_N + \sum_{\ell=0}^{\infty} \mathbf{A}_\ell \mathbf{A}_\ell' \right)^{-1} \mathbf{B}_L,$$

where $\mathbf{P}_L^\perp = \mathbf{I}_N - \mathbf{L}\mathbf{L}^T$, with $\mathbf{C}^-$ denoting the Moore–Penrose generalized inverse of the symmetric matrix $\mathbf{C}$, and $\mathbf{B}_L = \mathbf{P}_L^\perp \mathbf{B}$. In the above calculation it was used that, since $\mathbf{P}_L^\perp \mathbf{P}_L^\perp = \mathbf{P}_L^\perp$ and $\mathbf{P}_L^\perp (\sum_{\ell=0}^{\infty} \mathbf{A}_\ell \mathbf{A}_\ell') \mathbf{P}_L^\perp = \sum_{\ell=0}^{\infty} \mathbf{A}_\ell \mathbf{A}_\ell'$, it holds $(\mathbf{P}_L^\perp + \sum_{\ell=0}^{\infty} \mathbf{A}_\ell \mathbf{A}_\ell')^- = \mathbf{P}_L^\perp (\mathbf{I}_N + \sum_{\ell=0}^{\infty} \mathbf{A}_\ell \mathbf{A}_\ell')^{-1} \mathbf{P}_L^\perp$.

(4) Using the calculations in (3), the first term in (2.4) concerning the factor structure can be estimated by $\mathbf{B}^T \hat{\mathbf{L}}(\mathbf{I}_K + \hat{\mathbf{\Delta}})^{-1} \hat{\mathbf{L}}^T \mathbf{B}$. The second term concerning with the idiosyncratic time series component can be estimated using the estimation strategy for quadratic forms introduced in the previous sections.

2.5 Empirical Results

In this section, we carry out a set of simulation studies designed to show the effectiveness of the proposed methodology for estimating the MVF, when the observations form a time series. This is done in two settings – (i) when the observations together follow a stationary linear process model, following the basic structural assumptions outlined in Section 2.2; and (ii) when the observations follow a factor model structure with known factors, and where the idiosyncratic term constitutes linear process satisfying analogous assumptions. In addition, we demonstrate the effectiveness of the proposed strategy for estimating the distribution eigenvalues of the coefficient matrices of the linear process, even though the latter is not the primary focus of this paper.

In Section 2.5 we report results of MVF estimation as well estimation of the distribution of eigenvalues of coefficient matrices when the underlying data generating process is an MA(2) time series. In Section 2.5 we report results of MVF estimation when the underlying data generating

process is an AR(1) time series. In both cases, we estimate the process as an MA(2) process. Notice that when the data are generated from an AR(1), our model is misspecified, and an MA(2) model only fits an approximation to the AR(1) process. Finally, in Section 2.5 we demonstrate the performance of MVF estimation under the factor model described in Section 2.4.

We compare the performance of the proposed method, referred to as **LinShrink** (stands for shrinkage under linear process structure), for estimating the MVF with two other approaches. The first one, referred to as **Naive Estimate**, is based on simply replacing the population covariance with the sample covariance. The second method, which uses the shrinkage approach based on independent observations, as proposed in **?**, is referred to as **IndShrink**.

Estimation under MA(2) process

First we describe the simulation setting. We consider the setting where the two MA(2) coefficients, A_1 and A_2 are symmetric and simultaneously diagonalizable, with the joint bivariate spectral distribution given in Table 2.1 below. The common eigen-basis for A_1 and A_2 is chosen to be uniformly distributed on the space of $N \times N$ orthogonal matrices. The innovation process is taken to be i.i.d. vectors with independent $N(0, 1)$ entries.

Eigenvalue pair	Probability
$(0.1, 0.2)$	0.3
$(0.4, 0.5)$	0.3
$(0.7, 0.9)$	0.4

Table 2.1: Joint eigenvalue distribution of (A_1, A_2) for MA(2) process used in the simulation.

We consider two scenarios which are comparable in that $N/T = 1/3$: *Case 1:* $N = 1000$, $T = 3000$; *Case 2:* $N = 2000$, $T = 6000$. In each case, we use a square grid $\{(i/10, j/10) : 1 \le i, j \le 10\}$ as candidate eigenvalue pairs for estimation of the joint empirical distribution of eigenvalues of A_1 and A_2. For the mean variance frontier estimation, we choose the expected return μ to be the $N \times 1$ vector with j-th coordinate $\mu_j = 1 + j/N$ for all $j = 1, \ldots, N$.

We report the result of MVF estimation under *Case 1* in Figure 2.3. The left panel shows the plot of the variance of return against expected return, while the right panel shows the frequency distribution of the number of eigenvalue pairs receiving positive weights under the thresholding-based model selection approach described Algorithms 2.3.1 and 2.3.2 in Section 2.3. The true MVF is indicated by solid green line, while pointwise mean of LinShrink, IndShirnk and Naive Estimate are shown in solid blue, magenta and red curves, respectively. The broken curves of same colors represent $+/-$ one standard deviation bands. It can be seen that the LinShrink estimate is nearly unbiased, with a bit higher variability than the IndShrink estimate and the Naive Estimate, both of which are biased. The IndShrink estimate has smaller bias than the Naive Estimate, indicating that simply taking into account the dimensionality effect, while ignoring the temporal correlation, can still improve the estimate. The model selection performance shows that the proposed method largely avoids overfitting.
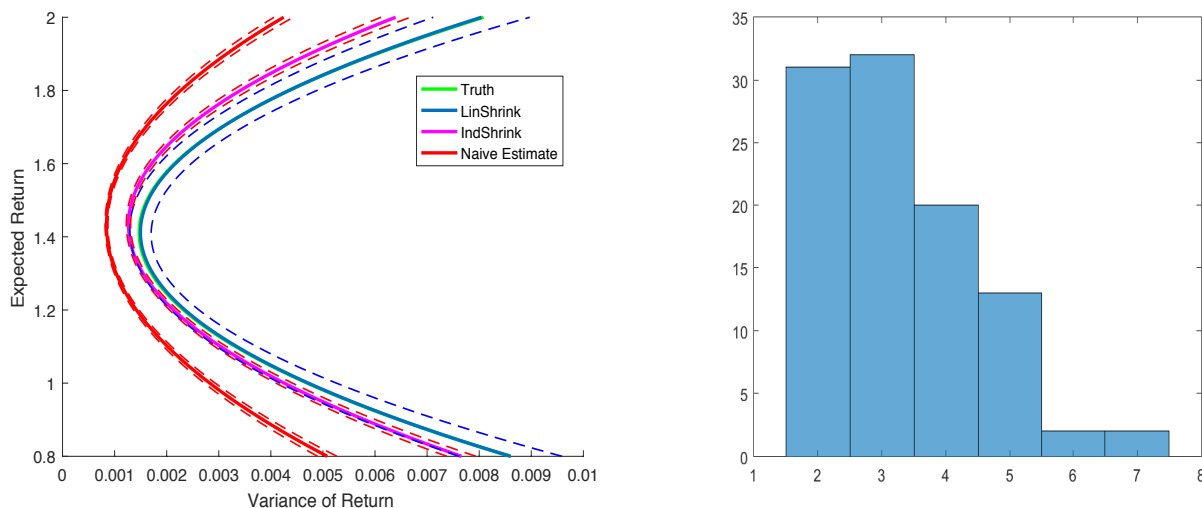
Figure 2.3: MVF estimation for MA(2) process with $N = 1000$, $T = 3000$. **Left panel:** True and estimated MVF (solid line), with one standard deviation band (broken line). LinShrink: blue; IndShrink: magenta; Naive estimate: red. **Right panel:** Frequency distribution of the number of eigenpairs chosen by the model selection procedure.

Results for MVF estimation under *Case 2* are reported in Figure 2.4. Even though the sample size is twice as before, the only noticeable qualitative change in the performance of the LinShrink method is a slight reduction in variability and essentially zero bias, as reflected by the near complete overlap of the blue and green solid lines. Also, the bias in estimates of the other two methods do not decrease, as would be expected, since the dimension-to-sample size ratio remains the same. The model selection performance of the proposed method seems to improve slightly, with fewer models with larger number of eigenvalue pairs getting selected compared with *Case 1*.

We also display the true and estimated cumulative distribution functions (CDF) of the marginal eigenvalue distributions of coefficient matrices A_1 and A_2 under *Case 2* in Figure 2.5. As can be seen from these plots, the estimated marginal CDFs are quite reasonable approximations to the true marginal CDFs.

Estimation under an AR(1) model

We now consider the case where the time series is an AR(1) process, with the symmetric AR coefficient matrix A having the eigenvalue distribution given in Table 2.2. The innovation process is again chosen to be i.i.d. with independent $N(0, 1)$ entries.

Recall that the proposed method is designed to estimate the eigenvalue distribution of MA processes. Therefore, we are using this example as test case for the effect of model misspecification. In our estimation procedure for MVF, we approximate the AR(1) process by an MA(2) process. As a way of restricting the class of models, and thereby increasing estimation effi-
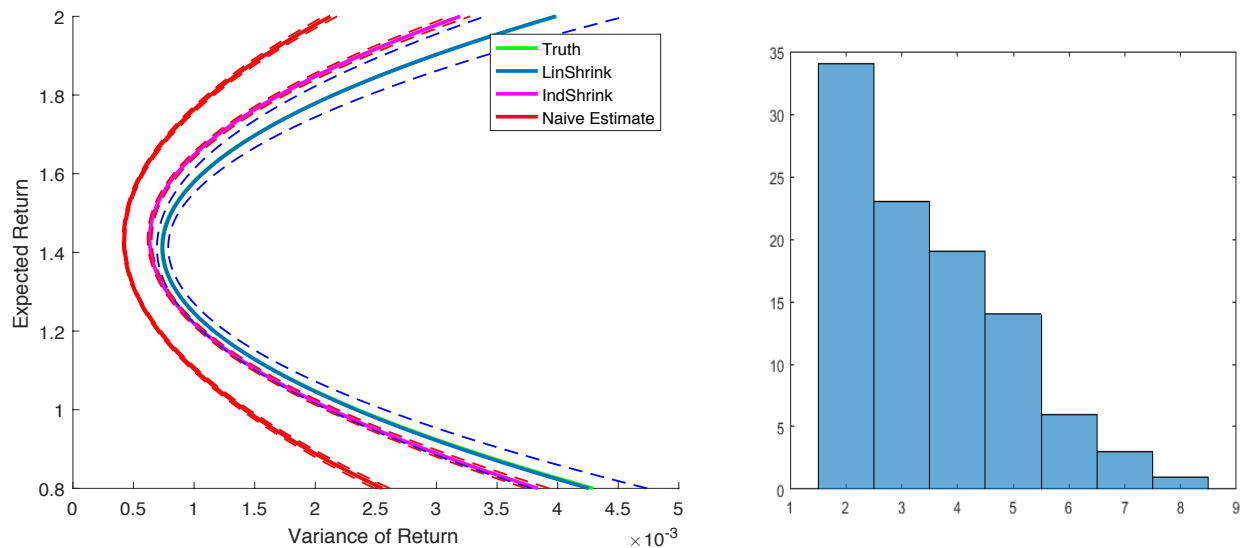
Figure 2.4: MVF estimation for MA(2) process with $N = 2000$, $T = 6000$. **Left panel:** True and estimated MVF (solid line), with one standard deviation band (broken line). LinShrink: blue; IndShrink: magenta; Naive estimate: red. **Right panel:** Frequency distribution of the number of eigenpairs chosen by the model selection procedure.
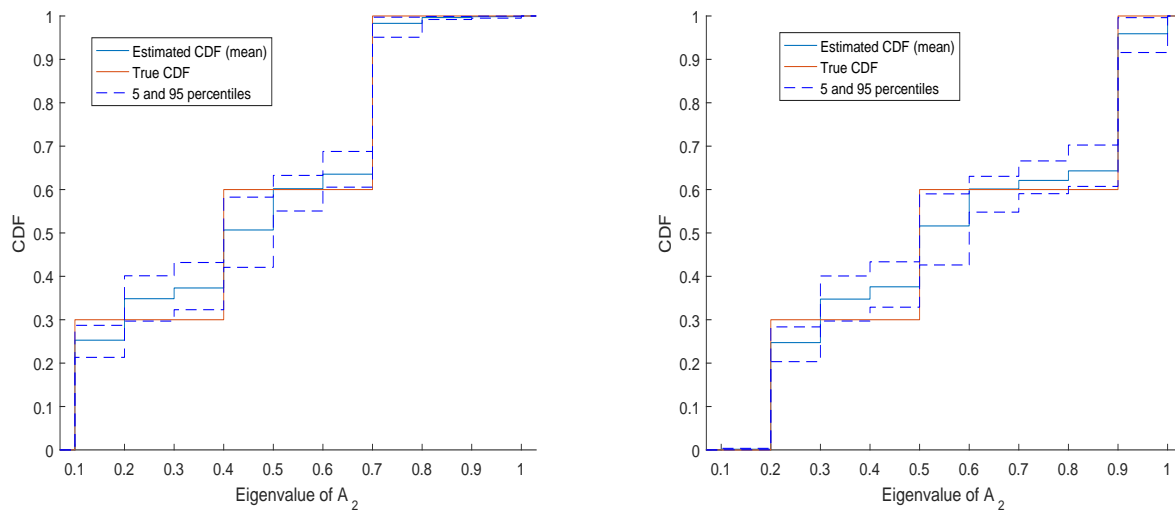

Figure 2.5: Estimation of CDF of the eigenvalue distribution of coefficient matrices for the MA(2) process with $N = 2000$, $T = 6000$. True CDF: solid red; Mean of estimated CDF: solid blue; 5-th and 95-th pointwise percentiles of estimated CDF: broken blue. **Left panel:** CDF of eigenvalue distribution of A_1; **Right panel:** CDF of eigenvalue distribution of A_2.

Eigenvalue pair	Probability
0.1	0.3
0.2	0.3
0.4	0.4

Table 2.2: Joint eigenvalue distribution of (A_1, A_2) for MA(2) process used in the simulation.

ciency, we choose an MA(2) representation derived by truncating the MA(∞) representation of the AR(1) process. Accordingly, the MA coefficients in our MA(2) representation are set as $A_1 = B$ and $A_2 = B^2$, for some unknown symmetric matrix B. Thus, while estimating the joint eigenvalue distribution of (A_1, A_2), we restrict the domain of the eigenvalue pairs to the grid $\{(j/10, (j/10)^2) : j = 1, \ldots, 10\}$. The parameter μ (expected return) is chosen to be the same as in the setting of MA(2) simulation in Section 2.5. The results for MVF estimation are displayed in Figure 2.6. What is apparent from this plot is that, now all three estimators – LinShrink, IndShrink and Naive Estimate – are biased, though the bias in the proposed LinShrink is significantly smaller compared to the other two methods. This bias in the proposed method is due to the bias in the approximation of the true AR(1) model by an MA(2) model. However, the relative performance of IndShrink and LinShrink, both of which are computed under model misspecification, also indicates that, with a finer approximation, for example by increasing the order of the MA process, the proposed should be able to obtain a nearly unbiased estimator of the MVF.
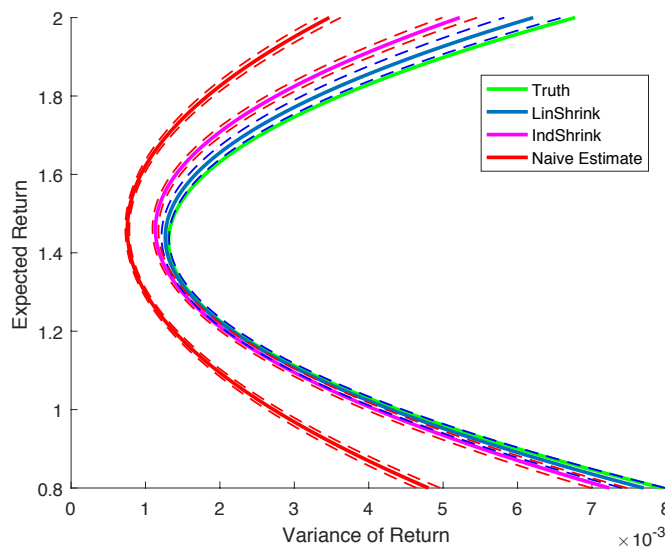

Figure 2.6: MVF estimation for an AR(1) process with $N = 1000, T = 3000$. True and estimated MVF (solid line), with one standard deviation band (broken line). LinShrink: blue; IndShrink: magenta; Naive estimate: red.

Estimation under a factor model

In this experiment, we assume that the returns Y_t's follow a factor model

$$Y_t = \mu_t + f_{1t}\mathbf{a}_1 + f_{2t}\mathbf{a}_2 + f_{3t}\mathbf{a}_1 + \varepsilon_t \tag{2.-1}$$

where $\mu_t \equiv 1$; $\mathbf{a}_k$, $k = 1, \ldots, 3$ are three randomly generated, mutually orthonormal vectors; and for each k, f_{kt} has i.i.d. $N(\nu_k, \sigma_k^2)$ entries with $\nu_k = 1.5$ for all k, and $\sigma_1 = 0.5$, $\sigma_2 = 0.625$ and $\sigma_3 = 0.7$. The time series $\{\varepsilon_k\}$ is assumed to follow the same MA(2) model as used in Section 2.5 and is independent of the f_{kt}'s. We obtain the estimates of the eigenvalue distributions of the coefficients of the MA(2) model by applying our spectrum estimation procedure to the residuals from the regression fit for $\{Y_t\}_{t=1}^T$, using the model (2.5), and treating f_{kt}'s as known. Then we apply the procedure described in Section 2.4 to estimate the MVF, using estimated mean vector. The result is shown in Figure 2.7. The relative ordering of the three methods, in terms of degree of bias in estimating the MVF, are similar as in the other cases considered above. But the difference between the estimates based on IndShrink and LinShrink is smaller compared the pure MA(2) time series case. The latter is due to the concentration of information in a few coordinates of the covariance matrix Σ as a result of the strong factor structure.
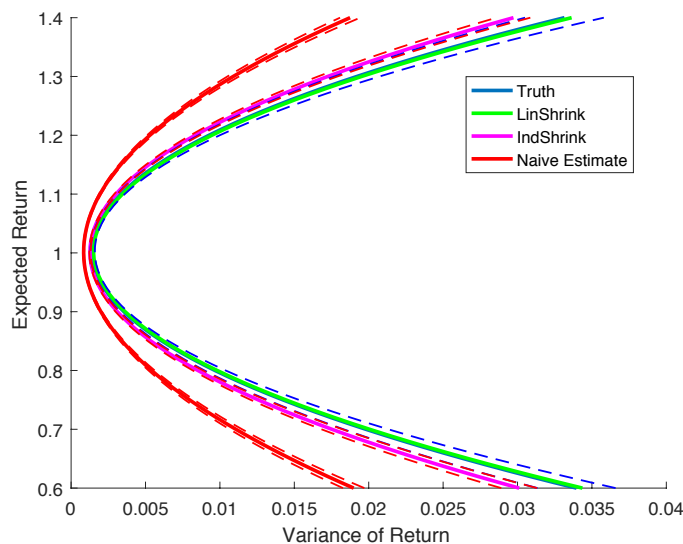

Figure 2.7: MVF estimation under a factor structure with MA(2) idiosyncratic terms and $N = 1000$, $T = 3000$. True and estimated MVF (solid line), with one standard deviation band (broken line). LinShrink: blue; IndShrink: magenta; Naive Estimate: red.

Conclusions

One important conclusion that can be derived from our numerical study is that even when the estimation of spectra of coefficients of the constituent linear process is not perfect – an admittedly difficult goal – the quality of estimation of MVF is still impressive, even when there is a model mismatch (as when we estimate the MVF involving an AR(1) process by approximating through an MA(2) process). More importantly, the results demonstrate the benefit of addressing the temporal dependence explicitly, as in our methodology, by way of obtaining a nearly unbiased estimate of the MVF when the model is correct. This is a significant improvement over the empirical MVF (i.e. based on using sample covariance matrix) and representative methods that only take into account the spatial (i.e., coordinatewise) dependence but assume independence of the observations across time.

2.6   Discussion

In this paper, we use a model for high-dimensional, stationary time series proposed by **?** to address the question of estimation of mean-variance frontier (MVF) within the framework of Markowitz portfolio optimization, where the returns are assumed to have a time series structure. This investigation is motivated by the established fact that empirical estimate of the MVF is known to be significantly biased when the dimensionality of the returns is not negligible compared to the sample size. The proposed method relies on the characterization of the limiting behavior of the empirical distributions of eigenvalues of symmetrized sample autocovariance matrices of the time series. We utilize the characterization established in **?** to formulate an estimator of the eigenvalue distribution of the coefficient matrices for high-dimensional, finite order, moving average (MA) processes with simultaneously diagonalizable coefficients. These estimates are then utilized as input in an optimization procedure aimed at estimation of quadratic forms involving the inverse population covariance matrix of the time series. The latter allows us to compute estimates of the MVF under two scenarios: (i) when the time series of returns belong to the class of linear processes considered in **?**; and (ii) when the returns follow a factor structure with known, finite-dimensional factors, and where the idiosyncratic terms are orthogonal to the factor space and follow the same linear process model as in (i). We perform a set of numerical simulations as a proof-of-concept, where we compare the performance of the proposed estimator of MVF with the empirical estimator, and a representative estimator of MVF proposed by **?**. The estimator demonstrates superior performance in terms of reducing the bias in estimating the MVF, thereby pointing to the key role played by the time series structure and its utilization in the proposed method.

There are several directions in which we plan to extend the current work. First, we aim to enhance the method of estimation of quadratic forms by implementing modifications that will allow us to tackle ARMA-type linear processes of a given order, where the coefficients satisfy a simultaneous diagonalizability condition. Secondly, we want to extend this estimation procedure to the setting of factor models with relatively strong, but unknown factors, with time-dependent idiosyncratic terms. Thirdly, we aim to carry out an extensive analysis based on data on stock prices

to obtain a meaningful estimate of the MVF and to quantify its uncertainty. Finally, we would like to carry out a thorough mathematical investigation aiming to establish consistency of the proposed procedure for finite order ARMA-type time series with the required structural assumptions.

Chapter 3

Do Expected Home Prices Affect Borrowers' Default Decision?

3.1 Introduction

Understanding mortgage borrowers' default behavior is important for both economic policies and pricing mortgage backed securities. In this paper I study how expected home prices affect borrowers' default decision.

Why should expected home prices affect default decisions? Because defaulters are locked out from the mortgage markets for a cooling-off period, in most cases 2-7 years. If a defaulter plans to buy a home after the cooling-off period, she would take into account the expected home prices in the next 2-7 years. The default decision is essentially comparing the two options: 1) keep paying down the remaining balance and stay as a homeowner; 2) default, suffer other losses including recourse, and can buy the next home at the home price after the cooling off period.

Figure 3.1 plots home prices in two CBSAs: Los Angeles and Riverside. The two CBSAs share similar home price history from 2003 to the end of 2007 but have different paths after 2008. Home prices in Los Angeles fell less and rebounded faster than home prices in Riverside. Consider two identical borrowers, one in Los Angeles the other one in Riverside. If they both defaulted on their mortgages in 2007, they could come back after the 2-7 years cooling off period and buy the next home between 2009 and 2014. Assume that they both had perfect foresight for future prices. The borrower in Riverside would have had more incentive to default than the borrower in Los Angeles. This is because the home prices in Riverside between 2009 and 2014 would be much lower than home prices in Los Angeles. The borrower in Riverside can better take advantage of the temporarily low home price after the cooling off period than the borrower in Los Angeles. The ratio of default probability curve in Figure 3.1 shows that borrowers in Riverside started to default much more often than borrowers in Los Angeles before 2007, when prices in the two CBSAs closely tracked each other. In this paper, I study how much of these lower default probabilities can be explained by the higher expected home price in Los Angeles. Treating prices as exogenous, I found that 1% higher expected home price lowers the default probability by 0.4%.
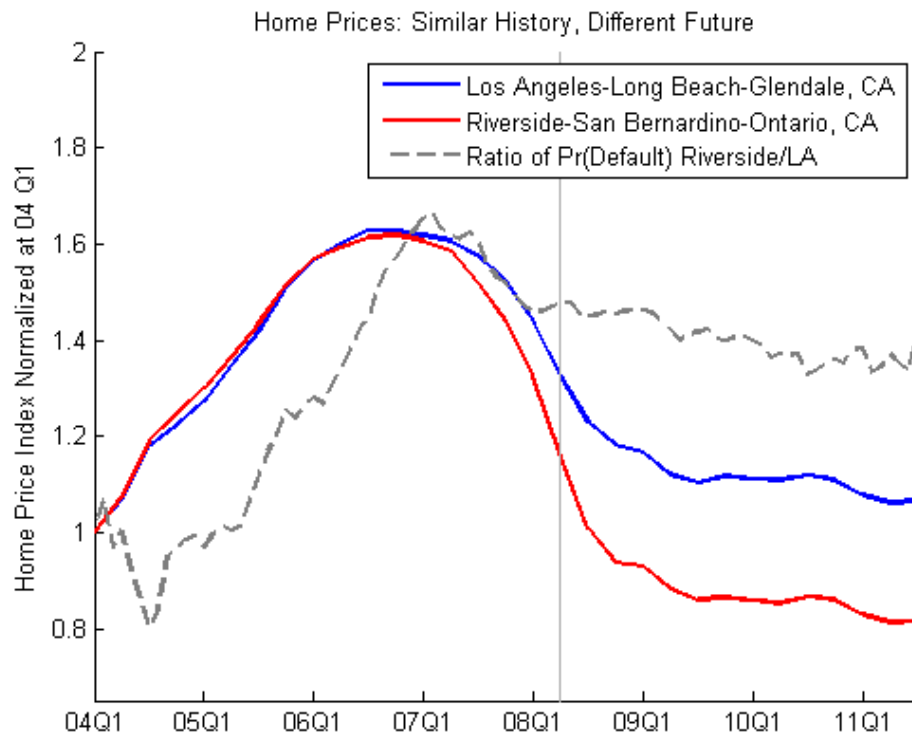
Figure 3.1: Home Prices in Los Angeles and Riverside

A confounding story is that borrowers in Riverside first defaulted more before 2008, and that is why home prices in Riverside fell more between 2008 and 2011 than Los Angeles. To account for the endogeneity of prices, I use the long run home price cyclicality instrument proposed in Palmer (2015). Sinai (2012) first observe that home price cyclicality are persistent over time. The long-run cyclicality is plausibly independent of shocks to the housing market in the 2000s. Using the long-run cyclicality instrument, I found that a 1% higher expected home price lowers the default probability by 0.8%.

Using variation in expected home prices, this paper studies how borrowers react to penalties and incentives in the cooling off period. When the expected home prices are higher, the penalty in the cooling period is larger and borrowers should default less often. Why should we care about how borrowers react to penalties in the cooling off period? First, the cooling off period is the largest penalty facing defaulters in non-recourse states. Even in recourse states, because recourse is rarely pursued (Fisher and Brueggeman (2010)), the penalty from the cooling off period is likely to be greater than the penalty from recourse. Thus it is interesting to compare how borrowers react to the threat from the cooling off period with how they react to the threat from recourse (Ghent and Kudlyak (2011)).

Second, the cooling off period is a flexible policy instrument. Studying how borrowers react to penalties from the cooling off period can guide us to optimally adjust the length of it. The

cooling off period is flexible in the two ways. First, it can be changed in reaction to changing economic environment. For instance, in August 2013, for defaulters who experienced at least 20 percent loss of income for 6 months or longer prior to default, FHA changed their cooling-off period from 3 years to 1 year (FHA (2013)). Changing recourse laws, on the other hand, can be relatively difficult and takes a longer time. Second, different borrowers can have different cooling off periods. Conforming loans require 2-7 years of good credit history. FHA loans, which are targeted more to low income borrowers, require 3 years of good credit from most borrowers.

Third, although borrowers have the right to strategically default and their mortgage interest rates already incorporate this option, many people find strategic default immoral. Eighty-two percent of respondents to the survey in Guiso et al. (2013) think it is morally wrong to engage in strategic default. Government subsidies also seem to screen out strategic defaulters. For FHA loans, only borrowers who experienced significant income loss are eligible for the shortened cooling off period (FHA (2013)). Are strategic defaulters bad borrowers? or they are sophisticated borrowers who, in normal times, behave better than others? I found that high credit score borrowers are more likely to react to both past price changes and future price changes than low credit score borrowers. This suggests that borrowers with high FICO scores are more likely to be strategic defaulters.

There is a large literature on mortgage borrowers' default behavior. In particular, there is an ongoing discussions on whether or not homeowners strategically default, *i.*e., choose to walk away from their mortgages even if they can afford mortgage payments. This paper tries to contribute to the literature by studying how borrowers strategically react to future home prices. There are two views on strategic defaults. The first view is that strategic defaults are rare and borrowers keep paying mortgage payments till they cannot. As evidence for this view, Bhutta et al. (2013) show that the median negative equity threshold for default is -67%. Gerardi et al. (2013) document that only 13.9% of defaulters have liquid or illiquid asset (including securities and automobiles) to make one month's mortgage payment. If we strict asset to liquid asset, only 6% of defaulters can make one mortgage payment. This suggests that in their sample, strategic defaulters cannot be more than 6% of all defaulters. On the other hand, there is a second view on strategic defaults, that borrowers strategically react to incentives and penalties of defaults. Guiso et al. (2013) estimate that 25% - 35% defaults in 2009-2010 were strategic, four times as many as the estimate in Gerardi et al. (2013). Mayer et al. (2014) document that homeowners strategically default to receive modification. Ghent and Kudlyak (2011) quantify how recourse deters some borrowers from defaulting. At the mean value of the default option for defaulted loans, borrowers are 30% more likely to default in non-recourse states. Furthermore, for homes appraised at $500,000$ to $750,000$, borrowers are twice as likely to default in non-recourse states.

The rest of the draft is organized as follows. Section 3.2 described the data and variables. Section 3.3 shows how expected home price growth affects default probability. Section 3.4 explores the effects from changing the cooling-off period. Section 3.5 presents our conclusion.

3.2 Data

In this section, I briefly describe the data sources used in this paper.

ABSNet Loan History Data. The main data source is ABSNet, which provides monthly loan performance and borrower information on privately securitized loans. The estimation sample is a 5% random sample of the first-lien subprime mortgages originated between 2003 and 2007. I restrict property types to "Single Family Residence", "Condominium" or "COOP". I also dropped mortgages with missing values for variables that we are interested in, including documentation type, loan purpose, ARM or not, interest only or not, balloon mortgage or not, original combined loan to value ratio, credit score, original loan to value ratio and original interest rate, resulting in a final dataset of over five million loan $\times$ month observations. Default date is defined to be the earliest of foreclosure date, REO date or liquidation date.

I censor all mortgage history to the first 52 months. This is because one of the explanatory variables is future home price change, more specifically, the log price change in the next 3 years. Home prices data are available through May 2015. Thus I can form the future price change variable till May 2012. The last cohort in my sample are mortgages originated in December 2007. For this cohort, I can only study the default probabilities in their first 52 months, *i.*e., between December 2007 and May 2012. All earlier cohorts are also censored to the first 52 months so that the earlier cohorts are not oversampled.

Table I reports the descriptive statistics. The average credit score is 621, below the national median score 720. By the 52nd month, 14% of mortgages have defaulted.

Zillow Home Value Index. For home price changes, I use zipcode level Zillow home value indexes. They cover over ten thousands zipcodes and are computed from estimated home values of over 70 millions homes.

Zillow Price to Rent Ratio. I control for price to rent ratio in all specifications as a proxy for utility difference between owning a home and renting the same home. It is widely observed and assumed in mortgage default models that people prefer being homeowners to being renters (Chatterjee and Eyigungor (2011); Hatchondo et al. (2014); Corbae and Quintin (2015b)). There are two utility losses from the cooling off period: missing the home price appreciation and being a renter instead of a homeowner. To isolate the effect of missing the home price appreciation, the utility loss from being a renter should be controlled for.

Next I show why price to rent ratio is a reasonable proxy for utility of being a homeowner. Assume that each home can provide constant housing service follow R and utility of being a homeowner U in each period. Also assume that people discount all future utilities using a constant discount rate β. Then the price of a home should satisfy

$$P = \sum_{t=0}^{\infty} \beta^t (R + U) = \frac{R + U}{1 - \beta}. \tag{3.1}$$

Price to rent ratio can be written as

$$\frac{P}{R} = \frac{R + U}{R(1 - \beta)} = \frac{1}{1 - \beta}\left(1 + \frac{U}{R}\right). \tag{3.2}$$

Thus price to rent ratio is a proxy for utility of being a homeowner per unit of housing service.

I use the Zillow zip code level price to rent ratio index. The index starts coverage in 2010. I use the average of the price to rent ratio between 2010 and 2015 as the zip code level proxy. Price to rent ratio is highly persistent. The correlation between the price rent ratios in any two periods between 2011 and 2015 is above 0.97. Thus I assume that the average price to rent ratio between 2010 and 2015 is a reasonable proxy for the price to rent ratio in my estimation sample period, between 2003 and 2015.

Unemployment Rates. To control for local labor market conditions, I use Metropolitan Statistical Area and Micropolitan Statistical Area unemployment rates from the Bureau of Labor Statistics (BLS) Local Area Unemployment Statistics series.

Long-run House Price Cyclicality. I use the long-run house price cyclicality instrument σ_g^P constructed by Palmer (2015), defined as the standard deviation of monthly changes in the CoreLogic repeat sales home price index from 1980-1995

$$\sigma_g^P \equiv \left(\frac{1}{T-1} \sum_{t \in \mathcal{T}} (\Delta HPI_{gt}) - \overline{\Delta HPI}_g)^2 \right)^{1/2}.$$

3.3 The Impact of Expected Home Prices on Default

In this section, I study how expected home prices affect probability of default. I first present results treating prices as exogenous. Then I explore how using the long-run home price cyclicality instrument affects my estimates. Finally I discuss the economic magnitude of the estimates.

Results Treating Prices as Exogenous

Observations in my sample are monthly default decisions and are thus in discrete time. Following Palmer (2015), I specify borrowers' default timing as a proportional hazard model in continuous time and then estimate it in discrete time using complementary log-log regression. Let $\lambda_{icg}(t)$ denote the latent instantaneous default probability for individual i from cohort c, location g with loan age t months. In a proportional hazard model, $\lambda_{icg}(t)$ is specified as

$$\begin{aligned} \lambda_{icg}(t) &\equiv \lim_{xi \to 0} \frac{\Pr\left(t - \xi < \tau \leq t | \tau > t - \xi, X\right)}{\xi} \\ &= \exp\left(X'_{icgt}\beta\right) \lambda_0(t), \end{aligned} \qquad (3.2)$$

where $\lambda_0(t)$ is the nonparametric baseline hazard function. Controls X_{icgt} include loan characteristics, borrowers characteristics, price to rent ratio and unemployment rate interacted with FICO score quantiles. The variables of interest are log home price changes in the past 12 months and in

Table 3.1: Summary Statistics

	mean	sd	min	max
CreditScore	620.83	60.37	395.00	850.00
DTI Missing	0.86	0.34	0.00	1.00
dti	4.84	13.40	0.00	96.75
DTI Missing	0.86	0.34	0.00	1.00
Owner Occupied	0.91	0.29	0.00	1.00
Combined LTV	0.82	0.14	0.00	1.96
OriginalInterestRate	7.83	1.57	0.99	21.75
Full Documentation	0.57	0.49	0.00	1.00
Cash Out Refinance	0.53	0.50	0.00	1.00
Adjustable Rate	0.54	0.50	0.00	1.00
Interest Only	0.13	0.34	0.00	1.00
Balloon	0.09	0.28	0.00	1.00
Has Second Lien	0.13	0.34	0.00	1.00
default	0.14	0.34	0.00	1.00
prepay	0.49	0.50	0.00	1.00
censor	0.37	0.48	0.00	1.00
Observations	254349			

the following 36 months interacted with FICO score quantile , *i.e.*, the covariates are

$$
\begin{aligned}
X'_{icgt}\beta &= \theta'_B W_i^{borrower} + \theta'_L W_i^{loan} + \delta_1 \Delta HPI_{t-12,t} \times 1_{\text{FICO quantile}} \\
&\quad + \delta_2 \Delta HPI_{t,t+36} \times 1_{\text{FICO quantile}} + \delta_3 \text{Unemployment}_{g,t} \times 1_{\text{FICO quantile}} \\
&\quad + \gamma_c + \alpha_g + \theta_{PR} \text{Price Rent Ratio} \\
\Delta HPI_{t-12,t} &= \log(HPI_{gt}) - \log(HPI_{g,t-1\ year}) \\
\Delta HPI_{t,t+36} &= \log(HPI_{g,t+3\ years}) - \log(HPI_{g,t})
\end{aligned}
$$

Assuming that the time varying covariates X_{icgt} are constants in each discrete time interval $[t-1, t]$, the continuous time model in (3.2) can be transformed to a discrete time model. In greater details, let $S(t)$ denote the survivor function and let $\Lambda(t) = -\log(S(t))$. $\Lambda(t)$ is also called the integrated hazard function because it satisfies the familiar identity

$$
\Lambda(t) = \int_0^t \lambda(\tau)d\tau \tag{3.-3}
$$

Using $\Lambda(t) = -\log(S(t))$ and identity (3.-3), the probability of survival between $t - 1$ and t conditional on that one survived the first $t - 1$ periods is

$$
\begin{aligned}
\Pr(\tau > t | \tau > t - 1) &= \frac{S(t)}{S(t - 1)} \\
&= \exp(\Lambda(t - 1) - \Lambda(t)) \\
&= \exp\left(-\int_{t-1}^{t} \lambda(\tau)d\tau\right)
\end{aligned}
$$

In general $\lambda(\tau)$ depends on $X_{icg\tau}$, which is time varying between $t - 1$ and t. Assuming that $X_{icg\tau}$ are constants when τ is between $t - 1$ and t. We have

$$
\begin{aligned}
\Pr(\tau > t | \tau > t - 1) &= \frac{S(t)}{S(t - 1)} \\
&= \exp(\Lambda(t - 1) - \Lambda(t)) \\
&= \exp\left(-X'_{icgt}\beta\right)\frac{1}{\lambda_0(t)},
\end{aligned}
$$

which is the complementary log-log model I estimate in discrete time.

Table 3.2 reports the estimation result of the proportional hazard model (3.2). All loan and borrower characteristics have the intuitive signs. Mortgages defaulted with higher probability if they were adjustable rate mortgages, had higher CLTVs or interest rates, or lacked full documentation. The signs on the unemployment rate interacted with credit score quantiles are counterintuitive. A one percentage point increase in unemployment rate decreases default hazard by roughly 1% to 5%. Also from the interactions between unemployment and FICO score quantile, a higher unemployment rate particularly lowers the default probability of low credit score borrowers compared to high credit score borrowers. This is somewhat different from what I expected.

Column 1 includes past price shock $\log(HPI_t)$ - $\log(HPI_{t-1\,year})$. A 1% larger home price decline in the past 12 months increases the default probability by 3.7%.

Column 2 adds the variable of interest, future home price growth or $\log(HPI_{t+3\,years})$ - $\log(HPI_t)$. The negative coefficient in front of future home price growth means a higher future home price lowers the default probability in the observation month. This supports the view that borrowers are strategic. For two identical borrowers who took out the same mortgage and also experienced the same past price shock (controlled by $\log(HPI_t)$ - $\log(HPI_{t-1\,year})$), the one with higher future home price growth ($\log(HPI_{g,t+3\,years}) - \log(HPI_{g,t})$) defaults with lower probability. This is in line with the story that high expected home price growth gives borrowers incentive to stay current on their mortgages. When expected home price growth is low, borrowers are more likely to default strategically to take advantage of the low future home price. The magnitude of the coefficient means that a 1% higher future home price lowers the default probability by 0.4%.

I further explore which subgroup of borrowers are more likely to react to expected future home prices. This is motivated by the facts that many people find strategic defaults immoral (Guiso et al. (2013)), and government subsidies aim to screen out strategic defaulters (FHA (2013)). Are strategic defaulters bad borrowers? or they are sophisticated borrowers who, in normal times, behave

Table 3.2: Future Price Changes and Default Probability

Unemployment	-0.039**	-0.017	-0.036**
	(0.017)	(0.014)	(0.015)
Unemployment $\times 1\_FICO \leq 560$	-0.125***	-0.130***	-0.092***
	(0.017)	(0.017)	(0.019)
Unemployment $\times 1\_560 < FICO \leq 620$	-0.075***	-0.078***	-0.046***
	(0.011)	(0.011)	(0.012)
Unemployment $\times 1\_620 < FICO \leq 680$	-0.035***	-0.037***	-0.025***
	(0.007)	(0.007)	(0.009)
Price Rent Ratio	-0.007***	-0.006***	-0.006***
	(0.002)	(0.002)	(0.002)
$\Delta HPI\_t - 12, t$	-2.900***	-2.887***	-3.784***
	(0.165)	(0.167)	(0.285)
$\Delta HPI\_t, t + 36$		-0.292***	-0.409***
		(0.099)	(0.117)
$\Delta HPI\_t - 12, t \times 1\_FICO \leq 560$			1.466***
			(0.297)
$\Delta HPI\_t - 12, t \times 1\_560 < FICO \leq 620$			1.242***
			(0.236)
$\Delta HPI\_t - 12, t \times 1\_620 < FICO \leq 680$			0.832***
			(0.192)
$\Delta HPI\_t, t + 36 \times 1\_FICO \leq 560$			0.472***
			(0.121)
$\Delta HPI\_t, t + 36 \times 1\_560 < FICO \leq 620$			0.316***
			(0.102)
$\Delta HPI\_t, t + 36 \times 1\_620 < FICO \leq 680$			-0.110*
			(0.066)
Obs	5058817	5058804	5058804
CBSA FE	y	y	y
Cohort FE	y	y	y
Loan Characteristics	y	y	y
Borrower Characteristics	y	y	y

Notes: Standard errors are in parentheses and are clustered by cbsa. * significant at 10 percent; ** significant at 5 percent; *** significant at 1 percent.

better than others? To study this, column 3 in Table 3.2 adds interactions between price changes and credit score quantiles. Borrowers are first divided into 4 groups according to their FICO scores: lower than 560, between 560 and 620, between 620 and 680, and above 680. 560 is around 25 percentile and 680 is around 75 percentile. The omitted group in column 3 is high credit score borrowers, with FICO scores above 680. The coefficient for $\Delta HPI_{t,t+36} \times I_{FICO \leq 560}$ is positive. This means that, borrowers with FICO scores lower than 560 are less likely to strategically react to expected future home price compared to high credit score borrowers. In fact the total effect of future home price growth on default probability of low credit score borrowers has the counterintuitive sign. The negative sum of coefficients for $\Delta HPI_{t,t+36}$ and $\Delta HPI_{t,t+36} \times I_{FICO \leq 560}$ suggests that low credit score borrowers default slightly more often when future home price is higher. These

results suggest that high credit score borrowers are more likely to engage in strategic defaults.

In column 3, I also control for interactions between $\Delta HPI_{t-12,t}$ and credit score quantiles. They show a similar pattern as in interactions between $\Delta HPI_{t,t+36}$ and credit score quantiles. Low FICO score borrowers are less likely to react to price changes in the last 12 months than high FICO score borrowers.

Notice that the patterns in interactions between price changes and FICO score quantiles are almost monotonic. Borrowers with FICO scores below 560 react less to price changes than borrowers with FICO scores between 560 and 620, who in turn react less than borrowers with FICO scores between 620 and 680. The only exception is that borrowers with FICO scores between 620 and 680 respond more to future home price growth than borrowers with FICO scores above 680.

I further explore if the above higher FICO score more reaction pattern holds in each cohort. Table 3.3 reports results from estimating the specification in column 3 in Table 3.2 on each cohort. The pattern that high FICO score react more to prices holds partially but does not hold perfectly in every cohort. For example, in the 2005 cohort, borrowers in the highest FICO quantile react least, instead of reacting most, to future home price growth. However all other FICO quantiles in the 2005 cohort follow the higher FICO score more reaction pattern.

In the Table 3.2, I assumed 3 years as the cooling off period and used $\Delta HPI\_t, t + 36$ as the future home price change variable. Table 3.4 repeats the specifications in Table 3.2, using $\Delta HPI\_t, t + 48$ as the future price change variable. We can see that the coefficients have the same signs and similar magnitudes as in Table 3.2.

Nonlinear Instrument Approach

As discussed above, there are plausible stories that the estimates in Table 3.2 are not causal. For instance, lower future home prices could be the result of more defaults, not the reason why borrowers defaulted more frequently ex-ante. I use the nonlinear instrument approach as in Palmer (2015). The instrument set for the future price change variable is the long-run cyclicality measure σ_g^P interacted with the calendar-month indicator variables. The first stage is

$$\Delta \log(HPI_{igct}) = \sum_s \pi_s \sigma_g^P 1(s = t + t_0(i)) + Z'_{2,icgt}\pi_2 + v_{icgt}, \tag{3.-8}$$

where $Z_{2,icgt}$ includes the same covariates as in the second stage. Palmer (2015) discussed the first stage and exclusion restriction for the long-run home price cyclicality instrument in great details. In summary, the instrument is highly correlated with price changes in the 2000s. It is uncorrelated with credit expansion but correlated with cyclicality of unemployment rate. CBSAs with more cyclical housing markets tend to have more cyclical unemployment rates. Thus I control for unemployment rate in all specifications.

Then I take the control function approach to estimate the second stage. Essentially, I estimate the hazard model (3.2) adding one more control variable, the estimated residual $\hat{v}_{icgt}$ from (3.-8)

Table 3.3: Effect of Future Price Changes by Cohort

$\Delta HPI\_t-12, t$	-0.615	-4.989***	-4.108***	-2.023***	-1.374***
	(1.547)	(0.902)	(0.349)	(0.393)	(0.498)
$\Delta HPI\_t, t+36$	0.469	-0.025	-0.190	-0.245	-0.166
	(0.451)	(0.310)	(0.139)	(0.149)	(0.217)
$\Delta HPI\_t-12, t \times 1\_FICO \leq 560$	-0.587	3.499***	2.054***	1.984***	0.152
	(1.303)	(0.914)	(0.398)	(0.388)	(0.620)
$\Delta HPI\_t-12, t \times 1\_560 < FICO \leq 620$	-0.371	2.064**	1.977***	1.410***	-0.024
	(1.431)	(0.848)	(0.295)	(0.374)	(0.553)
$\Delta HPI\_t-12, t \times 1\_620 < FICO \leq 680$	-0.926	0.796	1.054***	0.913***	-0.049
	(1.279)	(0.774)	(0.335)	(0.310)	(0.566)
$\Delta HPI\_t, t+36 \times 1\_FICO \leq 560$	-0.024	0.132	-0.105	-0.502***	0.131
	(0.517)	(0.339)	(0.187)	(0.123)	(0.377)
$\Delta HPI\_t, t+36 \times 1\_560 < FICO \leq 620$	-0.126	0.046	-0.189	-0.063	-0.185
	(0.485)	(0.298)	(0.152)	(0.097)	(0.182)
$\Delta HPI\_t, t+36 \times 1\_620 < FICO \leq 680$	0.163	-0.002	-0.428***	-0.253***	-0.646***
	(0.499)	(0.277)	(0.126)	(0.086)	(0.215)
Unemployment	-0.115*	0.014	0.089***	0.010	0.125***
	(0.069)	(0.058)	(0.024)	(0.016)	(0.035)
Unemployment $\times 1\_FICO \leq 560$	0.001	0.010	-0.028	0.024*	-0.013
	(0.052)	(0.045)	(0.022)	(0.014)	(0.021)
Unemployment $\times 1\_560 < FICO \leq 620$	0.042	0.015	-0.000	0.031***	-0.006
	(0.045)	(0.034)	(0.016)	(0.011)	(0.015)
Unemployment $\times 1\_620 < FICO \leq 680$	0.077*	0.024	-0.011	0.021*	-0.006
	(0.041)	(0.026)	(0.013)	(0.011)	(0.014)
Price Rent Ratio	-0.013***	-0.009***	-0.003	-0.002	0.003
	(0.003)	(0.002)	(0.003)	(0.002)	(0.003)
Obs	687536	1085086	1393921	1260993	266902
CBSA FE	y	y	y	y	y
Cohort FE	y	y	y	y	y
Loan Characteristics	y	y	y	y	y
Borrower Characteristics	y	y	y	y	y

Notes: Standard errors are in parentheses and are clustered by cbsa. * significant at 10 percent; ** significant at 5 percent; *** significant at 1 percent.

*i.*e., the covariates are

$$
\begin{aligned}
X'_{icgt}\beta &= \theta'_B W_i^{borrower} + \theta'_L W_i^{loan} + \delta_1 \Delta HPI_{t-12,t} \times 1_{\text{FICO quantile}} \\
&+ \delta_2 \Delta HPI_{t,t+36} \times 1_{\text{FICO quantile}} + \delta_3 \text{Unemployment}_{g,t} \times 1_{\text{FICO quantile}} \\
&+ \gamma_c + \alpha_g + \theta_{PR}\text{Price Rent Ratio} + \kappa\hat{v}_{icgt}
\end{aligned}
$$

Table 3.5 reports the second stage results. The magnitude of coefficient $\Delta HPI\_t, t+36$ is larger than the magnitude treating prices as exogenous. A 1% higher future home price triggers 0.8% more defaults. The following section studies the economic magnitudes of these coefficients using two counterfactual price paths.

Table 3.4: Effect of Future Price Changes in 4 Years

Unemployment $\times 1\_FICO \leq 560$	-0.115***	-0.124***	-0.069***
	(0.019)	(0.020)	(0.023)
Unemployment $\times 1\_560 < FICO \leq 620$	-0.069***	-0.075***	-0.028*
	(0.013)	(0.013)	(0.016)
Unemployment $\times 1\_620 < FICO \leq 680$	-0.027***	-0.030***	-0.012
	(0.008)	(0.008)	(0.011)
Price Rent Ratio	-0.008***	-0.006**	-0.005**
	(0.002)	(0.002)	(0.002)
$\Delta HPI\_t - 12, t$	-3.605***	-3.772***	-4.922***
	(0.192)	(0.187)	(0.333)
Unemployment	-0.074***	-0.034*	-0.066***
	(0.021)	(0.019)	(0.021)
$\Delta HPI\_t - 12, t \times 1\_FICO \leq 560$			2.066***
			(0.345)
$\Delta HPI\_t - 12, t \times 1\_560 < FICO \leq 620$			1.635***
			(0.290)
$\Delta HPI\_t - 12, t \times 1\_620 < FICO \leq 680$			0.916***
			(0.223)
$\Delta HPI\_t, t + 48$		-0.443***	-0.563***
		(0.108)	(0.126)
$\Delta HPI\_t, t + 48 \times 1\_FICO \leq 560$			0.375***
			(0.105)
$\Delta HPI\_t, t + 48 \times 1\_560 < FICO \leq 620$			0.282***
			(0.083)
$\Delta HPI\_t, t + 48 \times 1\_620 < FICO \leq 680$			-0.042
			(0.061)
Obs	4345401	4345386	4345386
R squ			
CBSA FE	y	y	y
Cohort FE	y	y	y
Loan Characteristics	y	y	y
Borrower Characteristics	y	y	y

Notes: Standard errors are in parentheses and are clustered by cbsa. * significant at 10 percent; ** significant at 5 percent; *** significant at 1 percent.

Economic Magnitude

In this section, I construct two counterfactual price paths to show the economic magnitudes of coefficients. The idea is to construct two price paths with the same history up to May 2012 and different history between May 2012 and May 2015, one with high home price growth and the other one with low home price growth. Again, the reason why I choose May 2012 as the turning point is that the variable of interest is future price change in the following 3 years. May 2012 is the last month when this variable is available. I then consider two borrowers who took out the same mortgage in December 2007 (the last cohort in my sample). One borrower faces the high price

Table 3.5: Effect of Future Price Changes, Nonlinear IV Approach

$\Delta HPI\_t-12,t$	-2.796***
	(0.161)
$\Delta HPI\_t,t+36$	-0.770**
	(0.341)
Unemployment	0.020
	(0.030)
Unemployment $\times 1\_FICO \leq 560$	-0.139***
	(0.016)
Unemployment $\times 1\_560 < FICO \leq 620$	-0.082***
	(0.010)
Unemployment $\times 1\_620 < FICO \leq 680$	-0.040***
	(0.006)
Price to Rent Ratio	-0.006***
	(0.002)
Residuals	0.416
	(0.416)
Obs	3687232
R squ	
CBSA FE	y
Cohort FE	y
Loan Characteristics	y
Borrower Characteristics	y

Notes: Standard errors are in parentheses and are clustered by cbsa. * significant at 10 percent; ** significant at 5 percent; *** significant at 1 percent.

growth path and the other one faces the low price growth path. I compare the default probabilities of the two borrowers before May 2012, a period when they shared the same past price experience but had different expected prices in the future.

The blue curve plotted in Figure 3.2 is the Zillow national home price index between December 2007 and May 2015. Between May 2012 and May 2015, the national index increased by 17%. The standard deviation of zipcode level home price change between May 2012 and May 2015 is 16%. The high growth and low growth paths plotted in Figure 3.2 are constructed by:

$$HPI_{\text{high growth},t} - HPI_{\text{high growth},t-1} = \frac{17\% + \frac{1}{2}16\%}{17\%}\left(HPI_{\text{national},t} - HPI_{\text{national},t-1}\right)$$
$$HPI_{\text{low growth},t} - HPI_{\text{low growth},t-1} = \frac{17\% - \frac{1}{2}16\%}{17\%}\left(HPI_{\text{national},t} - HPI_{\text{national},t-1}\right).$$

such that the price change between May 2012 and May 2015 in the high growth path is one half standard deviation above the national price change.

The loan and borrower characteristics for the two hypothetical borrowers are set to be the average values in the estimation sample. This means that for the binary characteristic variables,
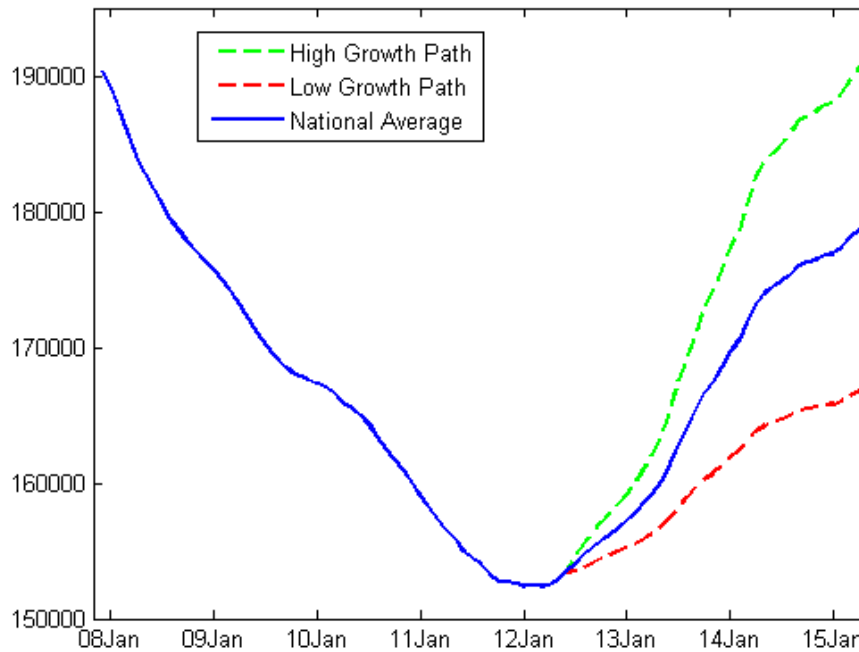
Figure 3.2: National Index and Two Counterfactual Price Paths

for example fixed rate or adjustable rate, the characteristics for the two hypothetical borrowers are non-integers.

I first consider the coefficients in Table 3.2 treating prices as exogenous. Figure 3.3 plots

$$\frac{\Pr\left(\text{Default in month } t | \text{Survived the first } t-1 \text{ months, low growth path}\right)}{\Pr\left(\text{Default in month } t | \text{Survived the first } t-1 \text{ months, high growth path}\right)} \tag{3.-12}$$

This ratio is equal to 1 in the first 16 months because both past price changes and future price changes in the following 3 years are identical for the two paths in the first 16 months. After the 17th month, the borrower facing low growth path starts to default with higher probabilities. In the 52nd month, the ratio is about 1.1.

Figure 3.4 plots the counterfactual cumulative default probabilities. There is almost no difference between the two paths. By the 52nd period, the low growth path has a cumulative default probability of 28.5%, slightly higher than 27.8% from the high growth path.

Figure 3.5 and Figure 3.6 plot the default probability ratio and cumulative default probabilities using the coefficients from the instrument variable approach. The default probability ratio reaches 1.15 in the 52nd, month. By the 52nd period, the low growth path has a cumulative default probability of 29.2%, slightly higher than 28.2% from the high growth path.
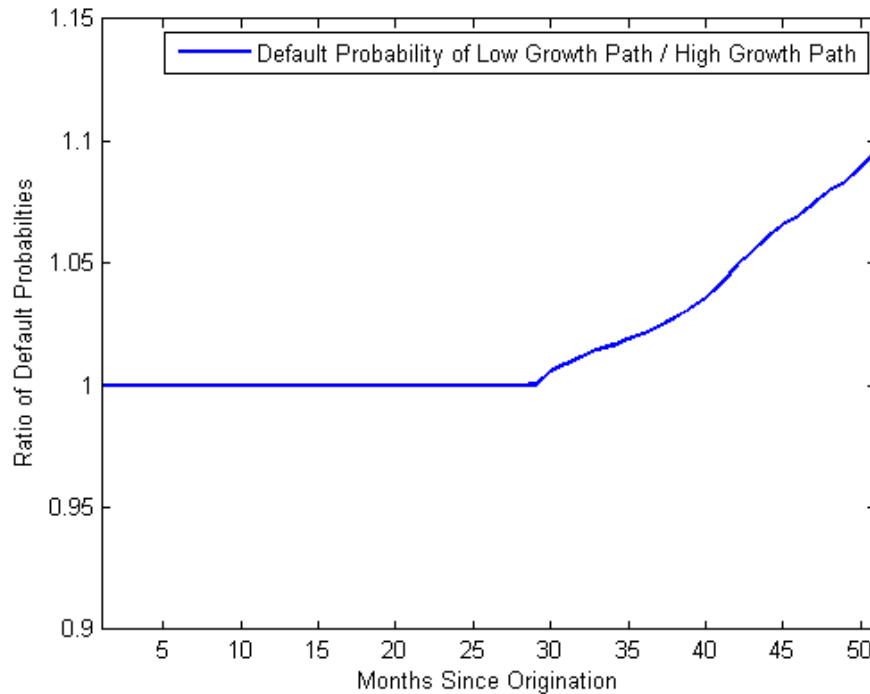
Figure 3.3: Ratio of Default Probabilties

3.4   Effects of Changing the Cooling off Period

Results in the previous section suggests that people do react to penalties from the cooling off period. As argued in Section 3.1, the cooling off period is a flexible policy instrument because it can be changed timely and different groups of borrowers can have different cooling off periods. In this section, we build a model to study the effect of changing the cooling off period.

Figure 3.7 summarizes the agents in the model. There are three types of sellers of homes: normal sellers, strategic defaulters and liquidity defaulters. Strategic defaulters are borrowers who defaulted when it is optimal for them to default. Liquidity defaulters were borrowers who were underwater and received liquidity shocks. These double triggers made them involuntarily default. After the cooling off period, defaulters come back to the housing market.

There are two effects from a longer cooling-off period. A longer cooling off period makes the default penalty harsher, which would deter some borrowers from strategic defaults. On the other hand, when borrowers do default, under a longer cooling-off period, they are locked out from the housing market longer. Locking out buyers depresses home prices longer, which makes more borrowers underwater and default when they receive liquidity shocks. The two effects work in two opposite ways: less strategic defaults and larger impact of each default, including more liquidity defaults. As we will show, the net effect also depends on the expected home price growth. In other words, which effect dominates depends on the local expected home price growth and varies by
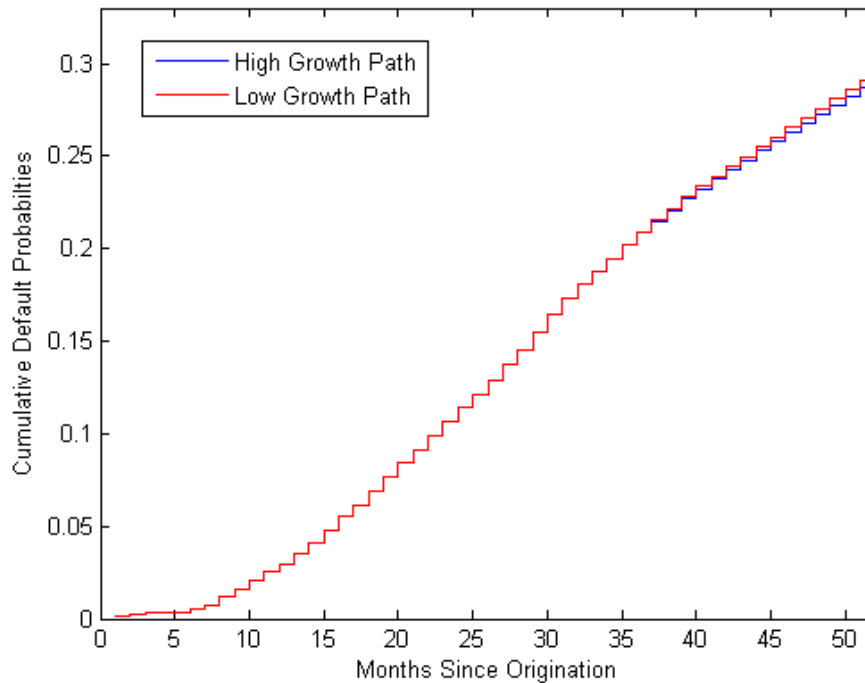
Figure 3.4: Counterfactual Default Probabilties

cities.

Discount rate is assumed to be β. Rent is omitted to be zero thus home prices are premium that people are willing to pay for being homeowners. Figure 3.8 shows demand and supply in a typical period. The blue curve is the downward sloping demand curve. The supply curve is also downward sloping because when home prices are lower, more borrowers are underwater and will default if they receive a liquidity shock. Thus the lower the home prices, the more REO homes are for sale. A percentage of the population are potential strategic defaulters. They strategically choose to default based on the the loss from the cooling off period and the gain from not having to pay for the remaining balance.

Figure 3.9 shows the demand, supply and normalized prices in one simulation. At time zero, there is a downward demand shock due to credit tightening. Price first fell and then rebounded and stabilized to a new long run level.

Figure 3.10 shows a similar simulation. The difference between Figure 3.10 and Figure 3.9 is that Figure 3.10 has increasing demand and price trends.

Figure 3.11 shows the effects of a longer cooling off period in constant and increasing demand markets. As the length of the cooling off period increases, there are less strategic default and more liquidity defaults. In a market with increasing demand, few people strategically default thus a longer cooling off period mainly increases liquidity defaults. In a market with constant demand, the more liquidity defaults from a long cooling off period is partially offset by the less strategic
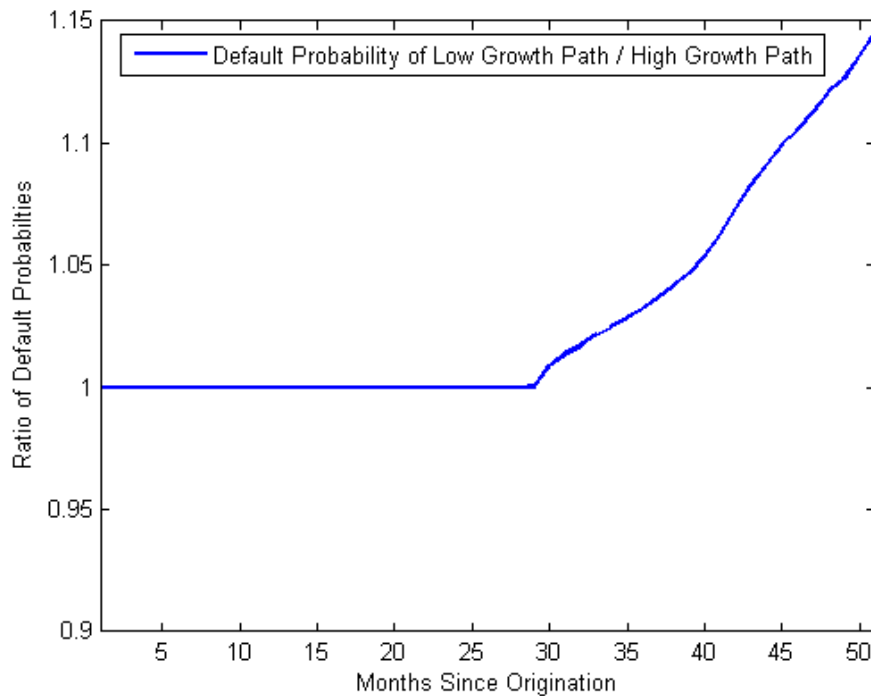
Figure 3.5: Ratio of Default Probabilties

defaults.

3.5 Conclusion

I study if future home prices affect borrowers' default decisions. I found that a higher future home price lowers default probability. High credit score borrowers are more likely to respond more to both past and future price changes than low credit score borrowers. The economic magnitudes are not large. A model is built to study the effect of a longer cooling off period. A longer cooling off period reduces strategic defaults and increases liquidity defaults. In high expected home price appreciation areas, the more liquidity defaults channel dominates. In low expected home price appreciation areas, the more liquidity defaults are partially offset by less strategic defaults.
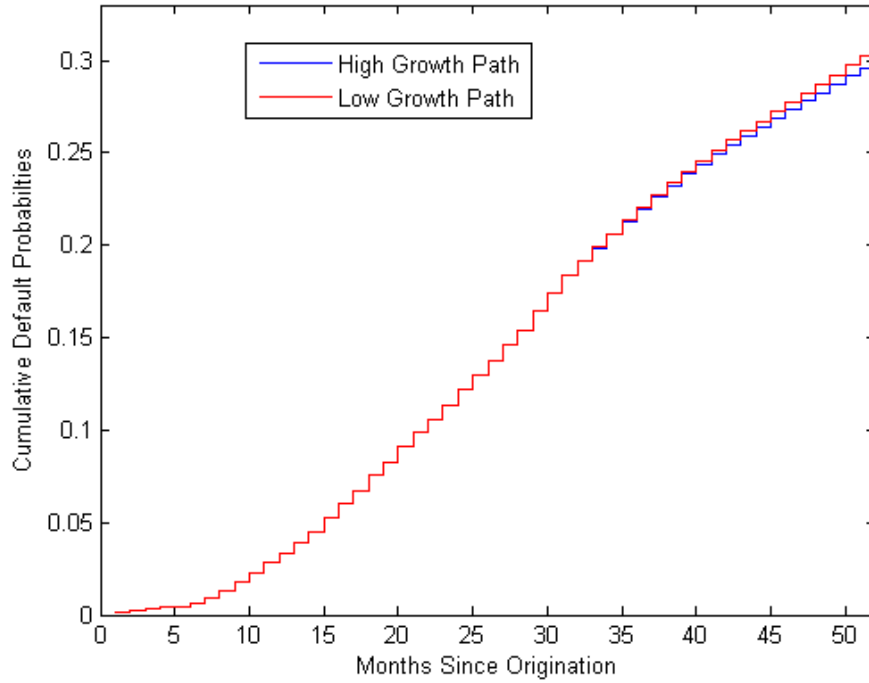
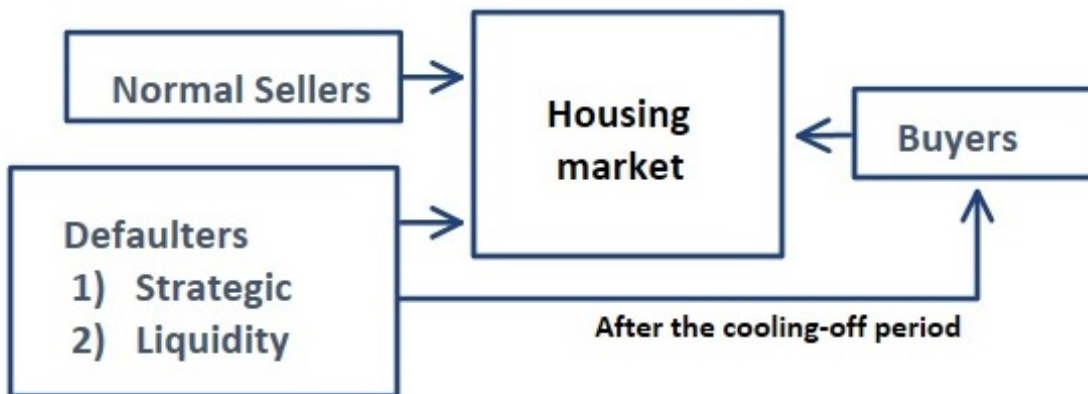Figure 3.6: Counterfactual Default Probabilties
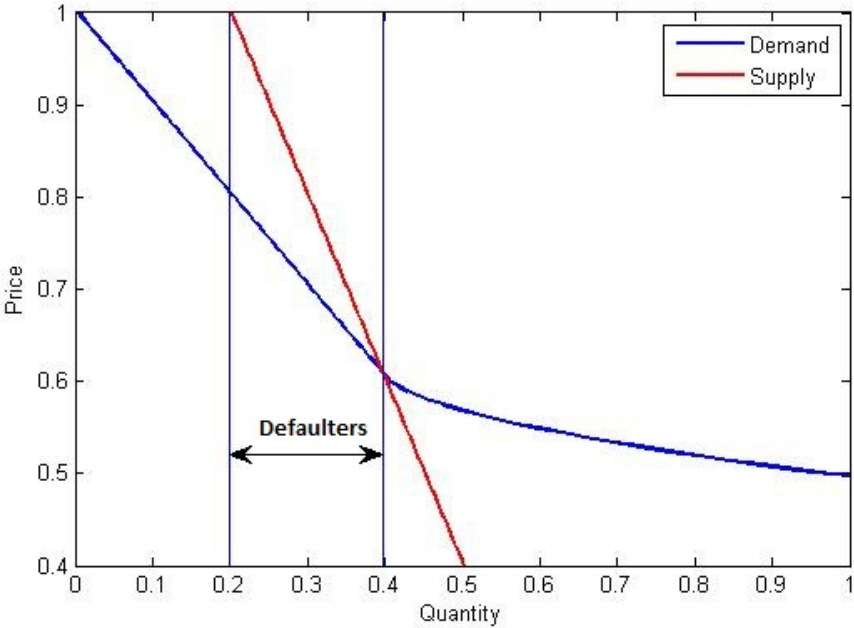

Figure 3.7: Agents in the Model

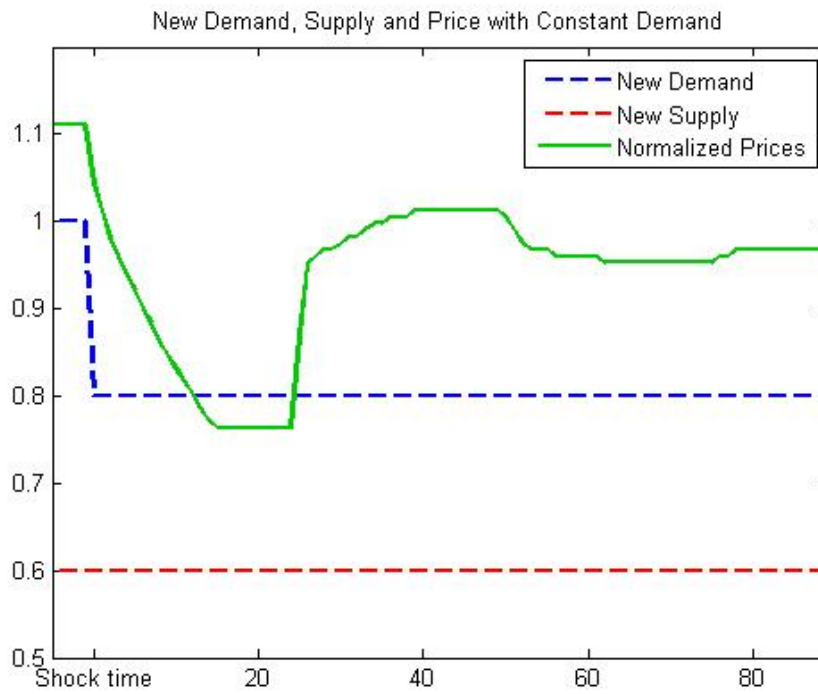Figure 3.8: Demand and Supply in a Typical Period

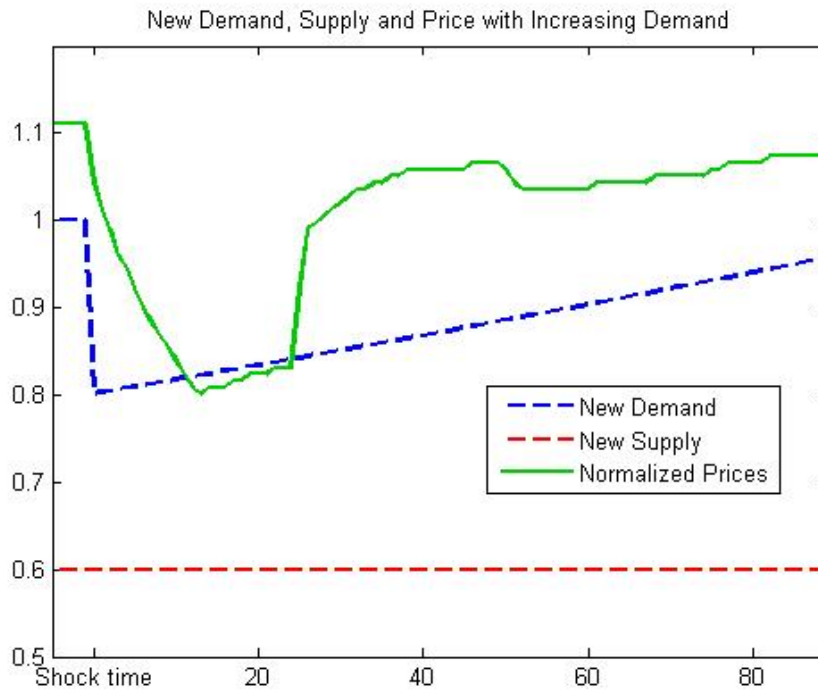Figure 3.9: Prices under Constant Demand

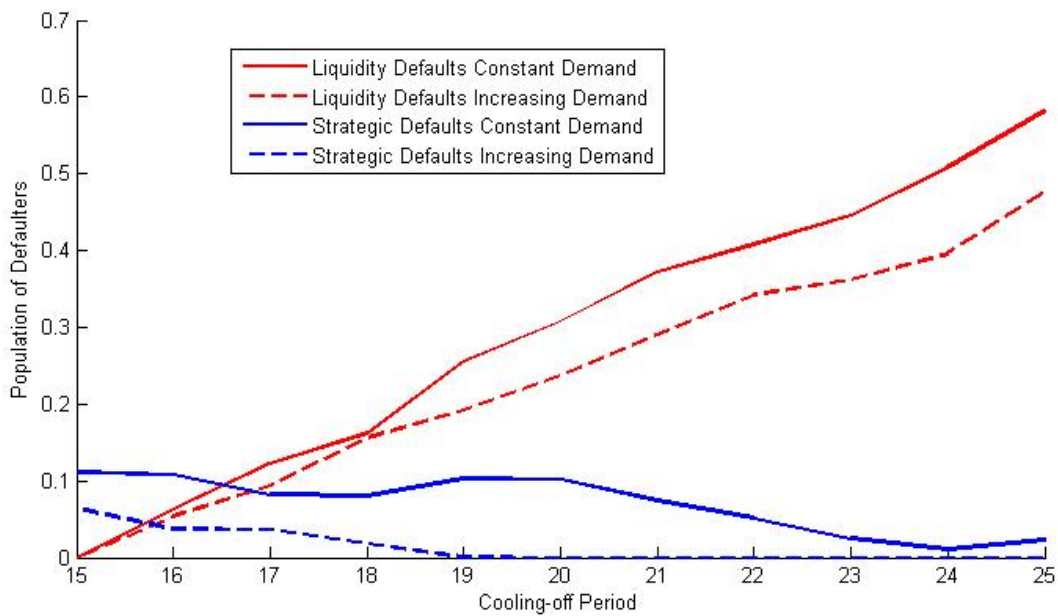Figure 3.10: Prices under Increasing Demand

Figure 3.11: Effects from a Longer Cooling-off Period

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