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Authors
Albergo, S.
Bieser, F.
Brady, F.P.
et al.

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Andrew S. Hirsch
Department of Physics, Purdue University
W. Lafayette, Indiana 47907

and

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IS THERE EVIDENCE FOR A LIQUID-GAS PHASE TRANSITION IN NUCLEAR MATTER?

ANDREW S. HIRSCH
Department of Physics, Purdue University
W. Lafayette, Indiana 47907, USA

and

S. ALBERGO[2], F. BIESER[6], F. P. BRADY[3], Z. CACCIA[2], D. A. CEBRA[3], A. D. CHACON[7], J. L. CHANCE[3], Y. CHOI[1], S. COSTA[2], J. B. ELLIOTT[1], M. L. GILKES[1], J. A. HAUGER[1], E. L. HJORT[1], A. INSOLIA[2], M. JUSTICE[5], D. KEANE[5], J. C. KINTNER[3], V. LINDENSTRUTH[4], M. A. LISA[6], U. LYNEN[4], H. S. MATIS[6], M. MCMAHAN[6], C. MCPARLAND[6], W. F. J. MÜLLER[4], D. L. OLSON[6], N. T. PORILE[1], R. POTENZA[2], G. RAI[6], J. RASMUSSEN[6], H. G. RITTER[6], J. ROMANSKI[2], J. L. ROMERO[3], G. V. RUSSO[2], H. SANN[4], R. SCHARENBERG[1], A. SCOTT[5], Y. SHAO[5], B. K. SRIVASTAVA[1], T. J. M. SYMONS[6], M. TINCKNELL[1], C. TUVÉ[2], S. WANG[5], P. WARREN[1], H. H. WIEMAN[6], and K. WOLF[7]

[1] *Purdue University, W. Lafayette, IN 47907*
[2] *Universita di Catania and Istituto Nazionale di Fisica Nucleare-Sezione di Catania, 95129 Catania, Italy*
[3] *University of California, Davis, CA 95616*
[4] *GSI, D-64220 Darmstadt, Germany*
[5] *Kent State University, Kent, OH 44242*
[6] *Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720*
[7] *Texas A and M University, College Station, TX 77843*
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ABSTRACT

The multifragmentation of gold nuclei at 1 GeV/nucleon has been studied using reverse kinematics. The moments of the resulting charged fragment distribution have been analyzed using methods borrowed from percolation theory. These moments provide clear evidence for critical behavior occurring in a system of about 200 nucleons. The critical exponents extracted from the data are close to those of liquid-gas systems.

1. Introduction

The breakup of large nuclei having excitation energies comparable to their total binding energy is known as nuclear fragmentation. It has long been known[1] that under suitable conditions a broad range of nuclei can result from such a breakup. Among the most hotly debated questions regarding this process is whether or not it involves critical behavior. Density fluctuations occurring in the neighborhood of a critical point would provide a natural explanation for the observed power law dependence of the fragment mass (or charge) yield[2]. In this paper, I present some recent results from a reverse kinematics multifragmentation experiment performed at the Lawrence Berkeley Bevalac by the EOS collaboration.

Charged particles resulting from 1 GeV/nucleon Au nuclei incident on a carbon target were identified using a time projection chamber (TPC)[3] for $1 \leq Z \leq 6$, a time-of-flight wall (TOF) for $7 \leq Z \leq 10$, and a multiple sampling ionization chamber (MUSIC)[4] for $11 \leq Z \leq Z_{beam}$. After all charged reaction products were identified, the number of nuclear fragments of each charge was determined. The total reconstructed charge, Z_{sum}, peaks at 79 with a full width at half maximum of 6. Only those events whose Z_{sum} was 79 ± 3 were selected for further analysis. Additional experimental details can be found in reference[5]. The analysis that follows is based on 9716 events that survive the above requirement.

2. Signals of Criticality

2.1. The k-Moments of the Distribution

For each event, we determine the multiplicity of charged fragments, m, and the number of charged fragments, n_z of nuclear charge Z. We then construct the k-moments of this distribution according to:

$$M_k(m) = \sum_Z Z^k n_Z(m) \tag{1}$$

Campi[6] was the first to suggest that the methods developed to study large percolation lattices may be relevant to the analysis of multifragmentation data. Bauer[7] also made significant observations regarding the applicability of the percolation analogy to nuclear fragmentation. In percolation theory the moments of the cluster distribution contain the signals for critical behavior[8]. Quantities that display divergent behavior in macroscopic systems still show a peaking behavior in systems containing one hundred or so constituents. In fact, it is well known in percolation theory how various quantities scale with system size[9]. In the analysis presented here, we have used the methods developed for determining percolation critical exponents to extract the critical exponents for nuclear matter from the moments of the fragment charge distributions.

We assume that m plays the same role as p in percolation. That is, the event multiplicity can be used as a linear measure of the distance from criticality. In a thermally driven phase transition, temperature would be the natural quantity with which to measure the distance from criticality. We will discuss this issue further in section 4.2.

We will refer to the region in m below the critical multiplicity, m_c, as the 'liquid' phase, and the region above m_c as the 'gas' phase. Following Stauffer[8], we will omit the biggest cluster (fragment), denoted Z_{max}, from the sum of Eq. 1 when we are on the liquid side of the phase transition. Physically, Z_{max} corresponds to the bulk liquid in an infinite system. The remaining clusters represent the uncondensed clusters in the gas. On the 'gas' side of the phase transition, there is no bulk liquid, and hence the Z_{max} is not omitted from the sum.

2.2. Critical Exponents

A macroscopic systems undergoing a continuous phase transition is characterized by a finite number of critical exponents. These exponents govern the singular behavior of various thermodynamic quantities and are the same for all systems of a given universality class[10]. The universality class is determined by the dimensionality of the space in which the system exists and the dimensionality of the order parameter. If we study large systems we find that:

$$M_2 \sim |\epsilon|^{-\gamma} \tag{2}$$

$$Z_{max} \sim |\epsilon|^{\beta} \tag{3}$$

$$n_Z \sim Z^{-\tau} \text{ for } m = m_c \tag{4}$$

where ϵ is the distance from criticality and is $p-p_c$, $T-T_c$, or $m-m_c$, for percolation, thermal systems, or nuclear fragmentation, respectively. The exponents are not all independent[11], since

$$\tau = 2 + \frac{\beta}{\beta+\gamma} . \tag{5}$$

In a fluid system, M_2 describes the isothermal compressibility which diverges at the critical point. In percolation it is the mean cluster size. In a fluid system the order parameter is the density difference between the liquid and gas phases. This quantity is nonzero only below the critical temperature and vanishes at the critical point. For the nuclear system, Z_{max} is the order parameter of phase transition. Eq. 4 describes the cluster distribution at the critical point. The singular behavior in each of Eqs. 2-4 is scale invariant, i.e. is given by a power law. Note also that the power law behavior holds not just at the critical point as in Eq. 4, but also in some neighborhood of the critical point as in Eqs. 2 and 3. Thus, if we can describe experimental data according to Eqs. 2-4, and the exponents so determined agree with those of a known universality class, and therefore necessarily obey Eq. 5, then we have gone a long way in demonstrating that critical behavior is present.

3. Determining Critical Exponents in Nuclear Multifragmentation

3.1. Finite Size Effects

In small systems the singular nature of Eqs. 2-4 is influenced by finite size effects. In percolation theory finite size effects can be offset by adjusting the value of p_c from its infinite lattice value until one obtains the power law behavior with the same value for γ in M_2 on both the liquid and gas sides of the phase transition. We have taken the same approach using multiplicity. We seek that value of m_c that gives the same value of γ for both the liquid ($m < m_c$) and gas ($m > m_c$) sides of Eq. 2. The value of γ can depend not only on the chosen m_c, but also on the region of ϵ used since finite-size distortions dominate as $\epsilon \rightarrow 0$, and signatures of critical behavior vanish for large ϵ, i.e., in the mean field regime.

3.2. The Method

The determination of the exponents was made by first selecting those values of m_c for which γ_{liquid} and γ_{gas} differed by no more than 10%. For each choice of m_c, the size of the fitted region on the liquid and gas sides of m_c were independently varied. Typically, at least 10 values of m were included in the individual fits to γ_{liquid} or γ_{gas} at a fixed m_c. An example of such a fit is shown in Fig. 1. If the difference between γ_{liquid} and γ_{gas} was 10% or less, this value of m_c was accepted. The distribution of all such values of m_c is peaked at 26. If we plot the difference between γ_{liquid} and γ_{gas} versus m_c, we see a minimum near 26. See Fig. 2. From Fig. 3 we see that when the values of γ_{liquid} and γ_{gas} are averaged and plotted versus m_c, they show little sensitivity to the choice of m_c when in the neighborhood of the minimum of Fig. 2. We have therefore taken m_c=26±1 for our subsequent analysis. In total there are 370 fitting regions that satisfy the gamma matching criterion for these m_c values.

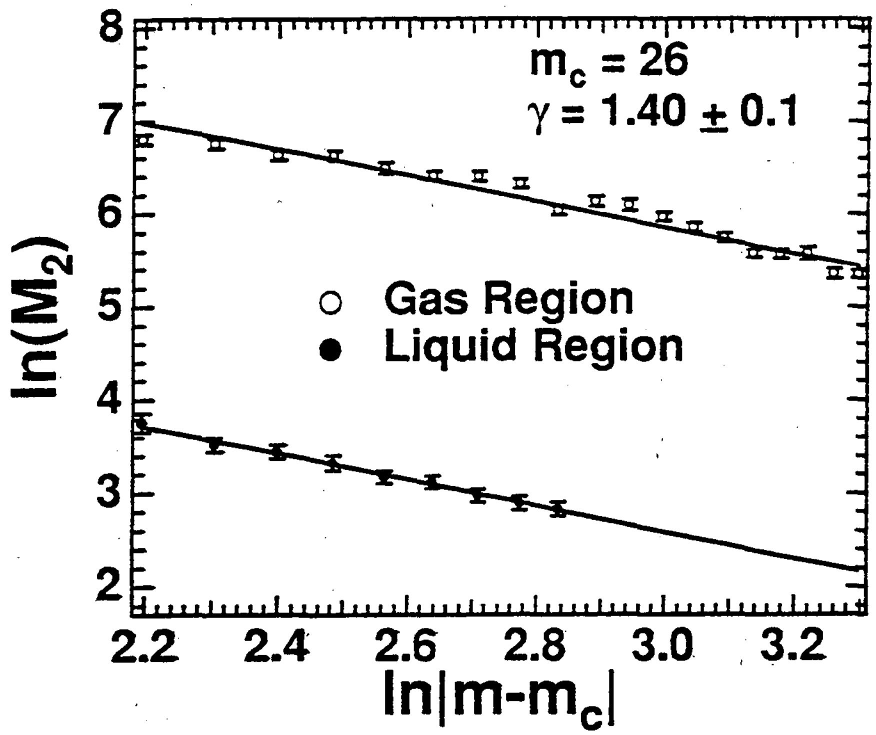

Figure 1. Example of the determination of the critical exponent γ for particular gas and liquid fitting regions.

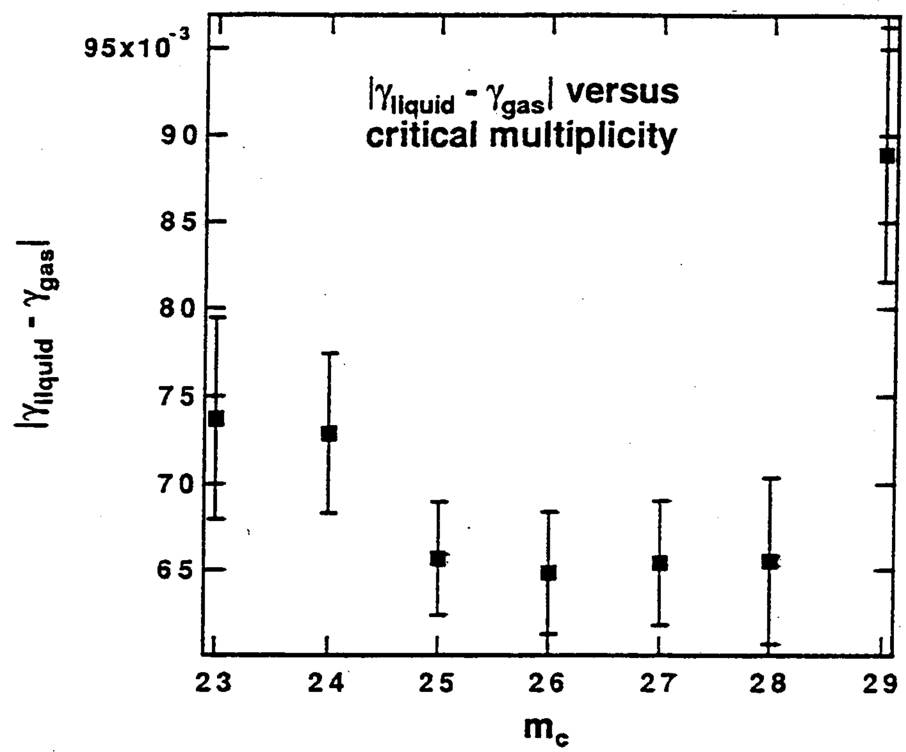

Figure 2. Difference between γ_{liquid} and γ_{gas} versus m_c.

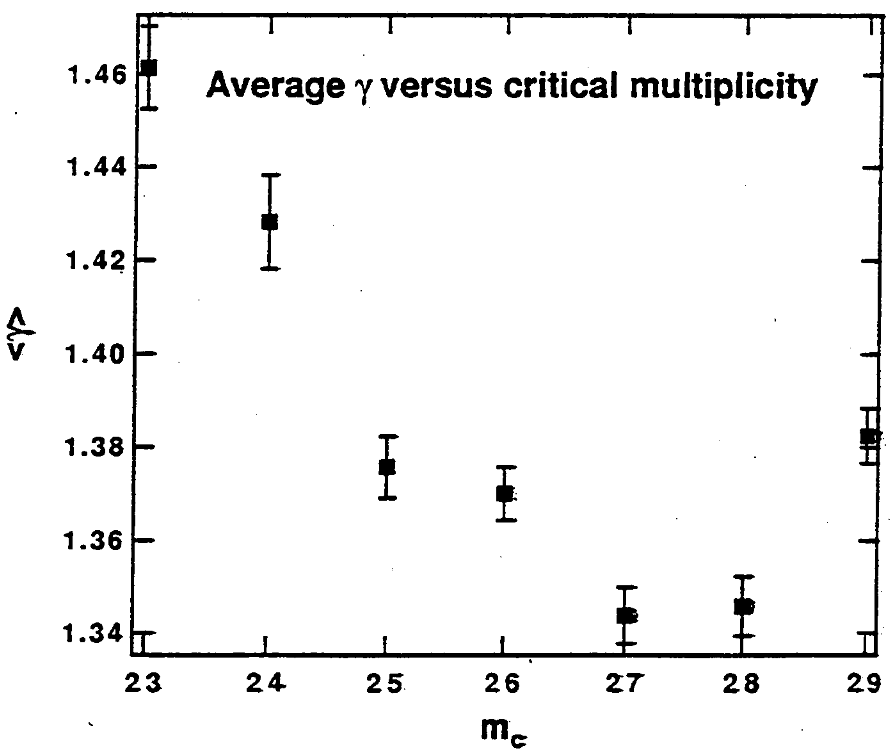

Figure 3. Average γ versus m_c.

The value of β was determined by a fit of Eq. 3 to the liquid side of m_c. See Fig. 4. It is important to note that the region fitted was identical to that used in determining each value of γ_{liquid} mentioned above. The exponent τ was determined from the slope of $\ln(M_3)$ vs $\ln(M_2)$, where we have used only the gas branch of the plot. The removal of the largest fragment in the liquid branch is a major perturbation on this correlation. This feature was also observed in our percolation simulations using a cubic lattice of 216 sites[12]. We also determined τ using Eq. 4 and obtained consistent results.

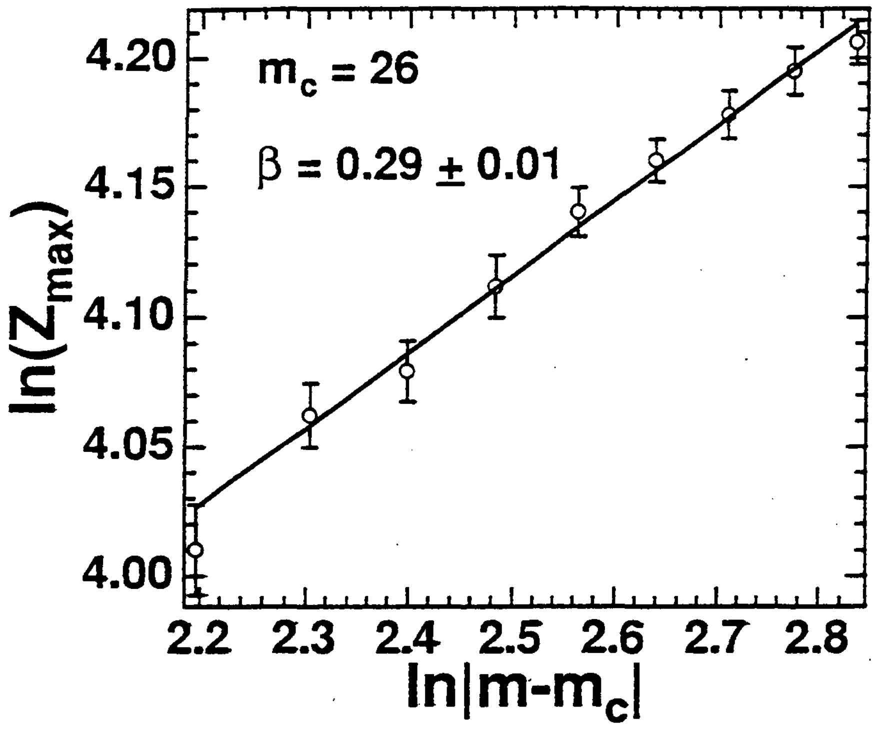

Figure 4. Example of the determination of the critical exponent β for a particular fitting region.

The values of the exponents obtained via this procedure are listed in Table 1. These values do not depend critically on our choice of a 10% slope matching criterion. Repeating the analysis requiring a more stringent 3% matching does not alter the values of the exponents. In order to judge the robustness of the exponent values with respect to Z_{sum}, we have also done the analysis for events with Z_{sum} = 72-75 or 83-86. The results are again statistically unaltered.

Table 1. Critical exponents for Au projectile fragmenation

Quantity	Value
γ	1.4±0.1
β	0.29±0.02
τ	2.14±0.06

4. Discussion

4.1. Comparison to other systems

As stated previously, the values of the critical exponents of systems belonging to a given universality class are the same. The exponent τ, however, shows little variation for different universality classes of the same dimensionality. For example, in fluid systems and in percolation, τ=2.21, and τ=2.18, respectively. Thus, we cannot use τ as a discriminator between universality classes. The exponents β and γ, however, differ significantly between universality classes. In Figure 5 we compare the exponents obtained from the Au fragmentation data to those for 3-dimensional percolation, fluid systems and a mean field treatment of fluid systems. The values of the critical exponents obtained from nuclear multifragmentation are remarkably close to those of fluid systems and are significantly different from the values obtained for both 3-dimensional percolation and the mean field treatment of liquid-gas systems.

4.2. Implications and Issues

Because the method described above minimizes finite size effects, the exponents so obtained are independent of system size. In principle, if one could perform a similar analysis for nuclear systems of different size, the results would be the same. In this sense, we have determined the critical exponents of nuclear matter.

Our analysis makes no explicit use of dynamical information. It is a cluster analysis performed in the spirit of those done for percolation lattices. The Fisher droplet model, which is used to describe the condensation in a fluid system near its critical point[13] is also a cluster picture that ignores dynamical information. However, in both standard percolation theory and in the Fisher droplet model, it is assumed that the system is in equilibrium. Having obtained critical exponents that obey the scaling relation of Eq. 5, we might be tempted to argue that our fragmenting system has had sufficient time to achieve at least a partial equilibrium. Additional evidence in support of this must come from dynamical information.

As we mentioned earlier, we have used multiplicity as a measure of the distance from criticality. If nuclear fragmentation is a thermally driven phenomenon, then in principle, we should use temperature as a measure of this distance. One can assign a temperature to the fragmenting projectile remnant by solving the following equation:

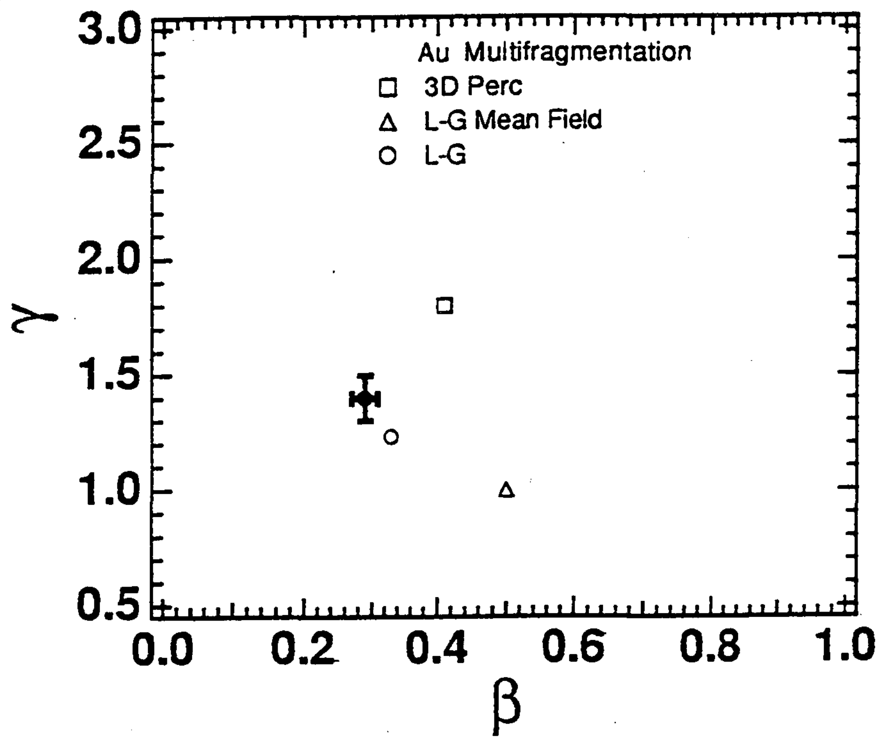

Figure 5. γ versus β extracted from Au multifragmentation compared to liquid-gas, 3D percolation, and liquid-gas mean field values.

$$E^*/A = \frac{1}{A_{meas}} \left[\sum_i (KE_i + Q_i) + 2nT \right] \qquad (6)$$

In Eq. 6, the left hand side is the excitation energy per nucleon of the projectile remnant, i.e., the system which is left after the initial, fast stage of the collision process is over. Quantities on the right hand side are computed in the rest frame of this remnant and exclude all light particles, mostly neutrons and protons, that are associated with the first stage of the reaction. The sum runs over all second stage reaction products. The number of neutrons, n, is estimated from the observed number of second stage protons and the neutron to proton ratio of gold. Q values are used only for fragments of charge less than 6. Assuming that following the first stage, the projectile remnant is an excited Fermi gas of nucleons at near normal density and temperature T, so that $\frac{E^*}{A} = T^2$over10 MeV, we can solve Eq. 6 for T. We can then plot T as a function of the event multiplicity. As seen in Fig. 6, we obtain a linear relation between T and m. Thus, we may be justified in using m as a linear measure of the distance from criticality.

The critical exponents extracted from a small 'sample' of nuclear matter, a gold nucleus, provide strong evidence that nuclear multifragmentation involves critical behavior. Fragments are formed over a wide range of remnant excitation energies, but power law behavior is exhibited over a limited range. This is characteristic of critical phenomena. If this picture of fragmentation is valid, then there are several implications.

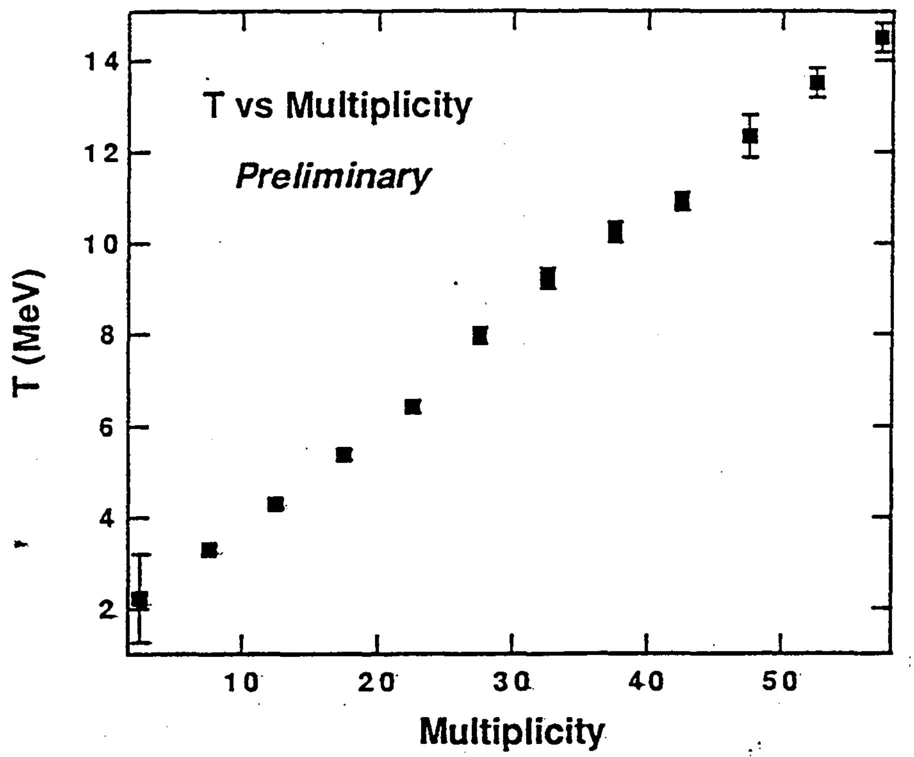

Figure 6. Temperature versus multiplicity

As mentioned previously, finite size scaling theory predicts how various quantities, such as the size of the largest fragment, scale with system size. In general, such scaling relations depend on critical exponents. By studying systems of different size, one can make an independent determination of some critical exponents. We are in the process of analyzing data from La and Kr fragmentation on carbon.

Scaling theory predicts that in the neighborhood of the critical point, a universal function, the scaling function, exists. Plotting the data (fragments of different charge) as a function of the scaling variable, one should observe a collapse of the data onto a universal curve. Furthermore, this curve is unique to a given universality class. Observation of such scaling for the nuclear fragmentation data would provide further evidence for critical behavior.

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