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Authors
Brown, Ian G.
Compiler, Alan B.
Kunkel, Wulf B.

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LAWRENCE RADIATION LABORATORY
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DISCLAIMER

RESPONSE OF A LANGMUIR PROBE IN A STRONG MAGNETIC FIELD*

Ian G. Brown,† Alan B. Compher,‡ and Wulf B. Kunkel

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

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ABSTRACT

A comparison was made of plasma density measurements obtained using cylindrical Langmuir probes and a high-sensitivity microwave interferometer in magnetic fields between 1 and 7 kG. The experiment was carried out in a current-free stream of ionized hydrogen with electron densities ranging from 3×10^9 to 4×10^{11} cm^{-3}. A set of probes of different sizes was used so that the ratio of probe diameter to ion gyroradius $2r_p/\rho_i$ covered the critical range from 1/5 to 5 or more. It was found that (1) the shape of the probe characteristics was not affected very much by the presence of the strong magnetic field; (2) the electron temperature derived from the slope seemed to be the true temperature; but (3) the apparent ion density inferred from the characteristic by using Laframboise's calculation, which applies in the absence of magnetic fields, was consistently low by a factor which depended primarily on the ratio r_p/ρ_i and seemed nearly independent of the Debye length. This reduction factor was considerably larger in the streaming plasma than in a stationary one, presumably because of the wake or "shadow" cast by the probe when r_p/ρ_i is not negligible.

I. INTRODUCTION

The use of Langmuir probes to measure electron density and temperature of plasmas is a well known technique which has received considerable theoretical[1] and experimental[1,2] attention. However, to date all analytic work has been restricted to plasmas with infinite or at least with sufficiently large ion gyroradii so that the ion collection current could be considered unaffected by any magnetic field.[3,4] Such an approximation has recently been verified in a magnetized alkali vapor plasma,[2] but there are many experimental situations where the effect of the magnetic field on the ion current undoubtedly cannot be neglected. Unfortunately, in that case ion collisions must be taken into consideration, even in the long-mean-free-path limit, so that it is not surprising that no general theory has as yet been developed. It was therefore deemed desirable to determine empirically how the presence of a strong magnetic field should be taken into account to obtain correct absolute density measurements.

We have made a comparison of density measurements obtained using a cylindrical Langmuir probe and a high-sensitivity microwave interferometer system. The experiments were performed in a hydrogen plasma in a magnetic field of from 1 to 7 kG, and covering a range of density from $\sim 3 \times 10^9$ cm^{-3} to $\sim 4 \times 10^{11}$ cm^{-3}. In the consideration of probes in a magnetic field, the field is usually classified as "strong" when $r_p/\rho_e \gg 1$ and $r_p/\rho_i \ll 1$, where r_p is the probe radius and ρ_e and ρ_i the electron and ion gyroradii respectively, and as "very strong" when both $r_p/\rho_e \gg 1$ and $r_p/\rho_i \gg 1$. The range covered in the present experiment is $0.4 < r_p/\rho_e < 20$ and $0.1 < r_p/\rho_i < 4$; we are thus operating in a magnetic field regime between strong and very strong, a case which has not previously been investigated either theoretically or experimentally.

In principle, it is quite possible to avoid in a given experiment the entire problem under study here by simply constructing a probe of sufficiently small radius. There are, however, physical limitations on this approach. The thermal capacity of a probe decreases with its size; this in turn places limits upon the maximum plasma density and temperature in which the probe may be used before incandescence, electron emission, and finally evaporation occur. Further a small radius implies a small collection current, which is more difficult to measure, and in the limit becomes buried in noise. A larger probe radius (within reasonable bounds imposed by other considerations) is clearly desirable, if it is indeed then possible to interpret the data with any accuracy. This is the problem we have attempted to attack in the series of experiments described here.

II. PLASMA SOURCE

The experiments were performed in the "COMPLEX" facility ("Cold Microwave Plasma Experiment"), (Fig. 1), which has been described elsehwere.[5] Briefly, it consists of a copper vacuum vessel, 11.6 cm i.d. immersed in a constant axial magnetic field of uniformity $\lesssim \pm 5\%$ for ~ 1 m of its length, with a 30-cm long microwave discharge region adjoining the main plasma vessel through a 5-cm aperture. The magnetic field strength is variable from 0 to ~ 7 kG in continuous operation, or to about double this value in a pulsed mode (on-time a few seconds). Microwave power at 10 GHz and $\lesssim 1$ kW cw is fed into the cavity via one of a number of possible coupling devices; for these experiments we have used a "Lisitano coil."[6] The magnetic field in the discharge chamber is adjusted so that there exists an electron cyclotron resonance region within it. The plasma thus created forms a column in the main chamber, of diameter limited either by the Lisitano coil or by the aperture, depending upon the precise magnetic field configuration, and is typically about 5 cm diameter. Hydrogen gas was used throughout, at a pressure typically near 5×10^{-5} torr; the vacuum base pressure is 5×10^{-8} torr. By varying the input microwave power, from 1 W to 1 kW, the plasma density can be controlled over roughly two orders of magnitude, from 3×10^{9} to 4×10^{11} cm^{-3}. The electron temperature is always from 5 to 15 eV, as measured by both a Langmuir probe and a three-grid electrostatic energy analyzer. The ion temperature is always much less; spectroscopic Doppler broadening measurements made on a helium plasma under similar conditions have shown that $T_i < 1$ eV, this limit being set by the instrumental limit of resolution.

When produced from one end, as described above, the plasma is found to be streaming along the magnetic field typically with a speed of $v_d \approx \frac{1}{2} \sqrt{kT_e/m_i}$. This has to be taken into consideration in the interpretation of the probe data.

III. MICROWAVE INTERFEROMETRY

We have used a high-sensitivity microwave interferometer system[7] capable of measuring phase shifts down to ~ 0.2 deg, corresponding to a peak (on axis) electron density of about 2×10^{9} cm^{-3}. Basically the system is of standard design, with the addition of a 100 kHz modulation impressed on the 35 GHz microwave signal, thus allowing the use of phase-sensitive detection and consequent high sensitivity.

A measurement of microwave phase shift yields a value for the total number of electrons integrated along the microwave beam path. In order to compare this with the electron density obtained from the Langmuir probe, it is necessary to know also the radial distribution of (relative) electron density. At each comparison "point" (magnetic field strength-electron density combination), we have made a complete radial scan of the plasma column with the Langmuir probe, thus obtaining the desired radial distribution function and allowing us to compute the density at a given radial position (we have chosen this to be on axis, $r = 0$) from the microwave phase shift. A further relevant consideration is the departure of the cylindrical plasma column from approximating a "plasma slab" as seen by the microwave beam. This effect can also be corrected for, once the relative distribution of electron density is determined.

Finally, the interferometer system, and a microwave-absorbing liner for the vacuum vessel which we have used, are fully described in Ref. 7.

IV. LANGMUIR PROBES

The probes we have used are cylindrical and are inserted into the plasma radially, i.e., with the probe length perpendicular to the magnetic field. All probes used were of tungsten of radius (for most of the work) $0.004 \leq r_p \leq 0.025$ cm and extending a distance $\ell_p = 0.3$ to 0.5 cm from a ceramic or glass sleeve $(15 \leq \ell_p/r_p \leq 100)$.

The probe characteristic (current vs voltage behavior) is plotted directly by an XY recorder using a low impedance swept voltage source which traces out the entire curve, from $V_{probe} = -90$ V to ~ +30 V, in a time of several seconds. Noise due to plasma fluctuations is filtered out with passive RC integrators. (It has been shown by Chen[8] that, so long as the fluctuation level is not too great, this averageing does indeed yield a reliable electron density and temperature measurement; our fluctuation level $\Delta i_{sat}/i_{sat} \lesssim 10\%$, which is sufficiently small.)

The interpretation of probe characteristics even in the absence of a magnetic field is a subject which is far from closed. Of the many theoretical papers written, the most thorough and therefore perhaps the most reliable is the work of Laframboise.[9] He has carried out a series of numerical computations applicable to the case of a Maxwellian plasma at rest and without magnetic field, and the results are presented in the form of suitably normalized curves of ion and electron currents as a function of probe voltage. We note that since we are operating in a density regime where $r_p \approx \lambda_D$, where λ_D is the (electron) Debye length, neither Langmuir's simple orbital motion theory[1] ($r_p \ll \lambda_D$) nor the Bohm formula[10] ($r_p \gg \lambda_D$) can reasonably be applied, and a more complete treatment, such as that of Laframboise, is required. Note also that since for us $T_i/T_e < 0.1$, the often assumed condition of zero ion temperature is well approximated. Since the shape of the probe characteristics were not significantly affected by the magnetic field in our work, we have used the results of Laframboise as our principal method for determining an "apparent" ion density from the probe characteristic.

Laframboise's results are quite general and are applicable to a great range of probe-plasma parameters encountered in laboratory plasma experiments. It is not possible to present his results in the form of a single equation covering the entire range of parameter space; each case should be considered individually in relation to his computer plots. For the parameter range covered in the present experiment, however, we can write an explicit expression for the ion density n_L: For a H_2^+ plasma, with $T_i/T_e < 0.1$, $r_p/\lambda_D \lesssim 3$,

$$n_L = 1.0 \times 10^8 \frac{i_L}{r_p \ell_p \sqrt{kT_e}} \text{ cm}^{-3}, \qquad (1)$$

where i_L is the ion saturation current (in μA) evaluated at a probe potential V_p such that $V_s - V_p = 10\ kT_e$ (as indicated in Fig. 2), r_p and ℓ_p are the probe radius and length (in mm), respectively, and kT_e is the electron "temperature" (in eV). As is usual, it is assumed that $r_p \ll \ell_p$. (Further useful information on the experimental application of Laframboise's results can be found in Refs. 11 and 12.)

Because of its simplicity, the Bohm formula is widely used to interpret probe characteristics, even though the conditions for its applicability may not be met. For this reason we have also used this method to make alternative estimates of the ion density. Thus,[10]

$$i_{sat} \simeq \frac{1}{2} n_i e \sqrt{\frac{kT_e}{m_i}}\, A, \qquad (2)$$

where A is the probe area. The inapplicability of the method in the present case is manifested by the lack of saturation of ion current with increasing negative probe potential (since $r_p \gg \lambda_D$ is not fulfilled) and the consequent uncertainty in the value of "i_{sat}" to be taken. We have evaluated the "saturation" ion current (a) at the estimated space potential V_s, yielding a density n_s, and (b) at an estimated "asymptotic" value i_A yielding a density n_A. The space potential is determined by the relationship given by Chen,[1] that for a cylindrical probe, $V_s - V_f \simeq 4kT_e$, where V_f is the floating potential. The manner in which the three density measurements are obtained from the probe characteristics is illustrated in Fig. 2.

None of the above methods takes into account the streaming or the presence of a magnetic field, while we are operating in a plasma wind and in a magnetic field regime between "strong" and "very strong." We attempt to examine the relationship between the zero-field theory and the data in a strong magnetic field, and to suggest in what manner the probe interpretation should be modified to include the effect of a nonnegligible magnetic field.

V. RESULTS

A. Electron Temperature Measurements

The electron temperature T_e is obtained from the probe characteristic by a measurement of the logarithmic slope in the transition region of the curve. We have not made a detailed study of the temperature so obtained as a function of magnetic field strength. We have, however, used a three-grid electrostatic energy analyzer[13] to provide an additional temperature measurement under a number of typical conditions. The analyzer is located at the end of the plasma column and is coaxial with the vessel and plasma axis; it thus measures the parallel (to the applied magnetic field) electron temperature. Although a complete survey was not performed, we can state that the analyzer and probe yield temperature values which are in agreement within the expected experimental uncertainty ($\lesssim \pm 50\%$). In a separate series of experiments, we have observed the propagation of ion-acoustic waves in a number of different rare gases. The measured wave velocity is always in good agreement with that predicted from the probe-measured electron temperature, providing another confirmation of electron temperature.[5] It is of interest to note that this insensitivity of the determination of T_e to the magnetic field is in fact predicted by the existing theoretical work for medium-strong fields,[3,4] and corroborates the experimental findings of Chen et al.[2]

For the purposes of the present experiment, the electron temperature is of relevance only as it enters into the determination of the electron density; it is a fortuitous circumstance that the parameters of our experiment result in only a very weak dependence of the probe-obtained density (by the method of Laframboise) upon electron temperature. An uncertainty of a factor of 2 in T_e produces typically an uncertainty in n_L of $\lesssim 5$ to 10%. This is a consequence of the slope of the ion "saturation" part of the probe characteristic and the dependence of $n_L \propto i_{sat}/\sqrt{T_e}$; as T_e increases, so also does the value of $|V_{probe}|$ at which i_{sat} is to be measured, and the value there of i_{sat}. For our probe characteristic the two effects very nearly cancel.

Also of interest is the observation that the electron energy distribution seems to be close to Maxwellian,[5] a property which our plasma may not have been expected to possess in view of the resonant nature of the electron heating mechanism. It is of importance, however, that the distribution indeed be Maxwellian in order that the probe theory of Laframboise be applicable. A typical temperature determination from the probe characteristic is shown in Fig. 3; that the distribution is Maxwellian is evidenced by the linear relationship between $\log(i - i_{ion})$ and V_{pr} over approximately two orders of magnitude variation in current. This conclusion is further verified by the energy analyzer data. It has been found that the range of current over which the linear $\log(i - i_{ion})$ vs V_{pr} relationship exists is greatest when the probe is located near the axis of the plasma, rather than near the edge of the column. For this reason all the density comparisons to be described have been made with probe density determined on axis.

B. Density Measurements

The density range available to our experiment is $3 \times 10^9 \lesssim n_e \lesssim 4 \times 10^{11}$ cm^{-3}, and the probe response is linear with density over this range, as is clear from Fig. 4; these data correspond to $0.4 \lesssim r_p/\lambda_D \lesssim 4$ and $r_p/\rho_i \approx 1.3$. The apparent density inferred from probe measurements by means of Laframboise's method is seen to be lower than the true density, but the correction necessary depends only very slightly on the density over at least the two orders of magnitude explored here. We conclude that under our conditions the ratio n_L/n_μ is nearly independent of the Debye length λ_D, and thus we feel justified in investigating the probe response as a function of magnetic field strength, at a fixed density, with the knowledge that our results should be applicable over a considerable density range.

At a fixed density of $\sim 5 \times 10^{10}$ cm^{-3} we have compared the on-axis densities measured by a probe and by the 8.6-mm interferometer. A scan of n(r) was taken for all measurements so as to obtained the on-axis density from the integrated interferometer reading, We have used a number of different probes, with tip diameters from 0.003 to 0.020 in., and have varied the magnetic field from 0.8 to 7 kG. Since, however, we cannot guarantee that the ion temperature (which determines the ion gyroradius) remains the same over wide variations of magnetic field, the results we show here are for a smaller range of field strength, $1.15 < B_0 < 3.85$ kG, over which range we believe the ion temperature does remain sufficiently constant. Typical results are shown in Fig. 5 where we have plotted the ratio n_L/n_μ as a function of magnetic field for various probe radii. It is clear that there is considerable spread in the different groups of data points, and it seems impossible to make a general statement of any functional dependence. Qualitatively the features are as might have been expected: for a given probe size and at a fixed plasma density the ion current decreases when the magnetic field B is made stronger. Conversely, for a given value of B the ratio n_L/n_μ decreases with increasing probe dimensions. Evidently, when the mean ion gyroradius ρ_i is comparable to the probe radius r_p, the ion saturation current depends on the magnetic field B as well as on r_p.

More specifically, we must expect that in the limit of an infinitely strong magnetic field all charged particles are constrained to move along the field lines. Therefore the effective collection area of the cylindrical probe in this limit is reduced by the ratio

$4r_p/2\pi r_p = 2/\pi$ if the particles approach the probe from both sides, and the by ratio $1/\pi$ if the particles all come from one side, as in our streaming plasma, so that the downstream side is "shadowed." When the magnetic field is finite, so that the most probable ion gyroradius ρ_i can no longer be neglected, we can proceed to argue that the effective width of the accessible flux tube is larger than $2r_p$, say $2r_p + \alpha\rho_i$. Here the unknown factor α then presumably should depend on the Debye length and on the collision mean free path for ions. In the limit of zero magnetic field, on the other hand, we should expect to recover fairly good agreement between our apparent density n_L inferred from our probe current with the help of Laframboise's analysis and the true value obtained from the microwave interferometer. This should be true even for our streaming plasma since in that case the shadowing effect of our small probes is negligible, and the subsonic flow velocity can only mildly perturb the ion motion towards a probe with strong negative potential of order 10 kT_e/e or more.[14] We therefore introduce a heuristic interpolation function for n_L/n_μ:

$$\frac{n_L}{n_\mu} \approx F(r_p,\rho_i) \equiv \frac{2r_p + \alpha\rho_i}{2\pi r_p + \alpha\rho_i} = \frac{r_pB + a}{\pi r_pB + a} , \qquad (3)$$

which goes to the expected limits for $B = 0$ and $B = \infty$ appropriate for our streaming plasma and for which the quantity a can be considered as an empirical constant, provided T_i is independent of B. Indeed, when the data of Fig. 5 are plotted against r_pB, as in Fig. 6, a systematic trend is recognizable, and even the form of $F(r_p,\rho_i)$ is suggested, as indicated by the solid line. The value of a used in the line drawn is

18 gauss cm, i.e., $\rho_i = 36/\alpha B$, which corresponds to molecular hydrogen ions near room temperature if $\alpha = 1$.

VI. DISCUSSION

It must be emphasized that Eq.(3) is really an empirical relation which may be used by experimentalists for a first-order correction of probe data. In particular, no physical significance should at this point be attached to the observation that a fair fit of the data is obtained for $kT_i = 0.03$ eV. The ion temperature was not measured in this experiment, and there is no reason why α should be unity in Eq. (3). In fact, the actual functional dependence of n_L/n_μ on r_p and ρ_i is undoubtedly much more complicated; and if our $F(r_p, \rho_i)$ represents a usable approximation, the findings shown in Fig. 4 indicate merely that the factor α can depend only very weakly on the Debye length and on the mean free paths.

Some of the scatter of our data in Fig. 6 may well be due to the possibility that the ion temperature varies a little between runs at different magnetic fields since the discharge power had to be adjusted to yield the predetermined plasma density. The greatest uncertainty, however, is undoubtedly caused by the fact that the ion species present in our plasma, instead of being 100% H_2^+ as is assumed in Eq. (1), actually always consists of a certain mixture of H^+, H_2^+, and H_3^+, and this mixture varies somewhat with discharge power. The experiment could have been carried out in helium to avoid this difficulty. But because of the importance of hydrogen in much of the recent work in strong magnetic fields, and in an attempt to minimize the gyroradii, we restricted our measurements to H_2.

Finally, it should be noted that a large part of the observed reduction in ion collection current in our interpretation is caused by the "shadow" (or depletion in the wake) that results when the plasma has a substantial streaming velocity along the strong magnetic field. We have verified this idea by adding an equivalent second discharge at the downstream end of the plasma column, enabling us to compare n_L/n_μ in double-ended and in single-ended operation. Interestingly, at our low-density, double-ended operation yielded only a negligible increase in this ratio, although n_μ itself nearly doubled as expected. Apparently, in this case the plasma consisted of two noninteracting streams so that the total density doubled, but the probe produced shadows for each of these streams. Only when the power was raised to yield densities above 10^{11} cm^{-3} did the ratio n_L/n_μ drastically increase when the second discharge was turned on. To stop the streaming by double-ended operation of the generating discharges, it is evidently necessary that the interaction between the two streams is sufficiently vigorous to randomize the particle velocities either by collisions or by unstable fluctuations. Unfortunately, our experiment becomes very awkward with discharges at both ends of the chamber so that the entire set of measurements presented in Figs. 5 and 6 could not be repeated in a plasma at rest. We can merely state that the observation we do have is consistent with the prediction that in very strong fields the ratio n_L/n_μ approaches the value of $2/\pi$ if the plasma is not streaming.

Finally, we have also used the simple Bohm formula, Eq. (2), to calculate apparent densities n_s and n_A in the manner described, i.e., based on the "saturation" currents i_s and i_A, as shown in Fig. 2.

Since these values are obtained by using quite dubious methods, we have not considered them in any detail, but simply remark that (a) n_s is always much too low, sometimes by almost an order of magnitude, and (b) n_A is usually quite close to n_μ (within a factor of about 2). These statements can only be strictly applicable to our particular parameters, and we can make no comment on the generality of these results. It is none the less interesting that the simplest possible probe interpretation results in a density surprisingly close to the true density.

FOOTNOTES AND REFERENCES

[*] Work performed under the auspices of the U. S. Atomic Energy Commission.

[†] Present address: Institut für Plasmaphysik, Garching bei München, Germany.

[‡] Present address: Arkon Scientific Laboratory, Berkeley, California.

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13. We are indebted to K. W. Ehlers for the design and construction of the electrostatic energy analyzer.
14. This statement is consistent with an analysis of the response of a directional Langmuir probe in a drifting plasma found in Martin Hudis, "Particle Flux Measurements in a Hollow Cathode Arc" (Ph. D. Thesis), MIT, Nov. 1969.

FIGURE LEGENDS

Fig. 1. The COMPLEX plasma facility, including a typical plot of on-axis magnetic field strength.

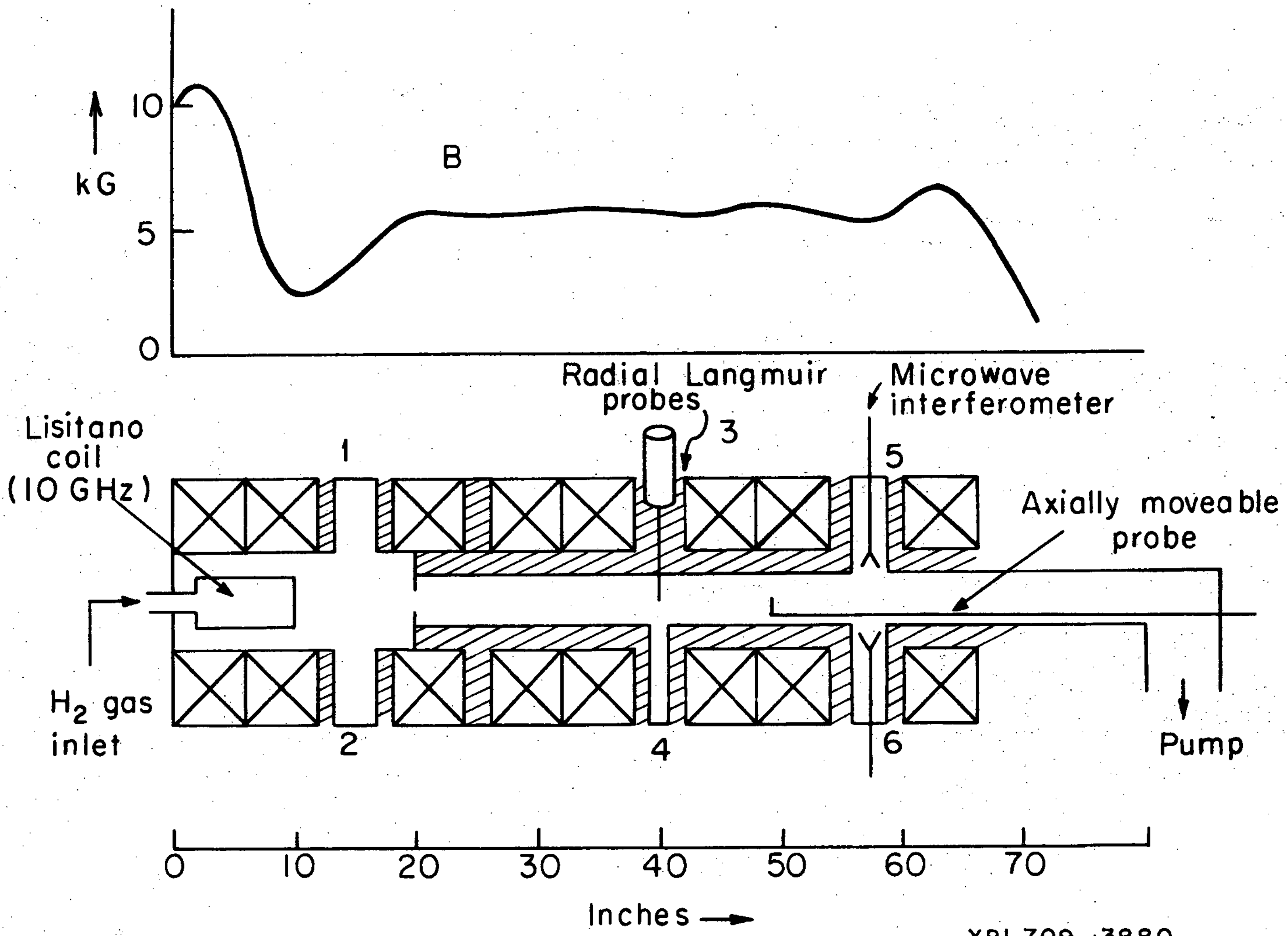
Fig. 2. Typical Langmuir probe characteristic, showing how the ion "saturation" current is obtained.

Fig. 3. Adjusted Langmuir probe current, $\ln(i - i_{sat})$, vs probe potential V_p for typical conditions.

Fig. 4. Probe-obtained density n_L vs interferometer-obtained density n_μ, showing the linearity of probe response with density.

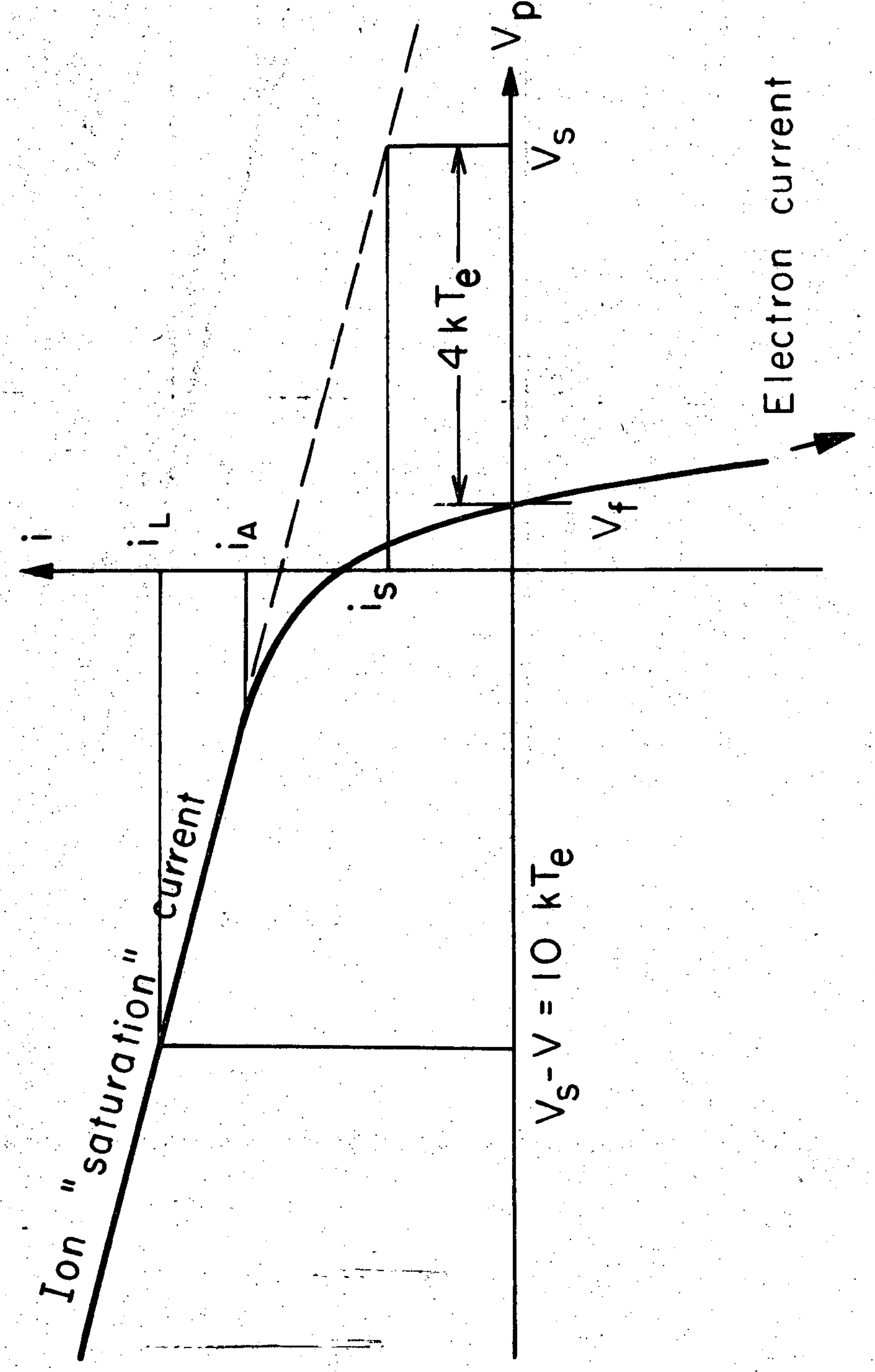
Fig. 5. The ratio n_L/n_μ vs magnetic field strength B for various probe radii r_p.

Fig. 6. The ratio n_L/n_μ vs the product r_pB. The solid curve is a plot of Eq. (3) with $\underline{a}$ = 18 gauss cm.



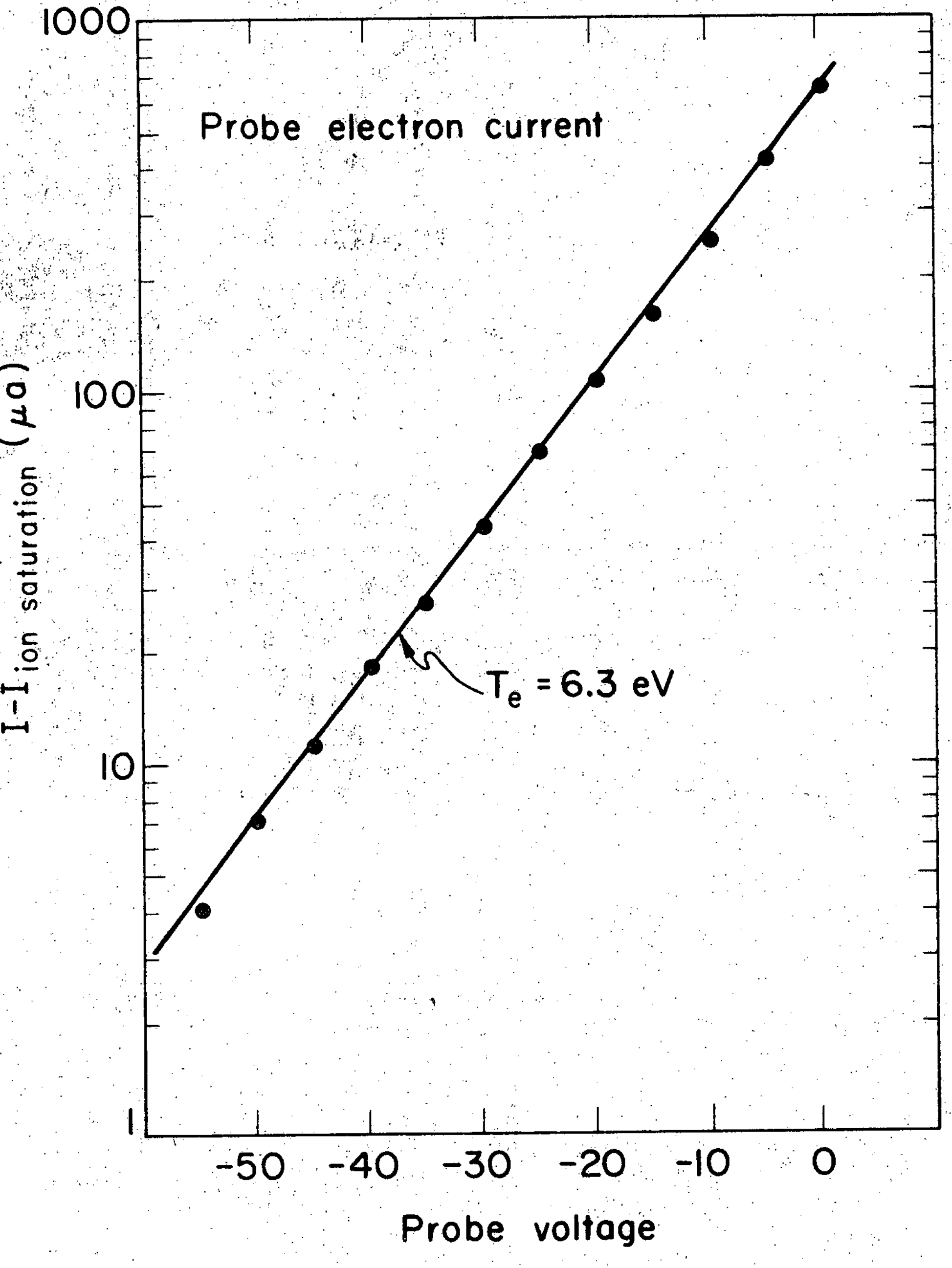
XBL 709-3880

Fig. 1



XBL709-3881

Fig. 2



XBL705-2814

Fig. 3

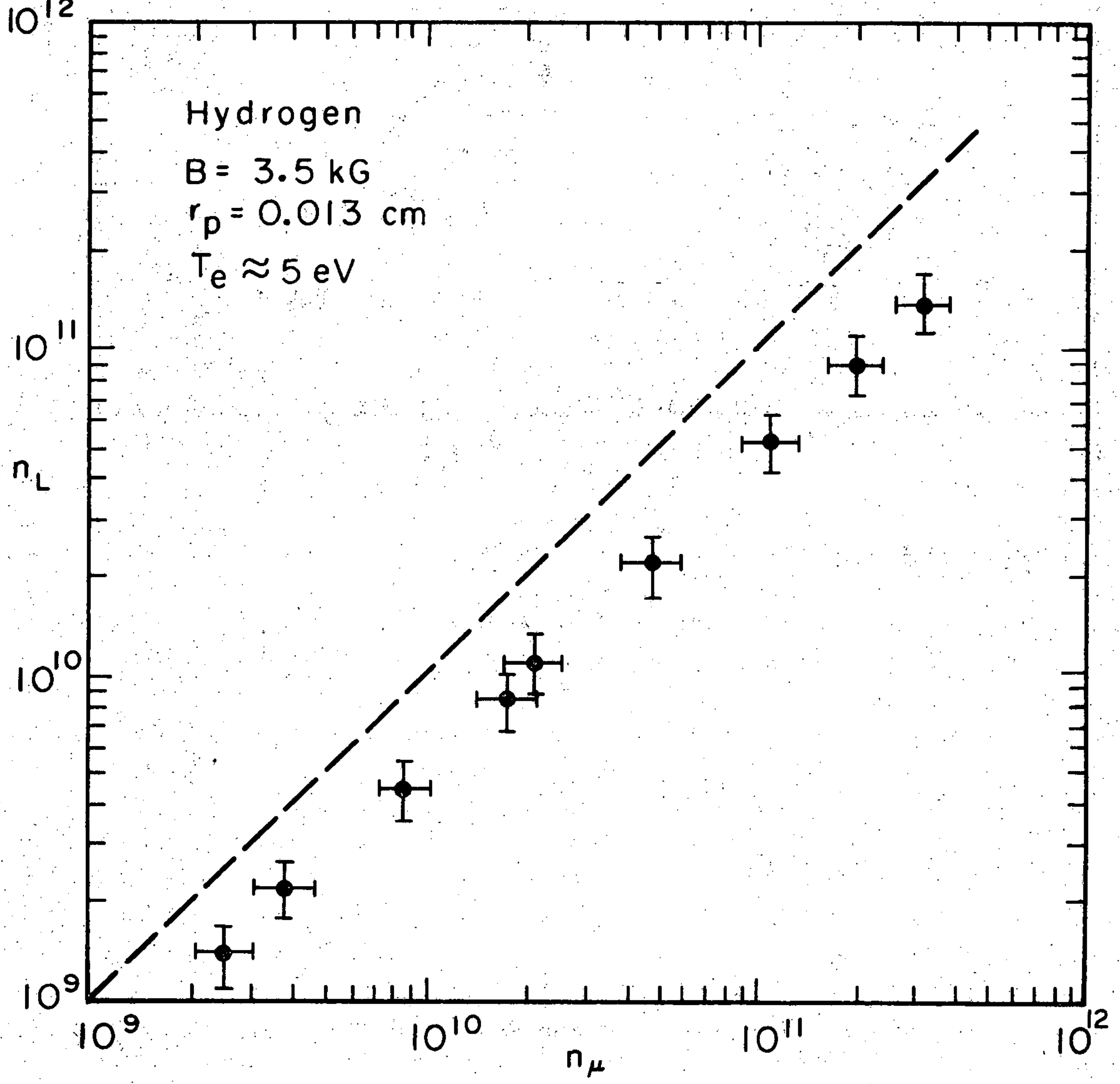


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Fig. 4

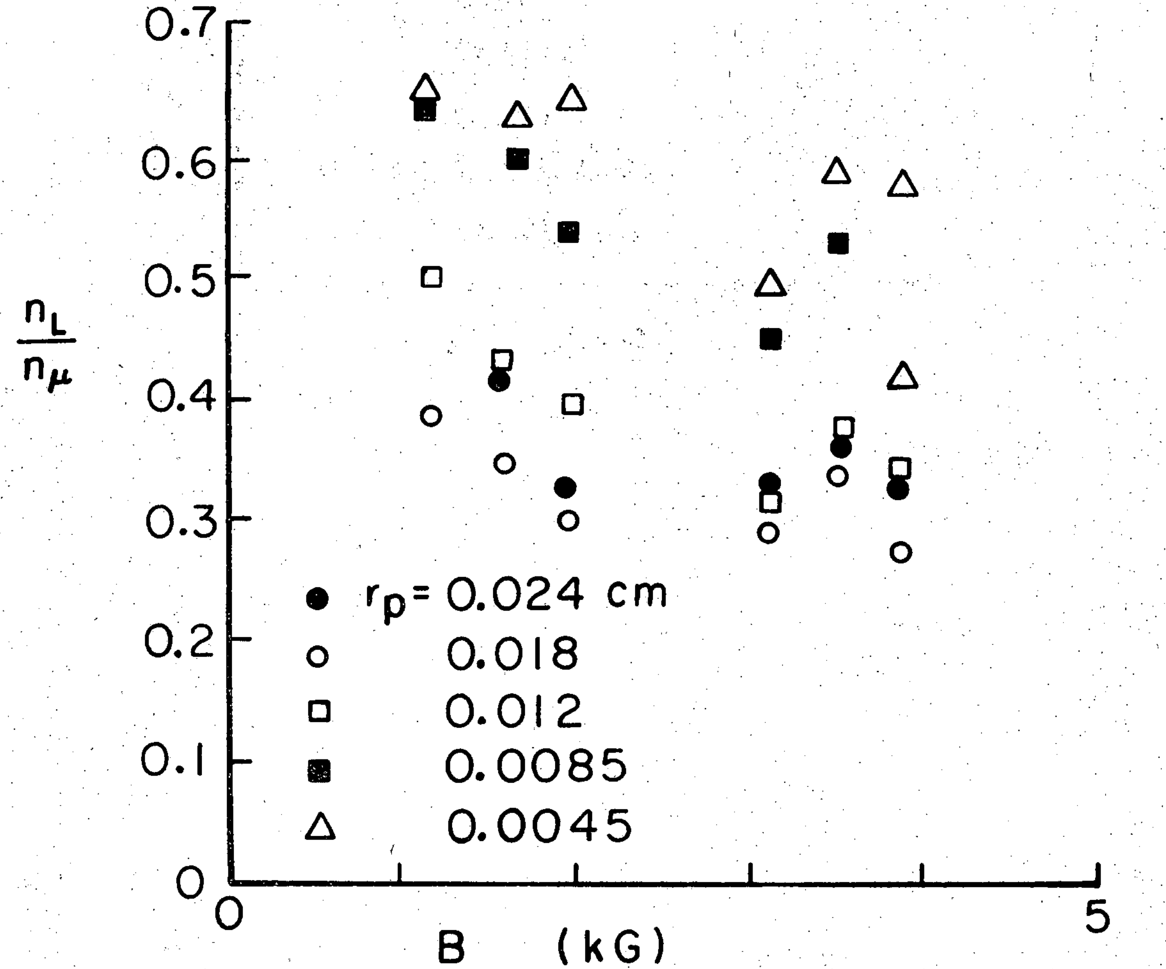

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Fig. 5

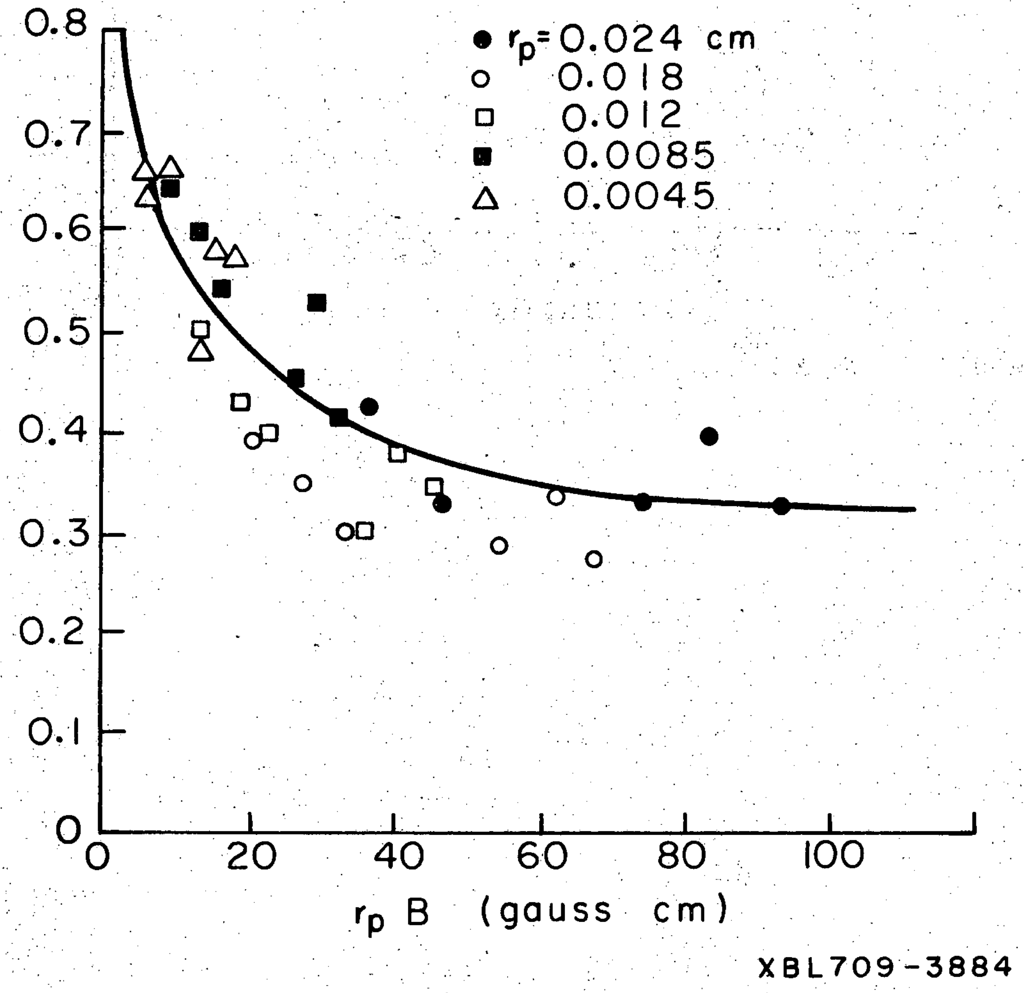
● r_p= 0.024 cm
○ 0.018
□ 0.012
■ 0.0085
△ 0.0045
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$\frac{n_L}{n_\mu}$
0
20
40
60
80
100
r_p B (gauss cm)
XBL709-3884

Fig. 6

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