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Comments on $\pi\pi$ Phase Shifts as Determined from the Peripheral Model*

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The determination of the S-wave $\pi\pi$ phase shifts $\delta_0{}^I$ at low energy from the analysis of $\pi N \rightarrow (2\pi)N$ is examined critically from the standpoint of the one-pion-exchange model with absorptive corrections. It is found that: (1) The value of $\delta_0{}^I$ depends strongly on the P-wave phase shifts, which cannot be unambiguously determined, at $m_{\pi\pi}<600$ MeV, by using a Breit-Wigner formula. (2) The ratio of the production density matrix elements ρ (with the $\pi\pi$ elastic scattering amplitudes factored out) depends strongly on $m_{\pi\pi}$ for $m_{\pi\pi}<600$ MeV. (3) The $(F-B)/(F+B)$ asymmetry shows a sizeable dependence on the momentum transfer t to the nucleon. It is concluded that more accurate data at low $m_{\pi\pi}$ are required in order to determine $\delta_0{}^I$ for $m_{\pi\pi}<600$ MeV. Tables of the $\rho(m_{\pi\pi},t)$ calculated from the absorption model for an incident-pion laboratory kinetic energy of 4 BeV are included. These could be directly applied to the data to obtain the low-energy $\pi\pi$ phase shifts.

THE determination of the S-wave $\pi\pi$ phase shifts $\delta_0{}^I(m_{\pi\pi})$ at low energy ($m_{\pi\pi}\lesssim 600$ MeV) is of considerable importance because of the following factors: (1) They enter into a variety of processes; in all of them either the theoretical understanding of the dynamics is somewhat shaky or more experimental data is needed, thus no unambiguous values of $\delta_0{}^I$ have been obtained from these experiments.[1] (2) From the theoretical standpoint there have been a number of predictions made by the utilization of current-algebra techniques together with low-energy theorems.[2] These predictions depend critically on the smallness of the $\pi\pi$ S-wave scattering lengths.

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[1] See, for example, P. Singer, Finnish Summer School, 1966 (to be published).

[2] See, for example, R. Dashen, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967), p. 51.

The production processes

$$\pi^- + p \rightarrow \pi^- + \pi^+ + n \quad (1)$$

$$\rightarrow \pi^- + \pi^0 + p \quad (2)$$

$$\rightarrow \pi^0 + \pi^0 + n \quad (3)$$

have been widely studied, using the (experimentally observed) peripheral nature of the interaction, to determine the $\pi\pi$ S- and P-wave amplitudes A_0 and A_1, mainly for $m_{\pi\pi}$ in the region of the ρ resonance.[3] The purpose of this article is to make a critical analysis of the possibility of using (1)–(3) to determine the $\pi\pi$ phase shifts at low $m_{\pi\pi}$. We use the one-pion-exchange

[3] W. D. Walker *et al.*, Phys. Rev. Letters **18**, 630 (1967), this paper contains references to earlier work; E. Malamud and P. E. Schlein, *ibid.* **19**, 1056 (1967).

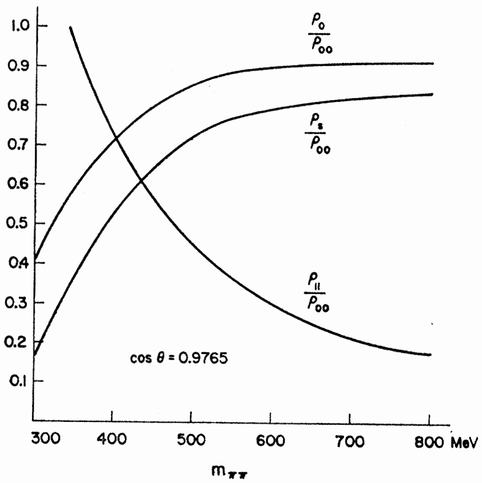

FIG. 1. Energy ($m_{\pi\pi}$) dependence of the ratios ρ_s/ρ_{00}, ρ_0/ρ_{00}, and ρ_{11}/ρ_{00}, computed at $\cos\theta_p=0.9765$ for $E_L=4$ BeV.

(OPE) model with absorptive corrections[4] to form the basis of our remarks. We find that: (1) The values of $\delta_0{}^I$ determined from processes (1) and (2) depend critically on δ_1. However, the P-wave $\pi\pi$ shift $\delta_1(m_{\pi\pi})$ is only known near m_ρ. (The determination of δ_1 away from m_ρ depends on the assumed energy dependence of the Breit-Wigner width Γ_ρ.) (2) An interesting model-independent analysis of (1) and (2) has been performed by Malamud and Schlein,[3] who assume that the ratios of production density matrix elements ρ [defined in Eq. (4)] with the $\pi\pi$ elastic scattering amplitudes A factored out, do not depend on $m_{\pi\pi}$. Our calculations with the absorption model support this assumption for $m_{\pi\pi}>600$ MeV. However, below this energy, we find an extremely strong $m_{\pi\pi}$ dependence. (3) The absorption model predicts a sizeable momentum transfer (to the nucleon) dependence for the ratios of the density matrix elements ρ. The phenomenological analysis of Malamud and Schlein[3] confirms this t dependence for $m_{\pi\pi}>600$ MeV. The experimental verification of our predicted t dependence of, e.g., the forward-backward asymmetry $(F-B)/(F+B)$ of the final pions in their c.m. system for $m_{\pi\pi}<600$ MeV, would give support to the absorption model and hence strengthen our conclusion (2). (See Fig. 2.)

TABLE I. Density matrix elements ρ, in μb/MeV, from the absorption model for processes (1)–(3) computed as functions of $m_{\pi\pi}$ with "partial" absorption. The upper line, for each energy, is for $1>\cos\theta_p>0.9765$. The lower line is for $0.9765>\cos\theta_p>0.909$.

$M_{\pi\pi}$ (MeV)	ρ_s	ρ_1	ρ_0	ρ_{11}	ρ_{10}	$\rho_{1,-1}$	ρ_{00}
300	0.164	−0.215	0.384	0.629	−0.509	0.054	0.938
	0.126	−0.257	0.596	1.216	−1.299	0.116	3.094
430	0.474	−0.214	0.587	0.215	−0.265	0.017	0.735
	0.372	−0.235	0.695	0.382	−0.458	0.073	1.372
525	0.638	−0.207	0.729	0.155	−0.235	0.013	0.838
	0.509	−0.220	0.782	0.264	−0.347	0.067	1.250
600	0.756	−0.199	0.838	0.126	−0.217	0.011	0.933
	0.612	−0.209	0.855	0.210	−0.295	0.062	1.234
675	0.844	−0.188	0.922	0.108	−0.202	0.009	1.011
	0.694	−0.200	0.917	0.179	−0.263	0.057	1.244
740	0.909	−0.177	0.985	0.095	−0.187	0.008	1.072
	0.760	−0.193	0.968	0.158	−0.241	0.052	1.261
800	0.952	−0.164	1.028	0.086	−0.172	0.007	1.115
	0.813	−0.185	1.008	0.143	−0.222	0.047	1.277
855	0.977	−0.151	1.054	0.078	−0.157	0.006	1.141
	0.854	−0.178	1.038	0.131	−0.207	0.042	1.289
910	0.985	−0.138	1.062	0.071	−0.143	0.005	1.150
	0.883	−0.170	1.060	0.121	−0.193	0.037	1.296

We conclude that a quite detailed analysis of very accurate data must be made in order to obtain a reliable set of phase shifts $\delta_0{}^I$ and δ_1 for $m_{\pi\pi}<600$ MeV. Unfortunately, the relatively small production cross sections[5] for low $m_{\pi\pi}$ make it difficult to obtain the necessary data.

We give the density matrix elements calculated from the absorption model as a function of $m_{\pi\pi}$ and t so that they may be directly used to obtain the $\pi\pi$ phase shifts when sufficiently accurate data on (1)–(3) become available in the region $m_{\pi\pi}\lesssim 600$ MeV.

We calculate processes (1)–(3) from the OPE model with absorptive corrections. Considering only S- and P-wave $\pi\pi$ scattering at low $m_{\pi\pi}$, we write the cross section as[6,7]

$$\begin{aligned} d\sigma= (1/4\pi)\{&\rho_s(t,m_{\pi\pi})|A_0(s)|^2+6\,\mathrm{Re}[A_0(s)A_1{}^*(s)] \\ &\times[\rho_0(t,m_{\pi\pi})\cos\theta_\pi-\sqrt{2}\rho_1(t,m_{\pi\pi})\sin\theta_\pi\cos\varphi_\pi] \\ &+9|A_1(s)|^2[\rho_{00}(t,m_{\pi\pi})\cos^2\theta_\pi+\rho_{11}(t,m_{\pi\pi})\sin^2\theta_\pi] \\ &-\sqrt{2}\rho_{10}(t,m_{\pi\pi})\sin\theta_\pi\cos\theta_\pi\cos\varphi_\pi \\ &-\rho_{1,-1}(t,m_{\pi\pi})\sin^2\theta_\pi\cos 2\varphi_\pi\} \\ &\times d\cos\theta_p dm_{\pi\pi} d\cos\theta_\pi d\varphi_\pi\,, \end{aligned} \quad (4)$$

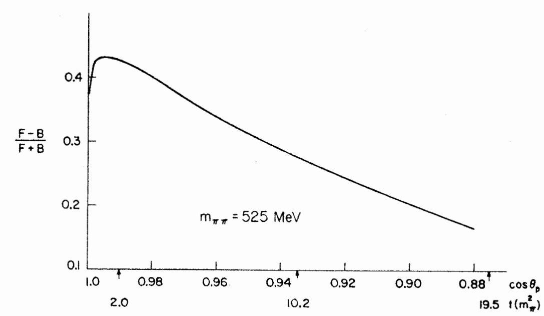

FIG. 2. Momentum transfer (t) dependence of the forward-backward asymmetry $(F-B)/(F+B)$ at $m_{\pi\pi}=525$ MeV, for $\delta_0{}^0=40°$, $\delta_0{}^2=-15°$, and $\delta_1=15°$.

[4] K. Gottfried and J. Jackson, Nuovo Cimento **34**, 735 (1964); L. Durand and Y. Chiu, Phys. Rev. **137**, B1350 (1965); M. Bander and G. Shaw, *ibid.* **139**, B956 (1965); **155**, 1675 (1967); L. J. Gutay *et al.*, Phys. Rev. Letters **18**, 142 (1967).

[5] See, for example, L. W. Jones *et al.*, Phys. Letters **21**, 590 (1966).

[6] We use units $\hbar=c=1$.

[7] We assume minimum off-mass-shell dependence of the $\pi\pi$ amplitude, namely that dictated by Feynman rules. The four-pion vertex for the particles in an l state is taken to be proportional to

$$\frac{1}{m^l{}_{\pi\pi}}\left(\frac{[m_{\pi\pi}{}^2-(m_\pi-\sqrt{t})^2][m_{\pi\pi}{}^2-(m_\pi+\sqrt{t})^2]}{m_{\pi\pi}{}^2-4m_\pi{}^2}\right)^{l/2} A_l(m_{\pi\pi})\,.$$

with

$$A_0=\frac{m_{\pi\pi}}{(m_{\pi\pi}^2-4m_\pi^2)^{1/2}}[C^0e^{i\delta_0^0}\sin\delta_0^0+C^2e^{i\delta_0^2}\sin\delta_0^2],$$

$$A_1=\frac{m_{\pi\pi}}{(m_{\pi\pi}^2-4m_\pi^2)^{1/2}}C^1e^{i\delta_1}\sin\delta_1,$$

where θ_π and φ_π are the polar and azimuthal (Yang-Treiman angle) angles, respectively, of the final two pions in their c.m. system, and θ_p is the production angle (linearly related to t) in the over-all c.m. system.

The isotopic spin factors are as follows:

$$\begin{array}{llll} \text{for (1),} & C^0=\tfrac{1}{3}\sqrt{2}, & C^2=1/3\sqrt{2}, & C^1=1/\sqrt{2}; \\ \text{for (2),} & C^0=0, & C^2=\tfrac{1}{2}, & C^1=\tfrac{1}{2}; \\ \text{for (3),} & C^0=\tfrac{1}{3}\sqrt{2}, & C^2=-\tfrac{1}{3}\sqrt{2}, & C^1=0. \end{array} \quad (5)$$

We present in Table I values of the ρ's calculated assuming the following: (i) The absorption of the final $(\pi\pi)N$ state is the same as for the initial πN state (which is obtained from analyzing the elastic scattering data), whereas for the ρ's in Table II (ii) there is total absorption in the relative $l=0$ state of the $(\pi\pi)N$ system.[8] The ρ's in Tables I and II correspond to an incident-pion laboratory energy E_L of 4 BeV. To present the t dependence of the $\rho(t,m_{\pi\pi})$ in a useful way, we have listed the ρ values integrated over two ranges of θ_p: (a) $0.9765<\cos\theta_p<1$ (or $|t|\lesssim 4m_\pi^2$) and (b) $0.909<\cos\theta_p<0.9765$ $(4m_\pi^2\lesssim|t|\lesssim 15m_\pi^2)$.

We expect that the physical situation should be between the cases (i) and (ii). In the region $m_{\pi\pi}\approx m_\rho$,

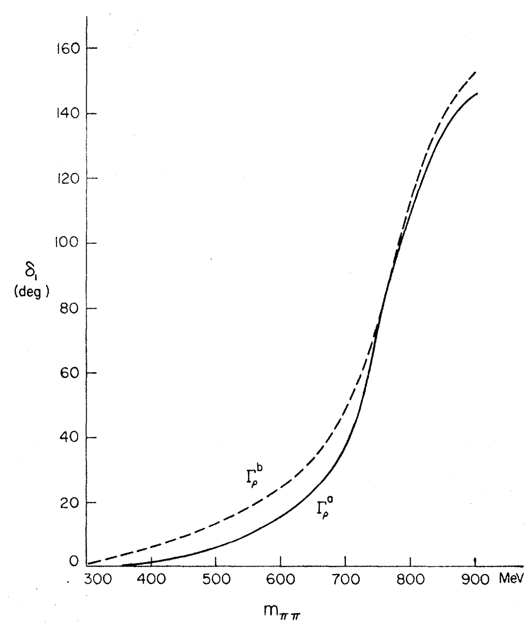

FIG. 3. $\delta_1(m_{\pi\pi})$ obtained by using different energy dependence of Γ_ρ. Curve a is for $\Gamma_\rho{}^a$ and curve b is for $\Gamma_\rho{}^b$ as defined by Eqs. (8).

[8] See, for example, M. Bander and G. Shaw, Ref. 4.

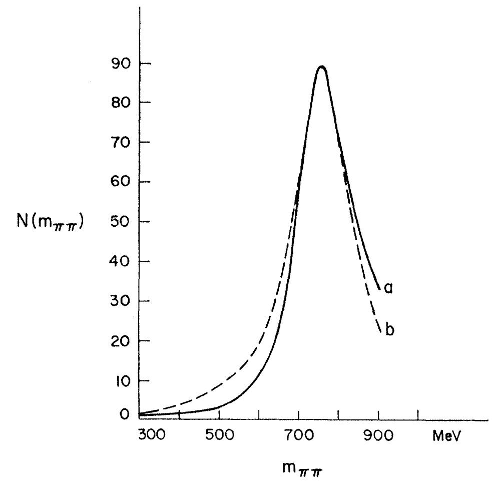

FIG. 4. Dipion mass distribution for process (1) assuming "partial" absorption. Full curve corresponds to δ_1 obtained from Eq. (8), (a). Dashed curve is for δ_1 obtained from Eq. (8), (b). δ_0^0 and δ_0^2 are taken from Walker *et al.*, Ref. 3.

we know that case (i) must be used to fit the data. We also observe that there is a considerable difference between the ρ's in Table I and II. However, the forward-backward asymmetry

$$\frac{F-B}{F+B}\equiv\frac{\bar\sigma(\theta_\pi<\frac{1}{2}\pi)-\bar\sigma(\theta_\pi>\frac{1}{2}\pi)}{\bar\sigma(\theta_\pi<\frac{1}{2}\pi)+\bar\sigma(\theta_\pi>\frac{1}{2}\pi)}=\frac{3\rho_0\,\mathrm{Re}(A_0{}^*A_1)}{\rho_s|A_0|^2+3(2\rho_{11}+\rho_{00})|A_1|^2} \quad (6)$$

[where $\bar\sigma$ is the cross section (4) integrated over θ_p and φ_π] calculated using Table I differs from that using Table II by $\lesssim 20\%$.

Malamud and Schlein have analyzed data on processes (1) and (2) by assuming that the density ρ

TABLE II. Density matrix elements ρ, in μb/MeV, from the absorption model for processes (1)–(3) computed as functions of $m_{\pi\pi}$ with "full" absorption. The upper line, for each energy, is for $1>\cos\theta_p>0.9765$. The lower line is for $0.9765>\cos\theta_p>0.909$.

$M_{\pi\pi}$ (MeV)	ρ_s	ρ_1	ρ_0	ρ_{11}	ρ_{10}	$\rho_{1,-1}$	ρ_{00}
300	0.156	−0.269	0.302	1.025	−0.522	0.089	0.599
	0.108	−0.306	0.349	2.084	−1.061	0.194	1.196
430	0.453	−0.269	0.521	0.352	−0.308	0.026	0.604
	0.318	−0.287	0.492	0.664	−0.464	0.114	0.793
525	0.610	−0.261	0.667	0.255	−0.283	0.019	0.735
	0.436	−0.273	0.593	0.464	−0.382	0.105	0.834
600	0.721	−0.251	0.776	0.208	−0.266	0.016	0.840
	0.526	−0.263	0.672	0.372	−0.341	0.096	0.886
675	0.803	−0.239	0.859	0.179	−0.250	0.014	0.924
	0.598	−0.255	0.736	0.317	−0.314	0.089	0.933
740	0.861	−0.225	0.919	0.158	−0.233	0.012	0.987
	0.657	−0.248	0.788	0.281	−0.294	0.081	0.974
800	0.900	−0.210	0.960	0.142	−0.217	0.011	1.031
	0.703	−0.241	0.829	0.254	−0.277	0.074	1.007
855	0.919	−0.195	0.983	0.130	−0.199	0.009	1.058
	0.739	−0.234	0.861	0.233	−0.262	0.067	1.033
910	0.922	−0.179	0.989	0.119	−0.182	0.008	1.067
	0.765	−0.226	0.883	0.217	−0.249	0.060	1.050

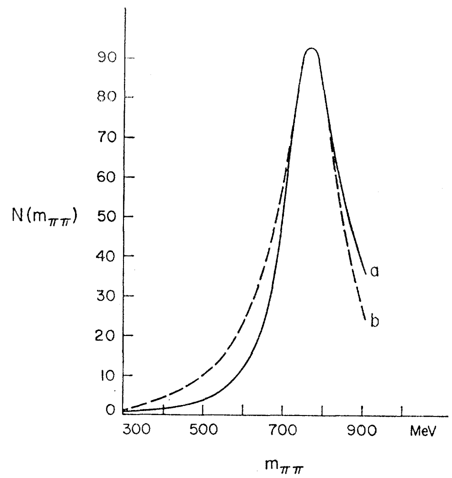

FIG. 5. Dipion mass distribution for process (1) assuming "full" absorption. Full curve corresponds to δ_1 obtained from Eq. (8), (a). Dashed curve is for δ_1 obtained from Eq. (8), (b). $\delta_0{}^0$ and $\delta_0{}^2$ are taken from Walker *et al.*, Ref. 3.

factors in (4) are independent of $m_{\pi\pi}$ near the mass of the ρ meson. We observe from Tables I and II that our model indeed supports this assumption. However, for low $m_{\pi\pi}$, the ρ's depend strongly on $m_{\pi\pi}$. Further, there is no simple dependence which could be factored out. We illustrate this in Fig. 1 where the ratios ρ_s/ρ_{00}, ρ_0/ρ_{00}, and ρ_{11}/ρ_{00} [which determine the $(F-B)/(F+B)$ asymmetry (6)] are plotted as a function of $m_{\pi\pi}$ [for a fixed $\cos\theta_p=0.9765$ and total absorption of the relative $(2\pi)N$ S wave]. Thus if the absorptive model is correct, the Malamud-Schlein analysis could not be carried out for $m_{\pi\pi}\lesssim 600$ MeV.

Malamud and Schlein scheme allows for a t dependence in the phenomenological density ρ elements. The t dependence of our calculated ρ's, averaged over the $m_{\pi\pi}$ interval they worked in, agree with their analysis. We illustrate the effects of the t dependence we obtain by plotting the $(F-B)/(F+B)$ asymmetry (6) at a mass $m_{\pi\pi}=525$ MeV. (See Fig. 2.)

Whereas the position and width of the ρ meson[9]

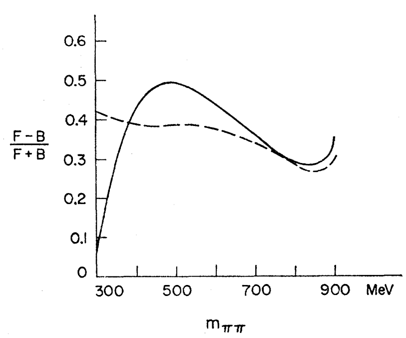

FIG. 6. Forward-backward asymmetry $(F-B)/(F+B)$ for process (1) assuming "partial" absorption. Full curve corresponds to δ_1 obtained from Eq. (8), (a). Dashed curve is for δ_1 obtained from Eq. (8), (b). $\delta_0{}^0$ and $\delta_0{}^2$ are taken from Walker *et al.*, Ref. 3.

[9] We use the values $m_\rho=765$ MeV and $\Gamma_\rho=150$ MeV. In the last few years the values quoted for Γ_ρ (as obtained from peripheral production processes) have varied from 120 to 150 MeV. More recently, an experiment $e^++e^-\rightarrow\pi^++\pi^-$ [Auslander *et al.*, Phys. Letters **25B**, 433 (1967)] gave the value 95 MeV. If this latter value holds up, an entire reevaluation of the peripheral mechanism should be in order. Possibly, an inadequate treatment of the background events is responsible for the discrepancy.

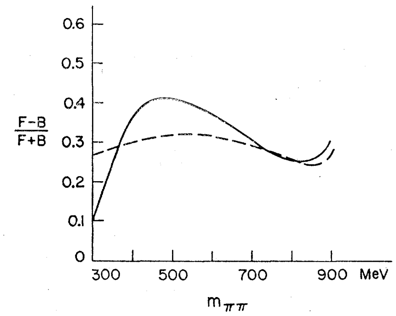

FIG. 7. Forward-backward asymmetry $(F-B)/(F+B)$ for process (1) assuming "full" absorption. Full curve corresponds to δ_1 obtained from Eq. (8), (a). Dashed curve is for δ_1 obtained from Eq. (8), (b). $\delta_0{}^0$ and $\delta_0{}^2$ are taken from Walker *et al.*, Ref. 3.

determine A_1 for $m_{\pi\pi}\sim m_\rho$, the detailed behavior of A_1 away from the peak is not known. To illustrate this ambiguity, we take two different forms for the energy dependence of the Breit-Wigner width Γ_ρ (see Ref. 10):

$$A_1(m_{\pi\pi})=\frac{m_{\pi\pi}}{(m_{\pi\pi}{}^2-4m_\pi{}^2)^{1/2}}\frac{m_\rho\Gamma_\rho(m_{\pi\pi})}{m_\rho{}^2-m_{\pi\pi}{}^2-im_\rho\Gamma_\rho(m_{\pi\pi})}, \quad (7)$$

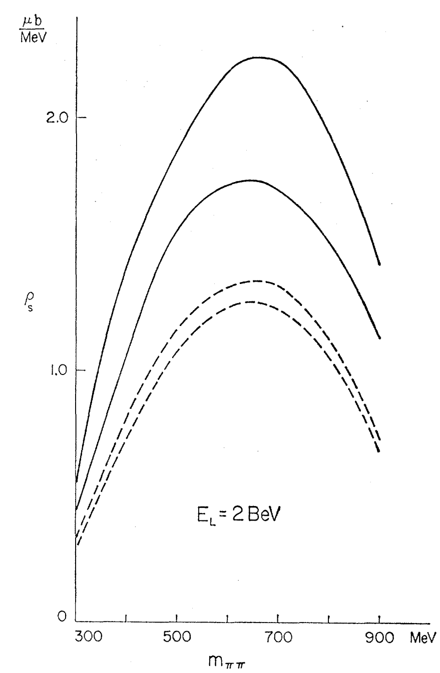

FIG. 8. ρ_s (μb/MeV) as a function of $m_{\pi\pi}$ and t for $E_L=2$ BeV. Full curves are for $\cos\theta_p<0.9765$. Dashed curves are for $\cos\theta_p>0.9765$. Upper curves of both pairs correspond to "partial" absorption; lower curves to "full" absorption.

[10] These are extreme forms of the energy dependence given by

$$\Gamma_\rho(m_{\pi\pi})\propto\frac{(m_{\pi\pi}{}^2-4m_\pi{}^2)^{3/2}}{m_{\pi\pi}}\frac{1}{1+\beta^2m_{\pi\pi}{}^2}.$$

with

$$\text{(a)}\quad \Gamma_\rho{}^a(m_{\pi\pi}) \propto \frac{(m_{\pi\pi}{}^2-4m_\pi{}^2)^{3/2}}{m_{\pi\pi}},$$
$$\text{(b)}\quad \Gamma_\rho{}^b(m_{\pi\pi}) \propto \left(\frac{m_{\pi\pi}{}^2-4m_\pi{}^2}{m_{\pi\pi}{}^2}\right)^{3/2}. \tag{8}$$

Figure 3 shows the phase shift δ_1 as a function of $m_{\pi\pi}$ corresponding to the forms (a) and (b) both having the same ρ width of 150 MeV. The $\pi\pi$ mass distribution

$$N(m_{\pi\pi}) = \int \sigma\, d\cos\theta_p d\cos\theta_\pi d\varphi_\pi, \tag{9}$$

and the asymmetry $(F-B)/(F+B)$ are very dependent on the values of δ_1. We illustrate this for process (1) in Figs. 4–7.

A number of different sets of $\delta_0{}^I$ have been obtained by a variety of methods.[3,11] We have not attempted to compare all these different sets of $\delta_0{}^I$ with experiment since the number of events at low $m_{\pi\pi}$ is quite limited. For example, it is not clear at all whether $(F-B)/(F+B)$ change sign at low $m_{\pi\pi}$ for processes (1) and (2). However, we hope that Tables I and II will be proven useful in distinguishing between the various $\delta_0{}^I$ (and δ_1) at low $m_{\pi\pi}$ when the data becomes sufficiently accurate.

Finally, there is a nontrivial dependence of the ρ's on the incident energy E_L. Since most of the relevant experiments have been done at $\sim$4 BeV, we have presented our results for this energy. However, an experiment determining the mass plot $N(m_{\pi\pi})$ for the $\pi^0\pi^0$ production process (3) have been done at $E_L\sim 2$ BeV.[12] Thus we give the density matrix elements $(\rho_s m_{\pi\pi})$ at this energy in Fig. (8).

We wish to thank Dr. Z. G. T. Guiragossian for helpful discussions concerning the experimental situation.

[11] H. J. Rothe, Phys. Rev. **140**, B1421 (1965); G. F. Chew, Phys. Rev. Letters **16**, 60 (1966); C. Lovelace, R. M. Heinz, and A. Donnachie, Phys. Letters **22**, 332 (1966); I. Fuji, *ibid.* **24B**, 190 (1967); University of Tokyo Report, 1967 (unpublished); M. G. Olsson, University of Wisconsin Report, 1967 (unpublished); J. R. Fulco and D. Y. Wong, Phys. Rev. Letters **19**, 1399 (1967); D. V. Shirkov, USSR Academy of Sciences, Novosibirsk Report, 1967 (unpublished).

[12] I. F. Corbett *et al.*, Phys. Rev. **156**, 1451 (1967).