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Systematic Underestimation of Maximum Crest Heights in Deep Water Using Surface-Following Buoys

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Abstract

Knowledge of the maximum wave elevation above mean sea level (as opposed to the wave height) is of primary importance in the design of large deep water structures. In particular, because of the hazards associated with waves breaking over the production equipment on offshore oil and gas platforms, it is typical for regulatory agencies to insist that positive clearance be maintained between the deck and the anticipated largest crest. The paper shows that the most commonly employed measurement device, the wave-following buoy, systematically underestimates the elevation of the largest waves and that these errors are significant. The magnitude of the correction is disclosed and, methodology is presented for correcting the measured time series (when it is available) or for making estimates of the appropriate correction in height from the statistics of the waves.

Introduction

Historically, field wave measurement programs have resulted in a condensed set of parameters describing the observations. Significant wave height (H_s), either directly derived from the elevation record, or estimated from its spectrum, has been the minimum reported value, often accompanied by the period of the peak in the energy spectrum (T_p) or some similar parameter. Because deep water waves appear to be approximately Rayleigh distributed, other height parameters have been readily estimated from H_s. Fixed platforms in deep water have been relatively rare until recent times, so that much of the deep water observations have been made from surface following buoys. The most often used device of this type is the Datawell Waverider buoy (Figure 1). Waverider observations have been compared with fixed platform measurements in a number of instances, for example Allender et al. (1989) and O'Reilly et al. (1996). These intercomparisons show that the Waverider provides essentially equivalent significant wave height and energy spectrum estimates when compared to wave staffs or pressure sensors. O'Reilly et al. found significant differences in the higher moments (skewness and curtosis), which suggests that all

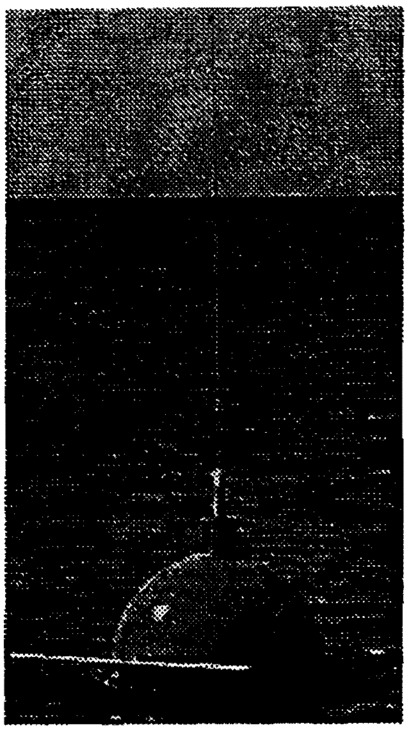

Figure 1 Waverider Buoy

of the details of the fixed platform elevation record may not be duplicated by the surface following buoy. Allender et al. noted that for high sea states, the Waverider underestimated the spectral energy above 0.3 Hz. This resulted in a response paper by Rademakers (1993) in which the theoretical differences in the elevation recorded by an orbit following device and a fixed instrument are described. However, Rademakers' analyses concentrated on the frequency domain, rather than the time domain effects.

To visualize the differences between a fixed-position wave observation and one which follows orbits, first assume that an orbit-follower would have, in deep water, a time history of vertical motion for a simple sinusoidal wave given by

$$z_o = a \cos(wt - kx) \qquad [1]$$

where z_o = the vertical motion as observed by an orbit-follower, a = the amplitude of the orbit, w = radial frequency, k = wave number, and x = horizontal distance from the center of the orbit.

in this case, x=0 because the observer maintains a fixed position relative to the center of the wave orbit.

An observer at a fixed geographic location at the nominal rest position of the buoy would experience, observing this same orbit-follower, a changing value of x because of its horizontal motion. Another way to visualize this difference is to assume that the measurement is made with a wave staff, which – instead of being fixed to the observer's platform – is constrained to move horizontally with a motion of x = a sin wt. Substituting x = a sin wt into eq. [1], we find that the time history of the fixed observation vertical motion is given by

$$z_f = a \cos(wt - ak \sin wt) \qquad [2]$$

where z_f = the vertical motion observed from a fixed platform mounted at the nominal rest position of the buoy.

Two cycles of z_o and z_f are plotted in Figure 2 for a sinusoid of exaggerated steepness , which shows that the fixed observation is a trochoid. Compared to the sinusoidal orbit-follower, the fixed observer sees a sharper, higher peak and a flatter, shallower trough. It is extremely important to recognize that, although both curves exhibit the same wave height, mean sea level is lower relative to the crest in the trochoid, resulting in an increase in the maximum crest elevation. In nature, of course, a fixed platform "sees" these same higher crests. It is therefore important that the platform designer understand the effects on maximum sea surface elevation when wave records are based upon surface-following buoy data. It is important to note that eq. [2] does not contain directional information. That is, the correction $z_f - z_o$ arising from a single wave component is independent of the direction of travel of that component. In addition to this difference, Peter Gerritzen

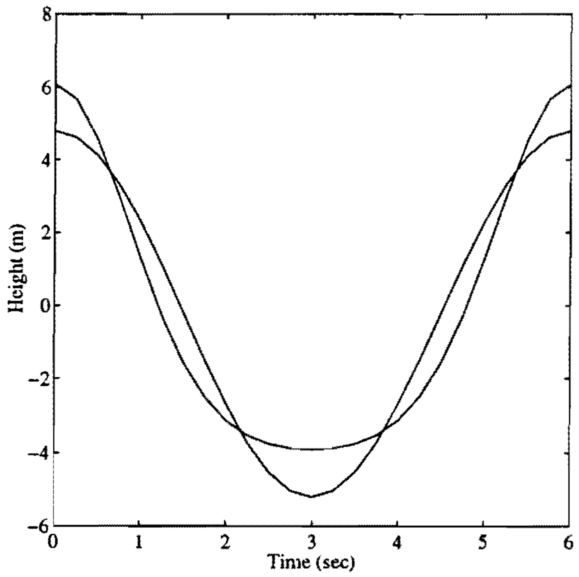

Figure 2– Equations 1 and 2 plotted for a sinusoid of exaggerated steepness to illustrate the differences in sea surface elevation observations between a moving and a fixed observer.

of Datawell pointed out in a personal communication that the Waverider contains a filter that distorts wave phases at all frequencies, but not the amplitudes at which there are significant values of energy. Because this has no effect on the estimates of the energy spectrum or its lowest order moments, no attempt has been made to remove the phase shifting from the elevation record.

Fixed platforms, or floating systems with restricted vertical response to waves such as Tension Leg Platforms or Spars, are typically designed so that the highest anticipated sea surface elevation will not result in impact or damage to the working superstructure. In the case of offshore oil and gas platforms, industry standards such as the American Petroleum Institute Recommended Practice 2A, or government regulations, dictate a minimum clearance between the deck and the highest wave elevation. For large platforms, increasing this clearance even a small amount results in very significant cost increases. It is therefore necessary for offshore engineers to have the best possible estimate of the maximum expected wave elevation above mean sea level or some other appropriate reference. This was recognized in Kriebel and Dawson (1993) which studied distributions of extreme crest heights in laboratory and ocean waves.

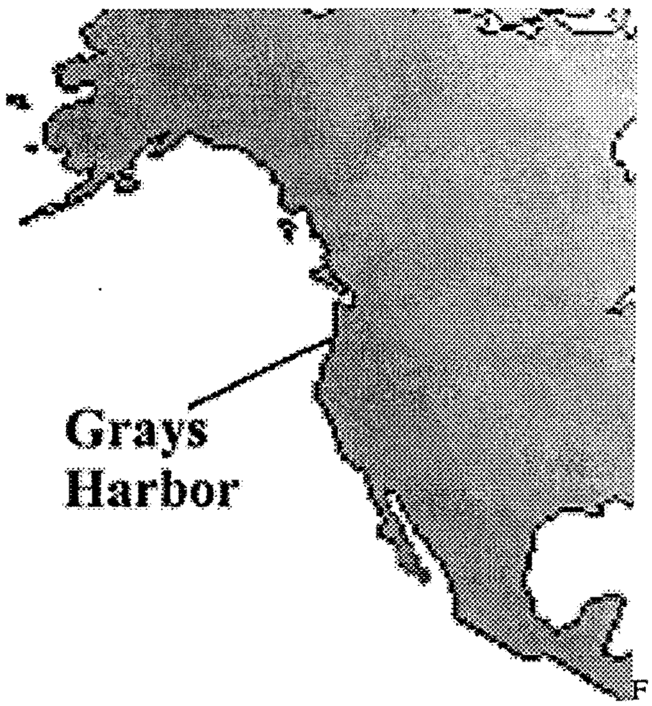

Figure 3 -- Grays Harbor Site

Tulin et al. (1996) raises questions about the ability of wave buoys to correctly measure extreme crest height. This conclusion was based upon physical and analytical modeling of extreme waves in Tulin's laboratory. This motivated the present study, in which a large data base of wave records taken with a Waverider buoy is corrected from orbit-following to fixed observations and the effects of the filter are investigated.

Data Source

A four-month-long series of measurements was obtained from a non-directional Datawell Waverider buoy moored at a depth of 42.6m, offshore Grays Harbor, Washington, on the Pacific Coast of the USA (see Figure 3). This unsheltered coastline is subject to a severe winter wave climate. The data were obtained from the archives of the Coastal Data Information Program (CDIP) at the Scripps Institution of Oceanography (SIO) (http://cdip.ucsd.edu). The CDIP wave data gathering network is described in Seymour et al. (1993.) The records were 8192 seconds in length with a 1Hz sampling rate (~2.25 hours.) A total of 860 records (almost 2000 hours) were examined, approximately evenly distributed over the first four months in 1991. To characterize the incident wave climate, the median significant wave height at Grays Harbor was 2m during that year and there were 4 intervals during 1991 when the significant wave height was in the range of 5.7m to 6m.

Methodology

The software filter used in the Waverider is a 3-pole Butterworth type with a cutoff frequency of 0.03247 Hz. The first stage in this research was to evaluate the effects of the filter on the surface elevation measurements. The filter high-passes the acceleration signal to remove all low frequency components. The simplest way to visualize the need for this is to consider the double integration in the frequency domain, where it is equivalent to dividing the spectral value by the square of the frequency. This clearly greatly amplifies any noise at frequencies close to zero and necessitates the high pass operation. A detailed description of the Butterworth filter can be found in Parks and Burrus (1987.) The filter characteristics were determined by using the Matlab routines *butter* and *freqz* which allow the calculation of amplitude and phase transfer functions. The time series of elevation was Fourier transformed using the Matlab routine *fft*. These filter functions were then applied in the reverse direction to the Fourier transform, effectively removing the effects of the filter on the elevation time series calculated by the buoy. No correction was made at frequencies lower than the cutoff frequency. The corrected time series was recovered with the Matlab routine *ifft*.

Each of the time series were subjected to this transformation and the statistics of the original and the corrected series were compared. For each time series in the pair, the significant wave height as well as the maximum and minimum elevations in the record were determined and recorded. The reverse-filtering had essentially no effect on the significant wave height, as shown in Figure 4. The period of the peak in the energy spectrum was obtained from the uncorrected Waverider record. A test was performed on all the data from one month which showed that none of the corrections made any change in the peak period estimate.

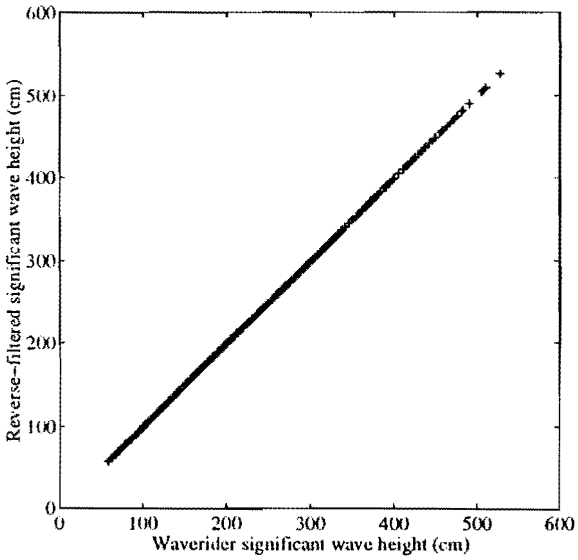

Figure 4 -- Comparison of significant wave heights for filtered and reverse-filtered records.

The second stage of the research was an investigation of the effects of buoy motion on the sea surface elevation that would have been seen at a fixed reference point, such as an offshore platform. Extending the concept of eq. [2] into a field of random waves, Rademakers (1993) shows that, after some simplifying assumptions, the elevation time history as recorded by a surface following buoy can be corrected to a fixed position observation with a term of the form

$$c(t) = \left(\sum_{n=1}^{\infty} k_n a_n \ \sin(\omega_n t + \phi_n) \right) \left(\sum_{m=1}^{\infty} a_m \sin(\omega_m t + \phi_m) \right) \qquad [3]$$

Note that Rademakers' approximation (eq. [3]) does not contain directional terms. As discussed above, each wave component – regardless of its direction in the x-y plane – contributes to the correction term in the z-direction as a function only of its amplitude, frequency and phase. Equation (3) was calculated using Matlab and the correction was applied to each time series in the total record. The same three parameters were then calculated and recorded for each of the revised elevation records as described above for the reversed filtering operation.

The final analysis involved combining both the filter and the buoy motion corrections, and calculating the fully corrected surface elevation history. Again, the same three parameters were found and recorded for each of the records. In this analysis, the elevation time series was corrected through reverse filtering prior to the calculation of equation [3].

Results

Correcting for the filter in the elevation record resulted in a significant change to the elevation time series, even though there was no appreciable change to the significant wave height, as shown in Figure 4. Figure 5 illustrates the distortion caused by phase shifting. To evaluate the effect of this on the maximum crest height in the record, the values of this parameter for both the filtered (Waverider output) and reverse-filtered records were found and a normalized error, ε, (positive error implies underestimation by the Waverider) calculated.

$$\varepsilon = \frac{\eta(reverse - filtered) - \eta(measured)}{H_S} \qquad [4]$$

These normalized errors are plotted against H_s in Figure 6. This plot suggested that the error was random and had a zero mean. This was tested by binning all of the errors into 10 height bins and then calculating the means and the standard deviations of each bin. The results of this effort, shown in Figure7, clearly establishes that the error is evenly distributed about zero for all values of significant wave height and that its maximum (3 sigma) expected value is about 15% of the significant height in the record. Similar analyses were performed plotting the errors against the period of the energy peak in the spectrum and against a steepness parameter. In both cases the error was observed to be randomly distributed about zero.

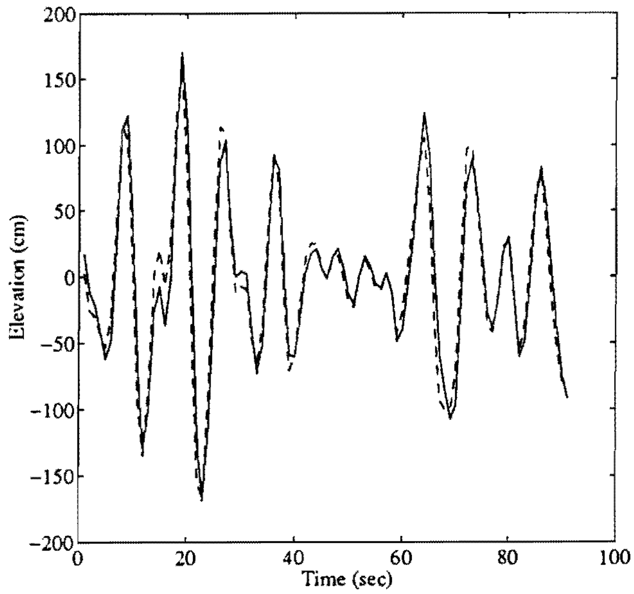

Figure 5 -- Typical distortion of the surface elevation record by the Waverider filter. Solid line is Waverider, dashed is correction by reverse-filtering.

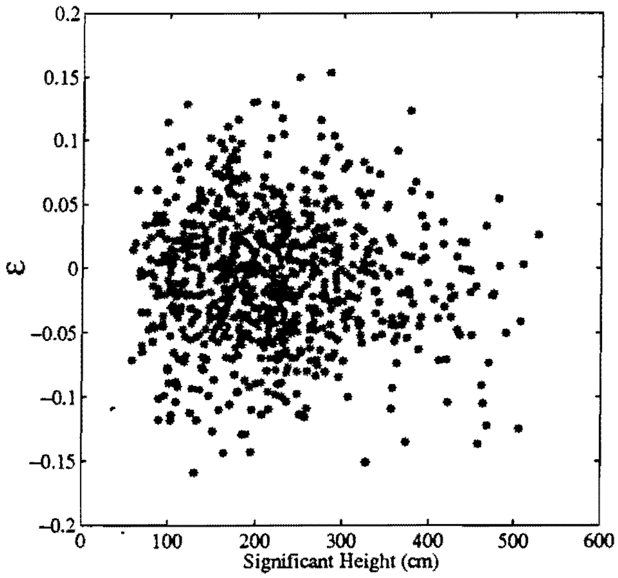

Figure 6 – Normalized filter-induced error plotted against significant height

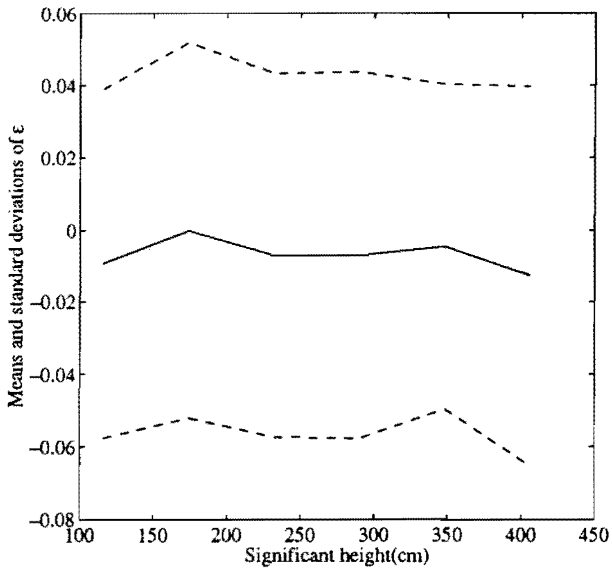

Figure 7 -- Means and standard deviations of the filter-induced normalized error (samples grouped in 10 bins.)

The next analyses involved isolating the effects of the buoy motion on the surface elevation as observed from a fixed platform. Filter effects were ignored at this time and the Waverider output was assumed to be the correct vertical excursion of the buoy. A revised maximum crest elevation for each record was calculated and the results from all the data were correlated against the observed significant wave height, the peak period of the spectrum and a steepness parameter defined by

$$S = kH_s \qquad [5]$$

The squared correlation coefficients of the corrections compared to the three parameters are shown in Table I.

Table I
Correlations between corrections for buoy motion to maximum crest elevation and various wave parameters

Parameter correlated	Squared correlation coefficient (r^2)
Steepness, S	0.47
Significant height	0.41
Period of peak energy	0.10

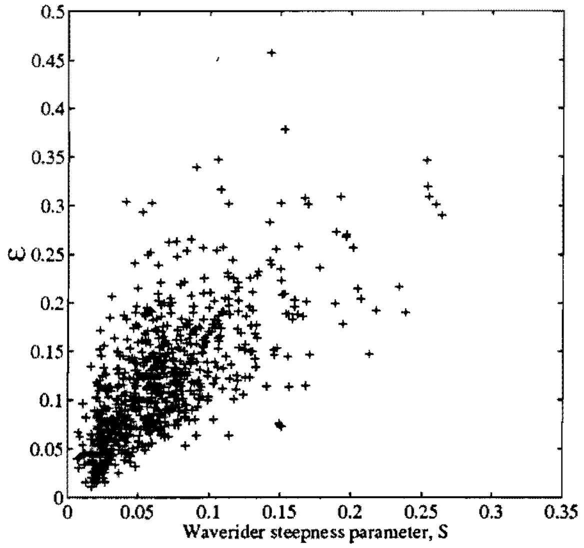

Figure 8 -- Normalized correction for buoy motion, ε, as a function of steepness parameter, S (eq. [5])

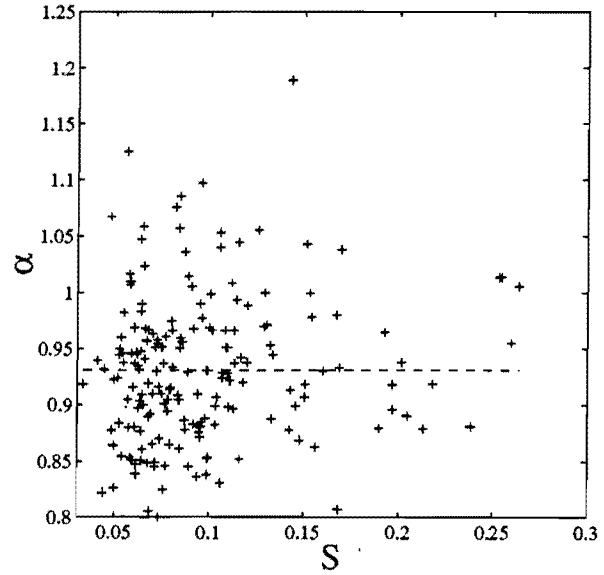

Figure 9 -- Normalized extreme crest height, α, for 15% highest steepness Waverider records, plotted against S. Dashed line represents the average level.

Steepness offers a slightly improved correlation compared to height, and the period is essentially uncorrelated. The correction, ε, is plotted against the steepness parameter,

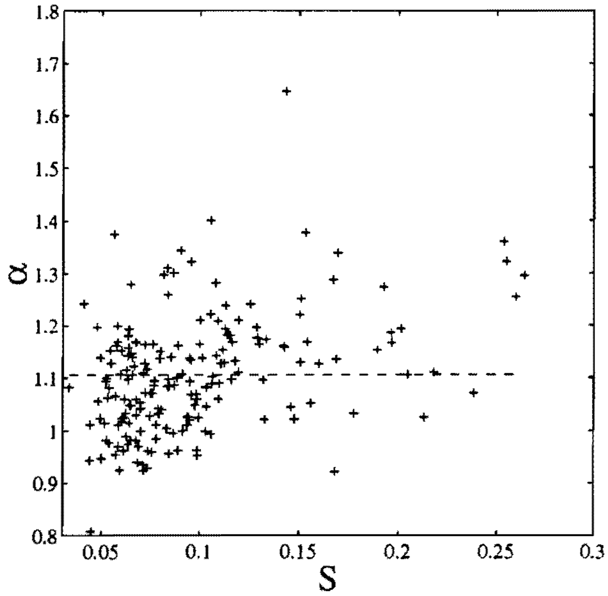

Figure 10 -- Normalized extreme crest heights corrected for buoy motions for 15% highest records, plotted against S. Dashed line is average.

S, in Figure 8. Although there was significant scatter in the data, due in some part to the effects of spectral shape that are not captured in the simple steepness parameter, the value of the correction trend to zero at low steepness and to higher values at values of S exceeding about 0.1.

It is clear from Figure 8 that many of the records had relatively low steepness. Investigation showed that the high steepness records corresponded almost completely with the highest significant wave heights. As these are the conditions of greatest interest here, a subset of the 15% highest steepness records (a total of 123) was selected. In Kriebel and Dawson, a ratio r/H_s was utilized, in which r is the maximum crest elevation in the record. This ratio will be referred to here as α for simplicity. The maximum values of α for the steep wave subset are plotted against S in Figure 9 and the average value of α is shown by the dashed line to be 0.95. The median peak period for these records was 11.25 seconds. This would allow for about 730 of these waves during the record such that the highest crest elevation probability would be about 0.0014. The field data reported in Kriebel and Dawson exhibited a mean α of about 1.1 at this probability, clearly illustrating the underprediction of the crest extreme by the wave-following buoy of about 16% on average. A similar plot, showing the corrected α is shown in Figure 10. The dashed line indicates the average value of 1.14, reasonably close to the Kriebel and Dawson findings.

A simplified model for predicting the effects of buoy motion on the maximum crest elevation as seen on a fixed platform is obtained from the average values of α from the observations and the corrected values, which suggests a correction of 20% for wave steepness greater than about 0.1. Because the maximum crest elevation is not always available from archived records, an estimate can be made directly from the significant wave height.

$$r = 1.14H_s \qquad [6]$$

Finally, the two effects were combined. The Waverider measurement was reverse-filtered to provide a corrected record of vertical position. This time series was then corrected for an observer at a fixed location. The results are shown in Figure 11. The $\alpha = 1.14$ line is plotted over the fully corrected results and plus and minus 15% limits are applied to it to account for the range of observed normalized error introduced by the filter (see Figure 6.) Figure 11 shows that for the full range of wave conditions, but particularly for the higher waves (H_s greater than 3m) that are of the greatest interest, the model of eq. [6], with limits taken from the filter analysis, satisfactorily encompasses most of the corrected elevations.

Conclusions

The software filter used in the Waverider buoys, while not affecting the energy spectrum or its lowest order moment, seriously distorts the phases and can result in an error in the maximum crest elevation in the record anywhere in the range of -15%to +15%. A wave-following buoy measures a different surface elevation history from that obtained with a fixed observation point at the nominal rest position of the buoy. The fixed observer sees higher, sharper crests and higher, broader troughs. Again, the energy spectrum is not significantly altered between the two observations. However, the value of the maximum crest elevation is changed significantly. For those records with the largest maximum crest elevations (3m to 6m), the Waverider values are underestimated from 0 to 38%, with the most likely value being 20%. These conclusions are based upon the assumption that the buoy is a true wave-follower. If the Waverider tends to avoid sharp crests in a short-crested sea, the underestimation can be even greater.

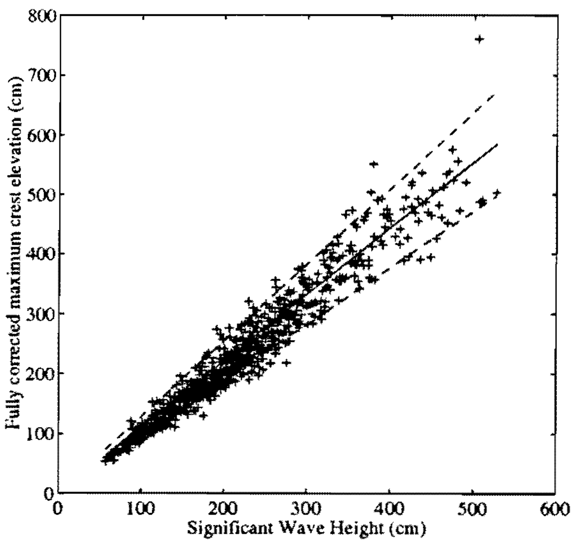

Figure 11 -- Results of combining both filter and buoy motion corrections to all of the records. The solid line is eq. [6] and the dashed lines are +/- 15% allowances for filter effect uncertainties.

Where access to the original Waverider elevation record can be gained, a method has been demonstrated for correcting this record so that it represents a realistic estimate of the actual time series. When the full record is not available, the investigator must assume a crest elevation distribution and estimate the highest crest elevation indirectly. A value of α of 1.14 for a probability of close to 0.001 is developed here and estimates of other probabilities can be found in Kriebel and Dawson (1993).

Acknowledgments

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