Title
MAXIMUM POWER FACTOR OF A RECTANGULAR PULSE

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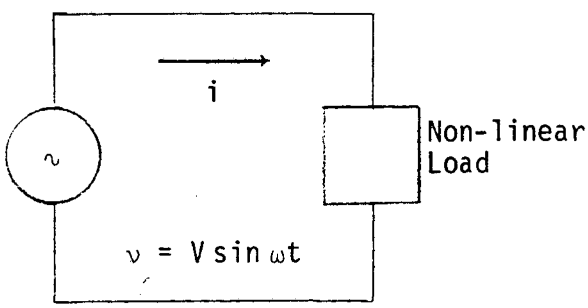
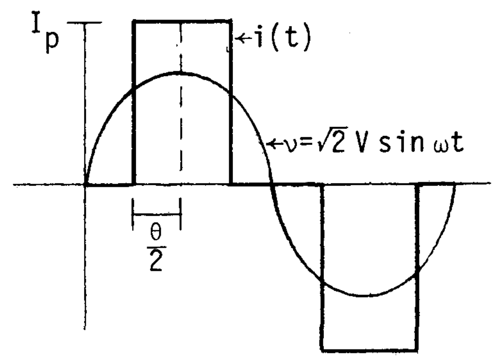
MAXIMUM POWER FACTOR OF A RECTANGULAR PULSE

Introduction

There has been a recent proliferation of electrical devices, such as the electronic ballast for fluorescent lamps, which draw a non-sinusoidal current from the three-phase, 60 Hz., power mains. The harmonic current components will reduce the power factor, sometimes called distortion power factor, and cannot be corrected by conventional means. In the case where the current waveform is a rectangular pulse, it would be interesting to know what is the maximum possible power factor.

Theory





power factor (pf) $= \frac{P}{VI}$

Because the voltage is sinusoidal, only the in-phase fundamental component of the current waveform will contribute to the power.

To find the fundamental component of the current, use Fourier analysis.

Generally, $i(t) = \frac{a_0}{2} + \Sigma a_n \cos n\omega t + \Sigma b_n \sin n\omega t$

where a_0 is the d.c. component,

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos n\omega t \, d(\omega t)$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin n\omega t \, d(\omega t)$$

Here, we need only b_1.

Let $x = \omega t$

$$b_1 = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin x\,dx = \frac{1}{\pi}\int_{90°-\frac{\theta}{2}}^{90°+\frac{\theta}{2}} I_p \sin x\,dx + \frac{1}{\pi}\int_{270°-\frac{\theta}{2}}^{270°+\frac{\theta}{2}} -I_p \sin x\,dx$$

$$= \frac{I_p}{\pi}\left|-\cos x\right|_{90°-\frac{\theta}{2}}^{90°+\frac{\theta}{2}} + \frac{(-I_p)}{\pi}\left|-\cos x\right|_{270°-\frac{\theta}{2}}^{270°+\frac{\theta}{2}}$$

$$= \frac{I_p}{\pi}\left[-\cos(90°+\frac{\theta}{2}) + \cos(90°-\frac{\theta}{2}) + \cos(270°+\frac{\theta}{2}) - \cos(270°-\frac{\theta}{2})\right]$$

$$= \frac{I_p}{\pi}\left[\sin\frac{\theta}{2} + \sin\frac{\theta}{2} + \sin\frac{\theta}{2} + \sin\frac{\theta}{2}\right]$$

$$= \frac{4}{\pi} I_p \sin\frac{\theta}{2},$$ the fundamental in-phase component peak value.

$$pf = \frac{P}{VI} = \frac{V_{rms} I_{fund}}{V_{rms} I_{rms}} = \frac{I_{fund}}{I_{rms}}$$

$$I_{fund} = \frac{b_1}{\sqrt{2}} = \frac{4}{\sqrt{2}\pi} I_p \sin\frac{\theta}{2}$$

$$I_{rms} = \sqrt{\overline{i^2(t)}} = \sqrt{\frac{\theta}{\pi} I_p^2} = \sqrt{\frac{\theta}{\pi}}\, I_p$$

$$pf = \frac{I_{fund}}{I_{rms}} = \frac{\frac{4}{\sqrt{2}\,\pi} I_p \sin\frac{\theta}{2}}{\sqrt{\frac{\theta}{\pi}}\; I_p} = \frac{4}{\sqrt{2}\,\pi}\,\frac{\sqrt{\pi}}{\sqrt{\theta}}\sin\frac{\theta}{2} = \frac{2\sqrt{2}}{\sqrt{\pi}}\,\frac{\sin\frac{\theta}{2}}{\sqrt{\theta}}$$

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To maximize pf, set $\frac{d(pf)}{d\theta} = 0$

$$\frac{d(pf)}{d\theta} = \frac{2\sqrt{2}}{\pi}\left[\tfrac{1}{2}\cos\frac{\theta}{2} \times \theta^{-\frac{1}{2}} + \sin\frac{\theta}{2}\left(-\tfrac{1}{2}\theta^{-\frac{3}{2}}\right)\right] = 0$$

$$\frac{1}{2}\theta^{-\frac{1}{2}}\cos\frac{\theta}{2} - \frac{1}{2}\theta^{-\frac{3}{2}}\sin\frac{\theta}{2} = 0$$

$$\theta\cos\frac{\theta}{2} = \sin\frac{\theta}{2}$$

$$\tan\frac{\theta}{2} = \theta$$

This condition holds when $\theta = 2.33$ radians, or 133.5°,

and $pf_{max} = \frac{2\sqrt{2}}{\sqrt{\pi}} \frac{\sin\frac{\theta}{2}}{\sqrt{\theta}} = \frac{2\sqrt{2}}{\sqrt{\pi}} \frac{\sin 66.75°}{\sqrt{2.33}} = 96\%$

Conclusion

An electrical device designed to draw a rectangular current pulse of width 133.5° will result in a power factor of 96%, which in practice will not cause 3-phase systems to have neutral overload problems due to triplen harmonics.

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