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Abstract

Admati, Demarzo, Hellwig, and Pfleiderer (ADHP, 2018) note that static models of optimal leverage have assumed firms have no prior debt. In this case, the leverage that maximizes firm value also maximizes value to the initial equity owners. However, using a simple two-period model with zero coupon debt and default possible only at maturity, ADHP prove two startling results: (i) when prior debt is extant, it will never benefit equity holders to retire debt, no matter how high the current leverage; and (ii) it will be in the equity owners' interest to issue sequential rounds of additional debt, until all the tax advantages of debt are exhausted: the "Leverage Ratcheting Effect" (LRE). An immediate conclusion is that one-round (static) models of optimal debt issuance with no prior debt provide poor guidance as to a firm's optimal leverage. We examine these contentions using an alternative model of debt, with rollover at a proportional rate m and average maturity = $1/m$, introduced in Leland (1994a). We show that when the average maturity of debt is substantially longer than 5 years, considerable further debt will indeed be issued, although issuance ceases well before tax benefits are exhausted. With 5-year average maturity, very little additional debt is issued under reasonable calibrations. With 3-year average maturity, no additional debt is issued and it may actually be optimal for the firm to buy back debt, in contradiction to the LRE. We explain why our model gives differing results.

Key Words: Capital Structure, Leverage Ratcheting Effect, Debt Maturity, Dynamic Tax Tradeoff Model

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1. Introduction

In a recent article, Admati, DeMarzo, Hellwig, and Pfleiderer (2018, ADHP)) pose a challenge to "traditional" models of capital structure: Whatever the current level of firm debt may be, it will never be in shareholders' interests to reduce it through debt buyback at fair market value. Instead, shareholders will choose to increase leverage even if it reduces firm value, eventually to a level where the tax benefits of debt are totally exhausted. The authors term this the "leverage ratchet effect" (henceforth, LRE).

This suggests that typical static or one-period models of optimal capital structure may have a serious flaw: while they indicate the leverage that optimizes equity value starting from an unleveraged initial situation, shareholders will issue further debt sequentially, perhaps in large amounts. This result holds even when the new debt must be junior to the initial debt issues. The authors conclude that "leverage choices based on static trade-off theory are therefore inherently unstable... the static trade-off theory of capital structure is unlikely to explain the capital structure of firms. The leverage ratchet effect implies that leverage begets more leverage."

The resistance to debt reduction on the open market has been noted previously by Black and Scholes (1973) and Leland (1994b, pp. 1246-7). But the impetus for equity holders to increase debt was not fully understood prior to ADHP.

A question remains as to the generality of the ADHP results across a range of debt models. ADHP uses a simple 1-period model to motivate the no-buyback and LRE results. The firm issues zero-coupon debt with default possible only at maturity. Default occurs when end-period firm value is less than debt face value, similar to the Black Scholes (1973) and Merton (1974) approach. This framework is used to

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motivate the key Propositions 1 ("Shareholder Resistance to Leverage Reduction") and 4 ("The Leverage Ratchet Effect"). [2]

This paper examines the same problem, also in the context where the firm and investors have no foresight that current-round debt choices will be followed by further rounds of debt issuance, even though those may occur.[3] We explore the magnitude of debt increases after the initial leverage choice is made, using the tools in Leland (1994a) where debt is committed (barring default) to be retired (and then replaced) at a constant rate that implies finite average maturity--a generalization of Leland (1994b) which considers infinite-maturity debt only.[4] We conclude that the LRE is not very large for ranges of initial leverage and average maturity approximating those of investment-grade firms, when assuming exogenous parameter calibration consistent with empirical findings (e.g. Feldhütter and Schaefer (2018)).[5] Indeed we even find that equity holders may voluntarily reduce debt, when debt has short maturity. Though the reductions are typically small, they nonetheless provide a counterexample to the assertion that it will never be optimal for equity holders to reduce debt. It also suggests the choice of shorter maturity, while typically costly in the Leland (1994a) model when there is a single debt issuance, may importantly protect bondholders from the LRE when subsequent debt issuance is possible. Our

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[2] In Section III, ADHP (2018) considers a model in which the firm has a constant earnings rate but is liquidated at a random time τ governed by an exogenous arrival rate λ (implying an average "maturity" of $1/\lambda$) at a random value independent of λ. Default probabilities in this approach are independent of the firm's leverage choice, although liquidation value will depend on debt's prior claim to principal value. DeMarzo and He (2016, DH) develops a model similar to Leland (1994a) but with no commitment to future debt retirement/issuance. Their model rules out discontinuous changes in debt (including if the firm starts with no debt), and assumes that the coupon rate c is independent of the total debt principal F, i.e. independent of leverage.

[3] "No foresight" coincides with ADHP's analysis through Section IIIA. We later argue that assuming foresight would only reduce the likelihood of further debt issuance.

[4] By "initial" leverage choice, we mean the first-round issuance of debt when the firm has no prior debt, as considered in most capital structure models.

[5] Choi, Hackbarth, and Zechner (2018) find that average (median) debt maturity outstanding, 2002-2012, is 6.35 (4.81) years, and when including loans average (median) maturity is 5.35 (3.91) years.

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analysis complements that of Dangl and Zechner (2016) that shorter-maturity debt can allow possible

debt reductions by incomplete rollovers, thereby reducing leverage.[6]

While not identical, the LRE bears resemblance to the "asset substitution effect (ASE)" in which equity

owners can raise the value of their claims by raising asset volatility once debt is in place, even when it

reduces the total firm value. In both these cases, agency costs arise because the value of previously-

issued debt can be reduced by *ex post* decisions of the firm, creating a wedge between equity value-

maximizing decisions and firm value-maximizing decisions.[7] It also bears resemblance to the "debt

overhang problem" first elucidated by Myers (1977).[8]

Agency problems may be reduced or eliminated by additional covenants or enforceable commitments in

the debt contract. Terms prohibiting use of derivatives, for example, may ameliorate the ASE. As ADHP

note, assigning absolute priority to pre-existing debt in default may reduce the LRE but not eliminate it,

as our results below confirm.[9] DeMarzo (2019) also notes further restrictive covenants reducing

operational or financial flexibility, relationship banking (where the preservation of previously-issued

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[6] In the models of Dangl and Zechner (2016) and DeMarzo and He (2016), debt will at most be reduced continuously at rate m, when all maturing debt (at rate m) is refunded using equity rather than debt. There will never be discrete debt reductions in these models.

[7] The agency costs associated with the ASE are explored in Leland and Toft (1996) and Leland (1998). They show that shorter maturity debt reduces asset substitution benefits to equity, and results in smaller agency costs. Our results below show that shorter maturity debt can also reduce (or eliminate) the incentive for sequential debt increases.

[8] Myers (1977) suggests shorter maturity can mitigate debt overhang costs. Diamond and He (2014) note that although shorter-term debt value is typically less sensitive to changes in asset value at time of issuance, the higher default boundary associated with refinancing shorter term debt can lead to greater debt overhang when cash flows deteriorate. Here we include changes in the (endogenous) default boundary with maturity, but assume investment decisions are fixed.

[9] Earlier theoretical papers show that LRE can be eliminated by requiring that current debt be retired if new debt is introduced. However, as an empirical matter, such covenants are rarely if ever observed. Renegotiation of debt can also "solve" the LRE, as discussed both in Leland (1994b, Section VIII) and ADHP (2018, p.157). Mao and Tserlukevich (2015) show difficulties in renegotiation with multiple creditors.

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debts' values may affect willingness to issue dilutive securities), and reputation building may limit additional debt issuance.

Here, we show debt maturity reduction can reduce or eliminate the LRE. While we do not examine explicitly the tradeoff between agency costs of the LRE and the additional potential default costs from shorter maturity, our analysis points the way towards doing so.

Our basic examination is quite rudimentary. Following ADHP (Sections II through IIIA), we first ask (in Round 1), "what is the optimal leverage when the firm starts with zero leverage," assuming no foresight of further debt offerings. This will generate "first round" or initial leverage L_1, the optimal leverage when there is no prior debt. This is identical to the optimal leverage considered in Leland (1994a, 1994b). Then, considering a second round of potential additional leveraging, we ask "what is the optimal leverage L_2 when starting with L_1?" We assume the first round happens immediately after the initial leveraging choice L_1, and there is no change in the underlying parameters including asset value (we later consider alternatives). We continue this process through many rounds, or until the increase in equity value is less than 0.01%. In all our examples, we assume absolute priority: debt issued in any round has claims to default value senior (up to principal value) to any subsequent debt issues.

Our principal results, based on a reasonable range of model parameters, are as follows:

➢ When the average maturity of debt is at least 10 years, the LRE is confirmed and may continue through many rounds of debt issuance.[10] However:

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[10] Our examples seek to be consistent with the range of empirically estimated parameters for asset volatility, payout ratios, tax advantages to debt, and default costs. We consider different debt maturities, and seek final parameter choices to generate an optimal leverage ratio in the initial round of issuance close to the BBB-rated average of about 40% (e.g. Huang and Huang (2012), Feldhütter and Schaefer (2018)).

- Additional rounds of debt offer relatively small increments to equity value beyond 2-3 rounds, and incur substantially higher borrowing costs (which may reduce the firm's debt rating).

- With reasonable cutoff points for further debt issuance (e.g. improvement of equity value of 0.01% or less), the ultimate total of debt issuance will be far from exhausting the net tax benefits of debt.

- Given fixed maturity, the LRE is likely to be greater when parameters lead to larger initial leverage. The LRE is also greater when initial leverage is the same and default costs are lower..

➤ The benefits of further debt issuance fall rapidly as maturity falls, and in our base case there are no benefits to even one round of additional debt when maturities are 3.6 years or less.[11]

➤ With the maturity of debt less than 3.6 years in the base case, it is advantageous for equity holders to reduce leverage from the original level through debt buybacks, contradicting the contention that it is never optimal to reduce debt. At the "neutral maturity" of 3.6 years, equity holders will not wish to change leverage from an initial leverage of 42%.

These results challenge the conclusions of ADHP's Propositions 1 and 4. We do not disprove ADHP's propositions, given their assumptions; but in our model of fully rolled-over debt, their Propositions 1 and 4 do not hold at shorter maturities.

An important difference in our models is that the firm is committed to maintaining a constant capital structure, barring endogenous default. This implies full debt rollover so that debt principal remains

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[11] We observe in our examples that, in comparison with ADHP (Section III.A.2), the "time inconsistency" in our model with no foresight is substantially reduced, and virtually disappears as maturity shortens.

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constant through time.[12] . In contrast with ADHP and DeMarzo and He (2016), but in common with

Dangl and Zechner (2016) and many other studies, our model also constrains the coupon to be set so

that debt initially sells at par value.

2. THE MODEL

Following Merton (1974), Leland (1994a) and many others consider a firm whose underlying asset value

V follows a risk-neutral geometric Brownian motion with

$$dV_t = \mu V_t dt + \sigma V_t dZ_t, \tag{1}$$

where $\mu = r - \delta$ is the risk-neutral drift of asset value, r is the risk-free rate of interest, and δ is the

payout ratio of asset value to security holders, all assumed to be constant. Volatility is given by σ, and

dZ_t is the increment to a standard Gauss-Wiener process.

The firm issues debt at time *t* = 0 with principal *P* and coupon rate *C*. The coupon rate *C* is set so the

bond initially sells at par. Debt has no explicit maturity date, but promises to retire extant debt principal

at par at the continuous proportional rate *m*, similar to a "sinking fund." As in Leland (1994a) we assume

that *m* is given exogenously, implying an average debt maturity of 1/*m*. Also by assumption, debt is fully

rolled over: the debt retired at any future time *s* is continuously replaced with debt having equal

principal and coupon. Thus total debt principal *P* and total coupon rate *C* remain constant through time,

barring default. Debt issued at any time will have claims in future default proportional to its remaining

principal.

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[12] Dangl and Zechner (2016) and DeMarzo and He (2016) consider the case where debt may not be fully rolled over by issuance of additional debt. Upward (but not downward) restructuring of debt principal, as in Goldstein, Ju, and Leland (2000), would not change the nature of our results.

Let V_0 denote asset value when the first round of debt is issued. The coupon rate C will be set so that the bond initially sells at par:

$$D(V_0, V_B, P, C, m) = P \tag{3}$$

where $D(V_0, V_B, P, C, m)$ is the market value of debt with parameters (P, C, m) when asset value $V = V_0$. At future times $t > 0$, almost surely $V \neq V_0$ and future debt will be rolled over at a premium (or discount) to par. The deficit/surplus from full rollover, at rate $m(P - D(V, V_B, P, C, m))$, will be borne entirely by shareholders.

2.1 Valuation of Debt and Equity

Default occurs when asset value first falls to a (time-independent) value $V_B < V_0$. The default event triggers a proportional loss α of asset value V_B at default. Recoverable value in default will therefore be $(1 - \alpha)V_B$. Upon default, this value (which will always be less than P) will go entirely to then-current debtholders through absolute priority. Equity will have zero value in default.[13]

Leland (1994a) shows that the value of debt in this model is given by

$$D(V, V_B, P, C, m) = \frac{C + mP}{r + m}(1 - q_1) + (1 - \alpha)V_B q_1 \tag{4}$$

where

$$q_1 \equiv \left(\frac{V}{V_B}\right)^{-x_1} \tag{4a}$$

and

$$x_1 = \frac{\left(r - \delta - \frac{1}{2}\sigma^2\right) + \left(\left(r - \delta - \frac{1}{2}\sigma^2\right)^2 + 2(r + m)\sigma^2\right)^{\frac{1}{2}}}{\sigma^2} \tag{5}$$

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[13] Debtholders having prior claims to equity is standard in structural models. Violations of absolute priority are considered (when $m = 0$) in Leland (1994a). With multi-period debt issuances, we assume time priority of earlier issues, including their rollover obligations.

q_1 is expected present value (discounted at the riskless rate r, with V following the risk neutral asset value process (1)) of $1, *exponentially declining through time at the rate m*, received at the time of first passage from current value V to the default boundary V_B. Thus the first term in (4) is the expected capitalized value of the cash flows to current debt, which decline exponentially through redemption at rate m, received until first passage time to the default boundary V_B. The second term of equation (4) is the expected present value of the current debt's claim to remaining asset value at default. We presume here that the default boundary is time independent, and later verify that it will indeed be so.

Following Leland (1994a, b), total firm value v at time t is given by asset value V, plus the expected present value of the tax benefits to debt, less the expected present value of potential default costs:

$$v(V,V_B,P,C,m) = V + \frac{\tau C}{r}(1 - q_2) - \alpha V_B q_2 \qquad (6)$$

where
$$q_2 = \left(\frac{V}{V_B}\right)^{-x_2} \qquad (6a)$$

and
$$x_2 = \frac{\left(r - \delta - \frac{1}{2}\sigma^2\right) + \left(\left(r - \delta - \frac{1}{2}\sigma^2\right)^2 + 2r\sigma^2\right)^{\frac{1}{2}}}{\sigma^2}. \qquad (7)$$

q_2 is expected present value of $1 (discounted at the riskless rate r but *not* declining exponentially), received at the time of first passage from V to V_B. The value of tax benefits is given by the second term in equation (6), and the value of potential default costs is given by the third term. These are invariant to debt maturity, except through the effects of maturity on V_B observed subsequently. Note that when $m = 0$ ("infinite" life debt), $q_1 = q_2$ *ceteris paribus*.

Firm value equals debt value plus equity value. Therefore equity value is given by

$$E(V,V_B) = v(V,V_B) - D(V,V_B) \qquad (8)$$

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where we have suppressed the debt parameter arguments (P, C, m) and the exogenous parameters (r, δ, σ).

2.2 The Endogenous Default Boundary

The endogenous default boundary V_B is assumed chosen by equity holders to maximize equity value, given the debt in place with parameters (P, C, m). As in Leland (1994a), it satisfies the smooth-pasting condition

$$\partial E(V, V_B)/\partial V|_{V=V_B} = 0 \tag{9}$$

with solution in Leland (1994a) given by

$$V_B = \frac{\frac{(C+mP)x_1}{r+m} - \frac{\tau C x_2}{r}}{1 + \alpha x_2 + (1-\alpha)x_1}. \tag{10}$$

3. Extension to Multiple Debt Issues: Sequential 1-period Optimal Decisions

The "static" model described above assumes that there is only one issuance of debt, the "initial debt." But the analysis is easily extended to sequential debt issuances, such as considered by ADHP.[14] Let $z = (1,...)$ index sequential rounds of debt issuance. The initial round 1 starts with no prior debt outstanding, but subsequent rounds start with all prior debt issues remaining. While we could allow equal priority for all issues, we choose to assign priority to claims in default to by order in the sequence. Thus round 1 debt must be fully repaid up to its principal value before any remaining assets (net of default costs) are paid to round 2 debt, which in turn has priority over subsequent issues. In all our examples, only the initial debt will recover positive amounts in default.

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[14] Alternative dynamic models, where prior debt must be retired before new debt is issued, are considered by Fischer, Heinkel, and Zechner (1989) and by Goldstein, Ju, and Leland (2001).

We focus the analysis on the situation where investors and the firm assume that each round of issuance is the last, and do not foresee that the firm may subsequently choose to issue additional debt. While clearly restrictive, this assumption is consistent with the sequential one-period analysis of ADHP and their development through Section IIIA.

In each round z, new debt is issued with principal value $P_z \geq 0$, coupon $C_z \geq 0$, and rollover rate m_z. We assume here that $m_z = m$ for all z, which assures that x_1 in equation (5) is the same for all rounds given the time independence of the riskfree rate r, payout rate δ, and volatility σ.[15] In determining optimal debt issuance in each round z, previous debt issuances are taken into account in the default boundary V_{Bz}, and the fraction of remaining value (if any) that the new debt will receive if default occurs, given debt priority rules. Consistent with sequential one-period maximization (and ADHP through Section IIIA), no anticipation of rounds beyond the current round z will affect the current leverage decision. We also assume initially that the underlying asset value is unchanged at each issuance round, i.e. $V_z = V_0$, where V_z is the asset value when debt is issued in round z, also as initially assumed in ADHP. Later, this assumption is relaxed.

3.1 Debt Values with Sequential 1-Period Optimization

Similar to the single-period debt value in (4) debt issued in the zth round will have value given by

$$D_z(V, V_{Bz}, P_z, C_z, m) = \frac{C_z + mP_z}{r + m}(1 - q_{1z}) + w_z(1 - \alpha)V_{Bz}q_{1z} \qquad (11)$$

for arbitrary asset value V, where P_z and C_z are the principal and coupon rate paid by debt issued in round z. V_{Bz} is the optimal default boundary given all debt up to and including the zth round, and w_z is the fraction of assets after default that is commanded by debt issued in the zth round. q_{1z} is given by

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[15] x_2 in equation (7) is also independent of m. However, q_1 and q_2 will typically depend upon m through V_B.

equation (4a) when $V_B = V_{Bz}$. Note w_z will depend on the amount of debt issued in round z and, through priority rules, on the debt previously issued in rounds $s = (1, …, z - 1)$. Because investors do not anticipate subsequent debt offerings, they do not anticipate further change in their default claims, even though those claims may well be diluted by future issues.

In each round, the coupon C_z is chosen so that the bond sells at par. Let $V = V_z$ be asset value when D_z is issued. Then for each z, C_z satisfies the condition

$$D_z(V_z, V_{Bz}, P_z, C_z, m) = P_z. \tag{12}$$

As noted earlier, for the moment we assume that $V_z = V_0,$ for all z.

Note that debt issued in any prior round s, $1 \le s < z$, will retain its coupon and principal (C_s, P_s) reduced by retirement at rate m through time. For simplicity and consistency with ADHP, we assume that rounds pass instantaneously, so there is effectively no reduction in coupon and principal of previously-issued debt between rounds and thus $V_z = V_0,$ for all z. But even with asset value V constant between rounds, debt value of prior rounds will have changed from its when-issued market value (and principal value) because in general $V_{Bz} \neq V_{Bs}$ and $w_s^z \neq w_s$, where w_z^s is the fraction of liquidation value $(1 - \alpha)$ V_{Bz} received in round z by the debt issued in round $s < z$.[16] Depending upon priority rules, w_s^z may differ from w_s because of additional debt issued in the subsequent rounds.

Debt issued at round $s < z$ will have subsequent value at time z

$$D_s^z(V, V_{Bz}, P_s, C_s, m) = \frac{C_s + mP_s}{r + m}(1 - q_{1z}) + w_s^z(1 - \alpha)V_{Bz}q_{1z}. \tag{13}$$

Observe that in general, $D_s^z(V, V_{Bz}, P_s, C_s, m) \neq P_s$ for all $s < z$.

We use the following notation for aggregate principal, aggregate coupon, and aggregate shares in default after z rounds of issuance:

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[16] For simplicity, we write w_s^s as w_s.

$$P_z^* = \sum_{s=1}^z P_s; \qquad C_z^* = \sum_{s=1}^z C_s; \quad D_z^* = \sum_{s=1}^z D_s^z; \quad w_z^* = \sum_{s=1}^z w_s^z = 1 \tag{14}$$

where the last equality follows from the fact that debt collectively at round z will have claim to the remaining asset value after default, $(1 - \alpha)V_{Bz}$.

It follows directly from equations (13) and (14) that aggregate debt value D_z^* at round z will have value

$$D_z^*(V, V_{Bz}, P_z^*, C_z^*, m) = \frac{C_z^* + mP_z^*}{r+m}\left(1 - q_{1z}\right) + (1 - \alpha)V_{Bz}q_{1z} \tag{15}$$

Again, we note that in general, $D_z^*(V, V_{Bz}, P_z^*, C_z^*, m) \neq P_z^*$ even when $V = V_z$, since in general $V_{Bz} \neq V_{Bs}$, $s < z$, and only the most recent debt issuance will sell at par.

3.2 Equity Value and Default Boundary at each Round

We can extend the single-round model of firm and equity value in a straightforward manner. At each round z, given V and V_{Bz}, the value of the firm is given by asset value plus the expected present value of tax benefits less default costs

$$v_z(V, V_{Bz}, P_z^*, C_z^*, m) = V + \frac{\tau C_z^*}{r}(1 - q_{2z}) - \alpha V_{Bz}q_{2z} \tag{16}$$

where (x_2, P_z^*, C_z^*) are given in equations (7) and (14) respectively, and q_{2z} is given by equation (6a) with $V_B = V_{Bz}$.

Equity value in round z is now given by

$$E_z(V, V_{Bz}, P_z^*, C_z^*, m) = v_z(V, V_{Bz}, P_z^*, C_z^*, m) - D_z^*(V, V_{Bz}, P_z^*, C_z^*, m) \tag{17}$$

Parallel to equations (9) and (10), the smooth pasting condition

$$\partial E_z(V, V_{Bz})/\partial V|_{V=V_{Bz}} = 0 \tag{18}$$

gives the default boundary V_{Bz} in round z:

$$V_{Bz} = \frac{\frac{(C_z^* + mP_z^*)x_1}{r+m} - \frac{\tau C_z^* x_2}{r}}{1 + \alpha x_2 + (1-\alpha)x_1} \tag{19}$$

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For our subsequent analysis it is useful to separate out the current debt (P_z, C_z) in the equations above.

From (14), it follows immediately that

$$P_z^* = P_{z-1}^* + P_z; \quad C_z^* = C_{z-1}^* + C_z; \quad D_z^* = D_{z-1}^* + D_z$$

Substituting these relationships into equation (17) yields the value of equity after zth round debt issuance P_z:

$$E_z(V, V_{Bz}, P_{z-1}^* + P_z, C_{z-1}^* + C_z, m) = v_z(V, V_{Bz}, P_{z-1}^* + P_z, C_{z-1}^* + C_z, m)$$

$$- D_{z-1}^*(V, V_{Bz}, P_{z-1}^*, C_{z-1}^*, m) - D_z(V, V_{Bz}, P_z, C_z, m) \qquad (20)$$

where we recall current debt has value that depends only on its principal and coupon from (12). Similarly, we may express (19) as

$$V_{Bz} = \frac{\frac{(C_{z-1}^* + C_z + m(P_{z-1}^* + P_z))x_1}{r+m} - \frac{\tau(C_{z-1}^* + C_z)x_2}{r}}{1 + \alpha x_2 + (1-\alpha)x_1} \qquad (21)$$

Note the value of previously-issued debt $D_{z-1}^*(V, V_{Bz}, P_{z-1}^*, C_{z-1}^*, m)$ does not depend directly on the current debt's principal and coupon (P_z, C_z), but does depend indirectly via the default boundary V_{Bz} as seen in (21).

4. Sequential 1-round Equity Value Optimization

Consistent with ADHP (through their Section IIIA), we assume equity holders will choose the amount of debt P_z in each round z to maximize the sum of resulting equity value plus debt proceeds P_z, subject to constraints (12) and (21) at $V = V_z$ and taking past debt obligations (P_{z-1}^*, C_{z-1}^*) as contractually fixed. Therefore equity holders in round z seek to

$$Maximize_{P_z, C_z} \{E_z(V, V_{Bz}, P_{z-1}^* + P_z, C_{z-1}^* + C_z, m) + D_z(V, V_{Bz}, P_z, C_z, m)\}, \qquad (22)$$

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subject to (12) and (21), given $V = V_z$.

Again, note that P_{z-1}^*, C_{z-1}^* are taken as given since they are determined in prior rounds.[17] Substituting from (20) into (22) gives the equivalent optimization problem that in each round z, equity seeks to

$$Maximize\ _{P_z,C_z}\ \{\ v_z(V, V_{Bz}, P_{z-1}^* + P_z\ , C_{z-1}^* + C_z, m)\ -\ D_{z-1}^*(V, V_{Bz}, P_{z-1}^* + P_z\ , C_{z-1}^* + C_z, m)\}\quad (23)$$

subject to (12) and (21), given $V = V_z$.

The optimal debt issuance policy $\{C_z, P_z\}$, and resulting $\{P_z^*, C_z^*\}$ are also given by the sequential maximization of (23), $z = \{1, \dots\}$.[18] Thus the optimal policy in each round maximizes the current value of the firm, less the value of total *previously issued* debt.

In the first round (z = 1), there is no prior debt (by assumption), i.e. $D_0^* = P_0^* = C_0^* = 0$. Therefore the firm chooses to issue debt principal P_1 that maximizes firm value v_1, with coupon C_1 chosen to make debt sell at par, and V_{B1} satisfying equation (10). The optimal leverage results in the first round are identical to those in Leland (1994a).

The firm no longer maximizes firm value in subsequent rounds, given positive debt issuance in the first round. Rather, it maximizes the sum of current firm value less the current value of previously issued debt. We follow this algorithm in constructing the examples below.

5. Examples of Sequential 1-Round Optimization: Debt Issuance

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[17] The maximand in (22) is not the value of the firm in round *z* > 1, because it includes only the value of current round debt D_z rather than total debt value D_z^*.

[18] Note this depends on the assumption of no foresight.

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We consider environments consistent with a firm issuing BBB-rated debt of different average maturities: 10 years, 5 years, and 3 years. [19] We choose exogenous parameters within reasonable ranges suggested by previous empirical studies, as noted in Table 1.

Table 1: Exogenous parameter ranges.

Parameter	High (Source)	Low (Source)
Asset volatility σ	28% Feldhütter and Schaefer (2018, Table 7)	22% Schaefer and Strebulaev (2008)
Payout rate δ	5.0% Feldhütter and Schaefer (2018, Table 6)	3.7% Feldhütter and Schaefer (2018, Table 7)
Default cost α	45% Glover (2016)	10% - 20% Andrade and Kaplan (1998)
Effective tax rate τ	25% He and Milbradt (2014)	15% Bhamra, Kuehn and Strebulaev (2010)
Risk-free rate r	5.60% Avg. 10-year Treasury yield 1950-2017	4.69% Avg. short-term riskless rate 1866-2008*
		* Giesecke, Longstaff, Schaefer and Strebulaev (2000, Table 4)

In all our examples, we assume the risk-free rate $r = 5.00\%$, asset volatility $\sigma = 25\%$ and a payout rate $\delta = 4.00\%$. The risk-neutral asset value drift $\mu = r - \delta = 1.00\%$ in equation (1). We further normalize initial asset value V_0 = 100 in all cases.

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[19] Choi, Hackbarth, and Zechner (2018) report an average (median) debt maturity of 5.15 (3.93) years, where average (median) bond maturity is 6.35 (4.81) years and average (median) loan maturity is 3.43 (2.92) years. Note that if a firm consistently issues debt with 10 year maturity and redeems and replaces it upon maturity, the average maturity of its outstanding debt will be 5 years (see e.g., Leland and Toft (1996)). While we could use the latter model in this analysis, the calculations are considerably more tedious.

Optimal first-round leverage depends on the chosen average debt maturity as well as the other exogenous parameters specified in Table 1. We choose the two remaining exogenous "tax tradeoff" parameters, the tax rate τ and the default cost (loss fraction of asset value at default) α, to generate an initial (first-round) leverage of about 38%, for each different average debt maturity. 38% is close to the average maturity of BBB-rated debt based on previous empirical work.[20]

For each example, we first consider 5 sequential rounds of debt issuance, using the algorithm in Section 4. We assume the sequential rounds occur with (essentially) zero time lag, and the asset value remains unchanged ($V_z = V_0 = 100$ for all z) between rounds. As previously noted, we follow ADHP in assuming that, at each round, the firm and investors do not anticipate further debt issuances or retirements.

While debt issuance in the current round is always assumed at par, because the default boundary will typically increase with additional debt issuance, the endogenous default boundary will typically increase and prior debt will sell below par. This is the agency problem noted in the introduction: prior obligations can be reduced in value by subsequent debt issuance. However, this is not the whole story: the increase in the endogenous default boundary caused by additional debt will also affect the present value of tax shields and default costs. Given the assumptions underlying the Leland (1994a) model, this dependence will importantly change the results derived by ADHP. In all our examples, we assume "absolute priority": debt issued in any round has claim to value in default that is senior (up to principal value) to any subsequent debt issue.

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[20] Estimates of BBB-rated firms' leverage include 38% by Feldhütter and Schaefer (2018), 43% by Huang and Huang (2012) and 28% by Rauh and Sufi (2010).

EXAMPLE 1: 10 year average maturity debt ($T = 10 \leftrightarrow m = 0.1$)

We choose tax rate $\tau = 20\%$ and default cost (loss of asset value upon default) $\alpha = 35\%$. In turn with our other parametric assumptions (common to all examples) of $rf = 5.00\%$, $\sigma = 25\%$ and $\delta = 4.00\%$, the Leland (1994a) optimal leverage solution is given by Round 1 of the following table. Note that initial leverage is close to the target 38% for BBB-rated debt, as found empirically by Feldhütter and Schaefer (2018), and the net benefits to leverage are 3.91%, bracketed by van Binsbergen, Graham, and Yang's (2010) estimate of 3.5% and Korteweg's (2010) estimate of 3.6% - 4.0%.

					EXAMPLE 1: 10 Year Debt									
					Tax Rate τ 20.0%		Default Cost α	35.0%						
	New Debt		New Debt											
	Issued	New Debt	Spread	Total Debt	Total Debt		% change in	Leverage	Equity	Prior Equity	Change to	% change	Tax	Default	Net Debt
Round	(at Par)	Coupon	(bps)	Principal	Value	Firm Value	Firm Value	(market)	Value	plus new debt	Equity	to equity	Benefits	Costs	Benefits
1	40.04	2.324	80	40.04	40.04	103.91	3.910%	38.53%	63.87	103.91	3.9104	3.9104%	6.65	2.74	3.91
2	6.87	0.485	206	46.91	46.51	103.66	-0.237%	44.86%	57.16	64.03	0.1548	0.2417%	7.47	3.80	3.66
3	4.20	0.316	251	51.11	50.34	103.36	-0.296%	48.71%	53.01	57.22	0.0602	0.1052%	7.92	4.56	3.36
4	2.84	0.223	286	53.95	52.88	103.08	-0.270%	51.30%	50.20	53.04	0.0285	0.0538%	8.20	5.12	3.08
5	2.04	0.166	313	56.00	54.67	102.84	-0.231%	53.16%	48.17	50.22	0.0152	0.0303%	8.38	5.54	2.84

We observe that the LRE effect is alive and well in this example, even though initial debt has absolute priority. Leverage rises from 38.5% in the first round to 54.5% after five rounds. After 10 rounds rounds), leverage rises further to 57.1%, and by the 50th round leverage reaches 60.5%, where it remains through the 100th round. At that limiting leverage, the net benefits to leverage are reduced from the initial level of 3.91% to 1.57%, but clearly are not reduced to zero.

Although the incremental leverage is quite large, the gains to equity from the leverage increases are small. The 6% growth in leverage in round 2 increases equity value (including current debt issuance) by less than 0.25%. The increment falls to 0.030% by round 5, and to 0.003% by round 10.

With each round, the required spread on additional debt rises rapidly from its initial 80 bps. Round 2 requires a spread of 206 bps, suggesting a substantial fall in debt rating if the firm issues even one round of additional debt. The spread rises to 389 bps in round 10, and reaches a limit of 445 bps. Whether firms would consider the issuance of additional debt in light of the small benefits to equity value and possible reputational costs is open to question.

EXAMPLE 2A: 5 year average maturity debt ($T = 5 \leftrightarrow m = 0.2$)

We now consider optimal sequential debt issuance when the average debt maturity (barring default) is 5 years. In Example 2A, we keep the tax rate and default cost fraction the same as in Example 1.

					EXAMPLE 2A: 5 Year Debt									
					Tax Rate τ	20.0%	Default Cost	35.0%						
Round	Debt Issued (at Par)	New Debt Coupon	New Debt Spread (bps)	Total Debt Principal	Total Debt Value	Firm Value	% change in Firm Value	Leverage (market)	Equity Value	Prior Equity plus new debt	Change to Equity	% change to equity	Tax Benefits	Default Costs	Net Debt Benefits
1	29.72	1.583	33	29.72	29.72	102.79	2.7904%	28.91%	73.07	102.79	2.7904	2.7904%	4.79	2.00	2.79
2	1.20	0.069	74	30.91	30.89	102.77	-0.0186%	30.06%	71.88	73.08	0.0040	0.0039%	4.93	2.16	2.77
3	0.18	0.011	75	31.10	31.07	102.77	-0.0038%	30.23%	71.70	71.88	0.0001	0.0001%	4.95	2.18	2.77
4	0.028	0.002	75	31.13	31.10	102.77	-0.0006%	30.26%	71.67	71.70	0.0000	0.0000%	4.96	2.19	2.77
5	0.004	0.000	75	31.13	31.10	102.77	-0.0001%	30.26%	71.66	71.67	0.0000	0.0000%	4.96	2.19	2.77

Here, we observe the LRE is played out almost completely after 5 rounds. The gains to additional leverage essentially are complete after 4 rounds, with even Round 3 producing a gain to equity value of less than 1 basis point. But perhaps this is due to the lower initial leverage in this example, considerably less at 28.9% than our "target" BBB-rating leverage of 38%. The lower initial leverage provides less debt to exploit by increasing subsequent leverage. Therefore we alter the tax and default cost parameters in Example 2B to generate a first round leverage close to the target of 38%.

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EXAMPLE 2B: 5 year average maturity debt ($T = 5 \leftrightarrow m = 0.2$), initial leverage preserved.

We raise the effective corporate tax rate from 20% to 25%. We also slightly reduce the default costs from 35% to 33%. These changes lead to initial Round 1 leverage of 38.0%, the average of BBB-rated firms found by Feldhütter and Schaefer (2018), and close to Round 1 leverage in Example 1.

	New Debt		New Debt			EXAMPLE 2B: 5 Year Debt, 38% initial leverage									
						Tax Rate τ	25.0%		Default Cost α	33.0%					
	New Debt		New Debt												
	Issued	New Debt	Spread	Total Debt	Total Debt		% change in	Leverage	Equity	Prior Equity	Change to	% change	Tax	Default	Net Debt
Round	(at Par)	Coupon	(bps)	Principal	Value	Firm Value	Firm Value	(market)	Value	plus new debt	Equity	to equity	Benefits	Costs	Benefits
1	39.73	2.244	65	39.73	39.73	104.51	4.508%	38.02%	64.78	104.51	4.5078	4.5078%	7.68	3.17	4.51
2	2.03	0.132	153	41.76	41.68	104.44	-0.067%	39.91%	62.76	64.79	0.0091	0.0087%	7.94	3.50	4.44
3	0.43	0.028	157	42.18	42.08	104.42	-0.018%	40.30%	62.34	62.76	0.0004	0.0006%	8.00	3.58	4.42
4	0.09	0.006	158	42.27	42.17	104.41	-0.004%	40.39%	62.24	62.34	0.0000	0.0000%	8.01	3.59	4.41
5	0.02	0.001	158	42.29	42.19	104.41	-0.001%	40.41%	62.22	62.24	0.0000	0.0000%	8.01	3.60	4.41

As expected, the higher initial leverage than in Example 2A implies somewhat greater amounts of additional debt issuance. But the differences are not large, and again the LRE is almost entirely played out after 5 rounds. The leverage ratio grows only from 38.0% to 40.5%, and equity increases in value by less than 2 bps. As in Example 1, there is a significant rise in the spread of subsequent debt issuances vs. the initial spread, again suggesting some reputational risk of a lower debt rating for the firm undertaking further leverage.

EXAMPLE 3A: 3 year average maturity debt ($T = 3 \leftrightarrow m = 0.33$), non-negative issuance

To keep initial leverage at about 38% with shorter maturity, we assume (as in the prior example) a tax rate of 25%, but reduce default costs to 25%. Optimal leverage in Round 1 is 38.3%. Optimal debt issuance is given below when we restrict $P \geq 0, C \geq 0$:

	New Debt Issued (at Par)	New Debt Coupon	New Debt Spread (bps)	Total Debt Principal	Total Debt Value	Firm Value	% change in Firm Value	Leverage (market)	Equity Value	Prior Equity plus new debt	Change to Equity	% change to equity	Tax Benefits	Default Costs	Net Debt Benefits
Round															
1	40.06	2.178	44	40.06	40.06	104.47	4.468%	38.35%	64.41	104.47	4.4676	4.4676%	7.22	2.75	4.47
2	0.00	0.000	N/A	40.06	40.06	104.47	0.000%	38.35%	64.41	64.41	0.0000	0.0000%	7.22	2.75	4.47
3	0.00	0.000	N/A	40.06	40.06	104.47	0.000%	38.35%	64.41	64.41	0.0000	0.0000%	7.22	2.75	4.47
4	0.00	0.000	N/A	40.06	40.06	104.47	0.000%	38.35%	64.41	64.41	0.0000	0.0000%	7.22	2.75	4.47
5	0.00	0.000	N/A	40.06	40.06	104.47	0.000%	38.35%	64.41	64.41	0.0000	0.0000%	7.22	2.75	4.47

EXAMPLE 3A: 3 Year Debt — Tax Rate τ 25.0%, Default Cost α 25.0%

Here, we see that it is never optimal to issue additional debt. The optimal issuance in Round 2 is strictly zero. Because Round 3's optimization problem is exactly the same as Round 2's, the firm will also issue zero additional debt, and by induction the firm will never increase leverage. This will be true for any set of (constant) initial parameters in which the firm does not increase leverage in Round 2.

In contrast with the earlier examples having longer maturities, the non-negativity constraint is strictly binding in this example. This raises the question of whether the firm would actually prefer to reduce leverage if it could.

EXAMPLE 3B: 3 year average maturity debt ($T = 3 \leftrightarrow m = 0.33$), negative issuance allowed

Simply removing the non-negativity constraint and optimizing sequentially generates the following results:

	New Debt Issued (at Par)	New Debt Coupon	New Debt Spread (bps)	Total Debt Principal	Total Debt Value	Firm Value	% change in Firm Value	Leverage (market)	Equity Value	Prior Equity plus new debt	Change to Equity	% change to equity	Tax Benefits	Default Costs	Net Debt Benefits
Round															
1	40.06	2.178	44	40.06	40.06	104.47	4.468%	38.35%	64.41	104.47	4.4676	4.4676%	7.22	2.75	4.47
2	-1.17	-0.071	N/A	38.89	38.91	104.49	0.024%	37.24%	65.58	64.41	0.0024	0.0023%	7.08	2.59	4.49
3	0.17	0.010	103	39.05	39.07	104.49	-0.003%	37.39%	65.42	65.58	0.0000	0.0001%	7.10	2.61	4.49
4	-0.02	-0.001	N/A	39.03	39.05	104.49	0.000%	37.37%	65.44	65.42	0.0000	0.0000%	7.10	2.61	4.49
5	0.00	0.000	103	39.03	39.05	104.49	0.000%	37.37%	65.44	65.44	0.0000	0.0000%	7.10	2.61	4.49

EXAMPLE 3B: 3 Year Debt (negative issues OK) — Tax Rate τ 25.0%, Default Cost α 25.0%

Indeed, we see that it is optimal to reduce debt in the second round. Thereafter there are mild fluctuations (issuance and reduction) in subsequent rounds, with the further adjustments fluctuating in

sign but leverage converging to a slightly lower amount than in the initial round. This clearly is in contrast with ADHP's result that, in their model, debt will never be reduced by equity holders.

Before explaining the reasons for the different conclusions, we first note that the debt reduction postulated here—while symmetric with debt issuance—is difficult to square with the typical means used to retire debt. Here the firm reduces both debt principal and coupon, but not in the same proportion. The coupon in this example makes the market value of the debt reduction equal to the principal reduction.[21] We now turn to a more traditional way of reducing leverage, using debt buybacks.

6. Reducing Leverage Through Debt Buybacks

Rather than assume that debt reduction can take place consistent with the modelling in Example 3B, we consider a more realistic debt reduction method. In this approach, the firm simply repurchases some of its currently outstanding debt, reducing both principal and coupon in the same proportion. The fractional debt rollover at rate m will continue for the debt that remains outstanding. The default boundary will decrease with this debt reduction, leading to an increase (per unit principal) in the market value of the remaining debt. The buyback price per unit principal of the debt must equal the higher market price of the post-buyback debt. This is consistent with ADHP's repurchasing assumption.

With such a proportional debt decrease, the value of tax benefits and default costs will be affected—not only because the total coupon and principal are reduced, but also because the endogenous default boundary changes. The value of the firm after debt repurchase (from equation (16)) will thus be altered,

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[21] One might imagine such a debt reduction as the firm *purchasing* fairly-priced debt (that would bear the coupon and principal of the proposed debt reduction). The resulting debt service (coupon and principal) would be the netting of the new debt principal and coupon from the original debt. But that purchased debt would then have to default under exactly the same circumstances as the firm, a rather bizarre proposition.

and surprisingly may actually increase. This is because while tax benefits will fall, default costs may fall even more. With short term debt, the potential increase in debt benefits may outweigh the increase in prior debt value. Later, we explain why the firm would not have decreased its initial round 1 debt if a lower debt amount would raise firm value.

Example 4A finds an optimal proportional buyback of 3.39%, reducing the original principal of 40.06 to 38.70—somewhat less than in Example 3B. This buyback maximizes the resulting value of equity after the cost of the buyback, at a price debtholders will demand when recognizing the higher value of debt per unit principal after the buyback. The coupon paid on this reduced amount of debt will fall by an equal percentage, from 2.18 to 2.10. The effects of the buyback are detailed in the Example 4A Buyback Summary below:

		EXAMPLE 4A: BUYBACK SUMMARY		V = 100	
			Percent Buyback		3.39%
			Round 1 Principal Repurchased		1.357
			Lowering Coupon by		0.074
	ROUND 1			AFTER BUYBACK	
Debt Principal	40.060				38.703
Debt Coupon	2.178				2.104
Default Boundary VB	32.602				31.497
Debt Value	40.060				38.748
V0	90.000			V1	90.000
Debt Value/Principal	1.000				1.0012
			Cost of Buyback		1.359
Spread (bps) on Market value	43.59	Spread (bps) on Market value			39.02
Tax Benefits	7.216	Tax Benefits			7.088
Default Cost	2.749	Default Cost			2.568
Firm Value	104.468	Firm Value			104.520
Equity Value	64.408	Equity Value			65.772
Market leverage	38.3%				37.1%
			Net Equity after Buyback Cost		64.413
			Buyback Net Benefit to Equity		0.0053
			Net Benefit (% of Round 1 Equity)		0.008%
q1	0.0284			q1	0.0254
q2	0.3372			q2	0.3262

With the lower coupon and principal, the default boundary will fall from 32.60 to 31.50, also by 3.39%, using equation (10) (or (19)). The market value of the remaining debt will fall to 38.75, using equation (4) with the new $P, C,$ and V_B, a fall of only 3.27%. Thus there is a rise in the market value of debt per unit principal P, from 1.0000 in Round 1 (reflecting debt selling at par) to 1.0012. This increase results from the decreased likelihood of default (lower) and is consistent with ADHP's observation that debt can only be repurchased at a price reflecting its *ex post* value per unit principal—which will be higher since default is less likely. Reflecting this higher cost, we multiply the principal reduction of 1.357 by the cost per unit principal (1.0012) to determine the buyback cost of 1.359.

After repurchase, the value of tax benefits will fall from 7.22 to 7.09, but default costs will fall by a larger amount, from 2.75 to 2.57. Firm value actually increases by 0.0526—again raising the interesting question, addressed in Section 7, as to why this lower amount of debt wasn't chosen in round 1. Obligations after the buyback include the remaining debt value plus the cost of the buyback, which exceed the former debt liability by 0.0473. The result is an increase in equity value of 0.0053, contradicting ADHP'a Proposition I. While the buyback benefits are quite small in this case (as were the further issuance benefits in Examples 2A and 2B), we can explore whether buybacks would be larger if the asset value of the firm had fallen between the initial round and the buyback. [22]

Example 4B considers the optimal buyback when the initial debt is again determined when asset value V_0 = 100 (and initial leverage is therefore the same as in Example 4A), but asset value V falls immediately to V_1 = 90 after the initial issuance. This results in lower Round 1 debt value, firm value, and equity value as can be seen in the Round 1 column. We assume buyback then occurs at this lower asset value.

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[22] For simplicity, we assume that virtually no time passes between initial round and buyback, despite the drop in asset value.

		EXAMPLE 4B: BUYBACK SUMMARY			V = 90		
				Percent Buyback	11.54%		
			Round 1 Principal Repurchased		4.624		
			Lowering Coupon by		0.251		
		ROUND 1			AFTER BUYBACK		
Debt Principal		40.064					35.440
Debt Coupon		2.178					1.927
Default Boundary VB		32.605					28.842
Debt Value		39.883					35.461
VO		90.000			V1		90.000
Debt Value/Principal		0.995					1.0006
					Cost of Buyback		4.627
Spread (bps) on Market value		43.61	Spread (bps) on Market value				41.25
Tax Benefits		6.822		Tax Benefits			6.438
Default Cost		3.045		Default Cost			2.391
Firm Value		93.777		Firm Value			94.046
Equity Value		53.894		Equity Value			58.585
Market leverage		42.5%					37.7%
			Net Equity after Buyback Cost				53.958
			Buyback Net Benefit to Equity				0.0640
			Net Benefit (% of Round 1 Equity)				0.119%
	q1	0.0397			q1		0.0269
	q2	0.3736			q2		0.3317

The optimal debt buyback of 11.54% in Example 4B is considerably greater than in Example 4. The buyback increases firm value by 0.270, whereas the incremental cost of remaining debt plus buyback increases by 0.206, resulting in the buyback benefiting equity holders by 0.064. The buyback raises the increment to equity value by a factor of 12 relative to Example 4A. This again contradicts ADHP's Proposition 1, which claims even when asset value falls and the firm becomes over-leveraged, it will never benefit equity to buy back debt. In fact, in Example 4B not only is leverage reduced, but reduced to 37.1%, a slight over-adjustment relative to the optimal leverage of 38.3%.[23] That is, optimal buybacks not only may occur, but imply leverage reversion back towards (and perhaps slightly beyond) the

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[23] Note 38.3% is the optimal initial leverage in Example 4A, and optimal initial leverage (as a percent) is invariant to the initial value of the firm when there is no prior debt. If there were a subsequent Round 3, it can be shown equity value now requires a small issuance of debt (0.029) with resulting leverage 37.4% and eventually (after small oscillations) converges to 37.3%.

original target leverage. Future exploration of this "leverage reversion to initial target" property seems warranted, but we do not consider it here.

7. What's going on?

Our analysis raises several questions:

1) Why do we get different answers than ADHP to the importance of the LRE, and to the existence of debt buybacks?

2) Why are the differences between predictions more pronounced for short-run debt than for long-run debt?

3) Why would a firm buy back debt (with no change in *V*) when first round debt is optimally chosen?

1) We first note that the ADHP and Leland (1994a) models differ in several important ways. ADHP (through Section II) assumes a single period model with zero coupon debt with exogenously fixed maturity and no continuation.[24] With zero coupon debt, there is no possibility of default prior to maturity. At maturity, default occurs if the (random) asset value at maturity is less than face value. In default, there is a cost that is quite generally specified. In contrast, default in our model is triggered only when the firm's asset value falls to the endogenous default boundary V_B, at a random future time. Our default costs are assumed proportional to asset value V_B at default.[25]

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[24] Recent work by DeMarzo and He (2016) considers a model with a constant coupon and redemption rate, without commitment to full rollover of debt levels (although there is commitment to coupon and debt redemption rates). There is always a loss to total firm value from leverage in their equilibrium model (DeMarzo and He (2016),p. 34).

[25] We have also considered default triggered by asset value falling to $V_b = kP$, a fraction $k \leq 1$ of the *principal* of currently outstanding debt. The debt recovery rate in default would then be (1 - α)*k*. Scaling *k* = 0.7, default costs α = 35%, and tax rate τ = 20% yields initial leverage of 38.0% and a recovery rate of 45.5%, very close our Example 4A. Allowing buybacks with this amended default trigger leads to an optimal buyback of 3.27%, slightly less than the buyback of 3.39% when the default trigger satisfies the smooth-pasting condition in equation (10).

Unlike in the ADHP approach, the default boundary in equation (10) depends not only on debt principal (face value), but also on the debt coupon—determined so debt sells at par—*and* on debt maturity. The default boundary in turn affects the value of both tax benefits and default costs in equation (6), and therefore the net benefits of debt. While in ADHP a decrease in debt (face value) always lowers net leverage benefits to equity, in our model the opposite can occur at short debt maturities. While tax savings will be reduced, default costs will fall more than proportionately, and equity will capture some of this gain as explained below.

Key to ADHP's conclusion that debt buyback will never benefit equity holders is the following contention: "All the benefits produced by the debt buyback, which in our model thus far come from reduced bankruptcy costs, accrue to existing debt holders." (ADHP, p. 156). While true in their single period framework which terminates when debt matures, it is not true in the full debt rollover model in Leland (1994a). From equation (4) we see that the present value of recovery of assets after default costs by current debtholders is given by $(1 - \alpha)V_B q_1$, where q_1 is given by (4a) and x_1 is given by (5). Recovery is reduced by the default cost fraction α, implying current debtholders bear a default cost burden $\alpha V_B q_1$.

But the present value of expected total default cost (including future rollovers) is $\alpha V_B q_2$ from equation (6), implying equity holders bear the remaining cost $\alpha V_B (q_2 - q_1)$.[26] Debtholders bear full

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[26] Recall that current debtholders from equation (4) receive $e^{-ms}(1 - \alpha)V_B$ of recoverable assets when default occurs at the random future time *s*, and thus bear a fraction e^{-ms} of total default costs αV_B. The remaining fraction $(1 - e^{-ms})$ of default costs are borne by subsequent debtholders as debt is rolled over. This remaining cost fraction will be passed along to equity holders when the debt that is redeemed at par is replaced by debt sold at the current market price, a price that reflects the default costs which the new debtholders will bear. The higher the rollover rate *m* (and lower the average maturity 1/*m*), the larger fraction of costs borne by the new debtholders and passed to equity holders through fair debt pricing. Hence, equity bears a larger fraction of default costs, the shorter the average maturity of debt being issued.

default costs in the Leland (1994a) model only when $\alpha V_B q_1 = \alpha V_B q_2$. But from (4a) and (6a), $q_1 = q_2$ only when $m = 0$ — the case of infinite life debt studied in Leland (1994a). When $m > 0$ and debt has finite average maturity, $q_1 < q_2$ and equity will bear a fraction $(q_2 - q_1)/q_2$ of the default costs. Leverage decisions that reduce these costs will benefit equity as well as debt holders.

2) The shorter the maturity debt, the greater is $(q_2 - q_1)/q_2$, the fraction of default costs that equity will bear, as can be observed from equations 4(a) and 6(a). [27] The larger this fraction, the more equity will benefit from a decline in debt and the resulting lower default costs. This explains why our results with short maturity debt differ from long maturity debt, and is a major difference between our approach and ADHP in which the role of maturity is not explicit.

3) The coupon and principal of debt after the buyback in Example 4 do indeed yield a higher firm value, as can be seen in the output below.

Maturity T in yrs. (=1/m) round 1	3.00
Asset Volatility σ	0.25
Riskless rate rf	0.050
Payout ratio del (δ)	0.040
Default cost fraction α	0.25
Effective corp. tax rate τ	0.250
Initial asset value V_0	100
m = 1/T	0.333
asset value drift μ = rf - del	0.010
DERIVED INPUTS	
rf+m (defined as _z1)	0.38
rf (defined as _z3)	0.05
x1 (defined here as _yy1)	3.18
x2 (defined here as _yy3)	0.97
Principal P	38.703
Coupon	2.104
Calculations	
Optimal default boundary VB	31.4972
Firm Value	104.5201
Equity	65.7192
Debt market value D	38.748

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[27] Using the parameters and V_B in Example 4, with 3 year debt ($m=1/3$), the percentage of default costs borne by equity is $(q_2 - q_1)/q_2 = 92\%$. With infinite life debt, this falls to zero for any set of parameters.

So why isn't this value-increasing debt structure chosen by equity in round 1? Importantly here, the bond is not selling at par, a constraint in our first-round maximization: note the market value D of debt exceeds the principal value P.

By assumption, the rate of debt rollover is mP. When $P < D$, the average rollover rate on debt with initial market value D is $m(P/D) < m$, implying a greater average maturity by a factor of $D/P > 1$ than when the bond sells at par. As the default boundary falls with longer average maturity, it is not surprising that firm value is higher.[28] The problem is resolved when debt is required to sell at par, and this constraint leads to the initial optimal leverage of 38.3% in Example 4A. In the debt buyback, however, both coupon and principal of the previously-issued debt are reduced in the same percentage, leading to prior debt value exceeding principal or par value. Observe that the opposite happens in the case when it is optimal to issue additional debt (e.g. Example 1), and the market value of previously-issued debt falls below par.

8. Some Comparative Statics

At what maturity would equity holders choose neither to increase nor to buy back debt? For debt with maturity less than this "neutral maturity," subsequent debt rounds will reduce leverage from its initial (Round 1) level. For maturities larger than the neutral maturity, leverage will be increased, consistent with the LRE. The higher in the sense that the LRE will be observed at all maturities greater than the neutral maturity, one can argue that parameter combinations with lower neutral maturities are more susceptible to leverage ratcheting. Although we do not consider optimal maturity here, concern with the

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[28] Maximizing firm value without the "sell at par" constraint gives the nonsense result that optimal debt principal should be zero and the coupon 51.7. Such an issuance is unlikely to find favor with firms, the IRS, and the accounting profession.

agency costs resulting from the LRE would induce firms with a lower neutral maturities to choose lower initial maturity, ceteris paribus.[29]

For our base case parameters in Example 4, maturity of 3.6 years exactly produces this balance, with leverage 42.3% in all rounds. The initial parameters affect the neutral maturity. Table 2 calculates the neutral maturities and resulting stationary leverage levels for a range of default costs and tax rates.

Table 2: Neutral debt maturities (yrs.) / stationary leverage (%).

		Default costs			
		15%	20%	25%	30%
	15%	2.1 / 33.7	2.3 / 27.7	2.5 / 23.4	2.5 / 19.6
Tax rate	20%	2.6 / 45.8	2.9 / 37.8	3.1 / 32.0	3.2 / 27.3
	25%	2.7 / 61.8	3.2 / 49.4	3.6 / 42.3	3.7 / 35.8

Several properties of the risk neutral debt maturity and resulting stationary optimal leverage can be observed. For any given default cost, the neutral debt maturity and the stationary optimal leverage increase with the tax rate. For any given tax rate, the neutral maturity increases and the stationary leverage falls as default costs rise. This supports the conclusion in Section 7 that default costs are an important determinant of the LRE: higher default costs lead to higher neutral maturities, and therefore the LRE is less significant.

There is a relatively wide range of maturities about the neutral maturity for which Round 2 changes are small. In the case where both the tax rate and default costs are 25%, with neutral maturity of 3.6 years,

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[29] Of course, this agency cost (and perhaps others) would need to be balanced with the net tax benefits of alternative maturity choices. As is well known, the Leland (1994a) model implies maximum leverage benefits when maturity is infinite—exactly the case when agency costs of the LRE are likely to be highest.

optimal Round 2 issuance will be less than 5% of initial debt for maturities up to 4.1 years, and repurchases will be less than 4% for any debt maturity less than 3.6 years.

With tax and default costs as above, but with asset volatility reduced from 0.25 to 0.20, the neutral maturity rises to 4.0 years, with constant leverage 45.8%. The higher (than 3.6) neutral maturity therefore implies the LRE is less pronounced for firms with lower asset volatility, ceteris paribus.[30]

In the Appendix, we consider the impact of rollover, issuance, and buyback transactions costs. The former marginally reduce the LRE (subsequent debt issuance is smaller/buybacks larger when rollover costs rise). But surprisingly, larger issuance costs seem to marginally increase the LRE and reduce the magnitude of buybacks. Unsurprisingly, buyback transactions costs reduce their size and may eliminate them entirely.

Clearly, we have only dipped into a full examination of parametric changes on subsequent debt issuance or buybacks. We leave this exploration for future work.

9. Conclusion

Using the model of debt with full rollover as in Leland (1994a), , we reach conclusions that differ from ADHP (2018)'s key Propositions 1 ("Shareholder Resistance to Leverage Reduction") and 4 ("The Leverage Ratchet Effect") as debt maturity shortens. For reasonable parameter calibrations and average debt maturities 5 years or less, we find little or no benefit for equity to ratchet up leverage to higher amounts. A firm sequentially maximizing equity value may even prefer to reduce leverage after initial

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[30] By reducing the tax rate to $\tau = 0.23$, the "no change" maturity is 3.75 years, with constant leverage 42.4% (close to the original), further support for the conclusion that the LRE is less pronounced when volatility is lower. "Ceteris paribus" is clearly a delicate issue in performing our comparative static exercises.

issuance, when using short term debt. With longer term debt, the LRE does persist, but even after many rounds of sequential debt issuance the firm leverage does not converge to where all tax advantages to debt are exhausted.

The reasons for the models' divergent predictions are explained in Section 7, and are based on the differences in the models' assumptions. A key difference is how default costs are shared between debt and equity. In the ADHP model, current debtholders realize all the benefits from reducing leverage and expected default costs. Equity holders resist buybacks in ADHP because debtholders fully realize these benefits. In the Leland (1994a) model, current debtholders realize all the benefits of lower default costs only when debt has infinite maturity—the case in Leland (1994b). As average debt maturity is shortened by a higher debt rollover rate m, *current* debtholders will bear a rapidly declining share of expected future default costs, with the remaining cost borne by future debtholders. But equity bears the cost of funding future debt through rollovers. Thus, as average debt maturity becomes shorter, equity bears an increasing share of future default costs. When maturity becomes short, equity can actually benefit from future buybacks which lower default costs.

Our model assumes that the debt retirement rate m (with resulting average maturity $1/m$) is exogenous and remains constant through all financing rounds.[31] In the Leland (1994a) model without agency costs, optimal maturity is infinite. Finite maturity has been shown in Myers (1977), Leland and Toft (1996), Diamond and He (2014) and elsewhere to mitigate agency costs between debt and equity holders. This paper suggests that finite maturity may also serve to reduce the agency costs associated with the LRE. Extending the Leland (1994a) model to include illiquidity costs that increase with maturity, which will

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[31] In the model introduced in ADHP (2018), Section III, debt also is issued with infinite maturity but the firm is liquidated at a random future date governed by an exogenous intensity parameter λ. Thus their model also implies an exogenous average maturity (=$1/\lambda$) that remains constant through financing rounds.

lead to endogenously-chosen finite maturity, is clearly necessary.[32] Integrating optimal maturity choice into the model should provide a more complete understanding of the relation between maturity and the leverage ratcheting effect.

While we have focused on examples of sequential debt issuance with no foresight, the model with foresight is likely to provide even fewer incentives to debt issuance beyond the initial (static) leverage choice: debtholders will demand higher interest rates when they foresee an increased likelihood of default from greater future debt.

APPENDIX: Transactions Costs

We define 3 potential transactions costs:

k_R rollover cost rate   (applied to rollovers, continuous at rate mP)

k_I issuance cost rate   (applied to debt principal issuance at each round, if any)

k_B buyback transactions cost rate (applied to debt buybacks per unit principal, if any)

We also define

$K_z^* $ = total issuance/buyback costs = $\sum_{s=1}^{z}(k_{Is} + k_{Bs})$, where

$k_{Is} = P_s k_i$ if $P_s > 0$; $k_{Bs} = -P_s k_B$ if $P_s < 0$.

The following formulas augment equations (16) and (19) for firm value and for the default boundary:

$$v_z(V, V_{Bz}, P_z^*, C_{Iz}^*, m, q_R, K_{Iz}^*) = V + \frac{(\tau C_z^* - mP_z^* q_R)}{r}(1 - q_{2z}) - \alpha V_{Bz} q_{2z} - K_{Iz}^* \text{(A.1)}$$

$$V_{Bz} = \frac{\frac{(C_z^* + mP_z^*)x_1}{r+m} - \frac{(\tau C_z^* - mP_z^* q_R)x_2}{r}}{1 + \alpha x_2 + (1-\alpha)x_1} \tag{A.2}$$

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[32] Recent studies of debt liquidity and optimal maturity include He and Xiong (2012), Chen, Xu, and Yang (2013), He and Milbradt (2014), Bruche and Segura (2017), and Chen, Cui, He, and Milbradt (2018)). Empirical studies by Longstaff, Mithal, and Neis (2005) and others have shown that liquidity spreads increase with debt maturity.

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Electronic copy available at: https://ssrn.com/abstract=3501914

Examples

Below, we consider examples of how varying combinations of transactions costs affect the firm's dynamic debt strategy. Parameters remain the same as in Examples 3 and 4 in the paper: riskless rate 5%, payout rate 4%, effective corporate tax rate 25%, default costs 25%, and debt maturity of 3 years.

Example 4C considers when rollover costs k_R (only) are positive at 0.50% of principal value. We note that, in comparison with Example 4A, initial leverage is lower at 35.0% and the optimal buyback percentage is slightly higher at 3.58% in Round 2. When rollover costs rise to 1.00%, initial leverage falls to 31.1%, but buyback remains almost unchanged (percentagewise) at 3.51%. More generally, an increase in rollover costs lowers initial leverage and marginally reduces subsequent issuance/increases buyback. [33]

Example 4D considers when issuance costs k_I (only) are positive at 0.50% of the principal. Again, leverage is slightly decreased from Example 4A. There is a small buyback in Round 2. When issuance costs rise to 1.00%, however, initial issuance in Round 1 drops to 36.3%. Because of lower initial leverage, no buybacks are optimal in Round 2.

Example 4E examines when both rollover and issuance costs are 0.50% each. Initial leverage is further reduced, and the optimal buyback of 0.98% is minimal.

When issuance costs now rise to 1%, no buyback in Round 2 is optimal, confirming the conclusion that issuance costs deter buybacks, and reduce subsequent debt issues when otherwise those would occur.

Finally, we see in Example 4F that even small buyback costs will importantly reduce the magnitude of debt buybacks. With no transactions costs, the optimal buyback is 3.39% in all the examples below. When there are proportional buyback transactions costs of only 0.25%, the benefit to equity of buybacks is reduced by 65%. And buybacks cease entirely in this example when buyback costs reach 0.40%.

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[33] We note that higher rollover costs will likely lead to shorter maturity. We do not consider such feedback effects here. With the example 2B parameters and T = 5 years, 2nd round issuance falls from 1.85 to 1.26 when $q_R = 0.50\%$.

EXAMPLE 4C: BUYBACK with transactions costs					
q_R =	0.50%	q_I =	0.00%	q_B =	0.00%
			Percent Buyback		3.58%
		Round 1 Principal Repurchased			1.296
		Lowering Coupon by			0.069
ROUND 1			AFTER BUYBACK		
Debt Principal	36.209				34.913
Debt Coupon	1.926				1.857
Default Boundary VB	29.750				28.685
Debt Value	36.209				34.945
V0	100.000			V1	100.000
Debt Value/Principal	1.000				1.0009
		Buyback cost w/o transactions costs			1.297
		Buyback cost w/transactions costs			1.297
Spread	31.86	Spread (bps) on Market value			28.35
Tax Benefits	6.658	Tax Benefits			6.519
Default Cost	2.295	Default Cost			2.136
Rolling Cost	0.835	Rolling Cost			0.817
Issuance Cost		Issuance Cost			
Firm Value	103.528	Firm Value			103.566
Equity Value	67.319	Equity Value			68.621
Market leverage	35.0%				33.7%
		Net Equity after Buyback Cost			67.324
		Buyback Net Benefit to Equity			0.0047
		Net Benefit (% of Round 1 Equity)			0.007%

EXAMPLE 4D: BUYBACK with transactions costs T = 3 years					
q_R =	0.00%	q_I =	0.50%	q_B =	0.00%
			Percent Buyback		1.17%
		Round 1 Principal Repurchased			0.456
		Lowering Coupon by			0.025
ROUND 1			AFTER BUYBACK		
Debt Principal	38.877				38.421
Debt Coupon	2.097				2.073
Default Boundary VB	31.624				31.253
Debt Value	38.877				38.436
V0	100.000			V1	100.000
Debt Value/Principal	1.000				1.0004
		Buyback cost w/o transactions costs			0.456
		Buyback cost w/transactions costs			0.456
Spread	39.49	Spread (bps) on Market value			38.02
Tax Benefits	7.053	Tax Benefits			7.009
Default Cost	2.589	Default Cost			2.529
Rolling Cost	0.000	Rolling Cost			0.000
Issuance Cost	0.194	Prior Issuance Cost			0.194
Firm Value	104.270	Firm Value			104.286
Equity Value	65.394	Equity Value			65.850
Market leverage	37.3%				36.9%
		Net Equity after Buyback Cost			65.394
		Buyback Net Benefit to Equity			0.0006
		Net Benefit (% of Round 1 Equity)			0.001%

EXAMPLE 4E: BUYBACK with transactions costs T = 3 years					
q_R =	0.50%	q_I =	0.50%	q_B =	0.00%
			Percent Buyback		0.98%
		Round 1 Principal Repurchased			0.341
		Lowering Coupon by			0.018
ROUND 1			AFTER BUYBACK		
Debt Principal	35.002				34.660
Debt Coupon	1.850				1.832
Default Boundary VB	28.747				28.467
Debt Value	35.002				34.668
V0	100.000			V1	100.000
Debt Value/Principal	1.000				1.0002
		Buyback cost w/o transactions costs			0.341
		Buyback cost w/transactions costs			0.341
Spread	28.52	Spread (bps) on Market value			27.64
Tax Benefits	6.489	Tax Benefits			6.451
Default Cost	2.145	Default Cost			2.104
Rolling Cost	0.818	Rolling Cost			0.814
Issuance Cost	0.175	Prior Issuance Cost			0.175
Firm Value	103.350	Firm Value			103.358
Equity Value	68.348	Equity Value			68.690
Market leverage	33.9%				33.5%
		Net Equity after Buyback Cost			68.349
		Buyback Net Benefit to Equity			0.0003
		Net Benefit (% of Round 1 Equity)			0.000%

EXAMPLE 4F: BUYBACK with transactions costs T = 3 years					
q_R =	0.00%	q_I =	0.00%	q_B =	0.25%
			Percent Buyback		1.21%
		Round 1 Principal Repurchased			0.485
		Lowering Coupon by			0.026
ROUND 1			AFTER BUYBACK		
Debt Principal	40.060				39.575
Debt Coupon	2.178				2.151
Default Boundary VB	32.602				32.207
Debt Value	40.060				39.592
V0	100.000			V1	100.000
Debt Value/Principal	1.000				1.0004
		Buyback cost w/o transactions costs			0.485
		Buyback cost w/transactions costs			0.487
Spread	43.59	Spread (bps) on Market value			41.92
Tax Benefits	7.216	Tax Benefits			7.171
Default Cost	2.749	Default Cost			2.683
Rolling Cost	0.000	Rolling Cost			0.000
Issuance Cost	0.000	Prior Issuance Cost			0.000
Firm Value	104.468	Firm Value			104.488
Equity Value	64.408	Equity Value			64.896
Market leverage	38.3%				37.9%
		Net Equity after Buyback Cost			64.411
		Buyback Net Benefit to Equity			0.0031
		Net Benefit (% of Round 1 Equity)			0.005%

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