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MESON EXCHANGE IN THE REACTION

$K^{+} + p \rightarrow K^{*} + N^{*}$

Berkeley, California

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ERRATUM

SUBJECT: UCRL-10799, "Meson Exchange in the Reaction $K^+ + p \rightarrow K^+ + N^*$," G. Goldhaber, W. Chinowsky, S. Goldhaber, W. Lee, and T. O'Halloran, May 13, 1964; published in Phys. Letters 6, 62 (1963).

The coupling constant g_{K^*} for the $K^+\pi^-$ mode is missing a Clebsh-Gordan coefficient of $(2/3)^{1/2}$. In Fig. 4, curves A and B should accordingly be scaled down by a factor 2/3. The form factor we have introduced to fit the data is still required. However, the best fit now occurs for $\Lambda = 2.9\ m_\pi$ (or $\Lambda^2 = 0.165\ \mathrm{GeV}^2$) instead of $\Lambda = 2.6\ m_\pi$ as quoted earlier.

We wish to thank Professor J. D. Jackson for bringing this omission to our attention

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Meson Exchange in the Reaction $K^{+} + p \rightarrow K^{*} + N^{*}$ (†)

Gerson Goldhaber, William Chinowsky, Sulamith Goldhaber,
Wonyong Lee,‡ and Thomas O'Halloran

Physics Department and Lawrence Radiation Laboratory
University of California, Berkeley, California

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In the study of the four-particle final states of the $K^{+}p$ interaction at 1.96 BeV/c[1] we find that these states are predominantly produced via "double resonance" production, i.e., $K^{+} + p \rightarrow K^{*}(890) + N^{*}_{33}(1238)$. Moreover, the experimental data support a spin-zero meson exchange, presumably a pion, for small four-momentum transfers ($\Delta^{2} \leqslant 25 m_{\pi}^{2}$). The experiment was carried out[2] in the Brookhaven National Laboratory 20-inch hydrogen bubble chamber[3] exposed in the Brookhaven-Yale separated beam.[4]

The four particles produced in the reaction $K^{+}p \rightarrow K\pi N\pi$ are in the $T=1$ state and occur in five possible charge combinations, viz.: (1) $K^{+}\pi^{-}p\pi^{+}$, (2) $K^{0}\pi^{0}p\pi^{+}$, (3) $K^{0}\pi^{+}n\pi^{+}$, (4) $K^{+}\pi^{0}n\pi^{+}$, and (5) $K^{+}\pi^{0}p\pi^{0}$. Of these only the first three are accessible to unique identification in the bubble chamber. We present in Table I the observed cross sections for these reactions. Also shown in the table are the branching ratios expected on the assumption that the nucleon and pion are always produced in an isotopic spin $T = 3/2$ state, and that the K meson and pion are always produced in a $T = 1/2$ state. It is seen that the data are in good agreement with the branching ratios. In fact, the $T = 1/2$ and $T = 3/2$ states can be identified with the well-known[6] $K^{*}(890)$ and $N^{*}(1238)$ resonances, respectively.

It is thus indicated that "double resonance" production, which corresponds to "quasi-two-particle" production, plays an important role in these processes.

We have found it convenient to represent the four-particle production processes discussed here in terms of the production of a pair of "two-particle composites" with invariant masses m_x and m_y (in the overall c.m. system). The kinematical limits in this representation are particularly simple, namely, they form a right angle isosceles triangle for which the length of each leg is given by $Q = W - \sum_{i=1,4} m_i$. Here W is the total energy in the overall c.m. system and m_i, $i=1,4$ are the masses of the four outgoing particles.

The phase-space distribution is given by $\phi \propto \frac{1}{W}\int k_x k_y p_0 \, dm_x \, dm_y$, where the integral extends over the triangle (see Fig. 1) which we will call phase-space triangle (PST). Here k_x and k_y are the momenta in the c.m. of the composites x and y, respectively, and p_0 is their momentum in the overall c.m. system. It is noteworthy that along each of the three sides of the PST one of the factors in the integrand vanishes.

There are three ways (channels) in which the final-state particles can be "paired" off into two-particle composites for each of the charge states (1), (2), and (3). Direct evidence for double-resonance production follows from the details of the events in charge state (1), which represents the largest sample of events, and permits unambiguous assignment of the resonance states.

In Figures 1a, b, and c, we show the distribution of events for the various possible "two particle" composites in charge state (1), viz:

$$K^{*0}\ (K^+ + \pi^-) + N_{33}^{*++}\ (p + \pi^+) \tag{1a}$$

$$(K^+ + \pi^+) + N_{33}^{*0}\ (p + \pi^-) \tag{1b}$$

$$\rho^0\ (\pi^+ + \pi^-) + \qquad (p + K^+) \tag{1c}$$

The corresponding three PST's, as well as the projections on the respective mass axes, are also shown together with the calculated phase-space distributions. Channel (1a) corresponds to double-resonance production.

Defining events with $840 \leq M_{K^+\pi^-} \leq 940$ to lie within the K^* resonance and events with $1130 \leq M_{p\pi^+} \leq 1300$ to lie within the N^*_{33} resonance, we find 64% of all the events to lie within the double resonance, yielding a cross section $\sigma(K^*N^*) = 1.1 \pm 0.2$ mb. Channel (1b) corresponds to single-resonance formation in the $p\pi^-$ channel. This is a small effect and occurs in only about 10% of the events. (See Fig. 1b.) No evidence for a resonance in the $K\pi$ $T = 3/2$ system is observed. In channel (1c), ρ^0 production is energetically possible but is strongly suppressed by phase-space factors. No evidence for ρ^0 production was observed, neither is there any evidence for a positive-strangeness Hyperon (K^+p) in the $T = 1$ state. It is noteworthy that the reflection of the dominant resonances for channel (1a) do not give rise to appreciable deviations from phase-space predictions in the other two channels.

In an earlier communication we have shown the spin and parity of the K^* to be 1^-, from the observed anisotropy in the angular distribution of the K^* decay products.[1] As was pointed out this distribution, for small momentum transfers, is in agreement with the predictions of the one-pion-exchange model (OPE). According to this model the angular distribution of the outgoing K meson, in the K^* rest system with respect to the incoming K direction, is simply $f(\alpha) = \cos^2\alpha$, independent of the K^* production angle. In Fig. 2 we show a scatter diagram of the distribution of $\cos\alpha$ plotted against the square of the four-momentum transfer Δ^2.[7] Those distributions of events with $\Delta^2 \leq 25\, m_\pi^2$, shown in the projected plot (II) of Fig. 2, are in excellent agreement with the prediction for single-pion exchange. A least-squares fit of the form $f(\alpha) = a + b \cos\alpha + c \cos^2\alpha$ yields $a = 0.1 \pm 0.4$, $b = 0.07 \pm 0.08$, and

$c = 1.0 \pm 0.14$. For larger Δ^2, the distribution becomes more isotropic. The fit in this case gives $a = 1.11 \pm 0.18$, $b = -0.06 \pm 0.23$, and $c = 1.0 \pm 0.5$. This indicates that the one-pion exchange dominates the reaction up to a value of $\Delta^2 \approx 25\ m_\pi^2$, while other diagrams contribute when the momentum transfer becomes larger.

For further corroboration of the role of the one-pion exchange, we show in Fig. 3 the distributions in the Treiman-Yang angle.[8] This distribution is in agreement with isotropy both for the lower- and higher-momentum-transfer events. This result is consistent with the exchange of a spin-zero particle, but provides no independent evidence for it.

Finally, we compare the experimental differential cross section in the c.m. system for the double-resonance region with the results of a calculation based on the one-pion-exchange model. The calculation due to S. Berman[9] takes explicit account of the spins 1 and 3/2 of the two resonances produced. The calculated differential cross section is

$$\left(\frac{d\sigma}{d\Omega}\right)_{c.m.} = \frac{\delta}{6m^{*2}M^{*2}\,W^2}\ \frac{g_{K^*}^2}{4\pi}\ \frac{g_{N^*}^2}{4\pi}\ \left(\frac{1}{\Delta^2+m_\pi^2}\right)^2$$

$$\times \left\{[\Delta^2+(m^*+m)^2]\,[\Delta^2+(m^*-m)^2]\right\}\left\{[\Delta^2+(M^*+M)^2]^2[\Delta^2+(M^*-M)^2]\right\}, \quad (6)$$

where the coupling constants

$$g_{K^*}^2/4\pi = \Gamma_{K^*}\,m^{*2}/p_K^3 \approx 1.8 \quad (\text{for } K^+\pi^- \text{ mode only})$$

and $$(g_{N^*}^2/4\pi) = (3/2)\,\Gamma_{N^*}\,M^{*2}/p_N^3 \approx 24.0$$

are determined from the decay of the K^* and N^* respectively,

and $$\delta = \left(\frac{(W^2+M^{*2}-m^{*2})^2-4M^{*2}W^2}{(W^2+M^2-m^2)^2-4M^2W^2}\right)^{1/2}\cdot\frac{1}{(M^*+M)^2-m_\pi^2}$$

Here W is the total c.m. energy; m^*, M^*, m, and M are the masses of the K^*, N^*, K, and N, respectively; and p_K and p_N are the momenta of the K^* and N^* decay products in their respective c.m. systems. In this equation the last two factors in brackets result from summing over final-state spin directions of the K^* and N^* respectively.

The experimental distribution, including all double-resonance events,[10] is shown in Fig. 4. The three solid curves A, B, and C represent attempts to fit the data with pion-exchange models.[11] Curve B is obtained by evaluating Eq. (6) with $\Delta^2 = -m_\pi^2$ in the spin factors so that the momentum-transfer dependence is contained only in the propagator, viz.: $1/(\Delta^2 + m_\pi^2)^2$. The resulting equation corresponds then to the form originally proposed by Chew and Low.[12] Curve A gives the results of evaluating Eq. (6), including the proper spin factors. Comparison with the experimental data shows that this calculation gives too high a value for the cross section and does not reproduce the experimental angular distribution. To obtain a quantitative fit to the data we multiply Eq. (6) by a form factor, $F^2(\Delta^2)$. The exact choice of this form factor is somewhat arbitrary.[13] We have chosen a one-parameter expression similar to the nucleon form factor with the condition that $F(\Delta^2) \to 1$ as $\Delta^2 \to -m_\pi^2$, viz.: $F(\Delta^2) = (\Lambda^2 - m_\pi^2)/(\Lambda^2 + \Delta^2)$. By adjusting the parameter Λ, we have obtained the fit shown in Fig. 4, Curve C. The resulting value of Λ is $\Lambda = 2.6\, m_\pi$. It should be re-emphasized here that agreement with the OPE model is expected to hold only up to Δ^2 of $25 m_\pi^2$, we therefore consider the apparent fit to the data for $\Delta^2 \geq 25 m_\pi^2$ somewhat fortuitous.

We wish to take this opportunity to thank the many members of the staff of the Brookhaven National Laboratory for their very helpful attitude in making this experiment possible. In particular, we would like to express our appreciation to Dr. Hildred Blewett, Dr. Hugh Brown, Dr. Ed Hart, Dr. Ralph Shutt, Dr. James Sanford, Mr. Julius Spiro, Dr. Medford Webster, and the AGS operating crew. We also wish to thank Dr. Samuel Berman and Dr. Hans-Peter Duerr, Dr. Gyo Takeda, and Mr. Bertram Schwarzchild for a number of helpful discussions. Finally, this work would not have been possible without the active help and interest of Lawrence Radiation Laboratory scanning, measuring, and computing personnel.

Table I. Cross sections for the various charge-state combinations in the reaction $K^+ + p \rightarrow K\pi p\pi$ at 1.96 BeV/c

Charge combination	K^*	N^*	σ exptl. (mb)	Number of events observed	Number of events corrected[a]	Probability from I-spin composition	Normalized	Experimental result Normalized
(1)	$K^+\pi^-$	$p\pi^+$	1.7 ± 0.2	435	435	1/2	1	1
(2)	$K^0\pi^0$	$p\pi^+$	1.3 ± 0.2	110	330	1/4	0.72	0.76 ± 0.15
(2')	$K^0\pi^+$	$p\pi^0$				1/9		
(3)	$K^0\pi^+$	$n\pi^+$	0.33 ± 0.1	27	81	1/18	0.11	0.19 ± 0.05
(4)	$K^+\pi^0$	$n\pi^+$	Unmeasurable	---	---	1/36	0.06	---
(5)	$K^+\pi^0$	$p\pi^0$	Unmeasurable	---	---	1/18	0.11	---

[a] Corrections for the "invisible" decay modes of the K^0 have been made.

FOOTNOTES

† Work done under the auspices of the U. S. Atomic Energy Commission.

‡ Present address: Physics Department, Columbia University, New York, N. Y.

1 W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Lee, and T. O'Halloran, Phys. Rev. Letters 9, 330 (1962).

2 In this work we utilized a modification of the geometrical reconstruction (PANG) and kinematical fitting (KICK) programs of the Alvarez Group: J. P. Berge, F. T. Solmitz, and H. D. Taft, Rev. Sci. Instr. 32, 538 (1961); A. H. Rosenfeld and J. M. Snyder, Rev. Sci. Instr. 33, 181 (1962).

3 R. I. Louttit, in Proceedings of the International Conference on Instrumentation for High-Energy Physics, Berkeley, California, September, 1960 (Interscience Publishers Inc., New York, 1961), p. 117.

4 C. Baltay, J. Sandweiss, J. Sanford, H. Brown, M. Webster, and S Yamamoto, in Proceedings of the High-Energy Instrumentation Conference, CERN, 1962 (to be published).

5 It must be noted that experimentally we observe a superposition of (2) and (2'). The agreement between the observed cross-section ratios and the expected values from isotopic spin combinations assumes no interference between the two reactions.

6 M. H. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters 6, 300 (1961).

7 Here we are plotting the actual Δ^2 values computed for each event. In the

subsequent discussion, however, we use a "quasi particle" approximation for the resonances in which the cosine of the K^* production angle $\cos\theta_{K^*}$ is uniquely related to Δ^2. We have checked this procedure with a scatter plot of Δ^2 versus $\cos\theta_{K^*}$, and find the events to lie within a narrow band along the curve corresponding to definite resonance masses.

[8] S. B. Treiman and C. N. Yang, Phys. Rev. Letters 8, 140 (1962).

[9] Samuel Berman (Stanford University), private communication.

[10] There is very little evidence for nonresonant background in the double-resonance region. If we consider the channel 1a we can ascribe the events to $K^{*0} + N^{*++}$, 64%; $K^+ + \pi^- + N^{*++}$, 25%; $K^{*0} + p + \pi^+$, 5%; and $K^+ + \pi^- + p + \pi^+$ nonresonant, 6%.

[11] We have also calculated the differential cross section, using the equations of Salzman and Salzman [Phys. Rev. 120, 599 (1960)]. We get results consistent with curve A, Fig. 4, when we take into account the p-wave nature of both resonances.

[12] G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

[13] Recent analysis of data on pion production in p-p collisions has also shown the necessity of introducing a form factor into the OPE model in order to fit the experimental differential cross-section data. See, for example, E. Ferrari and F. Selleri, Nuovo Cimento 27, 1450 (1963).

FIGURE CAPTIONS

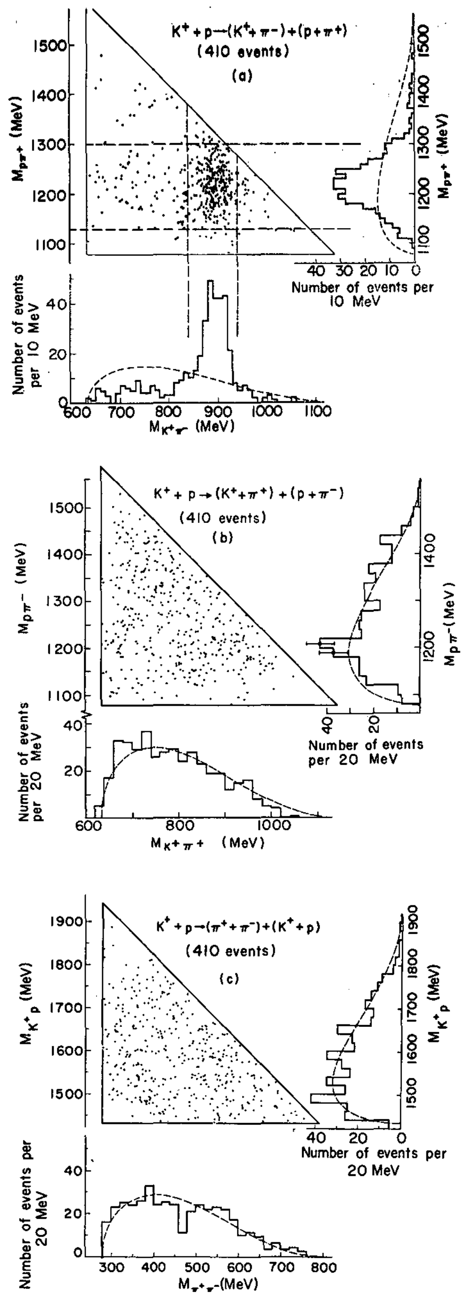
Fig. 1. Scatter diagrams of the effective mass distributions for the various two-particle composites in the reaction $K^+ + p \rightarrow K^+\pi^- p\pi^+$. The triangles delineate the kinematical limits; the dashed curves on the mass projections correspond to phase-space calculations without dynamic effects. It should be noted that the scale for mass projection is not the same in (a) as in (b) and (c).

Fig. 2. A scatter plot of the four-momentum transfer squared, Δ^2, versus the $K\pi$ scattering angle α. The events shown are those lying inside the "double-resonance rectangle" whose mass limits are given in the text. Here the angle α is defined as the angle between the incoming and outgoing K meson, <u>both</u> in the K^* c.m. system. The projections of the distributions in $\cos\alpha$ are given below the scatter plot, in region I for $\Delta^2 > 25\ m_\pi^2$ and in region II for $\Delta^2 \leq 25\ m_\pi^2$. The projection on the Δ^2 axis is shown on the right. The curves correspond to a fit with $f(\alpha) = a + b \cos\alpha + c \cos\alpha$. The coefficients for the two regions are given in the text.

Fig. 3. The Treiman-Yang angle distribution. The definitions of the angles, which correspond to a test for symmetry around the exchange particle axis, are given on the figure. The shaded region corresponds to the "double-resonance rectangle" whose mass limits are given in the text. The data are folded around the 0° - 180° axis, which takes into account parity conservation in strong interaction.

Fig. 4. The differential cross section for events lying in the "double-resonance rectangle" region. Curves A, B, and C refer to the OPE reaction, (A) as given in Eq. (6), (B) with the spin factors

evaluated on the mass shell, and (C) with a form factor, but with the spin factors evaluated in the physical region. The Δ^2 scale in this figure was computed by taking the masses of the K^* and N^* as fixed at their respective resonance values.



MUB-1790

Fig. 1

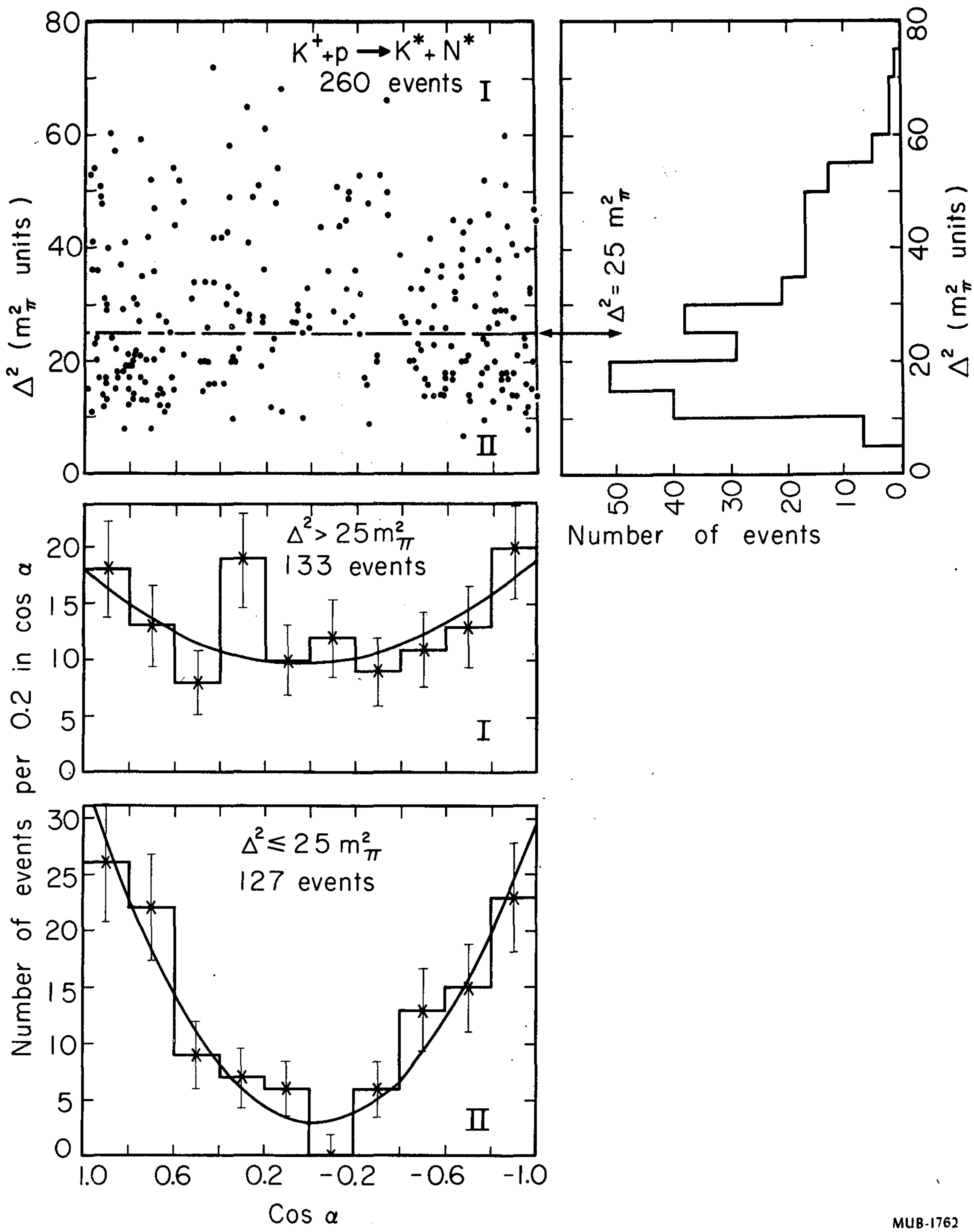

K⁺+p → K* + N*
260 events
I
II
Δ² = 25 m_π^2
Δ² (m_π^2 units)
Number of events
Δ² > 25 m_π^2
133 events
Δ² ≤ 25 m_π^2
127 events
Number of events per 0.2 in cos α
Cos α
MUB-1762

Fig. 2

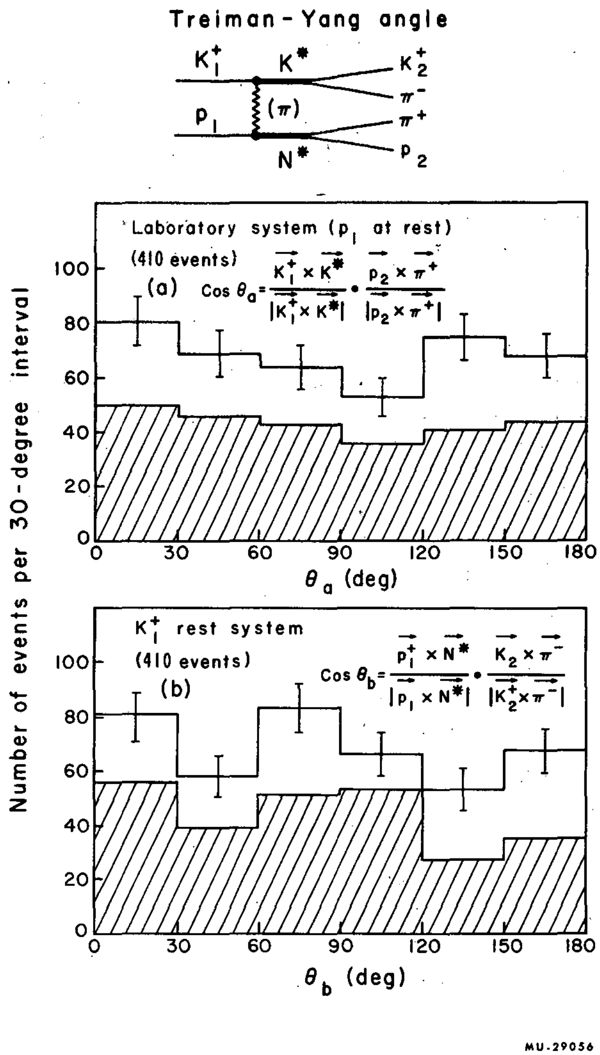
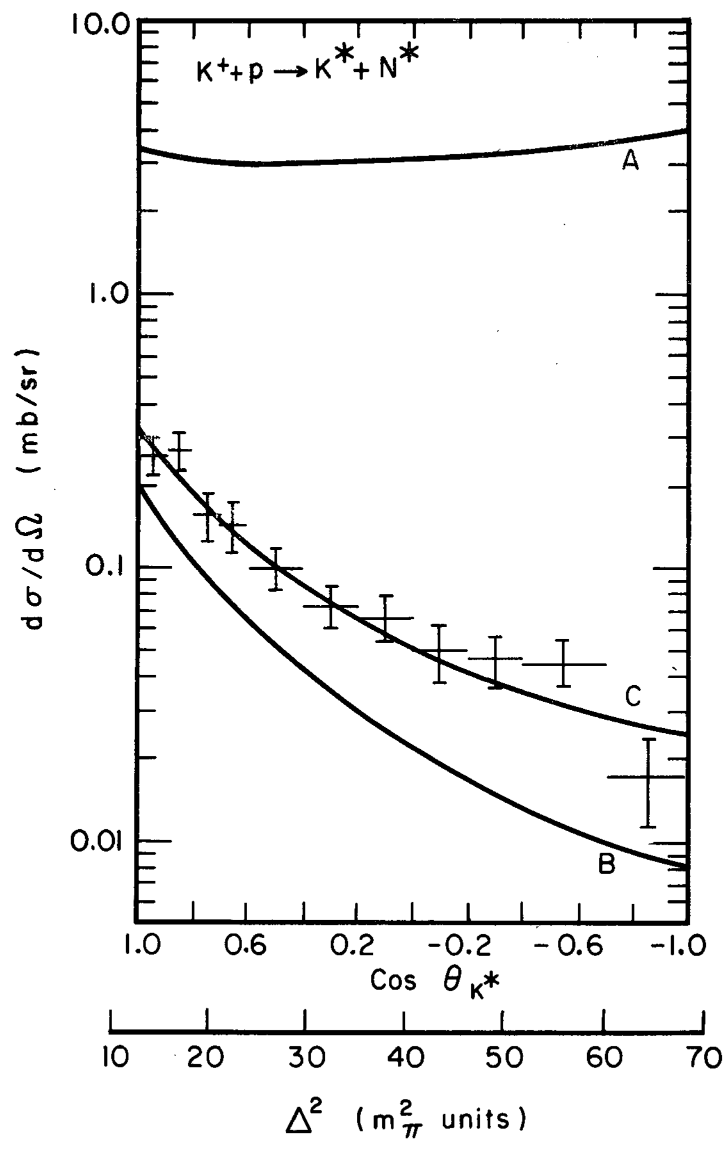

Treiman - Yang angle
Laboratory system (p_1 at rest)
(410 events)
(a)
$\cos\theta_a = \frac{\vec{K}_1^+ \times \vec{K}^*}{|\vec{K}_1^+ \times \vec{K}^*|} \cdot \frac{\vec{p}_2 \times \vec{\pi}^+}{|\vec{p}_2 \times \vec{\pi}^+|}$
θ_a (deg)
K_1^+ rest system
(410 events)
(b)
$\cos\theta_b = \frac{\vec{p}_1^+ \times \vec{N}^*}{|\vec{p}_1 \times \vec{N}^*|} \cdot \frac{\vec{K}_2 \times \vec{\pi}^-}{|\vec{K}_2^+ \times \vec{\pi}^-|}$
θ_b (deg)
Number of events per 30-degree interval
MU-29056

Fig. 3



10.0
1.0
0.1
0.01
$K^+ + p \rightarrow K^* + N^*$
A
B
C
$d\sigma/d\Omega$ (mb/sr)
1.0
0.6
0.2
-0.2
-0.6
-1.0
Cos θ_{K^*}
10
20
30
40
50
60
70
Δ^2 (m_π^2 units)
MU-30148

Fig. 4