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Essays on Theoretical and Empirical Macroeconomics

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Essays on Theoretical and Empirical Macroeconomics

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Tasneem Raihan

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Acknowledgments

Although my PhD journey has been an enthralling one, to be candid, when I decided to pursue a PhD in Economics little did I know about its challenges! It is only after I joined the PhD program at the University of California, Riverside that I started to truly assimilate them. However, I have been fortunate enough to have a bunch of amazing people who have stuck with me through thick and thin and helped me fulfill my dream against all the challenges.

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To my parents who inspire me to dream big.

ABSTRACT OF THE DISSERTATION

Essays on Theoretical and Empirical Macroeconomics

by

Tasneem Raihan

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Dr. Marcelle Chauvet, Chairperson

This dissertation is a collection of eclectic topics of interest in macroeconomics. The first chapter augments a medium-scale DSGE model with progressive income taxes levied on interest income, and shows how its interaction with a positive government expenditure shock may lead to increased overall tax revenues. Since with interest income tax government has an additional source of revenue, following the shock government's debt obligation is lower than the one implied by the existing models with no tax on interest income.

The second chapter re-examines the "natural resource curse" hypothesis popularized by the cross-sectional study of Sachs and Warner (1995). This chapter provides evidences against this hypothesis using the same set of variables used in Sachs and Warner (1995). Both static and dynamic panel methods are utilized to overcome omitted variable bias generally present in cross-sectional regressions. The results show that resource abundance proxied by primary commodity exports share of GDP (SXP) has no statistically significant negative impact on growth. Time fixed effects point out that

the debt crisis of 1980s drives the apparent negative relation between SXP and growth in cross-sectional regressions.

The third chapter seeks to identify the point in time when inflation uncertainty actually started to decline in the US, and to examine the performance of a two-regime Markov Switching-GARCH (MS-GARCH) model forecasting inflation uncertainty in the U.S. Results indicate that the switch to the low volatility regime happened approximately between April, 1979 and mid-1983. This time frame coincides with the period of aggressive monetary policy changes implemented by the then Fed chairman Paul Volcker. In addition, for a shorter horizon, normally distributed MS-GARCH forecasts and, for a longer horizon t-distributed MS-GARCH forecasts appear superior.

The final chapter seeks to predict the US recessions over the last three decades using the Treasury term spread data by employing a novel non-parametric approach called Dynamic Time Warping (DTW). Although compared to all parametric and non-parametric methods it is computationally much simpler, it has successfully signaled recessions as early as six months before the onsets of the actual recessions of 1990-1991, 2001, and 2007-2009. Compared to other non-parametric methods, DTW raises significantly fewer false recession signals.

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Chapter 1

Interest Income Tax and Government Spending: A Medium-scale DSGE Model Approach

1.1 Introduction

In most economies and particularly in advanced ones, tax revenue, which is used to finance public expenditures and investments and to pay off debts, constitutes a major proportion of government revenue. For example, in the U.S, approximately 89% of the federal government's revenue emanate from income tax which includes tax on interest income as well. Although tax revenue is an integral component for deciding the course of fiscal policies and their ramifications, a theoretical analysis of the exact direction of the relationship between tax revenue and public expenditure shock has surprisingly been limited in the literature.

On the other hand, to the extent pertinent empirical evidences are available, they are rather scant and inconclusive. For instance, using a mixed structural VAR/event-study approach to the post-war data of the U.S., Blanchard and Perotti (2002) documented a rise in net taxes following an expansionary government spending shock. They hypothesized that the increase in tax revenue owes to increased output following the positive shock. More recently, Mountford and Uhlig (2009) employed VAR with sign restrictions to investigate the effects of fiscal policies and they found that an expansionary government spending shock decreases tax revenue, albeit in a statistically non-significant manner.

As far as existing theoretical analyses are concerned, neoclassical macroeconomic theory suggests that government spending depresses private consumption by creating a negative wealth effect. This negative wealth effect emanates from consumers' apprehension of a future tax rise. A detailed explanation of this mechanism is provided in Baxter and King (1993). In addition to a decline in private consumption, private investment also decreases due to a crowding-out effect of increased government spending. Conclusions drawn from New Keynesian or DSGE models augmented with various frictions are also aligned with these results (see for example Smets and Wouters (2003)).[1] As an implication of these results, government revenues earned through distortionary taxes imposed on private consumption and investment tend to decline following a positive fiscal shock taking the form

[1] Galí et al. (2007) is an exception which allows for both Ricardian and non-Ricardian households, sticky prices and imperfectly competitive labor market to generate an increase in consumption following an expansionary government spending shock.

of a temporary increase in government spending. This ultimately leads to a fall in the overall tax revenue (see Costa Junior (2016) for example).

However, almost all of these models ignore the tax on interest income. The only exception is Stähler and Thomas (2012). Incorporating tax on interest income in a DSGE model is important for at least two reasons. The first reason is that it helps capture the real world tax raising mechanism. For example, interest earned on all U.S. Treasury securities is fully taxable at the federal level.[2] In fact, in the U.S. income earned from bonds are taxed at the same rate as ordinary income. This implies that the interest income will be taxed at the taxpayer's top marginal tax rate. For example, if a taxpayer falls in the 35% income tax bracket, then all of his or her interest income will also be taxed at the same rate. Since at least 50% of households in the U.S. who we characterize as Ricardian households are believed to have access to the financial market, and also since they tend to save a portion of their income possibly in the form of bonds, it is imperative to take into account the taxes that are raised from Ricardian's interest income.[3]

The second reason we should include tax on interest income is that it will allow us to arrive at a more complete theoretical analysis of the implications of fiscal policies such as, a temporary increase in government spending or a temporary reduction in labor income tax. Both of these fiscal tools are frequently used by policymakers when economies are faced with downturns. For example, to alleviate the recession of the early 1990s, President George Bush issued an executive order to lower the amount of income taxes that were being withheld from paychecks. As we will show in this paper, incorporating taxes on

[2]Municipal bond is an exception. They are generally non-taxable.
[3]Mankiw (2000) indicates this estimate of the fraction of Ricardian households in the economy.

interest income has important implications for overall tax revenues when the government conducts expansionary fiscal policies. Although several DSGE models have been developed to examine the impact of fiscal shocks on various macroeconomic variables, to the best of our knowledge, this paper is the first to analyze the impact on total tax revenues in the presence of a tax on interest income.

The main results of this paper can be summarized as follows: following an expansionary shock to government consumption, tax revenues from private consumption and investment indeed fall. However, under the scenario of a progressive income tax levied on interest income, overall tax revenues rise due to an increase in tax revenue from interest income which offsets the decline in revenues from private consumption and investment taxes. This increase in tax revenues from interest income owes to the increase in the issue of government bonds. The latter is caused by the government's attempt at maintaining a balanced budget when faced with a decline in public coffers after a positive spending shock. We further show that in comparison with an economy with no tax on interest income, government's debt obligation is much lower in an economy with progressive income tax on interest income. This result holds because now the government has an alternative source of revenue which is automatically set in motion whenever the government triggers an expansionary spending shock.

The organization of the paper is as follows: Section 1.2 presents the DSGE model incorporating progressive taxes on interest income, and also the log-linearized version of the model. This section also discusses the solution method used. Section 1.3 analyzes the results. Finally, Section 1.4 concludes.

1.2 Model

We assume that the economy is a barter one and is comprised of four different but interrelated sectors: (i) households (ii) retail firms and intermediate goods producing firms (iii) fiscal authority and (iv) monetary authority. Our basic model follows the one developed in Costa Junior (2016). In a nutshell, similar to Christiano et al. (2005) the model embeds Calvo-style nominal price and wage contracts, habit formation in preferences for consumption, adjustment costs in investment, and variable capital utilization. In what follows, we present the objective function of each sector and their maximization problems.

1.2.1 Household sector

There is a continuum of households indexed by $j \in [0, 1]$. A fraction ω_R has access to the financial market and acts as Ricardian-agents. As a result, they maximize their utilities intertemporally. Rest of the households characterized as non-Ricardians or rule-of-thumb households, indexed by $NR \in [\omega_R, 1]$ do not have access to the financial market and therefore, they neither can save nor can borrow. They simply consume current available income.

Each Ricardian household maximizes the following intertemporal utility function in terms of consumption $((C_{R,t}))$ and labor $(L_{R,t})$:

$$\max_{C_{R,t}, K^P_{t+1}, U_t, I^p_t, B_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_{R,t} - \phi_c C_{R,t-1})^{(1-\sigma)}}{1-\sigma} - \frac{L_{R,t}^{1+\varphi}}{1+\varphi} \right] \tag{1.1}$$

subject to,

$$P_t(1+\tau_t^c)(C_{R,t}+I_t^P)+\frac{B_{t+1}}{R_t^B}+\left(B_t-\frac{B_t}{R_{t-1}^B}\right)\tau_t^l=W_tL_{R,t}(1-\tau_t^l)+R_tU_tK_t^P(1-\tau_t^k)$$
$$-P_tK_t^P\left[\Psi_1(U_t-1)+\frac{\Psi_2}{2}(U_t-1)^2\right]+B_t+\omega_RP_tTR_t \quad (1.2)$$

with the law of motion of capital,

$$K_{t+1}^p=(1-\delta)K_t^P+I_t^P\left[1-\frac{\chi}{2}\left(\frac{I_t^P}{I_{t-1}^P}-1\right)^2\right] \quad (1.3)$$

where $E, K, B, W, 1/R^B, U, I^P, TR, P$ denote respectively the expectation operator, private capital, bond, nominal wage, bond discount factor, the degree of capital utilization, private investment, government transfers, and general price level. As regards the parameters, β, ϕ_c, σ, φ, δ, χ and ω_R represent respectively intertemporal discount factor, habit persistence, relative risk aversion coefficient, marginal disutility with respect to supply of labor, capital depreciation rate, sensitivity of investment with respect to adjustment cost, and percentage of Ricardian households. Besides these, $\Psi(U)$ denotes the cost of setting a level U of utilization rate. In addition, τ^c, τ^l and τ^k refer respectively to, tax on consumption and investment, tax on labor income and tax on capital income. Notice that consistent with the way interest income is taxed in the U.S., we have imposed the same labor income tax, τ^l on interest income given by $B_t - B_t/R_{t-1}^B$.

Maximizing equation (1.1) subject to equations (1.2) and (1.3) yields the following first order conditions:

$$\lambda_{R,t}=\frac{(C_{R,t}-\phi_cC_{R,t-1})^{-\sigma}}{P_t(1+\tau_t^c)}-\phi_c\beta\frac{(E_tC_{R,t+1}-\phi_cC_{R,t})^{-\sigma}}{P_t(1+\tau_t^c)} \quad (1.4)$$

$$Q_t=\beta E_t(1-\delta)Q_{t+1}+\lambda_{R,t+1}R_{t+1}U_{t+1}(1-\tau_{t+1}^k) \quad (1.5)$$

$$
\begin{aligned}
&- \lambda_{R,t+1}P_{t+1}\left[\Psi_1(U_t-1)+\frac{\Psi_2}{2}(U_t-1)^2\right] \\
&\frac{R_t}{P_t} = \left(\frac{1}{1-\tau_l^k}\right)[\Psi_1+\Psi_2(U_t-1)] && (1.6) \\
&\lambda_{R,t}P_t(1+\tau_t^c) - Q_t\left[1-\frac{\chi}{2}\left(\frac{I_t^p}{I_{t-1}^P}-1\right)^2-\chi\frac{I_t^P}{I_{t-1}^P}\left(\frac{I_t^P}{I_{t-1}^P}-1\right)\right] \\
&= \chi\beta E_t\left[Q_{t+1}\left(Q_{t+1}\frac{I_{t+1}^P}{I_t^P}\right)^2\left(\frac{I_{t+1}^P}{I_t^P}-1\right)\right] && (1.7) \\
&\frac{\lambda_{R,t}}{R_t^B} = \beta E_t\lambda_{R,t+1}\left[\left(\frac{1}{R_t^B}-1\right)\tau_{t+1}^l+1\right] && (1.8)
\end{aligned}
$$

where λ is the Lagrange multiplier on equation (1.2) and represents the marginal utility of income. Q_t is the Lagrange multiplier on equation (1.3) and represents the shadow price of private capital. It has the interpretation of Tobin's Q. Note that equations (4) and (8) together result in the dynamic consumption Euler equation. Equation (6) and (7) give respectively the demand for installed capacity and the demand for investment by Ricardians.

Non-Ricardian households solves the following optimization problem:

$$\max_{C_{NR,t}} E_t\sum_{t=0}^{\infty}\beta^t\left[\frac{(C_{NR,t}-\phi_c C_{NR,t-1})^{(1-\sigma)}}{1-\sigma}-\frac{L_{NR,t}^{1+\varphi}}{1+\varphi}\right] \tag{1.9}$$

subject to the following budget constraint which ignores capital investments and bonds:

$$P_t(1+\tau_t^c)C_{NR,t} = W_tL_{NR,t}(1-\tau_t^l)+(1-\omega_R)P_tTR_t \tag{1.10}$$

The first-order condition for the non-Ricardian households can be written as:

$$\lambda_{NR,t} = \frac{(C_{NR,t} - \phi_c C_{NR,t-1})^{-\sigma}}{P_t(1+\tau_t^c)} - \phi_c \beta \frac{(E_t C_{NR,t+1} - \phi_c C_{NR,t})^{-\sigma}}{P_t(1+\tau_t^c)} \tag{1.11}$$

Determination of wages

We assume that both Ricardian and non-Ricardian households supply differentiated labor in a market structure of monopolistic competition. This service is sold to a representative firm that aggregates these different types of labor (L_j) into a single labor input (L) using the following technology:

$$L_t = \left(\int_0^1 L_{j,t}^{\frac{\psi_W - 1}{\psi_W}} dj \right)^{\frac{\psi_W}{\psi_W - 1}} \tag{1.12}$$

where ψ_W is the elasticity of substitution between differentiated jobs and L_j, t is the amount of differentiated labor supplied by household j. Each type of labor j receives a wage $W_{j,t}$.

The labor-aggregating firm maximizes its profit in the following manner:

$$\max_{L_{j,t}} W_t L_t - \int_0^1 W_{j,t} L_{j,t} dj \tag{1.13}$$

The maximization problem in equation (1.13) together with the equation (1.12) yields the following demand equation for differentiated labor j:

$$L_{j,t} = L_t \left(\frac{W_t}{W_{j,t}} \right)^{\psi_W} \tag{1.14}$$

A little algebraic manipulation after substituting equation (1.14) in equation (1.12) yields the following function for the aggregate wage level:

$$W_t = \left(\int_0^1 W_{j,t}^{1-\psi_W} dj \right)^{\dfrac{1}{1-\psi_W}} \tag{1.15}$$

We assume that in each period $1-\theta_W$ households, chosen independently and at random, optimally define their wages. The remaining households, θ_W follow a Calvo style wage stickiness rule as in Calvo (1983). To be precise, they keep the same wage level as the previous period i.e. $W_{j,t} = W_{j,t-1}$.

As there is no distinction between labor offered by Ricardians ($x = R$) and non-Ricardians ($x = NR$), the problem of definition of wages is singular for both group of households and can be written as the following maximization problem:

$$\max_{W_{j,t}^*} E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \left\{ \frac{L_{x,j,t+i}^{1+\varphi}}{1+\varphi} - \lambda_{t+i} \left[-W_{j,t}^* L_{x,j,t+i} \right] (1-\tau_{t+i}^l) \right\} \tag{1.16}$$

After substituting equation (1.14) in the above equation and with a bit of algebraic manipulation, the first order condition yields the following equations for the definitions of optimal wages for Ricardian and non-Ricardian households respectively:

$$W_{j,t}^* = \left(\frac{\Psi_W}{\Psi_W - 1} \right) E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \left[\frac{L^{\varphi}{}_{R,j,t+i}}{\lambda_{R,t+i}(1-\tau_{t+i}^l)} \right] \tag{1.17}$$

$$W_{j,t}^* = \left(\frac{\Psi_W}{\Psi_W - 1} \right) E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \left[\frac{L^{\varphi}{}_{NR,j,t+i}}{\lambda_{NR,t+i}(1-\tau_{t+i}^l)} \right] \tag{1.18}$$

Given the above two equations, the aggregate nominal wage rule can be derived from equation (1.15) as follows:

$$W_t = \left[\theta_W W_{t-1}^{1-\psi_W} + (1-\theta_W) W_t^{*1-\psi_W} \right]^{\dfrac{1}{1-\psi_W}} \tag{1.19}$$

Determination of Aggregate Consumption and Labor

Since ω_R fraction of households represent Ricardians and $1-\omega_R$ represents non-Ricardians, aggregate consumption (C) and labor (L) can be determined in the following manner:

$$C_t = \omega_R C_{R,t} + (1-\omega_R) C_{N,R,t} \tag{1.20}$$

$$L_t = \omega_R L_{R,t} + (1-\omega_R) L_{NR,t} \tag{1.21}$$

1.2.2 Firms

We assume that the economy's production sector is comprised of two subsectors: an intermediate goods sector (wholesale firms) characterized by monopolistic competition, and a final goods sector (retail firms) characterized by perfect competition. A representative firm in the final goods sector buys a large variety of wholesale goods and aggregates them using a pertinent technology into a single good that will be sold in a perfectly competitive market.

Retail firms

A representative retail firm aggregates intermediate goods using a technology given by the following Dixit-Stiglitz aggregator function:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} \, dj \right)^{\frac{\psi}{\psi-1}} \tag{1.22}$$

where Y_t is the final product of a retailer in period t, and $Y_{j,t}$ is the jth intermediate good where $j \in [0,1]$. ψ denotes the elasticity of substitution between wholesale goods and takes a value greater 1.

With P_t as the nominal price of a retail product and $P_{j,t}$ as the nominal price of wholesale good j, the price of each wholesale good is taken as a given by retail firms. Therefore, the problem of the representative retail firms is maximizing its profit function as follows given equation (1.22):

$$\max_{Y_{j,t}} P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \tag{1.23}$$

The first-order condition for the above problem can be manipulated to obtain the following demand function for wholesale good j which is directly proportional to aggregate demand, Y_t and inversely proportional to its relative price level $(1/\frac{P_{j,t}}{P_t})$:

$$Y_{j,t} = Y_t \left(\frac{P_t}{P_{j,t}} \right)^{\psi} \tag{1.24}$$

A little algebraic manipulation after substituting equation (1.24) in equation (1.22) yields the pricing rule for final goods:

$$P_t = \left[\int_0^1 P_{j,t}^{1-\psi} dj \right]^{\dfrac{1}{1-\psi}} \tag{1.25}$$

Wholesale firms

The wholesale firm solves its problem in two stages. First, the firm determines the amount of capital and labor by minimizing its total production cost taking factor prices as given:

$$\min_{Lj,t,K_{j,t}} W_t L_{j,t} + R_t K_{j,t} \tag{1.26}$$

subject to the following production function for intermediate goods:

$$Y_{j,t} = A_t (U_t K^P_{j,t})^{\alpha_1} L^{\alpha_2}_{j,t} {K^G_{j,t}}^{\alpha_3} \tag{1.27}$$

where A_t, U_t, $K^P_{j,t}$ and $K^G_{j,t}$ represent respectively productivity, level of utilization of installed capacity (given by the relationship between the volume actually produced by the firm and what could be produced if the machines were operating at full capacity), private capital, and public capital. The parameters α_1, α_2 and α_3 capture the elasticity of the level of production with respect to respectively, utilized private capital, labor and public capital.

It is assumed that the law of motion of productivity follows a first-order autoregressive process, such that:

$$\log A_t = (1 - \rho_A) \log A_{ss} + \rho_A \log A_{t-1} + \epsilon_t \tag{1.28}$$

where A_{ss} is the value of productivity at the steady state, ρ_A is the autoregressive parameter of productivity, whose absolute value must be less than one to ensure the stationary nature of the process and $\epsilon_t \sim N(0, \sigma_A)$.

The first order conditions of the minimization problem defined in equations (1.26) and (1.27) are given by:

$$L_{j,t} = \alpha_2 MC_{j,t} \frac{Y_{j,t}}{W_t} \tag{1.29}$$

$$U_t K^P_{j,t} = \alpha_1 MC_{j,t} \frac{Y_{j,t}}{R_t} \tag{1.30}$$

Dividing equation (1.29) by equation (1.30):

$$\frac{L^P_{j,t}}{U_t Kj,t} = \frac{\alpha_2 R_t}{\alpha_1 W_t} \tag{1.31}$$

The left-hand side of the above equation is the marginal rate of technical substitution (MRTS) between labor and utilized capital. It measures the rate at which labor can be replaced by capital while maintaining a constant level of production. MRTS is equal to the economic rate of substitution (ERS) given by the right-hand side of the above equation. ERS measures the rate at which labor can replaced by capital while maintaining the same cost. Given equations (1.29) and (1.30) and the following expression for total cost:

$$TC_{j,t} = W_t L_{j,t} + R_t K_{j,t} \tag{1.32}$$

marginal cost (MC) can be written as:

$$MC_{j,t} = \frac{1}{A_t K^{G\alpha_3}_{jt}} \left(\frac{W_t}{\alpha_2}\right)^{\alpha_2} \left(\frac{R_t}{\alpha_1}\right)^{\alpha_1} \tag{1.33}$$

The second stage of the problem of the wholesale firm is defining the price of its goods. This firm decides the production level in each period according to the Calvo rule. In particular, we assume that in each period, a fraction $0 < 1 - \theta < 1$ of firms is randomly selected and allowed to optimally define the prices of its goods for the period. [4] The rest of the firms maintain the previous period's price i.e. $P_{j,t} = P_{j,t-1}$. An additional assumption that we make here for mathematical convenience is that wholesale firms have constant marginal costs. This allows us to express total cost (TC) by multiplying the quantity produced with the marginal cost.

Given the above assumptions, the problem of the wholesale firm that is capable of readjusting the price of its good is given by:

$$\max_{P^*_{j,t}} E_t \sum_{i=0}^{\infty} (\beta\theta)^i (P^*_{j,t} Y_{j,t+i} - TC_{j,t+i}) \tag{1.34}$$

Substituting equation (1.24) in the above equation and then taking the first-order condition results in,

$$P^*_{j,t} = \frac{\psi}{\psi - 1} E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{j,t+i} \tag{1.35}$$

where $\frac{\psi}{\psi-1}$ is the same gross frictionless price markup on the marginal cost that every wholesale firm sets. As a result, in all periods, $P^*_{j,t}$ is the same price for all the $1-\theta$ firms that optimally readjust their prices. Therefore, we can write $P^*_{j,t} = P^*_t$. The aggregate price level can be determined now from equation (1.24):

$$P_t = \left[\theta P_{t-1}^{1-\psi} + (1-\theta) P_t^{*1-\psi}\right]^{\frac{1}{1-\psi}} \tag{1.36}$$

[4] It can be shown that the expected duration between price changes is given by $\frac{1}{1-\theta}$.

1.2.3 Fiscal authority

The fiscal authority is responsible for taxing households and issuing debt to fund its expenses. Its expenses comprise public consumption or expenditure, G_t; public investment, I_t^G; and transfer of income to households, TR_t. We assume that the government does not issue currency. Therefore, the initial assumption of a barter economy remains valid. Given these assumptions, the government's budget constraint takes the following form:

$$\frac{B_{t+1}}{R_t^B} - B_t + T_t = P_t G_t + P_t I_t^G + P_t TR_t \tag{1.37}$$

where T is the total tax revenue given by,

$$T_t = \tau_t^c P_t (C_t + I_t^P) + \tau_t^l W_t L_t + \tau_t^k (R_t - \delta) K_t^P + \tau_t^l \left(B_t - \frac{B_t}{R_{t-1}^B} \right) \tag{1.38}$$

We assume that except labor income tax τ^l, all other fiscal policy instruments in $Z = \{G_t, I_t^G, TR_t, \tau_t^c, \tau_t^k, \tau_l\}$ that the government uses follow the same rule governed by,

$$\frac{Z_t}{Z_{ss}} = \left(\frac{Z_{t-1}}{Z_{ss}} \right)^{\gamma_Z} \left(\frac{B_t}{Y_{t-1} P_{t-1}} \frac{Y_{ss} P_{ss}}{B_{ss}} \right)^{(1-\gamma_Z)\phi_Z} S_t^Z \tag{1.39}$$

where $B_t / Y_{t-1} P_{t-1}$ is the ratio between public debt and GDP in period $t-1$, $Y_{ss} P_{ss} / B_{ss}$ is inverse of the long-run target of the previous ratio, ϕ_z measures the sensitivity of the corresponding instrument to deviations in the debt ratio from its long-run target. S_t^z is an exogenous shock to the instrument. Following Guo and Lansing (1998) and Mattesini and Rossi (2012), we specify the following form for the labor income tax rate, τ^l which admits a progressive tax schedule under certain conditions:

$$\tau_t^l = 1 - \eta \left(\frac{Y_{ss}}{Y_t}\right)^{\phi_n} S_t^{\tau_t^l} \tag{1.40}$$

where Y_t represents the individual household's current period taxable income, $Y_s s$ denotes the steady-state income. η determines the level of tax schedule, ϕ_n governs the slope of the tax schedule, and $S_t^{\tau_t^l}$ represents a labor income tax shock. It can be shown that whenever $\phi_n > 0$, households with taxable income above Y_{ss} face a higher tax rate than those with income below Y_{ss}. In this case, the tax schedule is referred to as progressive since the average tax increases in income. The flat tax schedule occurs when $\phi_n = 0$. Under this condition, the marginal tax rate equals the average tax rate.

Finally, the fiscal shock is represented by:

$$\log S_t^Z = (1 - \rho_Z) \log S_{ss}^Z + \rho_Z \log S_{t-1}^Z + \epsilon_{Z,t} \tag{1.41}$$

and the motion of stock of public capital K^G is governed by the following rule:

$$K_{t+1}^G = (1 - \delta_G) K_t^G + I_t^G \tag{1.42}$$

1.2.4 Monetary authority

We assume that the central bank adopts a Taylor-rule of the following form which seeks to attain two objectives: (i) price stability and (ii) economic growth:

$$\frac{R_t^B}{R_{ss}^B} = \left(\frac{R_{t-1}^B}{R_{ss}^B}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y_{ss}}\right)^{\gamma_Y}\right]^{1-\gamma_R} S_t^m \tag{1.43}$$

where γ_Y and γ_π are the sensitivities of the basic interest rate with respect to output, Y and inflation rate, π, respectively and γ_R is the smoothing parameter. Finally, S_t^m is the monetary policy shock which is given by the following first-order autoregressive process:

$$\log S_t^m = (1 - \rho_m)\log S_{ss}^m + \rho_m \log S_{t-1}^m + \epsilon_{m,t} \tag{1.44}$$

1.2.5 Equilibrium condition and log-linearization

The model's equilibrium condition states that total output produced in the economy is equal to the summation of private consumption, C_t; private investment, I_t^P; public investment, I_t^G, and government expenditure, G_t. This can be written as:

$$Y_t = C_t + I_t^P + I_t^G + G_t \tag{1.45}$$

We can now write out the reduced form of our New Keynesian model using equations that are log-linearized around a zero inflation steady-state. Note that in what follows, $x = (R, NR)$ distinguishes between Ricardians (R) and non-Ricardians (NR). Further note that $\widetilde{X}_t = \log X - \log X_{ss}$ represents the log of the variable X's deviation in relation to its steady state.

Household Lagrangian:

$$\widetilde{\lambda}_{x,t} + \widetilde{P}_t + \frac{\tau_{ss}^c}{1+\tau_{ss}^c}\widetilde{\tau}_t{}^c = \left[\frac{\sigma}{(1-\phi_c\beta)(1-\phi_c)}\right]\left[\phi_c\beta(E_t\widetilde{C}_{x,t+1} - \phi_c\widetilde{C}_{x,t}) - (\widetilde{C}_{x,t} - \phi_c\widetilde{C}_{x,t-1})\right] \tag{1.46}$$

Phillips equation for household wages:

$$\widetilde{\pi}_{W,t} = \beta E_t \widetilde{\pi}_{W,t+1} \left[\frac{(1-\theta_w)(1-\beta\theta_W)}{\theta_W} \right] \varphi \widetilde{L}_{x,t} - \widetilde{\lambda}_{x,t} + \left(\frac{\tau_{ss}^l}{1-\tau_{ss}^l} \right) \widetilde{\tau}_t^l \tag{1.47}$$

Gross wage inflation rate:

$$\widetilde{\pi}_{W,t} = \widetilde{W}_t - \widetilde{W}_{t-1} \tag{1.48}$$

Ricardian household's budget constraint:

$$\begin{gathered} P_{ss} C_{R,ss} \left[(\widetilde{P}_t + \widetilde{C}_{R,t})(1+\tau_{ss}^c) + \tau_{ss}^c \widetilde{\tau}_t^c \right] + P_{ss} I_{ss}^P \left[\left(\widetilde{P}_t + \widetilde{I}_t^P \right)(1+\tau_{ss}^c) + \tau_{ss}^c \widetilde{\tau}_t^c \right] + \\ \frac{B_{ss}}{R_{ss}^B} (\widetilde{B}_{t+1} - \widetilde{R}_t^B) - B_{ss} \tau_{ss}^l R_{ss}^B (\widetilde{B}_t + \widetilde{\tau}_l - \widetilde{R}_t^B) = W_{ss} L_{R,ss} \left[(\widetilde{W}_t + \widetilde{L}_{R,t})(1-\tau_{ss}^l) - \tau_{ss}^l \widetilde{\tau}_t^l \right] + \\ R_{ss} K_{ss}^P \left[\left(\widetilde{R}_t + \widetilde{K}_t^P \right)(1-\tau_{ss}^k) - \tau_{ss}^k \widetilde{\tau}_t^k \right] + B_{ss} \left[\widetilde{B}_t - \tau_{ss}^l (\widetilde{B}_t + \widetilde{\tau}_t^l) \right] \omega_R TR_{ss} \widetilde{TR}_t \end{gathered} \tag{1.49}$$

Tobin's Q:

$$\begin{gathered} \left(\frac{Q_{ss}}{\beta} \right) \widetilde{Q}_t = E_t \{ (1-\delta) Q_{ss} \widetilde{Q}_{t+1} + \lambda_{R,ss} R_{ss} U_{ss} (1-\tau_{ss}^k) \\ \left[\widetilde{\lambda}_{R,t+1} + \widetilde{R}_{t+1} + \widetilde{U}_{t+1} - \frac{\tau_{ss}^k}{1-\tau_{ss}^k} \widetilde{\tau}_{t+1}^k \right] - \lambda_{R,ss} P_{ss} \Psi_1 U_{ss} \widetilde{U}_{t+1} \} \end{gathered} \tag{1.50}$$

Demand for installed capacity:

$$\left(1 - \tau_{ss}^k \right) \frac{R_{ss}}{P_{ss}} \left[\widetilde{R}_t - \widetilde{P}_t - \left(\frac{\tau_{ss}^k)}{1-\tau_{ss}^k} \right) \widetilde{\tau_t^k} \right] = \Psi_2 U_{ss} \widetilde{U}_t \tag{1.51}$$

Demand for investments:

$$(1+\tau^c_{ss})\,\lambda_{R,ss}P_{ss}\left[\widetilde{\lambda}_{R,t}+\widetilde{P}_t+\frac{\tau^c_{ss}}{1+\tau_{ss}}\widetilde{\tau}^c_t\right]$$
$$-Q_{ss}\widetilde{Q}_t+\chi Q_{ss}(\widetilde{I}^P_t-\widetilde{I}^P_{t-1})=\chi\beta Q_{ss}(E_t\widetilde{I}^P_{t+1}-\widetilde{I}^P_t) \tag{1.52}$$

Law of motion of private capital:

$$\widetilde{K}^P_{t+1}=(1-\delta)\widetilde{K}^P_t+\delta\widetilde{I}^P_t \tag{1.53}$$

Public bond Euler equation:

$$1+\widetilde{\lambda}_{R,t}-\widetilde{R}^B_t=1+\widetilde{\lambda}_{R,t+1}+\widetilde{T}^B_{t+1} \tag{1.54}$$

where

$$\widetilde{\tau}^B_{t+1}=\frac{1}{1+\frac{\tau^l_{ss}}{R^B_{ss}}-\tau^l_{ss}}\left[\frac{\tau^l_{ss}}{R^B_{ss}}(\widetilde{\tau}_l-\widetilde{R}^B)-\tau^l_{ss}\widetilde{\tau}^l_{t+1}\right] \tag{1.55}$$

Aggregate Consumption:

$$C_{ss}\widetilde{C}_t=\omega_R C_{R,ss}\widetilde{C}_{R,ss}+(1-\omega_R)C_{NR,ss}\widetilde{C}_{NR,ss} \tag{1.56}$$

Aggregate Labor:

$$L_{ss}\widetilde{L}_t=\omega_R L_{R.ss}\widetilde{L}_{R,ss}+(1-\omega_R)L_{NR,ss}\widetilde{L}_{NR,ss} \tag{1.57}$$

Production technology:

$$\widetilde{Y}_t=\widetilde{A}_t+\alpha_1(\widetilde{U}_t+\widetilde{K}^P_t)+\alpha_2\widetilde{L}_t+\alpha_3\widetilde{K}^G_t \tag{1.58}$$

Problem of the firm's trade-off from equation (1.31)**:**

$$\widetilde{L}_t - \widetilde{U}_t - \widetilde{K}_t^P = \widetilde{R}_t - \widetilde{W}_t \tag{1.59}$$

Marginal cost

$$\widetilde{MC}_t = \alpha_2 \widetilde{W}_t + \alpha_1 \widetilde{R}_t - \widetilde{A}_t - \alpha_3 \widetilde{K}_t^G \tag{1.60}$$

New Keynesian Phillips Curve:

$$\widetilde{\pi}_t = \beta E_t \widetilde{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}(\widetilde{MC}_t - \widetilde{P}_t) \tag{1.61}$$

Equation (1.61) relates current inflation to current real marginal cost and expected future inflation. If we solve it forward imposing the condition that inflation does not explode, then we can write inflation as a present discounted value of real marginal costs:

$$\widetilde{\pi}_t = \frac{(1-\theta)(1-\beta\theta)}{\theta} \sum_{s=0}^{\infty} \beta^s E_t \widetilde{MC}_{t+s} \tag{1.62}$$

Gross inflation rate:

$$\widetilde{\pi}_t = \widetilde{P}_t - \widetilde{P}_{t-1} \tag{1.63}$$

Government budget constraint:

$$\frac{B_{ss}}{R_{ss}^B}(\widetilde{B}_{t+1} - \widetilde{R}_t^B) - B_{ss}\widetilde{B}_t + T_{ss}\widetilde{T}_t = P_{ss}G_{ss}(\widetilde{G}_t + \widetilde{P}_t) + P_{ss}I_{ss}^G(\widetilde{P}_t + \widetilde{I}_t^G) + P_{ss}TR_{ss}(\widetilde{P}_t + \widetilde{TR}_t) \tag{1.64}$$

Government tax revenues:

$$T_{ss}\widetilde{T}_t = \tau^c_{ss}P_{ss}\Big[C_{ss}(\widetilde{C}_t + \widetilde{P}_t + \widetilde{\tau}^c_t) + I^P_{ss}(\widetilde{I}^P_t + \widetilde{P}_t + \tau^c_t) + \tau^l_{ss}W_{ss}L_{ss}(\widetilde{W}_t + \widetilde{L}_t + \widetilde{\tau}^l_t)$$
$$+\tau^k_{ss}K^P_{ss}(R_{ss}(\widetilde{R}_t + \widetilde{K}^P_t + \widetilde{\tau}^k_t) - \delta(\widetilde{K}^P_t + \widetilde{\tau}^k_t)) + \tau^l_{ss}B_{ss}(\widetilde{\tau}^l_t + \widetilde{B}_t)\Big] \tag{1.65}$$

Law of motion of public capital:

$$\widetilde{K}^G_{t+1} = (1-\delta_G)\widetilde{K}^G_t + \delta\widetilde{I}^G_t \tag{1.66}$$

Rule for all fiscal policy instruments except income tax:

$$\widetilde{Z}_t = \gamma_Z\widetilde{Z}_{t-1} + (1-\gamma_Z)\phi_Z(\widetilde{B}_t - \widetilde{Y}_{t-1} - \widetilde{P}_{t-1} + \widetilde{S}^Z_t) \tag{1.67}$$

Progressive income tax rule:

$$\widetilde{\tau}^l = \frac{\eta\phi_n(\widetilde{W} + \widetilde{L})}{1-\eta} + \widetilde{S}^{\tau^l}_t \tag{1.68}$$

Equilibrium condition:

$$Y_{ss}\widetilde{Y}_t = C_{ss}\widetilde{C}_t + I^P_{ss}\widetilde{I}^P_t + I^G_{ss}\widetilde{I}^G_t + G_{ss}\widetilde{G}_t \tag{1.69}$$

Fiscal policy shock:

$$\widetilde{S}^Z_t = \rho_Z\widetilde{S}^Z_{t-1} + \epsilon_{Z,t} \tag{1.70}$$

We have used Dynare to solve the model presented in this section. All simulations and impulse response functions begin at the exact steady state computed by Dynare considering the initial values provided as mere approximations. A pure perturbation method as in Schmitt-Grohé and Uribe (2004) has been chosen to compute the quadratic approximation of the decision rules.

1.3 Results

We choose to calibrate the model rather than to estimate the parameters since the main objective of this study is to unravel the important impact that progressive income tax levied on interest income may have on overall tax revenues following an expansionary government consumption shock. As can be seen in Table 1, the calibration for macroeconomics variables is consistent with the existing literature.

First, let's consider the case of no tax on interest income. Figure 1.1 and 1.2 display the impulse-response functions of several macroeconomic variables corresponding to this case, following a positive government consumption i.e. expenditure shock. An expansionary shock to the government's current expenditure raises aggregate demand for output, Y. This in turn puts upward pressure on the general price level leading to an increase in inflation (PI). In response, according to the Taylor rule, the central bank raises the basic rate of interest R^B. This causes a crowding-out of private and public investment, I^P and I^G.

On the other hand, this particular expansionary fiscal shock imparts a negative wealth effect on households since they expect a rise in future taxes which would finance the current increase in government consumption. Accordingly, both Ricardian consumption, C^R and non-Ricardian consumption C^{NR} decline resulting in an overall decline in total consumption, C and naturally, in consumption tax revenue. This adversely affects the total tax revenue of the government which starts recovering only around the 10th quarter. As a response to this negative effect on public coffers, government resorts to debt-financing by issuing more bonds to public. Therefore, B rises in Figure 1.2. Also note that faced with a decline in welfare, households choose to work more by increasing their labor supplies, L^R and L^{NR}. A simultaneous fall in consumption and increase in labor supply indicates a strong substitution effect on households under no interest income tax.

Now let's focus on the case where we incorporate progressive income tax on interest income. The impulse responses corresponding to this case are displayed in Figure 1.3 and 1.4. Following a positive government expenditure shock, Ricardian households cut down their consumption to save more since they expect a rise in future taxes to fund today's increased government expenditure. They also increase their labor supply. Therefore, Ricardian households exhibit the existence of a strong substitution effect as in the previous case. On the other hand, non-Ricardian households have no such option of saving for future and therefore, in contrast with the previous case they enjoy the immediate increase in the real wage by immediately increasing their consumption (although marginally), and by working less. In other words, initially income effect is more dominant for the non-Ricardians when interest income tax is incorporated. Note that compared to the previous case where no tax on interest income was imposed, the increase in wages (W) in this case is much larger. However, as wage keeps declining over time, non-Ricardians start consuming less and supply more labor.

Notice that just as in the previous case with no interest income tax, overall consumption diminishes implying lower tax revenues from consumption. In response to the decline in public coffers, government decides to issue more bonds to raise funds. But this time, since a progressive income tax has been imposed on interest income, government earns extra revenue. This extra revenue must have offset the decline in consumption and investment tax revenues since total tax revenue, T has risen. In fact, total tax revenue follows the trend of total bond issues.

Another interesting result is that, with the introduction of tax on interest income, bond issue reaches a peak just below 0.004 which is much lower than the peak of approximately 0.01 attained under no interest income tax. This implies that under progressive tax on interest income, government's debt obligation is much lower than under no interest income tax. The reason is that under the scenario of an interest income tax, government earns extra tax revenue through the tax on bond returns which is not possible under the scenario of no interest income tax. Therefore, to finance its expenditure, the government needs to issue more debt in the latter case.

1.4 Conclusion

Governments using fiscal policies to stabilize output and employment, especially when the economy experiences a downturn is not a new phenomenon. Many countries including the U.S. have combined fiscal policies with lax monetary policies to combat the Great Recession. However, little is known about the impact of an expansionary government spending shock on total tax revenue, which is the major source of financing future public consumption and investment in advanced economies, such as the U.S or U.K. Existing New Keynesian analyses imply a negative relationship between total tax revenue and positive public spending shocks. However, they ignore the tax on interest income which can have a significant impact on total tax revenues through interacting with a positive public spending shock. Therefore, their model-implied impulse responses of tax revenues are biased.

To overcome the above-mentioned shortcoming of the existing DSGE models, this paper has augmented a medium-scale DSGE model with a progressive income tax on interest income. The result indicates that following an expansionary public spending shock, government issues bonds to Ricardian households in order to raise funds with a view to counteracting the decline in consumption and investment tax revenues. However, an increase in bond issue gives rise to a separate channel through which further tax revenues could be earned, which the existing models failed to take into account. The impulse response function implied by our model shows that total tax revenues rise after a positive public spending shock only when a progressive tax on interest income is incorporated within. This result is consistent with the empirical finding of Blanchard and Perotti (2002) who reported a rise in tax revenues following a positive government spending shock. Another interesting result is that, since with interest income tax government has an additional source of revenue, following an expansionary fiscal shock government's debt obligation is lower than the ones implied by the existing models with no tax on interest income.

Table 1.1: Calibrated parameter values

Parameter	Parameter definition	Calibrated value
σ	Relative risk aversion coefficient	2
φ	Marginal disutility with respect to the supply of labor	1.5
α_1	Elasticity of output with respect to private capital	0.3
α_2	Elasticity of output with respect to labor	0.6
α_3	Elasticity of output with respect to public capital	0.05
β	Discount factor	0.985
δ	Capital depreciation rate	0.025
θ	Price stickiness parameter	0.75
ψ	Elasticity of substitution among intermediate goods	8
θ_W	Wage stickiness parameter	0.75
ψ_W	Elasticity of substitution between differentiated labor	21
τ_{ss}^c	Consumption tax rate in steady state	0.16
τ_{ss}^l	Labor income tax rate in steady state	0.17
τ_{ss}^k	Capital income tax rate in steady state	0.08
ω_R	Fraction of Ricardian households in the economy	0.5
ϕ_c	Habit persistence	0.8
χ	Sensitivity of investments to adjustment cost	1
Ψ_1	Sensitivity of cost of under-utilization of maximum installed capacity 1	$(1+\tau_{ss}^c)(\frac{1}{\beta}-1+\delta)$
Ψ_2	Sensitivity of cost of under-utilization of maximum installed capacity 2	1
δ_G	Depreciation rate of public capital	0.025
γ_R	Interest rate persistence	0.79
γ_Y	Sensitivity of interest rate to GDP	0.16

Table 2.1: Calibrated parameter values (continued)

Parameter	Parameter definition	Calibrated value
γ_π	Sensitivity of interest to inflation	2.43
$\phi_{TR_{ss}}$	Government Transfer to GDP ratio	0.01
$\phi_{B_{ss}}$	Public debt to GDP ratio	1
$\phi^G_{I_{ss}}$	Public investment to GDP ratio	0.02
γ_G	Government consumption persistence	0
γ^G_I	Persistence of public investment	0.1
γ_{TR}	Persistence of income transfer	0.1
γ_{τ^c}	Persistence of consumption tax	0
γ_{τ^l}	Persistence of labor income tax	0
γ_{τ^k}	Persistence of capital income tax	0
ϕ_G	Government consumption to debt ratio	0
ϕ^G_I	Public investment to debt ratio	-0.1
$\phi_T R$	Government transfer to debt ratio	-0.1
ϕ_{τ^c}	Consumption tax to debt ratio	0
ϕ_{τ^l}	Labor income tax to debt ratio	0
ϕ_{τ^k}	Capital income tax to debt ratio	0
η	Labor tax schedule level	0.7
ϕ_n	Labor tax schedule slope	0.3

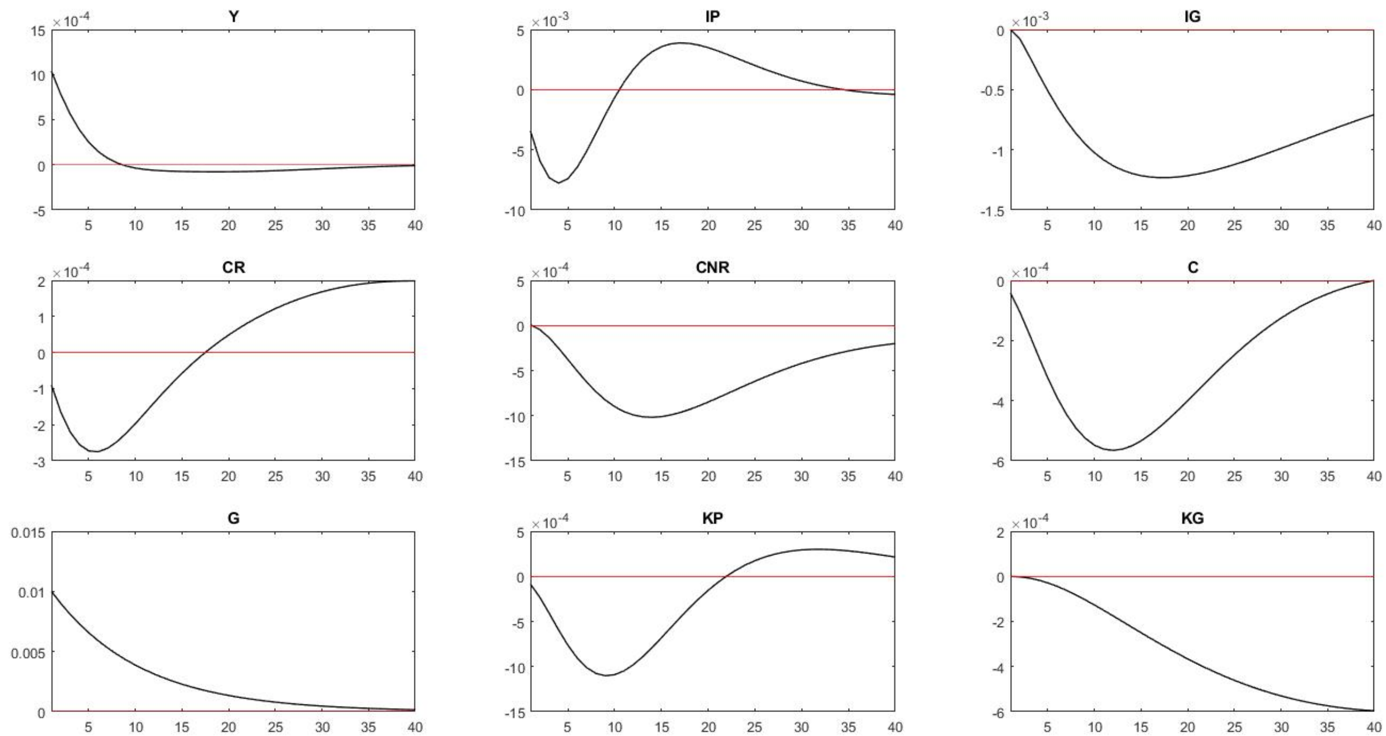


Figure 1.1: Impulse Responses to a 1 s.d. government consumption shock with no interest income tax

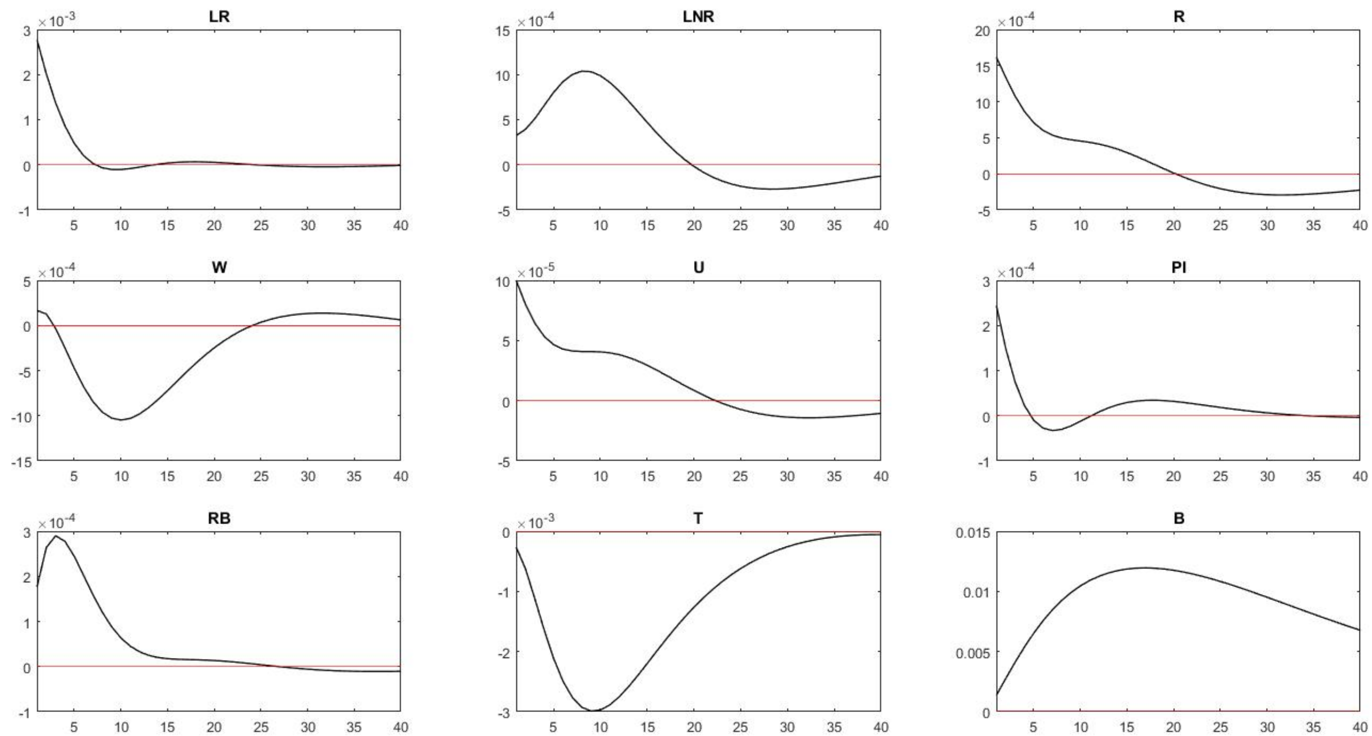


Figure 1.2: Impulse Responses to a 1 s.d. government consumption shock with no interest income tax

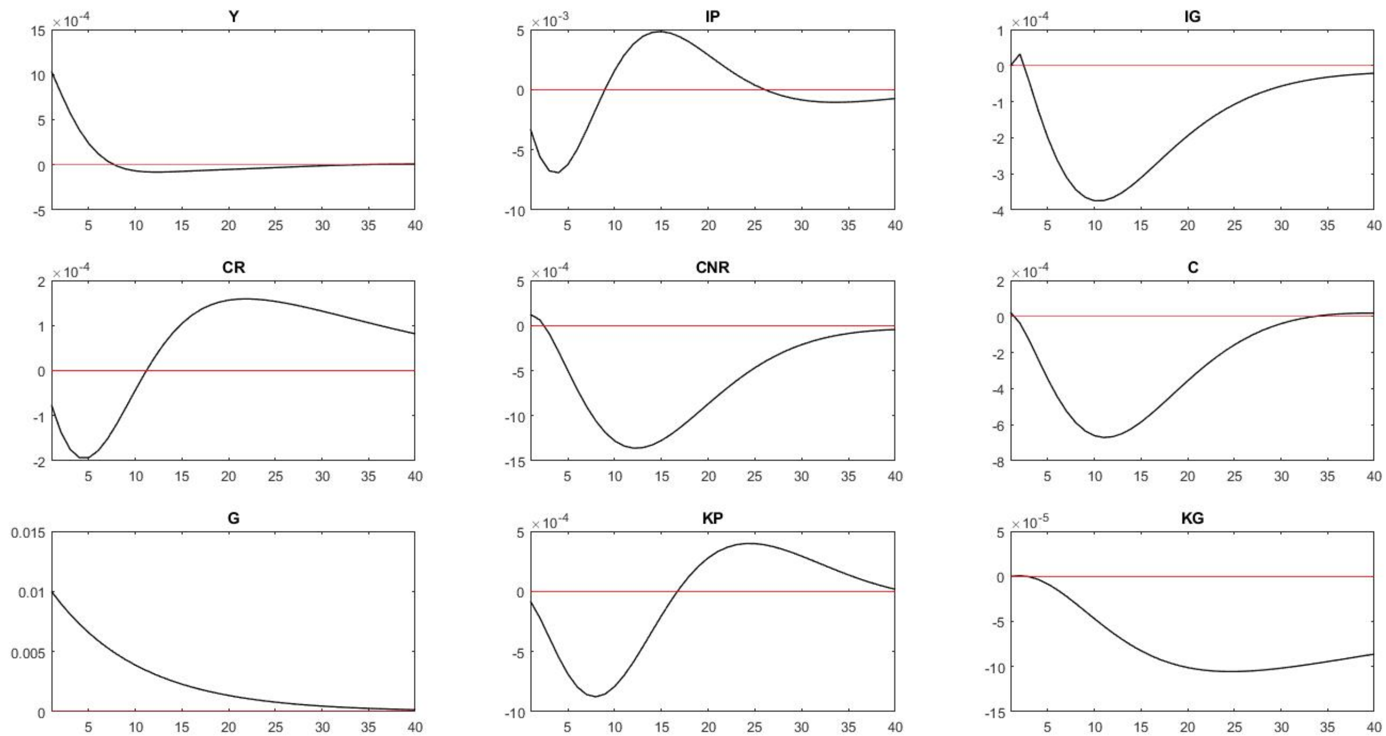


Figure 1.3: Impulse Responses to a 1 s.d. government consumption shock with interest income tax

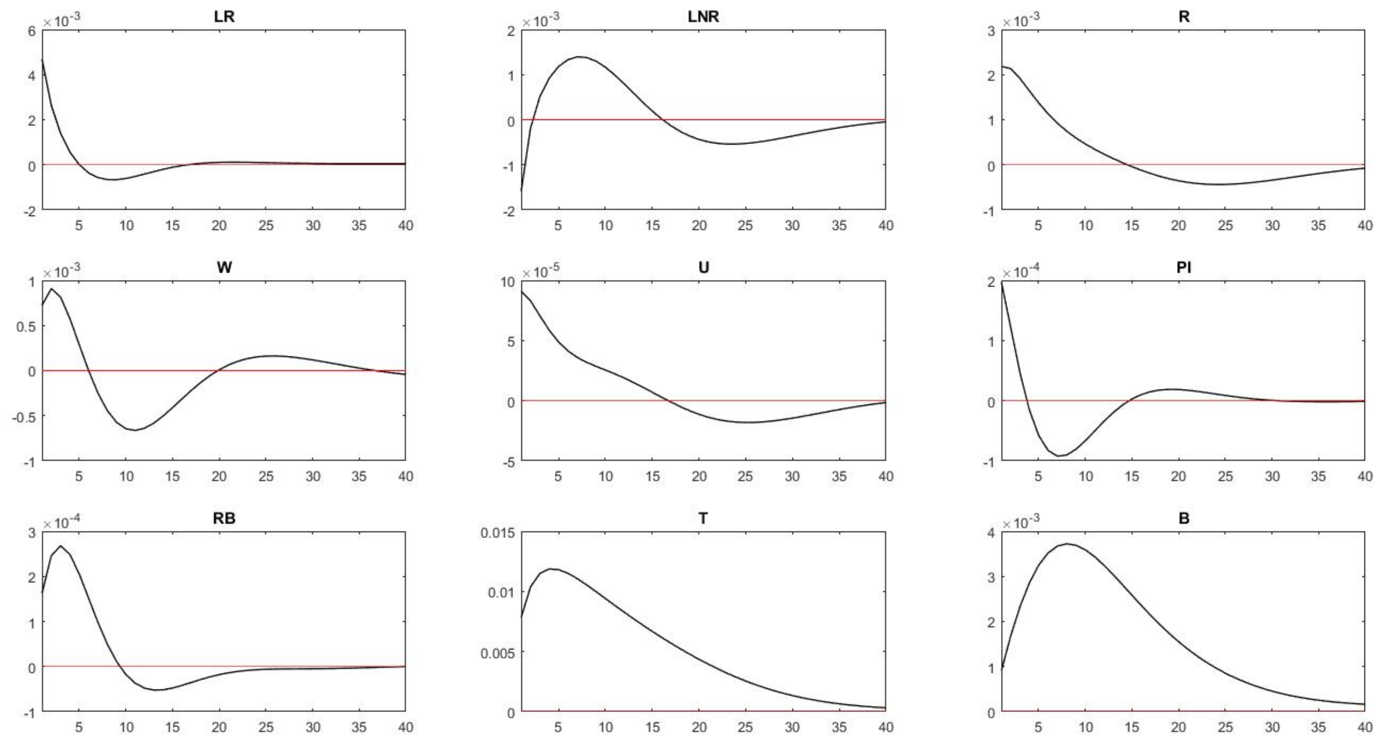


Figure 1.4: Impulse Responses to a 1 s.d. government consumption shock with interest income tax

Chapter 2

Debunking the Natural Resource Curse: A Panel Data Analysis

2.1 Introduction

The confounding finding in the economic literature that natural resource-rich countries tend to exhibit slower growth than resource-poor ones is known as the "natural resource curse". Although Auty and Warhurst (1993) is credited with the coining of this phrase, this very idea of an apparent curse gained traction following the work of Sachs and Warner (1995). Sachs and Warner (1995, 2001) used a relatively simple cross-sectional framework to convince that countries with higher primary commodity exports share of GDP in 1970 experience slower average growth over the next 20 years. After this finding, a whole new literature has developed to obtain a better understanding of the mechanism underlying this puzzle. While most of the subsequent studies have taken this finding as a given, only recently a few studies have challenged it. Those which challenged the resource curse, used variables different from the ones used in Sachs and Warner (1995, 2001). In this paper, we revisit Sachs and Warner (1995)'s original finding by closely following a set of variables used by them, albeit in a panel data framework.

The natural resource curse poses a conundrum since it goes against the conventional wisdom borne out of several historical accounts. Typically, natural resources are expected to increase wealth which in turn increases investment, and finally enhances growth rates. Relevant historical example includes the thriving steel industry driven economic growth in the U.S. following the discovery of rich iron ore deposits in the Great Lakes region in 1844. Britain and Germany also underwent similar experiences as they capitalized on their own iron ore deposit extractions. Specially, during a period when industrial revolution was necessitating the massive constructions of railroads, bridges and buildings, these countries endowed with abundant iron ore experienced rapid economic growth.

However, several countries' experiences from more recent periods are often cited as evidences which are in stark contrast with those of U.S., Britain and Germany. These examples include resource-abundant countries, such as Iran, Qatar, Kuwait, Iraq, Libya, Nigeria, Mexico, and Venezuela all of which failed to achieve rapid economic growth. Sachs and Warner (1995) also made specific mentions of Switzerland and Japan which exceeded the growth of resource-rich Russia in the nineteenth and twentieth century. However, between the two financial crises of 1998 and 2009 Russia demonstrated a strong average economic growth rate hovering around 5.6% which is much higher than those of Switzerland and

Japan for the same period (see Figure 2.1). Even post-crisis, the growth rate for Russia has been higher until before 2013. This impressive growth experience of Russia in addition to those of Norway and Botswana undermines the previous observation of Sachs and Warner (1995).

The main idea of Sachs and Warner (1995) was primarily motivated by a scatter plot similar to the one in Figure 2.2 which depicts for 104 countries a negative correlation between their economic growths per capita and natural resource abundances proxied by their primary commodity exports share of GDP (SXP). [1] Several hypotheses have been proposed in the literature to explain this negative correlation. The first and arguably the most popular hypotheses is the so-called “Dutch disease” which refers to the decline in the exports of other goods following the overvaluation of the national currency caused by a surge in natural resource exports (Corden, 1984). In addition, increased exchange rate volatility resulting from the recurrent booms and busts in the resource industry contributes to the reduction of total exports which include manufacturing and service exports (Gylfason, 2001). Ultimately, an overall decline in exports may reduce GDP growth (Frankel and Romer, 1999).

The second hypothesis relates the resource curse to rent-seeking analyses. Torvik (2002) developed a theoretical model which explained how natural resource abundance increases the number of entrepreneurs involved in rent-seeking activities and reduces the number of entrepreneurs engaged in productive activities. Rent-seeking is often associated with corruption which too can have an adverse impact on economic growth (Bhattacharyya

[1] Originally, Sachs and Warner (1995) had a scatterplot of 95 countries and graphed their annual growth rates between 1970-90 in relation to their primary commodity exports share of 1970’s GNP. In Figure 2.2, both x-axis and y-axis variables are averaged over the period 1971-1999.

and Hodler, 2010). The third hypothesis has argued that resource discovery leads to the weakening of institutions which in turn reduces growth at least in the developing countries (Alexeev and Conrad, 2011). On the other hand, other hypotheses have explored the negative associations between natural resource abundance and human capital accumulation (see for e.g. Gylfason (2001) and between resource abundance and prudent government policies specially with regards to savings and investment (see Atkinson and Hamilton (2003) in order to explain the negative relationship between resource abundance and growth.

Although Figure 2.2 documents a moderate negative correlation between SXP and growth, if we restrict the sample of countries to only those which have a minimum average SXP of 0.10 that corresponds approximately to the 50th percentile, the correlation becomes even more negative as can be seen in Figure 2.3. Although this is a simple correlation study, this is a good starting point to question the validity of the natural resource curse.

In this paper, we use panel data spanning the period 1971-1999 for 104 countries to reexamine the natural resource curse. Employing fixed-effect models, we show that SXP, the proxy variable for natural resource abundance used by Sachs and Warner (1995) and Gylfason and Zoega (2006) does not have a statistically significant impact on GDP growth. We also do not find any convincing evidence in a statistical sense, of a parabolic or u-shaped relationship between growth and SXP. This implies that there is no decreasing or increasing marginal effect of SXP. In addition, we found no evidence of a positive impact of SXP on growth for countries with very high values of SXP.

2.2 Literature review

A vast literature focused on exploring the linkage between natural resource abundance and economic growth has emerged since the 1980's. Here we will mainly focus on only a subset of them which are empirical rather than descriptive, and contain comparative growth analyses. Before Auty and Warhurst (1993) coined the phrase "natural resource curse", Gelb (1988) had discussed at length in the form of case studies, how oil rich economies formed less domestic capital than non-oil countries during the oil boom period of 1971-1983. However, none of the studies confirmed the resource curse on the basis of a worldwide empirical analysis until before Sachs and Warner (1995), which is considered as the seminal empirical investigation of the resource curse hypothesis. Sachs and Warner (1995) investigated the negative relationship between growth rates and natural resource abundance proxied by primary commodity exports share of GDP (SXP) within a cross-sectional regression framework. Using a sample of 71 countries, this study showed that countries with high SXP in 1970 tended to grow slowly during the subsequent 20-year period of 1970-1989. This result was further strengthened by Sachs and Warner (2001) which showed that the negative relationship remained even after the inclusion of various geography related variables as additional regressors.

Following the seminal work of Sachs and Warner (1995), a large volume of subsequent research was undertaken to examine the resource curse. For example, Gylfason et al. (1999) used both cross-sectional and panel regressions to show that primary product exports share of total exports had a smaller and less significant effect on growth than the primary labor share. In a later work, Gylfason (2001) employed SUR to the data of 85 countries to show that economic growth is inversely related to natural resource abundance. The proxy for resource abundance that they used was the share of natural capital in national wealth in 1994. They argued that resource abundance adversely affects growth by reducing the incentives to save and invest, as well as by hampering the development of financial institutions. Mehlum et al. (2006) used the data set from Sachs and Warner (1997) to establish a negative relationship between resource abundance and growth for countries with poor institutions which they called "grabber friendly institutions".

Similar results are presented by Ding and Field (2005) though unlike the above mentioned studies, they viewed SXP as a more appropriate measure of resource dependency than a measure of resource abundance. More recent econometric evidences in favor of the natural resource curse are produced in Nabli and Arezki (2012), Apergis and Payne (2014) and Kim and Lin (2017). While the first two focus on only Middle Eastern and North African (MENA) countries, the last one focuses only on developing countries. Nabli and Arezki (2012) found that although resource rich MENA countries maintained high level of income per capita, they exhibited relatively low economic growth as well as high macroeconomic volatility. Similar results with respect to growth are reported by Apergis and Payne (2014) for MENA countries for the period 1990-2003, though positive impact of oil abundance on growth is also noted after 2003 owing to improved institutional qualities. Kim and Lin (2017) used both SXP and natural resource rent as proxies for resource abundance along with heterogenous panel cointegration techniques. Their data set covered only the years from 1990 to 2012. Kangning and Jian (2006) Boyce and Herbert Emery (2011) are different from the previous studies as each carried out a within-country empirical study.

While the former used provincial data from China, the latter used the U.S. states data to find supporting evidence of the natural resource curse hypothesis.

Apart from these studies, there are other studies which found indirect negative growth impact of natural resources through variables which are thought to be closely related to economic growth such as education, institutions etc (see for example Gylfason (2001), Gylfason and Zoega (2006) and Atkinson and Hamilton (2003) among others). So far, the studies we have discussed have generally supported the natural resource curse hypothesis though we gave some glimpses of studies which opposed it. Now, we will focus solely on some recent studies which challenged this hypothesis by arguing that the negative impact of resource abundance in the existing literature is nothing more than a statistical mirage.

There are two main criticisms against the existing studies which provide evidences of the curse. The first is related to the way SXP (i.e. ratio between resource exports and GDP) is defined. Similar to Ding and Field (2005), the view of SXP as a measure of resource dependency has been supported by Brunnschweiler (2008) and Brunnschweiler and Bulte (2008) who proposed the logs of total natural capital and mineral resource assets in the year 1994 in US dollar per capita as a more appropriate measure of natural resource abundance. On the basis of these definitions, they found using a cross-sectional regression that resource dependence does not affect growth after controlling for resource abundance. A more recent study which maintained the same interpretation of SXP is Smith (2015). Instead of relying on SXP, it used a panel fixed-effects estimation framework to show that countries which were previously resource-poor experienced increased growth following the first discovery of a natural resource since 1950.

These studies also argued that the way SXP was defined creates potential endogeneity problem, which is the second criticism. For example, there might be a time-invariant factor independent of resource exports which may have a positive impact on GDP causing a lower SXP but a higher GDP growth, and thereby inducing a negative relationship between SXP and GDP growth. Since cross-sectional methods are incapable of handling this kind of endogeneity problem, panel data methods such as, fixed-effects models should be employed if SXP is used as an explanatory variable. Panel data methods are also better suited since it allows to control for year fixed effects. Accordingly, employing non-stationary panel methodologies on 53 oil exporting and importing countries' data, Cavalcanti et al. (2011) revealed that oil abundance has a positive growth impact. The proxy that they used for resource abundance was rent earned from oil production.

Overall, the studies which disproved the resource curse hypothesis did either of the following: used a proxy for resource abundance different from Sachs and Warner (1995), used cross-sectional regressions or used a different set of control variables. In contrast, in this paper, we use the same definition of the proxy variable for resource abundance, SXP as in Sachs and Warner (1995) albeit in a panel data framework. We also seek to stick to the same control variables as much as possible as in the seminal work of Sachs and Warner (1995). Doing these would allow for a better comparison of our work with those of Sachs and Warner (1995) and Sachs and Warner (2001). The paper which is the closest to what we present here is Manzano and Rigobon (2001). However, there are some important differences. First of all, our sample size for panel estimations is almost the double of theirs.

Second, unlike theirs we control for period fixed effects. Third, in addition to static panel data models, we employ dynamic panel data models which have become standard in modern studies on economic growth.[2]

2.3 Data and Methodology

We use the following panel fixed effect model to examine the relationship between natural resource abundance, SXP and economic growth rate:

$$Growth_{it} = \beta_0 + \beta_1 SXP_{it} + \mathbf{x'_{it}}\gamma + \alpha_i + \delta_t + u_{it} \tag{2.1}$$

where $Growth_{it}$ represents the average real GDP growth rate of a country i over the period $t-4$ to t. α_i represents country fixed effects and the δ_t denotes time fixed effects. The vector x contains a set of explanatory variables which includes Squared SXP, a measure of an economy's openness defined by Sachs and Warner (1995), Open, log investment per capita, a measure of how democratic a country is given by Polity, log government consumption and population growth. When we include previous period's GDP as initial GDP, our model takes the following dynamic panel fixed effect form, which we estimate using System GMM:

$$Growth_{it} = \beta_0 + \beta_1 SXP_{it} + \beta_2 Growth_{it-1} + \mathbf{x'_{it}}\gamma + \alpha_i + \delta_t + u_{it} \tag{2.2}$$

[2]See Durlauf, Steven et al. (2005) for a detailed discussion on the econometric methods used in growth studies.

2.4 Results

2.4.1 Main estimations

Before discussing the results from fixed-effect models, we present in Table 2.1 results from our regression using cross-sectional data on 104 countries. Although this is not an exact replication of Sachs and Warner (1995, 2001) and Gylfason (2001), it serves the purpose by showing how SXP appears as a statistically significant variable in the determination of real GDP per capita growth in a cross-sectional framework after controlling for the same basic variables as in Sachs and Warner (1995) and Gylfason (2001).[3] We regress for each country, real GDP growth per capita on SXP, log real GDP per capita in 1970, openness of economy proxied by a variable called Open as in (Sachs and Warner, 1995), and log investment per capita. All these variables except log real GDP per capita in 1970 are averaged over the years 1970-1990 in column 1, and averaged over the years 1970-1999 in column 2. Column 1's period refers to the one originally used by Sachs and Warner (1995). As can be seen from Table 2.1, regardless of sample periods, in a cross-sectional regression framework similar to those used in Sachs and Warner (1995, 2001) and Gylfason (2001), SXP as a proxy for natural resource abundance emerges as a statistically significant variable which has a negative impact on growth.

Just as most other cross-sectional regression models, regressions in Table 2.1 are subject to omitted variable bias since they are not controlling for country and year fixed-effects. Controlling for these fixed-effects is important in this analysis particularly for the

[3] In their original regressions, Sachs and Warner (1995) and Gylfason (2001) defined the independent variable SXP as the ratio of primary product exports to GDP in 1970, and to GDP in 1994 respectively. The former also includes a variable called Rule of Law which we exclude since it is available only from the year 1982.

way the proxy for natural resources, SXP has been defined. As discussed in Section 3, the use of SXP has been criticized in the literature since it may cause a subtle bias. For example, there may exist a time invariant factor such as a country's geography or climate, which if favorable, may lead to high income. As a result, for a low-income country with unfavorable geographic conditions, SXP will appear to be high, but GDP growth will be lower. This produces a negative relationship between SXP and growth in the cross-sectional model. The best way to control for this kind of time invariant factor is using fixed-effect models in a panel data setting. In what follows, we employ different specifications of fixed-effect models to see if SXP still has a statistically significant negative impact on growth.

In Table 2.2, we adopt a fixed-effect modeling approach to regress real GDP growth successively on a list of variables popular in the growth literature which contains SXP, squared SXP, a proxy for an economy's openness called Open originally used in Sachs and Warner (1995), log investment per capita, a democracy measure called Polity used in Bhattacharyya and Hodler (2010) and log government consumption. All variables but SXP are 5-year averages from 1971 to 1999. Data on SXP was only available for every 5 year interval. For example, we had data on SXP for the years 1970, 1975, and so on. Except in the first regression in 2.2, SXP appears to have a negative impact on growth. But the impact is not statistically significant.

Nevertheless, squared SXP has statistically significant positive coefficients in the third, fourth and fifth regression in Table 2.2. This indicates a possible U-shaped relationship between SXP and growth.s The minimum point on this U-shaped curve is achieved when SXP is around 0.24. This value roughly corresponds to the 90th percentile and only 10 countries have an average SXP greater than or equal to 0.24. Also, a joint significance test of the coefficients on SXP and Squared SXP yields insignificance. In addition and more importantly, the inclusion of log government consumption in the sixth regression renders the coefficient on squared SXP statistically insignificant. Therefore, we do not attach too much importance to this result, which indicate an apparent non-linear relationship between SXP and growth.

Table 2.2 extends Table 2.3's estimations by adding population growth and year dummies as additional regressors. SXP still maintains a statistically insignificant negative effect on growth in both of the regressions in Table 2.3. But consistent with the existing literature, capital investment and government consumption remain two statistically significant determinants of economic growth. Apart from the regressors in Tables 2.2 and 2.3, Sachs and Warner (1995, 2001) and Gylfason (2001) also included initial output per capita to account for growth convergence, as well as initial out per capita growth rate in their cross-sectional regressions. Including these variables as regressors in our panel data analyses will take us to the realm of dynamic models in order to overcome "Nickell" or dynamic panel bias (Nickell, 1981). In what follows, we focus on System GMM estimations to explore the relationship between growth and SXP in a dynamic panel model framework.

Table 2.4 displays the results from the estimations of two-step system GMM with Windmeijer-corrected standard errors and orthogonal deviations. Naturally, number of observations is lower under system GMM than previous fixed-effect models since the former starts from $t = 3$. In the first regression, we add previous period's log real GDP as an additional regressor, and in the second regression, we add previous period's growth rate too.

In both regressions, SXP and squared SXP are statistically insignificant. The estimations in Table 2.2, 2.3 and 2.4 provide evidence that once country and year fixed-effects are taken into account, SXP, which is a proxy for natural resource abundance loses its statistical significance unlike in the cross-sectional regressions typically used in Sachs and Warner (1995, 2001), Gylfason (2001) and Gylfason and Zoega (2006).

2.4.2 Robustness check

In this subsection, we check if our previous results remain valid if we alter our model specifications by introducing an alternative proxy for natural resource abundance. As discussed below, our previous findings remain tenable even with the use of an alternative proxy for resource abundance.

We substitute SXP with an alternative proxy for resource abundance, given by log resource rent per capita used in Bhattacharyya and Hodler (2010). The resources include energy, minerals and forestry. Formally, resource rent per capita for a country j is calculated using the following formula:

$$R_j = \frac{\sum r_{ij} q_{ij}}{P_j} \tag{2.3}$$

where R_j is the resource rent per capita of country j, r_{ij} is the resource rent per unit of output of a particular commodity i extracted in the country j, q_{ij} is the total quantity of the commodity i extracted in country j, and P_j is the total population of country j. r is estimated as the difference between a commodity's world price and the average extraction cost both expressed in US dollars. There are several reasons for choosing this proxy as an alternative to SXP. First, the way this is defined makes it directly connected to the total resource extraction in a country. Therefore, it should serve a as good proxy for natural resource abundance. Second, it overcomes the potential endogeneity concern associated with the measure SXP. Third, resource rent has been used as a proxy for natural resource abundance in other studies too, such as Bhattacharyya and Hodler (2010) and Ross (2006).

Table 2.5 and 2.6 are similar to Table 2.2 and 2.3 in terms of model specifications. However, Table 2.5 and 2.6 show that using log resource rent per capita instead of SXP in fixed-effect models does not change our results in a statistically significant manner from those of Table 2.2 and 2.3, which used SXP as a proxy for natural resource abundance.

2.4.3 What is driving the cross-sectional results?

As we have seen in Table 2.1, SXP appears as a statistically significant determinant in cross-sectional regressions. However, SXP loses its statistical significance in a panel data framework as shown in Table 2.2 and 2.3. Table 2.3 further reports that the period dummy 1985 which encompasses the periods from 1981 to 1985 has a negative and statistically significant impact on real GDP growth. This result leads to the next analysis reported in Table 2.8 where we conduct decade-by-decade cross-sectional regressions to examine if SXP remains as a statistically significant factor determining economic growth.

Table 2.8 reports three separate cross-sectional regressions for three decades: 1970s, 1980s and 1990s. It is evident from this table that SXP has a statistically significant negative impact on real GDP growth only in the decade of 1980s. This implies that there must be at least one factor specific to the decade of 1980s which is causing the negative relationship between SXP and real GDP growth rate in cross-sectional regressions. Indeed in the 1980s the developing world experienced such a severe debt crisis that this decade become known as “the lost decade”. During this decade, heavily indebted developing countries underwent significant economic slowdown. For instance, the average growth rate of Latin American countries plunged to 1.8% in the 1980s from 6% in the 1970s.

While we refer interested readers to Kaminsky and Pereira (1996) and Easterly (2001) for a more detailed account of the debt crisis in the 1980s that affected developing economies and to Mistry (1991) for the African experience in particular, here we just briefly explain the debt crisis to motivate the rest of our analyses. The decade just before 1980s saw two oil price shocks which led to enormous revenues from oil exports flowing into OPEC countries. These revenues denominated in US dollars were mainly deposited in foreign banks but the ones in the US. Therefore, they were basically euro-dollars deposits of OPEC countries. Due to lack of immediate prospects for investment in their domestic industrial projects, these sizable euro-deposits were funneled through the foreign commercial banks to a number of developing countries which heralded high growth prospects. Very soon countries such as, Mexico, Brazil, Korea, and several African countries became overburdened with external debt.

The lenders initially considered these borrowers relatively safe since some of these countries were enjoying large profits from primary commodity exports and some promised profitable investment projects. However, the bright prospect of the debtor economies soon turned bleak when with a view to combating inflation in the US, the newly appointed Federal Reserve Board Chairman Paul Volcker in 1979 started implementing some drastic contractionary monetary policies. In a nutshell, the contractionary monetary policy affected the debtor countries mainly in two ways: (i) increased interest rates in the US led to the rise in the debt service payment for debtor countries (ii) extremely tight monetary conditions in the US caused a slowdown in the industrial world, which played an important role in precipitating a sharp decline in commodity export prices and terms of trade of the debtor countries. [4] Sachs (1989) argues that the increased debt service payment disincentivized investment and therefore, growth in the debtor countries since a portion of the returns to investment were transferred to the foreign creditors.

Therefore, we run separate cross-sectional regressions for the decade 1980s in Table 2.9 where in addition to the previous regressors, we include percentage change in terms of trade in the first column's regression and add external debt to GDP ratio in the second column's regression. It seems that controlling for changes in terms of trade does not have a significant impact on the coefficient associated with SXP. However, regression in the second column shows that the inclusion of external debt to GDP ratio causes the variable SXP to lose its significance, although the newly included variable itself is not statistically significant.

Next we investigate if the inclusion of external debt to GDP ratio as an additional regressor in our fixed-effects model renders the period dummy of 1985 insignificant. The results of our investigation are presented in Table 3.10. According to the the regression in the first column, external debt to GDP ratio has a statistically significant but an economically insignificant negative impact on real GDP. Also notice that the inclusion of this ratio not only fails to render the negative coefficient on the

[4] Terms of trade is the relative price of imports in terms of exports. It is defined as the ratio of export prices to import prices, and can be interpreted as the amount of imported goods an economy can purchase for each unit of exported goods.

period dummy 1985 statistically insignificant, but also makes the negative coefficient on the period dummy 1980 statistically significant.

It has been already pointed out that during early 1980s, developing countries' interest burden increased owing to the Volcker shock in 1979. This might have been an additional factor affecting causing the slowdown in those countries' economies. Therefore, we add the log of interest on external debt as an additional control variable in our regression reported in the second column of Table 3.10. The coefficient on the newly added regressor has the expected negative sign. Although the impact of this variable on real GDP growth rate is statistically insignificant, both of the period dummies 1980 and 1985 become statistically insignificant now. This implies that these period dummies previously in Table 2.3 were capturing the effects of increased debt burdens of developing countries.

2.5 Conclusion

Despite the fact that natural resources constitute a country's wealth, abundance of this particular type of wealth is supposedly detrimental to the country's economic growth - this is the crux of the "Natural Resource Curse" hypothesis popularized by Sachs and Warner (1995). A huge literature developed after this, most of which instead of re-investigating the validity of the hypothesis mainly sought to pinpoint the channels through which abundance of natural resources could adversely affect economic growth. In this paper, we have challenged the resource curse hypothesis with the aid of data on 104 countries. Departing from the cross-sectional models as used in Sachs and Warner (1995), Sachs and Warner (1997) or in Gylfason et al. (1999), we have employed both static and dynamic panel fixed-effect models to conclude that primary commodity exports share of GDP (SXP) a proxy for natural resource abundance originally introduced by Sachs and Warner (1995) has no statistically significant impact on the real GDP per capita growth. We have also used an alternative proxy for resource abundance given by log resource rent per capita which nevertheless has not altered our main result with respect to the relation between SXP and economic growth. In addition, we also examined and rejected the possibility of a non-linear relationship between SXP and per capita growth.

Using time-effects in our panel data models we pinpointed the first half of the decade of the 1980s to be driving the apparent negative relation between SXP and GDP growth in the cross-sectional setting. The special circumstance which characterized 1980s was the severe debt crisis experienced by a number of developing countries. Thus to account for the debt crisis, we controlled for external debt to GDP ratio and interest on external debt in our static fixed-effect models. This rendered the coefficient on the period dummy capturing the first half of 1980s statistically insignificant. Furthermore, the debt crisis led to the decline in the commodity export prices and therefore, in terms of trade. However, controlling for the latter could not make the coefficient on SXP statistically insignificant in the cross-sectional regression for 1980s. But external debt to GDP ratio as an additional regressor succeeded in doing that. This result is consistent with the finding of Manzano and Rigobon (2001).

Finally, future work in this area should focus on re-examining the relationship between resource abundance and institutions and educational attainment in a panel data setting, and then contrast the results with those from existing cross-sectional studies.

Appendix

Variables: Definitions and Sources

SXP: Primary commodity exports divided by GDP. The data on primary commodity exports and GDP, measured in current US dollars were originally obtained from the World Bank. However, World Bank stopped releasing this data and therefore, data on SXP were obtained from Collier and Hoeffler (2004). The data is available at a 5 year interval from 1970 to 19995.

Open: The fraction of years during the period 1970-1995 in which the country is rated as an open economy according to the criteria in Sachs and Warner (1995).

Log investment pc: 5-year average of natural log of capital investment per capita. Yearly data collected from PWT 8.

Polity: A measure of democracy which takes a value between 0 and 1. It is computed in the manner as described in Bhattacharyya and Hodler (2010). Higher values imply better democratic institutions.

Log gov. consumption: 5-year average of natural log of government consumption. Yearly data collected from PWT 8.

Population growth: Collected from PWT 8.

External debt/GDP: 5-year average of the ratio between external debt and GDP. Yearly data collected from the World Bank database.

Percentage change in terms of trade: Yearly data on terms of trade are collected from the World Bank database. Next log deviations are computed to calculate percentage changes. Finally, 5-year averages are produced.

Log of interest on external debt: 5-year averages of natural log of interest on external debt. Yearly data are collected from the World Bank database.

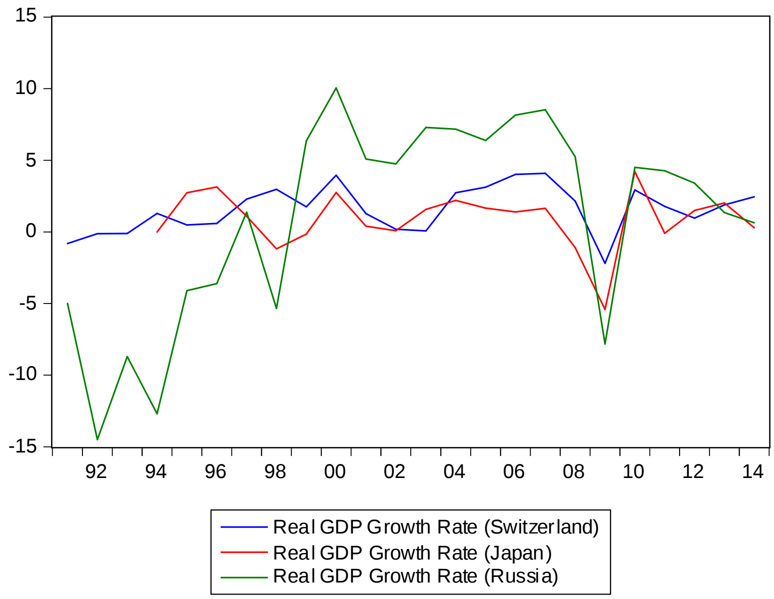


Figure 2.1: Time Series of Real GDP Growth Rates

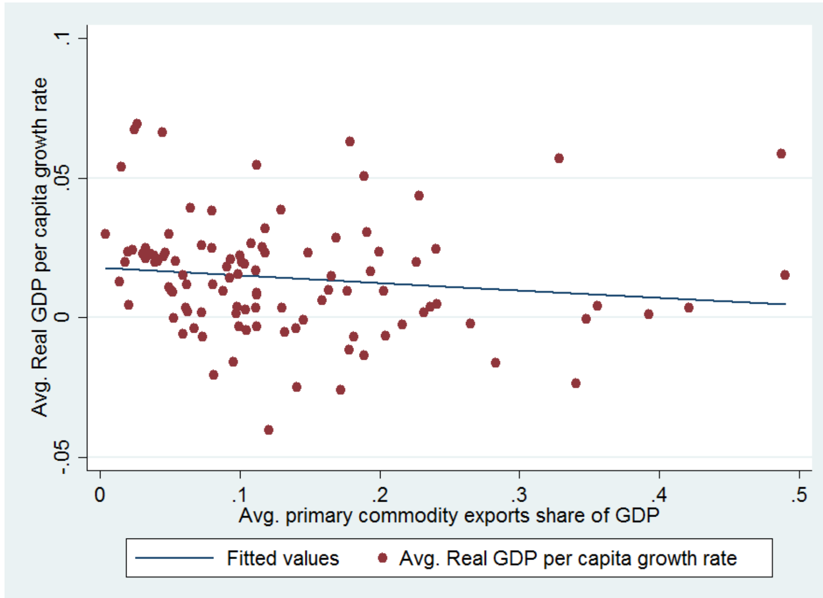


Figure 2.2: Full Sample: Average SXPs vs Average Real GDP Growth Rates

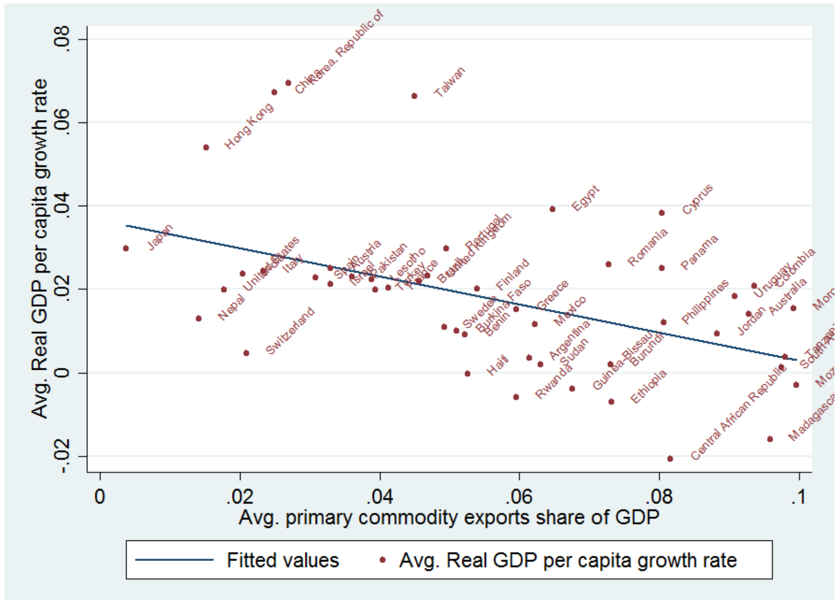


Figure 2.3: Restricted Sample: Average SXPs vs Average Real GDP Growth Rates

Restricted sample includes only those countries whose average SXP is less than or equal to the median SXP of 0.10.

Table 2.1: Regressions Using Cross-sectional Data

VARIABLES	(1) Real GDP pc Growth (1970-1990)	(2) Real GDP pc Growth (1970-1999)
SXP	−0.0431*	−0.0375*
	(0.0181)	(0.0159)
Initial Log Real GDP pc	0.000809	0.00152
	(0.00177)	(0.00155)
Open	0.000147*	0.000155*
	(5.80e-05)	(4.85e-05)
Log Investment pc (1970-1990)	0.0101*	0.00826*
	(0.00356)	(0.00312)
Constant	−0.0226	−0.0255*
	(0.0139)	(0.0121)
Observations	104	104
R-squared	0.189	0.228

Note: All the variables except Initial Log Real GDP pc in column 1 are averaged over the period 1970-1990, and in column 2 are averaged over the period 1970-1995. Standard errors in parentheses. * $p < 0.05$.

Table 2.2: Fixed-effect models estimation

VARIABLES	(1) RGDP Growth	(2) RGDP Growth	(3) RGDP Growth	(4) RGDP Growth	(5) RGDP Growth
SXP	0.0415	−0.0402	−0.114	−0.0978	−0.101
	(0.0309)	(0.0662)	(0.0783)	(0.0785)	(0.0775)
Squared SXP		0.136	0.242*	0.216	0.219*
		(0.0989)	(0.112)	(0.110)	(0.109)
Open			−0.00159	0.00176	0.00637
			(0.00413)	(0.00423)	(0.00460)
Log Investment pc				0.00121*	0.00117*
				(0.000393)	(0.000391)
Polity					−0.0226*
					(0.00946)
Log gov. consumption					
Constant	0.00886*	0.0155*	0.0225*	−0.00847	0.00442
	(0.00421)	(0.00617)	(0.00816)	(0.0109)	(0.0123)
Observations	518	518	453	453	448
R-squared	0.405	0.409	0.412	0.436	0.438

Note:All variables but SXP are 5-year averages for the period 1971 to 1995. Data on SXP was only available for every 5 year interval. Standard errors in parentheses. * $p < 0.05$.

Table 2.3: Fixed-effect models estimation (continued)

VARIABLES	(1) RGDP Growth	(2) RGDP Growth	(3) RGDP Growth
SXP	−0.0943	−0.0940	−0.101
	(0.0763)	(0.0767)	(0.0770)
Squared SXP	0.193	0.191	0.190
	(0.104)	(0.105)	(0.104)
Open	0.00734	0.00734	0.00840
	(0.00460)	(0.00460)	(0.00480)
Log Investment pc	0.00108*	0.00109*	0.00105*
	(0.000378)	(0.000385)	(0.000374)
Polity	−0.0229*	−0.0228*	−0.0165
	(0.00912)	(0.00915)	(0.00929)
Log gov. consumption	−0.0368*	−0.0370*	−0.0304*
	(0.00874)	(0.00872)	(0.00859)
Population Growth		−0.0993	−0.0513
		(0.377)	(0.365)
Year Dummy (1980)			−0.00584
			(0.00428)
Year Dummy (1985)			−0.0181*
			(0.00426)
Year Dummy (1990)			−0.00482
			(0.00452)
Year Dummy (1995)			−0.0108*
			(0.00468)
Constant	0.0860*	0.0879*	0.0779*
	(0.0237)	(0.0232)	(0.0223)
Observations	448	448	448
R-squared	0.472	0.472	0.507

Note: All variables but SXP are 5-year averages for the period 1971 to 1995. Data on SXP was only available for every 5 year interval. Standard errors in parentheses. * $p < 0.05$.

Table 2.4: System GMM estimations

VARIABLES	(1) RGDP Growth	(2) RGDP Growth
SXP	−0.00769	−0.0118
	(0.112)	(0.0927)
Squared SXP	−0.0146	−0.00582
	(0.153)	(0.130)
Open	0.0101	0.00947
	(0.00695)	(0.00620)
Log Investment pc	0.00123*	0.00116*
	(0.000522)	(0.000479)
Polity	−0.0136	−0.0132
	(0.0133)	(0.0122)
Log gov. consumption	0.00178	0.00261
	(0.00500)	(0.00495)
Population Growth	-2.011	-2.031
	(2.874)	(2.329)
Previous period's log real GDP	−0.00577	−0.00613
	(0.0223)	(0.0183)
Previous period's growth rate		0.0529
		(0.0952)

Table 2.4: System GMM estimations (continued)

VARIABLES	(1) RGDP Growth	(2) RGDP Growth
Year Dummy (1985)	-0.0137*	-0.0140*
	(0.00636)	(0.00599)
Year Dummy (1990)	0.000807	0.00153
	(0.00643)	(0.00551)
Year Dummy (1995)	-0.00707	-0.00702
	(0.0118)	(0.00903)
Constant	0.0767	0.0793
	(0.212)	(0.174)
Instruments	22	25
Hansen over-identification test statistic	12.97	12.58
Observations	355	355
Number of countries	96	96

Note: See 2.2's note for information on variables. Two-step system GMM with Windmeijer-corrected standard errors and orthogonal deviations are presented above. Standard errors in parentheses. * $p < 0.05$.

Table 2.5: Robustness check: Fixed-effect models estimation

	(1)	(2)	(3)	(4)	(5)
VARIABLES	RGDP Growth	RGDP Growth	RGDP Growth	RGDP Growth	RGDP Growth
Log Resource Rent per capita	0.00218	0.00145	0.00143	0.00159	0.00158
	(0.00197)	(0.00210)	(0.00204)	(0.00205)	(0.00196)
Open		−0.000963	0.00267	0.00667	0.00826
		(0.00434)	(0.00445)	(0.00482)	(0.00479)
Log Investment per capita			0.00136*	0.00131*	0.00114*
			(0.000431)	(0.000429)	(0.000404)
Polity				−0.0223*	−0.0235*
				(0.00986)	(0.00936)
Log gov. consumption					−0.0466*
					(0.0107)
Constant	−0.00772	−0.000499	−0.0331	−0.0216	0.0829*
	(0.0192)	(0.0210)	(0.0235)	(0.0237)	(0.0338)
Observations	475	425	425	422	422
R-squared	0.396	0.386	0.417	0.421	0.468

Note: All variables are 5-year averages for the period 1971 to 1995. Standard errors in parentheses. * $p < 0.05$.

Table 2.6: Robustness check

Fixed-effect models estimation (continued)

VARIABLES	(1) RGDP Growth	(2) RGDP Growth
Log Resource Rent per capita	0.00160	0.00210
	(0.00198)	(0.00216)
Open	0.00827	0.00945
	(0.00480)	(0.00495)
Log Investment per capita	0.00116*	0.00113*
	(0.000412)	(0.000409)
Polity	−0.0235*	−0.0163
	(0.00938)	(0.00981)
Log gov. consumption	−0.0467*	−0.0374*
	(0.0106)	(0.0108)
Population Growth	−0.114	−0.0646
	(0.394)	(0.385)
Year Dummy (1980)		−0.00644
		(0.00481)
Year Dummy (1985)		−0.0193*
		(0.00459)
Year Dummy (1990)		−0.00503
		(0.00495)
Year Dummy (1995)		−0.0112*
		(0.00495)
Constant	0.0849*	0.0627
	(0.0328)	(0.0334)
Observations	422	422
R-squared	0.468	0.507

Note: All variables are 5-year averages for the period 1971 to 1995. Standard errors in parentheses. * $p < 0.05$.

Table 2.7: Robustness Check Using System GMM Estimations

VARIABLES	(1) RGDP Growth	(2) RGDP Growth
Log resource rent per capita	−0.000490	−0.000152
	(0.00140)	(0.00127)
Open	0.0153*	0.0156*
	(0.00609)	(0.00571)
Log Investment per capita	0.00110*	0.000868*
	(0.000403)	(0.000432)
Polity	−0.00622	−0.00905
	(0.0129)	(0.0124)
Log gov. consumption	0.000216	0.00231
	(0.00550)	(0.00506)
Population Growth	−0.810	−0.761
	(0.783)	(0.753)
Previous period's log real GDP	−0.000408	−0.000787
	(0.00842)	(0.00788)
Previous period's growth rate		0.173
		(0.129)
Year Dummy (1985)	−0.0155*	−0.0147*
	(0.00478)	(0.00528)
Year Dummy (1990)	−0.00222	0.00157
	(0.00450)	(0.00553)
Year Dummy (1995)	−0.0115*	−0.00986*
	(0.00488)	(0.00473)
Constant	0.0123	0.00952
	(0.0630)	(0.0596)

Table 2.7: Robustness Check using System GMM Estimations (continued)

	(1)	(2)
Instruments	21	24
Hansen overidentification test statistic	21*	21.34*
Observations	336	336
Number of country	90	90

Note: See 2.2's note for information on variables. Two-step system GMM with Windmeijer-corrected standard errors and orthogonal deviations are presented above. Standard errors in parentheses. * $p < 0.05$.

Table 2.8: Cross-sectional Regressions for Different Decades

VARIABLES	(1) 1970s RGDP Growth	(2) 1980s RGDP Growth	(3) 1990s RGDP Growth
SXP	0.0109	−0.0735*	−0.0121
	(0.0206)	(0.0233)	(0.0208)
Initial Log GDP per capita	0.00128	0.00422*	0.00396*
	(0.00198)	(0.00188)	(0.00172)
Open	2.72e-05	0.000143*	3.35e-05
	(7.16e-05)	(6.77e-05)	(5.03e-05)
Log investment per capita	0.0224*	0.00354	0.0186*
	(0.00488)	(0.00475)	(0.00513)
Constant	−0.0561*	−0.0363*	−0.0757*
	(0.0172)	(0.0166)	(0.0154)
Observations	104	104	102
Adjusted R-squared	0.210	0.148	0.309

Standard errors in parentheses

* $p < 0.05$

Note: See 2.2's note for information on variables. Two-step system GMM with Windmeijer-corrected standard errors and orthogonal deviations are presented above. Standard errors in parentheses. * $p < 0.05$.

Table 2.9: Cross-sectional Regressions for 1980s Controlling for External Debt

VARIABLES	(1) Real GDP Growth 1980s	(2) Real GDP Growth 1980s
SXP	−0.0600*	−0.0470
	(0.0279)	(0.0315)
Log Initial GDP per capita	0.00202	−0.000552
	(0.00311)	(0.00361)
Open	0.000119	5.07e-05
	(8.67e-05)	(9.33e-05)
Log investment per capita	0.00789	0.00789
	(0.00566)	(0.00548)
Percentage change in terms of trade	0.00126	0.000718
	(0.000699)	(0.000734)
External debt/GDP		-8.43e-05
		(6.33e-05)
Constant	−0.0306	−0.00528
	(0.0225)	(0.0263)
Observations	72	62
Adjusted R-squared	0.123	0.073

Note: See 2.2's note for information on variables. Two-step system GMM with Windmeijer-corrected standard errors and orthogonal deviations are presented above. Standard errors in parentheses. * $p < 0.05$.

Table 2.10: Fixed-effects model Controlling for External Debt

VARIABLES	(1) RGDP Growth	(2) RGDP Growth
SXP	−0.0976	−0.0949
	(0.125)	(0.128)
Squared SXP	0.102	0.109
	(0.266)	(0.268)
Open	0.00149	0.000451
	(0.00602)	(0.00611)
Investment pc	0.00162*	0.00172*
	(0.000455)	(0.000454)
Polity	−0.0129	−0.0126
	(0.0107)	(0.0105)
Log gov. consumption	−0.0204	−0.0193
	(0.0103)	(0.0102)
Population Growth	0.471	0.489
	(0.430)	(0.429)
External debt to GDP ratio	-5.76e-05*	-6.73e-05*
	(1.95e-05)	(2.11e-05)
Log of interest on external debt		−0.00362
		(0.00335)
year = 1980	−0.0139*	−0.00946
	(0.00505)	(0.00609)
year = 1985	−0.0179*	−0.00839
	(0.00573)	(0.0106)
year = 1990	−0.00415	0.00662
	(0.00650)	(0.0119)
year = 1995	−0.00676	0.00442
	(0.00736)	(0.0120)

Table 2.10: Fixed-effects model Controlling for External Debt (continued)

	(1)	(2)
Constant	0.0379	0.0920
	(0.0322)	(0.0604)
Observations	263	263
Number of country	59	59
Adjusted R-squared	0.226	0.226

Note: See 2.2's note for information on variables. Two-step system GMM with Windmeijer-corrected standard errors and orthogonal deviations are presented above. Standard errors in parentheses. * $p < 0.05$.

Chapter 3

Modeling and Forecasting the US Inflation Uncertainty Using Markov Regime Switching Models

3.1 Introduction

Uncertainties of various macroeconomic and financial variables have garnered special attention of both academic researchers and practitioners because of the nontrivial role they play in influencing policy making and financial market decisions. For example, during the period 1979-1982, the Federal Reserve switched from targeting interest rates to using nonborrowed reserves as a monetary policy tool which led to unprecedented interest rate volatility. This rise in volatility might have distorted the relationship between nominal interest rates and other explanatory variables which are important ingredients in the policy making process (Gray, 1996). Another example of a macroeconomic variable which is susceptible to uncertainty is exchange rate. Financial market exploits exchange rate's volatility to determine the price of currency options which in turn is used for risk management. It is not difficult to find other variables that the portfolio managers, option traders and market makers all are interested in forecasting to either increase profit or hedge against risk. Hence, the importance of an accurate estimation and forecast of volatility cannot be overstated.

Such forecasts typically hinge on the stylized facts that high frequency time series data exhibit time-varying volatility and volatility clustering. The latter means that volatility periods of similar magnitude tend to cluster together. To capture these features, the most commonly used model in the literature is GARCH (Generalized Autoregressive Conditional Heteroskedasticity) first introduced by Bollerslev (1986) who generalized the idea of ARCH (Autoregressive Conditional Heteroskedasticity) by Engle (1982). Although GARCH models produces a better fit than a constant variance model and also yields good volatility forecasts as maintained by Andersen and Bollerslev (1998), there is a caveat. As Gray (1996) has argued these models maybe misspecified due to the reason that the structural form of conditional means and variances is relatively inflexible. In other words, the models are held fixed throughout the entire sample period and thus ignore possible structural changes in mean and variances. The latter may lead to estimated high persistence of individual shocks resulting in high volatility persistence as shown by Lamoureux and Lastrapes (1990). This high volatility persistence may be the reason behind excessive GARCH forecasts in volatile periods. To solve this problem, researchers have recently generalized the GARCH model by allowing for multiple regimes with varying volatility levels. This is called the Markov-Switching GARCH (MS-GARCH) model.

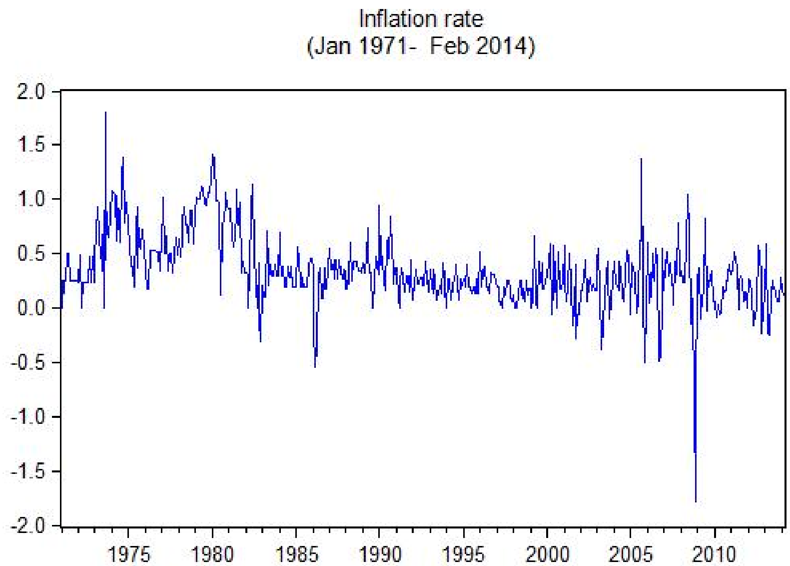
The main objectives of this paper are twofold: (i) find out when inflation volatility actually started to decline and (ii) examine the forecasting performance of a two-regime MS-GARCH model with respect to inflation uncertainty in the U.S over the period January 1971- March 2015 using multiple statistical loss functions. The first objective is pursued to address a gap in the existing literature which only reports the structural break date of several key macroeconomic variables including inflation and inflation volatility. Knowing the break date is of course important, but so is important to know when the process of the break had actually started. This information will allow us to identify the policy or policies which were most successful effecting the change. Also, this will give us an idea about the length of time required for a desired effect to take place following a policy change.

To fulfill the second objective, performances of two variants of an MS-GARCH model, one with normally distributed errors and another with t-distributed errors are juxtaposed with the performances of their standard non-regime switching counterparts. The existing literature so far has produced evidences on forecasting the volatility of exchange rates and stock returns using MS-GARCH. But surprisingly, the performance of MS-GARCH model forecasting inflation uncertainty has not been examined yet. It is important to put MS-GARCH to test to see how well it performs while forecasting inflation certainty for at least two reasons. First of all, it will shed light on the method's appropriate applicability in terms of forecasting. Second, inflation uncertainty is itself a very important macroeconomic variable which affects a society's welfare. It, in fact, was the first variable modeled using ARCH (Engle, 1982, 1983).

Moreover, obviously the same model cannot be expected to be equally good in characterizing and forecasting different variables. Therefore, testing MS-GARCH's forecasting capability with respect to different variables will yield a better understanding of the method's usefulness. The appropriateness of the application of MS-GARCH to inflation uncertainty can be primarily ascertained by eyeballing the data on U.S. inflation rate from 1971 to early 2014 (Figure 3.1). It seems that inflation rate was very volatile from early 1970s to mid 1980s. After that it remained relatively stable until before 2006 which coincides with the onset of the recent financial crisis. Therefore, a casual observation of the data suggests that the U.S. inflation rate might be characterized by at two regimes: a high volatility regime and a low volatility regime. While a standard GARCH model is not capable of distinguishing between these two regimes an MS-GARCH model is better suited at this task.

On the other hand, inflation uncertainty's being a variable of great interest to many parties is related to the general consensus that its future values are a major reason behind the welfare loss associated with inflation. Engle (1983) has argued that inflation uncertainty causes loss to risk averse economic agents even if the prices and quantities are perfectly flexible in all markets. It also distorts the efficiency of the current period's resource allocation decisions. In his Nobel lecture, Friedman (1977) has stressed that higher variability of inflation may even lead to decreased output, ceteris paribus. Inflation uncertainty's pervasive effect becomes specially more pronounced due to the use of

Figure 3.1



nominal contracts. This is because future price level uncertainty induces risk premia for long-term contracts and increases costs for hedging against inflation. Hence, in order to minimize hedging cost and loss of wealth, it is important to be able to forecast inflation uncertainty as accurately as possible.

After modeling US inflation volatility using both the regime-switching and non-regime switching versions of the GARCH model, several key results emerge. The paper finds that US inflation volatility can be characterized by two regimes, high volatility and low volatility regimes. In the high volatility regime, shock persistence is lower compared to the low volatility regime. However, the immediate impact of an individual shock is higher in the high volatility regime. There is evidence that the main source of volatility clustering in the high volatility regime is the persistence of the regime itself, not the persistence of an individual inflationary shock. The paper also finds that the regime switch of inflation uncertainty took place in mid 1983. This result is consistent with the general agreement in the literature that there was a structural break around 1984. A related but a novel finding of this paper is that the process of the regime switch started much earlier which is around April, 1979. This date is very close to when Paul Volcker was nominated as the chairman of the Board of Governors of the Federal Reserve System on July, 1979. The regime switching process seemed to have coincided with the aggressive monetary policy changes implemented by the newly appointed Fed chairman.

As regards forecasting performances, this paper provides evidences that for a forecast horizon of 1 to 5 months, a Markov regime-switching GARCH model with normally distributed errors performs better than both standard GARCH models and a Markov regime-switching GARCH model with t distributed errors. However, for longer horizons such as 8 to 12 months, a Markov regime-switching GARCH model with t distributed errors outperforms all other models.

The contribution of this paper is mainly twofold. This is the first paper which models US inflation uncertainty within a Markov regime-switching GARCH framework and thus uncovers inflation uncertainty's underlying regime-dependent characteristics. It is also the first attempt in the literature at forecasting US inflation uncertainty. The organization of the paper is as follows. Section 2 discusses the existing relevant studies in the literature. Section 3 describes the data and the methodology used. Then section 4 discusses the results. Finally, section 5 concludes.

3.2 Literature review

This paper is concerned with two strands of the literature. The first is Markov-switching GARCH models and the second is inflation uncertainty. Cai (1994) and Hamilton and Susmel (1994) are the first to extend the seminal idea of regime-switching parameters by Hamilton (1988b, 1989) to an ARCH specification to control for possible structural breaks which may bias the estimates. However, the authors have argued that regime-switching GARCH models are intractable and impossible to estimate due to the dependence of the conditional variance on previous regime-dependent conditional variances. In other words, the conditional variance at time t depends on the entire sequence of regimes up to time $t-1$. Since the number of possible regime paths grows exponentially with t, an econometrician, who does not observe regimes, will have to deal with a large number of paths to t. This renders the estimation of the likelihood function constructed by integrating over all possible paths, intractable for large sample sizes.

To remove path-dependence, Gray (1996) first proposed the idea of aggregating conditional variances from the two regimes at each time step as he developed a generalized regime-switching model of the short-term interest rate. This single regime-aggregated conditional variance is then used as the input to compute the conditional variance at the next step. To be precise, Gray's specification involves formulating the conditional variance equation in the GARCH(1,1) model in a regime-switching framework in the following manner:

$$h_{it} = \alpha_{0i} + \alpha_{1i}\varepsilon_{t-1}^2 + \alpha_{2i}h_{t-1} \tag{3.1}$$

where h_{it} denotes conditional variance at period t in regime $i = (1,2)$, and h_{t-1} is a state-independent average of past conditional variances. Gray (1996) makes use of the information observable at time $t-2$ to integrate out the unobserved regimes as follows:

$$h_{t-1} = E_{t-2}\{h_{it-1}\} = p_{1t-1}[\mu_{1t-1}^2 + h_{1t-1}] + (1-p_{1t-1})[\mu_{2t-1}^2 + h_{2t-1}] - [p_{1t-1}\mu_{1t-1} + (1-p_{1t-1})\mu_{2t-1}]^2 \quad (3.2)$$

where $p_{1t-1} = Pr(S_{t-1} = 1|I_{t-2})$ and I_{t-2} is the information available until time $t-1$. However, the main drawback of this model specification is that it is rather complicated to compute multi-period ahead volatility forecasts since this model does not make use of all the information. Dueker (1997a) also estimated Markov-switching models to forecast stock market volatility by adopting Kim's (1994) collapsing procedure to avoid the path-dependence problem. The collapsing procedure involves treating the conditional variance as a function of at most the most recent M values of the state variable S. Similar to Gray's specification, this method essentially leads to not using all the information. To use more observable information when integrating out the previous regime, alternative to equation (3.2) Klaassen (2002) proposed the following specification for the conditional variance:

$$h_{t-1} = E_{t-1}\{h_{it-1}|s_t\} = \mathrm{Var}(\pi_t|I_{t-1}) = \tilde{p}_{ii,t-1}[\mu_{it-1}^2 + h_{it-1}] + \tilde{p}_{ji,t-1}[\mu_{jt-1}^2 + h_{jt-1}] - [\tilde{p}_{ii,t-1}\mu_{it-1} + \tilde{p}_{ji,t-1}\mu_{jt-1}]^2 \quad (3.3)$$

where

$$\tilde{p}_{ji,t-1} = Pr(s_{t-1} = j|s_t = i, I_{t-2}) = \frac{p_{ji}Pr(s_{t-1=j}|I_{t-2})}{Pr(s_t = i|I_{t-2})} = \frac{p_{ji}p_{jt-1}}{p_{it}} \quad (3.4)$$

with $i, j = 1, 2$ and p_{ji} is the transition probability of switching from state j in period $t-1$ to state i in period t i.e. $p_{ji} = Pr(s_t = i|s_{t-1} = j)$. Equation 3.3 makes the distinction between Gray's and Klaassen's specification clear. It shows that Klaassen (2002) takes the information from the current state, s_t into account while calculating the conditional probability of the previous state being in a particular regime whereas, Gray (1996) incorporates information observable only at period $t-2$. Klaassen (2002) has argued that if regimes are highly persistent, current regime provides useful information about the previous regime and this information should be incorporated in the probability calculation. Another advantage of Klaassen's method is it provides a straightforward expression for the multi-step ahead volatility forecasts that can be calculated recursively as in standard GARCH models (Marcucci, 2005).

The second strand of the literature that this paper contributes to, as mentioned above is concerned with the importance and measures of inflation uncertainty. A vast literature has extensively analyzed these specially in the context of the inflation uncertainty's possible dependence on inflation rate and its potential harmful effect on real economic activity. For example, with regards to the latter, on the theoretical side some authors have pointed out that inflation uncertainty reduces the rate of investment by hindering long-term contracts (see Fischer and Modigliani 1978), or by increasing the option value of delaying an irreversible investment (Pindyck, 1991). Contrasting results are reported by Dotsey and Sarte (2000) who using a cash-in-advance constraint in their model show that inflation uncertainty may increase investment through its impact on precautionary savings. Motivated by these theoretical suggestions, a number of studies have empirically examined the relationship between inflation and other macroeconomic variables. But a measure of uncertainty needs to be employed to carry out these investigations.

Early studies use unconditional volatility measures as a proxy for uncertainty; for example Fischer (1981) employs the moving standard deviation of inflation. However, such measures fail to capture inflation uncertainty which is actually the variance of the stochastic, or unpredictable component of inflation rate (Grier and Perry, 1998). To clarify this point, suppose that agents have very little information about inflation. In this case, they may deem the future as highly uncertain even though econometricians observe small ex post variability. If however, agents possess adequate information in advance, then there may be very little uncertainty associated with large change in actual inflation (Evans, 1991). Therefore, higher variability does not necessarily imply higher uncertainty. Rather, it will imply higher uncertainty only if agents do not possess the relevant information to predict part of the increased variability (Kontonikas, 2004).

The second type of measures of uncertainty that has been used in the literature is based on surveys for instance, Survey of Professional Forecasters (SPF). SPF is a quarterly survey of professional forecasters' views on key economic variables. Studies that have used survey data to construct inflation uncertainty include Barnea et al. (1979), Melvin (1982), Holland (1995), Lahiri and Sheng (2010) among others. Typically, survey based measures summarize the dispersion of forecasts of individual forecasters at a point in time (see Giordani and Söderlind 2003 for different types of uncertainty measures based on survey data). However, Grier and Perry (1998) has argued that these measures do not provide information about individual forecaster's uncertainty about their own forecasts.

In a given time period, it is possible that each forecaster is extremely uncertain about inflation and yet submit very similar point estimates. This would lead to a significant underestimation of actual inflation uncertainty.

In contrast to these ad hoc measures of inflation uncertainty, GARCH provides a parametric technique to estimate a model of time-varying variance of stochastic innovations. This is a more sophisticated method than simply constructing a variability measure from past outcomes or from range of disagreement among individual forecasters at a point in time. With a view to examining the relationship between inflation and inflation uncertainty in the G7 countries., Grier and Perry (1998) employ an AR(12)-GARCH(1,1) model to estimate inflation uncertainty over the period 1948-1993. A similar study is conducted by Nas & Perry (2000) for Turkey which also measures inflation uncertainty using an ARMA-GARCH(1,1) model. In the context of the relationship between inflation uncertainty and real output, bi-variate GARCH models have been utilized to construct estimates of inflation uncertainty (see Grier et al., 2004; Bredin and Fountas, 2005; Fountas et al., 2006). However, none of these papers take into account structural shifts in their models which may ultimately lead to biased estimation of inflation uncertainty. This potential problem is partially addressed by Caporale et al. (2010a) who employ an AR(k)-GARCH(1,1) model with time-varying parameters only in the mean equation to estimate inflation uncertainty. But they do not incorporate regime shifts in the conditional variance model, parameters of which too are susceptible to such shifts.

With a view to accounting for structural changes in both the conditional mean and variance equations, Chang and He (2010) have first applied a bi-variate

Markov-switching ARCH model to analyze the relationship among inflation, inflation uncertainty and output growth using quarterly data from U.S. over the period 1960Q1-2003Q3. They have shown how allowing for possible regime switches culminates in uncovering effects or results that are either in contradiction with the conclusions from a single-regime GARCH model or are not captured by the latter at all. Nevertheless, to avoid the problem of path dependence this model omitted the potentially important GARCH term which could be used to parsimoniously represent a high-order ARCH process.

3.3 Data and methodology

This paper analyzes monthly U.S. inflation rates calculated as the differences in the log of monthly consumer price indices (CPI) collected from the Federal Reserve Economic Data (FRED). Monthly data has been chosen as opposed to quarterly ones since GARCH models are not well-suited for the latter ones. The sample period consists of two parts. The first part contains 518 observations from the period between January 1, 1971 and February 1, 2014. It is used for the purpose of in-sample estimation. The second part extends from March 1, 2014 to March 1, 2015 and is used for out-of-sample forecasting.

Figure 3.2 displays the histogram and Table 3.1 contains the descriptive statistics of the in-sample data. The mean inflation rate is small and around 0.34%. Both the histogram and the skewness coefficient suggest that U.S. monthly inflation rates are positively skewed. This implies that extreme positive inflation rates are more likely than extreme negative

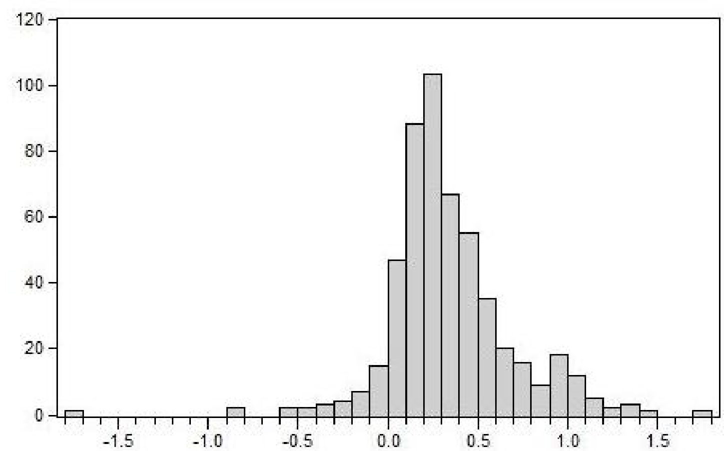


Figure 3.2: Histogram for monthly inflation rates from January 1, 1971 to February 1, 2014

Table 3.1: Summary statistics of monthly inflation rates

Statistic	Estimate
Mean	0.34
Median	0.28
Maximum	1.79
Minimum	-1.78
Standard deviation	0.33
Skewness	0.109
Kurtosis	7.37
Jarque-Bera	414.83*

Note: Inflation rates are reported in percentage terms for the sample period January 1, 1971 to February 1, 2014. *P-value = 0.

rates. However, the value of the skewness coefficient is not statistically significant at the 5% significance level.[1] On the other hand, positive excess kurtosis provides evidence of a fatter right tail. This result is statistically significant at the 5% significance level.[2] Overall, there is a strong indication of a non-normal distribution of inflation rates which is confirmed by a statistically significant large value of Jarque-Bera statistic.

We estimate four different types of GARCH(1,1) models. The first two are standard GARCH models, one with normally distributed errors and another with t-distributed errors to capture the potential fat-tailed behavior of the empirical distribution of inflation rate. Since our main focus is on volatility forecasting, we make use of a simplified GARCH model consisting of a mean equation of the following simple form:

$$\pi_t = \delta + \varepsilon_t \tag{3.5}$$

and a conditional volatility equation of the following form:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1} \tag{3.6}$$

where $\alpha_0 > 0$, $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$ to ensure a positive conditional variance. With a t-distribution, the probability density function of the innovations becomes:

$$f(\varepsilon_t) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi}\Gamma(\frac{\nu}{2})}(\nu-2)^{-\frac{1}{2}} h_t^{-\frac{1}{2}} \left[1 + \frac{\varepsilon_t^2}{h_t(\nu-2)}\right]^{-\frac{\nu+1}{2}} \tag{3.7}$$

The other two models are Markov-switching GARCH (MS-GARCH) models with two regimes, again one with normally distributed errors and another with t-distributed errors. We follow Klaassen's (2002) specification of MS-GARCH which consists of the

[1]Skewness coefficient/Standard error of skewness $= 0.109/\sqrt{6/518} = 1.01$ which is between -2 and $+2$.

[2]Excess kurtosis/Standard error of kurtosis $= 4.37/\sqrt{\frac{24}{518}} = 20.3 > 2$.

following conditional mean equation along with equations (3.1), (3.3) and (3.4):

$$\pi_t = \delta_i + \beta_{1i}\pi_{t-1} + \eta_t\sqrt{h_{it}} \tag{3.8}$$

where $i = 1, 2$ and η_t is an i.i.d process with zero mean and unit variance. Because of the absence of serial correlation in the monthly inflation rates, the m-step ahead volatility forecast at time T-1 can be computed in the following manner:

$$\widehat{h}_{T,T+m} = \sum_{\tau}^{m} \widehat{h}_{T,T+\tau} = \sum_{\tau=1}^{m}\sum_{i=1}^{2} Pr(s_\tau = i|I_{T-1})\widehat{h}_{iT,T+\tau} \tag{3.9}$$

where $\widehat{h}_{T,T+m}$ denotes the time aggregated volatility forecast for the next m steps calculated at time T, and $\widehat{h}_{iT,T+\tau}$ denotes the τ-step ahead volatility forecast in regime i made at time T that can be obtained recursively from the following:

$$\widehat{h}_{iT,T+\tau} = \alpha_{0i} + (\alpha_{1i} + \beta_{1i})E_T\{h_{iT,T+\tau-1}|s_{T+\tau}\} \tag{3.10}$$

This formula is analogous to the one derived for the standard, single-regime GARCH model and the probability to be used here to calculate the expected value comes from equation (3.4). Equation (3.9) suggests that the multi-step ahead volatility forecasts are computed as a weighted-average of the multi-step-ahead volatility forecasts in each regime estimated , where the weights are the prediction probabilities. Using the theory of Markov processes, to compute the volatility forecasts the filter probability at τ periods ahead $Pr\left(s_{t+\tau} = i|I_t\right) = p_{it+\tau} = M^\tau p_{it}$ is required where

$$M = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix} \tag{3.11}$$

The substantial simplification of the computation of the conditional variance due to the specification in equation (3.10) stands as one of the main advantages of Klaassen's MS-GARCH model over Gray's (1996) one. To estimate the Markov regime-switching model parameters, a quasi-maximum likelihood approach is undertaken with the aid of the ex-ante probability $p_{1t} = Pr\left(s_t = 1|I_{t-1}\right)$ which can be calculated from:

$$p_{1t} = p_{11}\left[\frac{f(\pi_{t-1}|s_{t-1}=1)(1-p_{1t-1})}{f(\pi_{t-1}|s_{t-1}=1)p_{1t-1}+f(\pi_{t-1}|s_{t-1}=2)(1-p_{1t-1})}\right] + (1-p_{22})\left[\frac{f(\pi_{t-1}|s_{t-1}=2)(1-p_{1t-1})}{f(\pi_{t-1}|s_{t-1}=1)p_{1t-1}+f(\pi_{t-1}|s_{t-1}=2)(1-p_{1t-1})}\right]. \quad (3.12)$$

Here $f\left(\cdot|s_t = i\right)$ denotes one of the possible conditional distributions from Normal and Student's t given that regime i occurs at time t. With the input in the previous equation the log-likelihood function can be written as:

$$l = \sum_{t=-R+w+1}^{T+w} log\left[p_{1t}f\left(\pi_t|s_t=1\right) + \left(1-p_{1t}\right)f\left(\pi_t|s_t=2\right)\right] \quad (3.13)$$

where $w = 0, 1, ..., n$. The maximimum likelihood estimates are obtained by maximizing equation (3.13) using quasi-Newton algorithm in the Matlab numerical optimization routines. The estimation is carried out on a moving window of 492 monthly observations.

In this paper, following Marcucci (2005) we evaluate the forecasting performances of competing models with respect to seven statistical loss functions which are listed below:

$$MSE_1 = n^{-1}\sum_{t=1}^{n}(\widehat{\sigma}_{t+1} - \widehat{h}^{1/2}_{t+1|t})^2 \quad (3.14)$$

$$MSE_2 = n^{-1}\sum_{t=1}^{n}(\widehat{\sigma}_{t+1} - \widehat{h}_{t+1|t})^2 \quad (3.15)$$

$$QLike = n^{-1}\sum_{t=1}^{n}(log\widehat{h}_{t+1|t} + \widehat{\sigma}_{t+1}\widehat{h}^{-1}_{t+1|t}) \quad (3.16)$$

$$
\begin{aligned}
\mathit{R2Log} &= n^{-1}\sum_{t=1}^{n}[log(\widehat{\sigma}^2_{t+1}\widehat{h}^{-1}_{t+1|t})]^2 && (3.17)\\
\mathit{MAD_1} &= n^{-1}\sum_{t=1}^{n}|\widehat{\sigma}_{t+1}-\widehat{h}^{1/2}_{t+1|t}| && (3.18)\\
\mathit{MAD_2} &= n^{-1}\sum_{t=1}^{n}|\widehat{\sigma}^2_{t+1}-\widehat{h}_{t+1|t}| && (3.19)\\
\mathit{HMSE} &= T^{-1}\sum_{t=1}^{T}(\widehat{\sigma}^2_{t+1}\widehat{h}^{-1}_{t+1|t}-1)^2 && (3.20)
\end{aligned}
$$

where $\widehat{\sigma}^2$ is an estimate of realized volatility and $\widehat{h}$ is volatility forecast from GARCH models. Equations (3.14) and (3.15) are loss functions based on typical mean squared error metrics. The loss function in equation (3.16) computes loss implied by a gaussian likelihood and is suggested by **?**. Equation (3.17) which is called the Logarithmic Loss Function, penalizes volatility forecasts asymmetrically in low volatility and high volatility periods (Pagan and Schwert, 1990). Loss functions in 3.18 and 3.19 are particularly useful as they are more robust to outliers than MSEs. However, these functions do not differentiate between over and under-predictions while applying the penalty. They are also sensitive to scale transformations. **?** have argued that MSE criterion might not be appropriate in heteroskedastic environment and therefore, suggested heteroskedasticity-adjusted MSE (HMSE) in equation (3.20).

In addition to the above statistical loss functions, two non-parametric measures of directional accuracy are also employed: (i) Success Ratio (SR) and (ii) Directional Accuracy (DA) test. These measures are generally aimed at computing the number of times a given model correctly predicts the directions of change of the actual volatility. As Marcucci (2005) has argued, directional accuracy of volatility forecasts bears special significance since they can be used as inputs to construct various trading strategies such as straddles. SR is defined as the fraction of the demeaned volatility forecasts that have the same direction of change as the corresponding demeaned actual volatility. Thus it measures the number of times the volatility forecast accurately captures the direction of the true volatility process. Formally, SR can be computed in the following manner:

$$SR = \frac{\sum_{j=1}^{m} I_{\{\bar{\sigma}_{t+j}\bar{h}_{t+j|t+j-1}\}>0}}{m} \tag{3.21}$$

where $I_{g>0}$ is an indicator function such that it takes the value of one when the function g is positive and zero otherwise.

The second test statistic, DA proposed by Pesaran and Timmermann (1992) is computed as follows:

$$DA = \frac{SR - SRI}{\sqrt{\mathrm{Var}(SR) - \mathrm{Var}(SRI)}} \tag{3.22}$$

where

$$\begin{aligned} SRI &= P\widehat{P} + (1-P)(1-\widehat{P}) \\ \mathrm{Var}(SR) &= m^{-1} SRI(1-SRI) \\ \mathrm{Var}(SRI) &= m^{-1}(2P-1)^2\widehat{P}(1-\widehat{P}) + m^{-1}(2\widehat{P}-1)^2 P(1-P) \\ &\quad + 4m^{-2}P\widehat{P}(1-P)(1-\widehat{P}) \end{aligned} \tag{3.23}$$

$$\begin{aligned} P &= m^{-1}\sum_{j=1}^{m} I(\bar{\sigma}_{t+j}) \\ \widehat{P} &= m^{-1}\sum_{j=1}^{m} I(\widehat{h}_{t+j|t+j-1}) \end{aligned} \tag{3.24}$$

$$I(g) = \begin{cases} 1 & \text{if } g > 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.25}$$

In words, P represents the fraction of times that $\bar{\sigma}_{t+j} > 0$ and $\widehat{P}$ gives the proportion of demeaned volatility forecasts that are positive. The square of the DA statistic has a χ^2 distribution with one degree of freedom. To compute equations (3.14) - (3.22), an estimate of realized volatilities, $\widehat{\sigma}^2$ is required. We compute that as squared inflation rates. This classical approach is used to calculate various financial series' realized volatilities including stock market returns.

3.4 Results

3.4.1 Single-regime GARCH

Estimation results of standard single-regime GARCH models with both normal and t-distributions are presented in Table 3.2. The t-statistics are calculated using asymptotic standard errors. Across the two models, all of the coefficients in the conditional mean and variance equations appear to be very similar and are statistically significant. Since the summation of the estimated ARCH and GARCH parameters, $\alpha_1 + \alpha_2 < 1$ for both models, the assumption of stationarity is satisfied though this violation is common when applying GARCH models on financial variables for e.g. short-run interest rates. Given these facts, it can be argued that at least the in-sample performance of standard GARCH models is quite good. Furthermore, in terms of log-likelihood, GARCH-t performs better than GARCH-n. This is not entirely unexpected since the histogram and summary statistics provided above suggested non-normality of inflation rate.

Also, notice that the estimated sum of α_1 and α_2 is relatively large which is indicative of high volatility persistence of individual shocks, as argued in the introduction.[3] For example, a shock of 1% to the inflation rate increases the conditional variance at times t+1 to t+5 by respectively 0.203, 0.135, 0.089, 0.059 and 0.039. Whether this high volatility persistence is spurious can be confirmed by estimating the regime-switching GARCH model. Further, the excess kurtosis of a t-distribution is given by $6/(\nu - 4)$ which gives a value of 3.97. This again confirms that the U.S. inflation rate exhibits fat-tailed behavior.

Parameters	GARCH-N	GARCH-t
δ	0.1106*	0.1084*
	(6.50)	(6.1425)
β_1	0.630*	0.629*
	(9.64)	(9.031)
α_0	0.008*	0.008*
	(2.86)	(2.855)
α_1	0.203*	0.232*
	(4.15)	(4.412)
α_2	0.663*	0.639*
	(10.34)	(9.261)
ν		5.51*
		(63.53)
Log-Likelihood	14.603	26.318

Table 3.2: Maximum Likelihood Estimates of Standard GARCH models with normal and t distributions

[3] $\alpha_1 + \alpha_2 = 0.87$ for normally distributed errors and $\alpha_1 + \alpha_2 = 0.847$ for t-distributed errors.

3.4.2 Markov-switching GARCH

Table 3.3 reports estimates of the Markov-switching GARCH models. The second and the third columns contain the results respectively for the models with normally distributed errors and t-distributed errors. As characterized by unconditional standard deviations σ_i, regime 1 has a slightly higher volatility than regime 2.[4] All of the coefficients in the conditional mean equation of both models appear statistically significant except the intercept term δ_2 in the second regime of MS-GARCH-N. But in the conditional variance equations, four of the total twelve parameters arise as statistically insignificant, three of which correspond to the MS-GARCH model with a t distribution. The t-statistics associated with α_{21} suggest that for both models in regime 1, the GARCH terms are probably not necessary, but in regime 2 they are useful. In fact, MS-GARCH with a t distribution suggests that unlike regime 2, regime 1 is characterized by a constant variance since both the ARCH coefficient α_{11} and the GARCH coefficient α_{21} are statistically insignificant. With respect to persistence, both MS-GARCH-N and MS-GARCH-t indicate lower value for regime 1 (higher volatility regime) than regime 2 (lower volatility regime).

The above results highlight the superior capability of Markov-switching GARCH models in identifying and distinguishing between different sources of volatility clustering. As Gray (1996) has argued, volatility clustering has two main sources. The first one is within-regime persistence and the second one is the persistence of regimes. The implication of regime persistence is that if the unconditional variance is higher in one regime than

[4]Regime-specific unconditional standard deviations are calculated as $\sigma_i = \sqrt{\alpha_{0i}/(1 - \alpha_{1i} - \alpha_{2i})}$ where $i = 1, 2$.

the other, then periods of high volatility tend to cluster together during episodes of high volatility-regime given that the regimes are persistent. This implies that for US inflation rates, volatility clustering in regime 1 is caused by the persistence of the high volatility regime and in regime 2 it is caused by both regime persistence and within-regime persistence. After all, the estimates of regime persistence as given by the transition probabilities p and q in Table 3.3 are both quite high and statistically significant.

The log-likelihood gives an initial idea of whether regime persistence is an important source of volatility persistence. For each error distribution, the log-likelihoods corresponding to the regime-switching models are higher than their single-regime counterpart. Hence, incorporating regimes can be an important mechanism to capture volatility clustering. Also as expected, estimates of persistence from standard GARCH models fall between the estimates from the high and low volatility regimes produced by the Markov-switching models. Another interesting result is that the immediate impact of an individual shock seems to be greater during the higher volatility regime (regime 1) as captured by higher values for the ARCH term in regime 1, α_{11} in comparison with the values for the ARCH term in regime 2, α_{12}. This means that for both Markov-switching models in the high volatility regime, inflationary shocks have a large immediate impact that dies out quickly. But the second regime's sensitivities to an individual shock are comparatively low and similar to the ones obtained under standard GARCH models.

The top panel in Figure 3.3 displays the time series plots of the smoothed, filter and ex ante probabilities that the inflation rate is in regime 1 at time t as estimated by the MS-GARCH-N model. MS-GARCH-t model also produces similar plots and therefore, they are not presented here. According to smoothed probabilities (blue dotted line), there was a 100% probability of the inflation rate being in the high volatility regime until April, 1979. Eventually, there was a switch to a low volatility regime around mid 1983. These results are consistent with the finding in the literature that inflation volatility was high in the 1970s but declined around 1984 during the period of Great Moderation (see Gordon (2007); Blanchard and Simon (2001); Stock and Watson (2002); Sensier and van Dijk (2004)). This consistency of result indicates the reliability of our choice of a simple AR(1) conditional mean equation in the Markov-switching GARCH model.

While the existing studies in the literature only report the break date of 1984, this paper is the first to present evidence on exactly when the process of structural break in inflation uncertainty started. According to the smoothed probability plot, the process started around April 1979 which marginally precedes the nomination of Paul Volcker to serve as the chairman of the Board of Governors of the Federal Reserve System on July, 1979. Upon the confirmation of the Senate, Paul Volcker took office on August 6, 1979 and started a series of contractionary monetary policies including shifting the Fed's focus to managing the volume of bank reserves from trying to manage the day-to-day level of the federal funds rate (Lindsey et al., 2013). Therefore, it can be argued that the process of volatility moderation closely followed the time frame of the drastic monetary policy changes implemented by the Fed under Paul Volcker. Nevertheless, to what extent Volcker's policy changes impacted inflation volatility or if they affected inflation volatility at all is a separate debate which we do not seek to settle here.

Table 3.3: Maximum Likelihood Estimates of Markov-switching GARCH models with normal and t distributions

Parameters	MS-GARCH-N	MS-GARCH-t
δ_1	0.162*	0.157*
	(9.41)	(9.31)
δ_2	0.638	0.471*
	(0.44)	(6.80)
β_{11}	0.739*	0.7706*
	(19.64)	(20.74)
β_{12}	0.330*	0.3458*
	(5.23)	(6.36)
α_{01}	0.039*	0.050*
	(3.52)	(3.72)
α_{02}	0.004*	0.005
	(1.98)	(1.66)
α_{11}	0.442*	0.488
	(2.95)	(1.12)
α_{12}	0.230*	0.196*
	(2.75)	(2.16)
α_{21}	0.025	0.005
	(0.82)	(0.23)
α_{22}	0.69*	0.708*
	(7.45)	(6.22)
p	0.997*	0.996*
	(226.61)	(215.79)
q	0.998*	0.998*
	(495.93)	(690.17)
ν		4.216*
		(9.84)
σ_1	0.27	0.31
σ_2	0.22	0.23
Log Likelihood	52.08	70.69

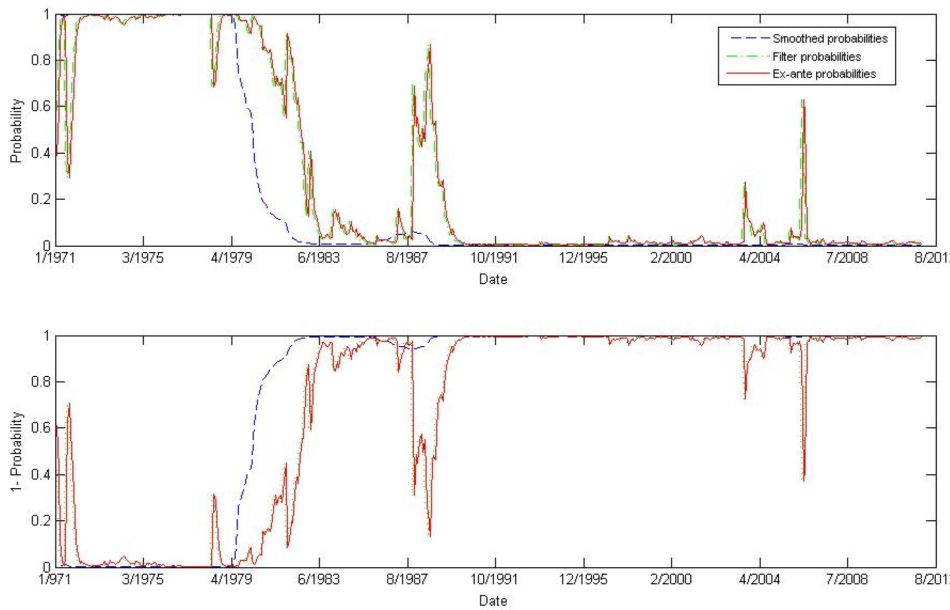


Figure 3.3: The top panel contains a time series plot of the smoothed, filter and ex ante probabilities that the inflation rate is in regime 1 at time t according to the MS-GARCH-N model. The bottom panel displays the same probabilities for regime 2.

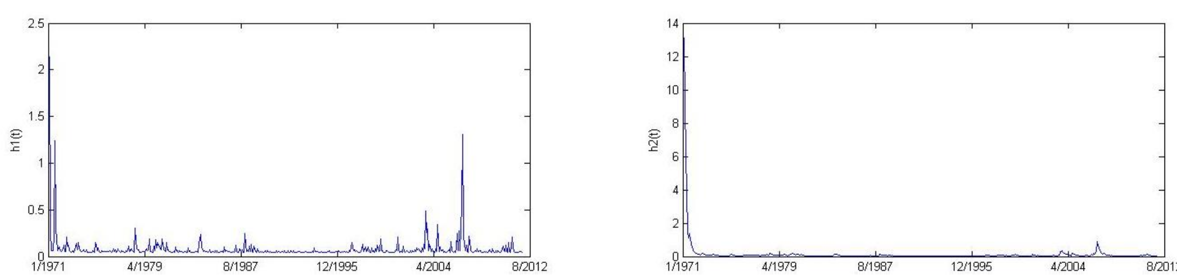


Figure 3.4: Conditional volatilities of US inflation rates over the period 1971-2012

As a final point before moving on to discuss in-sample goodness-of-fit statistics, both ex-ante and filter probabilities suggest occurrences of high volatility regimes between (i) late 1987 and late 1990 and (ii) around the onset of the 2007 recession. However, once information from the whole sample is taken into account by smoothed probabilities, it becomes clear that neither of these periods actually corresponds to high volatility regimes. Another alternative explanation based on ex-ante probabilities with respect to the period around the onset of the 2007 recession is possible. (Klaassen, 2002) has argued that some large shocks are are not persistent at all and have a rather "pressure relieving" effect. Since the within-regime persistence estimated in this paper for the high volatility regime is low, the effect of the shock to inflation volatility dies out quickly before switching to the low volatility regime. In that sense, the shock to the inflation volatility before the recession of 2007 imparted a "pressure relieving" effect. This is depicted in Figure 3.4 as a spike in the conditional volatility around the time of the recession in 2007.

3.4.3 In-Sample Goodness-of-Fit

First of all, it has to be clarified that testing the null hypothesis of a linear model or single-regime model against a regime-switching model is a non-trivial task. The difficulty mainly arises because conventional likelihood-based inference is invalid since the regime-staying probabilities remain as unidentified parameters under the null. This results in a likelihood ratio whose asymptotic distribution is not the usual χ^2 anymore and therefore, may lead to misleading conclusions (Klaassen, 2002). Although there are some papers which have sought to circumvent this problem (see for example Hansen (1992); Dufour and Luger (2017)), we do not seek to formally test for the significance of the second regime here.

Rather we only report some in-sample goodness-of-fit statistics in Table 3.4 as our main focus is on the forecasting performance.

It is evident from Table 3.4 that GARCH-N has the poorest performance of all. On the other hand, MS-GARCH-N that is, the Markov-switching model with normally distributed errors outperforms all other models by ranking first according to 7 out of 10 statistical loss functions. Based on the rest of the statistical loss functions, the MS-GARCH-t model ranks first which means that together the two Markov-switching models share between them 100% of the top places in the ranks. The superiority of the MS-GARCH-N model is consistent with the finding of Marcucci (2005) who examined the performance with respect to stock market volatility.

3.4.4 Out-of-Sample Forecasting Performance

One particular caveat about the previous section's results is that highly parameterized models tend to produce good in-sample fits. Therefore, one needs to be careful about the apparent superiority of Markov-switching models in terms of their in-sample performance since they are inherently highly parameterized. In contrast, out-of-sample tests are capable of controlling either possible over-fitting or over-parameterization problems (Marcucci, 2005). Therefore, in this section we examine and compare with each other the out-of-sample performances of the previous four variants of GARCH models in forecasting inflation volatility. Out-of-sample volatility forecasting performance is important also because of its relevance to researchers and practitioners.

Tables 3.5 to 3.10 report 1 to 12-month ahead inflation uncertainty forecasting performances in terms of the seven statistical loss functions defined in Section 3. They also report estimates for Success Ratio (SR) and Directional Accuracy (DA) test statistic. It is clear that MS-GARCH-N clearly outperforms all other models in forecasting inflation uncertainty 1 to 5-month ahead. For the same forecasting horizon, MS-GARCH-t ranks second best while GARCH-N fares worst. These rankings are consistent with in-sample performances found in Section 4.2. However, note that unlike for other models the DA test statistic for MS-GARCH-N is not statistically significant.[5] Nevertheless, MS-GARCH-N has the highest SR value for each forecast horizon from 1 to 5 months.

For forecast horizons of 6 and 7 months, both Markov-switching GARCH models have comparable performances. Standard GARCH models still perform worse than their regime-switching counterparts. From 8-month ahead horizon onward, MS-GARCH-t starts exceeding all other models in forecasting performance. In fact, for the 12-month ahead volatility forecasts, MS-GARCH-t ranks 1 in 6 out of 7 statistical loss functions. Also notice that beyond 5-month forecast horizon, MS-GARCH-N has a statistically significant DA test statistic. However, its performance clearly declines from 10-month forecast horizon onward when even standard GARCH-t performs better than MS-GARCH-N. In a nutshell, for short-term forecast horizon spanning 1 to 5-months, Markov-switching GARCH model with normally distributed errors (MS-GARCH-N) performs better than the other three GARCH models. But for longer horizons, MS-GARCH-t performs better in terms of out-of-sample forecasting evaluation.

[5] The square of DA test statistics for MS-GARCH-N are less than the 5% significance level χ^2 critical value 3.84. Therefore, we fail to reject the null hypothesis that forecasted conditional volatility cannot predict realized volatility.

3.5 Conclusion

Volatility of inflation rate or inflation uncertainty is as important a variable as the level of inflation rate. It has serious welfare loss implications for risk averse economic agents even if all the prices in the economy are fully flexible. Therefore, being able to forecast inflation uncertainty as accurately as possible is of paramount importance. Coupled with that is the fact that a casual “eyeballing” of the data on US inflation rates from 1971 to present suggests that its volatility might have undergone regime changes multiple times. Existing studies in the literature also confirm at least one structural break in 1984. Therefore, it might be appropriate to forecast inflation uncertainty using Markov-switching GARCH models which are capable of handling regime changes unlike standard GARCH models.

Modeling inflation uncertainty using regime-switching GARCH models also provides the opportunity to evaluate the forecasting performance of these models relative to standard ones. In this paper, we seize that opportunity to augment the existing evidences which already support regime-switching GARCH models' superior shorter horizon forecasting performance. However, those evidences are based on only stock market and exchange rate data. Following Marcucci (2005), this paper employs a broad set of statistical loss functions to evaluate the relative performances of Markov-switching GARCH models in forecasting US inflation uncertainty.

One of the first major findings of this paper is that a Markov-regime switching GARCH model consisting of a simple AR(1) conditional mean equation does remarkably well in identifying US inflation uncertainty's structural shift in the year 1984. This result is consistent with the general agreement in the literature on the break date. In addition, this paper has identified April, 1979 as the time when the regime switching process might have started before culminating in a complete switch in 1984. The whole switching process mirrors the time line which follows a specific period that starts from the nomination of Paul Volcker as the new chairman of the Federal Reserve System to his implementation of various drastic monetary policy initiatives until 1984.

Another important result of this paper is that in the high volatility regime, shock persistence is lower compared to the low volatility regime. But the immediate impact of an individual inflationary shock is higher in the high volatility regime. New evidences are presented which show that the main source of volatility clustering in the high volatility regime is caused by the persistence of the regime itself. Finally, a comparison of the forecasting performances of the four different GARCH models indicates that for a forecasting horizon of 1 to 5 months, a Markov regime-switching GARCH model with normally distributed errors (MS-GARCH-N) outperforms all other three models. However, for longer forecasting horizon such as 8 to 12 months, a Markov regime-switching GARCH model with t distributed errors (MS-GARCH-t) performs the best. For the same longer horizon, MS-GARCH-N performs poorly even compared to a standard GARCH model with t distributed errors.

The results and analyses of this paper can be extended in the future to explore the relationship between inflation and inflation uncertainty within a regime-switching framework. Also, forecasting exercises similar to the ones in this paper can also be carried out for other countries' inflation rates. It will be interesting to further evaluate the relative performances of Markov regime-switching GARCH models in the contexts of different economic settings.

Table 3.4: In-sample goodness-of-fit statistics

Model	NumPar	AIC	Rank	BIC	Rank	LogL	Rank	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank
GARCH-N	5	-0.03	4.00	0.02	4.00	11.23	4.00	0.06	3.00	0.12	3.00	-1.34	3.00	5.79	3.00	0.12	3.00	0.16	3.00	6.81	3.00
GARCH-t	6	-0.09	3.00	-0.04	3.00	28.71	3.00	0.06	2.00	0.12	2.00	-1.37	2.00	5.81	4.00	0.12	2.00	0.16	2.00	6.38	2.00
MS-GARCH-N	12	-0.16	2.00	-0.06	2.00	52.04	2.00	0.05	1.00	0.08	1.00	-1.43	1.00	5.37	1.00	0.11	1.00	0.15	1.00	6.12	1.00
MS-GARCH-t	13	-0.23	1.00	-0.12	1.00	70.69	1.00	0.07	4.00	0.23	4.00	-1.23	4.00	5.70	2.00	0.13	4.00	0.17	4.00	9.20	4.00

Note: NumPar is the number of parameters estimated in each model, AIC is Akaike Information Criterion calculated as $-2log(L)/T + 2k/T$ where k is the number of parameters and T is the total number of observations. BIC is the Bayesian Information Criterion or Schwarz Criterion calculated as $-2log(L)/T + (k/T)$. The rest of the statistical loss functions MSE_1, MSE_2, QLike, R2Log, MAD_1, MAD_2, and HMSE are defined in Section 3.

Table 3.5: Out-of-sample evaluation of one and two-month ahead volatility forecasts

1-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0048	4	0.0088	4	-2.7139	3	1.5208	2	0.1453	4	0.0594	4	1.2004	2	0.36	-3.9767
GARCH-t	0.003	3	0.0088	3	-2.9989	4	0.0026	1	0.1372	3	0.057	3	1.4516	3	0.38	-3.8858
MS-GARCH-N	0.0189	1	0.0042	1	-2.1548	2	5.2561	3	0.1116	1	0.0467	1	0.7784	1	0.51	-1.9727
MS-GARCH-t	0.0212	2	0.0047	2	-1.9906	1	5.6603	4	0.1208	2	0.0495	2	1.7038	4	0.44	-3.7334

2-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0048	4	0.0088	4	-2.7139	3	1.5208	2	0.1453	4	0.0594	4	1.2004	2	0.36	-3.9767
GARCH-t	0.003	3	0.0088	3	-2.9989	4	0.0026	1	0.1372	3	0.057	3	1.4516	3	0.38	-3.8858
MS-GARCH-N	0.0189	1	0.0042	1	-2.1548	2	5.2561	3	0.1116	1	0.0467	1	0.7784	1	0.51	-1.9727
MS-GARCH-t	0.0212	2	0.0047	2	-1.9906	1	5.6603	4	0.1208	2	0.0495	2	1.7038	4	0.44	-3.7334

Table 3.6: Out-of-sample evaluation of three and four-month ahead volatility forecasts

3-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0198	4	0.0409	4	-4.3265	4	0.5748	2	0.2155	4	0.1399	4	344.0068	4	0.41	-2.5931
GARCH-t	0.0157	3	0.0399	3	0.4847	1	0.4252	1	0.2064	3	0.1358	3	75.8866	3	0.44	-2.4503
MS-GARCH-N	0.0334	1	0.021	1	-0.9934	3	1.5323	3	0.1456	1	0.1107	1	0.4615	1	0.51	-1.4759
MS-GARCH-t	0.0366	2	0.0222	2	-0.9135	2	1.675	4	0.153	2	0.113	2	0.7898	2	0.49	-2.172

4-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0401	4	0.0706	4	-0.9952	3	-0.0826	1	0.2382	4	0.1849	4	2.1482	4	0.38	-2.7801
GARCH-t	0.0378	3	0.0692	3	-1.0669	4	-0.2367	2	0.2309	3	0.1808	3	1.8226	3	0.41	-2.627
MS-GARCH-N	0.0442	1	0.0344	1	-0.6642	2	1.3966	3	0.1661	1	0.1453	1	0.4674	1	0.49	-1.6718
MS-GARCH-t	0.0468	2	0.0355	2	-0.5998	1	1.4809	4	0.1806	2	0.1537	2	0.7232	2	0.41	-3.1542

Table 3.7: Out-of-sample evaluation of five and six-month ahead volatility forecasts

5-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.062	4	0.0991	4	-1.1493	3	0.0616	1	0.2661	4	0.232	4	6.9225	3	0.33	-3.1563
GARCH-t	0.0602	3	0.0971	3	-1.3805	4	0.0097	2	0.2593	3	0.228	3	10.4287	4	0.36	-2.9824
MS-GARCH-N	0.055	1	0.0509	2	-0.3972	2	1.3432	3	0.1818	1	0.1738	1	0.5602	1	0.49	-1.2888
MS-GARCH-t	0.0559	2	0.05	1	-0.36	1	1.3832	4	0.1988	2	0.1867	2	0.6854	2	0.41	-2.6436

6-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0756	4	0.1227	4	-0.5359	3	0.2616	2	0.2823	4	0.2706	4	3.3074	4	0.38	-2.4236
GARCH-t	0.0745	3	0.12	3	-0.6228	4	0.1868	1	0.2744	3	0.2644	3	3.0855	3	0.41	-2.2368
MS-GARCH-N	0.0657	2	0.0702	2	-0.1885	2	1.3152	3	0.199	1	0.2043	1	0.5717	1	0.44	-2.0503
MS-GARCH-t	0.0647	1	0.0667	1	-0.1724	1	1.321	4	0.2165	2	0.2209	2	0.6123	2	0.36	-3.4247

Table 3.8: Out-of-sample evaluation of seven and eight-month ahead volatility forecasts

7-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0858	4	0.1345	4	-0.6945	3	0.2278	1	0.2851	4	0.2874	4	5.0744	3	0.36	-2.6304
GARCH-t	0.0859	3	0.1313	3	-0.8356	4	0.2232	2	0.278	3	0.2813	3	7.102	4	0.38	-2.4333
MS-GARCH-N	0.0781	2	0.0974	2	-0.0201	2	1.3184	4	0.221	1	0.2453	1	0.5687	1	0.41	-2.2368
MS-GARCH-t	0.0742	1	0.089	1	-0.0175	1	1.2886	3	0.2302	2	0.2532	2	0.5944	2	0.33	-3.5687

8-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0936	4	0.1446	4	-1.5086	3	0.3767	1	0.2862	4	0.3043	4	41.4512	3	0.38	-2.133
GARCH-t	0.0934	3	0.1412	3	36.3074	4	1.9969	4	0.2802	3	0.2997	3	37865.0154	4	0.41	-1.913
MS-GARCH-N	0.0898	2	0.1295	2	0.1192	2	1.2825	3	0.246	2	0.2952	2	0.5445	1	0.38	-2.4236
MS-GARCH-t	0.0822	1	0.1131	1	0.1105	1	1.2208	2	0.2416	1	0.2842	1	0.5613	2	0.31	-3.719

Table 3.9: Out-of-sample evaluation of nine and ten-month ahead volatility forecasts

9-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0942	4	0.1558	3	0.438	4	0.6243	2	0.2902	4	0.3268	3	13.7268	4	0.36	-2.3527
GARCH-t	0.0932	3	0.1528	2	0.1126	3	0.5168	1	0.2854	3	0.3243	2	5.0627	3	0.38	-2.122
MS-GARCH-N	0.1019	2	0.1648	4	0.239	2	1.2903	4	0.2716	2	0.3487	4	0.5054	1	0.36	-2.612
MS-GARCH-t	0.0906	1	0.1386	1	0.2227	1	1.2054	3	0.2533	1	0.3154	1	0.5239	2	0.28	-3.8759

10-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.1064	3	0.1843	3	0.115	2	0.5126	1	0.2971	4	0.3571	3	4.9909	4	0.36	-2.3527
GARCH-t	0.1077	4	0.1828	2	-0.0347	1	0.5078	2	0.2943	3	0.3561	2	4.3118	3	0.41	-1.6576
MS-GARCH-N	0.1127	2	0.2041	4	0.3437	4	1.2571	4	0.2886	2	0.3941	4	0.4921	1	0.33	-2.8294
MS-GARCH-t	0.0975	1	0.1654	1	0.3198	3	1.1527	3	0.2617	1	0.3445	1	0.5012	2	0.28	-3.8759

Table 3.10: Out-of-sample evaluation of eleven and twelve-month ahead volatility forecasts

11-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.1136	2	0.2021	2	-0.1639	1	0.5188	1	0.2969	4	0.3738	2	7.9816	3	0.41	-1.8895
GARCH-t	0.1171	3	0.2033	3	-0.6314	4	0.6516	2	0.2955	2	0.3744	3	22.4232	4	0.41	-1.6576
MS-GARCH-N	0.1214	4	0.2446	4	0.4396	3	1.1945	4	0.296	3	0.4278	4	0.4719	2	0.33	-2.8294
MS-GARCH-t	0.1019	1	0.1909	1	0.4087	2	1.0713	3	0.2607	1	0.3615	1	0.4696	1	0.28	-3.8759

12-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.1192	3	0.2337	2	2.8951	4	1.2391	4	0.3038	3	0.4095	2	185.8821	4	0.33	-3.1633
GARCH-t	0.1184	2	0.2372	3	0.9242	3	0.8923	1	0.3012	2	0.4132	3	10.5512	3	0.31	-3.3477
MS-GARCH-N	0.133	4	0.2972	4	0.569	2	1.1636	3	0.3126	4	0.4771	4	0.5014	2	0.23	-4.6233
MS-GARCH-t	0.1086	1	0.225	1	0.5292	1	1.0204	2	0.2709	1	0.3968	1	0.4775	1	0.18	-5.8629

Chapter 4

Predicting US recessions: A Dynamic time warping exercise in Economics

4.1 Introduction

Among all macroeconomic phenomenon, recession has the most significant adverse welfare implications. It is accompanied by a loss of output, increase in unemployment, decrease in consumer confidence, and a decline in the overall well-being of people living in a country experiencing the recession. Sometimes these consequences become so pervasive that a full recovery of the economy to its normal state turns into a very prolonged process. For example, although the Great Recession officially ended in June 2009, the US GDP has not yet attained the level it should have been at now had there been no recession.

Because of pervasive repercussions ensuing a recession, accurate and timely prediction of recessions is of great interest. Consequently, an ever increasing body of economic literature has developed with a focus on forecasting recessions. Many of them are concerned with proposing and examining the efficacy of a number of recession indicators. Simultaneously, myriads of parametric models have also been employed and compared with each other in terms of their out-of-sample forecasting performances. In contrast, only a few non-parametric approaches have been adopted to predict recessions. In this paper, we seek to predict US recessions using Treasury term spread data with the aid of a non-parametric approach called Dynamic Time Warping (DTW).

The success of parametric models in predicting recessions depends heavily on the choice of explanatory variables and functional forms. Specifically, models need to be specified correctly such that they can incorporate possible structural breaks, and correctly identify their dates of occurrences in the data. Chauvet and Potter (2005) have argued that this is important since the probability of recessions is significantly affected by the consideration of breakpoints and their locations. However, pinpointing breakpoints is a non-trivial task. Therefore, we employ DTW which is a model-free approach, and does not require the identification of structural breaks to predict US recessions. In addition, instead of depending on multiple explanatory variables as is the case with most parametric model based studies, it relies only on a single variable, that is Treasury term spread. DTW is also computationally much simpler than other parametric and non-parametric methods, specially considering its remarkable success in predicting US recessions.

Our basic assumption is that effective leading indicators should exhibit similar patterns before each recession. Therefore, the approach that we adopt here seeks to exploit the similarity in pattern between two time series sequences. The most commonly used measure of similarity is Pearson's correlation coefficient. However, this coefficient has some serious drawbacks. For example, it is sensitive to the presence of outliers and fails to find similarity between two data series if they are out of phase. It also cannot be used to find similarity between two time series sequences of different lengths. Its poor performance in detecting dissimilarity between two trending but diverging sequences becomes evident when we consider two Wiener processes W_1 and $W_2 = W_1 + (0.002 \times t)$ where t is the time index. These two processes are displayed in Figure 4.1. Pearson's correlation coefficient yields a very high value of 0.92 despite the fact that these two sequences are dissimilar. To overcome these weaknesses of Pearson's correlation coefficient, we apply DTW which finds an optimal alignment between two time-series sequences under certain restrictions by warping the sequences in a non-linear fashion to match each other. Although it was originally developed in the 1970s as an algorithm to aid speech recognition, it has been successfully applied to other fields as well, such as, data mining and information retrieval (Müller, 2007; Begum et al., 2015). Unlike correlation coefficient, DTW enables the comparison between two data series of different lengths which adds to its advantages.

We employ two variants of DTW, one with a symmetric-step pattern and another with an asymmetric step-pattern. It turns out that the adoption of an asymmetric step-pattern produces better results than the symmetric-step pattern in terms of successfully predicting US recessions of 1990, 2001 and 2007. The success of DTW, particularly in

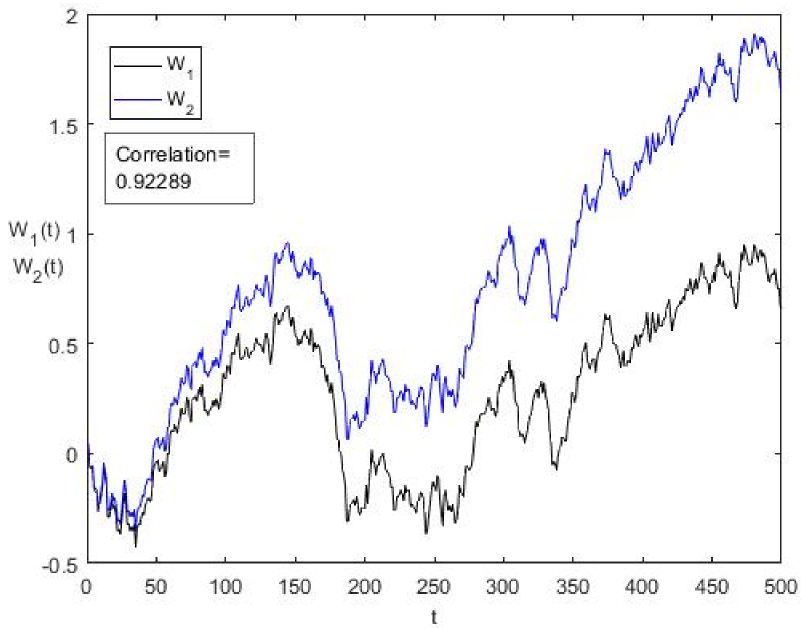


Figure 4.1: Diverging Wiener processes

predicting the recession of 1990 is noteworthy. This is because the recession of 1990 was triggered by unusual events such as the invasion of Kuwait, and therefore, most models using the yield curve found it difficult to signal this recession ahead of time such as Estrella and Mishkin (1998) and Stock and Watson (1993). The main contribution of this paper is that it employs a computationally much simpler non-parametric method than existing parametric and non-parametric methods to predict US recessions successfully.

The rest of the paper is organized as follows. Section 2 discusses the existing relevant literature, particularly on recession prediction. Section 3 presents a formal description of DTW. Then section 4 proceeds to explain the data used, and the steps involved in the practical implementation of DTW algorithm in predicting US recession. After that, results are discussed in section 5. Finally, section 6 concludes.

4.2 Literature Review

Wheelock et al. (2009) does an excellent survey of some important studies done between the years 1991 and 2008 concerned with forecasting recessions. This survey confirms that the majority of recession-forecasting studies estimate a probit model of the following type, in which the dependent variable is basically a binary variable. It takes a value of 1 to indicate recession periods and 0 to represent non-recession periods:

$$Pr(recession_t) = F(\alpha_0 + \alpha_1 S_{t-k}) \tag{4.1}$$

where F indicates the cumulative normal distribution function. According to this equation, a statistically significant α_1 implies that the recession indicator S_{t-k} is useful for forecasting a recession k periods ahead. Sometimes, additional explanatory variables in the form of a vector X_{t-k} are added to the above model in forecasting exercises to examine the usefulness of S_{t-k}:

$$Pr(recession) = F(\alpha_0 + \alpha_1 S_{t-k} + \alpha_2 X_{t-k}) \tag{4.2}$$

The main idea behind estimating equation (4.2) is that, if α_1 appears to be statistically significant in equation (4.1) but not in equation (4.2), then S_{t-k} actually does not have the explanatory power to predict recessions.

As already mentioned, numerous recession indicators have been discussed in the literature. But amongst all, treasury term spread stands out. Two of the earliest studies, Estrella and Hardouvelis (1991) and Estrella and Mishkin (1998) have shown using probit estimation that the term spread significantly outperforms other financial and macroeconomic variables in forecasting U.S. recessions.The latter uses quarterly US data from 1959:Q1 to 1995:Q1 and a probit model to compare the out-of-sample forecasting performance of various financial variables, including interest rates, interest rate spreads, stock prices and, monetary aggregates. Their results indicate that stock prices are useful predictors at one-to-three quarter horizons and are comparable to the Commerce Department's index of leading indicators. However, beyond one-quarter, the term spread i.e. the slope of the yield curve outperforms all other indicators.

The usefulness of domestic term spread in predicting recession has been confirmed for countries other than US too. For example, on the basis of German data for 1967-98 along with US data, Estrella et al. (2003) conclude that models that use the term spread to predict recessions are more stable compared to models forecasting continuous variables such as real activity or inflation. Similar evidences of term spread's usefulness have been produced by Gerlach and Bernard (1998) who using a probit model examined data from 1972-93 for eight industrialized countries. Those countries include Belgium, Netherlands, Canada, France, Germany, Japan and, the United Kingdom. They further add that foreign spreads provide limited information, except for Japan for which German spread is useful, and the UK for which US spread appears to be useful in predicting recessions. Another of their findings includes that the index of leading indicators is useful only for forecasting recessions in the immediate future. Moneta (2005) also reports superior performance of the term spread between 10-year government bonds and the 3-month interbank rate for the euro area as a whole. The finding holds true for individual euro countries as well.

Additional evidences from the European countries are reported in Sensier et al. (2004). Using a logit model, they conclude that domestic term spread helps forecast recessions well for Germany when used in conjunction with international variables. Forecasting performance for Italy is also enhanced once international variables, such as composite leading indicator and interest rates for Germany and US are introduced in the models. Several other studies too, such as Wright (2006) and King et al. (2007) highlight the superior performance of multivariate probit models which include extra variables in addition to the term spread, such as federal funds rate and corporate credit spread.

Other studies that employ probit models include Rosenberg and Maurer (2008) and Dotsey (1998). Decomposing the term spread into interest rate expectations and term premium, Rosenberg and Maurer (2008) find that although the term spread and the expectations component generally yield similar forecasts, between August 2006 and May 2007, the term spread model predicted a significantly higher recession probability than the expectations component model did. Dotsey (1998) on the other hand, while adds support to the effective forecasting performance of the term spread, also notes that the spread's forecasting performance has deteriorated in recent years.

Extensions to the probit model have also been explored in the recession forecasting literature. For example, embedding a probit forecasting model within a Markov-switching framework both Dueker (1997b) and Ahrens (2002) report more accurate estimates of recession probabilities. Chauvet and Potter (2005) consider Bayesian estimation of four different probit model specifications to compare forecasts of recessions: a time-invariant conditionally independent version, a business cycle specific conditionally independent model that takes into account multiple breakpoints, a time-invariant probit model with autocorrelated errors, and a business cycle specific probit model with autocorrelated errors. All specifications examined indicate that the yield curve predicts weak future economic activity in 2000-2001. However, the prediction strength differs substantially across the specifications. They also find that a probit model with business cycle specific innovation variance and an autoregressive component has a much better in-sample fit than a probit model.

Other studies which analyze probit model extensions include Kauppi and Saikkonen (2008) and Nyberg (2010). The former examines the predictive performance of various dynamic probit models. One of the variants allows the conditional probability of the binary response to depend on both its lagged values and on lagged values of the binary response. Extensions with interaction terms are also examined. They show that dyanamic probit models outperform the traditional static model in terms of both in-sample and out-of-sample predictions of U.S. recessions. They also confirm that the interest rate spread continues to be an important predictor. Nyberg (2010) provides similar results for US and Germany using the same methods as the previous study. He further reports that dynaminc probit models incorporating both US stock returns and German term spread outperform the models with only US term spread in predicting US recessions.

More recent studies that have sought to predict recessions include among several others Liu and Moench (2016), Berge (2015) and Gogas et al. (2015). Using Treasury term spread as the benchmark predictor of recessions, Liu and Moench (2016) reassess the predictability of U.S. recessions at horizons from three months to two years ahead for a large number of previously proposed leading-indicator variables. They employ an efficient probit estimator for partially missing data and assess relative model performance based on the receiver operating characteristic (ROC) curve. They find that adding six-month lagged observations of the Treasury term spread significantly improves the power of the probit model to predict recessions. They also show that at short forecast horizons, the annual return on the S&P500 index and at long forecast horizons, balances of margin debit at broker-dealers significantly enhance out-of-sample predictive ability.

Berge (2015) applies four model selection methods to the problem of predicting business cycle turning points: equally-weighted forecasts, Bayesian model averaged forecasts, and two models produced by the machine learning algorithm boosting. Although his models find it difficult to predict the 2001 recession, yield curve emerges again as a robust predictor of future turning points. He argues, however, that the best-performing models combine the slope of the yield curve with other macroeconomic information.

Thus far, all the studies examined primarily belong to the family of parametric methods. The use of non-parametric methods to predict recessions is by far minimal in the literature. To the best of our knowledge, only three studies have attempted to employ non-parametric methods to predict recessions. One of them is Gogas et al. (2015) who investigates the recession forecasting ability of the yield curve in terms of the U.S. real GDP cycle using a machine learning technique for classification, called Support Vector Machines. He uses data from a variety of short (treasury bills) and long term interest rates (bonds) for the period from 1976:Q3 to 2011:Q4 in conjunction with the real GDP for the same period. Despite the novelty in the approach and correctly forecasting all the recessions, his model erroneously provides 6 false recession flags. The other non-parametric study Filardo (2004) does even worse. The author uses declines in the Composite Leading Index for two consecutive months as an indicator of an imminent recession. Although this indicator successfully signals an imminent recession eight months before the actual NBER-denoted starting date, it has produced 19 false signals over the past four decades. The third study is Qi (2001) which is also the most successful one at least among all other non-parametric methods. It makes use of a neural network model along with the Treasury term spread data to successfully predict the recessions of 1973, 1980, 1981 and 1990. However, the computation of neural network models are more demanding than DTW for pattern recognition purpose.

The literature review makes three points clear: (i) Treasury term spread is generally a useful predictor of recession, sometimes by itself and sometimes in conjunction with other variables (ii)the evidence of the application of non-parametric method in the area of recession prediction is scant and associated with limited success except one (iii) there is no consensus on the type of the forecasting models to be used and additional variables to be included. This paper seeks to address these issues in the literature by employing Dynamic Time Warping (DTW) to predict US recessions. We will show how the application of DTW to the treasury term yield data alone is sufficient to correctly predict US recessions without raising many false flags. This will help remove the burden of searching for a suitable model specification and other variables that may be required to complement the recession forecasting ability of the treasury term spread.

4.3 Methodology

In this section, we will describe the fundamentals of Dynamic Time Warping (DTW). We will show how the basic algorithm works and also explore various constraints to achieve different goals. The following exposition roughly follows the choice of symbols used in Chapter 4 of Rabiner and Juang (1993) and the description in Chapter 4 of Müller (2007).

4.3.1 General definition

The objective of DTW is to compare two time-dependent sequences: a *query sequence*, $X = (x_1, x_2, ...x_N)$ of length $N \in \mathbb{N}$ and a $reference$ $sequence$, $Y = (y_1, y_2, ...y_M)$ of length $M \in \mathbb{N}$. In the following, to index the elements in X and Y we will use the symbols, respectively, $i = 1...n$ and $j = 1...m$. To compare the two different sequences, a local cost measure is required. This measure is also known as a local distance or dissimilarity measure. To obtain this measure, a non-negative, local dissimilarity function f is defined between any pair of elements x_i and y_j with the shortcut:

$$d(i, j) = f(x_i, y_j) \geq 0 \tag{4.3}$$

Typically, $d(i, j)$ is small i.e. low cost if x and y are similar to each other, otherwise $d(i, j)$ is large i.e. high cost. The most commonly employed distance function is the Euclidean distance, though other functions such as squared Euclidean, Manhattan, Gower coefficient etc are also available. In Cartesian coordinates, if $\mathbf{x} = (x_1, x_2, ..., x_n)$ and $\mathbf{y} = (y_1, y_2, ..., y_n)$ are two points in Euclidean n-space, then the distance d from x to y is given bythe following Euclidean distance function:

$$d(\mathbf{x},\mathbf{y}) = d(\mathbf{y},\mathbf{x}) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + ... + (x_n - y_n)^2} \tag{4.4}$$

Employing one of the distance functions mentioned above, the local cost measure for each pair of elements of the sequences X and Y is evaluated. This yields a cost matrix $C \in \mathbb{R}^{N+M}$ defined by $C(n,m) := d(i,j)$. The ultimate objective is to find next an alignment between X and Y such that the overall cost is minimal. As Müller (2007) put it, "Intuitively, such an optimal alignment runs along a 'valley' of low cost within the cost matrix C". More formally, finding an optimal alignment involves finding the warping curve $\phi(k)$, where $k = 1...T$:

$$\begin{aligned} &\phi(k) = (\phi_x(k), \phi_y(k)) \\ &\phi_x(k) \in \{1...N\} \\ &\phi_y(k) \in \{1...M\} \end{aligned} \tag{4.5}$$

The warping functions ϕ_x and ϕ_y remap the time indices of X and Y, respectively. Given ϕ, the average accumulated distortion between the warped time series X and Y is computed in the following manner:

$$d_\phi(X,Y) = \sum_{k=1}^{T} \frac{d(\phi_x(k), \phi_y(k)) m_\phi(k)}{M_\phi} \tag{4.6}$$

where $m_\phi(k)$ is a non-negative weighting coefficient which controls the contribution of each short-time distortion $d(\phi_x(k), \phi_y(k))$. Since this is usually related to the slope of the local path constraints which will be briefly discussed below, this is also known as slope weighting function. The denominator M_ϕ applies an overall normalization to the accumulated distortion to yield an average path distortion that is independent of the lengths of the two sequences being compared. The exact form of M_ϕ will be discussed in subsection 4.3.3. Finally, dynamic programming is applied to find the optimal alignment ϕ such that

$$D(X, Y) = \min d_\phi(X, Y). \tag{4.7}$$

4.3.2 Warping constraints

As can be imagined, the number of possible warping paths through the grid of the cost matrix is exponentially explosive unless the search space is reduced. This reduction is also necessary to ensure a proper time alignment between two sequences. Typical warping constraints that are considered necessary are as follows:

1. Boundary constraints
2. Monotonicity conditions
3. Local continuity constraints
4. Global path constraints
5. Slope weighting

Boundary constraints

The boundary constraints involve the imposition of the following conditions:

$$\phi_x(1) = \phi_y(1) = 1 \tag{4.8}$$

$$\phi_x(T) = N \tag{4.9}$$

$$\phi_y(T) = M \tag{4.10}$$

These ensure that the time sequences' beginning point and ending point match each other. As a result, the alignment does not consider partially one of the sequences. However, these constraints can be relaxed depending on the objective, for example partial time series matching. The basic idea of boundary constraints originated from the realization that speech patterns being compared usually have well-defined endpoints that mark the beginning and the ending frames of the pattern. Therefore, the endpoint information needed to be incorporated to obtain an accurate match.

Monotonicity conditions

Monotonicity conditions take the following form:

$$\begin{aligned} \phi_x(k+1) &\geq \phi_x(k) \\ \phi_y(k+1) &\geq \phi_y(k) \end{aligned} \tag{4.11}$$

The above conditions guarantee that time series' time ordering is preserved. These prevent the alignment path from going back in time. Essentially, negative slopes of a warping curve of the type as bounded within a gray circle in the left panel of figure 4.2 are ruled out.

Local Continuity Constraints

The basic objective of local continuity or step-size constraints is to ensure that no element in X and Y is omitted, otherwise potential loss of information may occur. Therefore, a discontinuous warping curve as in the right panel of figure 4.2 is ruled out. The shaded circle in the figure represents the hole in the curve. Generally, local continuity constraints can take various forms. Depending on the directions of matches between i and j

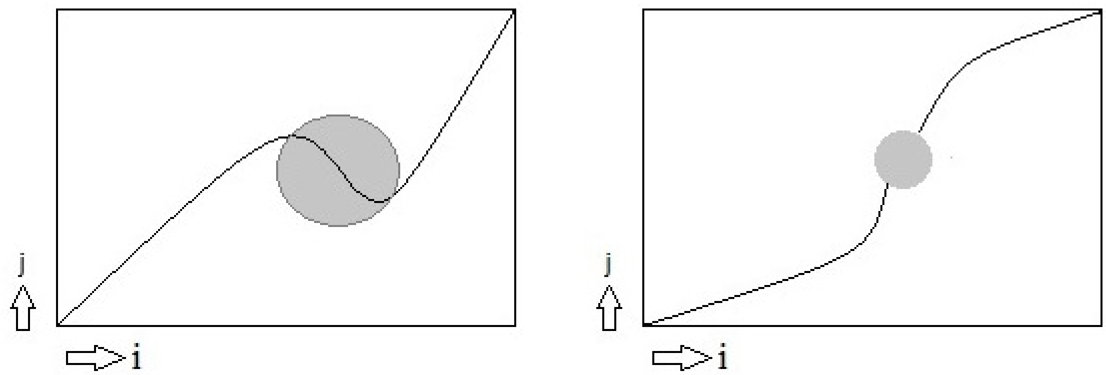


Figure 4.2: Monotonicity and continuity constraints

they allow, they can be categorized as symmetric or asymmetric. An example of symmetric local constraints as proposed by Sakoe and Chiba (1978) is as follows:

$$\begin{aligned}\phi_x(k+1) - \phi_x(k) &\geq 1 \\ \phi_y(k+1) - \phi_y(k) &\geq 1\end{aligned} \tag{4.12}$$

The above set of constraint is called symmetric because it allows an unlimited number of elements of the query X to match with a single element of Y, and vice versa. See the left panel of figure 4.3 for the step pattern admissible by the above symmetric local continuity constraint. In contrast, the right panel of figure 4.3 depicts an aysmmetric step pattern which allows multiple elements of the query sequence to match with the same element in the reference sequence, but not vice versa. Another way to think about it is that vertical alignment is prohibited. Numbers next to the path directions indicate the multiplicative weight m_ϕ for the local distance $d(i, j)$. Exactly how these numbers are obtained will be discussed in a later section.

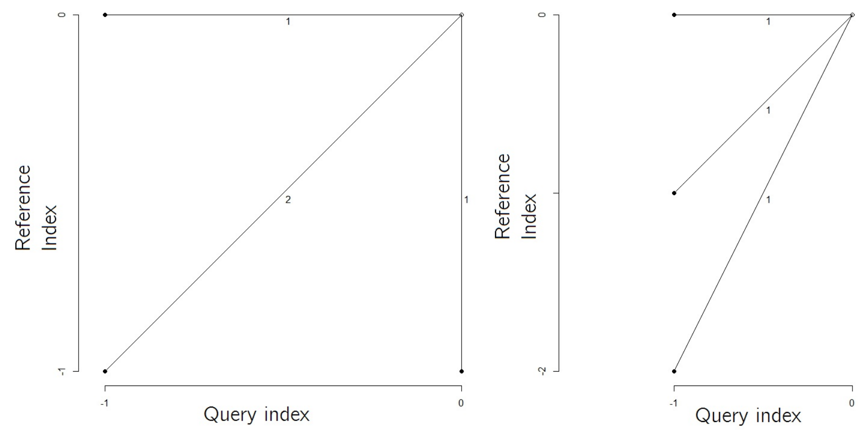


Figure 4.3: Symmetric and Asymmetric Step-patterns

Note: These figures are generated using the DTW package available in R by Giorgino (2009).

Global Path Constraints

In addition to the local path constraints, global path constraints or "windowing" can be applied to the warping functions to specify regions in the (i, j) plane where warping curves would not enter. It ensures that the warped curve is not too far away from the diagonal. Sakoe and Chiba (1978) proposed the following adjustment window condition such that time-axis fluctuation does not cause an excessive timing difference:

$$|\phi_x(k) - \phi_y(k)| \leq r \tag{4.13}$$

where r is an appropriate positive integer called window length. In figure 4.4, because of window length r the dotted warping curve is not allowed, but the solid warping curve within the window r is allowed. In this paper, we do not apply global path constraints. But interested readers can find more details in Sakoe and Chiba (1978) and Giorgino (2009).

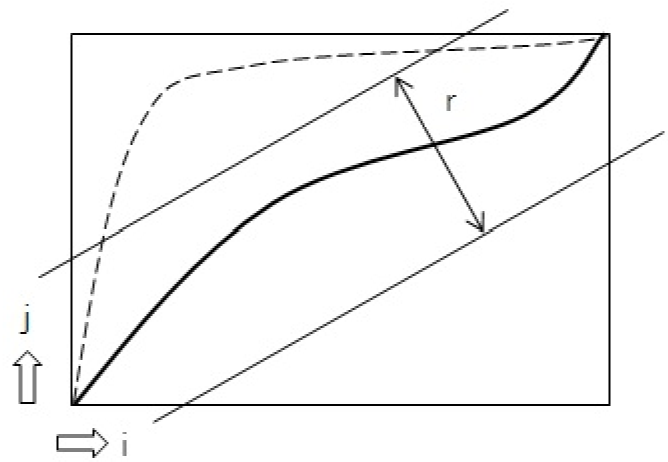


Figure 4.4: Global Path Constraints

Slope weighting

While the symmetric or asymmetric type of step pattern sets out the admissible directions in the alignment, the slope weighting function can be used to attach preferences to those directions. In other words, it allows us to attach different non-negative weights to vertical, horizontal, and diagonal directions based on our preferences. The idea is to have the least preferred direction receive the highest weight, and the most preferred direction the lowest weight. However, note that the actual warping path will be eventually determined according to equation (4.7).

Similar to the local continuity constraints, a number of heuristic slope weighting functions have been proposed in the literature. However, in this paper we will apply two of the weighting functions proposed by Sakoe and Chiba (1978). For the symmetric step pattern discussed above, we use the following weighting function:

$$m(k) = \phi_x(k) - \phi_x(k-1) + \phi_y(k) - \phi_y(k-1) \tag{4.14}$$

The above function admits an equal preference for alignments both in the vertical and horizontal direction which is also higher than the preference for an alignment in the diagonal direction. According to the left panel in figure 4.3, equation (4.14) attaches slope weights of 1,1 and 2 to the horizontal, vertical and diagonal transitions, respectively. For the asymmetric step pattern discussed above, we use the following weighting function which attaches an equal weight of 1 to all the directions (see the right panel in figure 4.3):

$$m(k) = \phi_x(k) - \phi_x(k-1) \tag{4.15}$$

The choice of the weighting functions in this paper is purely heuristic. It is not possible to determine a priori which function will yield a better result. We will apply both to compare their relative superiority.

4.3.3 Normalization

Normalization is done in order to compute an average per-step distance along the warping curve. This enables the comparison of alignments between two time sequences of different lengths. It is customary to define the normalizing factor M_ϕ as the sum of the components of the local weight.[1] Formally, it takes the following form:

$$M_\phi = \sum_{k=1}^{T} m(k) \tag{4.16}$$

Therefore, for the local weight in equation (4.14) the normalizing factor becomes:

$$\begin{aligned} M_\phi &= \sum_{k=1}^{T} [\phi_x(k) - \phi_x(k-1) + \phi_y(k) - \phi_y(k-1)] \\ &= \phi_x(T) - \phi_x(0) + \phi_y(T) - \phi_y(0) = N + M \end{aligned} \tag{4.17}$$

[1]For some local weighting functions, such as those of the types *min* and *max*, this definition makes the minimization problem in equation (4.7) unwieldy using recursive dynamic programming algorithm. For details, see Sakoe and Chiba (1978).

and for the local weight in equation (4.15) the normalizing factor becomes:

$$\begin{aligned} M_\phi &= \sum_{k=1}^{T} [\phi_x(k) - \phi_x(k-1)] \\ &= \phi_x(T) - \phi_x(0) = N \end{aligned} \tag{4.18}$$

4.3.4 Dynamic programming algorithm

To solve equation (4.7), recursive dynamic programming (DP) algorithms are very well-suited. We use the following algorithm for the symmetric step-pattern described above:

Initial condition: $g(1,1) = d(1,1)$

DP equation:

$$g(i,j) = \min \begin{bmatrix} g(i,j-1) & + & d(i,j) \\ g(i-1,j-1) & + & 2d(i,j) \\ g(i-1,j) & + & d(i,j) \end{bmatrix} \tag{4.19}$$

For the asymmetric step-pattern described above, the following change is made to DP-equation (4.19):

$$g(i,j) = \min \begin{bmatrix} g(i-1,j) & + & d(i,j) \\ g(i-1,j-1) & + & d(i,j) \\ g(i-1,j-2) & + & d(i,j) \end{bmatrix} \tag{4.20}$$

Note that we have not specified any restricting conditions for the global path constraints since we will not apply any windowing. The DP-equation, $g(i,j)$ must be recursively calculated in an ascending order with respect to coordinates i and j. The algorithm will run from the initial condition at $(1,1)$ up to (N,M).

Algorithm (4.19) is employed to produce the cost matrix in the left panel of figure 4.5 for the following two time series where X is the query series and Y is the reference series:

$$X = 3,2,2,1,2,3,1,3$$

$$Y = 2, 1, 2, 3, 3, 1, 2, 1.$$

The optimal path is denoted by the crooked line starting from the bottom-left corner (1,1) up to the top-right corner (8,8) and the measured distance is 4. Notice how the transition took place vertically from $(i, j) = (7, 6)$ to $(i, j) = (7, 7)$ instead of diagonally to $(i, j) = (8, 7)$, though both transitions occur between 0 and 1. The reason lies in the way our algorithm is defined which attaches a higher weight to the diagonal path resulting in a higher cost for that path. Therefore, the diagonal alignment is avoided in favor of the vertical alignment.

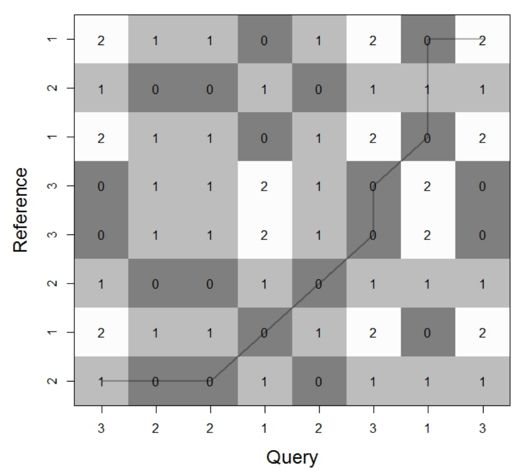


Figure 4.5: Cost matrix and the optimal warping curve with a symmetric step-pattern

4.4 Data and implementation of DTW

We use monthly data for Treasury term spread collected from the FRED database. Following Estrella and Trubin (2006), the treasury term spread is constructed by taking a difference between 10-year treasury constant maturity rate and secondary market 3-month treasury rate expressed on a bond-equivalent basis. Since FRED database provides the secondary market rate on a discount basis, the following formula is applied to convert the three-month discount rate into a bond-equivalent basis:

$$\textit{Bond-equivalent yield} = 100 \times \frac{\dfrac{365 \times \textit{discount yield}}{100}}{\dfrac{360 - 91 \times \textit{discount yield}}{100}} \tag{4.21}$$

3-month treasury constant maturity rate data are also available from FRED, but only from January 1982. However, data on both 10-year treasury constant maturity rate and secondary market 3-month treasury rate are available for a much longer period, from January 1953 up to now. Therefore, 3-month treasury constant maturity rate data are not used in this paper.

We apply DTW to find $k = 5$ time periods from historical data whose data patterns resemble most closely the pattern of some query data of a particular period of interest. This particular period of interest is chosen to represent a time frame of a reasonable length before a recession occurs. The choice of this query length is made with a heuristic approach. The basic idea is to pick a time frame which is long enough to capture some visibly distinct pattern underlying the query data. Through several experiments which are not presented here, it appears that a query length ranging from 16-24 months is reasonable for economic data. In the following, we summarize the basic steps to implement DTW with boundary constraints in our paper:

1. Select a query data X of length N and historical data Z of length H.

2. Compute distance D_1 using DTW between X and the first N elements of Z.

3. To find distances between X and all subsequent N elements of Z, run DTW algorithm $H-N$ times to calculate D_i where $i = 2, 3....(H-N+1)$. For example, $i = 2$ indicates the 2nd N elements of Z.

4. After obtaining all the D_is, find the minimum of all, D_k^{min} where $k = 1$.

5. Remove N number of D_is adjacent to left and N number of D_is adjacent to the right of D_1^{min}. This is done to ensure that the search for the next minimum D_i does not end up capturing almost the same historical time period shifted by 1 or two time indexes. As one can imagine, obtaining overlapping historical matches is not well-suited to our task of uncovering unique historical matches.

6. Repeat steps 4 and 5 to find 5 minimum values of D_is, i.e. D_k^{min} for $k = 2, ...5$, and obtain the time periods corresponding to them. These time periods reflect the 5 best historical matches for our query data X.

After obtaining historical matches, they are ranked in an ascending order of their DTW distances resulting in the best match being ranked one. Next, for each match we assign a probability of 1 if a recession occurs within 6 months of the matching period, otherwise assign 0. Then we calculate a weighted probability of a recession within 6 months. There are various methods available to calculate weights, the simplest being equal weights which do not take into account rank information. Several other methods for obtaining rank-based weights have been proposed by Stillwell et al. (1981), such as rank-sum, rank reciprocal and rank exponents. However, they are all ad hoc methods for generating weights.

Therefore, we employ a more systematic method called Rank Order Centroids (ROC) proposed by Barron (1992) to generate weights based on ranks. A comparative analysis of ROC is presented in Barron and Barrett (1996) where ROC emerges as a stable and superior weight selection method. It is a surrogate weighting method which converts ranks into values that are normalized on a $(0, 1)$ interval. ROC weights are computed from the vertices of the simplex, $w_1 \geq w_2 \geq ... \geq w_n \geq 0$, restricted to $\sum_{i=1}^{n} w_i = 1$. The defining vertices of this simplex are $\mathbf{e_1} = (1, 0, ..., 0)$, $\mathbf{e_2} = (1/2, 1/2, 0, ..., 0)$,..., $\mathbf{e_n} = (1/n, 1/n, ..., 1/n)$. The coordinates of the centroid which give the weights are obtained by averaging the corresponding coordinates of the defining vertices. Therefore, the centroid is given by,

$$C = (\frac{1}{n}\sum_{i=1}^{n}\frac{1}{i}, \frac{1}{n}\sum_{i=2}^{n}\frac{1}{i}, ..., \frac{1}{n}\sum_{i=n}^{n}\frac{1}{i}) \tag{4.22}$$

For example, for n=5 the weights are as follows:

$$\begin{aligned}
w_1 = & \frac{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}}{5} = 0.457 \\
w_2 = & \frac{0 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}}{5} = 0.256 \\
w_3 = & \frac{0 + 0 + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}}{5} = 0.157 \\
w_4 = & \frac{0 + 0 + 0 + \frac{1}{4} + \frac{1}{5}}{5} = 0.090 \\
w_5 = & \frac{0 + 0 + 0 + 0 + \frac{1}{5}}{5} = 0.040
\end{aligned} \tag{4.23}$$

4.5 Results

In this section we discuss the results obtained by employing DTW to identify 5 best matches from the historical data which are similar to the periods preceding the recessions

Match no.	Pr(Recession within 6 months)	Pr(Recession within 1 year)
1	1	1
2	0	0
3	0	1
4	1	1
5	0	0

Table 4.1: Probabilities of recessions within 6 and 12 months for historical matches for May 1988 - December 1989 using symmetric step-pattern DTW

that occurred, respectively, between (i) July 1990 - March 1991 (ii) March 2001 - November 2001 and (ii) December 2007 - June 2009.[2]

4.5.1 Historical matches for the period preceding the 1990-1991 recession

We select two query sequences with varying start and end dates preceding the recession of July 1990 - March 1991. The first query data we specify spans the period May 1988 - December 1989. The length of the query data is 5 quarters, and the query ends exactly 6 months before the recession starts in July 1990. Figure 4.7 displays the results yielded by a symmetric step-pattern DTW. The top panel plots the time series of DTW distances, and the middle panel displays the 5 best historical matches for the above specified query sequence. Historical matches are highlighted in red whereas the query sequence is in blue. The gray-shaded regions represent the NBER recession dates. The bottom panel in Figure 4.6 lists the dates of the 5 best historical periods. Except the second and the fifth matches, the rest precede a recession by less than a year. Table 4.1 contains the probabilities of recessions within 6 months and 12 months after the end date of each of the historical matches.

[2] All the estimations are carried out in R using the package called DTW by Giorgino (2009).

Next, in equations (4.24) and (4.25), using the probabilities in Table 4.1 along with the ROC weights, we compute the weighted probabilities of a recession within the next 6 and 12 months after the end date of our first specified query data, which is December 1989.

$$\Pr(\text{Recession within next 6 months}) = 0.457 + 0.09 = 0.457 \tag{4.24}$$

$$\Pr(\text{Recession within next 12 months}) = 0.457 + 0.157 + 0.09 = 0.709 \tag{4.25}$$

The computed probabilities in equations (4.24) and (4.25) correctly indicate a recession ahead of the end of our query data. They also indicate that once we broaden the horizon of prediction from within 6 months to 12 months, the probability of a recession becomes stronger by approximately 25 percentage points.

To examine if our prediction's accuracy can be further improved, we now apply DTW with an asymmetric step-pattern to the same query period May 1988 - December 1989. The historical matches obtained are displayed in Figure 4.7. Except the third and the fifth matches, all other matches indicate an imminent recession. The obtained matches are similar to the ones found in the symmetric case though the ranks of the matches have changed now. This change in the order of ranks will affect our weighted probability calculation. As before, we construct Table 4.2 with the probability of a recession within 6 and 12 months after the end date of each matched reference data. Using the probabilities in Table 4.2 along with ROC weights, we obtain the weighted probabilities of a recession within the next 6 and 12 months after the end date of our first specified query data as follows:

$$\Pr(\text{Recession within next 6 months}) = 0.457 + 0.256 = 0.713 \tag{4.26}$$

Match no.	Pr(Recession within 6 months)	Pr(Recession within 1 year)
1	1	1
2	1	1
3	0	0
4	0	1
5	0	0

Table 4.2: Probabilities of recessions within 6 and 12 months for historical matches for May 1988 - December 1989 using asymmetric step-pattern DTW

$$\text{Pr(Recession within next 12 months)} = 0.457 + 0.256 + 0.09 = 0.803 \qquad (4.27)$$

The estimated weighted probabilities from equations (4.26) and (4.27) provide evidence of a significant improvement in the strength of the prediction resulting from the adoption of an asymmetric step-pattern in comparison with a symmetric step-pattern.

The estimated weighted probabilities from equations (4.26) and (4.27) provide evidence of a significant improvement in the strength of the prediction resulting from the adoption of an asymmetric step-pattern in comparison with a symmetric step-pattern.

Now we will focus on our second query data which starts in August 1988 and ends in March 1990. As before, the length of the query is 5 quarters, and the query sequence ends exactly 3 months before the recession starts in July 1990. Figures 4.8 and 4.9 display the 5 best historical matches for the second query data obtained through DTW with a symmetric and an asymmetric step-pattern, respectively. Table 4.3 and 4.4 contain the probability of a recession within 6 and 12 months after the end date of each matched historical period.

Next using the probabilities in Table 4.3 (corresponding to asymmetric step-pattern) we obtain the weighted probabilities of recessions within 6 and 12 months

Match no.	Pr(Recession within 6 months)	Pr(Recession within 1 year)
1	1	1
2	0	1
3	0	0
4	0	0
5	1	1

Table 4.3: Probabilities of recessions within 6 and 12 months for historical matches for August 1988 - March 1990 using symmetric step-pattern DTW

Match no.	Pr(Recession within 6 months)	Pr(Recession within 1 year)
1	0	1
2	1	1
3	0	1
4	0	0
5	1	1

Table 4.4: Probabilities of recessions within 6 and 12 months for historical matches for August 1988 - March 1990 using asymmetric step-pattern DTW

after the query end data as follows:

$$\Pr(\text{Recession within next 6 months}) = 0.456 + .04 = 0.496 \quad (4.28)$$

$$\Pr(\text{Recession within next 12 months}) = 0.457 + 0.256 + 0.04 = 0.753 \quad (4.29)$$

And using the probabilities in Table 4.4 (corresponding to asymmetric step-pattern) we obtain the weighted probabilities of recessions within 6 and 12 months after the query end data as follows:

$$\Pr(\text{Recession within next 6 months}) = 0.256 + 0.04 = 0.296 \quad (4.30)$$

$$\Pr(\text{Recession within next 12 months}) = 0.457 + 0.256 + 0.157 + 0.04 = 0.91 \quad (4.31)$$

Surprisingly, although the second query data is 3 months closer to the actual recession start date of July 1990 than the first query data was, it provides a relatively weak signal of a recession within the next 6 months after the query end data, specially in the case of asymmetric step-pattern. However, the indication of a recession is much higher as before once the prediction horizon is widened from within 6 months to 12 months.

4.5.2 Historical matches for the period preceding the 2001 recession

As before, we specify several query data with varying start and end dates preceding the recession of March 2001 - November 2001. The first query data we specify spans the period June 1999 - September 2000. That is, it ends exactly 6 months before the recession starts in March 2001. The middle graph in figure 4.10 displays the 5 best historical matches for this period obtained using symmetric step-pattern DTW. Historical matches are highlighted in red while the query sequence is in blue. As usual, the gray-shaded regions represent NBER recession dates. The top panel plots the distances estimated by DTW between the query and each reference data.

The graph confirms that DTW has indeed been successful in matching the pre-recession query data with those from four other pre-recession periods. Only the second match corresponds to a historical period which does not precede any recession occurring within the next 2 years. Overall, from the perspective of visual confirmation the graph adds support to the efficacy of US treasury term spread in foreshadowing domestic recessions. Table 4.5 contains the probabilities of recessions within 6 months and 12 months of periods corresponding to the historical matches. Using the ROC weights above, we compute the weighted probabilities of a recession within the next 6 and 12 months, respectively, as

Match no.	Pr(Recession within 6 months)	Pr(Recession within 1 year)
1	1	1
2	0	0
3	0	1
4	0	0
5	0	0

Table 4.5: Probabilities of recessions within 6 and 12 months for historical matches for June 1999 - September 2000 from Symmetric DTW

follows:

$$\text{Pr(Recession within next 6 months)} = 0.457 \tag{4.32}$$

$$\text{Pr(Recession within next 12 months)} = 0.457 + 0.157 = 0.614 \tag{4.33}$$

The calculations in equations (4.32) and (4.33) show that once we widen the horizon of prediction from within 6 months to within 12 months, the probability of a recession improves by almost 15 percentage points. In fact, (4.33) suggests based on historical patterns that the probability of a recession occurring within a year from 01 September 2000 is higher than not occurring. Evidence of an actual recession taking place within half a year from 01 September 2000 corroborates our predictions' accuracy.

Figure 4.11 contains the historical matches for the same query period 01 June 1999- 01 September 2000, but obtained using the asymmetric step-pattern. Similar to the symmetric step-pattern, asymmetric step-pattern discovers 5 best historical matches for the query period preceding the recession of 2001. Only the third match does not precede any recession occurring within the next 2 years. Table 4.6 contains the probabilities of recessions within 6 months and 12 months of periods corresponding to the historical matches. Again, using the ROC weights above, we compute the weighted probabilities of a recession within

Match no.	Pr(Recession within 6 months)	Pr(Recession within 1 year)
1	1	1
2	0	0
3	0	0
4	0	1
5	0	1

Table 4.6: Probabilities of recessions within 6 and 12 months for historical matches for June 1999 - September 2000 from Asymmetric DTW

the next 6 and 12 months respectively as follows:

$$\Pr(\text{Recession within next 6 months}) = 0.457 \quad (4.34)$$

$$\Pr(\text{Recession within next 12 months}) = 0.457 + 0.090 + 0.040 = 0.587 \quad (4.35)$$

The weighted probabilities remain similar to the ones obtained previously using symmetric step-pattern. Although the probability of a recession within the next one year of September 2000 has slightly declined, it is still signaling as before a higher chance of a recession occurring than not.

Now, we specify our query data over the period 01 October 1999-01 December 2000. The length of the query here is 15 and the query ends 3 months before the recession of 2001. Figure 4.12 and 4.13 display the 5 historical matches for the query sequence using symmetric and asymmetric step-patterns respectively. Note that DTW with asymmetric step-pattern has picked up the period preceding the the recession of late 1973. But unlike other previous results, it fails to capture the similarity between the query data and the data preceding the recession starting in April 1960. Another way to put it is that the DTW with asymmetric step-pattern has traded the historical period preceding the 1960's recession with the period preceding the 1973's recession.

Tables 4.7 and 4.8 list the probabilities of recessions within 6 months and 12 months of periods corresponding to the historical matches obtained using DTW, respectively with a symmetric step-pattern and asymmetric step-pattern. Assigning the ROC weights to

Match no.	Pr(Recession within 6 months)	Pr(Recession within 1 year)
1	1	1
2	0	1
3	0	1
4	0	0
5	1	1

Table 4.7: Probabilities of recessions within 6 and 12 months for historical matches for 01 October 1999-01 December 2000 using Symmetric DTW

Match no.	Pr(Recession within 6 months)	Pr(Recession within 1 year)
1	1	1
2	0	1
3	1	1
4	0	1
5	0	0

Table 4.8: Probabilities of recessions within 6 and 12 months for historical matches for 01 October 1999-01 December 2000 using Asymmetric DTW

the probabilities in table 4.7, we compute the weighted probabilities of a recession within the next 6 and 12 months respectively as follows:

$$\begin{aligned} \text{Pr(Recession within next 6 months)} &= (0.457 \times 1) + (0.04 \times 1) \\ &= 0.497 \end{aligned} \tag{4.36}$$

$$\begin{aligned}\Pr(\text{Recession within next 12 months}) &= (0.457 \times 1) + (0.256 \times 1) \\ &\quad + (0.157 \times 1) + (0.04 \times 1) \\ &= 0.91\end{aligned} \tag{4.37}$$

Although the historical matches indicate a 0 probability of a recession within the next 3 months, they do provide a very strong signal of a recession within the next 12 months from the query end date i.e. December 2000. In the following, we compute the weighted probabilities of a recession within the next 6 and 12 months, respectively using ROC weights along with probabilities in table 4.8:

$$\begin{aligned}\Pr(\text{Recession within next 6 months}) &= (0.457 \times 1) + (0.157 \times 1) \\ &= 0.614\end{aligned} \tag{4.38}$$

$$\begin{aligned}\Pr(\text{Recession within next 12 months}) &= (0.457 \times 1) + (0.256 \times 1) \\ &\quad + (0.157 \times 1) + (0.09 \times 1) \\ &= 0.96\end{aligned} \tag{4.39}$$

Probabilities calculated in equations (4.38) and (4.39) are the highest amongst all the ones calculated so far. They provide strong indication for a recession within, respectively the next 6 and specially 12 months from the query end date of 01 December 2000. Additionally, equation (4.38) removes the uncertainty associated with probabilities of a recession within the next 6 months obtained in (4.26) and (4.30).

At this point, it seems appropriate to examine and compare the performance of Pearson's correlation coefficient in uncovering historical periods matching the one preceding the recession of 2001. Figure 4.19 displays those historical matches for the period 01 October 1999 - 01 December 2000. A visual inspection of the graph confirms the poor performance of the correlation coefficient relative to DTW. Correlation coefficient yields 5 historical matches all of which coincide with previous recessionary periods whereas the query data precedes the 2001 recession by 3 months. In contrast, historical matches obtained using DTW as in figures 4.12 and 4.13 provide a more accurate account since 9 out of 10 matches actually precede recessions just as the query data does.

4.5.3 Historical matches for the period preceding the 2007 recession

Similar to our analysis in the previous subsection, we specify two query data with varying start and end dates preceding the recession of December 2007 - June 2009. The first query data spans the period 01 March 2006 - 01 June 2007. It is of length 16 and ends 6 months before the start of the 2007 US recession. The middle graph in figure 4.14 contains 5 best historical matches for this query data obtained using DTW with the symmetric step-pattern. Although the first match corresponding to the minimum DTW measure does not precede any recession within the next 2 years, the rest of the matches either precede recessions occurring within 3-6 months, such as matches no. 2 and 3, or overlap with the recession, such as matches no. 4 and 5.

Table 4.9 lists the probabilities of recessions within 6 and 12 months of periods corresponding to the historical matches found in figure 4.14. Applying the ROC weights previously obtained in 4.23, we calculate the weighted probabilities of a recession within the next 6 and 12 months respectively as follows:

Match no.	Pr(Recession within 6 months)	Pr(Recession within 1 year)
1	0	0
2	1	1
3	1	1
4	1	1
5	1	1

Table 4.9: Probabilities of recessions within 6 and 12 months for historical matches for 01 December 2007-01 June 2009 using symmetric DTW

$$\begin{aligned} \text{Pr(Recession within next 6 months)} &= (0.256 \times 1) + (0.157 \times 1) \\ &\quad + (0.09 \times 1) + (0.04 \times 1) \\ &= 0.54 \end{aligned} \tag{4.40}$$

$$\begin{aligned} \text{Pr(Recession within next 12 months)} &= (0.256 \times 1) + (0.157 \times 1) \\ &\quad + (0.09 \times 1) + (0.04 \times 1) \\ &= 0.54 \end{aligned} \tag{4.41}$$

The estimated probabilities in equations (4.34) and (4.35) indicate a higher probability of a recession occurring than not within the next 6 and 12 months after the query end date. Actual data shows that a recession started indeed 6 months after the query data had ended in June 2007. Therefore, again we find evidence that historical matches obtained by applying DTW to Treasury yield spread data are very useful in predicting future US recessions.

Next, we apply DTW with the asymmetric step-pattern to the same query data. The 5 best historical matches for the query data obtained using this method are displayed in the middle graph in figure 4.15. In contrast with figure 4.14, figure 4.15 picks up the period preceding the recession starting in November 1973 (match no. 5), but cannot match

Match no.	Pr(Recession within 6 months)	Pr(Recession within 1 year)
1	0	0
2	1	1
3	1	1
4	1	1
5	1	1

Table 4.10: Probabilities of recessions within 6 and 12 months for historical matches for 01 March 2006 - 01 June 2007 using asymmetric DTW

the period preceding the recession of 1957. Otherwise, the rest of the matched periods are similar to the ones in figure 4.14. Table 4.10 contains the probabilities of recessions within 6 and 12 months of periods corresponding to the historical matches found in figure 4.15. As expected, table 4.10 is the same as table 4.9. Therefore, the weighted probabilities implied by table 4.10 should be equal to the ones obtained from equations (4.34) and (4.35). In other words, both symmetric and asymmetric step-patterns yield the same predictions for recessions.

Next, we specify our second query data of the same length as before but spanning the period 01 June 2006 - 01 September 2007. Applying DTW with the symmetric step-pattern, we obtain the 5 best historical matches for the query data as displayed in figure 4.16. The matched historical periods in this figure look very similar to the ones in figure 4.14, but much closer to the recession start dates. This time we construct table 4.11 which lists out the probabilities of recessions within the next 3, 6 and 12 months of the end dates of the periods corresponding to the historical matches. Using the probability weights calculated in equation (4.23), we estimate the weighted probabilities of recessions within

Match no.	Pr(Recession within 3 months)	Pr(Recession within 6 months)	Pr(Recession within 1 year)
1	0	0	0
2	1	1	1
3	1	1	1
4	1	1	1
5	1	1	1

Table 4.11: Probabilities of recessions within 3, 6 and 12 months for historical matches for 01 June 2006 - 01 September 2007 using symmetric DTW

the next 3, 6 and 12 months of the query data to be all equal to 0.54.[3] An actual recession starting in December 2007, which is within 3 months after the query end date confirms the prediction obtained through the symmetric step-pattern DTW application.

Finally, we employ DTW with the asymmetric step-pattern to uncover the 5 best historical matches for the second query period. Figure 4.17 presents these historical matches. Just as in the case of the query data ending 6 months before the recession of 2007 (see figure 4.14 and 4.15), figure 4.17 illustrates the uncovering of the period preceding the recession of 1973 as a historical match (see match no. 5) for the query data but fails to identify the period preceding the recession of 1957 as a match. Table 4.12 contains the probabilities of a recession within 3, 6 and 12 months after the periods associated with the historical matches found in figure 4.17. Using the ROC weights in equation (4.23), each of the weighted probabilities of a recession occurring within 3, 6 and 12 months after the end of the query data, can be estimated as a value equal to 0.744. In each case, the weighted probability is merely the summation of all the weights obtained in equation (4.23) less the second weight.

[3]Since the calculation is straightforward, and has been demonstrated several times already, we skip the calculation steps. Essentially, the weighted probability of 0.54 is obtained by summing up all the weights except the first one.

Overall, the results suggest that the strongest signal for an upcoming recession in 2007 is yielded by the asymmetric step-pattern DTW once combined with the query data ending 3 months before the recession starts.

Match no.	Pr(Recession within 3 months)	Pr(Recession within 6 months)	Pr(Recession within 1 year)
1	1	1	1
2	0	0	0
3	1	1	1
4	1	1	1
5	1	1	1

Table 4.12: Probabilities of recessions within 3, 6 and 12 months for historical matches for 01 June 2006 - 01 September 2007 using asymmetric DTW

4.5.4 Historical matches for the current period

Now that we have appreciated the usefulness of DTW in predicting US recessions, we shift our focus to the current economic phase US is in. Particularly, we look for 5 best historical matches for the period 01 March 2016 - 01 June 2017. As above, the query length set here is equal to 16 months, that is 4 quarters. The results obtained from a symmetric step-pattern DTW are depicted in figure 4.18. The closest match marked as 1 in the graph implies that if history is to repeat, then given our current economic state we are at least 6.5 years away from the next recession. While the rest of the historical matches do not signal a distance of this magnitude from the next recession, they do confirm that the next recession from now is at least more than a year away. The results remain very similar once DTW is implemented with an asymmetric step-pattern and therefore, is not presented here.

4.5.5 False flags

As discussed in the literature review, one of the major setbacks that non-parametric methods suffer from is that they raise frequent false flags signaling imminent recessions. For this reason, before concluding about the usefulness of DTW in conjunction with Treasury term spread data, it is imperative to look for the presence of similar symptoms in it. This can be accomplished by pinpointing multiple historical periods which are at least one year away from the next recession, and during which the behavior of Treasury term spreads appear to be similar to the ones as exhibited by actual pre-recession periods. To circumvent the challenge posed by the second condition, we examine multiple candidate periods with behavior seemingly similar or not.

Most of the candidates we consider cannot be matched with any historical periods in the vicinity of imminent recessions and therefore, we do not discuss them here. However, there are two suspect periods which raise false signals. The first one is January 1966 - December 1966 which overlaps with matches no. 1 and 2 that were incorrectly identified in figures, respectively 4.16 and 4.17. This period finds a maximum of 4 historical matches using DTW with symmetric step-pattern as displayed in figure 4.20. The first and the fourth match clearly do not precede any recession occurring in the next one year. However, the second and the third match do approach recessions. Using equation 4.22, it can be shown that the probability of a recession both within the next 6 and 12 months is equal to 0.42. Therefore, the probability is higher that a recession would not occur.

Another period that seems suspect is October 1997 - May 1999 since this overlaps with matches no. 4 and 5 falsely tagged in figures, respectively 4.12 and 4.13. Figure

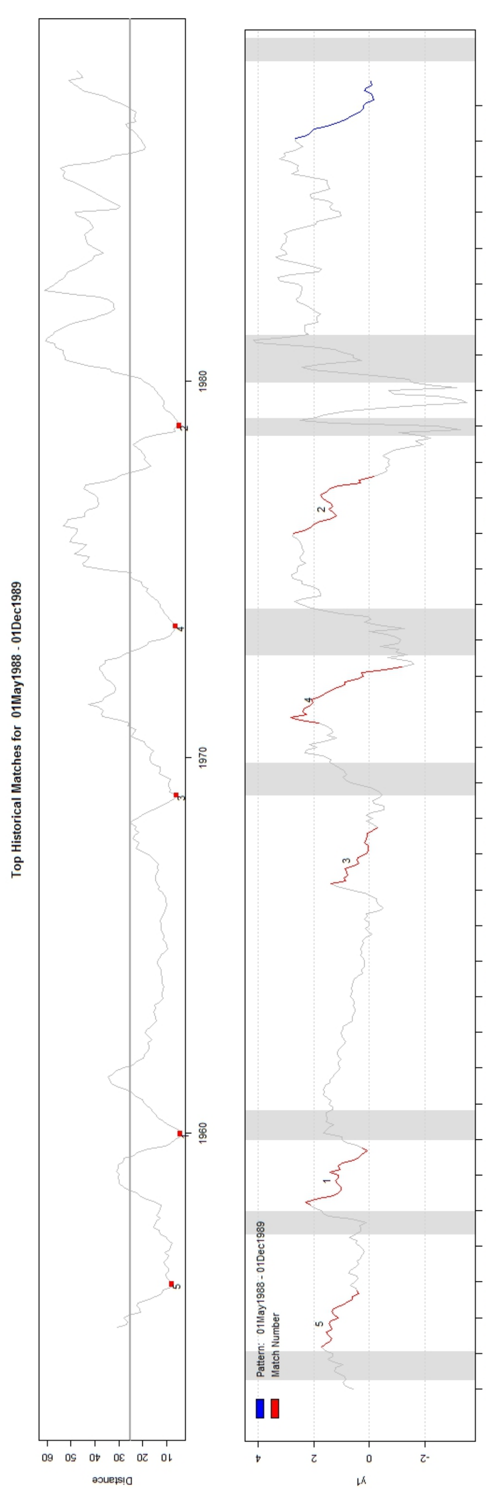
4.21 displays the historical matches for this period. Using equation 4.23, it can be shown that the probabilities of a recession within the next 6 and 12 months of the query period are 0.80 and 0.84, respectively. Although the calculated probabilities indicate a very high chance of an imminent recession, the query period of interest here was not followed by any actual recession within the next 6 or 12 months after May 1999. This implies that upon the implementation of DTW, among all the suspect periods only October 1997 - May 1999 raises a false flag signaling an imminent recession. In a nut shell, compared with the previous non-parametric studies, DTW yields a superior performance.

4.5.6 Conclusion

In this paper, we have adopted a novel non-parametric approach called Dynamic Time Warping to predict the US recessions of the last three decades using the Treasury term spread data. This method overcomes some of the pitfalls of existing parametric as well as non-parametric methods. For example, to successfully predict US recessions, this method does not require specifically modeling for structural breaks in the data. Also, compared to other methods, it is computationally much simpler.

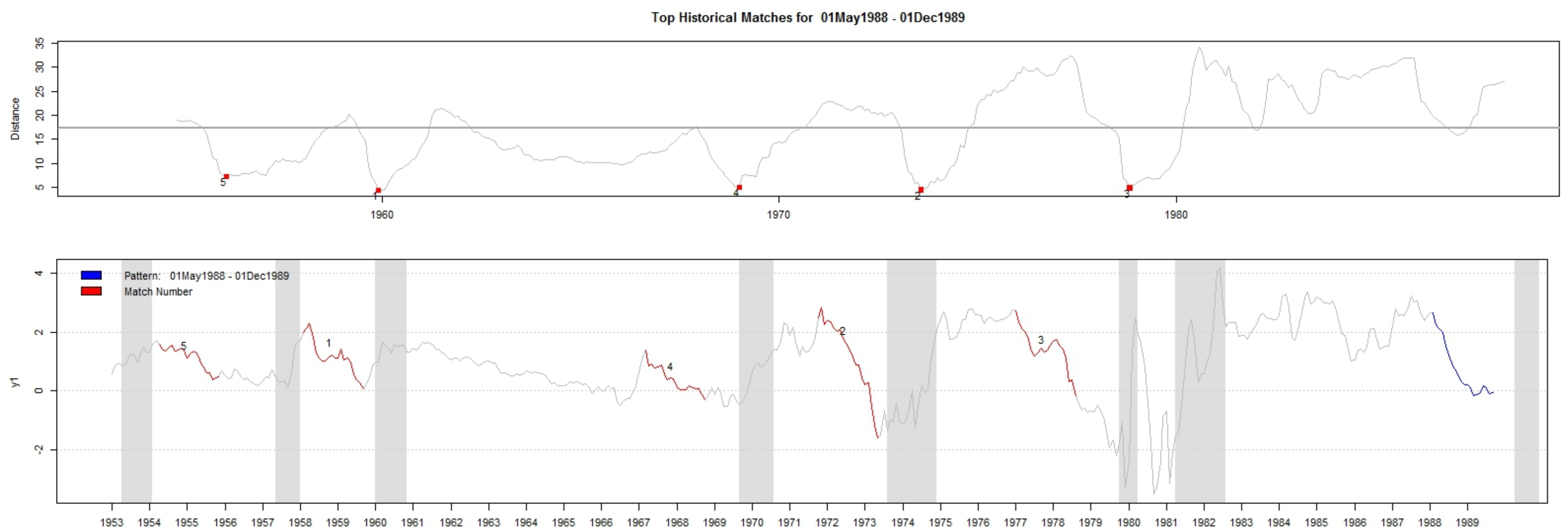
We employed two variants of DTW, one with a symmetric step-pattern and another with an asymmetric step-pattern. It turned out that the latter was more successful in finding historical matches for Treasury term spread data preceding a recession than the former one. As a result, DTW with an asymmetric step-pattern has been more efficient in predicting the US recessions of 1990-1991, 2001 and 2007-2009. DTW also indicates that given the current state of the economy as reflected in the Treasury term spread data, there is no recession lurking in the US at least within the next one year. Finally, the method used in this paper, DTW presents itself as an excellent potential tool to compare the efficiency of other existing leading indicators. Future research on DTW can proceed along this direction.

Appendix



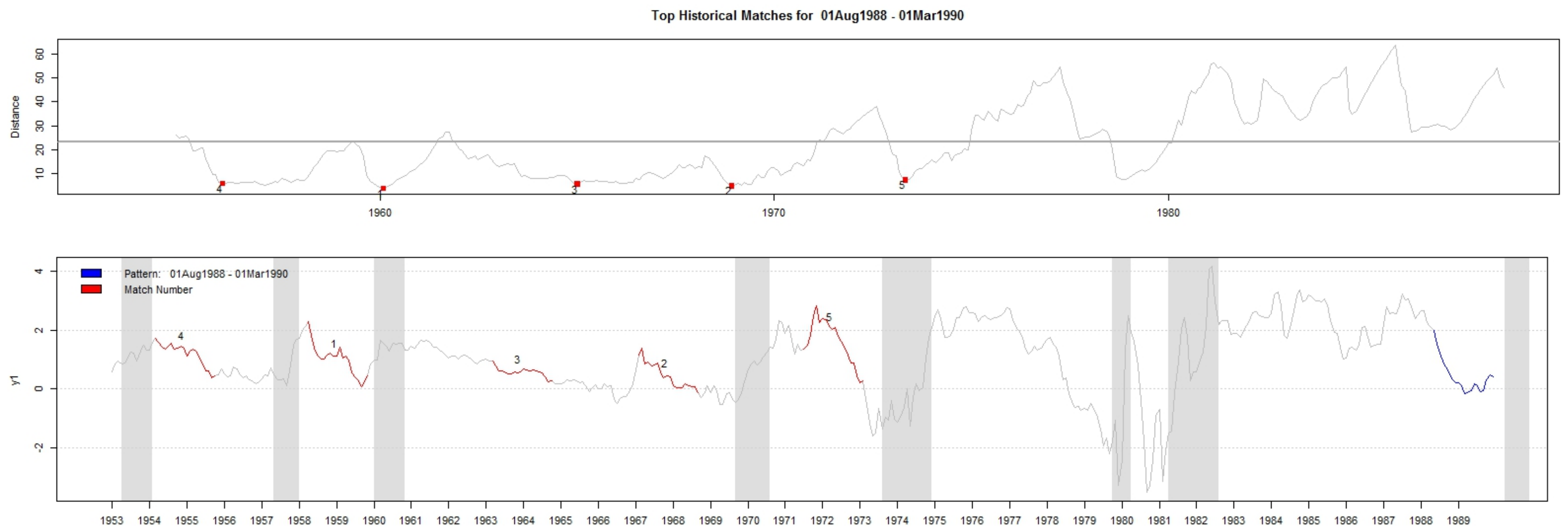
Match Number	Start	End
1	01 Jun 1958	01 Jan 1960
2	01 Apr 1977	01 Nov 1978
3	01 Jun 1967	01 Jan 1969
4	01 Dec 1971	01 Jul 1973
5	01 Jun 1954	01 Jan 1956
Query	01 May 1988	01 Dec 1989

Figure 4.6: 5 Historical periods similar to May 1988 - December 1989 using symmetric step-pattern DTW



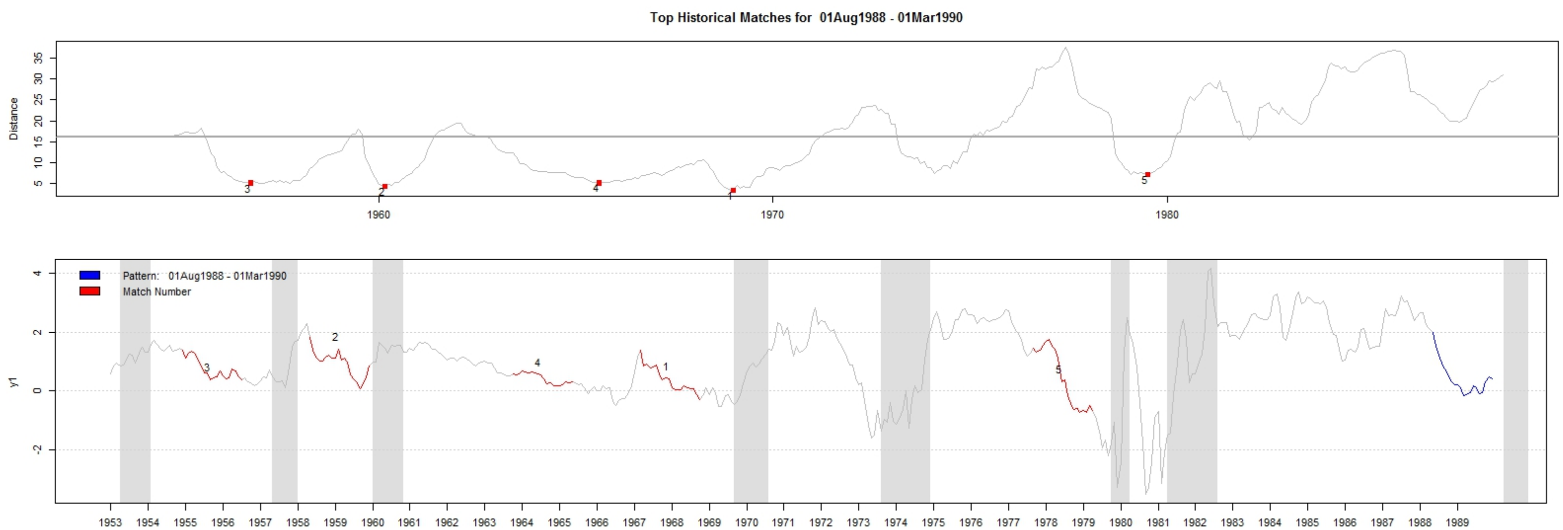
Match Number	Start	End
1	01 May 1958	01 Dec 1959
2	01 Jan 1972	01 Aug 1973
3	01 Apr 1977	01 Nov 1978
4	01 Jun 1967	01 Jan 1969
5	01 Jul 1954	01 Feb 1956
Query	01 May 1988	01 Dec 1989

Figure 4.7: 5 Historical periods similar to May 1988 - December 1989 using symmetric step-pattern DTW



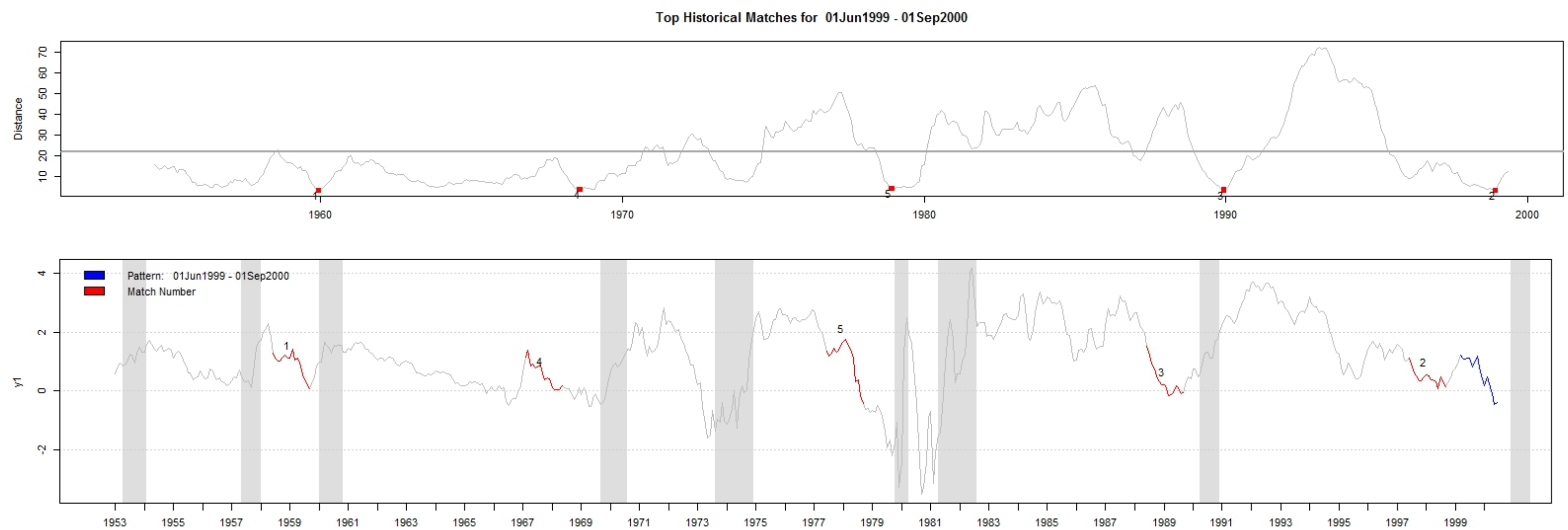
Match Number	Start	End
1	01 Jul 1958	01 Feb 1960
2	01 May 1967	01 Dec 1968
3	01 Jun 1963	01 Jan 1965
4	01 Jun 1954	01 Jan 1956
5	01 Oct 1971	01 May 1973
Query	01 Aug 1988	01 Mar 1990

Figure 4.8: 5 Historical periods similar to August 1988 - March 1990 using symmetric step-pattern DTW



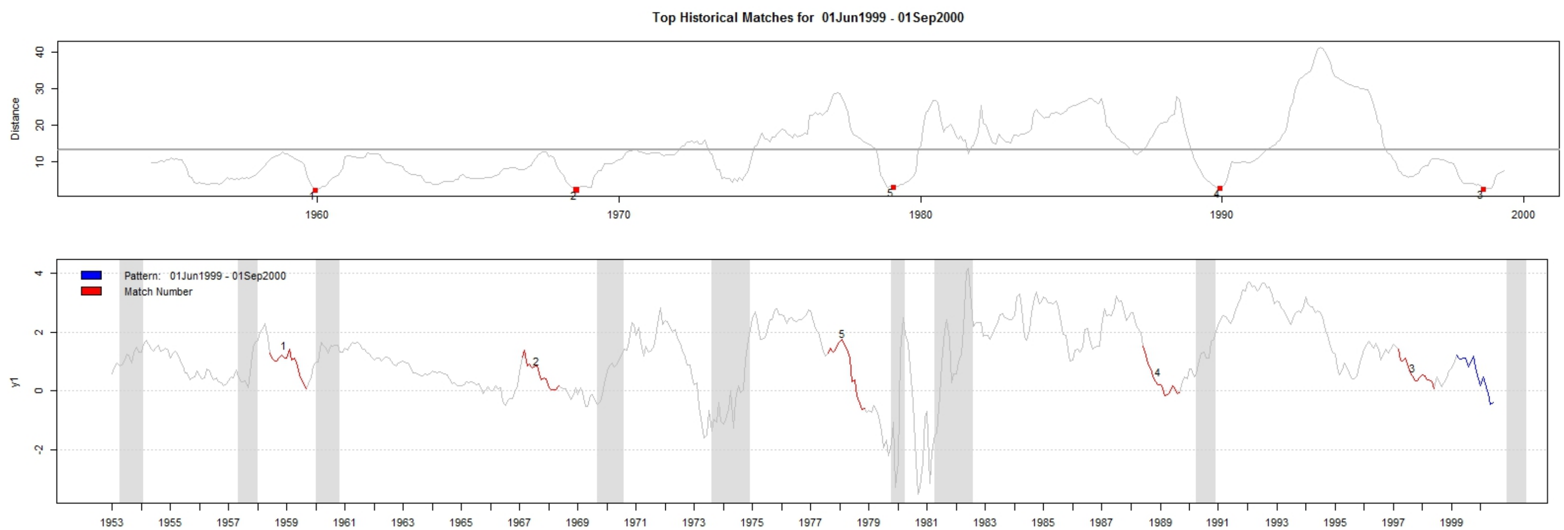
Match Number	Start	End
1	01 Jun 1967	01 Jan 1969
2	01 Aug 1958	01 Mar 1960
3	01 Mar 1955	01 Oct 1956
4	01 Jan 1964	01 Aug 1965
5	01 Dec 1977	01 Jul 1979
Query	01 Aug 1988	01 Mar 1990

Figure 4.9: 5 Historical periods similar to August 1988 - March 1990 using symmetric step-pattern DTW



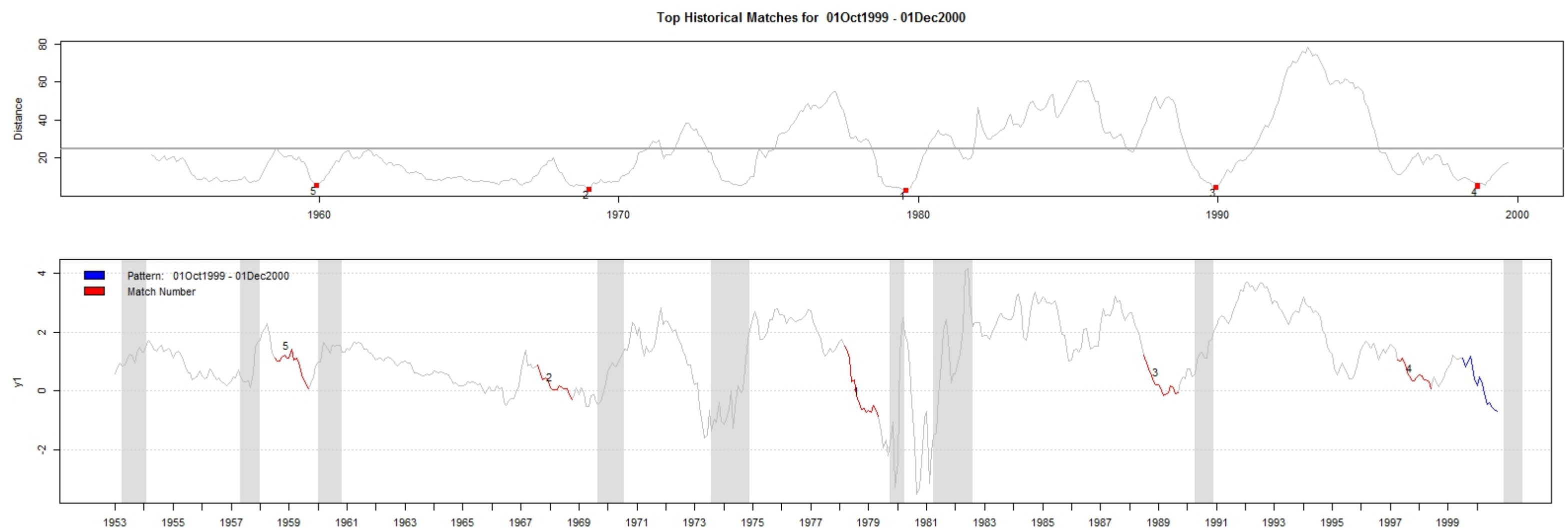
Match Number	Start	End
1	01 Sep 1958	01 Dec 1959
2	01 Sep 1997	01 Dec 1998
3	01 Sep 1988	01 Dec 1989
4	01 May 1967	01 Aug 1968
5	01 Sep 1977	01 Dec 1978
Current	01 Jun 1999	01 Sep 2000

Figure 4.10: 5 Historical periods similar to Jun 1999 - Sep 2000 using symmetric step-pattern DTW



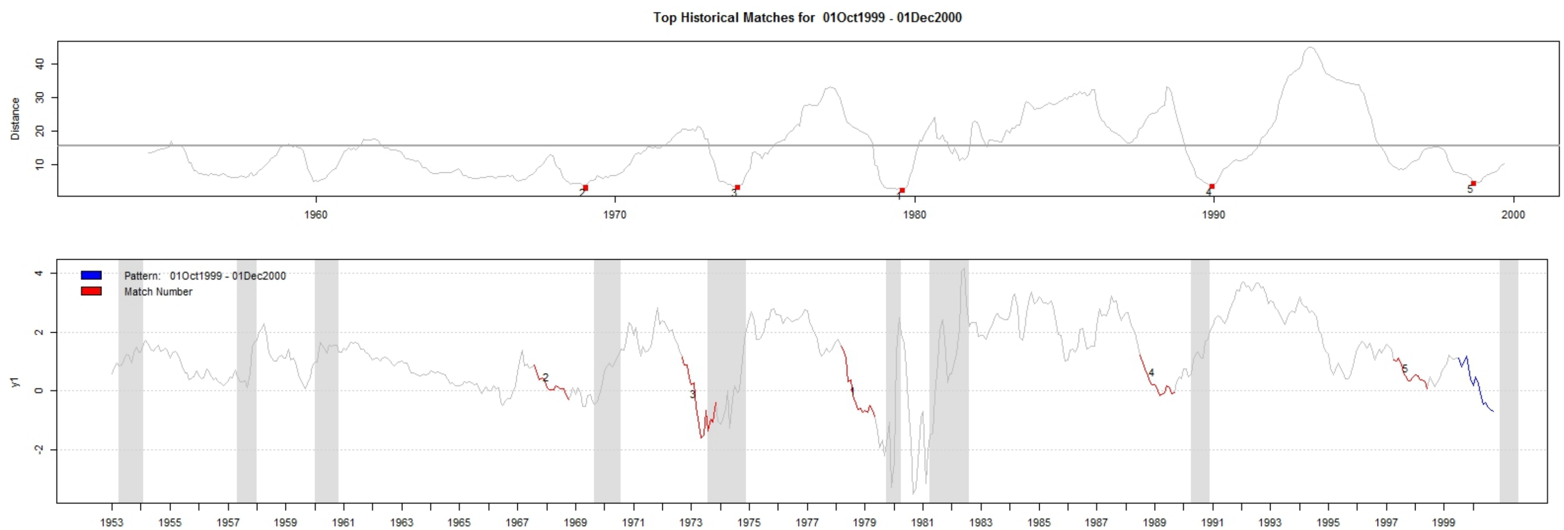
Match Number	Start	End
1	01 Sep 1958	01 Dec 1959
2	01 May 1967	01 Aug 1968
3	01 Jun 1997	01 Sep 1998
4	01 Sep 1988	01 Dec 1989
5	01 Nov 1977	01 Feb 1979
Current	01 Jun 1999	01 Sep 2000

Figure 4.11: 5 Historical periods similar to Jun 1999 - Sep 2000 using asymmetric step-pattern DTW



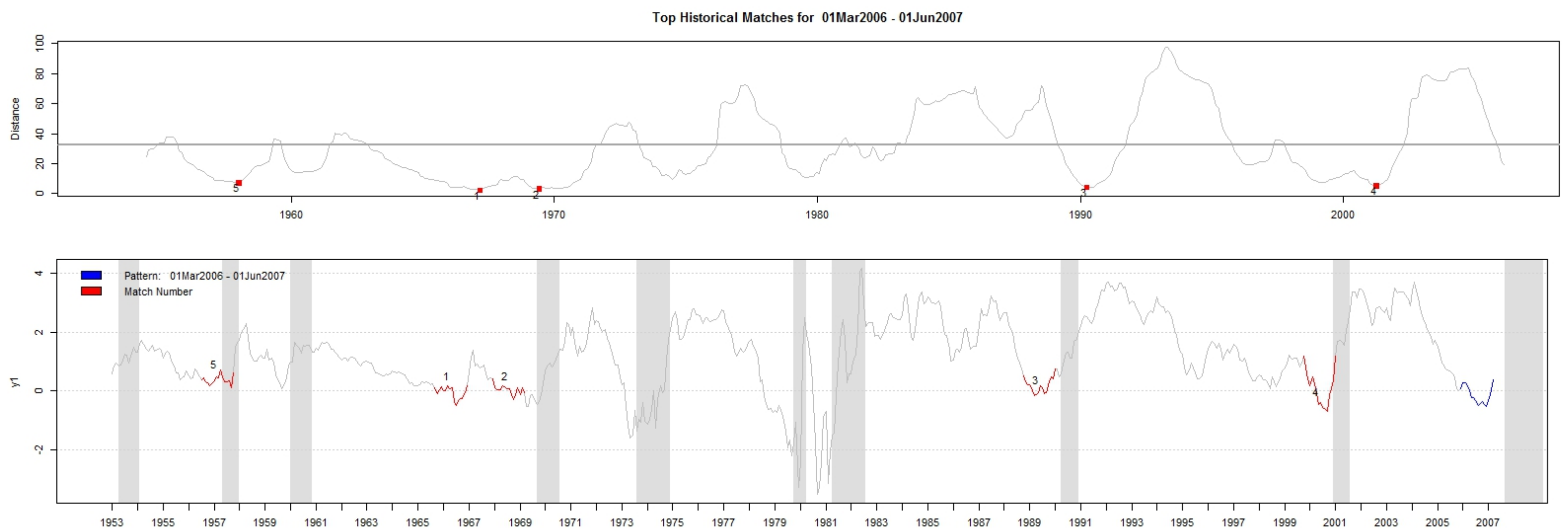
Match Number	Start	End
1	01 Jun 1978	01 Aug 1979
2	01 Nov 1967	01 Jan 1969
3	01 Oct 1988	01 Dec 1989
4	01 Jul 1997	01 Sep 1998
5	01 Oct 1958	01 Dec 1959
Current	01 Oct 1999	01 Dec 2000

Figure 4.12: 5 Historical periods similar to Oct 1999 - Dec 2000 using symmetric step-pattern DTW



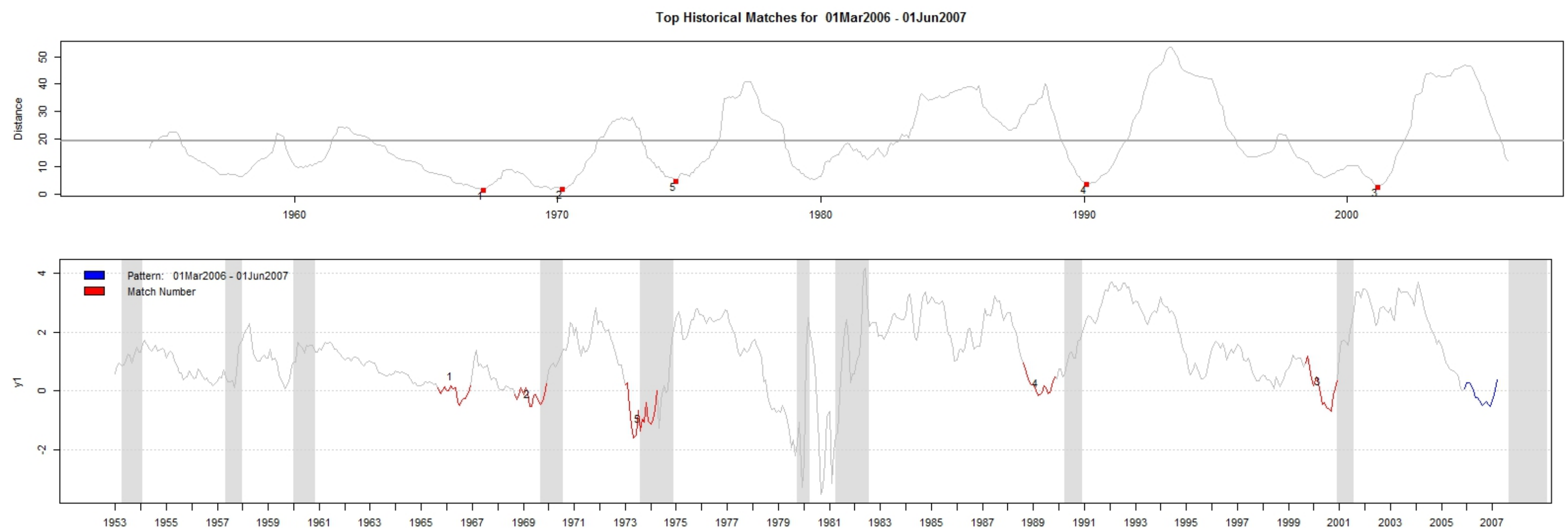
Match Number	Start	End
1	01 Jun 1978	01 Aug 1979
2	01 Nov 1967	01 Jan 1969
3	01 Dec 1972	01 Feb 1974
4	01 Oct 1988	01 Dec 1989
5	01 Jul 1997	01 Sep 1998
Current	01 Oct 1999	01 Dec 2000

Figure 4.13: 5 Historical periods similar to Oct 1999 - Dec 2000 using asymmetric step-pattern DTW



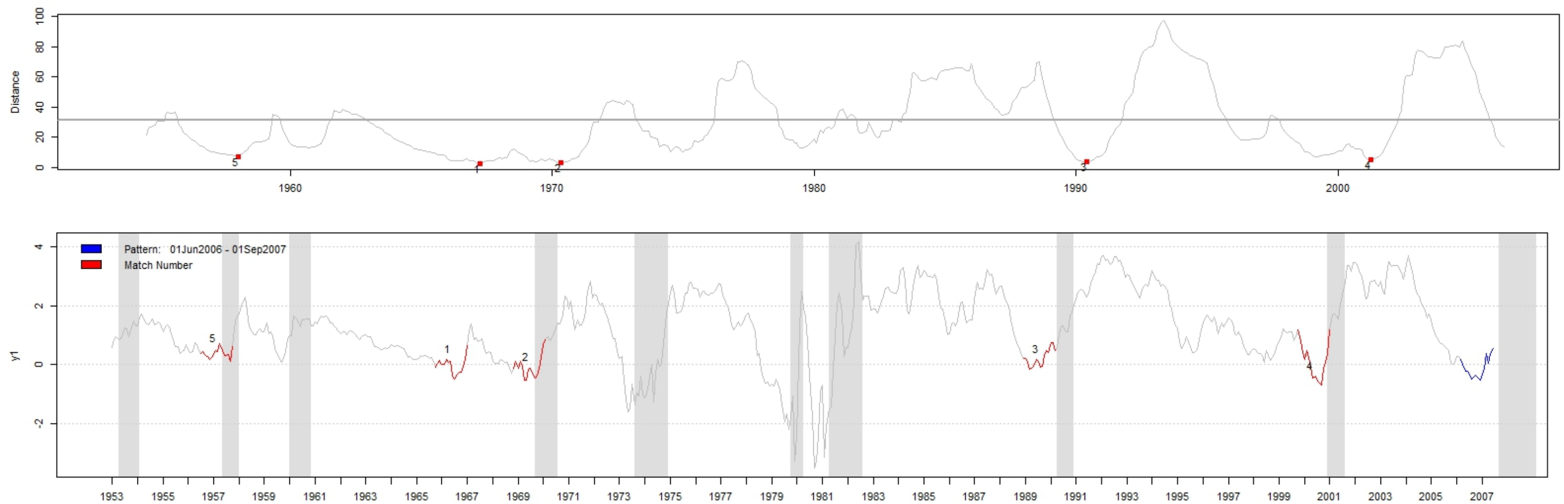
Match Number	Start	End
1	01 Dec 1965	01 Mar 1967
2	01 Mar 1968	01 Jun 1969
3	01 Jan 1989	01 Apr 1990
4	01 Jan 2000	01 Apr 2001
5	01 Oct 1956	01 Jan 1958
Current	01 Mar 2006	01 Jun 2007

Figure 4.14: 5 Historical periods similar to March 2006 - June 2007 using symmetric step-pattern DTW



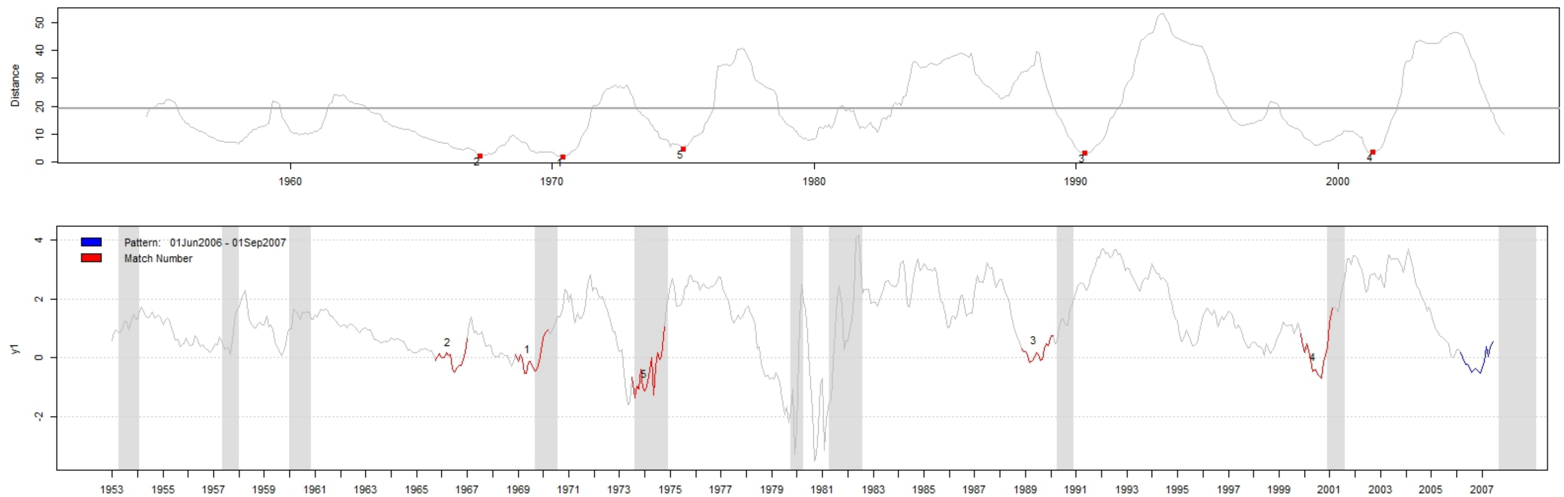
Match Number	Start	End
1	01 Dec 1965	01 Mar 1967
2	01 Dec 1968	01 Mar 1970
3	01 Dec 1999	01 Mar 2001
4	01 Nov 1988	01 Feb 1990
5	01 Apr 1973	01 Jul 1974
Current	01 Mar 2006	01 Jun 2007

Figure 4.15: 5 Historical periods similar to March 2006 - June 2007 using asymmetric step-pattern DTW



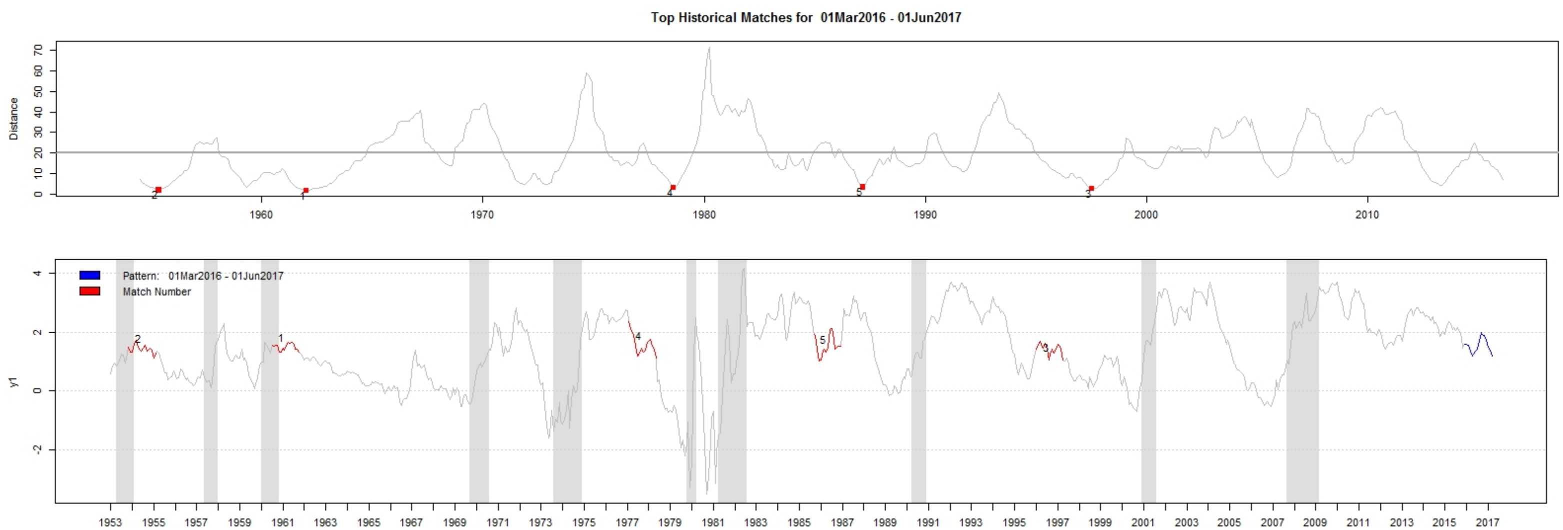
Match Number	Start	End
1	01 Jan 1966	01 Apr 1967
2	01 Feb 1969	01 May 1970
3	01 Mar 1989	01 Jun 1990
4	01 Jan 2000	01 Apr 2001
5	01 Oct 1956	01 Jan 1958
Current	01 Jun 2006	01 Sep 2007

Figure 4.16: 5 Historical periods similar to March 2006 - June 2007 using symmetric step-pattern DTW



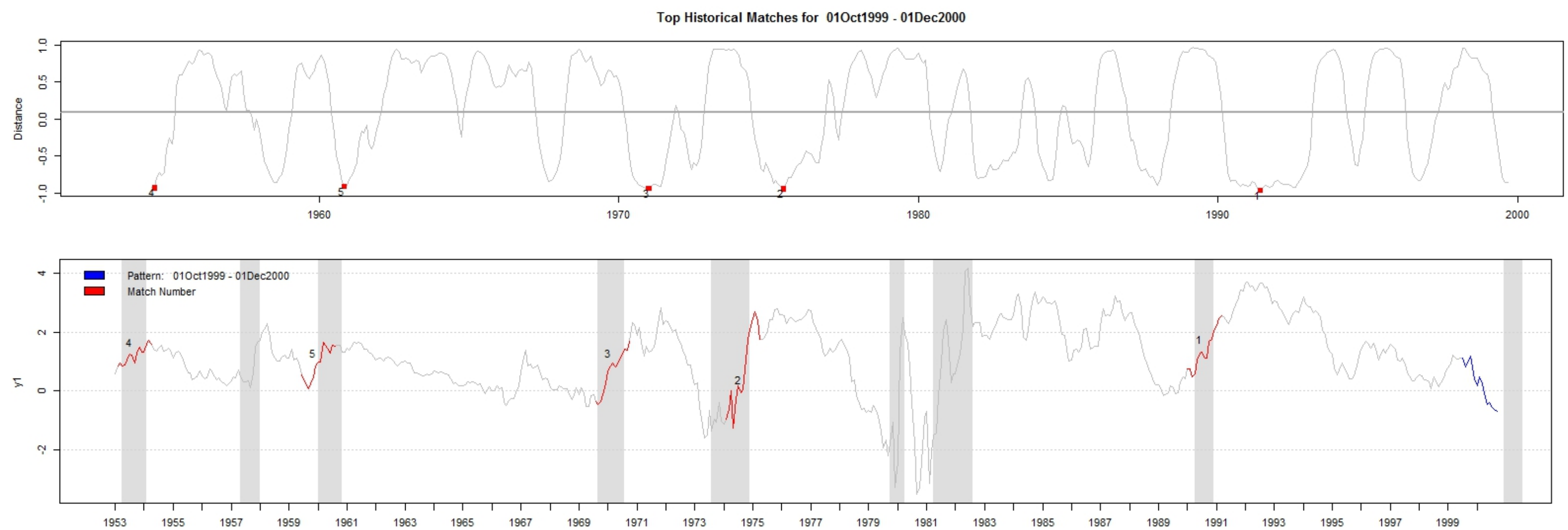
Match Number	Start	End
1	01 Mar 1969	01 Jun 1970
2	01 Jan 1966	01 Apr 1967
3	01 Feb 1989	01 May 1990
4	01 Feb 2000	01 May 2001
5	01 Oct 1973	01 Jan 1975
Current	01 Jun 2006	01 Sep 2007

Figure 4.17: 5 Historical periods similar to March 2006 - June 2007 using asymmetric step-pattern DTW



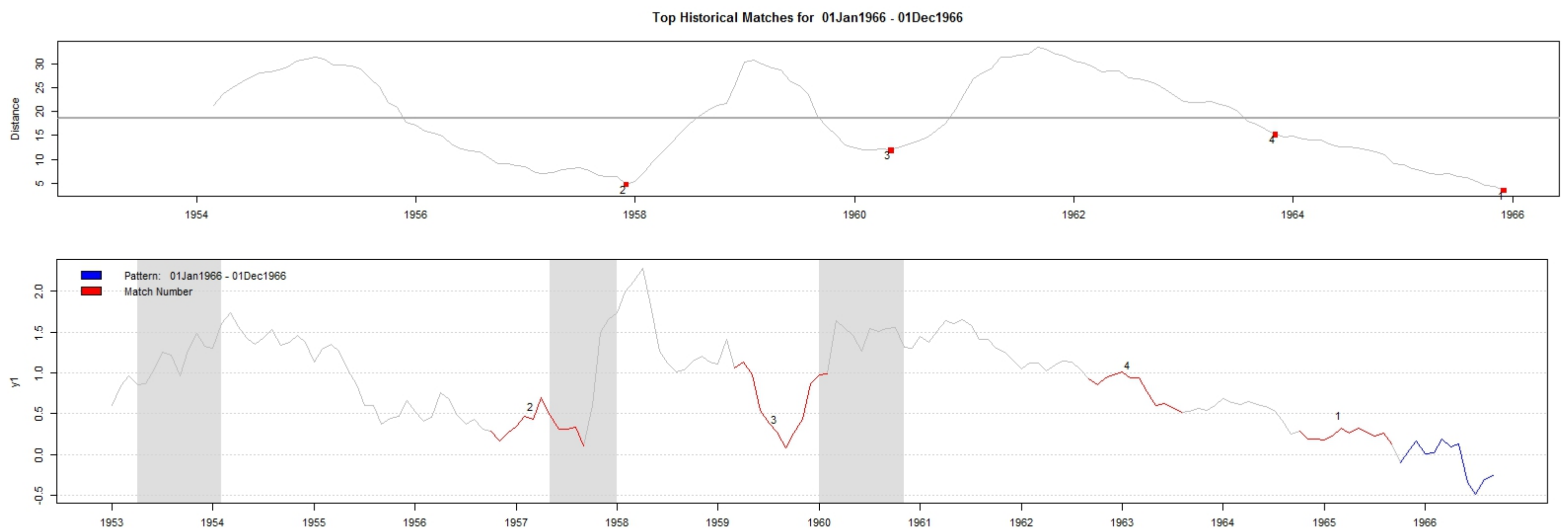
Match Number	Start	End
1	01 Oct 1960	01 Jan 1962
2	01 Feb 1954	01 May 1955
3	01 Apr 1996	01 Jul 1997
4	01 May 1977	01 Aug 1978
5	01 Dec 1985	01 Mar 1987
Query	01 Mar 2016	01 Jun 2017

Figure 4.18: 5 Historical periods similar to March 2016 - June 2017 using symmetric step-pattern DTW



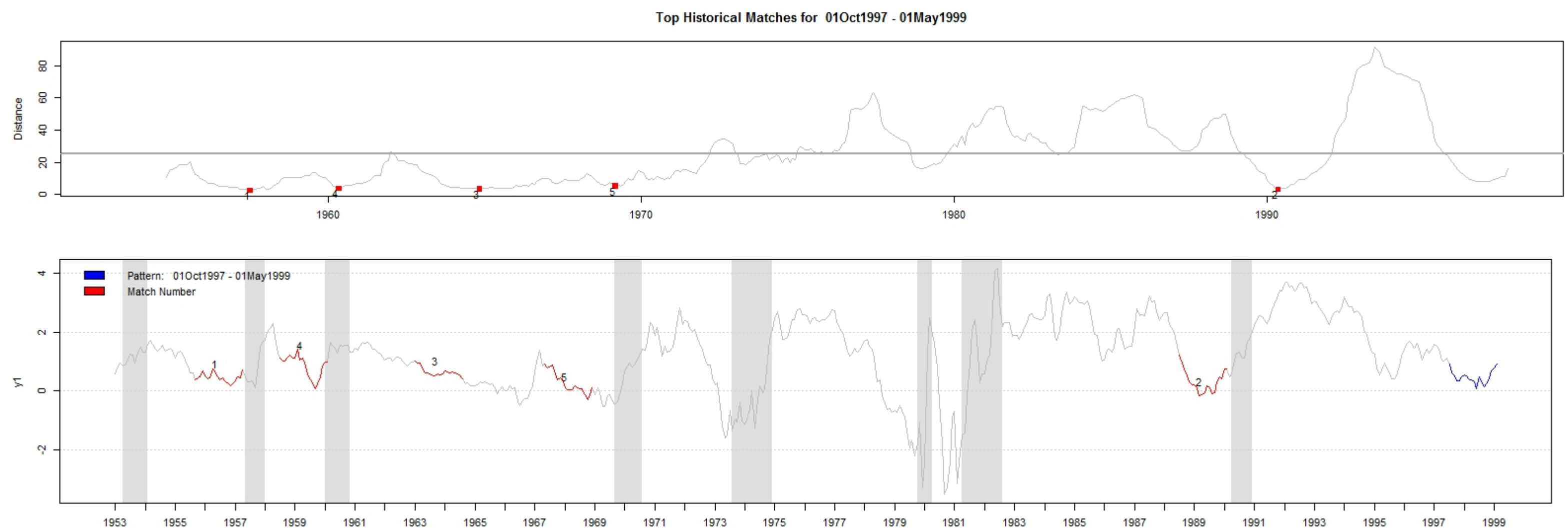
Match Number	Start	End
1	01 Apr 1990	01 Jun 1991
2	01 May 1974	01 Jul 1975
3	01 Nov 1969	01 Jan 1971
4	01 May 1953	01 Jul 1954
5	01 Sep 1959	01 Nov 1960
Query	01 Oct 1999	01 Dec 2000

Figure 4.19: 5 Historical periods similar to October 1999 - December 2000 obtained using Pearson's correlation coefficient



Match Number	Start	End
1	01 Jan 1965	01 Dec 1965
2	01 Jan 1957	01 Dec 1957
3	01 Jun 1959	01 May 1960
4	01 Dec 1962	01 Nov 1963
query	01 Jan 1966	01 Dec 1966

Figure 4.20: 4 Historical periods similar to Jan 1966 - December 1966 obtained using symmetric step-pattern DTW



Match Number	Start	End
1	01 Dec 1955	01 Jul 1957
2	01 Oct 1988	01 May 1990
3	01 Apr 1963	01 Nov 1964
4	01 Oct 1958	01 May 1960
5	01 Aug 1967	01 Mar 1969
Current	01 Oct 1997	01 May 1999

Figure 4.21: 4 Historical periods similar to October 1997 - May 1999 obtained using symmetric step-pattern DTW

Bibliography

Ahrens, R. (2002). Predicting recessions with interest rate spreads: a multicountry regime-switching analysis. *Journal of International Money and Finance*, 21(4):519–537.

Alexeev, M. and Conrad, R. (2011). The natural resource curse and economic transition. *Economic Systems*, 35(4):445–461.

Andersen, T. G. and Bollerslev, T. (1998). Answering the Skeptics: Yes, Standard Volatility Models do Provide Accurate Forecasts. *International Economic Review*, 39(4):885–905.

Apergis, N. and Payne, J. E. (2014). The oil curse, institutional quality, and growth in MENA countries: Evidence from time-varying cointegration. *Energy Economics*, 46:1–9.

Atkeson, A. and Ohanian, L. E. (2000). Are Phillips Curves Useful for Forecasting Inflation? *Federal Reserve Bank of Minneapolis Quarterly Review*, 25(1):2–11.

Atkinson, G. and Hamilton, K. (2003). Savings, Growth and the Resource Curse Hypothesis. *World Development*, 31(11):1793–1807.

Auty, R. (2002). *Sustaining development in mineral economies: the resource curse thesis*. Routledge.

Auty, R. and Warhurst, A. (1993). Sustainable development in mineral exporting economies. *Resources Policy*, 19(1):14–29.

Badeeb, R. A., Lean, H. H., and Clark, J. (2017). The evolution of the natural resource curse thesis: A critical literature survey. *Resources Policy*, 51:123–134.

Barnea, A., Dotan, A., and Lakonishok, J. (1979). The Effect of Price Level Uncertainty on the Determination of Nominal Interest Rates: Some Empirical Evidence. *Southern Economic Journal*, 46(2):609–614.

Barron, F. (1992). Selecting a best multiattribute alternative with partial information about attribute weights. *Acta Psychologica*, 80(1-3):91–103.

Barron, F. H. and Barrett, B. E. (1996). Decision Quality Using Ranked Attribute Weights. *Management Science*, 42(11):1515–1523.

Baxter, M. and King, R. G. (1993). Fiscal Policy in General Equilibrium.

Begum, N., Ulanova, L., Wang, J., and Keogh, E. (2015). Accelerating Dynamic Time Warping Clustering with a Novel Admissible Pruning Strategy. In *Proceedings of the 21th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, KDD '15, pages 49–58, New York, NY, USA. ACM.

Berge, T. J. (2015). Predicting Recessions with Leading Indicators: Model Averaging and Selection over the Business Cycle. *Journal of Forecasting*, 34(6):455–471.

Bhattacharyya, S. and Hodler, R. (2010). Natural resources, democracy and corruption. *European Economic Review*, 54(4):608–621.

Blanchard, O. and Perotti, R. (2002). An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output.

Blanchard, O. and Simon, J. (2001). The Long and Large Decline in U.S. Output Volatility.

Blinder, A. S. (2000). Central-Bank Credibility: Why Do We Care? How Do We Build It? *American Economic Review*, 90(5):1421–1431.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3):307–327.

Bollerslev, T., Engle, R. F., and Nelson, D. B. (1994). Chapter 49 Arch models. volume 4 of *Handbook of Econometrics*, pages 2959–3038. Elsevier.

Boyce, J. R. and Herbert Emery, J. (2011). Is a negative correlation between resource abundance and growth sufficient evidence that there is a resource curse? *Resources Policy*, 36(1):1–13.

Bredin, D. (2006). Inflation , inflation uncertainty , and Markov regime switching heteroskedasticity : Evidence from European countries. *Russell The Journal Of The Bertrand Russell Archives*, pages 0–22.

Bredin, D. and Fountas, S. (2005). Macroeconomic Uncertainty and Macroeconomic Performance: Are they related ? *The Manchester School*, 73:58–76.

Brunnschweiler, C. N. (2008). Cursing the Blessings? Natural Resource Abundance, Institutions, and Economic Growth. *World Development*, 36(3):399–419.

Brunnschweiler, C. N. and Bulte, E. H. (2008). The resource curse revisited and revised: A tale of paradoxes and red herrings. *Journal of Environmental Economics and Management*, 55(3):248–264.

Cai, J. (1994). A Markov Model of Switching-Regime ARCH. *Journal of Business & Economic Statistics*, 12(3):309–316.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.

Caporale, G. M., Onorante, L., and Paesani, P. (2010a). Inflation and inflation uncertainty in the euro area.

Caporale, M., Onorante, L., and Paesani, P. (2010b). Inflation and Inflation Uncertainty in the Euro Area.

Carey, M., Post, M., and Sharpe, S. A. (1998). Does Corporate Lending by Banks and Finance Companies Differ? Evidence on Specialization in Private Debt Contracting. *The Journal of Finance*, 53(3):845–878.

Cavalcanti, T. V. d. V., Mohaddes, K., and Raissi, M. (2011). Growth, development and natural resources: New evidence using a heterogeneous panel analysis. *The Quarterly Review of Economics and Finance*, 51(4):305–318.

Chang, K.-L. and He, C.-W. (2010). Does the Magnitude of the Effect of Inflation Uncertainty on Output Growth Depend on the Level of Inflation? *The Manchester School*, 78(2):126–148.

Chauvet, M. and Potter, S. (2005). Forecasting recessions using the yield curve. *Journal of Forecasting*, 24(2):77–103.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113(1):1–45.

Collier, P. and Hoeffler, A. (2004). Greed and grievance in civil war. *Oxford Economic Papers*, 56(4):563–595.

Corden, W. M. (1984). Booming Sector and Dutch Disease Economics: Survey and Consolidation. *Oxford Economic Papers*, 36(3):359–380.

Costa Junior, C. J. (2016). *Understanding DSGE models: Theory and Applications*. Vernon Press, Delaware.

Ding, N. and Field, B. C. (2005). Natural Resource Abundance and Economic Growths. *Land Economics*, 81(4):496–502.

Dotsey, M. (1998). The Predictive Content of the Interest Rate Term Spread for Future Economic Growth. *FRB Richmond Economic Quarterly*, 84(3):31–51.

Dotsey, M. and Sarte, P. D. (2000). Inflation uncertainty and growth in a cash-in-advance economy. *Journal of Monetary Economics*, 45(3):631–655.

Dueker, M. (1997a). Markov Switching in GARCH Processes and Mean- Reverting Stock-Market Volatility. *Journal of business & Economics Statistics*, 15(1):26–34.

Dueker, M. J. (1997b). Strengthening the Case for the Yield Curve as a Predictor of US Recessions. *Federal Reserve Bank of St. Louis Review*, (Mar):41–51.

Dufour, J.-M. and Luger, R. (2017). Identification-robust moment-based tests for Markov switching in autoregressive models. *Econometric Reviews*, 36(6-9):713–727.

Durlauf, Steven, N., Johnson, P. A., and Temple, J. R. (2005). Chapter 8 Growth Econometrics. *Handbook of Economic Growth*, 1:555–677.

Easterly, W. (2001). The Lost Decades: Developing Countries' Stagnation in Spite of Policy Reform 19801998. *Journal of Economic Growth*, 6(2):135–157.

Edwards, W. and Barron, F. H. (1994). SMARTS and SMARTER: Improved simple methods for multiattribute utility measurement. *Organizational behavior and human decision processes*, 60(3):306–325.

Eichengreen, B. (1992). *Golden Fetters: The Gold Standard and the Great Depression, 1919-1939*. Oxford University Press, Oxford.

Eisenstat, E. and Strachan, R. W. (2016). Modelling Inflation Volatility. *Journal of Applied Econometrics*, 31(5):805–820.

Engle, R. F. . (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation Author (s): Robert F . Engle Published by : The Econometric Society Stable URL : http://www.jstor.org/stable/1912773 REFERENCES Linked references ar. *Econometrica*, 50(4):987–1007.

Engle, R. F. (1983). Estimates of the Variance of U . S . Inflation Based upon the ARCH Model Author (s): Robert F . Engle Source : Journal of Money , Credit and Banking , Vol . 15 , No . 3 (Aug ., 1983), pp . 286-301 Published by : Blackwell Publishing Stable URL : http:. *Journal of Money , Credit and Banking*, 15(3):286–301.

Engle, R. F. and Bollerslev, T. (1986). Modelling the persistence of conditional variances. *Econometric Reviews*, 5(1):1–50.

Engle, R. F. and Patton, A. J. (2007). What good is a volatility model? *Forecasting Volatility in the Financial Markets*, 1:47–63.

Estrella, A. and Hardouvelis, G. A. (1991). The Term Structure as a Predictor of Real Economic Activity. *The Journal of Finance*, 46(2):555–576.

Estrella, A. and Mishkin, F. S. (1998). Predicting U.S. Recessions: Financial Variables as Leading Indicators. *The Review of Economics and Statistics*, 80(1):45–61.

Estrella, A., Rodrigues, A. P., and Schich, S. (2003). How Stable is the Predictive Power of the Yield Curve? Evidence from Germany and the United States. *Review of Economics and Statistics*, 85(3):629–644.

Estrella, A. and Trubin, M. (2006). The yield curve as a leading indicator: Some practical issues.

Evans, M. (1991). Discovering the Link Between Inflation Rates and Inflation Uncertainty. *Journal of Money, Credit and Banking*, 23(2):169–184.

Evans, M. and Wachtel, P. (2008). Inflation Regimes and the Sources of Inflation Uncertainty Author (s): Martin Evans and Paul Wachtel Source : Journal of Money , Credit and Banking , Vol . 25 , No . 3 , Part 2 : Inflation Uncertainty : A Conference Sponsored by the Federal Bank of Clev. 25(3):475–511.

Filardo, A. J. (2004). The 2001 US recession: what did recession prediction models tell us?

Fischer, S. (1981). Towards an understanding of the costs of inflation: II. *Carnegie-Rochester Conference Series on Public Policy*, 15:5–41.

Fischer, S. and Modigliani, F. (1978). Towards an understanding of the real effects and costs of inflation. *Weltwirtschaftliches Archiv*, 114(4):810–833.

Fountas, S. and Karanasos, M. (2007). Inflation, output growth, and nominal and real uncertainty: Empirical evidence for the G7. *Journal of International Money and Finance*, 26(2):229–250.

Fountas, S., Karanasos, M., and Kim, J. (2006). Inflation Uncertainty, Output Growth Uncertainty and Macroeconomic Performance. *Oxford Bulletin of Economics and Statistics*, 68(3):319–343.

Frankel, J. A. (2010). The natural resource curse: a survey. Technical report, National Bureau of Economic Research.

Frankel, J. A. and Romer, D. (1999). Does Trade Cause Growth?

Friedman, M. (1977). Nobel Lecture: Inflation and Unemployment. *Journal of Political Economy*, 85(3):451–472.

Galí, J., López-Salido, J. D., and Vallés, J. (2007). Understanding the Effects of Government Spending on Consumption. *Journal of the European Economic Association*, 5(1):227–270.

Gelb, A. H. (1988). *Oil windfalls: Blessing or curse?* Oxford university press.

Gerlach, S. and Bernard, H. (1998). Does the Term Structure Predict Recessions? The International Evidence. *SSRN Electronic Journal*.

Giordani, P. and Söderlind, P. (2003). Inflation forecast uncertainty. *European Economic Review*, 47(6):1037–1059.

Giorgino, T. (2009). Computing and visualizing dynamic time warping alignments in R: the dtw package. *Journal of statistical Software*, 31(7):1–24.

Gogas, P., Papadimitriou, T., Matthaiou, M., and Chrysanthidou, E. (2015). Yield Curve and Recession Forecasting in a Machine Learning Framework. *Computational Economics*, 45(4):635–645.

Gordon, R. J. (2007). Phillips Curve specification and the decline in US output and inflation volatility. In *Symposium on the Phillips Curve and the Natural Rate of Unemployment, Institut fur Weltwirtschaft, Kiel Germany, June*, volume 4.

Gray, S. F. (1996). Modeling the conditional distribution of interest rates as a regime-switching process. *Journal of Financial Economics*, 42(1):27–62.

Grier, K. B., Henry, Ó. T., Olekalns, N., and Shields, K. (2004). The asymmetric effects of uncertainty on inflation and output growth. *Journal of Applied Econometrics*, 19(5):551–565.

Grier, K. B. and Perry, M. J. (1998). On inflation and inflation uncertainty in the G7 countries. *Journal of International Money and Finance*, 17(4):671–689.

Guo, J.-T. and Lansing, K. J. (1998). Indeterminacy and Stabilization Policy. *Journal of Economic Theory*, 82(2):481–490.

Gylfason, T. (2001). Natural resources, education, and economic development. *European Economic Review*, 45(4-6):847–859.

Gylfason, T., Herbertsson, T. T., and Zoega, G. (1999). A MIXED BLESSING: Natural Resources and Economic Growth. *Macroeconomic Dynamics*, 3(2):204–225.

Gylfason, T. and Zoega, G. (2006). Natural Resources and Economic Growth: The Role of Investment. *The World Economy*, 29(8):1091–1115.

Hafedh, B. and Nooman, R. (2007). Why does private consumption rise after a government spending shock? *Canadian Journal of Economics/Revue canadienne d'économique*, 40(3):954–979.

Hamilton, J. (1988a). Rational-Expectations Econometric Analysis of Changes in Regime. *Journal of Economic Dynamics and Control*, 12(2-3):385–423.

Hamilton, J. D. (1988b). Rational-expectations econometric analysis of changes in regime. *Journal of Economic Dynamics and Control*, 12(2-3):385–423.

Hamilton, J. D. (1989). A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica*, 57(2):357–384.

Hamilton, J. D. (2010). A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle Author (s): James D . Hamilton Published by : The Econometric Society Stable URL : http://www.jstor.org/stable/1912559. *Society*, 57(2):357–384.

Hamilton, J. D. (2011). Calling recessions in real time. *International Journal of Forecasting*, 27(4):1006–1026.

Hamilton, J. D. and Susmel, R. (1994). Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics*, 64(1):307–333.

Hansen, B. E. (1992). The likelihood ratio test under nonstandard conditions: Testing the markov switching model of gnp. *Journal of Applied Econometrics*, 7(S1):S61–S82.

Holland, A. S. (1995). Inflation and Uncertainty: Tests for Temporal Ordering. *Journal of Money, Credit and Banking*, 27(3):827–837.

Kaminsky, G. L. and Pereira, A. (1996). The debt crisis: lessons of the 1980s for the 1990s. *Journal of Development Economics*, 50(1):1–24.

Kangning, X. and Jian, W. (2006). An Empirical Study of A Linkage Between Natural Resource Abundance and Economic Development [J]. *Economic Research Journal*, 1:78–89.

Kauppi, H. and Saikkonen, P. (2008). Predicting U.S. Recessions with Dynamic Binary Response Models. *The Review of Economics and Statistics*, 90(4):777–791.

Kim, C.-J. (1994). Dynamic linear models with Markov-switching. *Journal of Econometrics*, 60(1-2):1–22.

Kim, D.-H. and Lin, S.-C. (2017). Natural Resources and Economic Development: New Panel Evidence. *Environmental and Resource Economics*, 66(2):363–391.

King, T. B., Levin, A. T., and Perli, R. (2007). Financial Market Perceptions of Recession Risk.

Klaassen, F. (2002). EMPIRICAL Improving GARCH volatility forecasts with regime-switching GARCH. *Empirical Economics*, (27):363–394.

Kontonikas, A. (2004). Inflation and inflation uncertainty in the United Kingdom, evidence from GARCH modelling. *Economic Modelling*, 21(3):525–543.

Lahiri, K. and Sheng, X. (2010). Measuring forecast uncertainty by disagreement: The missing link. *Journal of Applied Econometrics*, 25(4):514–538.

Lamoureux, C. G. and Lastrapes, W. D. (1990). Persistence in Variance, Structural Change, and the GARCH Model. *Journal of Business & Economic Statistics*, 8(2):225–234.

Lindsey, D. E., Orphanides, A., and Rasche, R. H. (2013). The Reform of October 1979: How It Happened and Why. *Federal Reserve Bank of St. Louis Review*, pages 487–542.

Liu, W. and Moench, E. (2016). What predicts US recessions? *International Journal of Forecasting*, 32(4):1138–1150.

Mankiw, N. G. (2000). The SaversSpenders Theory of Fiscal Policy. *American Economic Review*, 90(2):120–125.

Manzano, O. and Rigobon, R. (2001). Resource curse or debt overhang? Technical report, National bureau of economic research.

Marcucci, J. (2005). Forecasting Stock Market Volatility with Regime-Switching GARCH models. *Studies in Nonlinear Dynamics & Econometrics*, 9(4).

Mattesini, F. and Rossi, L. (2012). Monetary Policy and Automatic Stabilizers: The Role of Progressive Taxation. *Journal of Money, Credit and Banking*, 44(5):825–862.

Mehlum, H., Moene, K., and Torvik, R. (2006). Institutions and the Resource Curse*. *The Economic Journal*, 116(508):1–20.

Melvin, M. (1982). Expected Inflation, Taxation, and Interest Rates: The Delusion of Fiscal Illusion. *The American Economic Review*, 72(4):841–845.

Mistry, P. S. (1991). African Debt Revisited: Procrastination or Progress? *African Development Review*, 3(2):99–148.

Moneta, F. (2005). Does the Yield Spread Predict Recessions in the Euro Area?*. *International Finance*, 8(2):263–301.

Mountford, A. and Uhlig, H. (2009). What are the effects of fiscal policy shocks? *Journal of Applied Econometrics*, 24(6):960–992.

Müller, M. (2007). *Information retrieval for music and motion*, volume 2. Springer.

Nabli, M. M. K. and Arezki, M. R. (2012). *Natural resources, volatility, and inclusive growth: Perspectives from the Middle East and North Africa.* Number 12-111. International Monetary Fund.

Nas, T. and Perry, M. (2000). Inflation, inflation uncertainty, and monetary policy in Turkey: 19601998. *Contemporary Economic Policy*, 18(2):170–180.

Nickell, S. (1981). Biases in Dynamic Models with Fixed Effects. *Econometrica*, 49(6):1417.

Nyberg, H. (2010). Dynamic probit models and financial variables in recession forecasting. *Journal of Forecasting*, 29(1-2):215–230.

Pagan, A. R. and Schwert, G. (1990). Alternative models for conditional stock volatility. *Journal of Econometrics*, 45(1-2):267–290.

Pesaran, M. H. and Timmermann, A. (1992). A Simple Nonparametric Test of Predictive Performance. *Journal of Business & Economic Statistics*, 10(4):461–465.

Pindyck, R. (1991). Irreversibility, uncertainty, and investment. *Journal of Economic Literature*, 29:1110–1148.

Qi, M. (2001). Predicting US recessions with leading indicators via neural network models. *International Journal of Forecasting*, 17(3):383–401.

Rabiner, L. R. and Juang, B. H. (1993). *Fundamentals of speech recognition.* PTR Prentice Hall, united sta edition.

Rosenberg, J. V. and Maurer, S. (2008). Signal or Noise? Implications of the Term Premium for Recession Forecasting.

Ross, M. (2006). A Closer Look at Oil, Diamonds, and Civil War. *Annual Review of Political Science*, 9(1):265–300.

Roszkowska, E. (2013). Rank Ordering Criteria Weighting Methods-a Comparative Overview. *Optimum: studia ekonomiczne*, (5 (65)):14–33.

Rother, P. (2004). Fiscal policy and inflation volatility. *ECB Working Paper No. 317*.

Sachs, J. D. (1989). *The Debt Overhang of Developing Countries*.

Sachs, J. D. and Warner, A. M. (1995). Natural resource abundance and economic growth. Technical report, National Bureau of Economic Research.

Sachs, J. D. and Warner, A. M. (1997). Sources of Slow Growth in African Economies1. *Journal of African Economies*, 6(3):335–376.

Sachs, J. D. and Warner, A. M. (1999). The big push, natural resource booms and growth. *Journal of Development Economics*, 59(1):43–76.

Sachs, J. D. and Warner, A. M. (2001). The curse of natural resources. *European Economic Review*, 45(4-6):827–838.

Sakoe, H. and Chiba, S. (1978). Dynamic programming algorithm optimization for spoken word recognition. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 26(1):43–49.

Schmitt-Grohé, S. and Uribe, M. (2004). Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control*, 28(4):755–775.

Sensier, M., Artis, M., Osborn, D. R., and Birchenhall, C. (2004). Domestic and international influences on business cycle regimes in Europe. *International Journal of Forecasting*, 20(2):343–357.

Sensier, M. and van Dijk, D. (2004). Testing for Volatility Changes in U.S. Macroeconomic Time Series. *The Review of Economics and Statistics*, 86(3):833–839.

Smets, F. and Wouters, R. (2003). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, 1(5):1123–1175.

Smith, B. (2015). The resource curse exorcised: Evidence from a panel of countries. *Journal of Development Economics*, 116:57–73.

Stähler, N. and Thomas, C. (2012). FiMod A DSGE model for fiscal policy simulations. *Economic Modelling*, 29(2):239–261.

Stillwell, W. G., Seaver, D. A., and Edwards, W. (1981). A comparison of weight approximation techniques in multiattribute utility decision making. *Organizational Behavior and Human Performance*, 28(1):62–77.

Stock, J. H. and Watson, M. W. (1993). A procedure for predicting recessions with leading indicators: econometric issues and recent experience. In *Business cycles, indicators and forecasting*, pages 95–156. University of Chicago Press.

Stock, J. H. and Watson, M. W. (1999). Forecasting Inflation. *Journal of Monetary Economics*, 44(2):293–335.

Stock, J. H. and Watson, M. W. (2002). Has the Business Cycle Changed and Why? *NBER Macroeconomics Annual*, 17:159–218.

Stock, J. H. and Watson, M. W. (2007). Erratum to Why Has U.S. In ation Become Harder to Forecast?. *Banking*, 39(1):3–33.

Torvik, R. (2002). Natural resources, rent seeking and welfare. *Journal of Development Economics*, 67(2):455–470.

Varga, B. (2013). Time Varying NAIRU Estimates in Central Europe. *Working Papers, Department of Mathematical Economics and Economic Analysis, Corvinus University of Budapest*, Varga, Bal(May):1–10.

Viner, J. (1952). America's aims and the progress of underdeveloped countries. *The Progress of Underdeveloped Areas*, pages 175–202.

Wang, G.-J., Xie, C., Han, F., and Sun, B. (2012). Similarity measure and topology evolution of foreign exchange markets using dynamic time warping method: Evidence from minimal spanning tree. *Physica A: Statistical Mechanics and its Applications*, 391(16):4136–4146.

Wheelock, D. C., Wohar, M. E., and Others (2009). Can the term spread predict output growth and recessions? A survey of the literature. *Federal Reserve Bank of St. Louis Review*, 91(5 Part 1):419–440.

Wright, J. H. (2006). The Yield Curve and Predicting Recessions. *SSRN Electronic Journal*.