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To Teach by Concept or by Procedure? Making the Most of Self-Explanations

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Abstract

Self-explanation prompts (prompts that encourage students to give explicit explanations for the content they study) have emerged as a promising method for promoting learning, but questions remain about how to maximize their utility. The current experiments investigated how instruction on concepts versus procedures affected the quality of self-explanations and subsequent learning outcomes when solving math equivalence problems (e.g. 7+3+9=7+_). Second through fifth-grade children learned about these problems under conditions varying on whether they were instructed on concepts or procedures (Experiment 1) and whether or not they were prompted for self-explanation after conceptual instruction (Experiment 2). The conceptual instruction group generated higher quality explanations and showed greater gains in conceptual knowledge and similar gains in procedural knowledge when compared to the procedural instruction group. No differences were found for the self-explanation manipulation. The results suggest that, in some circumstances, conceptual instruction is more efficient than procedural instruction and can improve knowledge enough to make self-explanation unnecessary.

Keywords: psychology, education, problem solving, skill acquisition and learning, instruction and teaching.

Introduction

Prompting students to generate explicit explanations of correct material and procedures (i.e. *self-explanation*) has emerged as a potentially powerful tool for promoting learning and transfer. While generating self-explanations has been shown to facilitate learning, less is known about how these explanations interact with different types of instruction. The current experiments examined a) whether the type of instruction affects conceptual knowledge, procedural knowledge, and explanation quality, and b) whether self-explanation prompts are beneficial when combined with conceptual instruction.

In their seminal study on the self-explanation effect, Chi, Bassok, Lewis, Reimann, & Glaser (1989) found that, when studying new material, the best learners spontaneously explained the material to themselves, providing justifications for each action in a solution sequence. Subsequent studies have shown that prompting students to self-explain leads to increased learning and transfer in domains ranging from arithmetic (Rittle-Johnson, 2006) to probability calculation (Große & Renkl, 2003) to biology text comprehension (Chi, DeLeeuw, Chiu, & LaVancher, 1994). Moreover, several of these experiments have shown causal effects for self-explanation prompts (e.g. Aleven & Koedinger, 2002; Chi et al, 1994; Siegler, 1995).

Nevertheless, there are substantial differences in explanation quality among individuals, and these differences are associated with differential learning outcomes (Chi et al, 1989; Renkl, 1997). Chi et al (1989) found that the explanations of successful learners tend to exhibit certain qualities. Namely, they (1) more frequently consider the goals of operators and procedures; (2) give more principle-based explanations; and (3) less frequently show illusions of understanding. Even when prompted, however, many participants continue to generate superficial explanations that fail to promote optimal learning (Renkl 1997; Chi et al, 1994). Indeed, a careful review of the literature reveals that prompting learners to self-explain sometimes fails to improve learning at all (e.g. Conati & Vanlehn, 2000; Große & Renkl, 2003).

It is also unclear how much of the self-explanation effect is due to simply increasing time on task. A majority of prior experimental studies held the number of examples or problems studied constant, resulting in students in the self-explanation condition spending more time on task (e.g. Chi et al 1994; Rittle-Johnson, 2001; Siegler, 1995). Only one published study has found a benefit for self-explanation while controlling for time on task, and it differed from typical self-explanation studies along several dimensions (e.g. students in the self-explanation condition were explicitly instructed to reference a glossary containing conceptual information and received feedback on the explanations (Aleven & Koedinger, 2002).

Thus, while the benefits of self-explanation prompts for learning and transfer have been documented and replicated, much work remains to be done in elucidating the conditions under which they are most effective. Investigating methods to promote higher quality explanations therefore seems critical for unleashing the full potential of self-explanation as a pedagogical tool.

One such unexplored avenue is the possibility that the type of instruction preceding prompts for self-explanation may influence subsequent explanation quality. We chose to compare procedural instruction against conceptual instruction because the historical debate over the comparative merits of the two remains unresolved (see Baroody & Dowker, 2003 or Rittle-Johnson & Siegler, 1998 for a summary). Furthermore, competing accounts of the mechanisms by which self-explanations operate suggest that different types of instruction might lead to different outcomes.

Importantly, the relations between conceptual and procedural instruction and the types of mathematical knowledge they support remain unclear (Rittle-Johnson & Siegler, 1998). In the present study, we assessed learning outcomes for both procedural (i.e. action sequences for

solving problems) and conceptual knowledge (i.e. explicit or implicit understanding of the principles that govern a domain).

The current experiments investigated these relations using math equivalence problems (i.e. problems of the type $7 + 3 + 9 = 7 + \_$). Math equivalence problems pose a relatively high degree of difficulty for elementary school children (70% of 4th and 5th graders fail to solve the problems according to Alibali, 1999). These problems directly tap the understanding of mathematical equivalence, which is a fundamental concept in arithmetic and algebra (Knuth, Alibali, McNeil, Weinberg & Stephens, 2005). Because they occupy an intermediate step between simple arithmetic and more advanced algebra, these problems provide a potentially fruitful field for exploration of the relations between conceptual and procedural knowledge in mathematical thinking more generally.

Recent experiments using this task have revealed two essential findings: first, that interventions can increase both procedural and conceptual knowledge in the domain; and second, that promoting one of the two types of knowledge can cause increases in the other (Perry, 1991; Rittle-Johnson & Alibali, 1999). Still, the question of whether one type of instruction is more productive in promoting overall learning and transfer remains to be answered.

Less still is known about how these types of instruction might interact with self-explanations. Different theories on the mechanisms of the self-explanation effect suggest that different types of instruction may differentially affect self-explanation quality and volume. Chi argues that self-explanations work primarily by facilitating revision of existing domain models (Chi et al, 1994). Based on this concept centered account, we might expect for conceptual instruction to lead to higher quality self-explanation. By both directly augmenting knowledge and directing attention to conceptual structure, conceptual instruction may help prime the type of analysis that self-explanation helps to drive. Thus, through its effects on knowledge and attention, conceptual instruction may augment the benefits of self-explanation prompts.

Alternatively, a limited-resources account might predict for procedural instruction to fare better. On this view, the capacity of working memory constrains all aspects of problem solving and analysis (Sweller, Van Merrienboer & Paas, 1998). Instruction offering a robust procedure may reduce the cognitive load involved in solving problems relative to conceptual instruction that places heavy demand on working memory. Thus, procedural instruction may free up resources to reflect on why a given solution method works. Prompts to self-explain may be necessary, however to employ these newly freed cognitive resources for the type of reflection that will promote deeper explanation and understanding (Rittle-Johnson, 2001).

Experiment 1

Method

Participants

Initial participants were 121 second- through fifth-grade children from a parochial school. Children correctly solving more than half of the mathematical equivalence problems targeted for intervention at pretest were excluded from the study (36% of 2nd graders, 58% of 3rd graders, 75% of 4th and 5th graders). The final sample consisted of 40 children (22 female). Children participated in the spring semester.

Procedure

Children completed a pretest, an intervention, an immediate posttest, and a delayed posttest. The written pretest and retention test were given in their classrooms, each in 30-min sessions. The one-on-one intervention and immediate posttest were given in one session lasting approximately 45 min.

Children were randomly assigned to either the *procedural instruction* (n = 22) or the *conceptual instruction* condition (n = 18). Children in the procedural instruction condition were taught an add-subtract procedure (e.g. *First add up all the numbers on the one side and subtract the amount on the second side*). Children in the conceptual instruction condition were given an explicit definition of the equal sign, focusing on its relational properties (e.g. *the things on both sides of the equal sign are **equal or the same***). No solution procedures were discussed in the conceptual instruction session. Instruction in both conditions took approximately six minutes, was centered around five sentences, and required student responses to ensure they were attending to instruction.

The remainder of the intervention session was the same for both conditions. Intervention problems were six standard mathematical equivalence problems with a repeated addend on both sides of the equation, and they varied in the position of the blank after the equal sign (e.g. $4+9+6 = 4 + \_$ and $3 + 4 + 8 = \_ + 8$). For each of the six problems, all children solved the problem, reported how they solved the problem, and received accuracy feedback. Children were then prompted to self-explain. They were shown the answers that two children at another school had purportedly given, one correct and one incorrect. The experimenter then asked the participants both how the other children had gotten their answers and why each answer was correct or incorrect.

Assessment

On the procedural knowledge assessment, there were two learning problems, each with the same format as the intervention problems, and six transfer problems of various formats (see Table 1). An accuracy score was calculated based on the percentage of problems children solved using a correct procedure, regardless of whether they made an arithmetic error.

The items on the conceptual knowledge assessment measured children's knowledge of two key concepts of equivalence problems – the meaning of the equal sign and the structure of equations. The five items asked students to define the equal sign, to reproduce four equivalence equations from memory after a 5s delay, to recognize the correct use of the equal sign in multiple contexts, to indentify the two sides of an equation, and to state the meaning of the equal sign in the context of a specific problem. All items were adapted from Rittle-Johnson (2006). Each of the five items was worth one point; scores were converted to percentages.

Data Analysis

To control for prior knowledge differences, pretest conceptual and procedural knowledge scores as well as grade level were included in all analyses as covariates. Preliminary analyses indicated that student grade level never interacted with condition, so this interaction term was not included in the final models.

Participants' explanations of why solutions were correct and incorrect were also coded as *procedural explanations* (e.g. explicitly referencing a correct procedure with no other rationale), *conceptual explanations* (e.g. referring to making the two sides of an equation equal), and *other* (e.g. referencing incorrect procedures or vague responses).

Results & Discussion

Pretest

Children began the study with some procedural and conceptual knowledge of mathematical equivalence (see Table 1). Though children were randomly assigned to condition, there was a difference at pretest in conceptual knowledge between the conceptual and procedural instruction conditions, $t(38) = 2.18$, $p = .036$.

Intervention

A repeated measures ANCOVA was conducted on accuracy scores during the intervention. Problem type (i.e. whether the blank came before or after the equal sign) was a within-subject factor and type of instruction was a between-subject factor. The control variables noted above were included to control for prior knowledge differences. There was a main effect for type of instruction such that accuracy was higher for the procedural instruction group than the conceptual instruction group, $F(1, 34) = 4.78$, $p = .036$, $\eta^2 = .123$.

To investigate effects of explanation quality, we conducted two one-way ANCOVAs, both with condition as a between-subject factor, but with frequency of conceptual explanations and frequency of procedural explanations as the dependent variables.

Children who were given conceptual instruction provided a conceptual rational on over half of all explanations ($M = .54$, $SD = .34$), while children in the procedural instruction condition ($M = .15$, $SD = .28$)rarely did so, $F(1,39) = 15.29$, $p < .001$, $\eta^2 = .287$. Accordingly, children in the procedural instruction condition provided a procedural rationale much more frequently ($M = .53$, $SD = .33$) than those in the conceptual condition ($M = .05$, $SD = .09$), $F(1,39) = 37.81$, $p = .001$, $\eta^2 = .499$. Type of instruction clearly influenced the quality of students' self-explanations.

Posttest and retention

As shown in Table 1, there was a general improvement in procedural knowledge from pretest to posttest, with a small decrement from posttest to retention test. Across conditions, children improved on both procedural learning and transfer problems from pretest to posttest.

A series of repeated measures ANCOVAs were conducted for procedural learning, procedural transfer, and conceptual knowledge scores, respectively, with time of assessment (posttest v retention) as a within-subject factor and type of instruction as a between-subject factor. Again, the aforementioned control variables were included as covariates to control for prior knowledge differences.

For procedural learning and transfer, there was no effect of condition, $F(1, 32) = .04$, $p = .843$, $\eta^2 = .001$ and $F(1, 32) = .93$, $p = .341$, $\eta^2 = .028$, respectively. Hence, while children in the conceptual instruction condition were never given explicit exposure to a solution procedure, conceptual instruction enabled students to generate and transfer correct solution procedures.

Table 1: Knowledge Assessment Score by Type of Instruction, Experiment 1

	Conceptual Knowledge Score			Procedural Knowledge – Trained Problems A + B + C = A + _ A + B + C = _ + C			Procedural Knowledge – Transfer Problems A + B + C = D + _; A + B + C = _ + D A + B − C = A + _; A + B − C = _ + C A + _ = A + B + C; _ + C = A + B + C		
	Pre-Test	Post-Test	Retention	Pre-Test	Post-Test	Retention	Pre-Test	Post-Test	Retention
Conceptual Condition	0.55	0.84	0.74	0.25	0.88	0.79	0.35	0.85	0.79
S.D	0.21	0.15	0.2	0.34	0.32	0.38	0.38	0.29	0.32
Procedural Condition	0.43	0.5	0.52	0.07	0.93	0.74	0.2	0.76	0.57
S.D.	0.2	0.23	0.2	0.18	0.18	0.42	0.29	0.26	0.42

For conceptual knowledge, there was a main effect of condition, with children in the conceptual condition performing significantly better, $F(1,32) = 16.13$, $p < .001$, $\eta^2 = .335$. This effect was over and above the main effect of conceptual knowledge at pretest, $F(1, 31) = 5.99$, $p = .020$, $\eta^2 = .158$. Hence, conceptual instruction led to equivalent gains in procedural knowledge and superior gains in conceptual knowledge compared to procedural instruction.

To ascertain whether or not explanation quality was predictive of learning, similar separate repeated measures ANCOVAs were conducted on procedural learning, procedural transfer, and conceptual knowledge scores, with the exception that this time the frequency of conceptual explanations was included in the analysis.

The frequency of conceptual explanations was predictive of learning outcomes on all three measures: for procedural learning problems, $F(1,31) = 7.71$, $p = .009$, $\eta^2 = .199$, procedural transfer problems, $F(1, 31) = 11.72$, $p = .002$, $\eta^2 = .274$, and for conceptual knowledge $F(1,31) = 13.69$, $p = .004$, $\eta^2 = .234$. This positive relation existed even after controlling for condition and prior knowledge, suggesting that neither the similarity in our criteria for conceptual explanations to our conceptual instruction nor prior conceptual knowledge accounted for this relation.

The data from Experiment 1 indicate that, when compared to procedural instruction, conceptual instruction promotes similar gains in procedural knowledge and superior gains in conceptual knowledge. Providing conceptually oriented self-explanations was also predictive of gains in all three outcomes.

Experiment 2

Experiment 1 demonstrated that quality of explanation varied by type of instruction and predicted learning. Because all students received the self-explanation prompts, however, it was unclear what role the prompts played in promoting learning. Prior research indicates that self-explanation improves procedural transfer independently of procedural instruction (Rittle-Johnson, 2006). Experiment 2 examined whether self-explanation prompts facilitated learning in combination with conceptual instruction or if the learning could be promoted by conceptual instruction alone.

Method

Participants

Initial participants were 98 third- through fifth-grade children from a parochial school. Children correctly solving more than half of the mathematical equivalence problems targeted for intervention at pretest were excluded from the study (31% of 3rd graders, 47% of 4th and 76% of 5th graders). One additional child was dropped from the study for failing to complete the intervention. The final sample consisted of 48 children (26 female). Children participated in the fall semester.

Procedure

The procedure was identical to that of Experiment 1 with the following exceptions. All children received the conceptual instruction provided in Experiment 1. Children were then randomly assigned to either a *self-explain* (n = 23) or *no self-explain* condition (n = 25). Children in the *self-explain* condition solved the same 6 problems and received the same prompts as those in Experiment 1. Children in the no self-explain condition were not prompted to explain and solved 12 problems instead of 6, to help control for time on task. Total time spent on the intervention problems was similar across *self-explain,* (M = 12.54 min, SD = 2.89 min) and *no self-explain* conditions (M = 12.01 min, SD = 4.60 min, t(46) = .473, p = .638). All assessments, scoring methods, and coding schemes were identical to those of Experiment 1.

Results & Discussion

Pretest

Children included in the study began with little knowledge of correct procedures for solving mathematical equivalence problems at pretest. There were no differences in accuracy across the different conditions (see Table 2).

Intervention

Because students solved different numbers of problems by condition, our intervention analysis was broken into two components. First, we compared the mean accuracy of

Table 2: Knowledge Assessment Score by Explanation Condition, Experiment 2

	Conceptual Knowledge Score			Procedural Knowledge – Trained Problems $A + B + C = A + \_$ $A + B + C = \_ + C$			Procedural Knowledge – Transfer Problems $A + B + C = D + \_; A + B + C = \_ + D$ $A + B - C = A + \_; A + B - C = \_ + C$ $A + \_ = A + B + C; \_ + C = A + B + C$		
	Pre-Test	Post-Test	Retention	Pre-Test	Post-Test	Retention	Pre-Test	Post-Test	Retention
Self-Explain Condition	0.27	0.84	0.74	0.17	0.57	0.52	0.13	0.50	0.49
S.D	0.17	0.15	0.2	0.29	0.43	0.46	0.28	0.38	0.44
No Self-Explanation Condition	0.24	0.5	0.52	0.10	0.54	0.62	0.12	0.48	0.55
S.D.	0.20	0.23	0.2	0.25	0.45	0.48	0.23	0.42	0.43

student performance on the first six problems in both conditions. There was no difference in accuracy for students in the *explain*, $M = 3.04$, $SD = 2.72$, and *no explain* conditions, $M = 2.8$, $SD = 2.7$, $t(46) = .310$, $p = .758$. Next, we compared the mean accuracy of student performance in the *no explain* condition on the last six problems of the intervention with their performance on the first six problems. There was a significant difference between performance on the first six, $M = 2.8$, $SD = 2.7$, and last six problems for students in the *no explain* condition, $M = 3.68$, $SD = 2.44$, $t(24) = 4.18$, $p < .001$). Thus, the additional practice seems to have helped students in the *no explain* condition, even though their time on task was similar to students in the explain condition.

Posttest and retention

As shown in Table 2, there was general improvement in conceptual and procedural knowledge from pretest to posttest, and little change from posttest to retention test.

A series of repeated measures ANCOVAs were conducted for the procedural learning, procedural transfer, and conceptual knowledge scores, with time of assessment (posttest v retention) as a within-subject factor and self-explanation prompt (present v absent) as a between-subject factor. Again, the prior knowledge control variables noted above were included as covariates.

There was no effect of condition on procedural learning, $F(1,41) = .489$, $p = .488$, $\eta^2 = .012$, procedural transfer, $F(1,41) = .244$, $p = .624$, $\eta^2 = .006$, or conceptual knowledge, $F(1,41) = .43$, $p = .239$, $\eta^2 = .034$. Thus, self-explanation prompts did not facilitate learning.

Although there was no main effect of condition, within the self-explain condition, the frequency of conceptual explanations was predictive of outcomes for procedural learning problems, $F(1,17) = 5.91$, $p = .026$, $\eta^2 = .258$, and showed a marginal trend for procedural transfer problems, $F(1, 17) = 3.11$, $p = .096$, $\eta^2 = .155$. In contrast to Experiment 1, the frequency of conceptual explanations was not predictive of gains in conceptual knowledge $F(1, 17) = .003$, $p = .954$, $\eta^2 = .000$.

In Experiment 2 we found no effect for self-explanation prompts on procedural learning and transfer or conceptual knowledge, and a limited effect of explanation quality on performance within the self-explanation condition. These data suggest that gains for students in the conceptual condition of Experiment 1 may have been due to the type of instruction independent of the self-explanation prompt. This suggests an alternative source for the variation in explanation quality that needs to be explained.

General Discussion

In Experiment 1, compared to procedural instruction, conceptual instruction on the meaning of the equal sign promoted similar procedural knowledge and superior conceptual knowledge when all students self-explained. Notably, students in the conceptual instruction group generated high quality explanations as well as correct solution procedures. In Experiment 2, there was no effect for self-explanation between conceptually instructed groups on measures of procedural or conceptual knowledge (given that no-explanation students had additional problem-solving practice). This suggests that the comparative advantage of the conceptual instruction group from Experiment 1 might be due to type of instruction alone. Taken together, the data lend support to two important conclusions: 1) that there is an asymmetry to the relations between conceptual and procedural knowledge; and 2) that there are constraints under which self-explanations may be effective, and that conceptual instruction can push these constraints, potentially rendering self-explanation prompts unnecessary.

Past research has suggested an asymmetric relationship between procedural and conceptual knowledge, with conceptual knowledge sometimes proving more facilitative of gains in procedural knowledge than vice versa (Rittle-Johnson & Alibali, 1999; Perry, 1991; Hiebert & Wearne, 1996). The current data support this contention. Increased conceptual knowledge, via direct instruction, led to gains in procedural knowledge. In contrast, increased procedural knowledge, via direct instruction, was less effective in promoting conceptual knowledge gains. Hence, conceptual instruction may be a more efficient method for improving both conceptual and procedural knowledge.

Moreover, type of instruction may constrain the benefits of prompting for self-explanation. In Experiment 2, prompts to self-explain did not lead to additional knowledge gains compared to conceptual instruction with additional problem-solving practice. Conceptual instruction, in this case, may be sufficient to promote conceptual and procedural knowledge acquisition with sufficient time on task, even without prompts to self-explain. We propose three methods by which conceptual instruction may render explanation prompts unnecessary.

As a first mechanism, in certain circumstances, conceptual instruction may help children build sufficiently rich mental models that self-explanation is not needed. Chi et al. (1994) argue that self-explanations operate by aiding students in the repair of faulty mental models. To the extent that conceptual instruction edifies existing mental models, it may leave less room for repair, reducing the effects of subsequent self-explanation prompts.

This proposed mechanism may help reconcile Rittle-Johnson's (2006) finding independent effects for self-explanation when using procedural instruction with the null findings of the current study. As was mentioned above, several studies show that procedural instruction alone is not always highly supportive of conceptual knowledge. If we equate mental model repair with increases in conceptual knowledge, it follows that procedural instruction might be less adept than conceptual instruction at promoting the repair of mental models.

Alternatively, conceptual instruction may render prompts unnecessary because conceptual instruction encourages more spontaneous self-explanation in the absence of explicit prompting. If conceptually instructed students must reference the knowledge that they do have (conceptual

knowledge) in order to generate procedures, they may effectively have to engage in self-explanation to generate procedures in the first place.

These proposed mechanisms are quite distinct. On the first view, conceptual instruction may render explanation prompts ineffective because there is little repair work left for self-explanation to do. On the second view, there is work for self-explanation to do, but conceptual instruction motivates spontaneous self-explanations without prompting. The first pathway posits that the effect of self-explanation prompts may be constrained by prior *knowledge* conferred by a given type of instruction while the second pathway suggests that the effect may be constrained by the *selective attention patterns* encouraged by a given type of instruction.

A third, more general, mechanism might partially account for the benefits of self-explanation; self-explanation generally increases time on task. We controlled for time on task by increasing the number of practice problems in the no-explanation condition. Given that prompting for self-explanations requires more time per problem than instruction without self-explanations, we should consider the tradeoffs in terms of constraints prompting for explanation imposes on the number of practice problems experienced.

In summary, in conjunction with self-explanation, conceptual instruction was more efficient than procedural instruction because it supported learning of both mathematical procedures and concepts about equivalence. In fact, conceptual instruction seemed to replace the benefits of self-explanation prompts. Conceptual instruction may sometimes be a more efficient means for supporting deep learning in mathematics than procedural instruction or self-explanation prompts. Whether these findings generalize to other mathematical tasks and to classroom settings are important areas for future research.

Acknowledgments

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