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A symposium on Recent Advances in Geotechnical Centrifuge Modeling was held on July 18-20, 1984 at the University of California at Davis. The symposium was sponsored by the National Science Foundation's Geotechnical Engineering Program and the Center for Geotechnical Modeling at the University of California at Davis.

The symposium offered an opportunity for a meeting of the International Committee on Centrifuges of the International Society for Soil Mechanics and Foundation Engineering. The U.S. participants also met to discuss the advancement of the centrifuge modeling technique in the U.S. A request is being transmitted to the American Society of Civil Engineers to establish a subcommittee on centrifuges within the Geotechnical Engineering Division.

FULFILMENT OF BOUNDARY CONDITIONS FOR SEISMIC SIMULATION

by

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1. INTRODUCTION

Experimental simulators where gravity is correctly scaled appear as both necessary and sufficient for a systematic study of structure - foundation - soil interaction during earthquakes. The primary aim is to solve as completely as possible the engineering problems. Only thus can the large sums involved with the models by justified. Solution of academic problems will come out in parallel.

While setting out experiments according to ad hoc codes, it is recognized that one of the main products of a systematic study would be a new code for seismic resistance. The present paper deals with bringing out the questions involved and giving some examples of partial solutions.

2. THE SYSTEM

In situ the structural system has three components : superstructure, foundations, infinite soil. In the model the soil is confined within a cell, which forms a forth component. The soil in the model represents a domain D in situ, the boundary of which is ∂D. During an earthquake velocities and tractions appear on ∂D, the compounded effect of the seismic energy coming from outside and the superstructure - foundation - soil energy comming from inside. A correct simulation requires the scaled velocities and tractions on the soil adjacent to the cell's walls.
The degrees of liberty (e.g. displacements) of the superstructure are grouped into internal (d) and foundation connection ones (Δ). The only excitation of the superstructure in the case of earthquakes are the reactions of the foundations R in terms of generalized forces for Δ.
Taking for simplicity the foundations as part of the soil component, the boundary values are Δ, R at the superstructure - soil junction. The soil's degrees of freedom on the rest of the boundary are called b and the related forces F_b .

3. INTERNAL BOUNDARY CONDITIONS

As far as the soil is concerned, the presence of the superstructure is equivalent to a boundary condition : the Δ, R relation. For example, if the superstructure is viscoelastic a relation between the Fourier transforms $\hat{\Delta}$, $\hat{R}$ is as follows :

$$-\omega^2 \begin{vmatrix} M_d & 0 \\ 0 & M_\Delta \end{vmatrix} \begin{matrix} \hat{d} \\ \hat{\Delta} \end{matrix} + i\omega \begin{matrix} C_d & \underline{C} \\ \underline{C}' & C_\Delta \end{matrix} \begin{matrix} \hat{d} \\ \hat{\Delta} \end{matrix} + \begin{matrix} K_d & \underline{K} \\ \underline{K}' & K_\Delta \end{matrix} \begin{matrix} \hat{d} \\ \hat{\Delta} \end{matrix} = \begin{matrix} 0 \\ \hat{R} \end{matrix}$$

putting $\underline{c} \cong 0$:

$$\begin{matrix} -\omega^2 M_d + i\omega C_d + K_d & \underline{K} \\ \underline{K}' & -\omega^2 M_\Delta + i\omega C_\Delta + K_\Delta \end{matrix} \begin{matrix} \hat{d} \\ \hat{\Delta} \end{matrix} = \begin{matrix} 0 \\ \hat{R} \end{matrix}$$

Solving by Gauss elimination :

$$(-\underline{K}' [-\omega^2 M_d + i\omega C_d + K_d]^{-1} \underline{K} - \omega^2 M_\Delta + i\omega C_\Delta) \hat{\Delta} = \hat{R}$$

In the frequency domain of interest, any other superstructure for which the above relation $(\hat{\Delta}, \hat{R})$ holds with good approximation can replace the original one, as far as the soil is concerned. Nor does it have to be an actual structure, because using a computer and an array of actuators, than at least in principle this $(\hat{\Delta}, \hat{R})$ relation can be implemented. In fact $f = m\ddot{x}$ is an arrangement for creating a force proportional to the second derivative of the displacement. This arrangement is "mass". Mechanical models can use a Guyan type approximation or a modal one. In the Guyan case $c_d \cong 0$ and $[-\omega^2 M_d + K_d]^{-1}$ is developped in series. The first order gives $K_d + \omega^2 M_d$

so $\hat{R} = (-\underline{k}' K_d \underline{k} - \omega^2 \underline{k}' M_d \underline{k} - \omega^2 M_\Delta + i\omega C_\Delta + K_\Delta)\hat{\Delta}$

which is the transform of $M_\Delta^* \ddot{\Delta} + C_\Delta \dot{\Delta} + K_\Delta \Delta = R(t)$

where $M_\Delta^* = M_\Delta + \underline{k}' M_d \underline{k}$

and $K_\Delta^* = K_\Delta - \underline{k}' K_d \underline{k}$

In the modal approximation only some of modes are retained and corrections added, resulting in a new superstructure. (The static loads must be correct.

4. EXTERNAL BOUNDARY CONDITIONS

The conditions for the rest of soil, i.e. for b and F_b are called external. Their role in the model is to insure the correct values of Δ and R. External conditions are devided into passive and active. The passive ones do not vary with time, e.g. free surface, rigid wall. The active ones are imposed by the experimenter.

In situ, an excitation of R_i causes signals at i and the other points j as the wave travels out. Some echoes come back later on from reflexions below. In the model there exist also reflexions form the walls. These perturbations distort radiation of energy from the structure to infinity, which is the main contribution to damping. So models of structures must be small relative to the cell. Howaver, very large structures are to be considered and on the centrifuge cell dimensions and length scale reduction are limited. The performance of the seismic simulator might be greatly improved by processing the signals digitally, either in real time or later on. The necessary codes should be designed together with the actuators and the cell to give the optimal result of the processed signals.

4.1. Some numerical tools

The soil is certainly not a linear system. However, corrections can be applied in a linear way, following a time honoured procedure going back to Froud.

A system of linear differential equations has a Green's function $G(t,\tau)$ which transforms the forcing term $x(t)$ to the solution $y(t)$:

$$y(t) = \int^t G(t,\tau)\, x(\tau)\, d\tau$$

If the equation has constant coefficients $G(t,\tau) = G(t-\tau)$ and $y = G * x$ (convolution product).

A property of the integral transformation by Green's function is that the transform acquires the smoothness of G. For example even if $x(t)$ is only piece-wise continuous and $G(t,\tau)$ has second continuous derivatives, then $y(t)$ has second continuous derivatives. Thus a convolution can be used to smooth function and a good example is the shock spectrum.

Several integral transformations exist which transform a function of time to a function of frequency (and inversely) ; a function of time to a function of an integer (and inversely) ; an analytic function to a function of an integer (and inversely) ; and finally a function of an integer on another one of its kind and inversely. In all those cases the following exists : if the integral transformation is $y = F[X]$, then :

$$F[X_1 * X_2] = (F[X_1]) \cdot (F[X_2])$$

It follows that $\log F[X_1 * X_2] = \log y_1 + \log y_2$. convolution is associative, so the graph of the logarithm of the transform, or Bode's diagram, is in fact a superposition of the graphs of the different factors of the convolution. This graph is accordingly amenable to procession by linear numerical algorithms which are normally called filters. One aim is to come out with $F[G]$ if G is not known. As $F[G^{-1}] = 1/F[G]$ an inverse convolution (called deconvolution) is possible giving $X = G^{-1} * y$. A second aim might be to first separate between the superimposed log graphs in order to recombine them premultiplied by weights, i.e. to make corrections. The corrected graph is the logarithm of the corrected signal's transform, so by exponentiation and inverse integral transformation the corrected signal is obtained.

The key to the efficiency of procedures is the fast discrete Fourier transform. If signals are sampled at T intervals ($t = nT$ when $n \in 0, 1, .. N-1$) they form vectors of N components. The components can be drawn on a drum, so that $X_o = X_N$, $X_1 = X_{N+1}$, etc... to give periodic functions. In exactly the same way to discrete Fourier transform is a vector of N components, drawn on the same drum. The fast transform algorithms executes the passage from one vector to the other rapidly. Convolutions are calculated by first putting the factors on a drum and then Fourier transforming.

4.2. Echoes

According to the principle of Huygens, the echoes can be looked upon as coming from secondary sources on the boundaries. The secondary emission $y_s(t)$ is given :

$$y_s = G_s * X(t - T_s)$$

where $X(t)$ is the source and T_s is the time of travel from the source to the boundary. The signal received is given by $y_r = G_r * y_s(t - T_r)$ when T_r is the time of travel from the secondary source to the receiver. In the present case, the sources and receivers are the points of connection between the foundations and the superstructure. The relation between the digital signals $(y_r)_n$ and X_n can be written as :

$$(y_r)_n = \sum_{k=1}^{M} \alpha_k X_{n-n_k}$$

α_k are distorsion factors, n_k is the delay of the echo n° k .

Let : $$\delta_n^{n_k} = \{ 0 , n \neq n_k \; ; \; 1 , n = n_k \}$$

then : $$G_n = \sum_{k=1}^{M} \alpha_k \delta_n^{n_k}$$

and : $$(y_r)_m = \sum_n G_{m-n} X_n \equiv (G * X)_m$$

(one echo)

An example taken from [1] :

$$G_n = \delta_n^0 + \alpha \delta_n^{\ell} , |\alpha| < 1 .$$

$$F[y_r] = F[X] F[G] ; \quad F[G] \equiv \sum_{n \in -\infty}^{\infty} G_n e^{-i\omega n} = 1 + \alpha e^{-i\omega \ell}$$

and has a period of $2\pi/\ell$.

Log F[G] has the same period. Taking log F[G] as the Fourier transform of a function g, the result is $g = \sum_{k=1}^{\infty} (-1)^{k+1} \alpha^k/k \ \delta_n^{k\ell}$ which is a sequence of pulses decaying as $|\alpha|^k/K$ at intervals equal to ℓ on the time index axis ($t = \ell T$). In parallel log F[X] is taken as the Fourier transform of ξ_n (i.e. $F[\xi] = \log F[x]$). If $\xi_n \neq 0$ in a different region on the index line, than the two can be separated (for example by dilating every n = k component). Ref. [1] gives examples of successful application of this procedure in acoustics and seismology.

The experimental procedure should be to apply pulses at point i and measure respons at all the points of connection. According to the last example separation of G will be manageable if the duration of the pulse is different from any time of travel from the different walls. In relation to the model this means that echo filtering will succeed better if non of the natural frequencies of the soil and wall system is predominant in either the model or the earthquake, as could have been expected.

The qualification of the cell for echoes during centrifugation might be quite expensive. For clays, once consolidated, tests could be carried out in the laboratory. For sands, the necessary rigidification pressure can be applied by a rubber membrane on the free surface. Such tests were carried out at CESTA on a large rigid steel cell, later used for Hydraulic Gradient seismic simulation. As the echoes intensity at the middle of the cell was founded to be relatively small, no filtering procedure was undertaken. It seems however interesting to see the form of the signals which are either horizontal (A_H) or vertical (A_V) accelerations at a depth of 10 cm. The dry sand was rigidified by vacuum. The source was a dropped metallic mass, of a very short duration (fig. 1, 2, 3, taken from [2]).

4.3. Non reflecting boundaries

At a shear wave velocity of 300 ms^{-1} and frequency of 300 Hz the wave length is 1 m. In order to be effective any "break-water" system must be at least one quarter wave thick. As this layer is on the outer perimeter, it consumes much volume and weight. Tests on the cell of fig.1 showed that a 2 cm layer of elastomer did not change anything.

By supporting the walls of the soil with mud pressure accross a rubber wall, the longidutinal wave motion energy is transmitted to the mud and can be arranged to be carried away hydraulically. This dissipates the pressure waves, which due to their larger phase velocity have larger wave length at equal frequencies. The rubber wall is a free surface for the transversal motion. This boundary thus separates between the transversal and longitudinal wave motions, a situation that might be convenient for echo corrections.

Wave energy at the boundary can be absorbed by boundary motion, in the same way that a ball bounced from a receding car becomes limp. For example a plane wave travelling from the origin according to $u = f(t - \frac{x}{c})$ would not be reflected at point $X = \ell$ if u is forced to be $f(t - \frac{\ell}{c})$ at that point. The same result can be obtained by changing the impedence at $X = \ell$ in a closed loop arrangement where u is measured at, say, $X = 0$. This installation exists in the 12 m diameter 105 m long shock tunnel of Gramat which simulates nuclear air blasts with very good results [3].

Writing once again the reflected signal y_r as $y_r = \int^t G(t,\tau) X(\tau) d\tau$ The form of G must be chosen to that the directionality and coherence of the echoes is destroyed. This can be achieved by introducing a random process into G both in time and location. In this case, the ordered wave energy is transformed to noise. The relatively small level of echoes in the described cell might be attributed to such a mechanism acting in the non linear non homogeneous sand.

4.4. Seismic generators

4.4.1. The desirable motion

The amount of information about real ground motion is scant. The bed-rock signal is filtered through the upper layers and once more through buildings at and near the accelerometer. Fig. 4 from centrifuge tests shows the difference between shock spectra at the same soil point with and without the structure. One result of the filtering is the absence of correlation between the 3 motion's components.

Wherus the basic motion variable is the velocity, (which times stress gives power) for technical reason it is the acceleration which is recorded. By the process of differentiation high frequency noise is greatly enhanced, and accelerograms are very difficult to compare.

At the moment a numerical filter is used to transform the acceleration A into a velocity V by the procedure of shock spectrum analysis. This filter corresponds to the physical system of one degree of freedom viscoelastic resonator. The viscosity adds to the smoothing effect of the convolution. The logarithmic scale further subdues variations, and many earthquakes look alike. Codes give very simple curves for spectra's envelops.

The filter used for the shock spectrum had the merit in the past that it could be calculated by analog methods. This merit no longer exists. A more satisfying procedure would be to take the Fourier transform, which is also a velocity, and smooth it up. A much better procedure would be to use the velocity, and compare it numerically to some master curves, by a computer procedure fixed by the codes. In the models 2 - 3 component motion can and should be produced. The codes give no indication how to combine them. One way to do it, is to use the free field motion to excite 3 - 4 degree of motion systems corresponding to either symetric or non symetric rigid bodies on the soil, representing the soil by spring - dashpot units. In the symetric case (no twist) the vertical motion is not coupled to the horizontal and rocking one so 2 x 2 matrices describe the system, in a way that can be hand calculated. Such an integration was carried out for free field motions on centrifuge. The results are seen in figure 5.

Actual velocity curve's comparison also takes in account the number of cycles. This is a very important parameter, both for liquefaction and for transfer of energy between different modes, a process which takes time. In situ the earthquake is a wave that has a given profil in the soil near the surface and which propagates at a certain velocity. This information is not available.

In problems related to liquefaction the stresses of the free field wave are important. They are not available. At least in principle an earthquake signal can be characterized by the stress field. This possibility exists for model tests.

Once a desired free field motion is given, the problem is to sustain it by boudary excitation.

4.4.2. Seismic generators

When the same momentum p is imparted to masses m_1 and m_2 in a collision, the energies imparted are $0.5p^2/m_1$; $0.5p^2/m_2$;

the smaller is the mass the greater is its share of energy. Efficiency of power requires models to be shaken against larger masses. For a shaking table the reactive mass is in the heavy foundation. Putting the experimental cell on such a table is feasible for models using the Hydraulic Gradient simulation. For models on a centrifuge shaking of the whole cell can be carried out when it is a small one fixed on a heavier support, with a substential loss in pay-load, or as it has been done to shake the cell against the centrifuge building or the centrifuge itself.

If part of the cell's boundary is shaken, than the accompanying mass has the volume of about L^3 when L is a typical dimension of the shaken part. This seems to be the correct way. The situation most similar to real earthquake is a running wave along the bottom of the cell, and containing both horizontal and vertical components. Such an installation would have to support the weight of the model during centrifugation and would have to be watertight. Thus it is estimated to consume quite a lot of pay-load, which however seems justified. The solution adopted at CESTA was to shake one side of a longish cell.

A tailored signal is a shaking system tuned by several tests so to give required free field as measured at different points in the soil. Once all is set up signals must be repetitive.

A servomechanism is a feed back controlled system which corrects itself automatically relative to a given signal. As a wave profil is the required quantity, a situation must exist where one variable controls the whole field. This variable can be the velocity on a rigid plate (e.g. the shaking table) or the pressure on a rubber membrane. It could be a field transducer reading. In that case the cell must be sufficiently large in order to eliminate the structure's filtering effect. Also the weighed sum of signals of several transducers can be the controlling variable. All this operation can be performed either by an analog computer or a digital one. Not all fields are controllable, especially in a non linear material. A servomechanism may loose control and become destructive, a disagreable situation on the centrifuge.

A tailored signal is dependent on the presetting of several physical fixtures, and its variability is somewhat limited, and setting takes time and labour.

A servomechanism is as variable as the library of curves it follows, of which there are actually not many. Thus in many cases a closed loop system acts in fact as part of a tailored signal generator, its main function being to assure repettivity. According to the last chapter the best control variable is the free field velocity. Apart from being the correct wave parameter, the scale of which is conserved, velocity has smooth signals which reduces the tolerance requirements on the servovalves.

4.5. Tailored pneumatic generators

A pneumatic seismic generator was tested at CESTA on the cell of fig. 1. This is a rigid cell made of steel. The soil sample is 1.8 m by 0.8 m by 0.5 m depth. This cell is currently in use at the Ecole Polytechnique for Hydraulice Gradient seismic simulation. A lighter cell of aluminium (1.3 m by 0.8 m by 0.4 m) is used on the CESTA centrifuge. In the tests to be described on the steel cell the soil was dry sand, rigidified by a pressure of 0.2 MPa on a thin rubber membrane spread over the soil's free surface.

The operation principle of a pneumatic generator is very much the same at that of a car engine. Energy in large quantities is stored either in compressed gas containers or as the chemical energy of combustibles. It is released according to a program either by valves or by electronic ignition. In the case of shock tubes the valves are metal membranes covering the tube's end. Membranes are either pierced mechanically or burst by small explosive charges, a system applied with great success in the Gramat shock tunnel.

The word tuning comes from musical instruments, and shows that an arrangement of cavities and openings will give repeatedly the same signal. This is because the boundaries are quite rigid relative to the air, and the opening large and not likely to be obstructed by debris. Added to this is the precise energy release rate of detonators and explosive pellets.

According to Helmholtz's model, cavities act as capacitances (or springs) the air volumes in the openings as inductances (or masses) and the resistances to flow in tubes as electric resistanced. Up to the shock range of pressures Mach's similitude is valied, (which is the same as for the soil model) : if stresses are conserved than velocity is conserved and the time scale equals the length scale. That means that doubling all the geometrical dimensions will double all the vibration periods. When the space is limited, as was the case for the Aluminium cell for the centrifuge, additional solid masses are included in the filter in order to reduce frequencies. Thus tuning of the filter is a quite simple operation. Once tuned, the signals are very repetitive.

Apart from general precautions in dealing with volumes of highly compressed gas, there are the special regulations for explosive handling. However, many combustibles are suitable which are not defined as explosives, being unsensitive. Sensitivity particularly means the amount of electric energy needed for ignition. There is no problem in providing suitable electric "sparks".

In the cells of CESTA explosive pellets and detonators are used, pre-packed inside cavities in a steel block, the function of which is to protect one charge from the others. The cost of the system is low and it has been used for years with no maintenance at all. Using shock tubes at the Ecole Polytechnique gave similar soil signals. The preparation of the test is more complicated than with explosives.

The quality of the signal was tested by 10 accelerometers placed in the sand at depths of 5 cm, 10 cm and 20 cm. The results showed that the wave filtered through the tapered part of the cell near the source to becomes regular. The lateral and longitudinal uniformity is shown in fig. 6 (spectra are given in fig. 7-10). The vertical uniformity was confirmed by comparison of signals at different depths. The wave traverses the cell attenuated first like a pressure wave, later on like a surface wave. Fig. 6 shows horizontal and vertical accelerations of similar amplitudes. Placing the generator at lowest level possible did not give very different results.

Some softening of the sand near the source was observed, and that zone was later strengthened by gravel.

5. CONCLUSIONS

An example was given of an apparently satisfactory boundary conditions settup for seismic simulation based on a tailored signal generated by explosions.

A rethinking of the whole problem has been discussed in order to make Centrifuge and Hydraulic Gradient models the major tools the should become in solving the engineering seismic problem. Such a procedure seems the more relevant because most of the large geotechnical centrifuges have quite similar features, so the same boundary condition arrangement could be used for them all.

Remark :

The centrifugal models described were built and tested by the Centre d'Etudes Scientifiques et Techniques d'Acquitaine (CESTA) of the Commissariat à L'Energie Atomique at Le Barp near Bordeaux.

The Hydraulic Gradient and shock tube models were constructed and tested in the Ecole Polytechnique.

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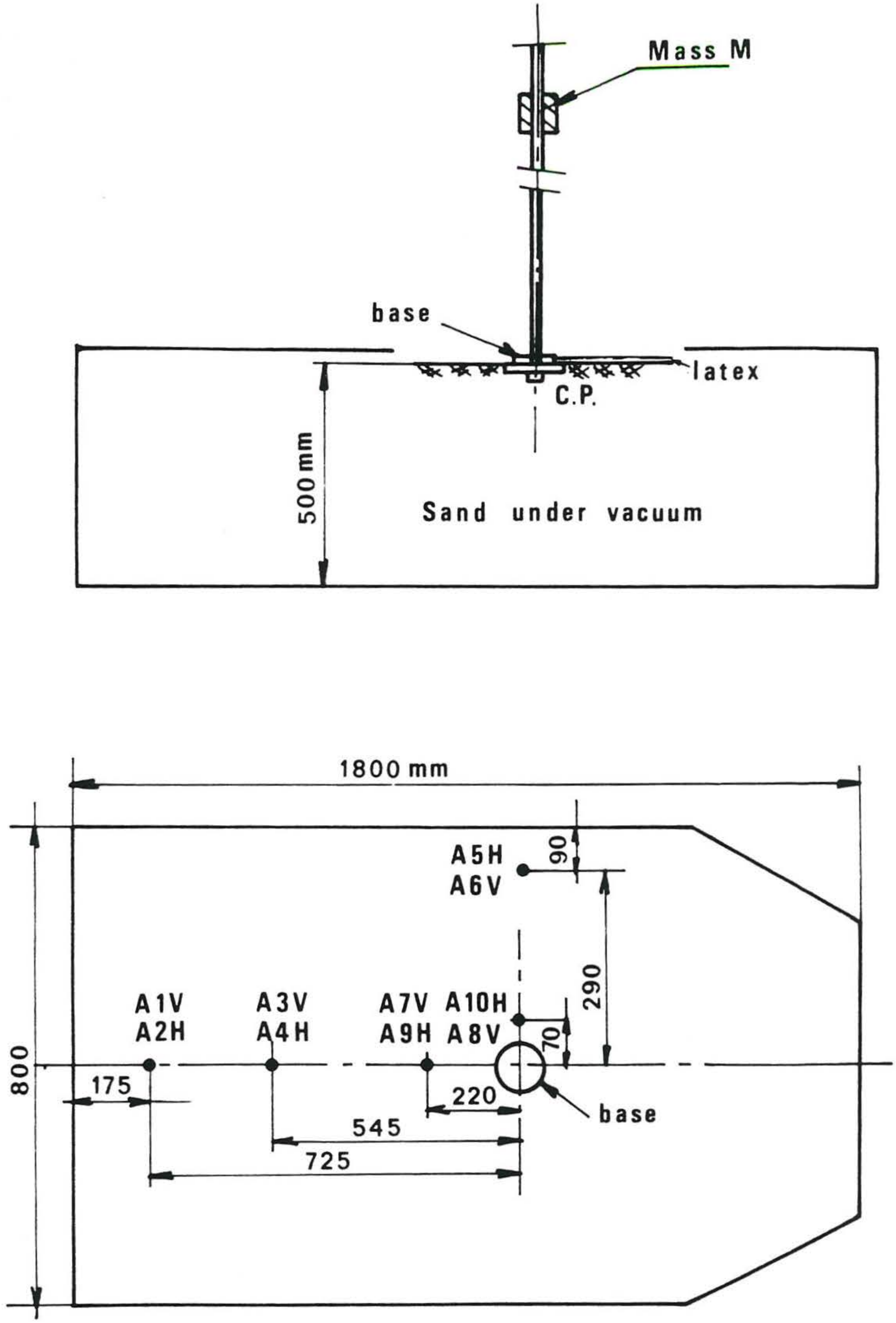


AH : acceleration - horizontal

AV : " - vertical

Fig. 1 Study of echoes by a falling mass

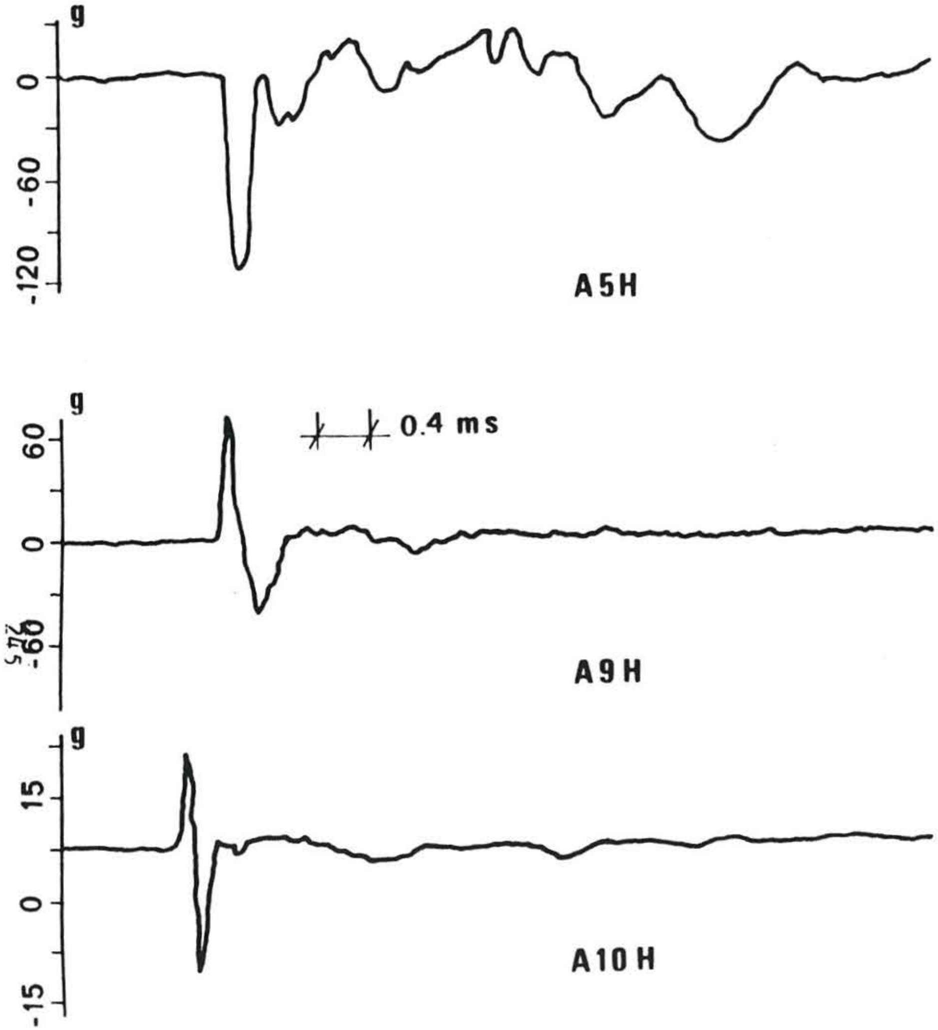


Fig. 2 Accelerations Depth 10 cm

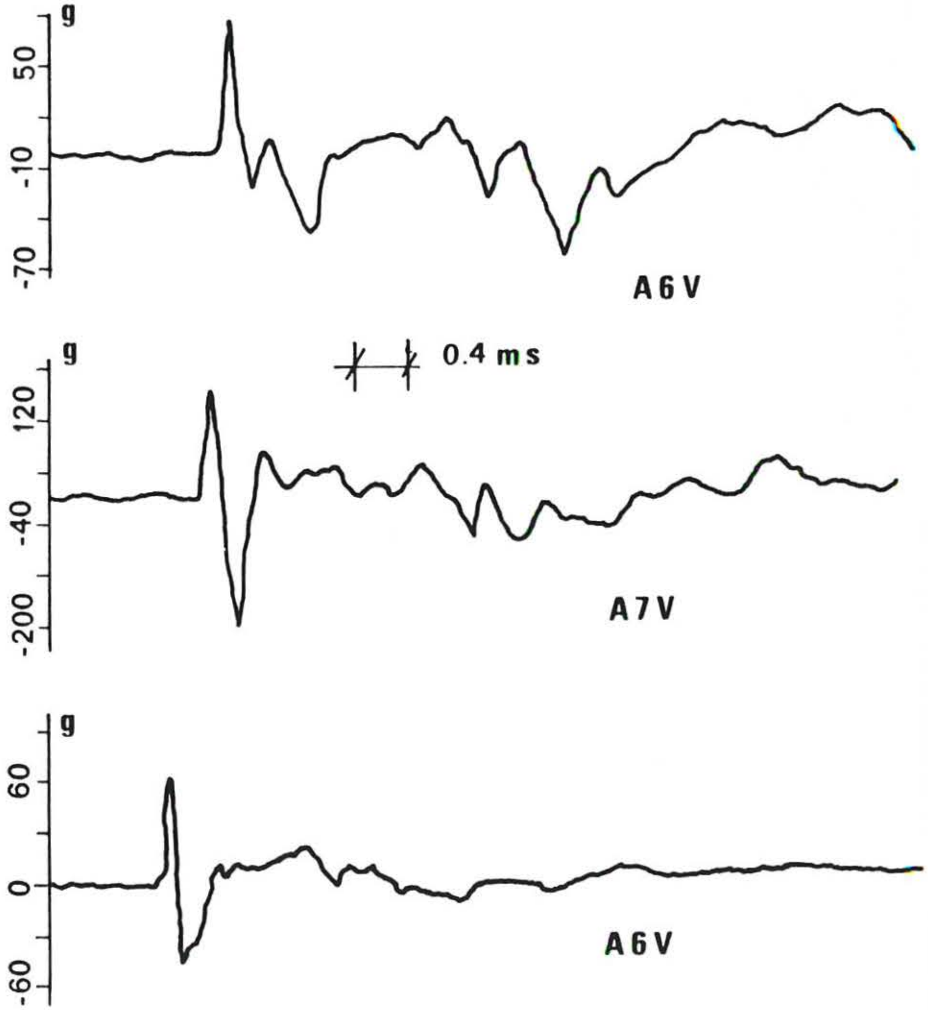


Fig. 3 Accelerations Depth 10 cm

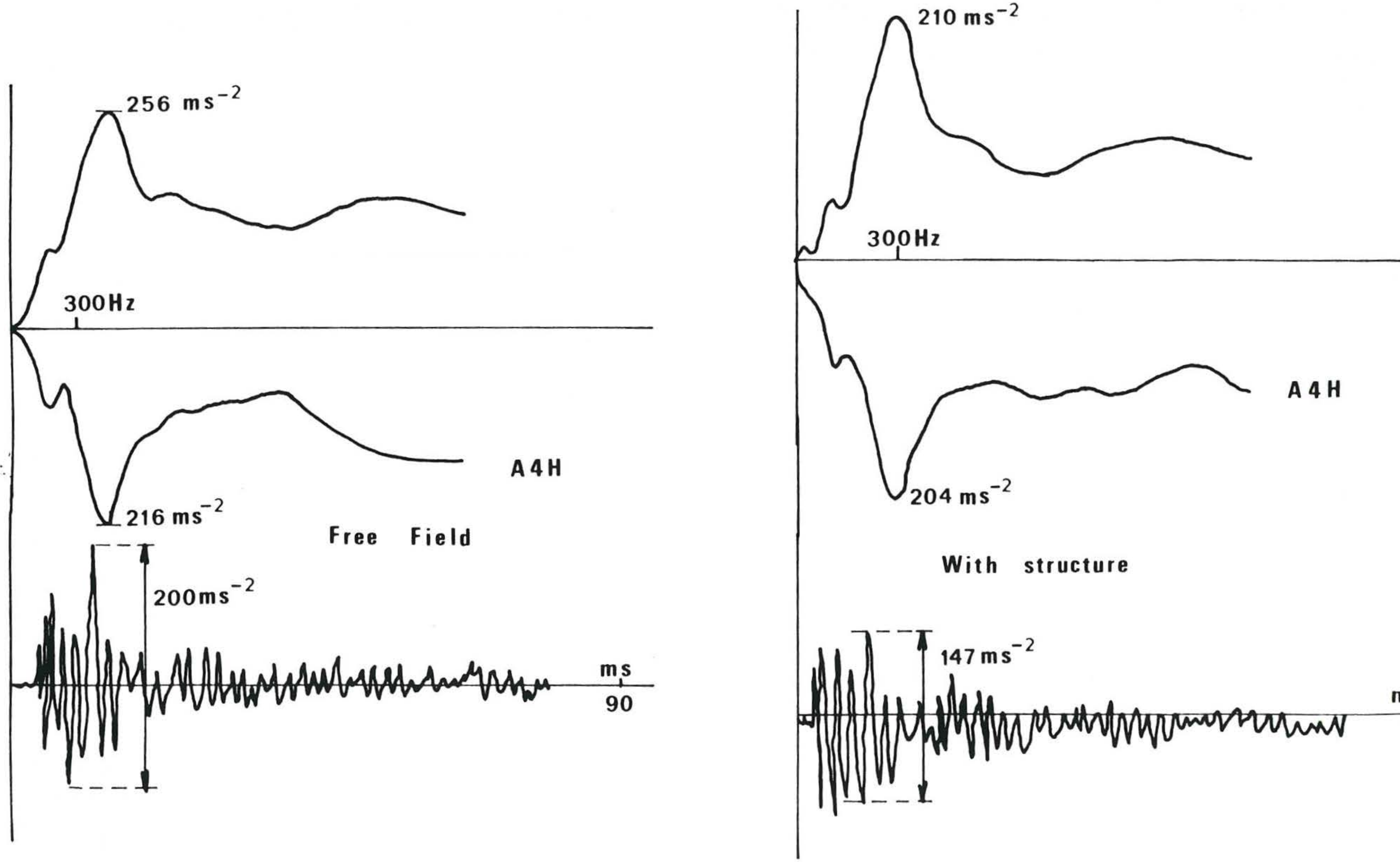


Fig.4 Influence of the structure

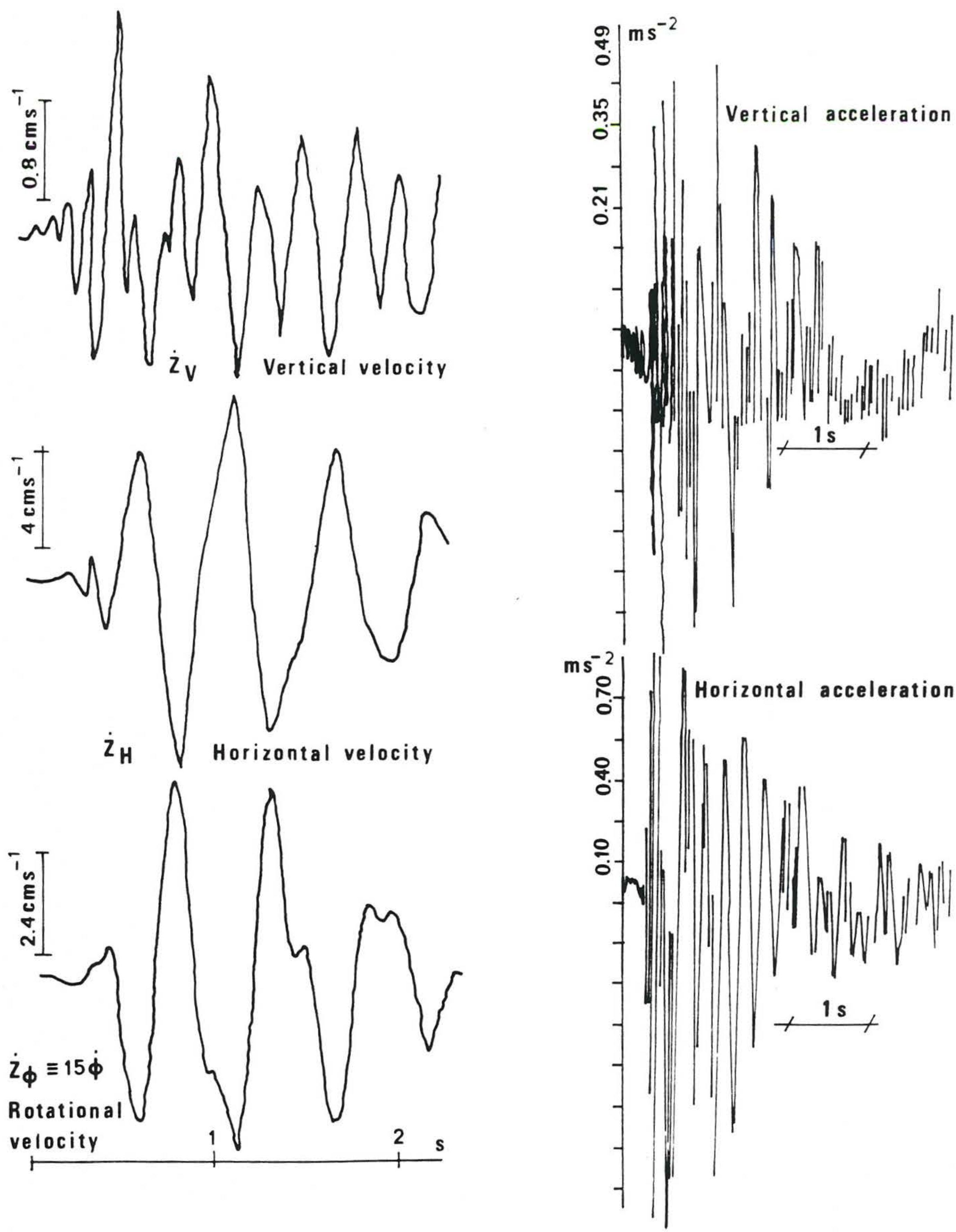


Fig. 5 Calculations of 3 deg. rigid structure

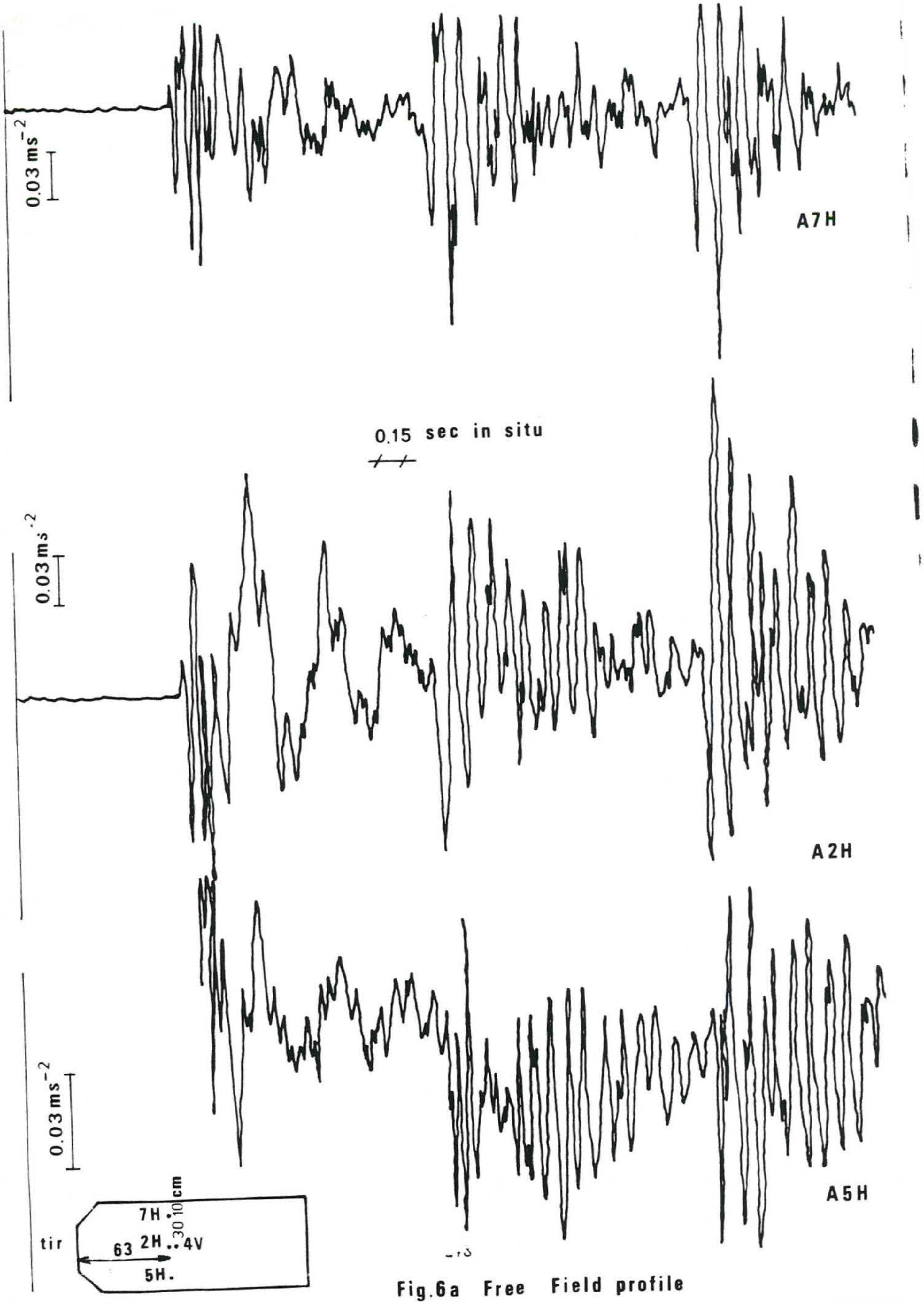


Fig.6a Free Field profile

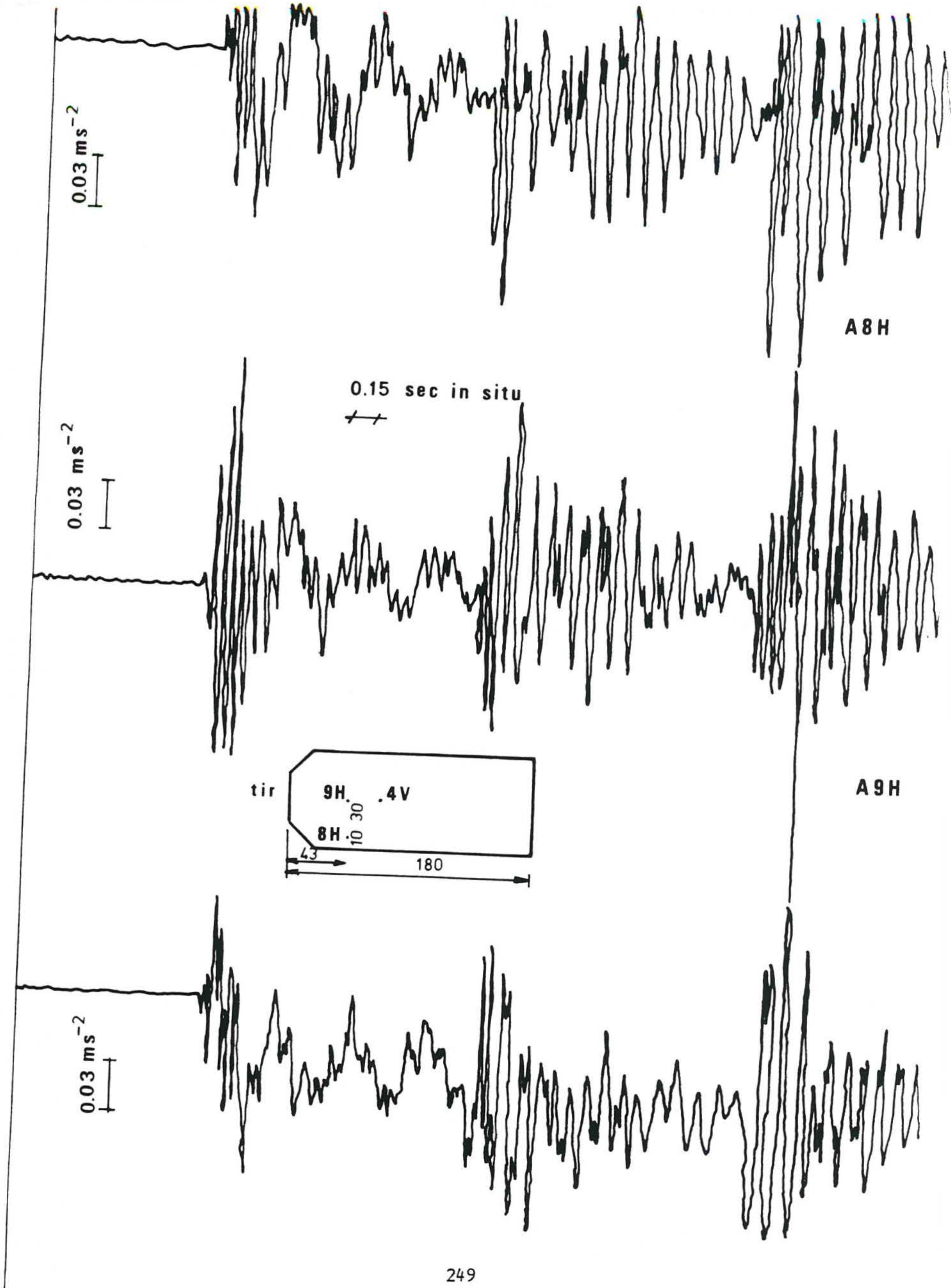


Fig. 6b Free field profile

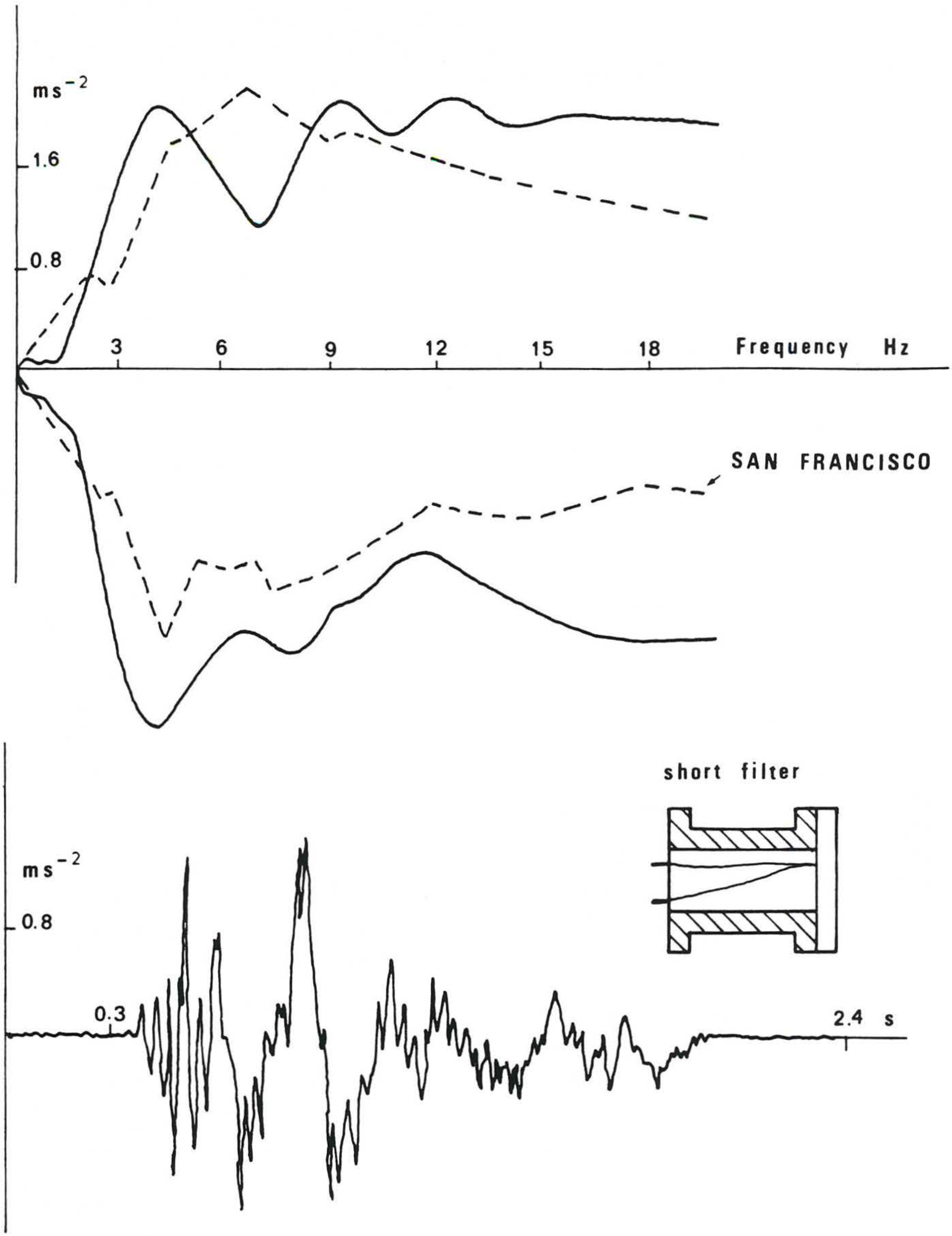
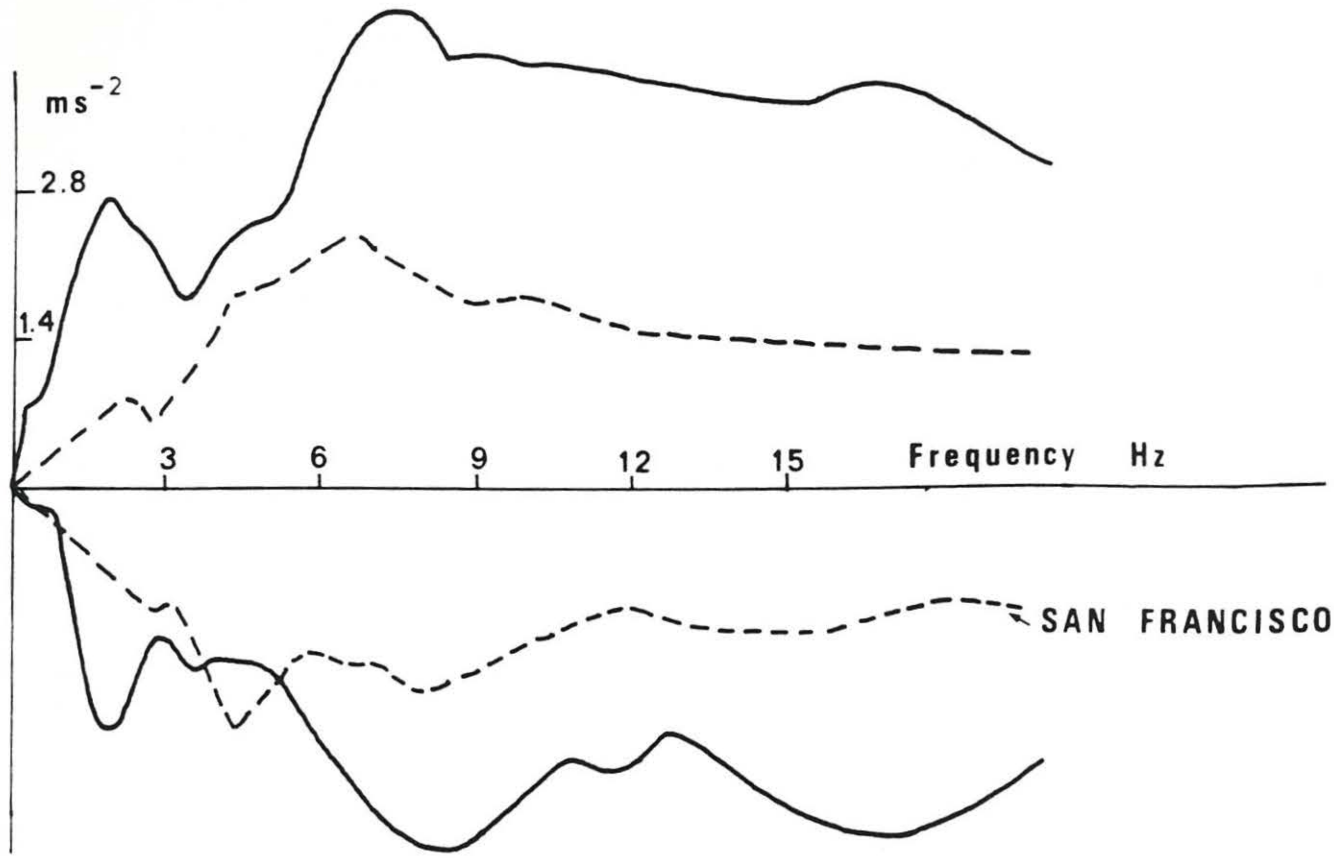


Fig.7 Spectrum of Fig.6
A1H



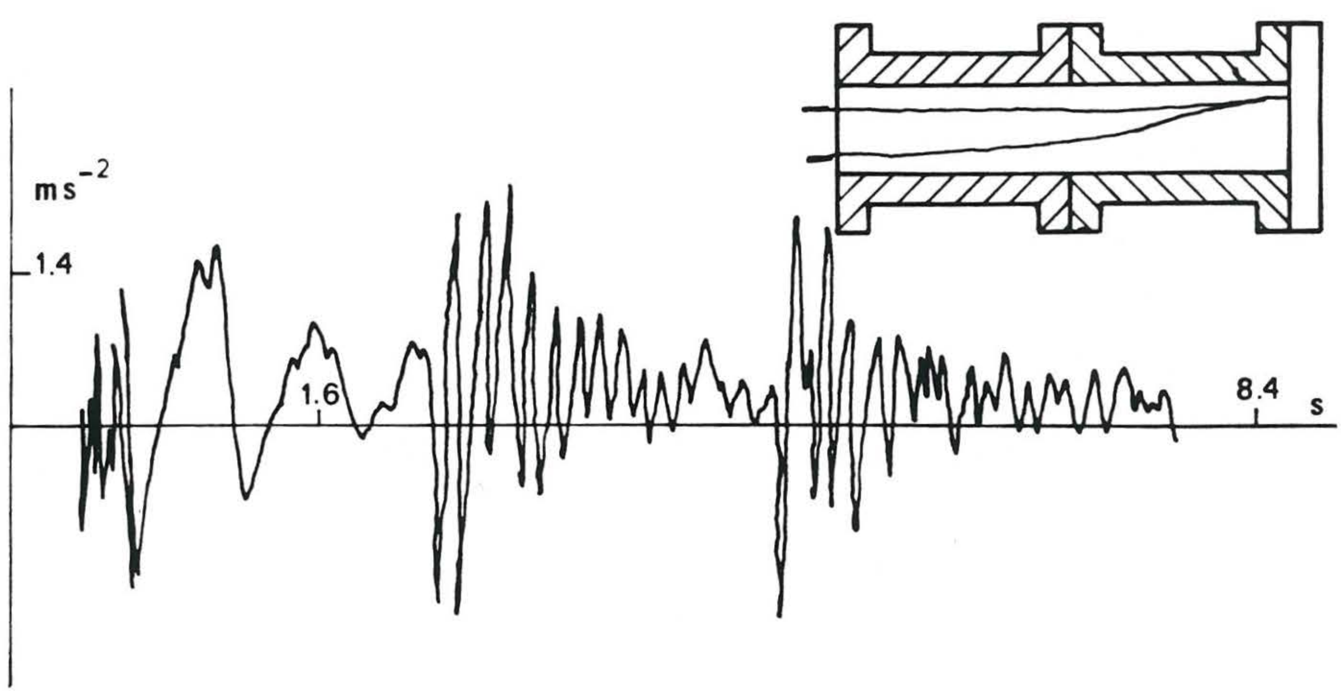


Fig. 8 Spectrum A2H

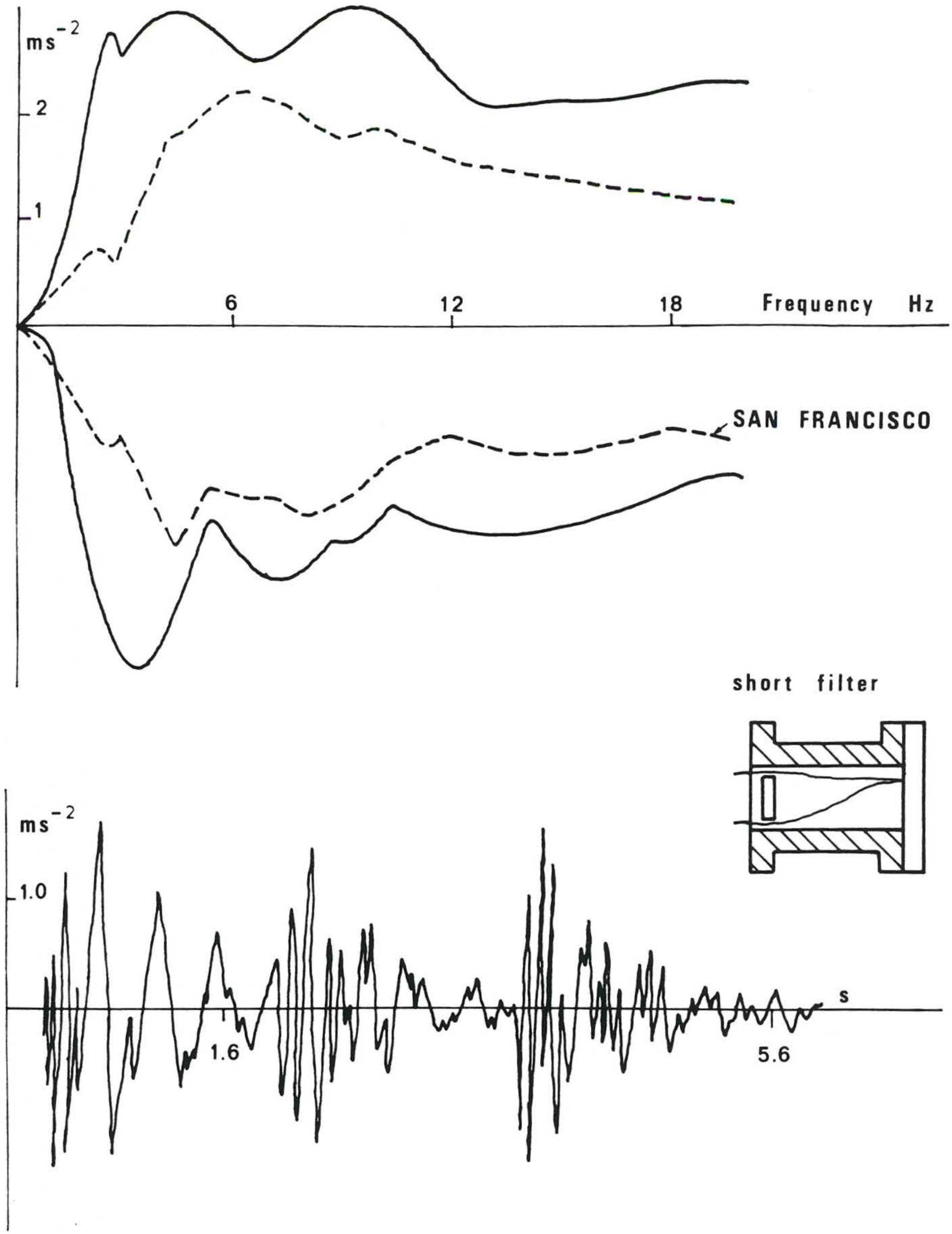


Fig. 9 Spectrum A2H

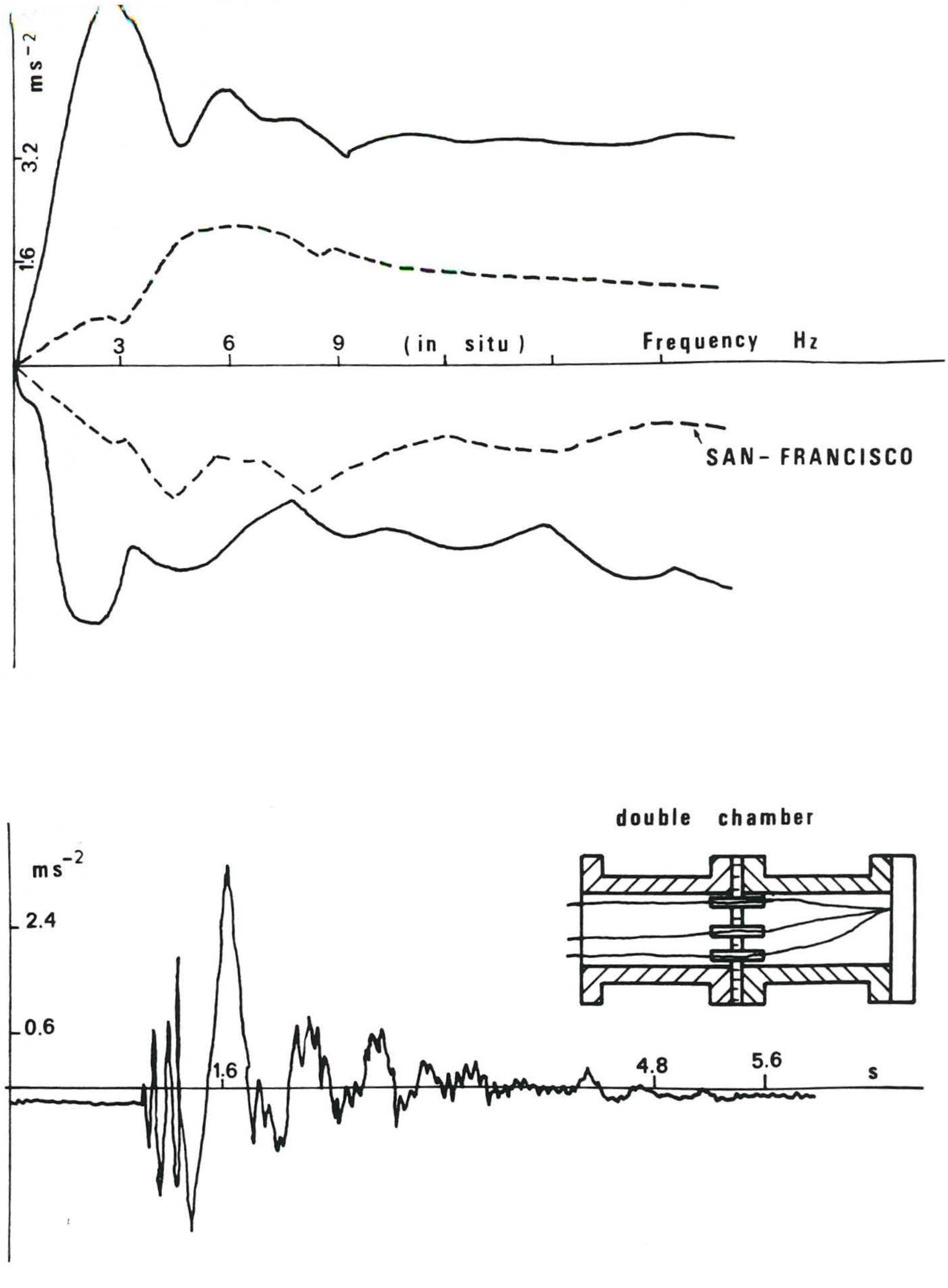


Fig. 10 Spectrum A 2H
(Damping 0.125 in all spectra)