Title
FRAGMENTATION OF HEAVY NUCLEI AT HIGH ENERGY

Authors
Lepore, Joseph V.
Riddell, Robert J.

Publication Date
1974-07-17

Submitted to Physical Review Letters

LBL-3086
Preprint c.1

FRAGMENTATION OF HEAVY NUCLEI AT HIGH ENERGY

Joseph V. Lepore and Robert J. Riddell, Jr.

July 17, 1974

Prepared for the U.S. Atomic Energy Commission
under Contract W-7405-ENG-48

FRAGMENTATION OF HEAVY NUCLEI AT HIGH ENERGY*

Joseph V. Lepore and Robert J. Riddell, Jr.

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

July 17, 1974

ABSTRACT

Fragmentation of high-energy nuclei is treated using the quantum mechanical sudden approximation. Nuclear wave functions necessary for the method are constructed using the shell model. Momentum distributions for ^{4}He particles from ^{16}O and ^{12}C and for protons and ^{6}Li from ^{12}C are determined. They are dominated by a Gaussian factor whose standard deviation is simply related to the shell model parameters of projectile and fragment.

In a series of remarkable experiments carried out over the past two years Beiser, Cork, Greiner, Heckman and Lindstrom[1] have observed the fragmentation of heavy nuclei on various targets at energies of 1.05 and 2.1 GeV per nucleon. They find that when a nuclear projectile ruptures on collision it gives rise to nuclei with velocities narrowly distributed about that of the projectile. All possible "long-lived" ($> 10^{-9}$ sec) fragments (compatible with the quantum numbers involved) are observed; the spread in velocity (rapidity) is of the order of the "Fermi momentum" for the projectile. They further find that the ratios of the cross sections for various fragments is independent of the target nucleus. It occurred to us that the process might be treated quantum mechanically using the "sudden" approximation.[2] This appeared to be the natural generalization of the analysis of the deuteron stripping reaction. Since wave functions to describe the nuclei are needed for such calculations, the choice of shell-model nuclear wave functions to carry out our program was then quite natural and those based on the harmonic oscillator potential offered the most hope for a simple analysis.

The essence of the sudden approximation is that the nuclear wave function changes only slightly during the period of interaction. The result of this assumption is that the amplitude for finding a system in a given state equals the overlap integral between that state and the original one. Thus for a process

$$P + T \rightarrow N + X \; , \tag{1}$$

(P represents projectile, T target, N the observed fragment, X the rest; the same letters will be used to represent the number of nucleons in each species) the amplitude for finding the final state is, in an obvious notation,

$$A_{PT}^{NX} = \int \Psi_N^{\,*}(\xi_1, \cdots \xi_N)\, \Psi_X^{\,*}(\xi_N, \cdots \xi_{P+T}) \Psi_P(\xi_1, \cdots \xi_P) \Psi_T(\xi_{P+1}, \cdots \xi_{P+T}) d\tau . \tag{2}$$

Our wave functions are not symmetrized between P,T and N,X, but a consideration of effects resulting therefrom, show these to be unimportant. In this note we assume that only nucleons are produced. The generalization required when meson production occurs will be given in a subsequent publication. The variables ξ_i represent an appropriate collection for description of the system. We shall find it convenient to use magnitude of momentum, angular momentum, and spin variables for these.

The fragmentation experiments are carried out in the manner of an "inclusive" measurement, in the language of high energy particle physics. Thus if we ask for the probability, W_{PT}^{N}, of finding a nuclear fragment N we have

$$W_{PT}^{N} = \sum_{X} \int \Psi_N^*(\xi_1,\cdots\xi_N)\Psi_X^*(\xi_{N+1},\cdots\xi_{P+T})\Psi_P(\xi_1,\cdots\xi_P)$$

$$\times \Psi_T(\xi_{P+1},\cdots\xi_{P+T})\Psi_N(\xi_1',\cdots\xi_N')\Psi_X(\xi_N',\cdots\xi_{P+T}')\Psi_P^*(\xi_1',\cdots\xi_P')$$

$$\times \Psi_T^*(\xi_{P+1}',\cdots\xi_{P+T}')d\tau \ .$$

Using the closure relation

$$\sum_{X} \Psi_X^*(\xi_{N+1},\cdots\xi_{P+T})\Psi_X(\xi_{N+1}',\cdots\xi_{P+T}') = \delta(\xi_{N+1}-\xi_{N+1}')\cdots\ \delta(\xi_{P+T}-\xi_{P+T}')$$

(where δ is the Dirac delta function), and integrating over the target variables we obtain our basic result:

$$W_{PT}^{N} = \int \Psi_N^*(\xi_1,\cdots\xi_N)\Psi_P(\xi_1,\cdots\xi_N,\xi_{N+1},\cdots\xi_P)\Psi_N(\xi_1',\cdots\xi_N')$$
$$\times \Psi_P^*(\xi_1',\cdots\xi_N',\xi_{N+1},\cdots\xi_P)d\tau \ . \qquad (3)$$

It is to be noted that we can only calculate relative cross sections using the sudden approximation since it does not lend itself to the calculation of a transition rate.

Theoretical justification of the use of the sudden approximation remains an interesting question. Our results, to be presented, apparently offer an a posteriori justification. An alternate point of view regarding Eq. (3) is that it could be written down immediately by simply asking for the probability that the observed nucleus, N, be found as a constituent of the projectile. Consideration of the issue of symmetrization between projectile and target show it to be unimportant.

The usual independent-particle shell model wave functions to be used in Eq. (3) must be modified to include a correct description of the center of momentum. We thus write

$$\Psi_N(\vec{p}_1, \cdots \vec{p}_N) = \prod_i \psi_i(\vec{p}_i - \vec{Q}_N/N)\ \delta\Big(\sum_{i=1}^{N} \vec{p}_i - \vec{Q}_N\Big) . \tag{4}$$

In this equation $\vec{Q}_N$ is the momentum of the nucleus "N". This type of wave function, when appropriately modified to take into account spin and the Pauli exclusion principle, will be used for the evaluation of the transition probability. A Slater determinant embodying these features is easily constructed.

Before giving the results of elaborate calculations involving such wave functions, we give a basic result which is independent of such complication. Note first that regardless of the state of a particle each harmonic oscillator wave function contains a Gaussian factor. To exhibit the aforementioned result we assume that each particle is in an S state and that spin may be neglected. All calculations are imagined to be done in the projectile rest frame. Upon inserting wave functions of the form of Eq. (5) into Eq. (3) we have

$$W_{PT}^{NX} = N_P^{\,2}\, N_N^{\,2} \int \prod_{i=1}^{N} \exp\left\{ -\alpha_N \left[(\vec{p}_i - \vec{Q}_N/N)^2 + (\vec{p}_i^{\,\prime} - \vec{Q}_N/N)^2 \right] - \alpha_P (p_i^{\,2} + p_i^{\prime 2}) \right\}$$

$$\times \prod_{i=N+1}^{P} \exp\left\{ -2\alpha_P p_i^{\,2} \right\} \delta\Big(\sum_{i=1}^{N} \vec{p}_i - \vec{Q}_N \Big) \delta\Big(\sum_{i=1}^{N} \vec{p}_i^{\,\prime} - \vec{Q}_N \Big)$$

$$\times\ \delta\Big(\sum_{i=1}^{P} \vec{p}_i \Big)\ \delta\Big(\sum_{i=1}^{N} \vec{p}_i^{\,\prime} + \sum_{i=N+1}^{P} \vec{p}_i \Big)\ d\tau ,$$

where N_P and N_N are normalization constants. We next introduce

$$\vec{q}_i = \vec{p}_i - \vec{Q}_N/N , \qquad i = 1, \cdots N$$

$$\vec{q}_i{}' = \vec{p}_i{}' - \vec{Q}_N/N , \qquad i = 1, \cdots N$$

$$\vec{\rho}_i = \vec{p}_i + \vec{Q}_N/(P - N) , \qquad i = N + 1, \cdots P$$

and find that

$$W_{PT}{}^N = N_P{}^2 N_N{}^2 \int \prod_{i=1}^{N} \exp\left\{-\alpha_N(q_i{}^2 + q_i'^2) - \alpha_P\left[(\vec{q}_i + \vec{Q}_N/N)^2 + (\vec{q}_i{}' + \vec{Q}_N/N)^2\right]\right\} \prod_{i=N+1}^{P} \exp\left\{-2\alpha_P\left(\vec{\rho}_i - \vec{Q}_N/(P - N)\right)^2\right\}$$

$$\times\, \delta\left(\sum_{i=1}^{N} \vec{q}_i\right) \delta\left(\sum_{i=1}^{N} \vec{q}_i{}'\right) \delta^2\left(\sum_{i=N+1}^{P} \vec{\rho}_i\right) d\tau .$$

It is to be noted that the square of a delta function representing overall momentum conservation appears in this formula. Such a factor inevitably appears when one is dealing with continuum wave functions. As is well known, one of these is commonly replaced by the volume of a box, and drops out of the calculation of relative probabilities. The dependence of $W_{PT}{}^N$ on the fragment's momentum is now easily found to be

$$W_{PT}{}^N = C \exp\left[-Q_N{}^2/2\sigma^2\right]. \tag{5}$$

In this expression the standard deviation is

$$\sigma = \left[N(P - N)/4\alpha_P P\right]^{1/2} . \tag{6}$$

These simple formulas give a good account of the experimental data. The Gaussian formula fits individual results and the observed standard deviation for production of N from P is in good accord with Eq. (6) as shown in Fig. 1.[3] It is to be noted that the only parameter is α_P, which is related to the harmonic oscillator force constant. The basic harmonic oscillator level spacing is chosen as[4]

$$\hbar \omega_0 = 45\, A^{-1/3} - 25\, A^{-2/3} \,. \qquad (7)$$

Figure 1 is plotted using a value of α_P which may be deduced from Eq. (7).

We now turn to the question of the evaluation of $W_{PT}{}^N$ using shell model wave functions which correctly take into account orbital and spin angular momentum and the Pauli exclusion principle. This is not an easy task because one encounters large numbers of terms in any calculation. Simplification of integrals because of rotational invariance is spoiled because of the need for dealing correctly with total momentum as in Eq. (5). One feature which remains in any calculation, as is easily seen, is the Gaussian factor of Eq. (5). In general this factor is multiplied by a polynomial in $\vec{Q}_N{}^2$ whose coefficients drop rapidly as the power of $\vec{Q}_N{}^2$ increases.

We have carried out four realistic calculations for the cases $^{16}O \rightarrow {}^4He$, $^{12}C \rightarrow p$, ^4He, or ^6Li. If we write

$$W_{PT}{}^N = C\, e^{-x^2}\, F(x^2) \,, \qquad (8)$$

where

$$x^2 = \vec{Q}_N{}^2 / 2\sigma^2 \,,$$

we find that F is given by:

(i) $$F(x^2) = 1 + 0.091\, x^2 + 3.5 \times 10^{-3}\, x^4 + 5.0 \times 10^{-5}\, x^6 + 1.4 \times 10^{-6}\, x^8 \qquad (^{16}O \rightarrow {}^{4}He)$$

(ii) $$F(x^2) = 1 + 0.154\, x^2 + 0.012\, x^4 + 4.0 \times 10^{-4}\, x^6 + 2.2 \times 10^{-5}\, x^8 \qquad (^{12}C \rightarrow {}^{4}He)$$

(iii) $$F(x^2) = 1 + 16\, x^2/9 \qquad (^{12}C \rightarrow p)$$

(iv) $$F(x^2) = 1 + 0.22\, x^2 + 0.023\, x^4 + 1.3 \times 10^{-3}\, x^6 + 7.7 \times 10^{-5}\, x^8 \qquad (^{12}C \rightarrow {}^{6}Li).$$

Only in the case of proton production is there a drastic modification of the Gaussian behavior. This result agrees with the experiments. In general, as the number of nucleons in projectile and fragment increases, higher coefficients in $F(x^2)$ become less significant.

We thank members of the Heckman group for many helpful discussions of their experiments.

FOOTNOTES AND REFERENCES

* This work was supported by the U. S. Atomic Energy Commission.

1. Beiser, et al., Bull. Am. Phys. Soc. 19, 518 (1974)

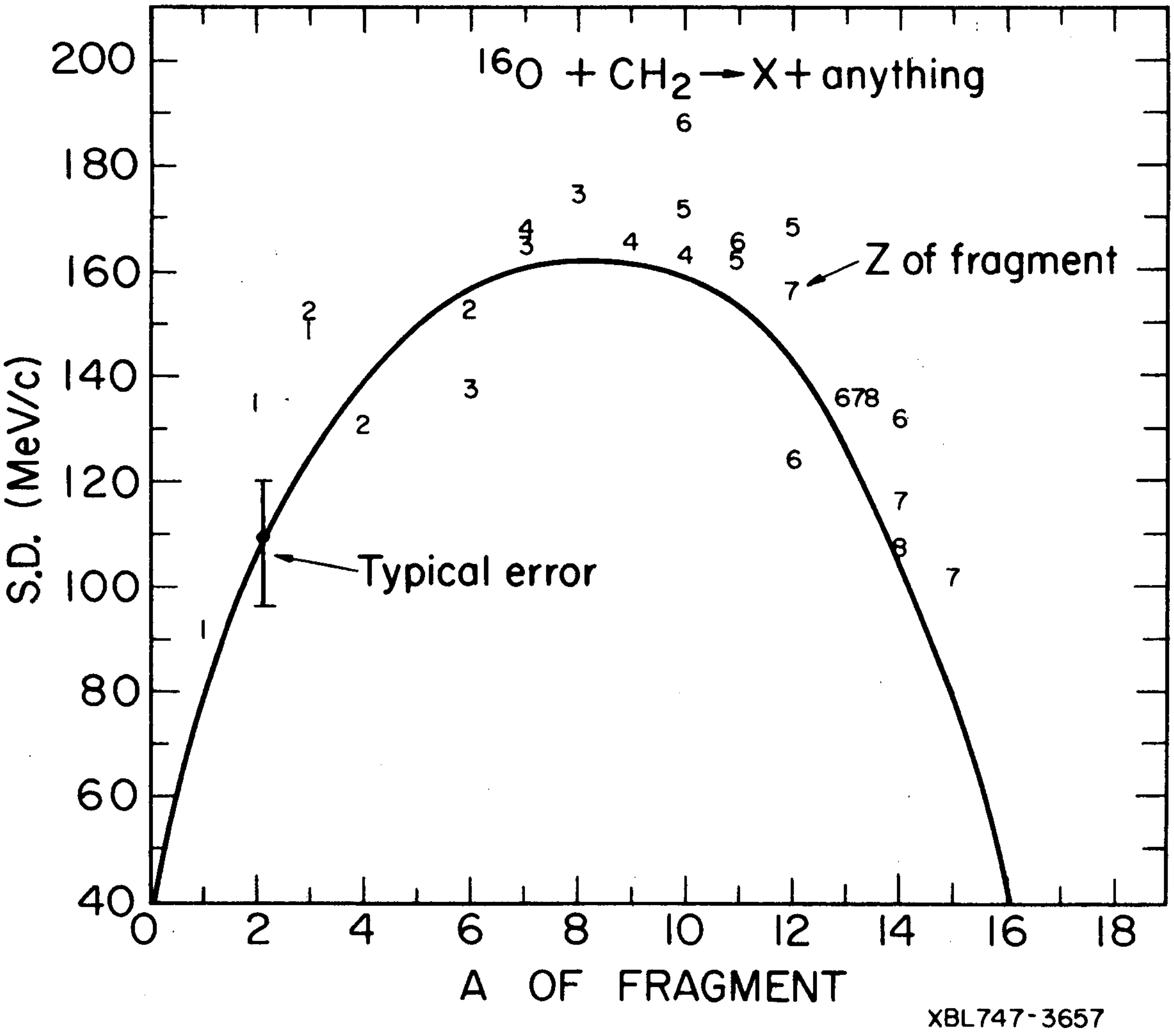
2. Compare L. I. Schiff, Quantum Mechanics, McGraw-Hill Book Co., New York, N. Y. (1949), p. 211.

3. Preliminary data presented by Beiser et al. at the High Energy Heavy Ion Summer Study, Lawrence Berkeley Laboratory, July 15-26, 1974.

4. G. F. Bertsch, The Practioner's Shell Model, American Elsevier Publishing Co., New York, N. Y. (1972), p. 33.

FIGURE CAPTION

Fig. 1. Comparison of Eq. (6) with preliminary data.



$^{16}O + CH_2 \rightarrow X +$ anything
Z of fragment
Typical error
S.D. (MeV/c)
A OF FRAGMENT
XBL747-3657

Fig. 1.

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720