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Depth Compensation for Pressure Transducer Measurements of Boat Wakes

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ABSTRACT

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Boat-generated waves are exceedingly difficult to parameterize because every wave in a wake has a slightly different period and size. In this study, several different sets of boat waves were measured using a vertical array of three pressure transducers (mounted at 0.44 m (PT 1), 1.44 m (PT 2), and 2.44 m (PT 3) below the mean water surface in 3.0 m water depth) and a capacitance-type wave gage ("wave wire"). Linear theory was used to correct the pressure signals for depth attenuation. The results were compared to surface fluctuations measured by the wave wire, which were presumed to be accurate. Wave period was estimated by calculating the period of the largest boat-generated wave (the "maximum" wave) and by obtaining average periods for the largest pair, trio, quartet (and so on) of consecutive waves in a given wake. Average instrument depth-to-wavelength (*d/L*) ratios were 0.15 (PT 1), 0.22 (PT 2), and 0.27 (PT 3), indicating that the use of linear wave theory should be applicable. Regression analysis indicated that the average r^2 values decrease with increasing instrument depth: 0.92 at PT 1, 0.59 at PT 2, and 0.40 at PT 3. The slope of the regression equation is closest to unity when the shortest wave period is used for depth-compensation. Average wave height underestimations for the corrected pressure transducer records are 15% at PT 1, 48% at PT 2, and 53% at PT 3. If only the shortest wave periods are considered, the maximum wave height underestimations are 6%, 31%, and 41% for PTs 1-3, respectively.

ADDITIONAL INDEX WORDS: *Linear wave theory, study design.*

INTRODUCTION

Pressure transducers are widely used to measure water surface fluctuations in lakes, oceans, and navigable waterways. It is well known that the pressure signals associated with the passage of short wavelength waves are attenuated with depth (SEIWELL, 1947). A compensatory correction is often necessary to reproduce the magnitude of surface fluctuation, especially when instrument depth is large relative to the wave length of the waves being measured (USACE, 1984).

According to linear wave theory, pressure records can be corrected to yield estimates of surface fluctuations, z_c, as follows:

$$z_c = \frac{z - \bar{z}}{K_p} \tag{1}$$

where z is the calibrated pressure transducer reading (transducer depth below the water level), overbar indicates a time average, and K_p is the pressure response factor:

$$K_p = \frac{\cosh k(h - z)}{\cosh(kh)} \tag{2}$$

where k is wave number and h is total water column depth. The following sequence of equations, presented by DEAN and DALRYMPLE (1984) and KAMPHUIS (2000), are used to obtain K_p:

$$k = 2\pi / L \tag{3}$$

where L is wavelength and is calculated using:

$$L = L_\infty \left[\tanh \frac{2\pi h}{L_\infty} \right]^{1/2} \tag{4}$$

where L_∞ is deep water wave length calculated using:

$$L_\infty = \frac{gT^2}{2\pi} \tag{5}$$

where g is acceleration due to gravity and T is wave period.

DRAPER (1957) suggested that short period waves attenuate more than Eqn. 1 predicts. Others (*e.g.*, HOM-MA *et al.*, 1966) have also recommended that Eqn. 1 be modified to include an empirical correction factor, N:

$$z_c = N \frac{z - \bar{z}}{K_p} \tag{6}$$

and present formulas to calculate N. If linear theory is assumed, a value of unity for N is appropriate (DEMIRBILEK and VINCENT, 2002).

There has been much debate in the literature about the accuracy of Eqn. 1 and the determination of N (*c.f.,* BISHOP and DONELAN, 1987; KUO and CHIU, 1994). GRACE (1978) presented results from ocean- and laboratory-generated waves that show N is close to unity for instrument depth-to-wavelength ratios (*d/L*) between about 0.10 and 0.23, respectively. SIMPSON (1969) found that Eqn. 1 adequately reproduced surface fluctuations when *d/L* was less than 0.40. ESTEVA and HARRIS (1970) compared the root-mean-square wave heights of two corrected pressure sensors with a wave wire, and found that the corrected records were on average 2% larger and 4% smaller than wave wire wave heights for the 'upper' and 'lower' mounted instruments, respectively. FOLSOM (1947) found that corrected pressure records were 10% smaller than electric point gage wave heights using Eqn. 1. TSAI *et al.* (2001) compared linear corrections with methods presented by KUO and CHIU (1994) and HASHIMOTO *et al.* (1997) and found that pressure records corrected using linear theory had wave heights closest to those measured with a wave wire.

All of this previous work has considered corrections for ocean or laboratory waves, usually with an assumption of a monochromatic wave field. Our concern is with boat wakes. The correction process for boat wakes is complicated because period and height are different for each wave in a wake. One approach has been to identify an "index" wave as representative of the entire wake event, where the index wave is usually the largest wave within a given wake (PARNELL, 1996; BAUER *et al*., 2002). It has been the practice to use linear wave theory to correct pressure transducer measurements of boat wakes (*e.g.,* ELLIS *et al*., 2002), but the accuracy of the surface-equivalent estimates is unknown. The boat wake variable that is critical for the application of Eqn. 1 is the local wavelength, which in turn depends on wave period and water depth. A challenge for characterizing boat wake wavelength and wave period is that there is a different period for each wave within the wake. The purpose of this study is to compare the depth corrected pressure records from an array of transducers to measurements obtained with a capacitance wave gage, and also to recommend a protocol for such corrections. Of particular importance is the selection of a representative wave period for the multiple waves in a boat wake.

METHODS

A vertical array of three pressure transducers (KPSI model 720T, 0-5 psi) were mounted on galvanized water pipe and suspended vertically from a bridge crossing Telephone Cut in the Sacramento River Delta near Stockton, California on 6 January 2003. Sensors were mounted 2.66 (PT 1), 1.66 (PT 2), and 0.66 (PT 3) meters above the bed where the total water depth averaged 3.0 meters. A co-located Brancker WG-50 capacitance-type wave wire was deployed to capture the water surface fluctuations directly. Wakes were generated with a 22-foot boat traveling at various speeds and distances from the array. Each wake was monitored for 40 seconds, a duration longer than the time required for the wake to pass the instrument array, at a sampling rate of 50 Hz. Four wake events, representative of the different wake forms generated during the study, are examined in this paper.

Instruments were calibrated in situ via progressive stepwise submergence. Output voltages at fixed water depths were recorded for one minute and averaged. Regression coefficients for the calibration relations were greater than 99.9% for all instruments.

Boat wakes within the 40-second pressure transducer time series were isolated by selecting only sequences of wave peaks and troughs that were greater than or equal to one standard deviation of the 40-second record. Wake records were extended temporally so that each wake begins and ends with a zero crossing. These wakes were matched with the same time span in the wave wire record.

Several estimates of wave period were derived for each wake. First, the maximum wave was identified as either a crest-trough-crest (CTC) or a trough-crest-trough (TCT) wave, whichever was larger. If the CTC and TCT maximum wave heights were equivalent, the single portion of the wave (*i.e.,* crest or trough) that deviated the most from mean water level governed the selection. All other waves in a wake were characterized as CTC or TCT in accordance with the selection of the maximum wave. The heights of consecutive waves within the specific wake were then ranked from largest to smallest. Wave periods were then calculated for the maximum wave and from the average periods for the largest pair, trio, quartet (and so on) of consecutive waves in a given wake.

Eqn. 1 was applied to each pressure transducer record for the four wakes using the different estimates of wave period. Linear regression was used to compare the corrected pressure sensor (dependent) and wave wire (independent) records. Maximum wave height, because of its common usage, was selected as the basis for comparing the pressure transducer data with the wave wire measurements by calculating a ratio (expressed as a percentage) between the wave wire and pressure transducer values (H_m *ratio*). A ratio of zero percent indicates equal wave height. A negative ratio indicates that the wave wire value is greater than the pressure transducer value, and vice versa.

RESULTS

Table 1 summarizes the results for wave period (T), wavelength (L), instrument depth-to-wavelength ratio (d/L), maximum wave height (H_m), and maximum wave height difference (H_m *ratio*). Wave periods for the four boat wakes range from 1.52 to 3.42 s. Generally, longer periods are associated with PT 3, the deepest instrument. Wavelength (L), calculated using Eqn. 4, vary from 3.51 to 16.06 m. Wavelength increases with increasing instrument depth, as does instrument depth to wave length ratios (d/L). Maximum wave height from the wave wire data varied from 0.180 m to 0.239 m. Values of H_m, obtained from the pressure transducer records, were all smaller than the corresponding wave wire measurements, with the differences increasing with water depth, even after depth correction. This is made apparent in the wave height ratios (H_m *ratios*) that indicate that the uncorrected pressure transducers underestimate maximum wave height by at least -39.5%, and up to -84.1%. H_m *ratios* for the uncorrected records average -46%, -80%, and -82% for PT 1, 2, and 3, respectively. Correcting the PT records improves the results substantially, reducing the range of underestimation to between -0.2% and -71.5%. Evaluation of H_m shows that the pressure transducer estimates for the maximum wave height are smaller than those measured by the wave wire by about 15% at PT 1, 48% at PT 2, and 53% at PT 3. Depth corrections made using the shortest wave period in a wake consistently yield the least negative H_m *ratio* values for that record, even though the surface fluctuations measured by the wave wire are underestimated by 6% at PT 1, 31% at PT 2, and 41% at PT 3.

Table 2 shows the results of the regression analysis. The values for r^2 and the y-intercept (b) are nearly identical for all cases, while the values of slope (m) change substantially. This implies that the latter is the most sensitive indicator of the effect of using different values for wave period in Eqn. 1. Depth corrections made using the shortest wave period within a wake always correspond to the slope (m) closest to 1.0, except in the case of Run 3 with PT 1, where the period of the maximum wave provides slightly better results.

DISCUSSIONS

This analysis demonstrates that uncorrected pressure transducer records will yield estimates of maximum wave height that are always substantially smaller than the actual water surface fluctuation (as indicated by the wave wire). The use of the shortest wave period within each wake produces the largest increase in H_m, but the resulting magnitude is still smaller than those measured by the wave wire. Moreover, Fig. 1, that illustrates what we consider to be the "best case" scenario, shows that the effects of correcting the pressure records are not uniform within the wake. At the beginning of each wake, the corrected estimates are greater than the actual surface fluctuations. The match is best toward the middle of the records (*i.e.,* at wave 2 in Runs 1 and 2, wave 3 in Run 3 and at wave 4 in Run 4), but the quality of matching degrades substantially as the wake passes, especially for the deep pressure transducers (*e.g.*, PT 3).

Maximum wave height ratios (Table 1) show that the shortest estimated wave period most accurately estimates the maximum wave as measured by the wave wire. Use of the shortest period also produces the best regression statistics. However, even the best case values of r^2 and the H_m *ratios* decrease substantially as the depth of the pressure transducers increases. For example, the average r^2 values for the best results in each run decrease

Table 1. *Summary statistics showing the wave period (T), wavelength (L), instrument depth-to-wavelength ratio (d/L), maximum wave (Hm) and percent difference of the wave wire and corrected PT maximum waves (Hm ratio). Values less than 0% indicate wave wire waves are greater that those estimated by the PT. UnCOR indicates un-corrected wave heights and corrected wave heights are shown according to the period used (e.g., T1 uses the period of the maximum wave).*

	PT 1					PT 2					PT 3				
	T (s)	L (m)	d/L	H_m (m)	H_m ratio (%)	T (s)	L (m)	d/L	H_m (m)	H_m ratio (%)	T (s)	L (m)	d/L	H_m (m)	H_m ratio (%)
RUN 1															
wire				0.239					0.239					0.239	
UnCOR				0.132	-45.0				0.054	-77.5				0.043	-81.8
T1	1.82	5.16	0.12	0.209	-12.4	2.42	8.99	0.18	0.132	-44.6	3.10	13.82	0.19	0.089	-62.7
T2	1.78	4.94	0.12	0.214	-10.5	2.09	6.79	0.24	0.189	-20.9	2.68	10.82	0.24	0.125	-47.9
T3	2.19	7.43	0.08	0.181	-24.5										
RUN 2															
wire				0.194					0.152					0.152	
UnCOR				0.100	-48.4				0.036	-76.5				0.027	-82.0
T1	1.50	3.51	0.16	0.182	-6.3	2.34	8.44	0.19	0.091	-40.0	3.32	15.37	0.17	0.050	-67.3
T2	1.79	5.00	0.11	0.159	-17.9	2.81	11.75	0.13	0.066	-57.0	2.86	12.11	0.21	0.066	-56.9
T3	1.95	5.92	0.10	0.142	-26.8	2.55	9.92	0.16	0.077	-49.6	2.58	10.11	0.25	0.086	-43.4
T4	1.83	5.22	0.11	0.149	-23.2										
T5	2.11	6.91	0.08	0.135	-30.4										
RUN 3															
wire				0.180					0.180					0.132	
UnCOR				0.085	-52.6				0.029	-84.1				0.025	-81.3
T1	1.60	3.99	0.16	0.161	-10.8	2.42	8.99	0.18	0.071	-60.5	2.90	12.40	0.21	0.050	-61.8
T2	1.56	3.80	0.16	0.166	-7.8	2.77	11.47	0.14	0.055	-69.3	2.69	10.89	0.24	0.061	-53.9
T3	1.65	4.23	0.15	0.155	-13.9	2.56	9.97	0.16	0.064	-64.7	2.50	9.55	0.27	0.076	-42.3
T4	1.77	4.86	0.13	0.143	-20.4	2.42	8.95	0.18	0.071	-60.4	2.42	8.95	0.29	0.086	-35.0
T5	1.90	5.62	0.11	0.134	-25.9	2.25	7.84	0.21	0.084	-53.5					
T6	1.81	5.11	0.12	0.140	-22.4	2.13	7.06	0.23	0.096	-46.5					
T7	2.04	6.49	0.10	0.126	-30.3										
RUN 4															
wire				0.190					0.190					0.172	
UnCOR				0.115	-39.4				0.035	-81.6				0.028	-83.6
T1	1.64	4.20	0.13	0.189	-0.2	2.08	6.72	0.23	0.117	-38.4	3.42	16.06	0.16	0.049	-71.5
T2	1.72	4.61	0.12	0.181	-4.6	1.88	5.51	0.28	0.157	-17.3	2.97	12.90	0.20	0.062	-63.9
T3	1.67	4.37	0.13	0.185	-2.2	2.03	6.39	0.24	0.125	-33.9	2.73	11.15	0.23	0.076	-56.1
T4	1.77	4.89	0.11	0.176	-7.1	2.18	7.40	0.21	0.103	-45.7	2.60	10.25	0.25	0.086	-49.9
T5	1.88	5.51	0.10	0.168	-11.5	2.40	8.85	0.18	0.083	-56.0	2.42	8.96	0.29	0.109	-36.6
T6	1.82	5.18	0.11	0.172	-9.3	2.28	8.03	0.19	0.093	-50.9					
T7	1.94	5.84	0.10	0.164	-13.5	2.21	7.57	0.21	0.100	-47.2					
T8	2.09	6.76	0.08	0.156	-17.7										

from 0.92 at PT 1 to 0.59 at PT 2, and to 0.40 at PT 3. The average values for the H_m *ratios* decreases with decreasing instrument depth: -15% at PT 1, -48% at PT 2, and -53% at PT 3.

The average instrument depth-to-wave length ratio (*d/L*) for PTs 1-3 calculated using the shortest periods for each wake are: 0.15, 0.22, and 0.27, respectively. The *d/L* ratios are all well within the limit of 0.4 recommended by SIMPSON (1969) as being acceptable for the applicability of linear theory-based corrections. The ratios for PTs 1 and 2 are also within the stricter 0.23 criterion suggested by GRACE (1978). Our study indicates that errors exceeding 10% of the maximum wave height occur with a ratio of 0.15 found at PT 1.

SUMMARY AND CONCLUSION

Regression analysis indicates that the best corrections for depth attenuation, based on linear wave theory (Eqn. 1-5), are produced when using the shortest boat wake wave period. A comparison of maximum wave heights between the corrected pressure records and the wave wire records also indicates that the best results are obtained when using the shortest period. As the depth-to-wavelength ratio increases, the ratio between the corrected PT and wave wire wave heights decreases, implying increasingly greater underestimate of the actual surface fluctuations.

This study suggests that the short wavelength waves that are typical of boat wakes generated by small recreational watercraft mandate that a depth correction be used with pressure transducer records, even when the instruments are installed close to the mean water surface. Even then, the corrected records will underestimate actual wave heights, despite values of *d/L* were well within the limits commonly ascribed for linear wave theory. This suggests that the waves within a boat wake are not easily amenable to analysis using linear theory, or alternatively, that values of *N* larger than unity, although not otherwise indicated, might be necessary for boat wake analysis.

Table 2. *Least-squares best fit line results for Runs 1-4: slope (m), y-intercept (b), and coefficient (r^2). The period used in the correction equations are signified with a T. The numbers following 'T' indicate the number of waves considered.*

	PT 1				PT 2				PT 3			
	T (s)	r^2	m	b	T (s)	r^2	m	b	T (s)	r^2	m	b
RUN 1												
T1	1.82	0.909	0.917	0.002	2.42	0.580	0.523	0.000	3.10	0.351	0.252	0.000
T2	1.78	0.910	0.937	0.002	2.09	0.581	0.747	0.001	2.68	0.351	0.352	0.000
T3	2.19	0.907	0.787	0.001								
RUN 2												
T1	1.50	0.894	1.049	0.002	2.34	0.606	0.549	0.000	3.32	0.268	0.166	0.000
T2	1.79	0.894	0.915	0.001	2.81	0.606	0.394	0.000	2.86	0.268	0.219	0.000
T3	1.95	0.893	0.813	0.001	2.55	0.606	0.461	0.000	2.58	0.268	0.287	0.000
T4	1.83	0.893	0.854	0.001								
T5	2.11	0.893	0.772	0.001								
RUN 3												
T1	1.60	0.913	0.991	0.001	2.42	0.517	0.380	0.000	2.90	0.410	0.325	0.000
T2	1.56	0.913	1.024	0.001	2.77	0.517	0.296	0.000	2.69	0.410	0.392	0.000
T3	1.65	0.913	0.956	0.001	2.56	0.517	0.339	0.000	2.50	0.410	0.490	0.000
T4	1.77	0.913	0.884	0.001	2.42	0.517	0.382	0.000	2.42	0.410	0.553	0.000
T5	1.90	0.913	0.823	0.001	2.25	0.517	0.448	0.000				
T6	1.81	0.913	0.862	0.001	2.13	0.517	0.514	0.000				
T7	2.04	0.912	0.773	0.001								
RUN 4												
T1	1.64	0.949	1.006	0.002	2.08	0.639	0.694	0.000	3.42	0.482	0.274	0.000
T2	1.72	0.949	0.962	0.002	1.88	0.639	0.933	0.001	2.97	0.481	0.348	0.000
T3	1.67	0.949	0.987	0.002	2.03	0.639	0.746	0.000	2.73	0.481	0.423	0.000
T4	1.77	0.949	0.937	0.001	2.18	0.639	0.612	0.000	2.60	0.481	0.482	0.000
T5	1.88	0.949	0.893	0.001	2.40	0.639	0.496	0.000	2.42	0.481	0.611	0.000
T6	1.82	0.949	0.915	0.001	2.28	0.639	0.554	0.000				
T7	1.94	0.949	0.873	0.001	2.21	0.639	0.595	0.000				
T8	2.09	0.948	0.831	0.001								

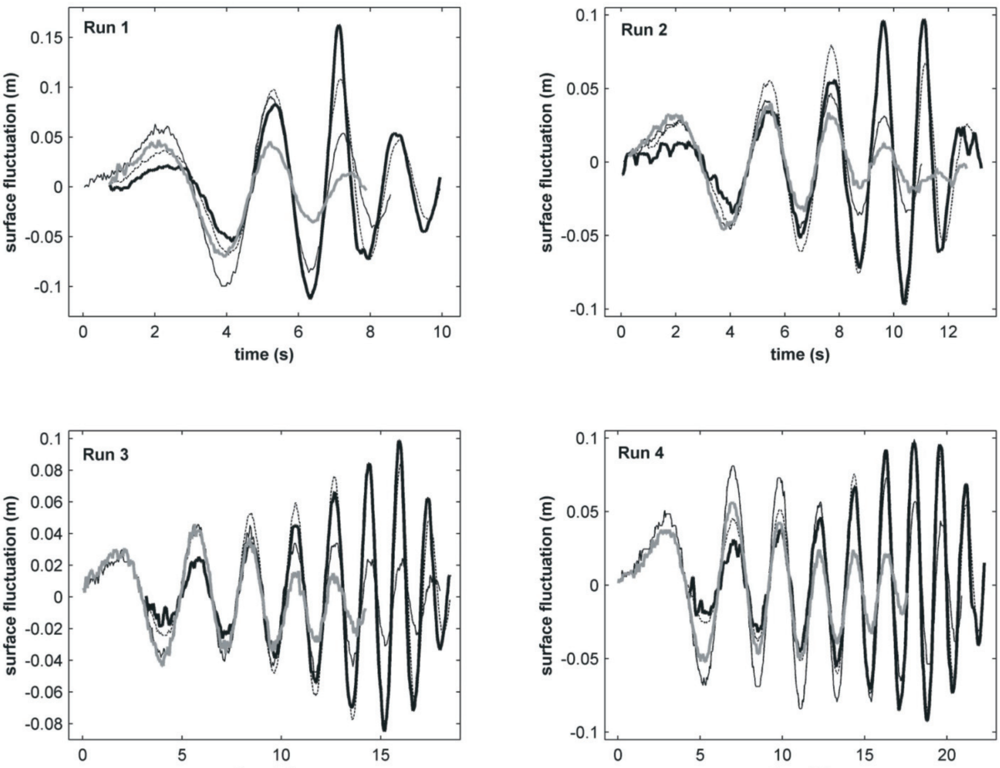

Figure 1. Estimated surface fluctuations from PT 1 (dash), PT 2 (black), PT 3 (gray), and wave wire (bold).

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