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Closure by Hugo A. Loaiciga,[8] Associate Member, ASCE and Miguel A. Mariño,[9] Member, ASCE

To clarify the points of view expressed by the original paper and Shrestha and Shrestha's discussion, it is necessary to explain the difference between censored and truncated distributions. Suppose a random variable is (left) truncated to the interval (δ, ∞). Then values below the truncation level δ are not observable. Objects that cannot be remotely sensed if their dimensions fall below the sensor's resolution are examples of size truncation. Censoring, on the other hand, purposely restricts a random variable to some specified interval. For example, a study of the lower tail of the distribution of streamflows might require the use of those order statistics that fall below certain threshold only (Loaiciga and Mariño 1988).

Let X_T and X_C represent truncated and censored random variables, respectively, and δ represent a (left) truncation or censoring, fixed, threshold. The probability density (or distribution) functions (f) of the left-truncated and censored variables are

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$$f_{X_T}(x) = f_x(x)/P(X \geq \delta), \quad x \geq \delta \quad (68)$$

$$f_{X_C}(x) = f_x(x) \quad \text{if } x > \delta \quad (69a)$$

$$f_{X_C}(x) = 0 \quad \text{if } x < \delta \quad (69b)$$

$$f_{X_C}(x) = P(X \leq \delta) \quad \text{if } x = \delta \quad (69c)$$

in which X = the original (i.e., not subject to truncation or censoring) random variable, and P = the cumulative probability. Notice that the censored random variable has a discontinuous probability distribution at $x = \delta(X_c = X$ if $X \geq \delta$, and $X_c = \delta$ if $X < \delta)$. From (68) and (69), it follows that the expected values of the truncated and censored random variables are

$$E(X_T) = \int_\delta^\infty x f_{X_T}(x)\, dx \quad (70)$$

$$E(X_C) = \int_\delta^\infty x f_X(x)\, dx + \delta \cdot P(X \leq \delta) \quad (71)$$

Eqs. (70) and (71) have been developed in Loaiciga et al. (1991).

Eq. (70) is the continuous analog of Shrestha and Shrestha's (60). Their expected values in (62) and (66) are, therefore, applicable to truncated interoccurrence times within the context of the recurrence of geophysical phenomena. The expected values presented in (12) and (23) of the original paper, on the other hand, are related to the expected value of censored interoccurrence times as defined by (71). To show this, let us rewrite (71) as follows:

$$E(X_C) - \delta \cdot P(X \leq \delta) = \int_\delta^\infty x f_x(x)\, dx \quad (72)$$

The right-hand side of (72) is the continuous analog of the right-hand side of (33) in the original paper. The term in the right-hand side of (72) may be defined as the expected value of the (excess) interoccurrence time (beyond the threshold δ) or, alternatively, as the expected value of the time remaining, beyond δ, till the next event. The expected values presented in (12) and (23) of the original paper are then based on expression (72) and are correct and so are the conclusions reached by them in their work.

In conclusion, the differences between the results of the original paper and those of Shrestha and Shrestha are explained by the underlying interpretations adopted in the papers. Both sets of results correctly describe their relevant problems. It is remarkable that a subtle change in the definitions of an expected value can introduce such divergent set of results, as seen in Figs. 3 and 4. The expected value in the original paper can be defined as the average time remaining, beyond δ until the next event. Shrestha and Shrestha's expected value is the average interoccurrence time between events when it is known that the interoccurrence time exceeds δ years.

In their discussion, Ashkar et al. added further references relevant to the statistical modeling of flood recurrence. It is clarified that in the analysis of the original paper, extremes include not only large-magnitude events, such as floods and earthquakes, but also events with very low amplitude, of which droughts are an example. Although the original paper recognized the value of carefully applied statistical techniques to model moderate interoccurrence arrivals, it advised against the limitations encountered in the statistical mod-

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eling of very infrequent events, and suggested that a more basic understanding of the process is called for in these circumstances. To illustrate this last point, Loaiciga et al. (1991) reconstructed streamflows in several river basins of the arid West using tree-ring chronologies for the last five centuries. With such an extended record, it was possible to accurately assess the recurrence of droughts lasting at least three years with below-median flow, something that has evaded the fanciest of statistical models [see, e.g., Mandelbrot and Wallis (1968)]. The solution of this drought problem was made possible by an understanding of the relationship of a proxy variable, tree-ring growth, to precipitation, which, in turn, determines runoff, rather than by tinkering with limited historical records. A second approach to the recurrence of events was provided by Keller and Loaiciga (1991) in the study of earthquake activity in the transverse ranges of California. By analyzing released seismic energy, fluid pressure buildup, tectonic shortening of a mountain range, and hydrogeologic properties of deep crystalline rocks, Keller and Loaiciga (1991) derived the average recurrence time of earthquakes with Richter magnitude 7.5 in the study area. This would have not been possible with the recorded earthquake incidence (a sample of size one for recorded measurements). The scientific approach to event recurrence can be found in many other fields, for example, in paleohydrologic studies (Ely and Baker 1985), and in other areas concerning rather exotic events, such as asteroidal impacts on Earth (Chapman 1989).

Appendix. References

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