LBNL-48969
Preprint

ERNEST ORLANDO LAWRENCE
BERKELEY NATIONAL LABORATORY

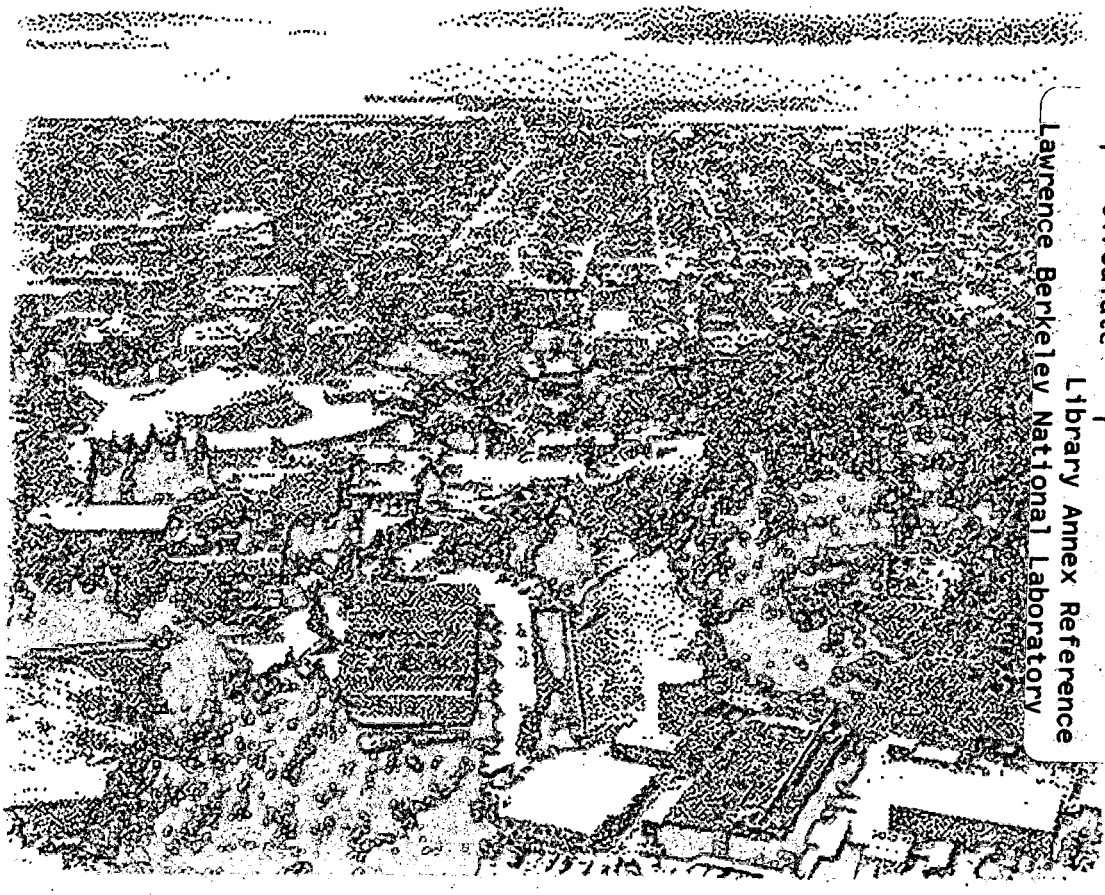
$SU(3)$ Breaking and $D^0 - \bar{D}^0$ Mixing

Adam F. Falk, Yuval Grossman,
Zoltan Ligeti, and Alexey A. Petrov

Physics Division

October 2001

Submitted to
Physical Review D



DISCLAIMER

LBNL-48969

$SU(3)$ Breaking and $D^0 - \bar{D}^0$ Mixing

Adam F. Falk,[1] Yuval Grossman,[2] Zoltan Ligeti,[3] and Alexey A. Petrov[4]

[1]Department of Physics and Astronomy, The Johns Hopkins University,
3400 North Charles Street, Baltimore, MD 21218

[2]Department of Physics, Technion-Israel Institute of Technology,
Technion City, 32000 Haifa, Israel

[3]Physics Division, Ernest Orlando Lawrence Berkeley National Laboratory,
University of California, Berkeley, CA 94720

[4]Department of Physics and Astronomy, Wayne State University,
Detroit, MI 48201

October 2001

This work was supported by the Director, Office of Science, Office of High Energy and Nuclear Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098, and by National Science Foundation Grant No. PHY-9970781, Israel Science Foundation Grant No. 237/01-1, US-Israel Binational Science Foundation Grant No. 2000133, and the Harry Werksman Research Fund.

JHU-TIPAC-2001-04
LBNL-48969
WSU-HEP-0101
hep-ph/0110317
October, 2001

$SU(3)$ Breaking and $D^0 - \overline{D}^0$ Mixing

Adam F. Falk,[1] Yuval Grossman,[2] Zoltan Ligeti,[3] and Alexey A. Petrov[4]

[1]*Department of Physics and Astronomy, The Johns Hopkins University*
3400 North Charles Street, Baltimore, MD 21218

[2]*Department of Physics, Technion–Israel Institute of Technology*
Technion City, 32000 Haifa, Israel

[3]*Ernest Orlando Lawrence Berkeley National Laboratory*
University of California, Berkeley, CA 94720

[4]*Department of Physics and Astronomy*
Wayne State University, Detroit, MI 48201

Abstract

The main challenge in the Standard Model calculation of the mass and width difference in the $D^0 - \overline{D}^0$ system is to estimate the size of $SU(3)$ breaking effects. We prove that D meson mixing occurs in the Standard Model only at second order in $SU(3)$ violation. We consider the possibility that phase space effects may be the dominant source of $SU(3)$ breaking. We find that $y = \Delta\Gamma/2\Gamma$ of the order of one percent is natural in the Standard Model, potentially reducing the sensitivity to new physics of measurements of D meson mixing.

I. INTRODUCTION

It is a common assertion that the Standard Model prediction for mixing in the $D^0 - \overline{D}^0$ system is very small, making this process a sensitive probe of new physics. Two physical parameters that characterize $D^0 - \overline{D}^0$ mixing are

$$x \equiv \frac{\Delta M}{\Gamma}, \qquad y \equiv \frac{\Delta\Gamma}{2\Gamma}, \tag{1}$$

where ΔM and $\Delta\Gamma$ are the mass and width differences of the two neutral D meson mass eigenstates, and Γ is their average width. The $D^0 - \overline{D}^0$ system is unique among the neutral mesons in that it is the only one whose mixing proceeds via intermediate states with down-type quarks. The mixing is very slow in the Standard Model, because the third generation plays a negligible role due to the smallness of $|V_{ub}V_{cb}|$ and the relative smallness of m_b, and so the GIM cancellation is very effective [1, 2, 3, 4, 5].

The current experimental upper bounds on x and y are on the order of a few times 10^{-2}, and are expected to improve significantly in the coming years. To regard a future discovery of nonzero x or y as a signal for new physics, we would need high confidence that the Standard Model predictions lie significantly below the present limits. As we will show, in the Standard Model x and y are generated only at second order in $SU(3)$ breaking, so schematically

$$x\,, y \sim \sin^2\theta_C \times [SU(3)\text{ breaking}]^2\,, \tag{2}$$

where θ_C is the Cabibbo angle. Therefore, predicting the Standard Model values of x and y depends crucially on estimating the size of $SU(3)$ breaking. Although y is expected to be determined by Standard Model processes, its value nevertheless affects significantly the sensitivity to new physics of experimental analyses of D mixing [6].

At present, there are three types of experiments which measure x and y. Each is actually sensitive to a combination of x and y, rather than to either quantity directly. First, there is the D^0 lifetime difference to CP even and CP odd final states [7, 8, 9, 10, 11], which to leading order measures

$$y_{CP} = \frac{\tau(D \to \pi^+K^-)}{\tau(D \to K^+K^-)} - 1 = y\cos\phi - x\sin\phi\,\frac{A_m}{2}\,, \tag{3}$$

where $A_m \equiv |q/p|^2 - 1$ (see Eq. (5) for the definition of the neutral D mass eigenstates), and ϕ is a possible CP violating phase of the mixing amplitude. Second, one can measure the time dependence of doubly Cabibbo suppressed decays, such as $D^0 \to K^+\pi^-$ [12], which is sensitive to the three quantities

$$(x\cos\delta + y\sin\delta)\cos\phi\,, \qquad (y\cos\delta - x\sin\delta)\sin\phi\,, \qquad x^2 + y^2\,, \tag{4}$$

where δ is the strong phase between the Cabibbo allowed and doubly Cabibbo suppressed amplitudes. A similar study for $D^0 \to K^-\pi^+\pi^0$ also would be valuable, with the strong phase difference extracted simultaneously from the Dalitz plot analysis [13]. Third, one can search for D mixing in semileptonic decays [14], which is sensitive to $x^2 + y^2$.

In a large class of models, the best hope to discover new physics in D mixing is to observe the CP violating phase, $\phi_{12} = \arg[M_{12}/\Gamma_{12}]$ (see the definitions (7) and (8) below), which is very small in the Standard Model. However, if $y \gg x$, then the sensitivity of any physical observable to ϕ_{12} is suppressed, since A_m is proportional to x/y and ϕ is to $(x/y)^2$, even

if new physics makes a large contribution to ΔM [6]. It is also clear from Eq. (4) that if y is significantly larger than x, then δ must be known very precisely for experiments to be sensitive to new physics in the terms linear in x and y. It may be possible to measure δ with some accuracy at the planned τ-charm factory CLEO-c [15, 16].

There is a vast literature on estimating x and y within and beyond the Standard Model; for a compilation of results, see Ref. [17]. Roughly, there are two approaches, neither of which give very reliable results because m_c is in some sense intermediate between heavy and light. The "inclusive" approach is based on the operator product expansion (OPE). In the $m_c \gg \Lambda$ limit, where Λ is a scale characteristic of the strong interactions, ΔM and $\Delta\Gamma$ can be expanded in terms of matrix elements of local operators [1, 2, 18]. Such calculations yield $x, y \lesssim 10^{-3}$. The use of the OPE relies on local quark-hadron duality, and on Λ/m_c being small enough to allow a truncation of the series after the first few terms. The charm mass may not be large enough for these to be good approximations, especially for nonleptonic D decays. An observation of y of order 10^{-2} could be ascribed to a breakdown of the OPE or of duality [18], but such a large value of y is certainly not a generic prediction of OPE analyses. The "exclusive" approach sums over intermediate hadronic states, which may be modeled or fit to experimental data [5, 19, 20]. Since there are cancellations between states within a given $SU(3)$ multiplet, one needs to know the contribution of each state with high precision. However, the D is not light enough that its decays are dominated by a few final states. In the absence of sufficiently precise data on many decay rates and on strong phases, one is forced to use some assumptions. While most studies find $x, y \lesssim 10^{-3}$, Refs. [21, 22, 23] obtain x and y at the 10^{-2} level by arguing that $SU(3)$ violation is actually of order unity, but the source of the large $SU(3)$ breaking is not made explicit.

In this paper, we compute the contribution to $\Delta\Gamma$ from $SU(3)$ breaking from final state phase space differences. This is a calculable source of $SU(3)$ violation, which enhances the rates to final states containing fewer strange quarks. In Sec. II we review the formalism of $D^0 - \overline{D}^0$ mixing. In Sec. III we give a general group theory proof that ΔM and $\Delta\Gamma$ are only generated at second order in $SU(3)$ breaking if $SU(3)$ violation enters these quantities perturbatively. In Sec. IV we discuss the estimates of $SU(3)$ breaking using the "inclusive" and "exclusive" analyses, and remind the reader of the shortcomings of each. Our main results are found in Sec. V, namely the calculation of $SU(3)$ breaking in $\Delta\Gamma$ from phase space effects in two-, three- and four-body final states. We find that such effects are very important, and can naturally account for $\Delta\Gamma/2\Gamma$ at the percent level. We extend the analysis to intermediate resonances in Sec. VI. In Sec. VII we present our conclusions and ask whether in light of our results it remains possible for the measurement of D mixing to probe new physics.

II. FORMALISM

We begin by reviewing the formalism for $D^0 - \overline{D}^0$ mixing. The mass eigenstates D_L and D_S are superpositions of the flavor eigenstates D^0 and $\overline{D}^0$,

$$|D_{L,S}\rangle = p\,|D^0\rangle \pm q\,|\overline{D}^0\rangle\,, \tag{5}$$

where $|p|^2 + |q|^2 = 1$. In the Standard Model CP violation in D mixing is negligible, as is CP violation in D decays both in the Standard Model and in most scenarios of new physics. From here on we will assume that CP is a good symmetry. Then $p = q$, and $|D_{L,S}\rangle$ become

CP eigenstates,

$$CP|D_\pm\rangle = \pm|D_\pm\rangle\,, \tag{6}$$

with the mass and width differences defined as $\Delta M \equiv m_{D_+} - m_{D_-}$ and $\Delta\Gamma \equiv \Gamma_{D_+} - \Gamma_{D_-}$. The off-diagonal element of the $D^0 - \overline{D}^0$ mass matrix can be expressed as

$$M_{12} = \langle\overline{D}^0|\mathcal{H}_w^{\Delta C=2}|D^0\rangle + \mathrm{P}\sum_n \frac{\langle\overline{D}^0|\mathcal{H}_w^{\Delta C=1}|n\rangle\langle n|\mathcal{H}_w^{\Delta C=1}|D^0\rangle}{m_D^2 - E_n^2}, \tag{7}$$

$$\Gamma_{12} = \sum_n \rho_n \langle\overline{D}^0|\mathcal{H}_w^{\Delta C=1}|n\rangle\langle n|\mathcal{H}_w^{\Delta C=1}|D^0\rangle\,, \tag{8}$$

where the sum is over all intermediate states, P denotes the principal value, and ρ_n is the density of the state n. The first term in Eq. (7) comes from the local $|\Delta C| = 2$ operators (box and dipenguin), which affect M_{12} only. The second term comes from the insertion of two $|\Delta C| = 1$ operators. There is a contribution of this type to both M_{12} and Γ_{12}.

One can then express y in two equivalent ways, either as a sum over the states that are common to D^0 and $\overline{D}^0$,

$$y = \frac{1}{2\Gamma}\sum_n \rho_n \left[\langle D^0|\mathcal{H}_w|n\rangle\langle n|\mathcal{H}_w|\overline{D}^0\rangle + \langle\overline{D}^0|\mathcal{H}_w|n\rangle\langle n|\mathcal{H}_w|D^0\rangle\right], \tag{9}$$

or as the difference in the decay rates of the two mass eigenstates

$$y = \frac{1}{2\Gamma}\sum_n \rho_n \left[|\langle D_+|\mathcal{H}_w|n\rangle|^2 - |\langle D_-|\mathcal{H}_w|n\rangle|^2\right]. \tag{10}$$

A similar pair of expressions can be written for x,

$$\begin{aligned} x &= \frac{1}{\Gamma}\left[\langle\overline{D}^0|\mathcal{H}_w|D^0\rangle + \mathrm{P}\sum_n \frac{\langle D^0|\mathcal{H}_w|n\rangle\langle n|\mathcal{H}_w|\overline{D}^0\rangle + \langle\overline{D}^0|\mathcal{H}_w|n\rangle\langle n|\mathcal{H}_w|D^0\rangle}{m_D^2 - E_n^2}\right], \\ &= \frac{1}{\Gamma}\left[\langle\overline{D}^0|\mathcal{H}_w|D^0\rangle + \mathrm{P}\sum_n \frac{|\langle D_+|\mathcal{H}_w|n\rangle|^2 - |\langle D_-|\mathcal{H}_w|n\rangle|^2}{m_D^2 - E_n^2}\right]. \end{aligned} \tag{11}$$

Note that x and y are generated by off-shell and on-shell intermediate states, respectively.

III. $SU(3)$ ANALYSIS OF $D^0 - \overline{D}^0$ MIXING

We now prove that $D^0 - \overline{D}^0$ mixing arises only at second order in $SU(3)$ breaking effects. The proof is valid when $SU(3)$ violation enters perturbatively. This would not be the case, for example, if D transitions were dominated by intermediate states or single resonances close to threshold. As we will see explicitly in Secs. V and VI, in such cases it is sometimes possible for $SU(3)$ violation to be enhanced substantially. Yet other than in these exceptional situations, treating $SU(3)$ violation perturbatively seems to us to be a mild assumption.

The quantities M_{12} and Γ_{12} which determine x and y depend on matrix elements with the general structure

$$\langle\overline{D}^0|\,\mathcal{H}_w\mathcal{H}_w\,|D^0\rangle\,, \tag{12}$$

where in this section we let $\mathcal{H}_w$ denote specifically the $\Delta C = -1$ part of the weak Hamiltonian. Let D be the field operator that creates a D^0 meson and annihilates a $\overline{D}^0$. Then the matrix element may be written as

$$\langle 0|\, D\, \mathcal{H}_w \mathcal{H}_w\, D\, |0\rangle\,. \tag{13}$$

Let us focus on the $SU(3)$ flavor group theory properties of this expression.

Since the operator D is of the form $\bar{c}u$, it transforms in the fundamental representation of $SU(3)$, which we will represent with a lower index, D_i. We use a convention in which the correspondence between matrix indices and quark flavors is $(1,2,3) = (u,d,s)$. The only nonzero element of D_i is $D_1 = 1$. The $\Delta C = -1$ part of the weak Hamiltonian has the flavor structure $(\bar{q}_i c)(\bar{q}_j q_k)$, so its matrix representation is written with a fundamental index and two antifundamentals, H_k^{ij}. This operator is a sum of irreducible representations contained in the product $3 \times \overline{3} \times \overline{3} = \overline{15} + 6 + \overline{3} + \overline{3}$. In the limit in which the third generation is neglected, H_k^{ij} is traceless, so only the $\overline{15}$ (symmetric on i and j) and 6 (antisymmetric on i and j) representations appear. That is, the $\Delta C = -1$ part of $\mathcal{H}_w$ may be decomposed as $\frac{1}{2}(\mathcal{O}_{\overline{15}} + \mathcal{O}_6)$, where

$$\begin{aligned}
\mathcal{O}_{\overline{15}} &= (\bar{s}c)(\bar{u}d) + (\bar{u}c)(\bar{s}d) + s_1(\bar{d}c)(\bar{u}d) + s_1(\bar{u}c)(\bar{d}d) \\
&\quad - s_1(\bar{s}c)(\bar{u}s) - s_1(\bar{u}c)(\bar{s}s) - s_1^2(\bar{d}c)(\bar{u}s) - s_1^2(\bar{u}c)(\bar{d}s)\,, \\
\mathcal{O}_6 &= (\bar{s}c)(\bar{u}d) - (\bar{u}c)(\bar{s}d) + s_1(\bar{d}c)(\bar{u}d) - s_1(\bar{u}c)(\bar{d}d) \\
&\quad - s_1(\bar{s}c)(\bar{u}s) + s_1(\bar{u}c)(\bar{s}s) - s_1^2(\bar{d}c)(\bar{u}s) + s_1^2(\bar{u}c)(\bar{d}s)\,,
\end{aligned} \tag{14}$$

and $s_1 = \sin\theta_C \approx 0.22$. The matrix representations $H(\overline{15})_k^{ij}$ and $H(6)_k^{ij}$ have nonzero elements

$$\begin{aligned}
H(\overline{15})_k^{ij}: &\quad H_2^{13} = H_2^{31} = 1\,, & H_2^{12} = H_2^{21} = s_1\,, \\
&\quad H_3^{13} = H_3^{31} = -s_1\,, & H_3^{12} = H_3^{21} = -s_1^2\,, \\
H(6)_k^{ij}: &\quad H_2^{13} = -H_2^{31} = 1\,, & H_2^{12} = -H_2^{21} = s_1\,, \\
&\quad H_3^{13} = -H_3^{31} = -s_1\,, & H_3^{12} = -H_3^{21} = -s_1^2\,.
\end{aligned} \tag{15}$$

We introduce $SU(3)$ breaking through the quark mass operator $\mathcal{M}$, whose matrix representation is $M_j^i = \mathrm{diag}(m_u, m_d, m_s)$. Although $\mathcal{M}$ is a linear combination of the adjoint and singlet representations, only the 8 induces $SU(3)$ violating effects. It is convenient to set $m_u = m_d = 0$ and let $m_s \neq 0$ be the only $SU(3)$ violating parameter. All nonzero matrix elements built out of D_i, H_k^{ij} and M_j^i must be $SU(3)$ singlets.

We now prove that $D^0 - \overline{D}^0$ mixing arises only at second order in $SU(3)$ violation, by which we mean second order in m_s. First, we note that the pair of D operators is symmetric, and so the product $D_i D_j$ transforms as a 6 under $SU(3)$. Second, the pair of $\mathcal{H}_w$'s is also symmetric, and the product $H_k^{ij} H_n^{lm}$ is in one of the representations which appears in the product

$$\begin{aligned}
\Big[(\overline{15} + 6) \times (\overline{15} + 6)\Big]_S &= (\overline{15} \times \overline{15})_S + (\overline{15} \times 6) + (6 \times 6)_S \\
&= (\overline{60} + \overline{24} + 15 + 15' + \overline{6}) + (42 + 24 + 15 + \overline{6} + 3) + (15' + \overline{6})\,.
\end{aligned} \tag{16}$$

A straightforward computation shows that only three of these representations actually appear in the decomposition of $\mathcal{H}_w \mathcal{H}_w$. They are the $\overline{60}$, the 42, and the $15'$ (actually twice, but with the same nonzero elements both times). So we have product operators of the form

$$\begin{aligned}
DD &= \mathcal{D}_6\,, \\
\mathcal{H}_w \mathcal{H}_w &= \mathcal{O}_{\overline{60}} + \mathcal{O}_{42} + \mathcal{O}_{15'}\,,
\end{aligned} \tag{17}$$

where the subscript denotes the representation of $SU(3)$.

Since there is no $\overline{6}$ in the decomposition of $\mathcal{H}_w\mathcal{H}_w$, there is no $SU(3)$ singlet which can be made with $\mathcal{D}_6$, and no $SU(3)$ invariant matrix element of the form (13) can be formed. This is the well known result that $D^0 - \overline{D}^0$ mixing is prohibited by $SU(3)$ symmetry.

Now consider a single insertion of the $SU(3)$ violating spurion $\mathcal{M}$. The combination $\mathcal{D}_6\mathcal{M}$ transforms as $6 \times 8 = 24 + \overline{15} + 6 + \overline{3}$. Note that there is still no invariant to be made with $\mathcal{H}_w\mathcal{H}_w$. It follows that $D^0 - \overline{D}^0$ mixing is not induced at first order in $SU(3)$ breaking.

With two insertions of $\mathcal{M}$, it becomes possible to make an $SU(3)$ invariant. The decomposition of $\mathcal{D}\mathcal{M}\mathcal{M}$ is

$$\begin{aligned} 6 \times (8 \times 8)_S &= 6 \times (27 + 8 + 1) \\ &= (60 + \overline{42} + 24 + \overline{15} + \overline{15}' + 6) + (24 + \overline{15} + 6 + \overline{3}) + 6 \,. \end{aligned} \tag{18}$$

There are three elements of the 6×27 part which can give invariants with $\mathcal{H}_w\mathcal{H}_w$. Each invariant yields a contribution proportional to $s_1^2 m_s^2$. As promised, $D^0 - \overline{D}^0$ mixing arises only at second order in the $SU(3)$ violating parameter m_s.

IV. ESTIMATING THE SIZE OF $SU(3)$ BREAKING

We now turn to review some general estimates of the size of $SU(3)$ breaking effects. These effects can be approached from either an inclusive or an exclusive point of view. It is instructive to see how $SU(3)$ violation appears in each case.

A. "Inclusive" approach

An elegant and concrete estimate of how $SU(3)$ violation enters x and y is the short distance analysis, first applied to $D^0 - \overline{D}^0$ mixing by Georgi [1] and later extended by other authors [2, 18]. We review it briefly, both to establish the contrast with our approach and to recall the results. Let Λ be a scale characteristic of the strong interactions, such as m_ρ or $4\pi f_\pi$. In the limit $m_c \gg \Lambda$, the momentum flowing through the light degrees of freedom in the intermediate state is large and an operator production expansion (OPE) can be performed. For example, one can write

$$\Gamma_{12} = \frac{1}{2m_D} \mathrm{Im} \langle \overline{D}^0 | i \int \mathrm{d}^4x \, T\left\{ \mathcal{H}_w^{\Delta C=1}(x) \, \mathcal{H}_w^{\Delta C=1}(0) \right\} | D^0 \rangle \,, \tag{19}$$

where $\mathcal{H}_w^{\Delta C=1}$ is the $|\Delta C| = 1$ effective Hamiltonian. In the OPE, the time ordered product in Eq. (19) can be expanded in local operators of increasing dimension; the higher dimension operators are suppressed by powers of Λ/m_c.

The leading contribution comes from the dimension-6 $|\Delta C| = 2$ four-quark operators corresponding to the short distance box diagram,

$$\begin{aligned} O_1 &= \bar{u}_\alpha \gamma_\mu P_L c_\alpha \, \bar{u}_\beta \gamma_\mu P_L c_\beta \,, \qquad O_1' = \bar{u}_\alpha P_L c_\alpha \, \bar{u}_\beta P_L c_\beta \,, \\ O_2 &= \bar{u}_\alpha \gamma_\mu P_L c_\beta \, \bar{u}_\beta \gamma_\mu P_L c_\alpha \,, \qquad O_2' = \bar{u}_\alpha P_L c_\beta \, \bar{u}_\beta P_L c_\alpha \,, \end{aligned} \tag{20}$$

where $P_L = \frac{1}{2}(1 - \gamma_5)$. If one neglects QCD running between M_W and m_c, in which case O_2 and O_2' do not contribute, one finds the simple expressions

$$\Delta M_{\mathrm{box}} = \frac{2}{3\pi^2} X_D \, \frac{(m_s^2 - m_d^2)^2}{m_c^2} \left[1 - \frac{5}{4} \frac{B_D'}{B_D} \frac{m_D^2}{(m_c + m_u)^2} \right] , \tag{21}$$

ratio	4-quark	6-quark	8-quark
$\Delta M/\Delta M_{\rm box}$	1	$\Lambda^2/m_s m_c$	$(\alpha_s/4\pi)(\Lambda^2/m_s m_c)^2$
$\Delta\Gamma/\Delta M$	m_s^2/m_c^2	$\alpha_s/4\pi$	$\beta_0\,\alpha_s/4\pi$

TABLE I: The enhancement of ΔM and $\Delta\Gamma$ relative to the box diagram at various orders in the OPE. Λ denotes a hadronic scale around $4\pi f_\pi \sim 1\,\text{GeV}$.

$$\Delta\Gamma_{\rm box} = \frac{4}{3\pi} X_D \frac{(m_s^2-m_d^2)^2}{m_c^2}\frac{m_s^2+m_d^2}{m_c^2}\left[1-\frac{5}{2}\frac{B_D'}{B_D}\frac{m_D^2}{(m_c+m_u)^2}\right]. \tag{22}$$

where $X_D = V_{cs}^2 V_{cd}^2 G_F^2 m_D B_D f_D^2$, and $B_D^{(\prime)}$ are bag factors for $O_1^{(\prime)}$, normalized to one in vacuum saturation. Including leading logarithmic QCD effects enhances this estimate of $\Delta\Gamma$ by approximately a factor of two [24]. Eqs. (21) and (22) then lead to the estimates

$$x_{\rm box} \sim \text{few} \times 10^{-5}\,, \qquad y_{\rm box} \sim \text{few} \times 10^{-7}\,. \tag{23}$$

Neglecting m_d/m_s, Eq. (22) is proportional to m_s^6. This factor comes from three sources: (i) m_s^2 from an $SU(3)$ violating mass insertion on each quark line in the box graph; (ii) m_s^2 from an additional mass insertion on each line to compensate the chirality flip from the first insertion; (iii) m_s^2 to lift the helicity suppression for the decay of a scalar meson into a massless fermion pair. The last factor of m_s^2 is absent from Eq. (21) for ΔM; this is why at leading order in the OPE $y_{\rm box} \ll x_{\rm box}$. Higher order terms in the OPE are important, because the chiral suppressions can be lifted by quark condensates instead of by mass insertions, allowing ΔM and $\Delta\Gamma$ to be proportional to m_s^2. This is the minimal suppression required by $SU(3)$ symmetry, as we proved in Sec. III.

The order of magnitudes of the resulting contributions are summarized in Table I. In the first line, the contributions to ΔM are normalized to $\Delta M_{\rm box}$; in the second line, the contributions to $\Delta\Gamma$ are normalized to ΔM at each order. The contribution of 6-quark operators to ΔM is enhanced compared to the 4-quark operators by $\Lambda^2/m_c m_s$. This can be as much as an order of magnitude, if we identify the hadronic scale Λ as $4\pi f_\pi$ [25]. The second chiral suppression can also be lifted, but only at the price of adding a hard gluon, so the contribution of 8-quark operators to ΔM compared to the 6-quark operators is $(\alpha_s/4\pi)(\Lambda^2/m_c m_s)$, which is of order unity.[1] In the case of $\Delta\Gamma$, higher dimension operators are even more important [18]. A 6-quark operator, including a hard gluon to give an on-shell intermediate state, lifts both a chiral suppression and the helicity suppression. The 8-quark operators require a second intermediate particle to contribute to $\Delta\Gamma$, which can be obtained by splitting the gluon already present for ΔM into a quark pair [18], only costing a factor of $\beta_0\,\alpha_s/(4\pi) \sim 1$, where $\beta_0 = 11 - \frac{2}{3}n_f = 9$. Thus, the dominant contributions to x are from 6- and 8-quark operators, while the dominant contribution to y is from 8-quark operators. With some assumptions about the hadronic matrix elements, the resulting estimates are

$$x \sim y \lesssim 10^{-3}\,. \tag{24}$$

[1] We disagree with Ref. [18], in which it was claimed that x and y can arise at first order in m_s. Such contributions were claimed to come from pseudogoldstone loops which diverge in the infrared. However, there are no such divergences because the π, K and η are coupled derivatively. Such a contribution would also be in conflict with our proof in Sec. III that D mixing is second order in $SU(3)$ violating effects.

It is a general feature of OPE based analyses that $x \gtrsim y$. We emphasize that at this time these methods are useful for understanding the order of magnitude of x and y, but not for obtaining reliable quantitative results. For example, to turn the estimates presented here into a systematic computation of x and y would require the calculation of almost two dozen nonperturbative matrix elements.

B. "Exclusive" approach

A long distance analysis of D mixing is complementary to the OPE. Instead of assuming that the D meson is heavy enough for duality to hold between the partonic rate and the sum over hadronic final states, here one assumes that D transitions are dominated by a small number of exclusive processes, which are examined explicitly. This is particularly interesting for studying $\Delta\Gamma$, which depends on real final states in D decays.

For a long distance analysis, it is useful to express the width difference directly in terms of observable decay rates. From Eq. (9), we find

$$y = \sum_n \eta_{\rm CKM}(n)\, \eta_{CP}(n)\, \cos\delta_n \sqrt{\mathcal{B}(D^0 \to n)\, \mathcal{B}(\overline{D}^0 \to n)}\,, \tag{25}$$

where δ_n is the strong phase difference between the $D^0 \to n$ and $\overline{D}^0 \to n$ amplitudes. In decays to many-body final states, the strong phases may have different values in different regions of the Dalitz plot, in which case the sum is supplemented by an integral over the Dalitz plot for each final state. The CKM factor is $\eta_{\rm CKM} = (-1)^{n_s}$, where n_s is the number of s and $\bar{s}$ quarks in the final state. For example, $\eta_{\rm CKM}(K^+K^-) = +1$ and $\eta_{\rm CKM}(K^+\pi^-) = -1$. The factor $\eta_{CP} = \pm 1$ is determined by the CP transformation of the final state, $CP|f\rangle = \eta_{CP}|\bar{f}\rangle$, which is well-defined since $|f\rangle$ and $|\bar{f}\rangle$ are in the same $SU(3)$ multiplet. This factor is the same for the whole multiplet. For example, $\eta_{CP} = +1$ for the decays to K^+K^-, and therefore to all decays into two pseudoscalars. For states where different partial waves contribute with different CP parities, η_{CP} is determined separately for each partial wave. For example, $\eta_{CP}(\rho^+\rho^-) = +1$ for $\rho^+\rho^-$ in a relative s or d wave, and -1 in a p wave. Finally, it is convenient to assemble the final states into $SU(3)$ multiplets and write

$$y = \sum_a y_a\,, \qquad y_a = \eta_{CP}(a) \sum_{n\in a} \eta_{\rm CKM}(n) \cos\delta_n \sqrt{\mathcal{B}(D^0 \to n)\, \mathcal{B}(\overline{D}^0 \to n)}\,, \tag{26}$$

where a indexes complete $SU(3)$ multiplets. By multiplets we refer to the $SU(3)$ representation of the entire final state, not of the individual mesons and baryons.

In practice, we cannot use Eq. (26) to get a reliable estimate of y, since the doubly Cabibbo suppressed rates have large errors, and there are very little data on strong phase differences. To proceed further, we would be forced to introduce model dependent assumptions about the amplitudes and/or their strong phases. For example, in two-body D decays to charged pseudoscalars ($\pi^+\pi^-$, π^+K^-, $K^+\pi^-$, K^+K^-), the $SU(3)$ violation can enter through the decay rates or the strong phase difference. We know experimentally that in some of these rates the $SU(3)$ breaking is sizable; for example $\mathcal{B}(D^0 \to K^+K^-)/\mathcal{B}(D^0 \to \pi^+\pi^-) \simeq 2.8$ [26]. Such effects were the basis for the claim in Ref. [21] that $SU(3)$ is simply inapplicable to D decays. In contrast, we know very little about the strong phase δ which vanishes in the $SU(3)$ limit; Ref. [27] presented a model calculation resulting in $\cos\delta \gtrsim 0.8$, but it is also

possible to obtain much larger values for δ [22]. Using Eq. (26), the value of y_a corresponding to the U-spin doublet of charged π and K is

$$y_{\pi K} = \mathcal{B}(D^0 \to \pi^+\pi^-) + \mathcal{B}(D^0 \to K^+K^-) - 2\cos\delta\sqrt{\mathcal{B}(D^0 \to K^-\pi^+)\,\mathcal{B}(D^0 \to K^+\pi^-)}\,. \tag{27}$$

The experimental central values, allowing for D mixing in the doubly Cabibbo suppressed rates, yield $y_{\pi K} \simeq (5.76 - 5.29\cos\delta) \times 10^{-3}$ [6]. For small δ there is an almost perfect cancellation even though the ratios of the individual rates significantly violate $SU(3)$. In the "exclusive" approach, x is obtained from y by use of a dispersion relation, and one generally finds $x \sim y$.

At this stage, one cannot use the exclusive approach to predict either x or y. Any estimate of their sizes depends on computing $SU(3)$ breaking effects. While this problem is not tractable in general, one source of $SU(3)$ breaking in y, from final state phase space, can be calculated with only minimal and reasonable assumptions. We will estimate these effects in the next section.

V. $SU(3)$ BREAKING FROM PHASE SPACE

We now turn to the contributions to y from on-shell final states. There is a contribution to the D^0 width difference from every common decay product of D^0 and $\overline{D}^0$. In the $SU(3)$ limit, these contributions cancel when one sums over complete $SU(3)$ multiplets in the final state. The cancellations depend on $SU(3)$ symmetry both in the decay matrix elements and in the final state phase space. While there are certainly $SU(3)$ violating corrections to both of these, it is extremely difficult to compute the $SU(3)$ violation in the matrix elements in a model independent manner.[2] However, with some mild assumptions about the momentum dependence of the matrix elements, the $SU(3)$ violation in the phase space depends only on the final particle masses and can be computed. In this section we estimate the contributions to y solely from $SU(3)$ violation in the phase space.[3] We will find that this source of $SU(3)$ violation can generate y of the order of a percent.

The mixing parameter y may be written in terms of the matrix elements for common final states for D^0 and $\overline{D}^0$ decays,

$$y = \frac{1}{\Gamma}\sum_n \int [\mathrm{P.S.}]_n \,\langle \overline{D}^0|\,\mathcal{H}_w\,|n\rangle\langle n|\,\mathcal{H}_w\,|D^0\rangle\,, \tag{28}$$

where the sum is over distinct final states n and the integral is over the phase space for state n. Let us now perform the phase space integrals and restrict the sum to final states F which transform within a single $SU(3)$ multiplet R. The result is a contribution to y of the form

$$\frac{1}{\Gamma}\,\langle \overline{D}^0|\,\mathcal{H}_w \Big\{ \eta_{CP}(F_R) \sum_{n\in F_R} |n\rangle\rho_n\langle n| \Big\} \mathcal{H}_w\,|D^0\rangle\,, \tag{29}$$

---

[2] The $SU(3)$ breaking in matrix elements may be modest even in cases such as $D \to K^+K^-$ and $D \to \pi^+\pi^-$, for which the ratio of measured rates appears to be very far from the $SU(3)$ limit [28].

[3] The phase space difference alone can explain the large $SU(3)$ breaking between the measured $D \to K^*\ell\bar{\nu}$ and $D \to \rho\ell\bar{\nu}$ rates, assuming no $SU(3)$ breaking in the form factors [29]. Recently it was shown that the lifetime ratio of the D_s and D^0 mesons may also be explained this way [30].

where ρ_n is the phase space available to the state n. In the $SU(3)$ limit, all the ρ_n are the same for $n \in F_R$, and the quantity in braces above is an $SU(3)$ singlet. Since the ρ_n depend only on the known masses of the particles in the state n, incorporating the true values of ρ_n in the sum is a calculable source of $SU(3)$ breaking.

This method does not lead directly to a calculable contribution to y, because the matrix elements $\langle n|\mathcal{H}_w|D^0\rangle$ and $\langle\overline{D}^0|\mathcal{H}_w|n\rangle$ are not known. However, CP symmetry, which in the Standard Model and almost all scenarios of new physics is to an excellent approximation conserved in D decays, relates $\langle\overline{D}^0|\mathcal{H}_w|n\rangle$ to $\langle D^0|\mathcal{H}_w|\overline{n}\rangle$. Since $|n\rangle$ and $|\overline{n}\rangle$ are in a common $SU(3)$ multiplet, they are determined by a single effective Hamiltonian. Hence the ratio

$$y_{F,R} = \frac{\sum_{n\in F_R}\langle\overline{D}^0|\,\mathcal{H}_w|n\rangle\rho_n\langle n|\mathcal{H}_w\,|D^0\rangle}{\sum_{n\in F_R}\langle D^0|\,\mathcal{H}_w|n\rangle\rho_n\langle n|\mathcal{H}_w\,|D^0\rangle} = \frac{\sum_{n\in F_R}\langle\overline{D}^0|\,\mathcal{H}_w|n\rangle\rho_n\langle n|\mathcal{H}_w\,|D^0\rangle}{\sum_{n\in F_R}\Gamma(D^0\to n)} \tag{30}$$

is calculable, and represents the value which y would take if elements of F_R were the only channel open for D^0 decay. To get a true contribution to y, one must scale $y_{F,R}$ to the total branching ratio to all the states in F_R. This is not trivial, since a given physical final state typically decomposes into a sum over more than one multiplet F_R. The numerator of $y_{F,R}$ is of order s_1^2 while the denominator is of order 1, so with large $SU(3)$ breaking in the phase space the natural size of $y_{F,R}$ is 5%.

In this analysis, phase space is the only source of $SU(3)$ violation which we will include. Of course, there are other $SU(3)$ violating effects, such as in matrix elements and final state interaction phases. The purpose of our calculation is to explore the rough size of $SU(3)$ violation in exclusive contributions to y. We assume that there is no cancellation with other sources of $SU(3)$ breaking, or between the various multiplets which occur in D decay, that would reduce our result for y by an order of magnitude. This is equivalent to assuming that the D meson is not heavy enough that duality can be expected to enforce such cancellations.

We begin by computing $y_{F,R}$ for D decays to states $F = PP$ consisting of a pair of pseudoscalar mesons such as π, K, η. We neglect $\eta-\eta'$ mixing throughout this analysis, and we have checked that this simplification has a negligible effect on the numerical results. Since PP is symmetric in the two mesons, it must transform as an element of $(8\times8)_S = 27+8+1$. In principle, there are three possible amplitudes for $D^0 \to PP$, one with the pair in a 27 and $\mathcal{H}_w$ in a $\overline{15}$,

$$A_{27}(PP_{27})^{km}_{ij}H^{ij}_k D_m\,, \tag{31}$$

one with the pair in an 8 and and $\mathcal{H}_w$ in a $\overline{15}$,

$$A_8^{\overline{15}}(PP_8)^k_i H^{ij}_k D_j\,, \tag{32}$$

and one with the pair in an 8 and and $\mathcal{H}_w$ in a 6,

$$A_8^6(PP_8)^k_i H^{ij}_k D_j\,. \tag{33}$$

However, the product $H^{ij}_k D_j$ with (ij) symmetric (the $\overline{15}$) is proportional to $H^{ij}_k D_j$ with (ij) antisymmetric (the 6), and the linear combination $A_8 \equiv A_8^{\overline{15}} - A_8^6$ is the only one which appears. Thus there are effectively two invariant amplitudes. There is no $SU(3)$ invariant amplitude to produce the final state in an singlet. Note that since we are assuming $SU(3)$ symmetry in the matrix elements, such final states do not appear in our analysis.

It is straightforward to use these invariants in Eq. (30) to compute $y_{F,R}$. As an example, for $y_{PP,8}$ we obtain

$$\begin{aligned} y_{PP,8} \;=\; s_1^2 \Big[& \frac{1}{2}\,\Phi(\eta,\eta) + \frac{1}{2}\,\Phi(\pi^0,\pi^0) + \frac{1}{3}\,\Phi(\eta,\pi^0) + \Phi(\pi^+,\pi^-) + \Phi(K^+,K^-) - \frac{1}{6}\,\Phi(\eta,K^0) \\ & - \frac{1}{6}\,\Phi(\eta,\overline{K}^0) - \Phi(K^+,\pi^-) - \Phi(K^-,\pi^+) - \frac{1}{2}\,\Phi(K^0,\pi^0) - \frac{1}{2}\,\Phi(\overline{K}^0,\pi^0) \Big] \\ & \times \Big[\frac{1}{6}\,\Phi(\eta,\overline{K}^0) + \Phi(K^-,\pi^+) + \frac{1}{2}\,\Phi(\overline{K}^0,\pi^0) + \mathcal{O}(s_1^2) \Big]^{-1} , \end{aligned} \tag{34}$$

where $\Phi(P_1,P_2)$ is the phase space integral for the decay into mesons P_1 and P_2. In a two-body decay, $\Phi(P_1,P_2)$ is proportional to $|\vec{p}\,|^{2\ell+1}$, where $\vec{p}$ and ℓ are the spatial momentum and orbital angular momentum of the final state particles. For $D^0 \to PP$, the decay is into an s wave. It is straightforward to compute the required ratios from the known pseudoscalar masses,

$$y_{PP,8} = -0.0038\, s_1^2 = -1.8 \times 10^{-4} , \qquad y_{PP,27} = -0.00071\, s_1^2 = -3.4 \times 10^{-5} . \tag{35}$$

These effects are no larger than one finds in the inclusive analysis. This is not surprising, since as in the parton picture, the final states are far from threshold.

Next we turn to final states of the form PV, consisting of a pseudoscalar and a vector meson. Note that three-body final states $3P$ can resonate through PV, and so are partially included here. In this case there is no symmetry between the mesons, so in principle all representations in the combination $8 \times 8 = 27 + 10 + \overline{10} + 8_S + 8_A + 1$ can appear. For simplicity, we take the quark content of the ϕ and ω respectively to be $\bar{s}s$ and $(\bar{u}u + \bar{d}d)/\sqrt{2}$, and consider only the combination which appears in the $SU(3)$ octet. We have checked that reasonable variations of the $\phi - \omega$ mixing angle have a negligible effect on our numerical results. For each representation, there is a single invariant, up to the same degeneracy for the 8 as in the PP case. Along with the analogues of Eqs. (31)–(33) with coefficients B_{27} and $B_8 \equiv B_8^{\overline{15}} - B_8^6$, we have the new invariants

$$B_{10}(PV_{10})_{ijk} H_m^{ij} D_n \epsilon^{kmn} \tag{36}$$

for $\mathcal{H}_w$ in a $\overline{15}$, and

$$B_{\overline{10}}(PV_{\overline{10}})^{ijk} H_i^{lm} D_j \epsilon_{klm} \tag{37}$$

for $\mathcal{H}_w$ in a 6. It turns out that these two invariants are proportional to each other. As before, the $SU(3)$ singlet final state is not produced.

Both because one of the particles is more massive, and because the decay is now into a p wave, the phase space dependence is stronger than for the PP final state. We obtain the ratios

$$\begin{aligned} y_{PV,8_S} &= 0.031\, s_1^2 = 0.15 \times 10^{-2} , & y_{PV,8_A} &= 0.032\, s_1^2 = 0.15 \times 10^{-2} , \\ y_{PV,10} &= 0.020\, s_1^2 = 0.10 \times 10^{-2} , & y_{PV,\overline{10}} &= 0.016\, s_1^2 = 0.08 \times 10^{-2} , \\ y_{PV,27} &= 0.040\, s_1^2 = 0.19 \times 10^{-2} . \end{aligned} \tag{38}$$

For any representation of the final state, the effects are less than one percent.

For the VV final state, decays into s, p and d waves are all possible. Bose symmetry and the restriction to zero total angular momentum together imply that only the symmetric

Final state representation		$y_{F,R}/s_1^2$	$y_{F,R}$ (%)
PP	8	-0.0038	-0.018
	27	-0.00071	-0.0034
PV	8_S	0.031	0.15
	8_A	0.032	0.15
	10	0.020	0.10
	$\overline{10}$	0.016	0.08
	27	0.040	0.19
$(VV)_{s\text{-wave}}$	8	-0.081	-0.39
	27	-0.061	-0.30
$(VV)_{p\text{-wave}}$	8	-0.10	-0.48
	27	-0.14	-0.70
$(VV)_{d\text{-wave}}$	8	0.51	2.5
	27	0.57	2.8

TABLE II: Values of $y_{F,R}$ for two-body final states. This represents the value which y would take if elements of F_R were the only channel open for D^0 decay.

$SU(3)$ combinations appear. Because some VV final states, such as ϕK^*, lie near the D threshold, the inclusion of vector meson widths is quite important. Our model for the resonance line shape is a Lorentz invariant Breit-Wigner normalized on $0 \leq m < \infty$,

$$f(m; m_R, \Gamma_R) = N(m_R, \Gamma_R) \frac{m\,\Gamma_R}{(m^2 - m_R^2)^2 + m^2\Gamma_R^2}, \tag{39}$$

where m_R and Γ_R are the mass and width of the vector meson, and m^2 is the square of its four-momentum in the decay. For s wave decays, we find the ratios

$$y_{VV,8} = -0.081\, s_1^2 = -0.39 \times 10^{-2}, \qquad y_{VV,27} = -0.061\, s_1^2 = -0.30 \times 10^{-2}, \tag{40}$$

while for p wave decays we find

$$y_{VV,8} = -0.10\, s_1^2 = -0.48 \times 10^{-2}, \qquad y_{VV,27} = -0.14\, s_1^2 = -0.70 \times 10^{-2}, \tag{41}$$

and for d waves,

$$y_{VV,8} = 0.51\, s_1^2 = 2.5 \times 10^{-2}, \qquad y_{VV,27} = 0.57\, s_1^2 = 2.8 \times 10^{-2}. \tag{42}$$

With these heavier final states and with the higher partial waves, we see that effects at the level of a percent are quite generic. The vector meson widths turn out to be quite important; if they were neglected, the results in the p- and d-wave channels would be larger by approximately a factor of three. The finite widths soften the $SU(3)$ breaking which otherwise would be induced by a sharp phase space boundary. We have checked that our results are not very sensitive to variations in the line shape used to model the vector meson widths. Again, $4P$ and PPV final states can resonate through VV, so they are partially included here. Our results for two-body final states are summarized in Table II.

As we go to final states with more particles, the combinatoric possibilities begin to proliferate. We will consider the final states $3P$ and $4P$, and for concreteness require that

the pseudoscalars be found in a totally symmetric 8 or 27 representation of $SU(3)$. This assumption is convenient, because the phase space integration is much simpler if it can be performed symmetrically. These final states should be representative; we have no reason to believe that this choice selects final state multiplets for which phase space effects are particularly enhanced or suppressed. Note that $3 \times (\overline{15} + 6)$ contains no representation larger than a 27.

In contrast to the two-body case, for three-body final states the momentum dependence of the matrix elements is no longer fixed by the conservation of angular momentum. The simplest assumption is to take a momentum independent matrix element, with all three final state particles in an s wave. In that case, we find

$$y_{3P,8} = -0.48\, s_1^2 = -2.3 \times 10^{-2}\,, \qquad y_{3P,27} = -0.11\, s_1^2 = -0.54 \times 10^{-2}\,. \tag{43}$$

Note that the $SU(3)$ violation is smaller for the larger multiplets, as more final states enter the sum. It may be that the 8 is in some sense an unusually small representation for three or more particles, and that this mode enhances the $SU(3)$ violation by providing fewer distinct final states among which cancellations can occur. The enhancement of $y_{3P,8}$ over $y_{3P,27}$ is not a peculiarity of s wave decays. We have also considered other matrix elements; for example, if one of the mesons has angular momentum $\ell = 1$ in the D^0 rest frame (balanced by the combination of the other two mesons), then the ratios become

$$y_{3P,8} = -1.13\, s_1^2 = -5.5 \times 10^{-2}\,, \qquad y_{3P,27} = -0.074\, s_1^2 = -0.36 \times 10^{-2}\,. \tag{44}$$

Alternatively, we could imagine introducing a mild "form factor suppression," with a weight such as $\Pi_{i\neq j}(1 - m_{ij}^2/Q^2)^{-1}$, where $m_{ij}^2 = (p_i + p_j)^2$, and $Q = 2\,\mathrm{GeV}$ is a typical resonance mass. The result then changes to

$$y_{3P,8} = -0.44\, s_1^2 = -2.1 \times 10^{-2}\,, \qquad y_{3P,27} = -0.13\, s_1^2 = -0.64 \times 10^{-2}\,. \tag{45}$$

Finally, we have studied the final state with four pseudoscalars, with the mesons in an overall symmetric 8 or a symmetric 27. We take a momentum independent matrix element. There are actually two symmetric 27 representations; we call the 27 the representation of the form $R^{ij}_{kl} = [M^i_m M^m_k M^j_n M^n_l + \text{symmetric} - \text{traces}]$ and the $27'$ the one of the form $R^{ij}_{kl} = [M^i_m M^m_n M^n_k M^j_l + \text{symmetric} - \text{traces}]$. Then we find

$$\begin{aligned} &y_{4P,8} = 3.3\, s_1^2 = 16 \times 10^{-2}\,, \qquad y_{4P,27} = 2.2\, s_1^2 = 11 \times 10^{-2}\,. \\ &y_{4P,27'} = 1.9\, s_1^2 = 9.2 \times 10^{-2}\,. \end{aligned} \tag{46}$$

Here the partial contributions to y are very large, of the order of 10%. This is not surprising, since $4P$ final states containing more than one strange particle are close to D threshold, and the ones containing no pions are kinematically inaccessible. There is no reason to expect $SU(3)$ cancellations to persist effectively in this regime. Our results for $3P$ and $4P$ final states are summarized in Table III.

Formally, one could construct y from the individual $y_{F,R}$ by weighting them by their D^0 branching ratios,

$$y = \frac{1}{\Gamma} \sum_{F,R} y_{F,R} \Big[\sum_{n \in F_R} \Gamma(D^0 \to n) \Big]\,. \tag{47}$$

However, the data on D decays are neither abundant nor precise enough to disentangle the decays to the various $SU(3)$ multiplets, especially for the three- and four-body final states.

Final state representation		$y_{F,R}/s_1^2$	$y_{F,R}$ (%)
$(3P)_{s\text{-wave}}$	8	-0.48	-2.3
	27	-0.11	-0.54
$(3P)_{p\text{-wave}}$	8	-1.13	-5.5
	27	-0.07	-0.36
$(3P)_{\text{form-factor}}$	8	-0.44	-2.1
	27	-0.13	-0.64
$4P$	8	3.3	16
	27	2.2	9.2
	$27'$	1.9	11

TABLE III: Values of $y_{F,R}$ for three- and four-body final states.

Final state	fraction
PP	5%
PV	10%
$(VV)_{s\text{-wave}}$	5%
$(VV)_{d\text{-wave}}$	5%
$3P$	5%
$4P$	10%

TABLE IV: Total D^0 branching fractions to classes of final states, rounded to nearest 5% [26].

Nor have we computed $y_{F,R}$ for all or even most of the available representations. Instead, we can only estimate individual contributions to y by assuming that the representations for which we know $y_{F,R}$ to be typical for final states with a given multiplicity, and then to scale to the total branching ratio to those final states. The total branching ratios of D^0 to two-, three- and four-body final states can be extracted from Ref. [26]. The results are presented in Table IV, where we round to the nearest 5% to emphasize the uncertainties in these numbers. Close to half of all D^0 decays are accounted for in this table; the rest are decays to other modes such as PPV, decays to states with $SU(3)$ singlet mesons, decays to higher resonances, semileptonic decays, and other suppressed processes. Based on data in the channel $\overline{K}^{0*}\rho^0$, the VV final state is dominantly CP even, consistent with an equal distribution between s and d wave decays (although favoring a small s wave enhancement).

We estimate the contribution to y from a given type of final state by taking the product of the typical $y_{F,R}$ found in our calculation with the approximate branching ratios given in Table IV. Such estimates are necessarily crude, but they are sufficient to give a sense of the order of magnitude of y which is to be expected. While in most cases the contributions are small, of the order of 10^{-3} or less, we observe that D^0 decays to nonresonant $4P$ states naturally contribute to y at the percent level. The reason for such unusually large $SU(3)$ violating effects in y is that approximately 10% of D^0 decays are to final states for which the complete $SU(3)$ multiplets are not kinematically accessible.

It should be noted that for D decays to final states so close to threshold, our argument that D mixing is second order in $SU(3)$ violation is inapplicable, because its underlying

assumption that $SU(3)$ violation enters perturbatively is not met. In particular the proof fails near D threshold, if the decay is either to weakly decaying final states or to hadrons with widths Γ which are smaller than m_s. In either case, the phase space available for the decay can vary rapidly on the scale of m_s, spoiling the analytic expansion. For decays to hadronic resonances, Γ/m_s is a small parameter which is not analytic as $m_s \to 0$. For decays to long lived mesons, the Θ-functions which fix the boundaries of phase space are not analytic functions of their arguments, which in turn depend on m_s through the masses of the final state hadrons. In this way, the generic m_s^2/m_D^2 suppression is lifted and we find larger $SU(3)$ violation in y just at the point that the conditions of the proof are not satisfied. We will see a similar failure of $SU(3)$ cancellations when we study D mixing induced by resonances in Sec. VI.

We have not considered all possible final states which might give large contributions to y. In particular, the branching ratio for $D^0 \to K^- a_1^+$ is $(7.3 \pm 1.1)\%$ [26], even though this final state is quite close to D threshold. Unfortunately, the identities of the $SU(3)$ partners of the $a_1(1260)$, which has $J^{PC} = 1^{++}$, are not well established. While it is natural to identify the $K_1(1400)$ as the corresponding strange axial vector meson, and the $f_1(1285)$ as the analogue of the ω, there is no natural candidate for the $\bar{s}s$ analogue of the ϕ. The size of y_{PV^*} is quite sensitive to this choice, as well as to the value taken for the poorly measured width of the a_1. If we take the $\bar{s}s$ state to be the $f_1(1420)$, and $\Gamma(a_1) = 400\,\mathrm{MeV}$, we find $y_{PV^*,8_S} = 1.8\%$. If instead we take the $f_1(1510)$, we find $y_{PV^*,8_S} = 1.7\%$. With $\Gamma(a_1) = 250\,\mathrm{MeV}$, these numbers become 2.5% and 2.4%, respectively. Although it is clear that percent level contributions to y are possible from $SU(3)$ violation in this channel, the data are still too poor to draw firm conclusions.

On the basis of this analysis, in particular as applied to the $4P$ final state, we would conclude that y on the order of a percent would be completely natural. Anything an order of magnitude smaller would require significant cancellations which do not appear naturally in this framework. Cancellations would be expected only if they were enforced by the OPE, that is, if the charm quark were heavy enough that the "inclusive" approach were applicable. The hypothesis underlying the present analysis is that this is not the case.

VI. $SU(3)$ BREAKING FROM NEARBY RESONANCES

One interesting feature of the D^0 is that there are excited mesons with masses close to m_D. As a result, it would not be unnatural for K resonances to play an important role in D decays. This possibility has already been explored in the literature [20, 27, 31, 32] . Here we explore $SU(3)$ breaking in the resonance contribution to D mixing.

We are interested in the process $D^0 \to R \to \overline{D}^0$, where R is a resonance with mass m_R and width Γ_R. Only spin zero resonances are relevant. The contribution of a single state to the D mass and width differences is given by

$$y_R^{\rm res} = \eta_R \frac{|H_R|^2}{\Gamma} \frac{\Gamma_R}{(m_D^2 - m_R^2)^2 + m_D^2 \Gamma_R^2}, \qquad x_R^{\rm res} = \eta_R \frac{2|H_R|^2}{\Gamma\, m_D} \frac{m_D^2 - m_R^2}{(m_D^2 - m_R^2)^2 + m_D^2 \Gamma_R^2}. \quad (48)$$

where $|H_R|^2 \equiv \langle \overline{D}^0 | \mathcal{H}_W | R \rangle \langle R | \mathcal{H}_W | D^0 \rangle$ parameterizes the couplings of R to D^0 and $\overline{D}^0$, and η_R is the CP eigenvalue of the $SU(3)$ multiplet within which the resonance resides. If we assume the absence of direct CP violation in D decays, then $\langle \overline{D}^0 | \mathcal{H}_W | R \rangle$ may be related to

$\langle R|\mathcal{H}_W|D^0\rangle$ by $SU(3)$. The ratio

$$\frac{x_R^{\rm res}}{y_R^{\rm res}} = \frac{2(m_D^2 - m_R^2)}{m_D\,\Gamma_R}\,, \tag{49}$$

is independent of H_R. Significant contributions to x and y from the resonance R are possible only if $m_D^2 - m_R^2 \lesssim m_D\Gamma_R$.

As a concrete example, consider the $K^*(1950)$, a positive parity excited kaon which, because of its large width, may play an important role in mediating $D^0 \to K^-\pi^+$. Fitting the $K^*(1950)$ contribution to the observed $D \to K\pi$ rates, one finds that resonance mediation could be as large as the usual quark tree amplitude [27]. We can estimate an upper bound on the contribution of $K^*(1950)$ to y by assuming that the resonance is completely responsible for $D \to K\pi$. The limit is given by

$$\frac{|\Delta\Gamma|}{\Gamma} \le \frac{\langle\overline{D}^0|\mathcal{H}_W|K_H\rangle\langle K_H|\mathcal{H}_W|D^0\rangle}{\langle D^0|\mathcal{H}_W|K_H\rangle\langle K_H|\mathcal{H}_W|D^0\rangle} \times \frac{\mathcal{B}(D^0\to K\pi)}{\mathcal{B}(K_H\to K\pi)}\,, \tag{50}$$

where we denote the $K^*(1950)$ by K_H. With $\mathcal{B}(D^0 \to K\pi) \simeq 6\%$ and $\mathcal{B}(K_H \to K\pi) \simeq 52\%$, we find $|y| \le 0.06\, s_1^2 \simeq 3\times 10^{-3}$. If D mixing is mediated by a resonance, then we expect x and y to be roughly of the same size.

This upper bound is too generous, because we have not included the suppression from $SU(3)$ cancellations. Note that our proof of Sec. III, that $SU(3)$ violation appears only at second order in m_s, applies only so long as $m_s \ll \Gamma_R$. While this must be true in the limit $m_R \sim m_D \to \infty$, in which case Γ_R scales as m_c, the ratio m_s/Γ_R may not be small for resonances near the physical D mass. Therefore, $SU(3)$ cancellations may be less effective for resonances than for real final states.

The resonances in question fall into a positive parity 8 of $SU(3)$, consisting of states which we will denote (π_H, K_H, η_H). If these states were degenerate and had equal widths, their contributions to D mixing would cancel. A measure of the actual effectiveness of this cancellation is the contribution of the entire multiplet relative to that of the K_H. The $SU(3)$ partners of the $K^*(1950)$ have not been conclusively identified. Instead of speculating, we will explore the efficiency of $SU(3)$ cancellations qualitatively by taking the simple mass relations

$$m_{\pi_H} = m_{K_H} - m_s\,, \qquad m_{\eta_H} = m_{K_H} + \frac{1}{3}\,m_s\,, \tag{51}$$

and assuming that the widths of the π_H and η_H are the same as $\Gamma(K_H) \simeq 200\,\text{MeV}$. Then

$$y^{\rm res} = y_{K_H}^{\rm res} - \frac{1}{4}\,y_{\pi_H}^{\rm res} - \frac{3}{4}\,y_{\eta_H}^{\rm res}\,. \tag{52}$$

For $m_s = 150\,\text{MeV}$, we find $y^{\rm res}/y_{K_H}^{\rm res} = 0.27$. The cancellations are somewhat less effective for $x^{\rm res}$, with $x^{\rm res}/x_{K_H}^{\rm res} = 0.50$. We see that even for the $K^*(1950)$, likely to be the most favorable for inducing a large effect, $SU(3)$ cancellations reduce the contributions to $x^{\rm res}$ and $y^{\rm res}$. We conclude that it would be quite unlikely for resonances to make a contribution to y at the level of one percent.

VII. CONCLUSIONS

The motivation most often cited in searches for $D^0-\overline{D}^0$ mixing is the possibility of observing a signal from new physics which may dominate over the Standard Model contribution.

But to look for new physics in this way, one must be confident that the Standard Model prediction does not already saturate the experimental bound. Previous analyses based on short distance expansions have consistently found $x, y \lesssim 10^{-3}$, while naïve estimates based on known $SU(3)$ breaking in charm decays allow an effect an order of magnitude larger. Since current experimental sensitivity is at the level of a few percent, the difference is quite important.

In this paper we have performed a general $SU(3)$ analysis of the contributions to y. We proved that if $SU(3)$ violation may be treated perturbatively, then $D^0 - \overline{D}^0$ mixing in the Standard Model is generated only at second order in $SU(3)$ breaking effects. Within the exclusive approach, we identified an $SU(3)$ breaking effect, $SU(3)$ violation in final state phase space, which can be calculated with minimal model dependence. We found that phase space effects alone can provide enough $SU(3)$ breaking to induce $y \sim 10^{-2}$. Large effects in y appear for decays to final states close to D threshold, where an analytic expansion in $SU(3)$ violation is no longer possible.

We believe that this is an important result. Despite the large uncertainties, this is the first model independent calculation to give y close to the present experimental bounds. While some degree of cancellation is possible between different multiplets, as would be expected in the $m_c \to \infty$ limit, or between $SU(3)$ breaking in phase space and in matrix elements, it is not known how effective these cancellations are. The most reasonable assumption in light of our analysis is that they are not significant enough to result in an order of magnitude suppression of y. Therefore, any future discovery of a D meson width difference should not by itself be interpreted as an indication of the breakdown of the Standard Model.

However, our analysis does not amount to a Standard Model calculation of y. First, we have considered only $SU(3)$ breaking from phase space, and have not included any symmetry breaking in the matrix elements. Second, we have not calculated the contributions from all final states. Had we done so, we would still need very precise experimental data in order to disentangle the various $SU(3)$ multiplets and combine the results into an overall value of y. Third, we have assumed that the charm quark is not heavy enough for duality to enforce significant cancellations between the various nonleptonic D decay channels, although some degree of cancellation is to be expected.

The implication of our results for the Standard Model prediction for x is less apparent. While analyses based on the "inclusive" approach generally yield $x \gtrsim y$, it is not clear what the "exclusive" approach predicts. The effect of $SU(3)$ breaking in phase space in x is softer than in y, so one would expect $x < y$ from our analysis. Thus if $x > y$ is found experimentally, it may still be an indication of a new physics contribution to x, even if y is also large. On the other hand, if $y > x$ then it will be hard to find signals of new physics, even if such contributions dominate ΔM. The linear sensitivity to new physics in the analysis of the time dependence of $D^0 \to K^+\pi^-$ is from $x' = x\cos\delta + y\sin\delta$ and $y' = y\cos\delta - x\sin\delta$ instead of x and y. If $y > x$, then δ would have to be known precisely for these terms to be sensitive to new physics in x.

There remain large uncertainties in the Standard Model predictions of x and y, and values near the current experimental bounds cannot be ruled out. Therefore, it will not be easy to find a clear indication of physics beyond the Standard Model in $D^0 - \overline{D}^0$ mixing measurements. We believe that at this stage the only robust potential signal of new physics in $D^0 - \overline{D}^0$ mixing is CP violation, for which the Standard Model prediction is very small. Unfortunately, if y is larger or much larger than x, then the observable CP violation in $D^0 - \overline{D}^0$ mixing is necessarily small, even if new physics dominates x. Therefore, searching

for new physics and CP violation in $D^0 - \overline{D}^0$ mixing should aim at precise measurements of both x and y, and at more complicated analyses involving the extraction of the strong phase in the time dependence of doubly Cabibbo suppressed decays.

Acknowledgments

It is a pleasure to acknowledge helpful discussions with Yossi Nir, Helen Quinn and Martin Savage. We thank the Aspen Center for Physics for hospitality while portions of this work were completed. A.F. was supported in part by the U.S. National Science Foundation under Grant PHY–9970781, and is a Cottrell Scholar of the Research Corporation. Y.G. was supported in part by the Israel Science Foundation under Grant No. 237/01-1, and by the Technion V.P.R Fund – Harry Werksman Research Fund. Z.L. was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098. The work of Y.G. and Z.L. was also supported in part by the United States–Israel Binational Science Foundation (BSF) through Grant No. 2000133. A.P. thanks the Cornell University Theory Group, where part of this work was performed.

---

[1] H. Georgi, Phys. Lett. B297, 353 (1992).
[2] T. Ohl, G. Ricciardi and E. H. Simmons, Nucl. Phys. B403, 605 (1993).
[3] A.A. Petrov, Phys. Rev. D56, 1685 (1997).
[4] A. Datta and M. Khambakhar, Zeit. Phys. C27, 515 (1985).
[5] J. Donoghue, E. Golowich, B. Holstein and J. Trampetic, Phys. Rev. D33, 179 (1986).
[6] S. Bergmann, Y. Grossman, Z. Ligeti, Y. Nir and A.A. Petrov, Phys. Lett. B486, 418 (2000).
[7] E.M. Aitala *et al.*, E791 Collaboration, Phys. Rev. Lett. 83, 32 (1999).
[8] J.M. Link *et al.*, FOCUS Collaboration, Phys. Lett. B485, 62 (2000).
[9] D. Cronin-Hennessy *et al.*, CLEO Collaboration, hep-ex/0102006.
[10] K. Abe *et al.*, BELLE Collaboration, BELLE-CONF-0131.
[11] B. Aubert *et al.*, BaBar Collaboration, hep-ex/0109008.
[12] R. Godang *et al.*, CLEO Collaboration, Phys. Rev. Lett. 84, 5038 (2000).
[13] G. Brandenburg *et al.*, CLEO Collaboration, Phys. Rev. Lett. 87, 071802 (2001).
[14] E.M. Aitala *et al.*, E791 Collaboration, Phys. Rev. D57, 13 (1998).
[15] J. P. Silva and A. Soffer, Phys. Rev. D61, 112001 (2000).
[16] M. Gronau, Y. Grossman and J. L. Rosner, Phys. Lett. B508, 37 (2001).
[17] H.N. Nelson, hep-ex/9908021.
[18] I. Bigi and N. Uraltsev, Nucl. Phys. B592, 92 (2000).
[19] F. Buccella, M. Lusignoli and A. Pugliese, Phys. Lett. B379, 249 (1996).
[20] E. Golowich and A.A. Petrov, Phys. Lett. B427, 172 (1998).
[21] L. Wolfenstein, Phys. Lett. B164, 170 (1985).
[22] P. Colangelo, G. Nardulli and N. Paver, Phys. Lett. B242, 71 (1990).
[23] T.A. Kaeding, Phys. Lett. B357, 151 (1995).
[24] A.A. Petrov, hep-ph/0009160: E. Golowich, "CP violation in the charm sector," *Workshop on CP Violation, Adelaide, Australia, July 3-8, 1998.*
[25] A. Manohar and H. Georgi, Nucl. Phys. B 234, 189 (1984).

[26] D.E. Groom *et al.*, Particle Data Group, Eur. Phys. J. C15, 1 (2000).
[27] A.F. Falk, Y. Nir and A.A. Petrov, JHEP 12, 019 (1999).
[28] M.J. Savage, Phys. Lett. B257, 414 (1991).
[29] Z. Ligeti, I.W. Stewart and M.B. Wise, Phys. Lett. B420, 359 (1998).
[30] S. Nussinov and M.V. Purohit, hep-ph/0108272.
[31] E. Golowich, Phys. Rev. D24, 676 (1981).
[32] M. Gronau, Phys. Rev. Lett. 83, 4005 (1999).

Ernest Orlando Lawrence Berkeley National Laboratory
One Cyclotron Road ⁞ Berkeley, California 94720

Prepared for the U.S. Department of Energy under Contract No. DE-AC03-76SF00098