

Lawrence Berkeley National Laboratory

Recent Work

Title

NUMERICAL SIMULATION OF FLUID INJECTION INTO DEFORMABLE FRACTURES

Permalink

<https://escholarship.org/uc/item/6zx6267r>

Authors

Noorishad, J.
Doe, T.W.

Publication Date

1981-05-01



Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA, BERKELEY

EARTH SCIENCES DIVISION

RECEIVED
LAWRENCE
BERKELEY LABORATORY

AUG 31 1981

Submitted to Journal of Geophysical Research

LIBRARY AND
DOCUMENTS SECTION

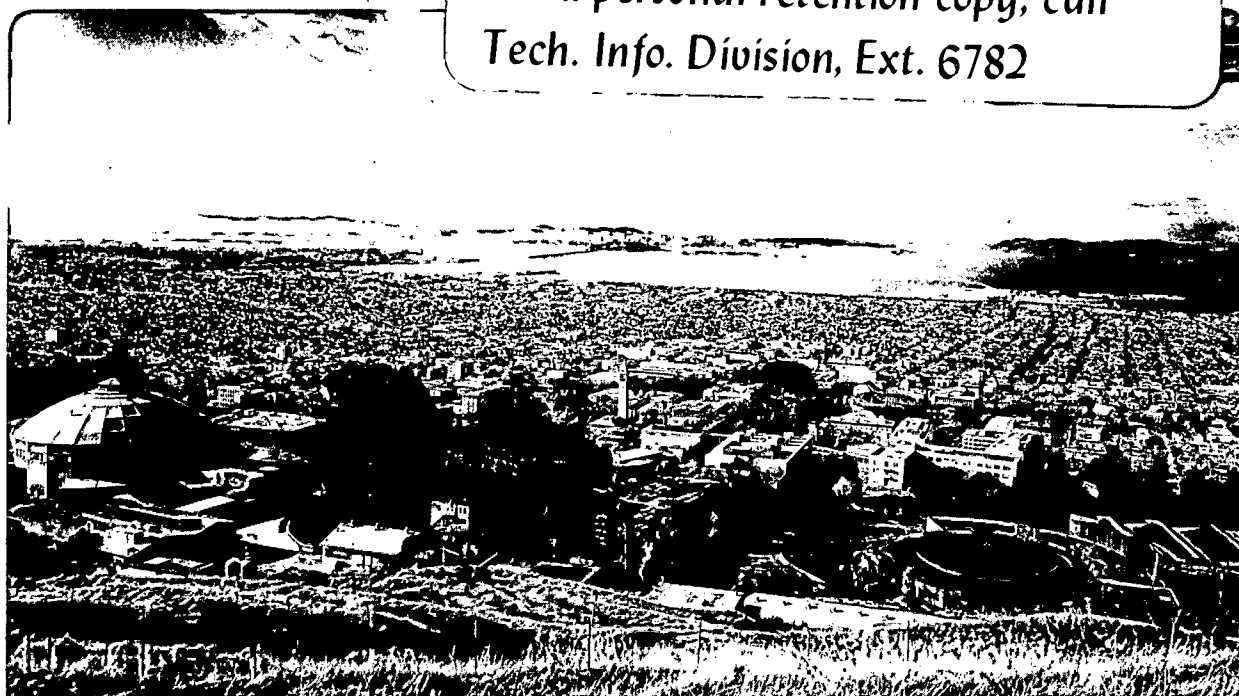
NUMERICAL SIMULATION OF FLUID INJECTION INTO DEFORMABLE
FRACTURES

J. Noorishad and T.W. Doe

May 1981

TWO-WEEK LOAN COPY

*This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 6782*



LBL-12833
-2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

NUMERICAL SIMULATION OF FLUID INJECTION
INTO DEFORMABLE FRACTURES

J. Noorishad and T. W. Doe

Lawrence Berkeley Laboratory

University of California, Berkeley, CA 94720

ABSTRACT

The problem of fluid flow from a well into a horizontal fracture has been studied using a nonlinear coupled hydromechanical finite element model. Such approach is required due to the strong nonlinearity and anisotropy of deformation and fluid flow moduli introduced by the presence of the fracture. The resulting nonuniform total stress field invalidates the application of conventional fluid flow techniques to the problem. Stable convergent solution from the above analysis are verified by a coupled steady state algorithm. Results from the two cases; constant head injection and constant flow rate injection, depart significantly from that which would be predicted from conventional well test solutions. For example, in the transient flow rate case, flow rate initially declines, as predicted by the noncoupled solution; however, at later time as the pressure front has advanced radially, the flow rate increases until steady flow conditions are achieved due to a constant pressure outer boundary.

INTRODUCTION

Injection of fluids into fractured rocks is a common practical concern in many phases of civil, mining, and petroleum engineering. In civil and mining engineering the fluid injection problem arises in grouting dam foundations, pressure tunnels, and in permeability testing. In petroleum engineering, the injection of fluids is of concern in water flooding, well treatment and hydraulic fracturing.

The traditional means of predicting injection flow rate or well pressure behavior is by use of solutions of the general transient equations of fluid flow in porous or equivalent porous media [Dewiest, 1968; Snow 1968]. Availability of numerical techniques and computers has extended this analysis capability and made the simulation of fluid flow in fractured and/or porous media a possible task. In these treatments the deformability of the medium is thought to be represented through the concept of specific storage. This assumption, inherently requires the existence of a uniform total stress field unaffected by temporal and spacial variations of the fluid pressure field [Terzaghi, 1925]. Such conditions generally, prevail for regional fluid flow problems in saturated porous continua within acceptable range of approximation. However, the requirements cannot be met where rapid variations of pressure field are taking place or when the media possess strong nonlinearity and anisotropy with regard to different deformation moduli and fluid flow properties [Snow, 1968; Noorishad, 1971; Gale, 1975]. Therefore, the traditional fluid flow treatments may no longer be applicable in such situations. Development of the general theory of consolidation by Biot [1941] has provided the basis on which more realistic attempts of predicting fluid flow behavior of deformable media have been made [e.g. Ghaboussi and Wilson, 1971]. In

this study a new coupled hydromechanical finite element technique, developed by Ayatollahi et al. [1981] and generalized by Noorishad et al. [1981], is used to simulate a more realistic behavior of a deep lying fracture subject to fluid injection under constant head and constant flow conditions. We have found that the form of transient well bore pressure or flow rate curve is basically different from conventional treatments. In verifying these results the fluid flow equation of the coupled phenomena is reformulated in an approximate manner and a finite element algorithm for the coupled steady state phenomena is worked out.

FIELD EQUATIONS OF COUPLED HYDROMECHANICAL PHENOMENA

Field equations originally formulated by Biot [1941] in his general theory of consolidation are the basis for the hydromechanical analysis of the fractured porous media [Ayatollahi et al. 1981]. These relationships for the isotropic continuum portions of the medium include the stress-strain equation, the fluid flow law, and the law of static equilibrium as follows:

$$\tau_{ij} = 2\mu e_{ij} + \lambda \delta_{ij} \delta_{kl} e_{kl} + \alpha \delta_{ij} P \quad (1)$$

$$\zeta = -\alpha \delta_{ij} e_{ij} + \frac{1}{M} P = -\alpha e + \frac{1}{M} P$$

$$\frac{\partial \zeta}{\partial t} = \nabla \cdot \frac{k}{\eta_l} (\nabla P + \rho_l g \nabla z) \quad (2)$$

$$\frac{\partial \tau_{ij}}{\partial x_j} + \rho_s f_i = 0 \quad (3)$$

where

- τ_{ij} = solid stress tensor
- e_{ij} = solid strain tensor
- e = bulk dilatation equal to $e_{11} + e_{22} + e_{33}$
- P = fluid pressure

- δ_{ij} = Kronecker delta function
 α = Biot's coupling coefficient
 M = Biot's storativity coefficient
 ζ = fluid volume strain
 $\underline{k}$ = intrinsic permeability tensor of porous parts
 η_l = liquid dynamic viscosity
 ρ_l = fluid mass density
 ρ_s = average porous solid density
 f_i = body force components
 μ, λ = Lamé's elasticity constants

Similarly for the fracture portion of the medium, the corresponding field equations are

$$\tau_i = \delta_{ij} C_{jk} \bar{U}'_k + \alpha \delta_{i2} P \quad (4)$$

$$\zeta = -\alpha \frac{\bar{U}'_2}{2b} + \frac{1}{M} P$$

$$\frac{\partial \zeta}{\partial t} = \nabla \frac{k}{\eta} (P + \rho_l g z) \quad (5)$$

$$\tau_i \Big|_{US} - \tau_i \Big|_{LS} = 0 \quad (6)$$

Equation (4) is the stress strain relationship in which τ_i stands for normal and tangential stress, C_{jk} are the components of fracture moduli (stiffness) matrix, $\bar{U}'_k$ represents the average net tangential or transversal deformation, and $2b$ is the initial fracture aperture. Coefficient α and M are equivalent Biot constants for fractures and k is the fracture permeability. Equation (5) represents the fracture fluid flow law and equation (6) balances the forces across the upper and lower faces of the fractures.

The above field equations for the porous solid and the fracture along with initial and boundary conditions completely define the mixed initial boundary value problem of fluid flow in a deformable fractured porous rocks [Noorishad et al. 1981].

SOLUTION APPROACH

Complexity of the phenomena of flow of fluids in deformable fractured rocks practically inhibits analytical solution attempts. Only simple problems involving linear isotropic porous media have been solved. Extension of the existing variational finite element method for linear elastic porous media [Ghaboussi and Wilson, 1971] by Ayatollahi et al. [1981] and its generalization by Noorishad et al. [1981] provides the numerical formulation of the problem as follows:

$$\begin{aligned}\underline{K} \underline{U} + \underline{C} \underline{P} &= \underline{F} \\ \underline{C}^T \underline{U} + \underline{E} \underline{P} + l^* \underline{H} \underline{P} &= -l^* \underline{Q}\end{aligned}\tag{7}$$

where

$\underline{K}$ = structural stiffness matrix of the fractured media

$\underline{C}$ = coupling matrix

$\underline{E}$ = storativity matrix

$\underline{H}$ = fluid conductivity matrix

$\underline{F}$ = nodal force vector

$\underline{Q}$ = nodal flow vector

In above equation $\underline{U}$ and $\underline{P}$ represent the nodal displacement and pressure vectors and l^* stands for time integration.

Transient Solution

A predictor corrector scheme is used to obtain the time marching solution of equation (7). The development of the above equations and the details of the solution are explained in references 5 and 1.

Steady State Solution

Simple modification of equation (8) offers a way of directly determining steady state solutions for coupled hydromechanical problems. Considering the basic assumptions regarding the nature of the dependent variables [Finlayson, 1972], the term $l^* \underline{H} \underline{P}$ in equation (7) could be replaced by $\underline{H} t_{st} \underline{P}_{st}$ for linear problems. In the latter expression t_{st} equals a very large time value and $\underline{P}_{st}$ represents the steady state pressure to be found. Implementing this operation in equation (7) results in the following equation

$$\underline{K} \underline{U}_{st} + \underline{C} \underline{P}_{st} = \underline{F} \quad (8)$$

$$\underline{C}^T \underline{U}_{st} - (\underline{E} + t_{st} \underline{H}) \underline{P}_{st} = -t_{st} \underline{Q}$$

A single direct solution of equation (8) provides the steady state results for coupled hydromechanical problems in porous elastic solids.

For nonlinear materials, such as fractured rocks, equation (8) is nonlinear on the account of the dependency of $\underline{K}$ and $\underline{H}$ matrices on the dependent variables. Therefore, the term $l^* \underline{H} \underline{P}$ can only be replaced by $t_{st} \underline{H}_{st} \underline{P}_{st}$ which appears to make the solution complicated. However, the same formulation as in equation (8) in which $\underline{K}$ and $\underline{H}$, representing the first approximations of final values of $\underline{K}_{st}$ and $\underline{H}_{st}$ can be used to start an iteration loop in which all nonlinear quantities are updated until convergence to true value of $\underline{P}_{st}$ and $\underline{U}_{st}$ is achieved. At this point, the nonlinear structural stiffness matrix $\underline{K}$ and the system conductivity matrix $\underline{H}$ will have taken their final steady state values.

This steady state modification, implemented in the computer code, provides a useful option for prompt assessment of the late time results of the transient problems such as the ones considered below.

APPLICATION TO FLUID INJECTION PROBLEMS

The problem considered here, as sketched in Figure 1 is that of a horizontal fracture in a low permeability, rigid rock (e.g. granite) located at a depth of 100 meters. A 0.05 m radius well intersects the fracture at its center. It is assumed that the fracture encounters a region of high permeability capable of maintaining a constant head at a radial distance of 150 m from the well. The rock has a density of 2600 kg/m^3 . The hydrologic system is at ground level hydrostatic equilibrium before fluid injection takes place. Other system properties are shown in Table 1. To simulate this problem by a two dimensional model certain assumptions have to be made. The overburden is replaced by a 10 m slab of rock with a load on its upper surface equal to the remaining 90 m of overburden. Due to the very small deformations involved not much accuracy is lost by ignoring the tangential tractions which might be mobilized at the top of the modeled slab. To insure static equilibrium, corresponding in situ stress and initial pressures are input in the model. The problem is assumed to have axial symmetry, and its outer boundary is restricted to vertical movement only. The fracture is modeled by a joint element [Goodman et al., 1968] with initial stiffness of $K_s = 0.5 \text{ GPa/m}$, $K_m = 1.6 \text{ GPa/m}$ and assumed to have a linear behavior in the load-deformation range under consideration. To represent the rock rigidity, the Young's modulus is assumed to have a value of 70 GPa. The rock is of sufficiently low permeability (10^{-20} m^2), such that under the hydraulic conditions and the time periods considered the flow into the rock itself would be confined to the 10 m slab of the model.

Constant Head Injection

Steady state analysis - the system is pressurized by injection under 50 m of constant differential head. Note that the injection head is less

Table 1. Data Used for Various Analysis of Fracture Injection Problem.

Material	Property	Value
Fluid	Mass density, ρ_l	$9.80 \times 10^2 \text{ kg/m}^3$
	Compressibility, β_p	$5.13 \times 10^{-1} \text{ GPa}^{-1}$
	Dynamic viscosity, η_l	$2.80 \times 10^{-4} \text{ N sec/m}^2$
Rock	Young's modulus, E_s	70.0 GPa, 0.7 GPA
	Poisson's ratio, ν_s	0.25
	Mass density, ρ_s	$2.5 \times 10^3 \text{ kg/m}^3$
	Porosity, ϵ	0.15
	Intrinsic Permeability, k	10^{-20} m^2
	Biot's constant, M	1.47GPa, 14.0 GPa*
	Biot's coupling constant, α	1.0, 0.0*
Fractures	Initial normal stiffness, K_n	1.60 GPa/m
	Initial tangential stiffness, K_s	0.50 GPa/m
	Cohesion, C_o	0.0
	Friction angle, δ	30°
	Initial aperture, b	10^{-4} m
	Porosity, ϵ	0.50
	Biot's constant, M	1.47 GPa, 14.0 GPa*
	Biot's constant, α	1.0,0.0*

*Used in the uncoupled case.

than one quarter of the overburden stress on the fracture. The steady state pressure profile in the fracture is plotted in figure 2. The resulting steady state flow rate is $0.381 \text{ m}^3/\text{sec}$.

In a second run the calculated flow rate above is used to simulate a constant flow rate injection problem. The results of this run plots precisely on the pressure profile curve of figure 2 as is expected, thus adding confidence to the soundness of the steady state algorithm.

Transient analysis - To simulate the transient behavior of the system in the above problem, the coupled finite element technique (Noorishad et al. 1981) is used. The resulting values of the well flow rates are plotted in figure 3. The curve exhibits an initial exponential decline lasting about 10 seconds followed by a lengthy slow rise continuing for about a 1000 seconds until a steady state trend of much longer duration takes over. The early time behavior follows the familiar pattern of the constant injection head fluid flow problems. To delineate this behavior, comparison needs to be made between fluid flow equations of the coupled (hydromechanical) problem and that of the uncoupled or equivalent non-deformable problem. In the coupled theory, the flow equation as expressed in equation (2) or equation (6) is of the following form:

$$\nabla \cdot \underline{k} \nabla (P + \rho_g z) = \frac{\partial}{\partial t} \left(-\alpha e + \frac{1}{M} \right) P$$

Assuming solid grains are incompressible, the Biot coefficient M can be replaced by inverse products of porosity ϵ and fluid compressibility β_p , resulting in the following presentation of the above equation:

$$\nabla \cdot \underline{k} \nabla (P + \rho_g z) = \frac{\partial}{\partial t} (-\alpha e + \epsilon \beta_p) P \quad (10)$$

Under the condition of uniform total stresses in the fluid flow region, such as those assumed in one dimensional theory of consolidation, it is possible to write:

$$\tau' = -P \quad (11)$$

where τ' is the effective normal stress in the fracture. Using the definition of compressibility one easily finds

$$\beta_r = -e/P \quad (12)$$

notice that τ' , P and e are incremental in nature and represent deviations from initial state. therefore, using e from equation (12) in equation (10) yields

$$\nabla \cdot k \nabla T = \frac{\partial}{\partial t} (\beta_r + \epsilon \beta_p) P \quad (13)$$

which is the same as the familiar fluid flow equation with $(\beta_r + \epsilon \beta_p)$ representing the constant specific storage parameters S_g . Thus, under specific conditions, the true fluid flow behavior of a deformable porous elastic continua may comparably be obtained by a fluid flow analysis alone, using the conventional fluid flow equation. The traditional approach has also been employed for analysis of fluid flow behavior of nonlinear material, such as fractured rocks, by using an equivalent specific storage for fractures defined as $S_g^f = 1/(2bK_n)$ [e.g. Snow 1968]. Such extensions of the concept of specific storage may hold valid for extensively fractured rocks, with frictionless fractures, under special circumstances. However, the combination of anisotropy and nonlinearity due to presence of fractures and the consequent pressure-coupled total stress field, invalidate application to fluid flow problems in most fractured rocks. To demonstrate this point, the problem under consideration is solved in uncoupled manner (i.e. fluid flow analysis alone) using an

equivalent storage value of $S_S^f = \frac{1}{2bK_n}$ equal to 0.06 MPa^{-1} (neglecting the much smaller fluid compressibility contribution). The results, checked also by Jacob-Lohman [1951] analytic solution, plot much higher than the coupled analysis curve indicating need for a much smaller specific storage value in the analysis. With some trial and error, using a specific storage of 2.0 GPa^{-1} , an uncoupled solution, which plots closely under the coupled curve, is obtained. These solutions and comparison with the coupled analysis results point to the fact that although the fracture tends to behave in regard to fluid flow in the very early period, like an equivalent porous material, its storage is not represented by the $1/(2b.K_n)$, but rather by the deformation behavior of the rock-fracture system. This porous media-like behavior is rapidly violated in later time by the effects of deformation on the intrinsic fracture permeability, necessitating a coupled stress-flow analysis of such problems. Consequently, the early behavior of the coupled results and the fact that it follows almost the same general pattern as the fluid flow solution for some time (although, no direct way of prediction exist as shown earlier), can only be attributed to the fact that fracture permeability does not change much during early period. However, as the pressure front expands into the fracture, the rock deforms. The increase in flow due to fracture deformation counteracts the decrease in flow caused by increasing flow path length. These two effects are balanced at about 10 seconds, after which the fracture deformation effect dominates until the pressure front reaches the boundary and steady flow is achieved. This part of the transient flow rate behavior, which is exaggerated in the log scale, lasts almost a 1000 seconds. Final behavior of the model is marked by small but consistent oscillations about a constant value of about $0.381 \times 10^{-3} \text{ m}^3/\text{sec}$. The steady state flow rate obtained earlier closely approximates the final results of the transient analysis.

The above transient analysis has been repeated for a much softer rock, having a Young's modulus of 0.7 GPa. The same general behavior, though much less striking as shown in figure 3, is observed. It appears, in such cases involving soft rock, small variations of inflow rate and rapid stabilization should be expected.

Constant flow rate injection

A transient wellbore pressure analysis for a constant flow rate injection problem has been made using the same system geometries as described above for the constant head case. The analyses are made using a flow rate of 0.381×10^{-3} m³/sec, and both deformable and non deformable conditions are applied for the fracture. In the latter case, a storage coefficient of 2.0 GPa⁻¹ (as in the earlier problem) is used. The transient pressure curves are shown in Figure 4. As with the transient flow rate case, the results of the conventional fluid flow and the coupled stress and fluid flow analysis follow the same pattern for the very early behavior but deviate as pressure front advances radially in the fracture. In the deformable fracture, the increase in permeability with time is accompanied by decreasing wellbore pressure until steady flow is finally reached.

CONCLUSION

The primary conclusion of this work is that pressure dependence of permeability through fracture deformation may have a major effect on the behavior of fluid injection phenomena. In well testing simulations, the inclusion of deformation results in drastic changes in the form of transient flow rate or pressure curves that may invalidate use of conventional well test type curves. The deformation of the fracture depends on the state of the effective stress tensor in the rock rather than the pressure alone. Hence,

the deformation of the fracture--and the accompanying deviations from nondeformable media type curves--occurs when the pressure front from the injection has advanced some distance from the well.

Unfortunately, there is not any solution technique available for verifying the coupled stress-flow model. However, using an approximate reformulation of the flow equation in the coupled method, an explanation for early time behavior of the flow rate, in the constant head injection problem, is presented. An important by-product of this attempt is that fluid flow analysis of fractured rocks can not generally be performed by employing the conventional methods which use the concept of specific storage. Definition of fracture specific storage, based on its deformation moduli [Snow, 1963], is not realistic even under favorable conditions.

Employing an algorithm developed here, the steady state coupled hydromechanical problem is solved directly and the same steady flow rate and pressure profile as those resulting from the transient analyses are obtained. This correspondence can be offered as evidence for the validity of the modeling technique.

A basic result of our modeling effort in constant pressure injection is that a time lag exists between the application of the pressure and the outset of fracture deformation in very rigid rocks. Aside from well test applications, this time lag may have implications for stability of dam foundations or induced seismicity under reservoirs. The time lag effect would suggest that potential problems may exist when traditional analysis results indicate otherwise.

Existence of mixed and geometrically complex boundary conditions and variable rock properties will naturally alter the behavior of pressure and flow rate from those described in the simple models presented in this paper.

Nonetheless, parametric coupled hydromechanical studies may provide a good insight into such rock behaviors.

ACKNOWLEDGEMENT

This work was supported by the Assistant Secretary for Nuclear Energy, Office of Waste Isolation of the U. S. Department of Energy under contract W-7405-ENG-48. Funding for this project is administered by the Office of Nuclear Waste Isolation at Battelle Memorial Institute.

REFERENCES

- Ayatollahi, M.S., J. Noorishad and P.A. Witherspoon, A finite-element method for stress and fluid flow analysis in fractured rock masses, submitted to Journal of Engineering Mechanics Div., ASCE, 1981.
- Biot, M.A., General theory of three-dimensional consolidation, Journal Applied Physics, v. 12, pp. 155-164, 1941.
- Dewiest, R., On the storage coefficient and the equation of groundwater flow, Journal of Geophysical Research, v. 71, pp. 1117, 1122, 1966.
- Jacob, C.E., and S. Lohman, Nonsteady flow to a well of constant drawdown in an extensive aquifer, Trans. Am. Geophys. Union, v. 33 no. 4, 1952.
- Gale, J.E., A Numerical Field and Laboratory Study of Flow in Rocks with Deformable Fractures, Ph.D. Thesis, University of California, Berkeley, 1975.
- Ghaboussi, J., and E.L. Wilson, Flow of Compressible Fluids in Porous Media, SESM Report No. 72-12, University of California, Berkeley, 1971.
- Goodman, R.E., R.L. Taylor and T. Brekke, A model for the mechanics of jointed rock, Journal of Soil Mechanics and Foundation Division, ASCE, v. 94, no. SM3, 1968.
- Finlayson, B.A., The Method of weighted Residual and variational Principles, Academic Press, New York, 1972.
- Narasimhan, T.N., and P.A. Witherspoon, Numerical model for saturated-unsaturated flow in deformable porous media, 1. Theory, Water Resources Research, v. 13, no. 3, pp. 657-664, 1977.
- Noorishad, J., M.S. Ayatollahi, and P.A. Witherspoon, Coupled stress and fluid flow analysis of fractured rocks, submitted to International Journal of Rock Mechanics and Mining Sciences, 1981.
- Noorishad, J., P.A. Witherspoon and T.L. Brekke, A method For Coupled Stress and Flow Analysis of Fractured Rock Masses, Geotechnical Engineering Publication No. 71-6, University of California, Berkeley, 1971.
- Snow, D.T., Fracture deformation and changes of permeability and storage upon changes of fluid pressure, Quarterly Colorado School of Mines, v. 63, no. 1, pp. 201, 1968.
- Terzaghi, K., Erdbaumechanik auf Bodenphysikalischer Grundlage, Leipzig, F. Deuticke, 1925.

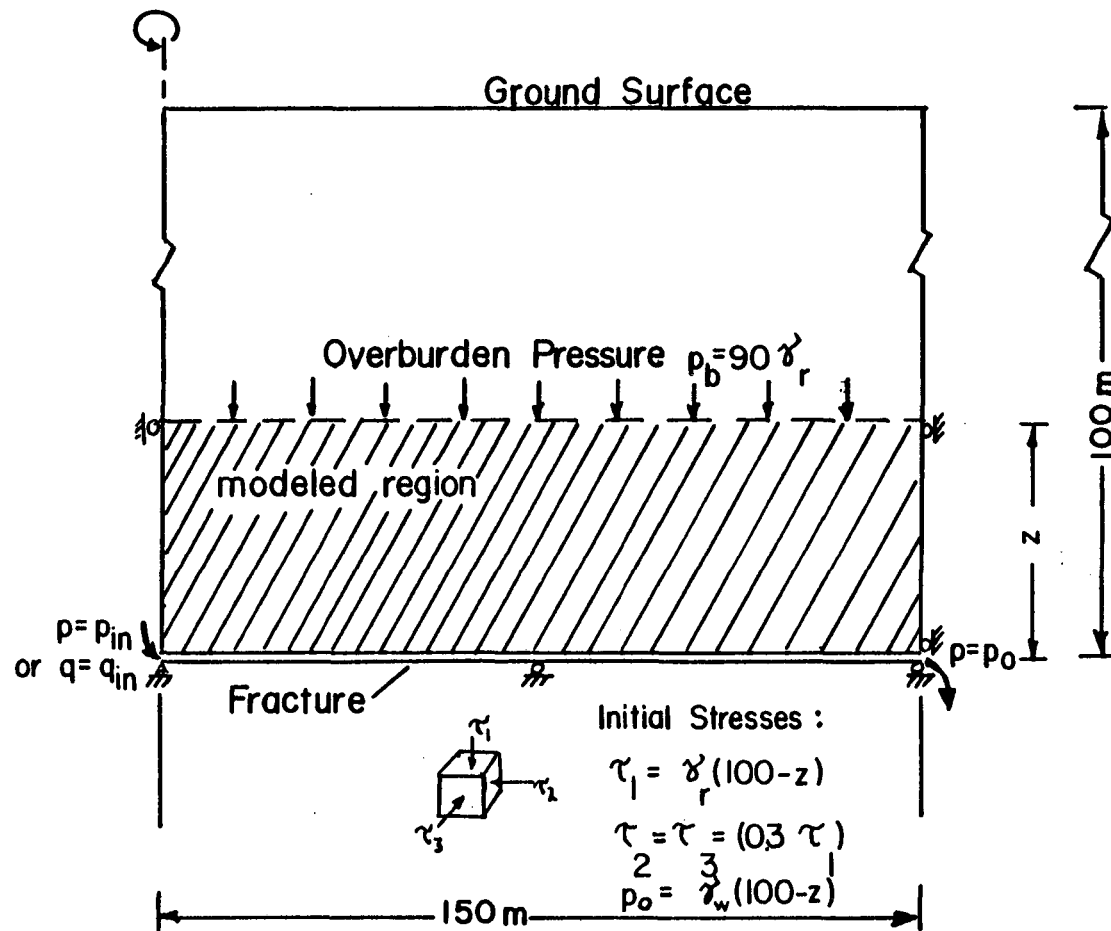


Fig. 1. Schematic geometry of the model used in injection problem.

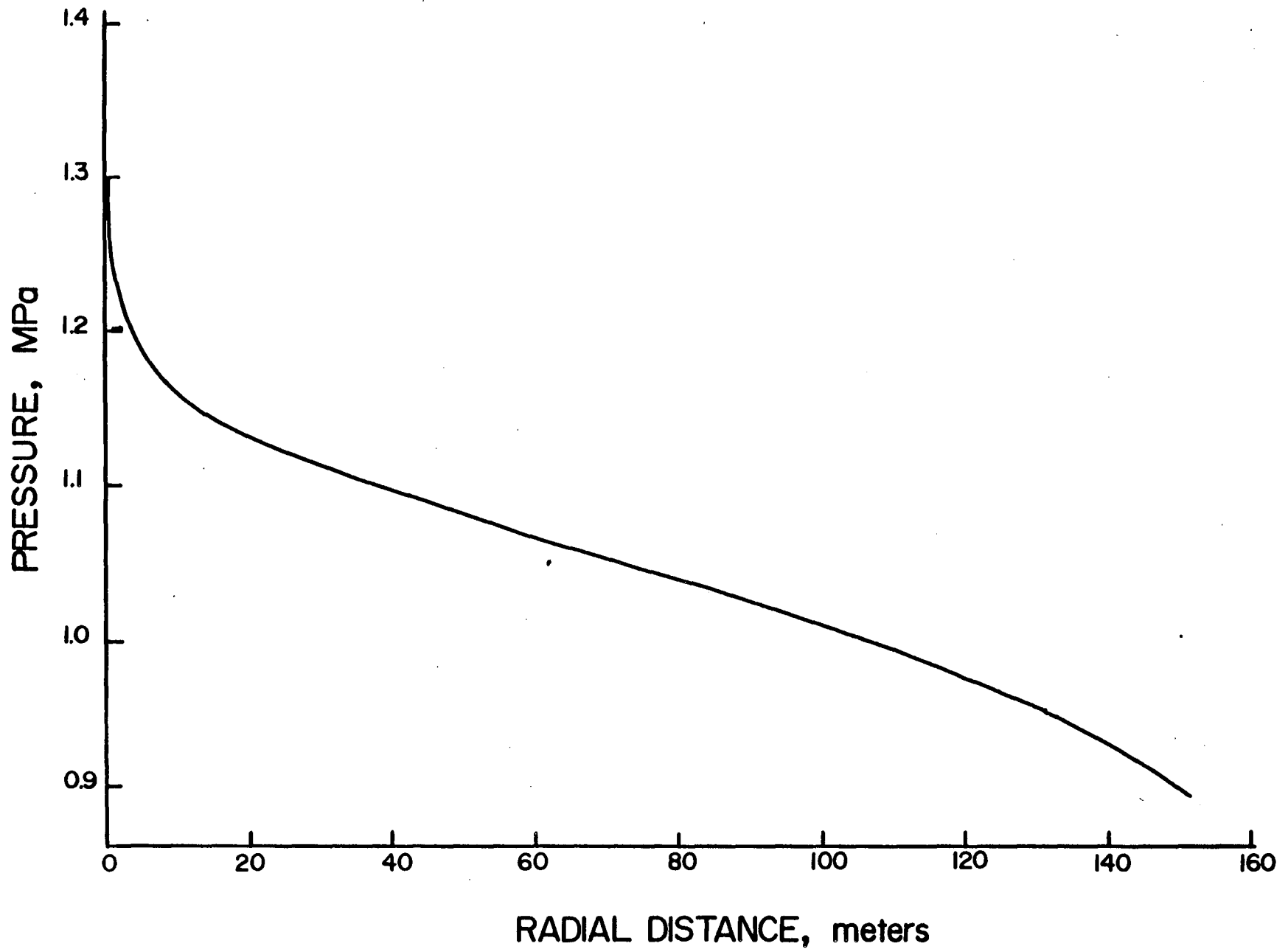


Fig. 2. Steady state pressure distribution along the fracture for injection at constant head of 50 m and injection rate of $3.8 \times 10^{-3} \text{ m}^3/\text{sec}$.

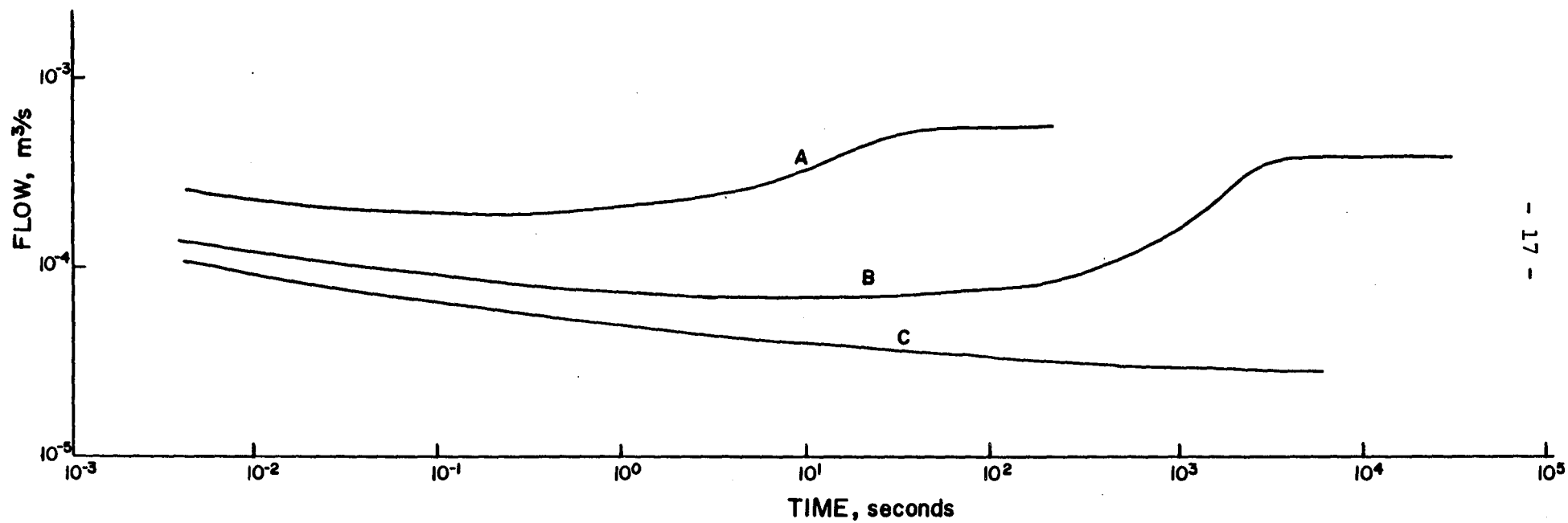


Fig. 3. Transient well flow rate versus time for (a) nondeformable fracture, (b) deformable fracture overlain by rigid rock ($E=7.0$ GPa) and (c) deformable fracture overlain by soft rock ($E=0.7$ GPa).

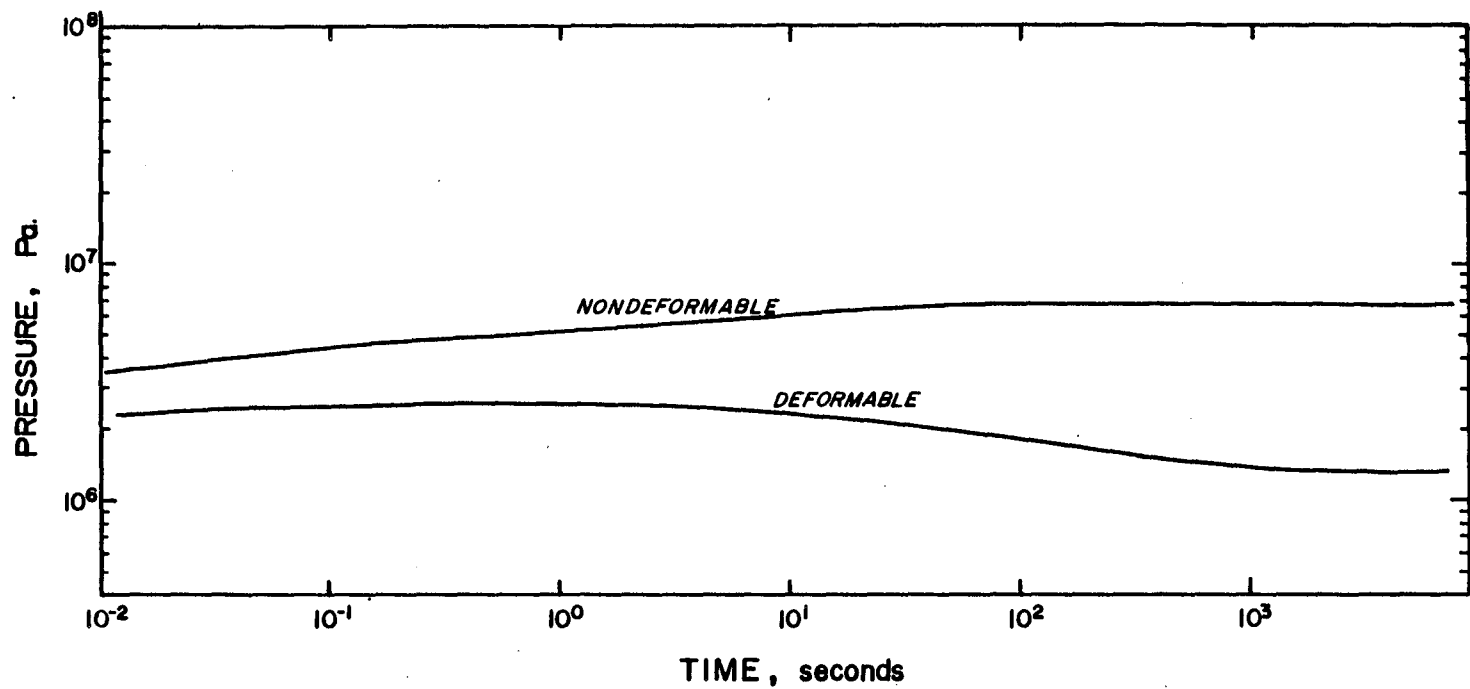


Fig. 4. Transient well pressure versus time for (a) nondeformable fracture and (b) deformable fracture overlain by rigid rock ($E=70$. Pa).

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720