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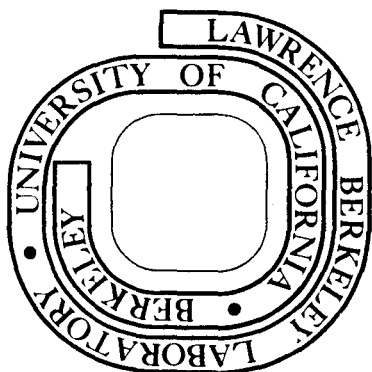
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FACTORIZATION IN RELATIVISTIC HEAVY-ION SCATTERING[†]

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We have calculated the total cross sections for relativistic nucleus-nucleus scattering in the Glauber theory. We propose a simple factorization relation for heavy ions. We also discuss the Gribov and Udgaonkar - Gell-Mann Regge-Pomeron factorization limits for heavy ions.

- - -

In this note, we discuss factorization in the Regge theory [1,2] as applied to relativistic heavy-ion scattering. The factorization hypothesis greatly simplifies the discussion of asymptotic behavior of the total cross sections. Assuming that only the Pomeron trajectory is important at high energies, we would have the following factorization relation for the total cross sections (denoted as σ_{AB}) among particle A and B:

$$\frac{(\sigma_{AB})^2}{\sigma_{AA} \sigma_{BB}} = 1 \tag{1}$$

where A and B may also refer to nuclei. A direct application of (1) to nucleus-nucleus scattering led to the Gribov paradox [3], which states that, by substituting $\sigma_{nA} = 2\pi R_A^2$ in (1), the total cross section σ_{AA} is given as

$$\sigma_{AA} = \text{constant} \times R_A^4 \tag{2}$$

[†]Work performed under the auspices of the U. S. Atomic Energy Commission.

which is in contradiction with the expected behavior $\sigma_{AA} \propto R_A^2$ for a particle with radius R_A . This paradox may be solved by assuming that there is only surface interaction between the nuclei and that the interaction radius of two nucleons becomes much larger than the radius of the nucleus [3]. At extremely high energies, Udgaonkar and Gell-Mann also show that $\sigma_{nA} \propto R^3$ and therefore $\sigma_{AA} \propto R^6$ [4]. For relativistic heavy-ion scattering at a few GeV/nucleon, our calculations show that Regge factorization does not hold and thus the Gribov limit will not be achieved.

We base our discussions on the results obtained from an optical model calculation of the total cross sections for heavy ions at about 2 GeV/nucleon incident energy.

The Glauber theory for the nucleus-nucleus scattering has been developed by Czyż and Maximon [5]. Its analogue in particle physics is generally referred to as the Chou-Yang droplet model [6], whose application to the nucleus-nucleus scattering has been discussed [7]. Here we follow the Glauber theory. The elastic scattering amplitude $F_{AB}(q^2)$ is given by

$$F_{AB}(q^2) = \frac{ik}{2\pi} \int d^2\vec{b} \exp[i\vec{q}\cdot\vec{b}] \{1 - \exp[ix \int d^2\vec{b}' T_A(\vec{b}') T_B(\vec{b} - \vec{b}')]\} \quad (3)$$

where q is the momentum transfer and k is the incident momentum. The two-dimensional densities $T(\vec{b})$ are related to the nuclear density distribution $\rho(\vec{r})$ by

$$T_A(\vec{b}) = \int_{-\infty}^{\infty} \rho_A(\vec{r} \equiv \vec{b} + \vec{z}) dz, \quad (4)$$

where $\rho_A(\vec{r})$ is normalized to unity. The interaction parameter x is related to the nucleon-nucleon total cross section σ_{nn} by

$$x = \frac{(1 - i\alpha)}{2} AB \sigma_{nn} \quad , \quad (5)$$

where A and B are the mass numbers of the colliding nuclei, α is the ratio of real to imaginary parts of the nucleon-nucleon elastic scattering amplitude.

In our calculation we use $\sigma_{nn} = 44.5$ mb and $\alpha = -0.2$. The nuclear density distribution $\rho(\vec{r})$ is taken to have the spherical Woods-Saxon form. The parameters we need in the calculations are listed in ref. 8.

From the numerical results to be shown later, we have found that the total cross sections, obtained through the optical theorem from eq. (3), are qualitatively similar to those obtained in a sharp cutoff (i.e., black-sphere) model. To simplify our discussion, we therefore wish to state our main conclusion in terms of the sharp cutoff model. We first define the factorizability Γ_{AB} for A-B scattering as

$$\Gamma_{AB} = \frac{(\sigma_{AB})^2}{\sigma_{AA} \sigma_{BB}} \quad , \quad (6)$$

and use the result of the sharp cutoff model for σ_{AB} ,

$$\sigma_{AB} = 2\pi r_0^2 (A^{1/3} + B^{1/3})^2 \quad , \quad (7)$$

where r_0 is a radius parameter. The simple result of eq. (7) follows from the assumption that the absorption processes occur whenever the two nuclei contact, each with a radius $R_A = r_0 A^{1/3}$. From eqs. (6) and (7), we have the factorization relation for the nucleus-nucleus scattering:

$$\Gamma_{AB} = \frac{(1 + \gamma_{AB})^4}{16\gamma_{AB}^2} \quad (8)$$

where γ_{AB} is the ratio of the radii of the two nuclei,

$$\gamma_{AB} = R_A/R_B \quad (9)$$

This heavy-ion factorization relation is the main conclusion of this work, from which we may predict the factorizability of any nucleus-nucleus scattering. The Gribov paradox may also be solved by using eq. (8), as to be shown later.

It is interesting first to observe the implication of eq. (8) as a "size" correction to the usual factorization hypothesis in the elementary particle scattering. By taking the proton-proton total cross section σ_{pp} and the pion-proton total cross section $\sigma_{\pi p}$ as 38 mb and 25 mb, respectively, we obtain the pion-pion total cross section $\sigma_{\pi\pi} = 16.45$ mb from the usual ($\Gamma_{\pi\pi} = 1$) factorization hypothesis. To use eq. (8), we assume $\gamma_{\pi p} = (\sigma_{\pi\pi}/\sigma_{pp})^{1/2}$ and obtain the value of $\sigma_{\pi\pi}$ to be 14.7 mb. This comparison is meaningful since the Chou-Yang model also assumes pointlike interactions between the constituents, and will not predict factorization unless the colliding particles are of the same size.

We now show the justification of our parameterization in eqs. (7) and (8) from our numerical results. In table 1, we show the total cross sections and the factorizabilities for each scattering. We also list the factorizabilities as obtained from the sharp cutoff model, eq. (8), which may be taken to be a limiting case of an idealized nucleus-nucleus scattering process. To be consistent in our comparison, we have used $\gamma_{AB} = (\sigma_{AA}/\sigma_{BB})^{1/2}$ with the values of σ_{AA} and σ_{BB} obtained from the Glauber calculation as listed in the table. We note that our total cross sections are obtained by using the nuclear densities determined from electron scattering without any modification for the finite range nature of the strong interactions [8]. The actual values for σ_{AB} may be much larger, especially for low-mass nuclei.

In fig. 1, we have plotted the total cross section as a function of the effective radius $R_{\text{eff}} = R_A + R_B$. The total cross section follows the simple prescription of eq. (7) quite well, especially for large nuclei; we have chosen $r_0 = 1.25$ fm. This general dependence of the total cross sections from the Glauber calculation is the main justification of our factorization relation of eq. (8). The deviations for light nuclei from the curve simply show the effects of neglecting the finite range of the strong interaction, as discussed above.

The slopes of the forward scattering amplitudes obey the factorization relation of eq. (8). It is not surprising, since, in the sharp cutoff model, the factorization of the slope is directly related to the factorization of the total cross section. That is, we have the logarithmic derivative as

$$\left. \frac{1}{|F_{AB}(q^2)|} \frac{d|F_{AB}(q^2)|}{dq^2} \right|_{q^2=0} = \frac{\sigma_{AB}}{16\pi} \quad (10)$$

Finally we consider the Gribov paradox. From eqs. (6) and (8), we may write the nucleus-nucleus scattering cross section σ_{AA} in terms of the nucleon-nucleon cross section σ_{nn} as

$$\sigma_{AA} = \frac{(\sigma_{nA})^2}{\sigma_{nn}} \left[\frac{16(R_A/r_n)^2}{(1 + R_A/r_n)^4} \right] \quad (11)$$

where we have taken $\gamma = R_A/r_n$ for the nucleon-nucleus scattering, and r_n is a characteristic length associated with the nucleon. For $R_A \gg r_n$, we may replace $(1 + R_A/r_n)$ by R_A/r_n in eq. (10) and obtain, by substituting $\sigma_{nA} = 2\pi R_A^2$,

$$\sigma_{AA} \sim \frac{64\pi^2 r_n^2}{\sigma_{nn}} R_A^2 \quad (12)$$

which has the usual dependence on the radius of the colliding objects. At extremely high energies, we may take the range of nucleon interaction to be much larger than the size of the nucleus, so that $r_n \gg R_A$ in eq. (11), we have the Udagonkar - Gell-Mann limit, where $\sigma_{AA} \sim (64\pi^2/\sigma_{nn} r_n^2) R_A^6$. The Gribov limit is obtained by further assuming only surface interactions.

It is also rather important to point out that our results are not in contradiction with the results of Gribov or of Udagonkar and Gell-Mann. The discussions in refs. 3 and 4 are based on a multiperipheral model, where the radius of interaction grows logarithmically with energy, and their limits are valid only when the radius of interaction is as large as the size of the nucleus; this corresponds to incident energy far beyond 10^9 GeV/nucleon [4]. At these energies all the particles (elementary and nuclear) interact with long-range forces and we have the usual factorization. A true "nuclear democracy" [9] may then exist when the sizes of all the strongly-interacting particles are small when compared to the range of the strong interaction.

Contrary to the high-energy asymptopia, our factorization relation, eq. (8), is derived with an explicit assumption in the Glauber theory that the interaction radius is much smaller than the nuclear radius [5]. This is the reason that the relative sizes of the colliding objects remain important since the scattering process is still dominated by the geometrical properties of the objects. The Glauber theory, with approximate accommodation for the increase in strong interaction range, will reproduce factorization only at extremely high energies. It is also well known that only the Born term factorizes in the Glauber-eikonal formalism (or the Chou-Yang model) [10]; the importance of the nuclear multiple scattering contributes to Regge cuts (or absorptive corrections) which destroy the factorization.

We conclude that there will be no Regge factorization in the nucleus-nucleus scattering as long as the sizes of the colliding objects remain much larger than the range of nucleon-nucleon interaction. This remark is based on the Glauber theory; however, it would be unlikely for the Glauber theory to be grossly invalid at the Bevatron energies. We have also concluded that the Gribov or the Udgaonkar - Gell-Mann limits will not be realized in the relativistic nucleus-nucleus scattering. It is clear that the features discussed in this note will not depend much on the model. The essential features are the short-range interaction and the large sizes of the colliding objects; these are generally true in the relativistic heavy-ion scattering. There are also other Glauber calculations by Fishbane and Trefil [11], using Gaussian nuclear density distributions. (We have used the Woods-Saxon density.) The general results are qualitatively the same. Their main conclusion that the factorization relation (1) holds approximately for nucleus-nucleus scattering is only due to the fact that the values of γ_{AB} usually are not very far from unity, and should not be taken to mean that the Pomeron role is dominant, since the relation (8) also holds at much lower energies, say, at 10 MeV/nucleon [12].

Finally, if we accept any constituent model for the elementary particles, such as the partons, the factorization (1) could imply that the parton-parton interaction has a range larger than the sizes of the elementary particles. At this limit, the Chou-Yang model must be modified to account for the finite-range interaction in order to reproduce the factorization.

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12. See, for example, J. S. Blair, Phys. Rev. 108 (1957) 827.

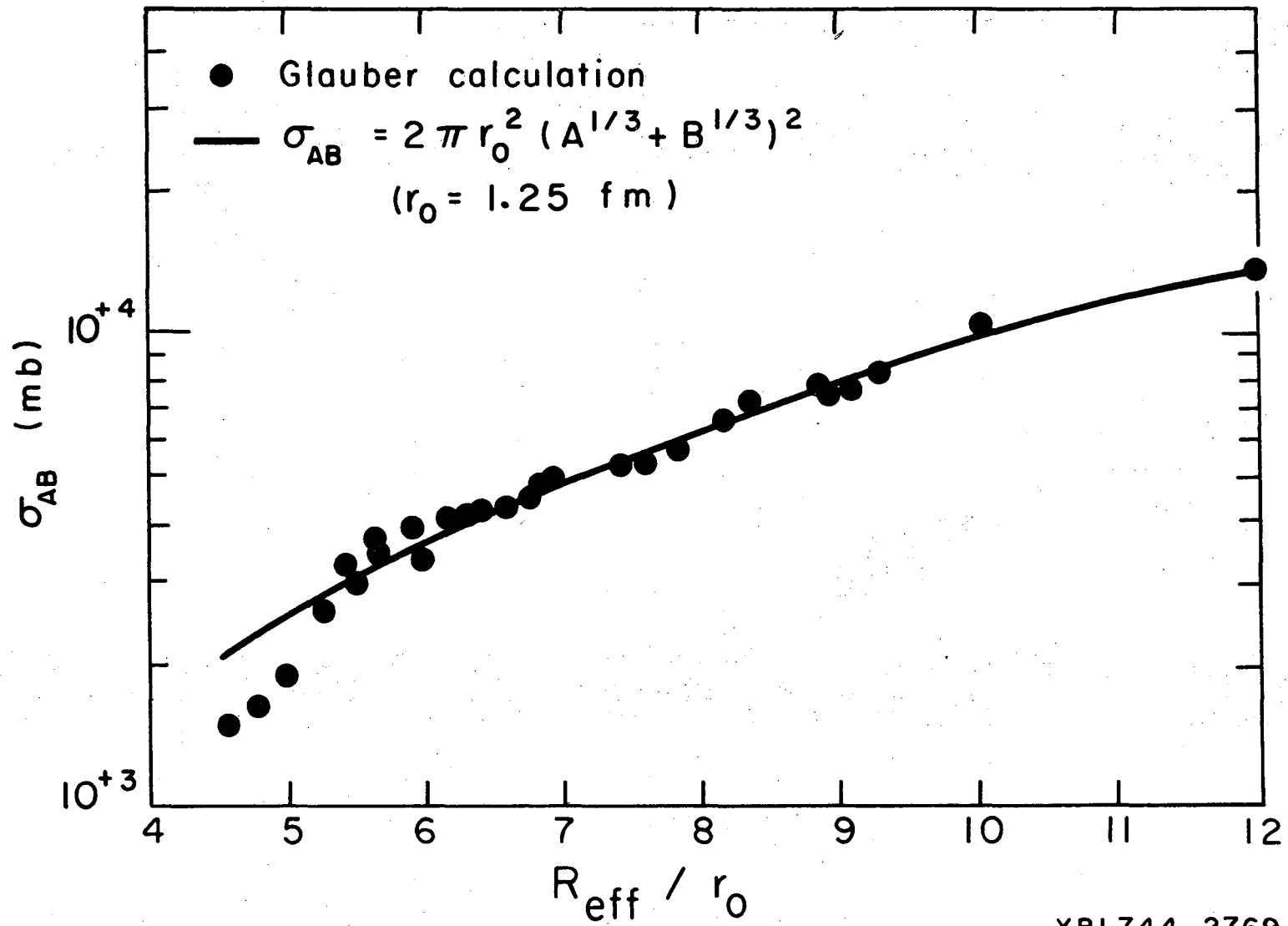
Table 1. Total Cross Section and Factorizability. Nucleus-nucleus total cross sections (fm^2) calculated in the optical model. The averaged nucleon-nucleon cross section $\bar{\sigma}_{nn} = 44.5 \text{ mb}$ and $\alpha = -0.2$ (see eq. (5)). The nuclear density is of a Fermi distribution. The factorizability (the lower left table) is defined by eq. (6). The values in the parentheses are from eqs. (8) and (9).

| B \ A | 4 | 12 | 16 | 27 | 32 | 40 | 88 | 208 |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|------|
| 4 | 35.2 | 89 | 104 | 164 | 200 | 221 | 315 | 502 |
| 12 | 1.52 (1.29) | 148 | 165 | 260 | 321 | 341 | 443 | 659 |
| 16 | 1.63 (1.41) | 0.97 (1.01) | 189 | 298 | 369 | 389 | 493 | 720 |
| 27 | 2.34 (1.80) | 1.40 (1.08) | 1.44 (1.04) | 326 | 405 | 424 | 523 | 754 |
| 32 | 2.78 (2.03) | 1.70 (1.14) | 1.76 (1.08) | 1.01 (1.01) | 409 | 428 | 530 | 762 |
| 40 | 2.93 (2.21) | 1.66 (1.18) | 1.69 (1.11) | 1.15 (1.02) | 0.94 (1.00) | 474 | 577 | 819 |
| 88 | 3.64 (2.98) | 1.71 (1.40) | 1.66 (1.28) | 1.08 (1.10) | 0.88 (1.05) | 0.91 (1.03) | 775 | 1051 |
| 208 | 5.18 (4.44) | 2.12 (1.81) | 1.98 (1.61) | 1.26 (1.29) | 1.03 (1.20) | 1.02 (1.15) | 1.03 (1.04) | 1383 |

Figure Caption

Fig. 1. The total cross section (in fm^2) of the nucleus A and B, plotted against the effective radius R_{eff} in units of r_0 . The points represent the results of the Glauber theory; the values for the total cross sections are from table 1. The solid line is a fit to the results of the calculation, by choosing $r_0 = 1.25$ fm in eq. (7).

Fig. 1



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