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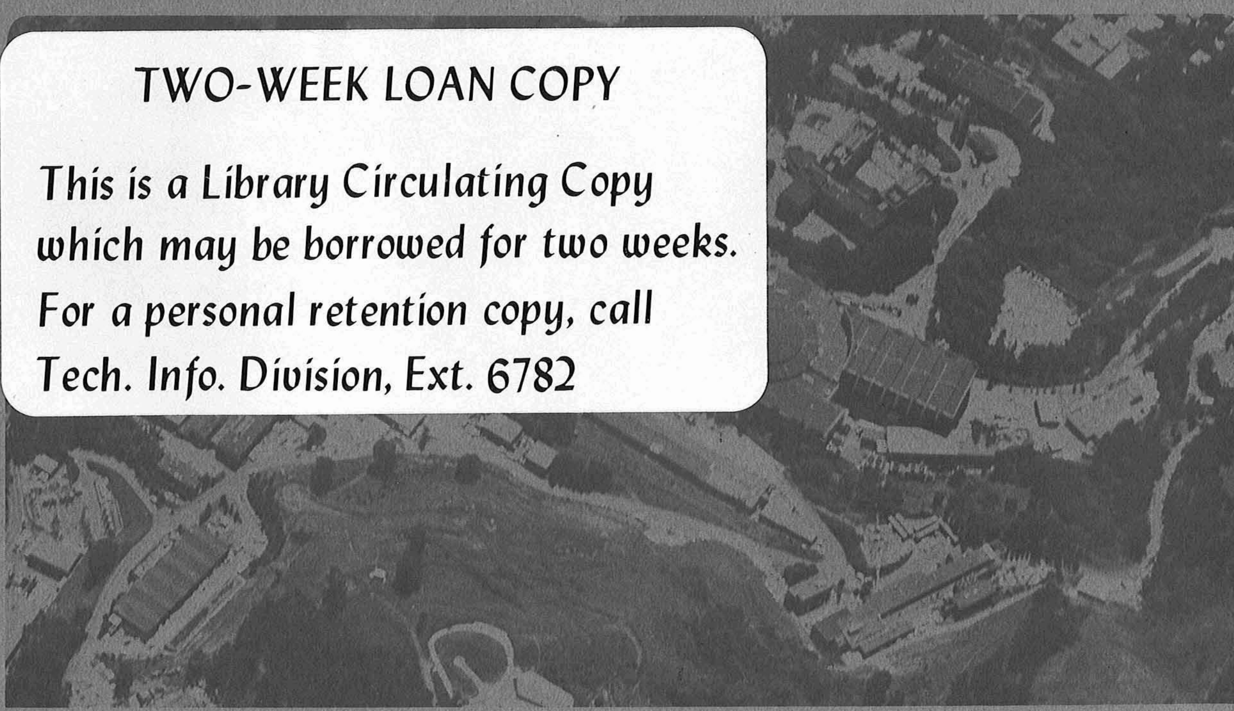

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ABSTRACT

We discuss here an attempt to simulate the beam-beam limit effect in the SPEAR I 1.5 GeV storage ring located at SLAC. A summary discussion is given of the models used and the results obtained. Remarks are made concerning the difficulties encountered in this simulation problem.

INTRODUCTION

One of the principal problems present in colliding beams storage rings is the limit on luminosity (counting rates) caused by what is commonly called the "beam-beam" limit. At this limit an increase in beam strength leads to a decrease in luminosity due to excessive growth in the size of one of the beams. Also, the stored beam is eventually lost due to a shortening of the beam lifetime. It was to investigate this beam-beam limit that the computer program PEP was developed.[1]

In this note results from that study are summarized. The study was carried out over a period of time and it is not possible to cover in detail all the lessons learned and results obtained. A somewhat more complete survey and references to background material can be found in [2] and some recent detailed results in [3]. In this note are discussed the basic, original model and results obtained from it (Model I), an updated model that leads to better agreement with measured data (Model II), and some more recent results based on a variant of the second model (Model III). In conclusion, a number of remarks are made that are derived from the experience gained during the course of this study.

It should be emphasized that these calculations were meant, in so far as possible, to simulate a real storage ring. This is in contrast to other types of studies where a specific nonlinear equation is studied with the object of understanding the properties of its solution space. This simulation attitude strongly influenced the model development and the presentation and interpretation of results.

MODEL I

Since the exact modeling of a colliding beam storage ring is not possible, a simplified model was developed. This simulation model, called Model I here, concentrated on the collective effect of one beam upon the particles in the other beam as the two beams intersected in the field-free interaction regions of an e^+e^- storage ring. Thus, the simulation consisted of two bunched beams; a weak beam, beam 1, and a strong beam, beam 2.

The strong beam was defined analytically by a charge distribution. Initially this distribution was Gaussian in the variable x, y, z with r.m.s. beam widths σ_x, σ_y, σ_z. The strong beam was assumed to be highly relativistic, traveling parallel to the z axis, and stationary in the sense that its charge distribution was not a function of the number of collisions it experienced with the weak beam. The three-dimensional character of the strong beam and the rapid change of the beta functions in the interaction region were included in the calculation by discretizing the interaction region along the z axis. See Figure 1B, 1C, also Table 1.

The weak beam, also assumed to be highly relativistic, was initially defined by drawing samples of test particles from a Gaussian distribution. For the weak beam the effects of beam growth and damping in the transverse x,y plane due to quantum radiation were included. Also, the x motion was coupled into the y motion in a manner that represented the natural coupling present in the ring.

The basic ring structure between interaction regions was represented by a linear transformation. See Figure 1A. All energy loss effects, other than quantum noise pertubations, were ignored. Thus, all collisions for each test particle were time independent in the sense that a test particle was either always early, on time, or late when it arrived at the interaction region. The only forces considered were the basic Lorentz force of the strong beam bunch acting on the weak beam test particle, the random noise due to quantum radiation, and a balancing damping effect. The latter two effects were adjusted to yield the correct (experimentally measured) beam size. In order to compare the calculated results with experimentally measured results, a luminosity calculation was included.

The simulation using Model I was done by setting the model parameters such as the machine tunes (ν_x,ν_y); the beam distribution sizes $(\sigma_x,\sigma_y,\sigma_z)$; the beam strengths and quantum noise parameters to values that represented the machine being simulated and then following the evolution of the weak beam charge distribution as a function of the number of interactions with the strong beam.

The model was originally developed with the goal of simulating the e^+e^- machine SPEAR I operating at SLAC. The details included, such as the beta function variation in the interaction region and the quantum noise, were those that were thought to be important for correctly representing the beam-beam effect in that machine. For a fuller discussion of these modeling details the reader should consult [2].

RESULTS I

The results presented here are a summary of some of the main conclusions that were derived from computer runs made using the CDC program PEP based on Model I. A somewhat fuller discussion can be found in [2].

The natural time unit to use for this calculation was the transverse quantum radiation damping time which was about 66×10^3 μ sec., or 170,000 interactions. In order that the calculations could be carried out over a number of damping times, the machine parameters were scaled by a factor of ten to 15 GeV. At that energy, the damping time was only 66 μ sec. or 170 interactions. This scaling was carried out in a manner that left the small amplitude tune shift the same for the scaled and unscaled machine. [2]

There were essentially two types of results calculated; single test particle results and statistical results obtained by choosing a sample of test particles to represent the weak beam. The single particle results were similar to those obtained using a sample.

Our basic conclusions from runs using Model I can be summarized as:

1. the beam strengths needed to exhibit beam blow up were unreasonably high and did not correlate with experimentally obtained results;
2. quantum noise and damping, although contributing adversely to a beam that showed growth, did not in themselves seem to trigger or cause the observed growth;
3. Model I must be missing some necessary details since it tended to show only stable results for beam strengths that were known to cause growth in SPEAR I.

These calculations also showed that it was necessary to calculate fields in a smooth accurate manner in order to eliminate the introduction of numerically spurious results that caused artificial beam growth. They also showed that beam shape was a factor that influenced the results. Flat beams grew at tune shifts that were different from those of round beams. For example, the small amplitude linear force tune shift for the separation of stable and unstable beams was about $\Delta\nu_x$=.136 for a round beam but appeared more like $\Delta\nu_y$=.302 for a flat beam. Thus, an understanding of the beam-beam limit in a particular machine would require that the beam be correctly modeled.

The results also showed that when strengths were such that beams were stable, they would exhibit stable behavior over a large number of interactions. We had single particle runs on the order of 10^6 interactions that tended to support this. When they were unstable they exhibited fast growth.

In Figures 2 and 3 are shown the type of results obtained from Model I calculations. In each case luminosity is plotted versus the number of interactions. Figure 2 is for a flat beam with parameters chosen to represent the scaled SPEAR I machine, whereas a round beam was used in the calculations shown in Figure 3. The fluctuations were attributed to the small sample size and smoothed for larger samples. As noted above, a rough or inaccurate impulse (field) calculation shows numerical blow up in Figure 2 where in fact the more accurate, smooth calculation shows none. Both Figures 2 and 3 show stable beam, no luminosity loss, at beam strengths that were known experimentally to cause beam loss in SPEAR I. The small amplitude linear force tune shifts are a measure of the interaction force. The blow up exhibited in Figure 3 is at an unreasonably high interaction force level.

MODEL II

The original model had failed to produce results that correlated with experimental measurements. In general, it had shown no beam growth when in fact there actually was. The basic model was, therefore, expanded to include the effect of momentum errors $\Delta p/p$. [4] This was done by including an early late timing effect and also a phase modulation in the linear transfer matrices. Thus, as the test particles arrived at the interaction region, those for which $\Delta p/p \neq 0$ were sometimes early, on time, or late as their position in the weak beam bunch changed. Also, they were transferred between interaction regions using a transfer matrix with appropriately updated the tunes.

At the same time a new charge distribution was used for the collective effect of the strong beam on the weak beam. See Table 1. The closed-form solution for this distribution as derived by Dr. Smith [4,5] was about a factor of 5 faster than the evaluation of the bi-gaussian by numerical quadrature. The model was also refined to include the fact that the beta functions which determined the strong beam shape in the interaction region depended on the beam strength.

RESULTS II

Runs were made on a parameter set that corresponded as closely as possible to the 1.5 GeV SPEAR I machine running with about 8.2 ma of current. This beam intensity corresponded to beams with 4.0×10^{10} particles per bunch, linear tune shifts of $\Delta\nu_x = .059$ and $\Delta\nu_y = .072$

and was one of the higher intensity runs for which SPEAR I results were available.

With the inclusion of longitudinal motion, the relevant damping time was the longitudinal quantum radiation damping time which was about 85,000 interactions. Single test particle runs were made over this length of time using the unscaled machine parameter set. Exploratory runs showed that off-momentum particles oscillating on the order of $3\sigma_z$ axis showed significant amplitude growth within 85,000 interactions, that is within one longitudinal damping time τ_z.

Because of these preliminary results, a series of runs that covered a $5\sigma_x$ by $10\sigma_y$ area in the initial value space were made. All test particles were started with zero slope. These runs were made with $\Delta p/p=0$ and $\Delta p/p \propto 3\sigma_z$. Results of these runs were saved for later analysis and plotting.

The results that are shown in Figure 4 are plots of the maximum excursion A_x that was achieved by the test particle that started initially at the values (x^0,y^0). The ordinate is the initial vertical displacement y^0, the abscissa the initial horizontal displacement x^0, the number amplitude A_x that the test particle would have in the ring at the time of the plot. All values have been divided by a measure of the beam width, σ_x for x^0 and A_x, and σ_y for y^0.

The plots are given for a number of interactions N that correspond to .001, 0.5, and 1.0 damping times. Those points that don't have an amplitude number had no test particle tracked for that initial value. Due to the expense of generating these results, not all points in the $5\sigma_x$ by $5\sigma_y$ space shown were selected for tracking.

The interpretation of these plots is limited to rather elementary conclusions. The plot on the bottom is for $\Delta p/p=0$ and the one on the top for $\Delta p/p \alpha 3\sigma_z$. The cross-hatched area has particles that do not exceed $1\sigma_x$ in amplitude, the next area is for those that do not exceed $3\sigma_x$, and the darker region for those that do not exceed $6\sigma_x$ in amplitude. The evolution of these areas is shown for the damping times $t/\tau_z = 0.0, 0.5, 1.0$.

It is seen that the $\Delta p/p=0$ plots are relatively stationary; however, this is not the case for $\Delta p/p \propto 3\sigma_z$. The region with A_x between $3\text{-}6\sigma_x$ shrinks and outside this region oscillations are at a large amplitude. There are many values at $19\sigma_x$ and some at $20\sigma_x$.

The same type of results are shown in Figure 5 for the maximum amplitude A_y in the y plane. For $\Delta p/p=0$ the results, although not as clear cut, tend to show that the beam is relatively stationary when synchrotron oscillations are not included. Again, when $\Delta p/p \propto 3\sigma_z$ the $3\text{-}6\sigma_y$ region shrinks and outside that region particles increase their amplitude of oscillation and there is at least one point out to $28\sigma_y$.

These results tend to indicate that not including synchrotron oscillations in the original model was a serious omission since their inclusion leads to beam growths of significant magnitude. Previous results obtained from Model I suggest that the inclusion of quantum noise and damping would have made these growths greater. Luminosity calculations might, however, not show much beam loss since not much growth occurred within 3σ and most of the beam is contained within that region. Aperture restrictions for SPEAR I were about $47\sigma_x$ and $30\sigma_y$, so clipping was not experienced. However, from a lifetime or diffusion point of view there are points in the initial distribution space $5\sigma_x$ by $5\sigma_y$ by $3\sigma_z$ that are getting close to the y place wall. Long tails grow for particles that are at $3\sigma_z$ and for these particles the growth of the tails of the distribution is rather fast and large. Thus, it might very well be that a lifetime calculation, if one could perform it, would show a short beam lifetime. Obviously, the results given here are not complete enough to draw any definite conclusions.

MODEL III

In response to a renewed interest in the original beam-beam calculations, runs were made using a program WEA10 developed by Dr. Laslett of LBL. [6] This program is based on a model, called Model III here, that is similar to Model II. It does a somewhat more restricted simulation than was done using the program PEP and Model II. In particular, Model III is an impulse calculation and does not take into account the longitudinal axis. Also $\Delta p/p=0$ appears only in the transfer matrix elements, no timing effects are included, all interactions take place at the symmetry point of the interaction region. It does, however, have the same coupling of the x motion into the y motion that was used in Model II. No quantum noise is considered. Since no use was made of this effect in Model II calculations, its omission in Model III is not important.

RESULTS III

Many single test particle runs were made using Model III. These results are described in detail in [3]. The main interest here is to note how these results relate to the original runs.

As closely as possible the original SPEAR I parameter set was used as input. Since only a strength parameter is used in WEA10, the small amplitude tunes were used as a measure of the beam strength and they were set so that $\Delta\nu_x$ was the same as previously. Because no longitudinal beam length was included, $\Delta\nu_y$ did not turn out to be the same as before. There was in fact no way that this model would give the same tune shifts as Model II.

An initial test particle at $4\sigma_x$, $3\sigma_y$ was chosen since this test particle had previously exhibited large growth in amplitude within 85,000 interactions. The particle was run to 84,000 interactions and no noticeable growth in amplitude was observed. For all practical purposes the selected particle showed regular behavior with no growth.

By sampling the initial value space it was possible to find an initial value that did exhibit growth and studies were made on its behavior as $\Delta p/p$ was varied and also as the synchrotron oscillation frequency ν_s was varied. These results are also reported on in [3]. However, the interesting result obtained from Model III was that a change in the model which on the surface looked rather slight caused a completely different behavior of a particular initial valued test particle. Thus, it would be necessary to redo the $5\sigma_x$ by $5\sigma_y$ sample set to see if the results were qualitatively the same with Model III. A lack of time and the cost of such runs prevented that from being done.

GENERAL REMARKS

The results summarized in this report were obtained trying to numerically simulate the beam-beam effect as it actually occurred in an e^+e^- storage ring. The first attempt, Model I, although it contained many details of the interaction process, did not produce results that correlated with experimentally measured values. It also appeared that the inclusion of some details like quantum radiation/damping were not necessary and in fact only made understanding the results more difficult.

The inclusion of synchrotron oscillations and tune variation due to momentum errors, Model II, gave beam growths that were large at beam strengths that were close to the experimentally determined limits. However, the results were limited to a $5\sigma_x$ by $5\sigma_y$ region evaluated at $0\sigma_z$ and $3\sigma_z$. The calculations needed to adequately populate a full three-dimensional space, much less the full six-dimensional phase space, are time consuming. It would be a very large task to do a complete set of runs over a range of parameters, such as beam strengths and synchrotron frequencies, and then analyze the results.

The third set of runs, derived from Model III and only mentioned here [3], point out the perplexing fact the behavior of a specific orbit is i) very sensitive to slight parameter changes and ii) model dependent.

Thus, it seems that before conclusions can be reached a rather dense sampling of initial value space must be done and the behavior of the sample investigated. Also, since different models give different orbits, it would appear that only the total sample behavior can have any real validity and that two models would be judged equivalent if the total sample behavior were equivalent, regardless of what individual orbits did.

One of the guide lines that should be followed is to remember that the storage rings are analogue models and results must always be checked against them or there is no way of knowing whether the numerical calculations reflect anything that relates to a real machine.

It also appears important to correctly model the beam shapes that are in the machine being simulated. Although this is difficult, idealized beam shapes can lead to erroneous results. In particular, sharp edges on charge distributions and doubly valued charge distributions are to be avoided unless they really exist. Round beams are not like flat beams. The distribution suggested by Dr. Smith [4,5] is both numerically fast and physically reasonable.

The change in the function across the interaction region and the finite bunch length must be considered. A check that results are correct must be made when simple impulse calculation is used. If the changes in the beta function and bunch shape are significant, a simple impulse calculation will not suffice. At least for the simulation of SPEAR I, a longitudinal discretization was necessary.

The problem of trying to simulate and understand the beam-beam effect is a rather perplexing one. From a throretical viewpoint, it is interesting to construct a "model", choose a set of parameters, and then explore the properties of specific orbits, either numerically, analytically, or with a combination of both. However, from a practical point of view, none of the parameters are known exactly and further what the model should be is not obvious. The calculated results are not only model dependent, but small parameter changes can lead to qualitatively different behavior of individual orbits. [3]. Thus, it may make little sense to infer results from individual orbits and only the collective sample behavior may be meaningful. But the sample size is of necessity small compared to beam bunches which for the unscaled SPEAR I machine were on the order of 10^{10} particles per bunch. So gross qualitative conclusions are eventually arrived at from results obtained using a very small sample.

This state of affairs necessitates the checking of calculations against experimental results and trying to build a computational model that accurately reflects a real machine. It was this approach that was taken in developing Model I and II. The results obtained with Model II are a somewhat encouraging indication that such a model can be built. Unfortunately, a simulation calculation is expensive in time, effort, and money. Also, it is not obvious that the model extrapolates to another, different machine.

What is really needed is a theory that would exhibit as of a function of machine parameters what happens to all orbits. This is what the usual linear orbit theory does. It would be nice if somehow in the present nonlinear problem equations could be obtained that show globally how all solutions behave with respect to some of the relevant machine parameters. Just what these "solutions" represent is left open. However, they should be relatable to experimentally measured beam quantities.

REFERENCES

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TABLE I

Charge Distribution For Nonlinear Fields

1. Bi-Gaussian

$$p = \frac{\lambda}{2\pi\sigma_x\sigma_y} \, e^{-\frac{1}{2}\left[\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right]}$$

2. L. Smith

$$p = \frac{\lambda}{2\pi\sigma_x\sigma_y} \left[1 + \frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right)\right]^{-2}$$

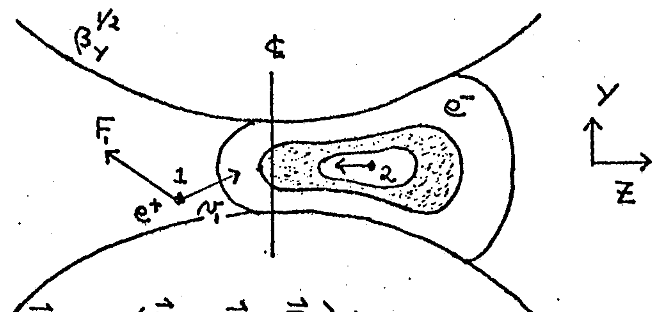
$\lambda = \pi/2$

A. LINEAR TRANSFER MATRIX

$$M = \begin{bmatrix} \cos\mu & \beta \sin\mu \\ -\frac{1}{\beta}\sin\mu & \cos\mu \end{bmatrix} \; ; \quad \mu = \frac{2\pi\nu}{N}$$

$$\nu = \frac{1}{2\pi}\int_0^C \frac{ds}{\beta} \qquad X_i = M_{-L/2}\, M\, M_{-L/2}\, X_f$$

B. BEAM IN INTERACTION REGION



$$\vec{F}_1 = e_1(\vec{E}_1 + \vec{v}_1 \times \vec{B}_2) + \cdots$$

$$v/c \sim 1$$

C. DISCRETIZED INTERACTION REGION

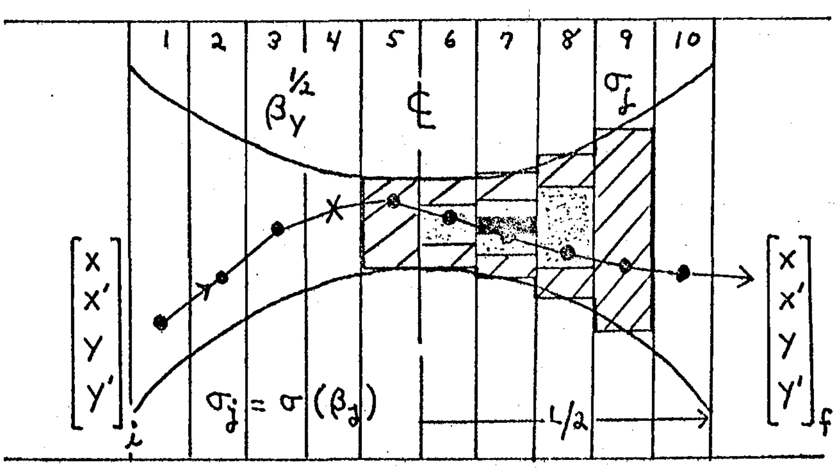


FIGURE 1

Beam-Beam Simulation Model

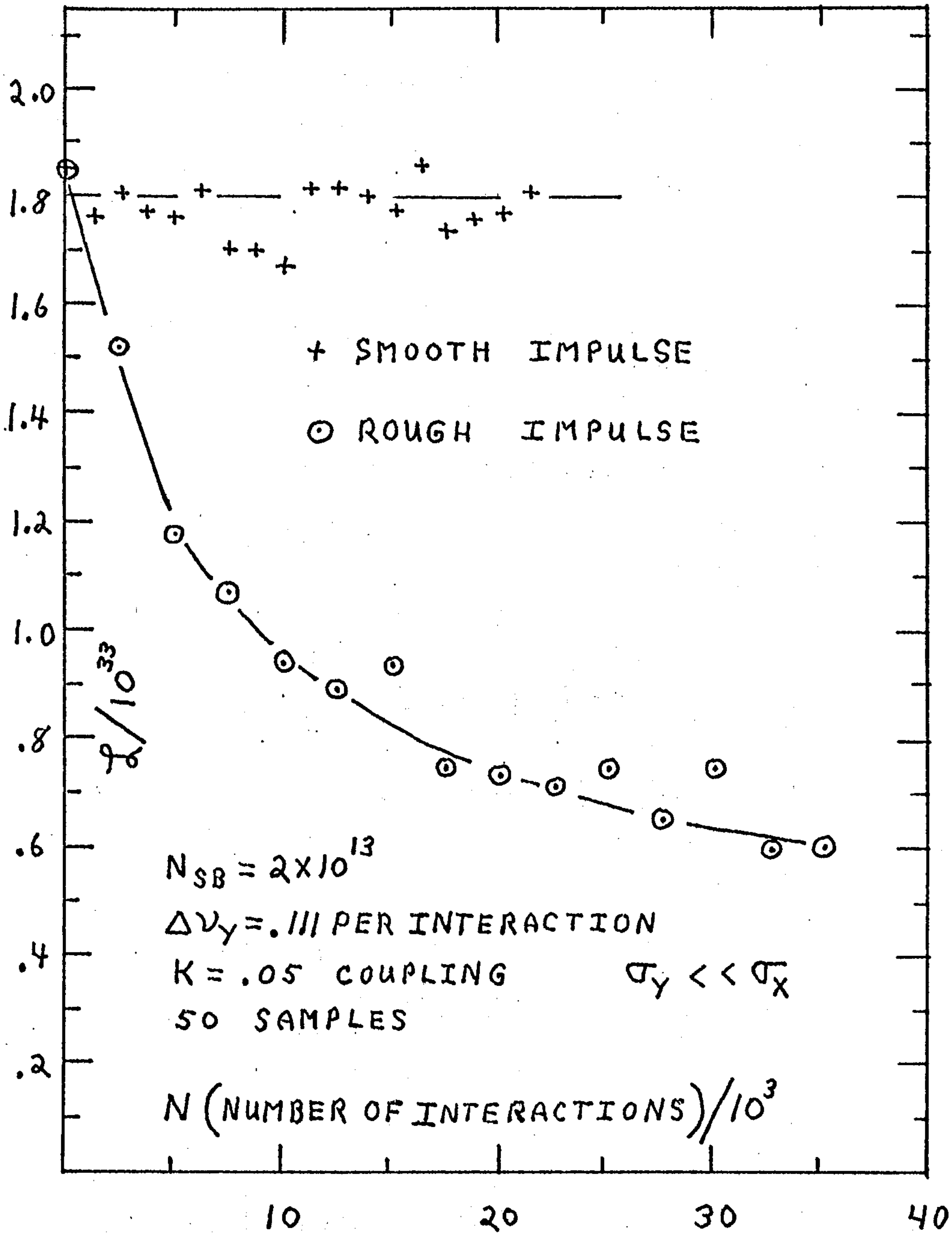


FIGURE 2

SPEAR I Scaled to 15 GeV
Flat Beam Luminosity Calculations

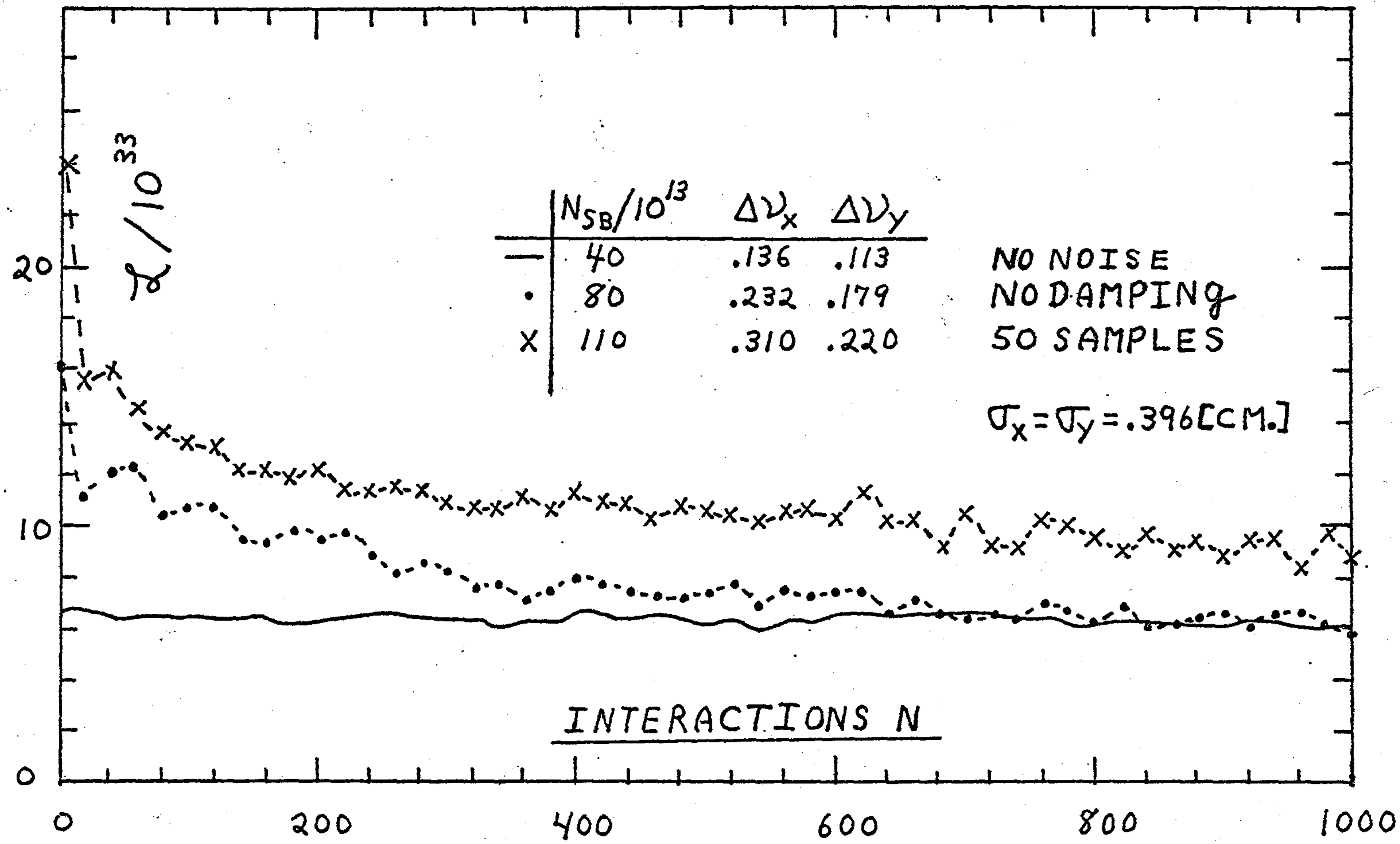
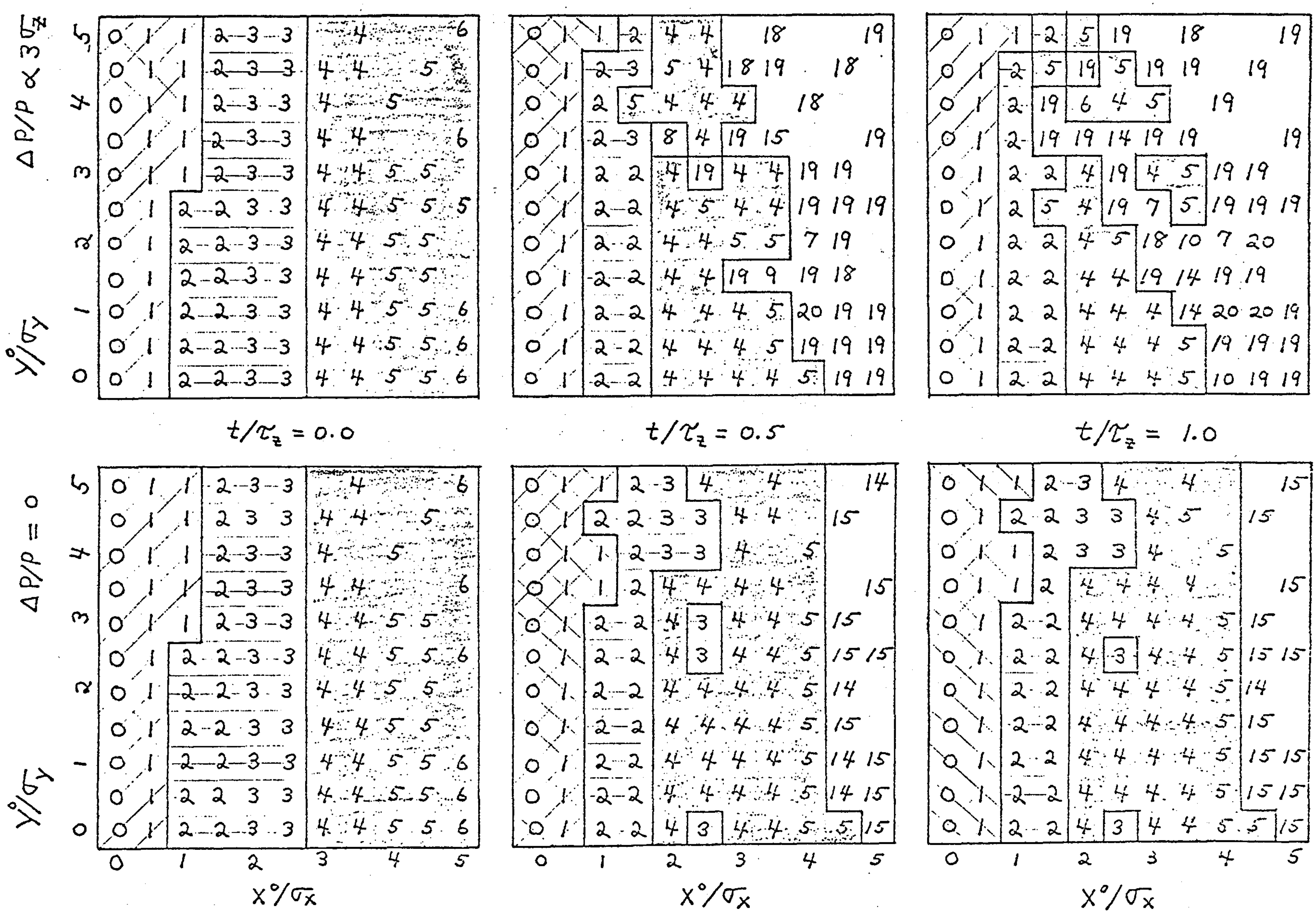


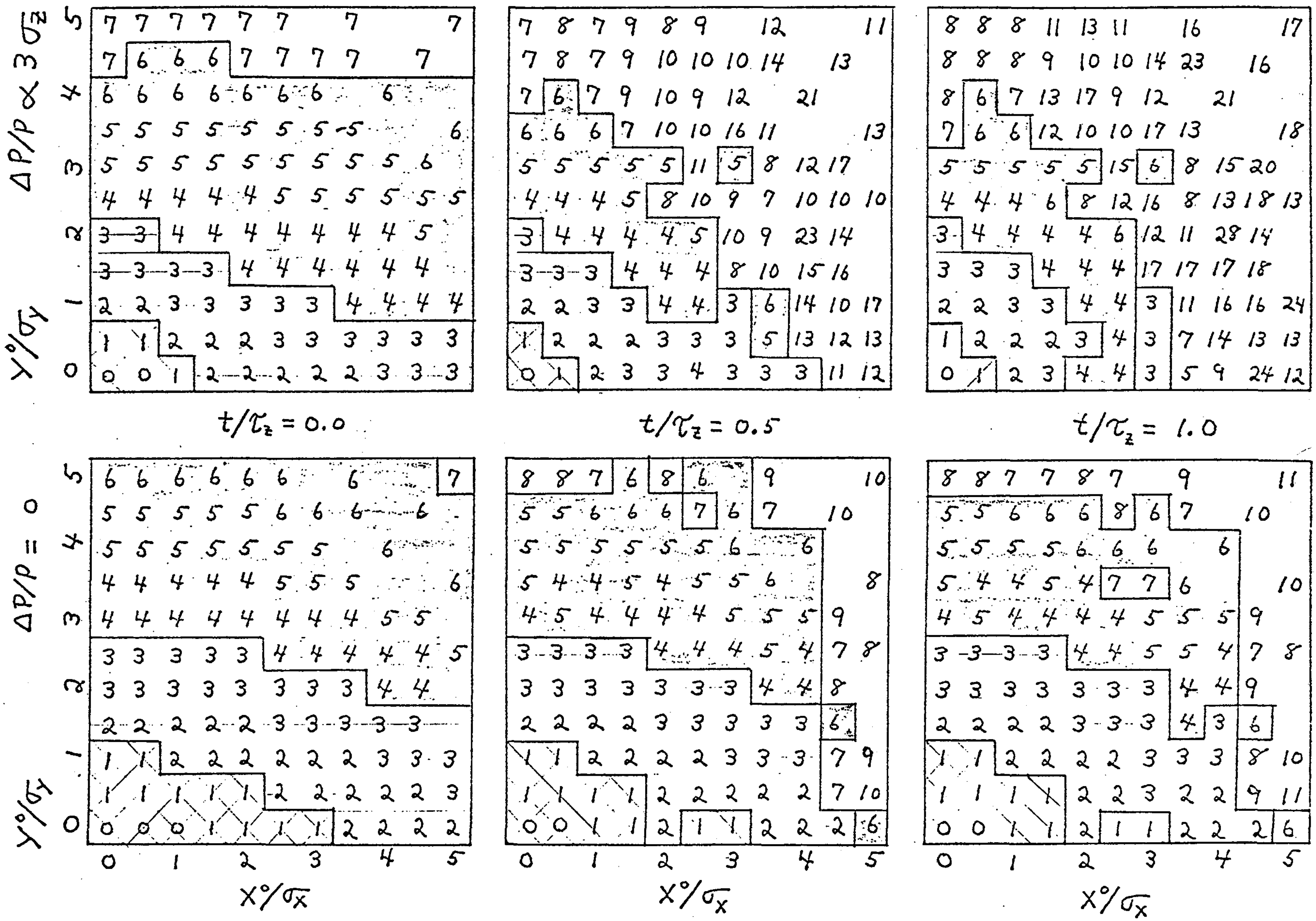
FIGURE 3

SPEAR I Scaled to 15 GeV
Round Beam Luminosity Calculations



SPEAR I at 1.5 GeV
Flat Beam Maximum x-plane Amplitude Plots

FIGURE 4



SPEAR I at 1.5 GeV
Flat Beam Maximum y-plane Amplitude Plots

FIGURE 5

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