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Taking Advantage of Multi-User Diversity in OFDM Systems

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***Abstract*—A multi-user diversity approach, called Opportunistic Interference Management (OIM), is considered in connection with Orthogonal Frequency Division Multiplexing (OFDM) systems. The OIM is applied to each consecutive group of OFDM sub-channels that are highly correlated. We use the OIM to transmit information in Q_g consecutive sub-channels to d users in parallel. The expected parallel transmissions per group of sub-channels is then computed. The results show that the maximum expected parallel transmissions is achieved almost surely when enough mobile users exist.**

I. Introduction

Opportunistic Interference Management (OIM) [5] is a multiuser diversity technique that exploits fading in wireless channels in order to overcome interference. A mobile station (MS) antenna that possesses a strong channel with one base station (BS) antenna, and weak channels with the remaining BS antennas, will be able to receive data from the strong channel while other BS antennas can simultaneously transmit to other users without the need of any active interference management technique due to the good signal-to-interference-plus-noise ratio (SINR). This is done by selecting appropriate thresholds for the signal-to-noise ratio (SNR) and the interference-to-noise ratio (INR) for transmitting and interfering antennas respectively. If two mobile stations satisfy the OIM condition, and have strong channels with different base station antennas, then they can receive data simultaneously without significantly interfering with each other. Thus, the effects of fading become beneficial in the interfering channels.

OFDM is a method of efficiently utilizing a given bandwidth by subdividing the channel into many overlapping, orthogonal, flat-fading sub-channels of equal bandwidth. This is done by exploiting the properties of frequency-selective fading channels, and adding an appropriate cyclic prefix to the transmitted data stream in order to avoid intersymbol interference (ISI). When this technique is used in multi-antenna systems, each transmitting antenna can transmit data to multiple receiver antennas during the same transmission period using different sub-channels. In this paper within each group of adjacent sub-channels, we use multi-user diversity for data transmission.

More specifically, we investigate the application of the OIM technique to OFDM systems by considering the high correlation between consecutive sub-channels [6]. Since OFDM has a flat-fading model for each sub-channel, the OIM technique is directly applicable. Due to this high correlation between sub-channels, if a mobile station (MS) antenna satisfies the OIM condition, there is a high probability that the MS antenna satisfies the OIM condition in the neighboring sub-channels. In order to apply the OIM technique [8] to OFDM systems efficiently, we consider groups of sub-channels, and only transmit using the OIM technique if a MS antenna satisfies the OIM condition with the same BS antennas for the entire group of sub-channels. Hence, in each group, we can transmit data from a set of BS antennas to a set of MS antennas within the same group of sub-channels, so long as each BS antenna sends data to only one MS antenna, and each selected MS antenna receives data from only one BS antenna. As in the original OIM technique [8], the expected value of the multiplexing gain $D = \mathrm{E}\,[\![d]\!]$ is computed analytically and compared to simulation results.

The rest of the paper is organized as follows. In section II, the related work is described and section III focues on preliminaries. We derive the probability of satisfying the OIM condition in section IV. Section V demonstrates the simulation results and the paper is concluded in section VI.

II. Related Work

A multi-user diversity technique was first applied to wireless channels by Knopp and Humblet [1], where they showed that if the channel with the highest SNR is selected, all of the power should be allocated to the user associated with this channel. There have been several techniques that take advantage of multi-user diversity. For example, Viswanath et al. [2] used opportunistic beamforming in cellular system models to achieve the gains of true beamforming without the transmitter needing to employ any further coding beyond that of single antenna transmissions. Other techniques in [3], [4] have also taken advantage of multi-user diversity.

The multi-user diversity technique used in this work is called Opportunistic Interference Management, or OIM. This technique was developed by Wang et al. in [5], and is proven to scale similar to DPC capacity in the downlink of wireless cellular systems asymptotically.

III. Preliminaries

A. Opportunistic Interference Management

Consider a multi-antenna system with N_T BS antennas and N_R MS antennas. Let the channel have flat fading, so that the entire channel can be described by the equation

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{w}. \tag{1}$$

where $\boldsymbol{y}, \boldsymbol{w} \in \mathbb{C}^{N_R \times 1}$, $\boldsymbol{x} \in \mathbb{C}^{N_T \times 1}$, and $\boldsymbol{H} \in \mathbb{C}^{N_R \times N_T}$. Also for $H_{i,j} = \{\boldsymbol{H}\}_{i,j}$, then $H_{i,j} \sim \mathcal{CN}(0, \sigma^2)$. If $i \neq k$ or $j \neq l$, we have $\mathrm{E}[\![H_{i,j}H^*_{k,l}]\!] = 0$ and $\boldsymbol{w} \sim \mathcal{CN}(\boldsymbol{0}, N_0\boldsymbol{I})$.

A mobile user MS_i is said to have the OIM condition if it has a strong channel (high SNR) with a base station antenna BS_{j_s} and a weak channel (low SNR) with the remaining base station antennas BS_{j_w}, $j_w \neq j_s$. A channel is considered strong if the SNR is above a certain threshold SNR_{th}, and weak if the SNR is below a threshold INR_{th}. This condition is summarized as

$$\begin{aligned} \mathrm{SNR}_{i,j_s} &\geq \mathrm{SNR}_{th} \\ \mathrm{INR}_{i,j_w} &\leq \mathrm{INR}_{th}, \forall j_w \neq j_s. \end{aligned} \tag{2}$$

It can be seen that, in the event that a MS antenna satisfies the OIM condition, the probability that the strong channel is with BS_{j_1} is the same as if the strong channel is with BS_{j_2}. This is a consequence of the independence between the channels. Therefore, as long as the channels between different antennas are independent, the probability distribution of the multiplexing gain d is found by the approach in the appendix.

B. Orthogonal Frequency Division Multiplexing

Consider an OFDM wireless communication system with N_T BS antennas and N_R MS antennas. The channel between antennas BS_j and MS_i is a frequency-selective channel with impulse response that is described by at most L i.i.d. channel taps $\{h_{i,j}[0], h_{i,j}[1], \ldots, h_{i,j}[L-1]\}$, such that $h_{i,j}[l] \sim \mathcal{CN}(0, \sigma^2_{i,j}[l])$, $\forall i = 1, \ldots, N_T$, and $j = 1, \ldots, N_R$. We also assume the channel remains almost constant over a transmission period T. Further, $h_{i_1,j_1}[l_1]$ and $h_{i_2,j_2}[l_2]$ are independent for all $i_1 \neq i_2$ or $j_1 \neq j_2$. Each $h_{i,j}[l]$ is a circularly symmetric complex Gaussian process, such that $\Re\{h_{i,j}[l]\}$ and $\Im\{h_{i,j}[l]\}$ are independent [7].

In order to take advantage of the frequency-selective channel, an OFDM system sends data symbols over the channel in separate sub-channels. Suppose that each symbol X_k is allocated in sub-channel k. Then, a block of Q data symbols $\{X[0], X[1], \ldots, X[Q-1]\}$ is transmitted over Q sub-channels during one transmission period. The block of Q data symbols undergo a Q-point inverse discrete Fourier transform (IDFT) to create Q time-domain symbols $\{x[0], x[1], \ldots, x[Q-1]\}$. The Q-point IDFT is given by

$$x[m] = \frac{1}{Q}\sum_{k=0}^{Q-1} X[k] \exp\left[j\frac{2\pi km}{Q}\right]. \tag{3}$$

After applying a cyclic prefix and transmitting the time-domain symbols, the channel gain between BS_j and MS_i in the k^{th} sub-channel is the DFT of the time-domain channel with frequency k as $H_{i,j}[k]$. Therefore, the received symbol vector in subchannel k has the flat-fading model

$$\boldsymbol{Y}[k] = \boldsymbol{H}[k]\boldsymbol{X}[k] + \boldsymbol{W}[k], \tag{4}$$

where $\boldsymbol{Y}[k]$ is the vector of received symbols at the mobile users, $\boldsymbol{X}[k]$ is the vector of transmitted symbols in sub-channel k, $\boldsymbol{H}[k]$ is the channel matrix in subchannel k, and $\boldsymbol{W}[k]$ is the additive receive noise vector in subchannel k.

Clearly, each sub-channel has the same structure that was used for OIM, i.e. a flat-fading model as in Eq(1). Therefore, it is easy to take advantage of this by using OIM. However, the matter of subchannel correlation needs to be considered, since different sub-channels, though orthogonal, are not independent of each other. Taking this into consideration, the conditions by which a mobile user will receive information using OIM will be proposed and analyzed.

C. Channel Estimation Considerations in OFDM Systems

In order to determine the channel conditions in the OFDM sub-channels, estimates of the sub-channels must be obtained at the receiver and sent to the transmitter via a feedback channel. The channel estimates of the sub-channels are obtained by inserting pilot tones into the transmitted symbol streams at the transmitter into specific sub-channels, which are known to the receiver. The known pilots are received in the known sub-channels at the receiver, and then a MMSE estimate of the sub-channel gains are computed [10]–[13]. In our work, it is assumed that via the channel estimates, the SNR conditions all of the sub-channel between all of the BS-MS antenna pairs are known at the transmitter via feedback.

IV. Probability of Satisfying the OIM condition for Group of $Q_g \leq L$ sub-channels

An OFDM system with Q sub-channels is divided into groups Q_g consecutive sub-channels. It is known that approximately $Q/2L$ sub-channels are strongly correlated [6]. If Q_g is set so that $Q_g < Q/2L$, then there is a strong probability that if one sub-channel possesses the OIM condition for a set of BS-MS pairs, then the entire group of Q_g sub-channels possess the OIM condition for the same set of BS-MS pairs. We are interested to compute the probability that Q_g consecutive sub-channels satisfy the OIM condition. Indeed, we can see that the event of satisfying this condition is independent between different BS-MS antenna pairs, due to the time-domain channel vectors being independent between different antenna pairs. If we call the event of MS_i satisfying the OIM condition as A, then the probability of event A is the product of the SNR conditions for each BS antenna with respect to MS_i.

The distribution of the magnitude-squared (or power) of sub-channel gains, from which we can determine the SNR distribution of the sub-channels, must be found for consecutive sub-channels. Each sub-channel gain $H_{i,j}[k]$ is a circularly symmetric Gaussian RV. Let the vector $\boldsymbol{h}_{i,j} = [h_{i,j}[0], h_{i,j}[1], \ldots, h_{i.j}[L-1]]^T$ be the channel vector from

BS_j to MS_i, and the coefficients Q-length DFT for sub-channel k be $\boldsymbol{v}_k = [1, e^{-j2\pi k/Q}, \ldots, e^{-j2\pi(L-1)/Q}]$. For now we drop the antenna indexes, knowing that we refer to a specific BS-MS antenna pair. Then the covariance between $H[k_1]$ and $H[k_2]$ is found by

$$\begin{aligned} \text{E}\left[\!\left[H[k_1]H^*[k_2]\right]\!\right] &= \text{E}\left[\!\left[\boldsymbol{v}_{k_1}\boldsymbol{h}\boldsymbol{h}^H\boldsymbol{v}_{k_2}^H\right]\!\right] \\ &= \boldsymbol{v}_{k_1}\text{E}\left[\!\left[\boldsymbol{h}\boldsymbol{h}^H\right]\!\right]\boldsymbol{v}_{k_2}^H \\ &= \sigma^2\boldsymbol{v}_{k_1}\boldsymbol{v}_{k_2}^H. \end{aligned} \tag{5}$$

If the DFT matrix for the group of Q_g consecutive sub-channels $k, k+1, \ldots, k+Q_g-1$ is $\boldsymbol{V}_k = [\boldsymbol{v}_k^T, \boldsymbol{v}_{k+1}^T, \ldots \boldsymbol{v}_{k+Q_g-1}^T]^T$, then the vector of Q_g consecutive sub-channel gains is $\boldsymbol{G}_k = [H[k], H[k+1], \ldots, H[k+Q_g-1]]^T = \boldsymbol{V}_k\boldsymbol{h}$. Therefore, the covariance matrix is $\boldsymbol{C}_G = \sigma^2\boldsymbol{V}_k\boldsymbol{V}_k^H$. Notice that the matrix $\boldsymbol{V}_k\boldsymbol{V}_k^H$ is the same for all groups of Q_g sub-channels. The joint distribution of $\boldsymbol{G}_k \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{C}_G)$.

In applying the OIM conditions to the groups of OFDM sub-channels, the SNR in each sub-channel for the same pair of BS-MS antennas must meet the same threshold condition. So the joint distribution of the SNR of Q_g consecutive sub-channels analyzed and found from knowing the Q_g dimensional complex region which contains either the SNR threshold condition or INR threshold condition.

If we normalize the AWGN sub-channel noise ($N_0 = 1$), the event that BS_j and MS_i have Q_g consecutive sub-channels with power less than or equal to the INR_{th} is event $A_{\text{INR}} = \{P_{i,j}[k] \leq \text{INR}_{th},\ P_{i,j}[k+1] \leq \text{INR}_{th},\ \ldots,\ P_{i,j}[k+Q_g-1] \leq \text{INR}_{th}\}$. Let $H[k] = \alpha[k] + j\beta[k]$. We know that $\alpha[k]$ and $\beta[k]$ are independent for any $k = 0, \ldots, Q-1$. Define the Q_g dimensional complex region $\mathcal{S}_{\text{INR}} = \{\boldsymbol{z} = [z_0\ z_1\ \ldots\ z_{Q_g-1}]^T \in \mathbb{C}^{Q_g \times 1} : |z_i| \leq \sqrt{\text{INR}_{th}},\ \forall i = 0, \ldots, Q_g-1\}$. It follows that the probability of event A_{INR} for any BS-MS antenna pair is

$$\begin{aligned} &\Pr\{A_{\text{INR}}\} \\ &\quad= \Pr\{\boldsymbol{G}_k \in \mathcal{S}_{\text{INR}}\} \\ &\quad= \int_{\mathcal{S}_{\text{INR}}} \frac{1}{\pi^{Q_g}|\boldsymbol{C}_{\boldsymbol{G}}|} \exp\left[-\boldsymbol{z}^H\boldsymbol{C}_G^{-1}\boldsymbol{z}\right] d\boldsymbol{z} \end{aligned} \tag{6}$$

where $|\boldsymbol{C}_G| = \det(\boldsymbol{C}_G)$.

Similarly, the event where Q_g consecutive sub-channels have a power above the threshold SNR_{th} is $A_{\text{SNR}} = \{P_{i,j}[k] \geq \text{SNR}_{th}, P_{i,j}[k+1] \geq \text{SNR}_{th}, \ldots, P_{i,j}[k+Q_g-1] \geq \text{SNR}_{th}\}$. Define the Q_g dimensional complex region $\mathcal{S}_{\text{SNR}} = \{\boldsymbol{z} = [z_0\ z_1\ \ldots\ z_{Q_g-1}]^T \in \mathbb{C}^{Q_g \times 1} : |z_i| \geq \sqrt{\text{SNR}_{th}},\ \forall i = 0, \ldots, Q_g-1\}$. Then the probability of event A_{SNR} for any BS-MS antenna pair is

$$\begin{aligned} &\Pr\{A_{\text{SNR}}\} \\ &\quad= \Pr\{\boldsymbol{G}_k \in \mathcal{S}_{\text{SNR}}\} \\ &\quad= \int_{\mathcal{S}_{\text{SNR}}} \frac{1}{\pi^{Q_g}|\boldsymbol{C}_{\boldsymbol{G}}|} \exp\left[-\boldsymbol{z}^H\boldsymbol{C}_G^{-1}\boldsymbol{z}\right] d\boldsymbol{z} \end{aligned} \tag{7}$$

Once the probabilities of the SNR and INR thresholds are found, the probability for a MS antenna satisfying OIM condition with the BS antennas, which we will call event A, is

$$\Pr\{A\} = \binom{N_T}{1}\Pr\{A_{\text{SNR}}\}(\Pr\{A_{\text{INR}}\})^{N_T-1} \tag{8}$$

V. Theoretical and Simulation Results

Now that the probability of event A is known, the distribution of the multiplexing gain d is found in the same manner as the original OIM paper. Details of how to formulate the conditional distribution of d given a certain number of MS antennas n can be found in the appendix. Then, the expected multiplexing gain $D = \text{E}[\![d]\!]$ is found using Theorem 1 in [8]. Therefore,

$$D = N_T\left(1 - \left(1 - \frac{\Pr\{A\}}{N_T}\right)^{N_R}\right). \tag{9}$$

Figure 1. shows the performance of this scheme. For simplicity, both the time-domain channel taps and AWGN sub-channel noise have been normalized. The OIM conditions from (2) simulated for $\text{SNR}_{th} = 10$, 20, and 30, and $\text{INR}_{th} = 2$. For this simulation, we used 512-FFT, 8 taps wireless channel and using 4 sub-channels for each group. The number of BS antennas are 3 and each MS has only one antenna. The theoretical plot of D using (9) and the plot of the channel simulation can be seen to be approximately the same. Therefore, for any given number of BS and MS antennas, OIM conditions, and sub-channel group size, we can determine what the expected multiplexing gain is for the system using equations (6,7,8,9). Note that in the original OIM without OFDM, we require more than 30000 nodes to achieve multiplexing gain of two, while by using OFDM, we need less than 1500 nodes to achieve the same multiplexing gain. As we showed in [9], by utilizing a modified version of antenna selection we could reduce that number from 30000 nodes to less than 200. We predict that by combining antenna selection as in [9] with OFDM technique, we can reduce the required number of mobile users to a very small number which makes OIM a very practical approach for taking advantage of multiuser diversity in wireless cellular networks. We will investigate this approach in the future.

VI. Discussion and Future Work

By using the fact that OFDM turns a frequency-selective fading channel into parallel flat-fading sub-channels, we were able to show that the multi-user diversity technique OIM can be applied to groups of sub-channels. Thus, we can transmit information to a given number of users within the group of sub-channels using point-to-point communication, so long as the base-station knows the sub-channel conditions. By using this approach, the expected multiplexing gain within a group of sub-channels can be found deterministically for a given number of base-station antennas and mobile users. It can be seen that if the number of resolvable channel taps increases, it allows for a larger group size to be considered, such that the OIM condition will still be satisfied with a good probability.

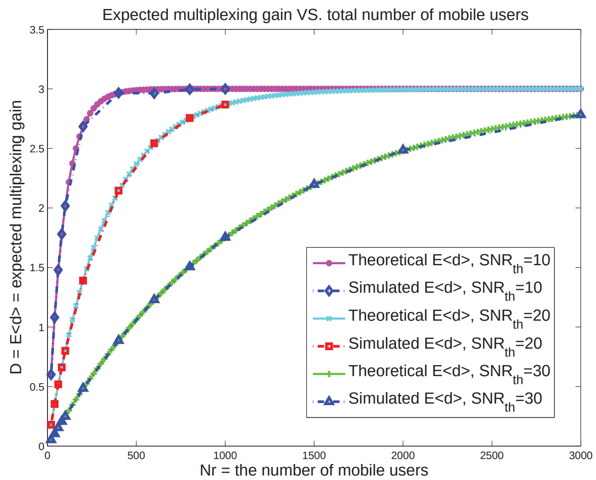

Fig. 1. This plot shows the performance of the expected multiplexing gain when $Q = 512$, $L = 8$, $Q_g = 4$, and $N_T = 3$

An investigation of the system and OIM parameters is needed to know how to achieve a given desired performance. Furthermore, we can use an antenna-selection algorithm similiar to [9] in order to increase the multiplexing gain per sub-channel group requiring smaller number of mobile users. We can soften the conditions set in (2), as well as those set by the regions $\mathcal{S}_{\text{INR}}$ and $\mathcal{S}_{\text{SNR}}$. The latter can be done by allowing the sub-channel SNR's to satisfy the OIM condition on average, so long as the SINR in each group of sub-channels is still good.

Another matter to consider is the optimum selection for the group size of consecutive sub-channels in order to maximize the expected multiplexing gain per group. This seems to be dependent on the number of resolvable channel taps, the length of the DFT, and the OIM conditions. Knowing the optimum group size for these given parameters is important in order to not waste the available bandwidth. This may also lead to better selection of DFT size and selection of thresholds for OIM conditions.

VII. Appendix

A. Distribution of Multiplexing Gain d as a Function of N_R

In current base stations, the antennas can be arranged in a manner such that, when considering the channel received at a mobile station, the channels from the different base station antennas are considered statistically independent. In other words, the channel gain h_{i_1,j_1} from one BS_{i_1} to a MSj_1 antenna has the same distribution of the channel gain h_{i_2,j_2} from BS_{i_2} to MSj_2, but they are independent random variables when either $i_1 \neq i_2$ or $j_1 \neq j_2$. Suppose that there are the events A_{SNR} and A_{INR}, which are such that when a mobile station has the event A_{SNR} with one BS antenna and A_{INR} with the remaining BS antennas, we say that the MS antenna is eligible for OIM selection. The OIM eligibility event A has a probability $\Pr\{A\} = \binom{N_T}{1}\Pr\{A_{\text{SNR}}\}(\text{PR}\{A_{\text{INR}}\})^{N_T-1}$. Satisfying the OIM contidtion has a Bernoulli distribution, with $p = \Pr\{A\}$. Since the probability of satisfying this event A for one MS antenna is independent of satisfying it for another MS antenna, the probability distribution for the number of MS antennas N that satisfy this event out of the total of MS antennas N_R is

$$\Pr\{N = n\} = \binom{N_R}{n}(\Pr\{A\})^n(1 - \Pr\{A\})^{N_R-n}. \quad (10)$$

This event does not consider the multiplexing gain, because more than one MS antenna can have a strong channel with the same BS antenna. The distribution of the multiplexing gain is found sequentially. Consider the event that there are n MS antennas satisfying the OIM condition. If there are N_T BS antennas, the total number of scenarios where n MS antennas satisfy the OIM eligibility event is $(N_T)^n$. This is because the strong channel can land in any one of N_T BS antennas, for each of the n MS antennas. Each one of these scenarios has the same probability of occuring, so in order to find the distribution of the multiplexing gain, we must just find the number of scenarios where a specific multiplexing gain occurs, and divide it by the total number N_T^n.

The distribution should be found sequentially. Since $1 \leq n \leq N_R$, we begin considering the probability of having a multiplexing gain of $d = 1$. For n MS antennas, the only way that a multiplexing gain of 1 can occur is when all of the MS antennas have the strong channel with the same BS antenna. So, all of the MS users can have a strong channel with one BS antenna only one way. Let's call $C(1) = 1$. This can happen a total of $\binom{N_T}{1}$ different ways. Let's call this number $C(1)$. So the probability of having a multiplexing gain of $d = 1$ is

$$\Pr\{d = 1|N = n\} = \binom{N_T}{1}C(1)\frac{1}{N_T^n}. \quad (11)$$

If the multiplexing gain $d = 2$, the strong channels for all of the n MS antennas have to all occur with only 2 out of the N_T BS antennas. Each MS antenna can have the strong channel with either of the 2 BS antennas. Therefore, there are a total of 2^n ways that the MS antennas have strong channels with within the two BS antenas. Furthermore, the two BS antennas can be selected a total of $\binom{N_T}{2}$. However, this does not discount the number of ways in which all of the BS antennas have strong channels with the same BS antenna out of the two. This occurs 2 ways out of the possible 2^n ways. So if we subtract that number, the total number of ways of having a strong channel within those 2 BS antennas is $C(2) = 2^n - 2$, and hence the total number of ways to have a multiplexing gain $d = 2$ is $\binom{N_T}{2}C(2)$. Therefore, the probability of having a multiplexing gain of $d = 2$ when $N = n$ is

$$\Pr\{d = 2|N = n\} = \binom{N_T}{2}C(2)\frac{1}{N_T^n}. \quad (12)$$

Likewise, we consider a multiplexing gain of $d = 3$ when $N = n$ MS antennas have satisfied the OIM condition. For 3 BS antennas, there are 3^n ways that n MS antennas can have strong channels within these 3 BS antennas. From this there are $\binom{3}{2}C(2)$ ways that the n MS users can have the condition with only 2 of the 3 BS antennas, and $\binom{3}{1}$ ways that they can have strong channels with only one of the 3 BS antennas. The number $C(3) = (3^n - (3)C(2) - 3) = 3^n - 3(2^n) + 3(1)$ is the number of ways that all 3 out of 3 BS antennas have at least one strong channel. So the total number of ways that there is a multiplexing gain $d = 3$ is $\binom{N_T}{3}C(3)$. Hence,

$$\Pr\{d = 3 | N = n\} = \binom{N_T}{3} C(3) \frac{1}{N_T^n}. \quad (13)$$

Generally, the number of ways that a group of N_T BS antennas and N_R MS antennas can have a multiplexing gain of $d = m$ given that $N = n$ MS antennas have satisfied the OIM condition is

$$\begin{aligned} C(m) &= m^n - \sum_{q=1}^{m-1} \binom{m}{q} C(q) \\ &= \sum_{q=1}^{m} \binom{m}{q} (-1)^{m-q} q^n. \end{aligned} \quad (14)$$

Hence, the probability of having $d = m$ given that $N = n$ is

$$\Pr\{d = m | N = n\} = \binom{N_T}{m} C(m) \frac{1}{N_T^n}. \quad (15)$$

Therefore, the distribution of the multiplexing gain is given by

$$\begin{aligned} &\Pr\{d = m\} \\ &= \sum_{n=1}^{N_R} \Pr\{d = m, N = n\} \\ &= \sum_{n=1}^{N_R} \Pr\{d = m | N = n\} \Pr\{N = n\} \\ &= \sum_{n=1}^{N_R} \binom{N_T}{m} C(m) \frac{1}{N_T^n} \\ &\quad \cdot \binom{N_R}{n} (\Pr\{A\})^n (1 - \Pr\{A\})^{N_R - n}. \end{aligned} \quad (16)$$

It is shown in [8], that the expected value of d is given by (9).

References

[1] R. Knopp and P. Humblet, "Information Capacity and Power Control in Single-Cell Multiuser Communications," Proc. of IEEE ICC, Seattle, Washington, USA, June 18-22 1995.

[2] P. Vizwanath, D. Tse, R. Laroia, "Opportunistic Beamforming using Dumb Antennas," *IEEE Transaction on Information Theory*, Vol. 48, No. 6, pp. 1277-1294, June 2002.

[3] M. Sharif and B. Hassibi, "A Comparison of Time-Sharing, DPC, and Beamforming for MIMO Broadcast Channels with Many Users," *IEEE Transaction on Communications*, Vol 55, No. 1, pp. 11-15, January 2007.

[4] A. Tajer and X. Wang, "Opportunistic Multi-Antenna Downlink Transmission with Finite-Rate Feedback," *Proc. of Allerton 2008*, Monticello, Illinois, USA, September 23-26, 2008.

[5] Z. Wang, M. Ji, L. Dritsoula, H. Sadjadpour, JJ. Garcia-Luna, "On the Minimum Side Information of the MIMO Broadcast Channel," European Wireless Conference, 2010, pp. 26-33

[6] D. Tse and P. Viswanath, "Point-to-Point Communication: detection, diversity, and channel uncertainty," in Fundamentals of Wireless Communication, Cambridge University Press, 2005, ch.3, pp.49-110.

[7] X. Wang and V. Poor, "Advanced Signal Processing for Coded OFDM Systems," in Wireless Communication Systems: Advanced Techniques for Signal Reception, First Printing, Upper Saddle River, CA, Prentice Hall Professional Technical Reference, 2004 ch. 10, sec.1-2, pp.551-555.

[8] M. Kiskani, Z. Wang, H.R. Sadjadpour and JJ. Garcia-Luna, "Opportunistic Interference Management - A New Approach for Multi-Antenna Downlink Cellular Networks," UCSC Technical Report, soe.ucsc.edu/~mohsen/Publications.htm.

[9] L. Dritsoula et al., "Antenna Selection for Opportunistic Interference Management in MIMO Broadcast Channels," Signal Processing Advances in Wireless Communication, 2010, pp.1-5.

[10] F. Dowla, "Handbook of RF and Wireless Technologies," Elsevier, 2004, Chapter 10.

[11] Y.M. Abdelkader, E.A. Jamal, "Optimum Spacing Design for Pilots in OFDM Systems over Multipath Fading Channels," Scientific Research - Comunications and Network, 2010.

[12] S. Coleri et al., "Channel Estimation Techniques Based on Pilot Arrangement in OFDM Systems," IEEE Transactions on Broadcasting, Vol. 48, No. 3, September 2002.

[13] S.K. Kim, Y.H. Lee, "3-Dimensional MMSE Channel Estimation in Multi-Antenna OFDM Systems," Third International Conference on Digital Telecommunications, IEEE Computer Society, 2008.