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Pricing of Software Services

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Abstract

We analyze and compare *fixed-fee* and *usage-fee* software pricing schemes – in fixed-fee pricing, all users pay the same price; in usage-fee pricing, the users' fees depend on the amount that they use the software (e.g., the user of an online-database service might be charged for each data query). We employ a two-dimensional model of customer heterogeneity – specifically, we assume that customers vary in the amount that they will use the software (*usage heterogeneity*) and also in their *per-use valuation* of the software.

To understand the performance of these pricing schemes and their sensitivity to the competitive environment in which they are used, we look at a number of different scenarios: a monopolist offering just one of these schemes, a monopolist offering a choice of pricing schemes, and several duopoly scenarios. We characterize and compare the equilibria that arise in these scenarios and provide insights into optimal pricing strategies.

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1 Introduction

In this paper we analyze and compare *fixed-fee* and *usage-fee* software pricing schemes – in fixed-fee pricing all users pay the same price; in usage-fee pricing the users’ fees depend on the amount that they use the software (e.g., the user of a database-based system might be charged for each data query). Fixed-fee pricing is today’s dominant pricing scheme, and our interest in usage-fee pricing stems from the following:

1. The software services industry is currently acquiring scale in the form of Application Service Providers (ASPs) that offer industrial software such as CRM (e.g., Salesforce.com) to small and medium sized firms. The technical feasibility of usage monitoring, a prerequisite for usage-fee pricing, is assured by the fact that the World Wide Web is the primary delivery platform used by ASPs, and firms are currently offering usage-fee pricing (e.g., KnowledgePoint). Nonetheless, usage-fee pricing in this industry is still in a nascent stage.¹
2. In contrast to 1, usage-fee pricing is relatively common among computing infrastructure service providers. Some notable examples of firms that have pursued such a strategy in this space are Jamcracker and HP. In other areas such as media licensing, firms offer quite different pricing schemes; for example, Apple iTunes charges customers 99 cents for every song downloaded while Realplayer’s Rhapsody charges a fixed monthly fee with an unlimited number of downloads
3. In a sense, usage-fee pricing is a rebirth of an old business model that was once prevalent in the software industry. Back in the days when all computers were huge and expensive, time-sharing on large IBM mainframes was common practice, so firms implemented usage-fee pricing schemes. This business model became less important as computer hardware got a lot cheaper and faster. Recently, the spread of complex enterprise software has again increased the costs of software deployment and maintenance, so enterprise software firms such as SAP are turning to usage-fee pricing to reach small and medium size customers.

An interesting question is how do consumers behave in this setting? Unlike many other service industries in which consumers pay a price based on resource utilization (also referred to in the literature as access industries), two consumers may use a software package for the same amount of

¹<http://www.comptia.org/sections/ssg/research.asp>

time but may differ in the relative importance of their work. Compare a long but frivolous web surfing session by one consumer versus a short search on a high quality industrial database. While a usage meter shows a low number in the former case, the latter consumer is probably willing to pay much more. Similarly, some music lovers value a small collection of specific songs while others prefer quantity and selection over specific tastes.

These consumer heterogeneity issues introduce several modeling and analytical issues. First, we believe that this setting begs for a 2-dimensional representation of consumer heterogeneity and this is in contrast to the preponderance of existing literature. Second, the performance of these pricing schemes are sensitive to the competitive environment in which they are used; to better understand this, we look at a number of different scenarios: a monopolist offering just one of these schemes, a monopolist offering a choice of pricing schemes, and several duopoly scenarios. We characterize and compare the equilibria that arise in these scenarios and provide insights into optimal pricing strategies.

Our models assume “customer usage heterogeneity” meaning that some customers will use the product more than others. The literature on pricing in the face of this form of heterogeneity has roots in economics via Oi(1971) who shows that a truly discriminating two part tariff globally maximizes monopoly profits by extracting all consumer surpluses. A stream of literature that analyzes different pricing mechanisms under different conditions flows from this basic source. The Oi model is extended by Schmalensee(1981) who analyzes the case of profit constrained welfare maximization. Phillips and Battalio(1983) investigate the situation where buyers can substitute between visits and also consumption between visits. Hayes(1987) shows that two part tariffs act as a form of insurance in environments with uncertainty and hence is offered by firms even in a competitive setting. Other forms of pricing can also be mapped into this framework, most notably quantity discounts (see Dolan(1987)). Fixed-fee pricing in the face of customer usage heterogeneity is also known as “buffet pricing” as in Nahata et al(1999). Also related is Bashyam(2000) who models competition in business information services markets between a fixed-price two-part tariff, Sundararajan (2002) who compares a monopolist’s fixed-fee versus a usage-fee in the presence of network externalities, and Wu et al(2002).

In comparing fixed- and usage-fees it becomes important to recognize that customers may also be heterogeneous with respect to their “per use” valuation of the product and that this likely exists in combination with customer usage heterogeneity. We thus incorporate both types of heterogeneity, and this two-dimensional model of customer heterogeneity is a distinctive feature of this paper.

Other than Bashyam(1996), which focuses on a technology choice problem, we are unaware of any other papers that employ such a model.

Recently emerging is research into pricing of various kinds of access services. Jain et al(1999) develop a model for cellular phonecall prices and find the optimal pricing strategy over time. Essegai et al(2002) compare two part tariffs with fixed-fee pricing and usage pricing for access service industries under conditions of customer usage heterogeneity and limited capacity. Danaher(2002) conducts a market experiment to compare different two part pricing packages for new subscription services.

There is also a growing literature around ‘economics of software’ which studies various economic issues related to software, highlighting its distinctive features and unique production process. Boehm(1981) and Kemerer(1987) address software cost estimation. Richmond et al(1992) and Whang(1992) tackle contract design for custom software development. Whang(1995) addresses pricing and bidding for custom software projects in the presence of learning effects for the developers. Snir and Hitt(2003) and Carr(2003) investigate bidding behavior of software vendors in online auctions for software development projects. Conner and Rumelt(1991) analyze piracy protection strategies for a software firm.

In short, this paper adds to the literature on software pricing by comparing these different pricing schemes in a richer model of customer heterogeneity than previously seen. It contributes to the access industry pricing literature since the software services industry is part of the online access industry. It also adds to the software economics literature by tackling economic issues related to the latest developments in the software industry.

The paper is organized as follows. Section 2 defines the model. Sections 3 and 4 apply the model to monopoly and duopoly settings respectively, and the body of the paper concludes with section 5. Section 6 contains proofs and derivations.

2 Model definition

Again, we analyze equilibrium behavior in scenarios in which one (section 3) or two (section 4) firms offer combinations of fixed and usage-based pricing schemes to heterogenous customers who

self-select which price (if any) to pay. This section defines the general model and notation.

Customers and product: To model the two types of heterogeneity discussed earlier, each customer is described by a two dimensional vector (α, β) in which α denotes the utility that the customer will derive from a single use of the product and β represents the frequency with which the customer will use the product. A scalar U parameterizes the products quality or functionality, so an “ (α, β) -customer” enjoys utility of $\alpha\beta \cdot U$ from purchasing the product.

The set of potential customers is modelled as an atomless spread of (α, β) pairs distributed evenly over the $[0, 1] \times [0, 1]$ square. Thus, if M is the size of the potential market, then any market of size $x \cdot M$ corresponds to a fraction x of the area of this square. To simplify however, the market is normalized by setting M equal to one; this is just scaling and does not sacrifice generality. Purchasing decisions follow naturally; customers are assumed to self-select whatever purchasing option maximizes this utility net of price.

Fixed-fee pricing: This is today’s ubiquitous pricing scheme; buyers pays a common price P_f , and receive unlimited use of the product. Disregarding any other pricing options, purchasing is worthwhile for an (α, β) -customer if

$$\alpha\beta \cdot U - P_f > 0. \quad (1)$$

Figure 1 illustrates. Customers who share the same value of $\alpha \cdot \beta$ derive the same utility from the product, so hyperbolae on the unit square become lines of “iso-utility.” These are the curves in figure 1(a); each iso-utility line is a locus of customers with identical purchasing behavior under fixed-fee pricing. The P_f price chosen by the firm then segments the customers along one of these hyperbolae as shown in figure 1(b). In that figure, the boundary between the “do not purchase” segment and the “pay fixed-fee” segment is the curve $\alpha \cdot \beta = \frac{P_f}{U}$.

Usage-fee pricing: In this pricing scheme, the customer pays P_u for each use. Since she uses the product with frequency β , her total payments are $\beta \cdot P_u$ versus utility of $\alpha\beta U$, so this scheme is worthwhile to her if

$$\beta (\alpha U - P_u) > 0.$$

This is analogous to (1) but results in a very different structure as illustrated by figure 2. Iso-utility lines are now vertical, and the firm’s selection of a particular value of P_u segments the market along

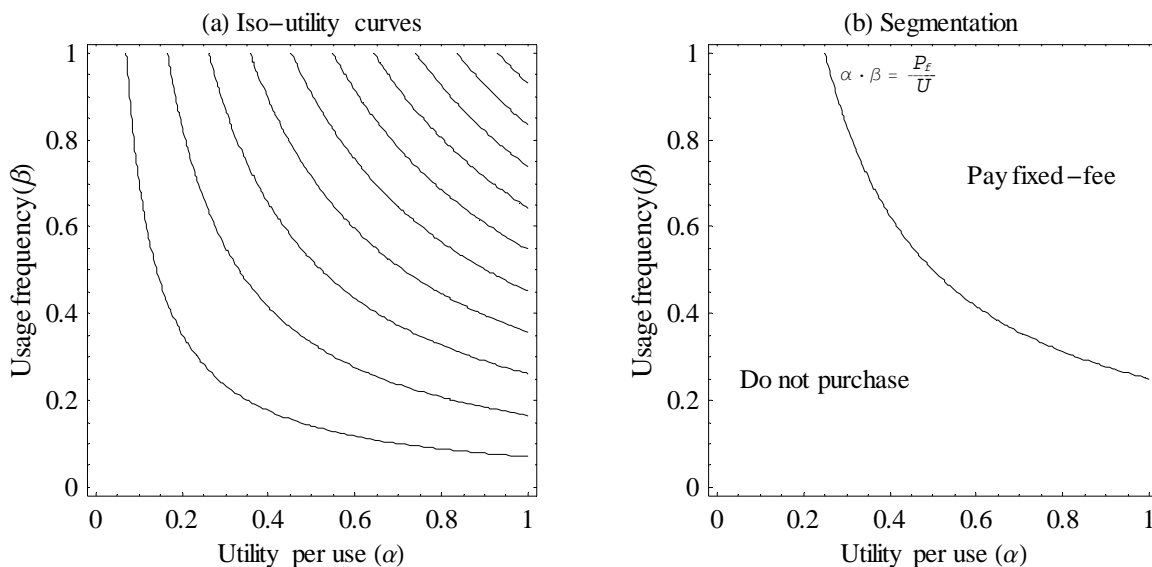


Figure 1: Fixed pricing: (a) iso-utility curves, and (b) segmentation

$$\alpha = \frac{P_u}{U}.$$

Costs and revenues: With fixed-fee pricing, a firm’s profits are P_f times the area above the segmenting hyperbola. With usage-fee pricing, revenues accrue on a per-use basis and the firm additionally incurs costs of c_u dollars per-use to cover, for example, costs of metering and monitoring.

Equilibrium: The first stage of this “game” is the pricing decision; customer purchases then follow. The solution concept employed is a standard Nash equilibrium between the set of potential customers and the firm or firms supplying the software. However, to better distinguish between the models with and without inter-firm competition, we use the term “optimal” for the price that maximizes a monopolist’s profits² and reserve “equilibrium” for the duopoly cases.

²This is admittedly an abuse of the standard taxonomy – more precisely, the price establishes an equilibrium between the monopolist firm and the customers.

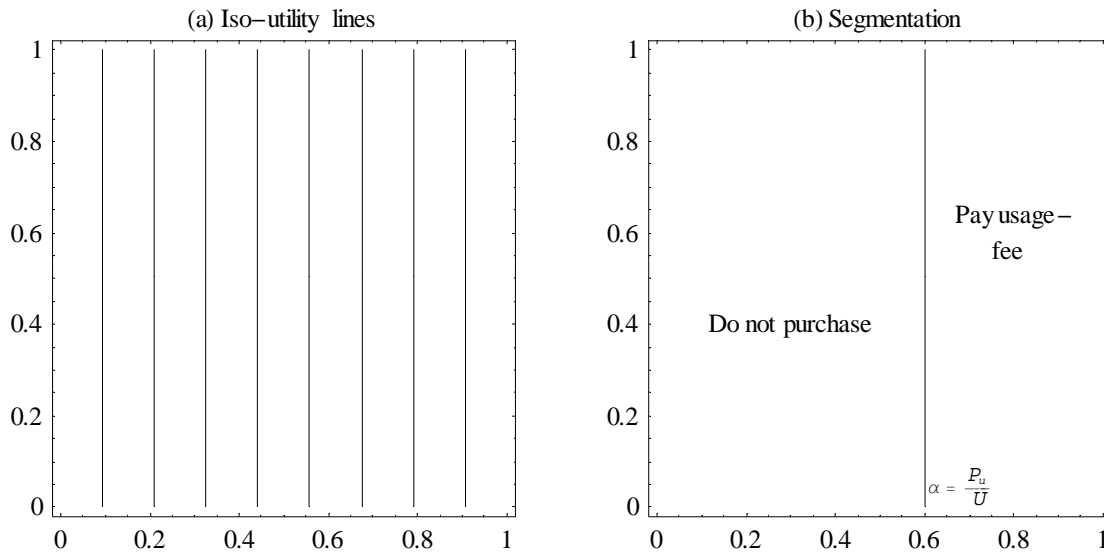


Figure 2: Iso-utility lines and segmentation: metered pricing

3 Monopoly analysis

In this section we consider a single firm in isolation – that is, a monopolist. For this firm, we analyze three pricing scenarios: fixed-fee pricing (section 3.1), usage-fee pricing (3.2), and fixed- and usage-fee pricing offered simultaneously (section 3.3).

3.1 The fixed-fee monopolist:

Referring to figure 1(a), given fixed-fee P_f and product quality U , all customers with (α, β) above the $\alpha\beta = \frac{P_f}{U}$ hyperbola have $\alpha\beta \cdot U - P_f > 0$, the size of the fixed-fee segment is

$$q_f = \left(1 - \frac{P_f}{U}\right) - \int_{\frac{P_f}{U}}^1 \frac{P_f}{\beta \cdot U} d\beta = \left(1 - \frac{P_f}{U}\right) + \frac{P_f}{U} \ln\left(\frac{P_f}{U}\right),$$

and the firm's profits π_f are

$$\pi_f \triangleq P_f \cdot q_f.$$

The following lemma is a handy result that facilitates optimization of π_f and is also useful later.

Lemma 1 *If a non-negative function g has $g(a) = g(b) = 0$ and $g''' > 0$. Then, on the interval (a, b) , the function g has a maximum and no (other) local maxima.*

Under this model of customer heterogeneity and after disallowing negative prices, the optimal P_f is guaranteed to fall in the interval $[0, U]$. Evaluating π_f over this interval,

$$\pi_f|_{P_f=0} = \pi_f|_{P_f=U} = 0, \quad \text{and} \quad \pi_f|_{0 < P_f < U} > 0.$$

Differentiating π_f gives

$$\frac{d\pi_f}{dP_f} = 1 - \frac{P_f}{U} + 2\frac{P_f}{U} \ln\left(\frac{P_f}{U}\right), \quad \frac{d^2\pi_f}{dP_f^2} = \frac{1}{U}(1 + 2\ln\left(\frac{P_f}{U}\right)), \quad \text{and} \quad \frac{d^3\pi_f}{dP_f^3} = \frac{2}{P_f \cdot U}.$$

This strictly positive third derivative (over $(0, U)$) together with (15) allows use of the lemma which guarantees a unique and π_f -optimizing solution to the first order optimality condition $\frac{d\pi_f}{dP_f} = 0$. The first derivative $\frac{d\pi_f}{dP_f}$ has roots of the form $P_f = \frac{-1}{2y}$ where y is a solution to $\frac{-1}{2\sqrt{e}} = y \ln(y)$; only the root in the open interval $(0, U)$ is relevant, and it identifies this monopolist's optimal price, profits, and segment size:

$$P_f^* = 0.285 U, \quad \pi_f^* = 0.102 U, \quad q_f^* = 0.357 \quad (2)$$

3.2 The usage-fee monopolist

Illustrated by figure 2, when a monopolist offers a usage-fee (instead of a fixed-fee) the customer segment that uses the product is the area to the right of $\alpha = \frac{P_u}{U}$. The size of this segment is $1 - \frac{P_u}{U}$, and it is the total number of uses (or transactions) by this segment that determines the firm's costs and profits. Specifically, the usage-fee monopolist's profit (denoted π_u) is

$$(P_u - c_u) \cdot \left(1 - \frac{P_u}{U}\right) \cdot \left(\int_0^1 \beta d\beta\right) \text{ which equals } \frac{1}{2} \cdot (P_u - c_u) \cdot \left(1 - \frac{P_u}{U}\right)$$

This is concave, and first order optimality conditions give optimal price and profits:

$$P_u^* = \frac{1}{2} \cdot \left(1 + \frac{c_u}{U}\right) \cdot U \quad \text{and} \quad \pi_u^* = \frac{1}{8} \cdot \left(1 - \frac{c_u}{U}\right)^2 \cdot U. \quad (3)$$

The simple forms of (2) and (3) facilitate comparison of the two pricing schemes. As given in the proposition below, the monitoring/metering cost determines which scheme is most profitable. For simplicity of exposition, denote c to be the monitoring cost normalized by product value; that is,

$$c \triangleq \frac{c_u}{U}.$$

Proposition 1 *A monopolist optimally selects the usage-fee pricing scheme if and only if the monitoring cost c is less than 0.097; above this threshold the fixed-price scheme is optimal.*

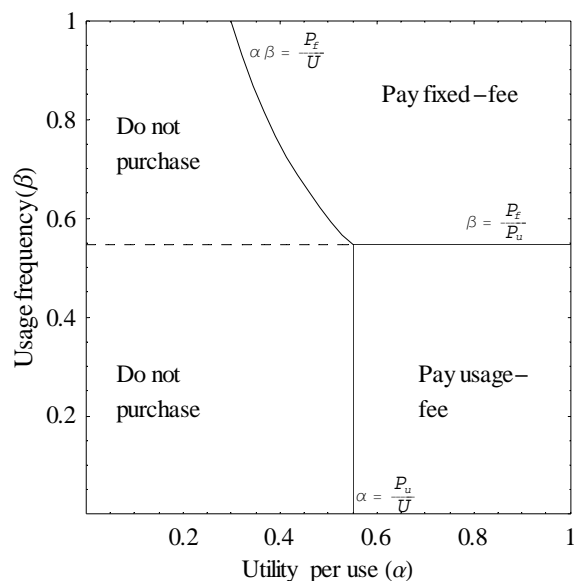


Figure 3: Segmentation for dual-scheme monopoly

Intuitively, usage-fee pricing dominates when monitoring costs are low, and this is reflected in many real world settings, some far removed from the traditional software arena. For example, at a ski resort it would be costly and difficult (not to mention cold and miserable) to manually sell and collect a ticket each time a skier mounts a chairlift, so it is unsurprising that resorts have mostly relied on per-day pricing. However, information systems now enable automated monitoring and billing, so a number of resorts now offer customers a choice of paying a fixed-fee or paying on a per-ride basis. The question then becomes, when it is feasible to offer both schemes simultaneously, what prices should the firm choose, and what are the benefits of offering a second pricing scheme?

3.3 The “dual-fee” monopolist

We now consider a monopolist who offers customers a choice of fixed- or usage-fees. The model is otherwise unchanged – the cost c_u is incurred for all usage-fee transactions, and the same value service is offered under both schemes (there is no versioning). Figure 3 illustrates the segmentation that results when both options are offered – although there is just one firm, and thus just one profit function to be maximized, the two pricing mechanisms “compete” in the sense that each truncates the other’s segment along the horizontal line $\beta = \frac{P_f}{P_u}$ – this is the locus of indifference between the fixed- and usage-fee schemes.

P_f is chosen by the “pay fixed-fee” segment of figure 3, and q_f , the size of this segment, is

$$q_f = \int_{\frac{P_f}{U}}^1 \left(1 - \frac{P_f}{\alpha U}\right) d\alpha - \int_{\frac{P_u}{U}}^1 \left(\frac{P_f}{P_u} - \frac{P_f}{\alpha U}\right) d\alpha = 1 - \frac{P_f}{P_u} + \frac{P_f}{U} \ln\left(\frac{P_f}{P_u}\right).$$

P_u is chosen by the “pay usage-fee” segment, and this then number of times this segment uses the software is

$$\int_{\frac{P_u}{U}}^1 \int_0^{\frac{P_f}{P_u}} \beta d\beta d\alpha = \left(\frac{P_f}{P_u}\right)^2 \left(\frac{1}{2} - \frac{P_u}{2U}\right)$$

Altogether, the dual-fee monopolists profits, denoted π_d , are

$$\pi_d = \begin{cases} P_f \cdot \left(1 - \frac{P_f}{P_u}\right) + \frac{P_f^2}{U} \ln\left(\frac{P_f}{P_u}\right) + \frac{1}{2} \cdot (P_u - c_u) \cdot \left(1 - \frac{P_u}{U}\right) \cdot \left(\frac{P_f}{P_u}\right)^2 & \text{if } P_f < P_u \\ \frac{1}{2} \cdot (P_u - c_u) \cdot \left(1 - \frac{P_u}{U}\right) & \text{otherwise} \end{cases} \quad (4)$$

The objective of the firm is to set prices P_f and P_u in order to maximize profit: $\max_{P_f, P_u} \{\pi_d\}$

Proposition 2 *In a dual-fee monopoly:*

(i) *A unique price pair (P_f^*, P_u^*) maximizes profits.*

(ii) $P_u^* = \frac{(1-c) + \sqrt{(1-c)^2 + 16c}}{4} \cdot U$

(iii) $P_u^* > P_f^*$

The optimal usage-fee (P_u^*) is given in the proposition above, but the optimal fixed-fee (P_f^*) is not available in closed form. Proceeding numerically, profits by segment and in total are shown in figure 4. An immediate observation is the optimality of offering a choice of pricing schemes. Intuitively, offering two schemes simultaneously increases the “size of the pie” through market segmentation, and this has the greatest benefit when monitoring costs are roughly at the $c = .097$ level that appeared in proposition 1.

Looking at the limiting cases, as c goes to 1 the equilibrium degenerates to the fixed-fee monopoly. On the other hand, as c goes to 0, the equilibrium degenerates to a usage-fee pricing monopoly.

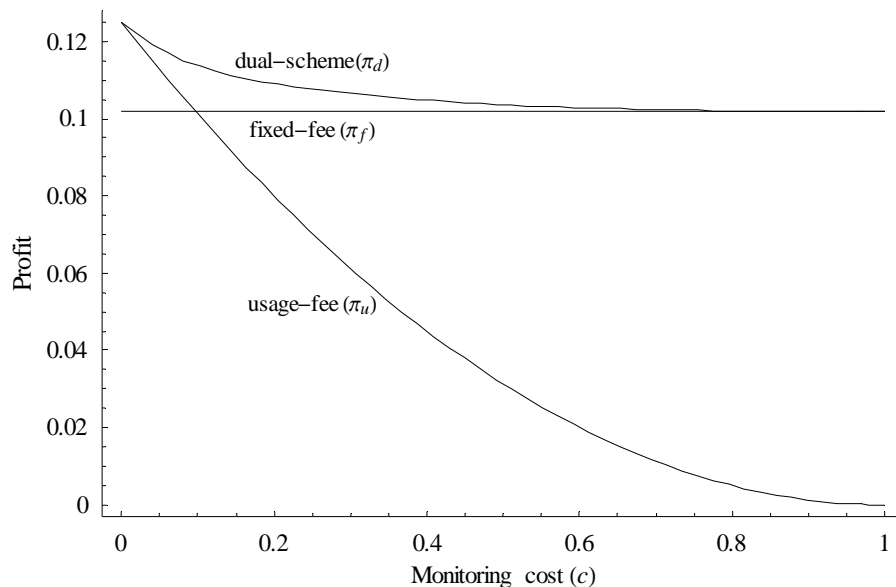


Figure 4: Profit for monopoly strategies

4 Duopoly analysis

We now consider scenarios with competition between two firms, and we again consider three scenarios: (1) One firm offers fixed pricing, the other firm offers usage-fee pricing, and the firms' products are undifferentiated. (2) Same as the previous scenario except that the products are now differentiated. (3) Both firms offer usage-fee pricing; products are again differentiated.

4.1 Dual-fee Duopoly, undifferentiated products

Here, one firm offers fixed-fee pricing and the other offers usage-fee pricing, and both offer the same value product (i.e., both offer the same U). From the customers' perspective, the model remains basically unchanged – for a given price pair (P_f, P_u) the customers will make the same selection as in the previous section, so segmentation remains as shown in figure 3. What does change is that the model is no longer profit optimization by a single firm with a single objective function. Rather, the competitors each have their own objectives, and we solve for equilibrium rather than optimal prices. The profit functions for the two firms are:

Fixed price firm:

$$\pi_f = \begin{cases} P_f \cdot \left(1 - \frac{P_f}{P_u}\right) + \frac{P_f^2}{U} \ln\left(\frac{P_f}{P_u}\right) & \text{if } P_f < P_u \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Usage-fee firm:

$$\pi_u = \begin{cases} \frac{1}{2} \cdot (P_u - c_u) \cdot \left(1 - \frac{P_u}{U}\right) \cdot \left(\frac{P_f}{P_u}\right)^2 & \text{if } P_u > P_f \\ \frac{1}{2} \cdot (P_u - c_u) \cdot \left(1 - \frac{P_u}{U}\right) & \text{otherwise} \end{cases} \quad (6)$$

An equilibrium is a (P_u, P_f) pair that simultaneously satisfies the firms' respective objectives:

$$\max_{P_u} \{\pi_u\} \text{ and } \max_{P_f} \{\pi_f\}$$

The next lemma characterizes the “best response” prices and is used to develop the equilibrium results given in the immediately following proposition.

Lemma 2 *At equilibrium:*

(i) *For any usage-fee P_u , the fixed price firm selects a strictly lower price*

(ii) *For any fixed price P_f , the usage-fee firm selects:*

$$P_u = \begin{cases} \frac{2cU}{1+c} & \text{if } P_f < \frac{2cU}{1+c} \\ P_f & \text{if } \frac{2cU}{1+c} \leq P_f \leq \frac{(1+c)U}{2} \\ \frac{(1+c)U}{2} & \text{if } P_f > \frac{(1+c)U}{2} \end{cases}$$

Proposition 3 (i) *There exists a unique equilibrium in pure strategies.*

At equilibrium,

(ii) $P_u = \frac{2cU}{1+c}$

(iii) *Fixed-price profit increases with the monitoring cost.*

(iv) *There exists an interval bounded below by $c = 0$ for which the usage-fee profit increases with the monitoring cost.*

(v) *There exists an interval bounded above by $c = 1$ for which the usage-fee profit decreases with the monitoring cost.*

(vi) *In the limit as monitoring cost approaches zero: the market is fully covered, each firm gets exactly half of the market, and $\pi_f = \pi_u = 0$.*

Noting that the firms compete through price-setting and that the firms' products are identical, one might expect to see the "Bertrand result" that the firms are unprofitable at equilibrium. It is thus interesting to observe that both firms' equilibrium profits are actually strictly positive in this model (for all $c \in (0, 1)$). The key here is that the Bertrand result depends on a complete lack of differentiation. In this duopoly however, the fact that the two firms offer different pricing schemes provides a form of differentiation that results in a profitable equilibrium.

Equilibrium results for the dual-fee duopoly are illustrated in figure 5.³ In this figure, we see that both firms' profits are lower than their monopolist counterparts (cf. figure 4). This is not surprising; we should expect competition to reduce profits. What is more notable is that the usage-fee firm's duopolists are so drastically reduced from those of the usage-fee monopolist.

Also changed from the previous section is that the usage-fee profits (π_u) are no longer monotonic in the monitoring cost c . Rather, that firm's profits first increase and then decrease with c (as anticipated by proposition 3(iv) and (v)). The fact that π_u can increase as its costs increase is somewhat surprising, but it has a straightforward explanation – the increase in c has the effect of reducing the degree of competition between the two firms, and this is beneficial to both competitors. This does not continue indefinitely however; for c greater than about 0.2 the negative effects of increasing c dominates and π_u begins to fall. As c gets large, π_u disappears and the fixed-fee firm becomes essentially a monopolist.

Figure 6 compares the dual-fee monopolist's profits to the combined profits of the dual-fee duopolists. As is inevitable⁴, the monopoly outperforms the duopoly. Another observation is that, as c increases, the monopolist's profits always decrease,⁵ but the duopoly profits actually increase.

³As for the dual-scheme monopolist, the equilibrium fixed price is unavailable in closed form. Numerical analysis provides the equilibrium prices and profits shown in these figures.

⁴The monopolist sets P_f and P_v to jointly optimize π_d thereby guaranteeing this result.

⁵Proof that the monopoly π_d decreases with c is straightforward – since π_d decreases with c for any (P_f, P_v) pair, it must decrease when the prices are optimally chosen.

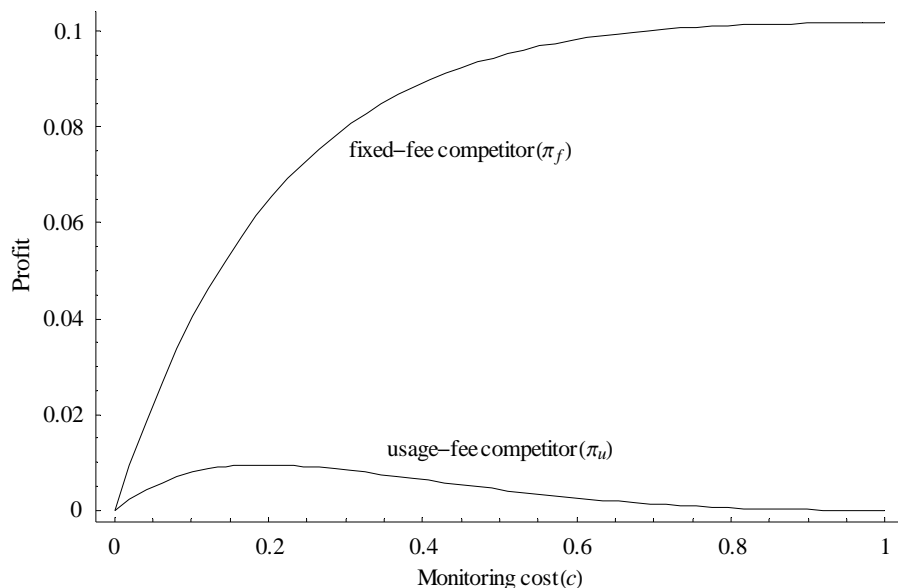


Figure 5: Competitor's profits: dual scheme duopoly

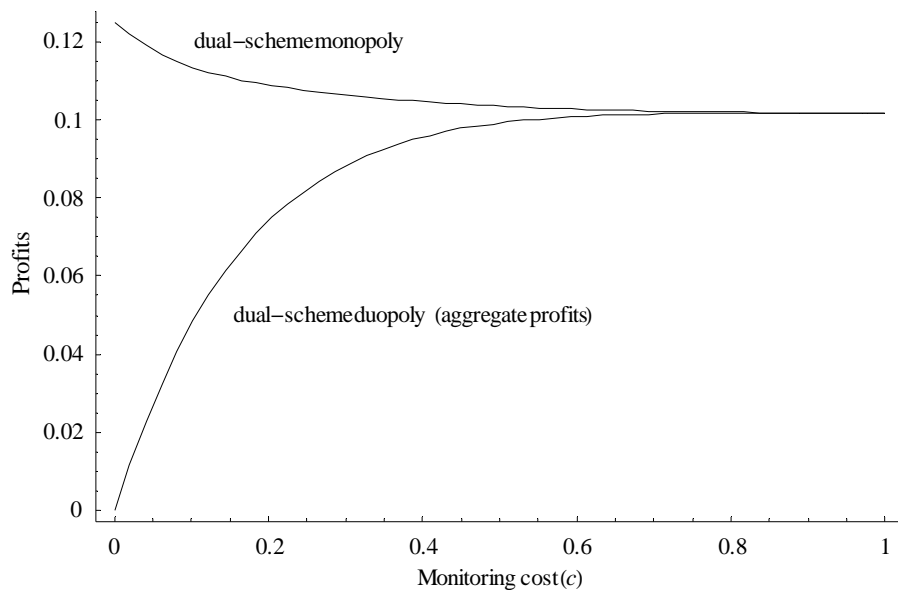


Figure 6: Aggregate duopoly profit vs dual scheme monopoly profit

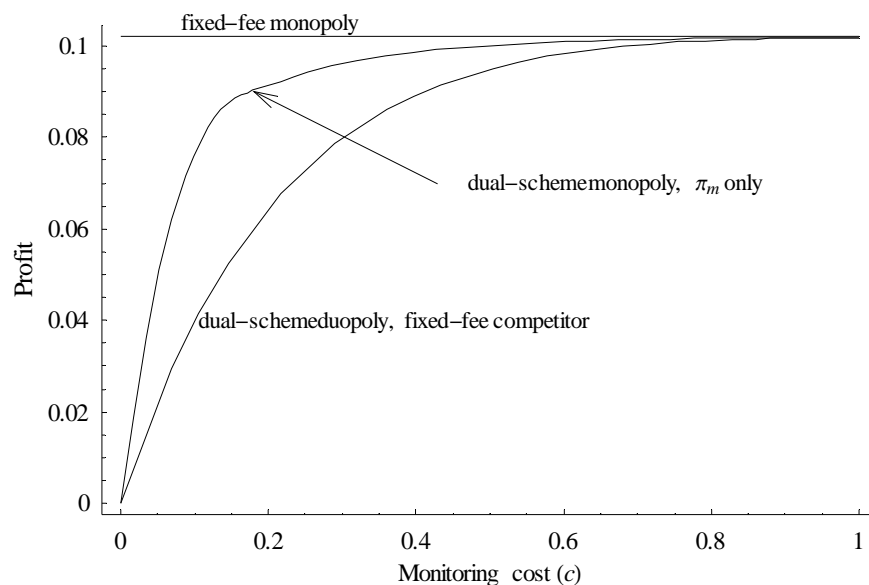


Figure 7: Fixed price profit under different settings

Figure 7 compares the performance of fixed-fee pricing in the three scenarios in which it has thus far appeared. The fixed-fee generates the highest profits when there is no competition and it is the only alternative offered. It generates lower profits when offered in combination with a usage-fee by a dual-fee monopolist, and it performs worst in the competitive duopoly. When the cost to monitor usage-fee transactions is large however, the situation essentially becomes a fixed-fee monopoly in each of these scenarios.

Analogous to figure 7, figure 8 compares the performance of fixed-fee pricing in different scenarios, and comparison of these two figures highlights the differences between the two pricing schemes. The ordering of these curves in figure 7 is the same as the analogous curves in figure 8, but a key difference is the relatively poorer performance of the usage-fee in the duopoly setting since profit is low not only at *high* monitoring cost (because the fixed-fee duopolist acquires monopoly power) but also at *low* monitoring cost (because intensity of competition increases).

While this numerical study reveals important details, a broader conclusion emerges regarding the usage-fee pricing scheme. While the usage-fee pricing scheme is optimal for a monopoly firm at low monitoring cost, it is highly sensitive to competition against a fixed-fee competitor.

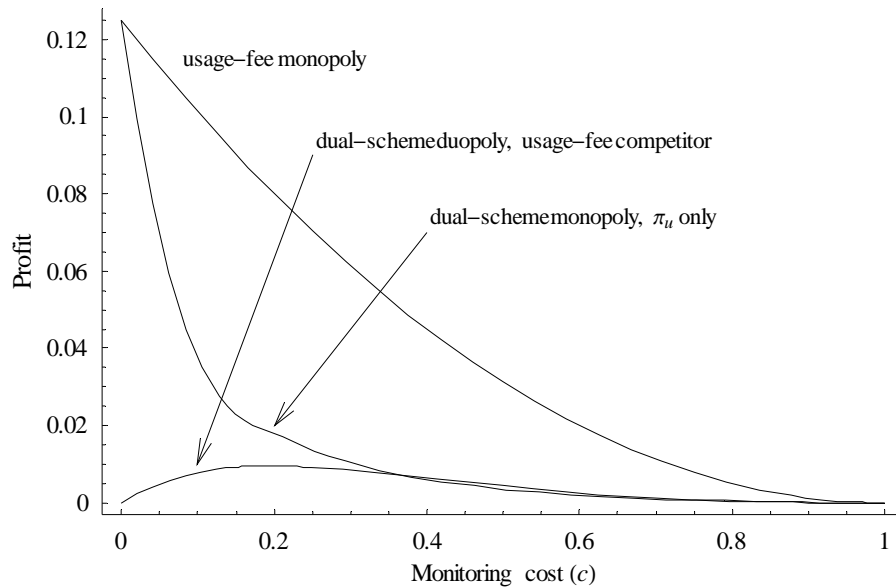


Figure 8: Metered price profit under different settings

4.2 Dual-fee duopoly, differentiated products

In several real world contexts in the software industry a software vendor offers a higher value product at a fixed-fee while an ASP offers a lower value service with a usage-fee pricing scheme. For instance, in the CRM (customer relationship management) space, Siebel is an established software vendor offering a product of higher value geared towards large firms while Salesforce.com is a online service provider targeting small and medium sized firms. To model such a scenario, we relax the assumption of undifferentiated products; now, U_f and U_u denote the values of the services offered by the fixed- and usage-fee firms respectively. With a differentiated product, the boundary between the two market segments is no longer a constant β ; instead the line of indifference is a curve $\beta = \frac{P_f}{\alpha(U_f - U_u) + P_u}$. Unlike the previous scenarios, two cases are now possible:

- Case A: $\frac{P_f}{U_f} < \frac{P_u}{U_u}$ – This case gives segmentation illustrated by the first graph in figure 9, and we refer to this as *structure A*. In structure A, the set of fixed-fee customers is adjacent to the set of non-purchasing customer types.
- Case B: $\frac{P_f}{U_f} \geq \frac{P_u}{U_u}$ – This case gives segmentation illustrated by the second graph of figure 9, and we refer to this as *structure B*. In structure B, the fixed-fee and the do-not purchase sets are strictly disjoint.

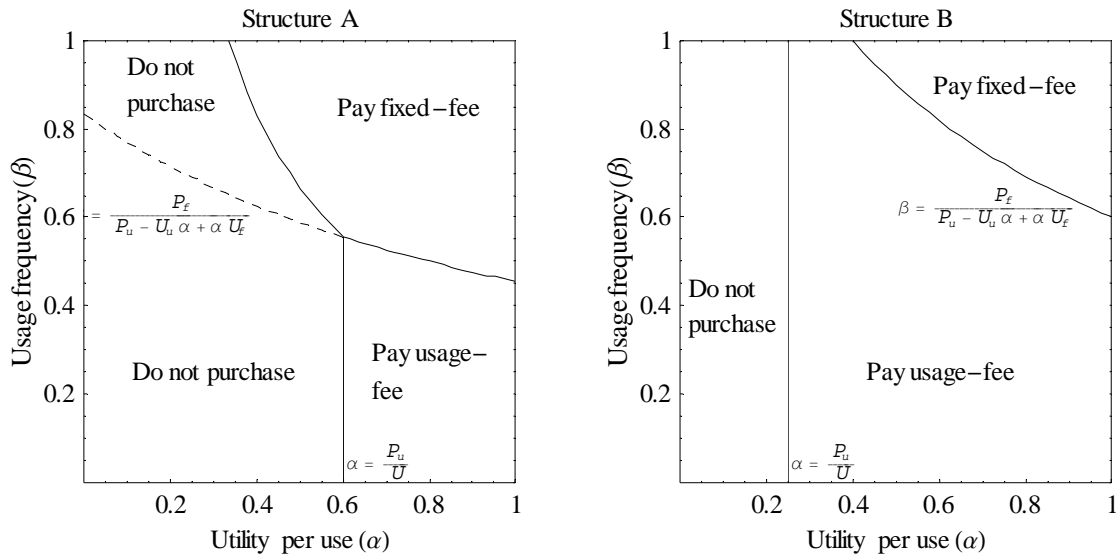


Figure 9: Market segmentation in a differentiated duopoly

In order to capture the level of differentiation, we introduce a parameter γ that is defined by

$$\gamma \triangleq \frac{U_u}{U_f}.$$

We use the same definitions for π_f and π_u as before and derive the profit functions for both firms under both cases. The exact derivation is provided in the appendix (page 27). The fixed-price firm's profit is:

$$\pi_f = \left\{ \begin{array}{ll} P_f \cdot (1 - P_f) + \frac{P_f^2}{U_f} \ln \left(\frac{U_u}{U_f} \cdot \frac{P_f}{P_u} \right) - \frac{P_f^2}{U_f - U_u} \ln \left(\left(\frac{U_u}{P_u} \right) \cdot \left(\frac{U_f - U_u + P_u}{U_f} \right) \right) & \text{if } P_f < \frac{P_u}{\gamma} \\ P_f \cdot \left(1 - \frac{P_f - P_u}{U_f - U_u} \right) - \frac{P_f^2}{U_f - U} \ln \left(\frac{U_f - U_u + P_u}{P_f} \right) & \text{if } P_f \geq \frac{P_u}{\gamma} \end{array} \right\} \quad (7)$$

The usage-fee firm's profit is:

$$\pi_u = \left\{ \begin{array}{ll} \frac{1}{2} \cdot (P_u - c_u) \cdot (U_u - P_u) \cdot \frac{P_f^2}{P_u U_f (P_u + U_f - U_u)} & \text{if } P_u > \gamma P_f \\ \frac{1}{2} \cdot (P_u - c_u) \left(\left(\frac{P_f - P_u}{U_f - U_u} - \frac{P_u}{U_u} \right) + \left(\frac{P_f}{U_f - U_u} \right) \left(\frac{U_f - U_u + P_u - P_f}{U_f - U_u + P_u} \right) \right) & \text{if } P_u \leq \gamma P_f \end{array} \right\} \quad (8)$$

Next we state a proposition relating to the equilibrium of this duopoly.

Proposition 4 Suppose (P_f, P_u) is an equilibrium price pair in pure strategies

If $\frac{P_f}{U_f} < \frac{P_u}{U_u}$ (case A):

$$(i) P_u = \frac{c\gamma + \sqrt{c(1-\gamma+c\gamma)}}{1+c\gamma} \cdot U_u \text{ with } c \triangleq \frac{c_u}{U_u}.$$

(ii) There is no other equilibrium in pure strategies having structure A

If $\frac{P_f}{U_f} \geq \frac{P_u}{U_u}$ (case B):

$$(iii) P_f = \frac{(1-\gamma)(2-0.78\gamma)+0.39\gamma(1-\gamma-\gamma c)+\gamma c}{3.5(2-0.78\gamma)} \cdot U_f, P_u = \frac{0.39(1-\gamma-\gamma c)+c}{2-0.78\gamma} \cdot U_u$$

(iv) There is no other equilibrium in pure strategies having structure B

This proposition greatly restricts the equilibrium possibilities but leaves open the question of whether there are 0, 1 or 2 equilibria. Analytical conditions for uniqueness and existence are cumbersome and not available in closed form. However, for any given c and γ , they can be computed numerically. Below we state the conditions for existence of the equilibria aided by the following definitions:

$$\left. \begin{array}{l} \rho_f^A(P_u) \triangleq \arg \max_{P_f} \pi_f \text{ subject to } P_f \leq \frac{P_u}{\gamma} \\ \rho_f^B(P_u) \triangleq \arg \max_{P_f} \pi_f \text{ subject to } P_f \geq \frac{P_u}{\gamma} \end{array} \right| \begin{array}{l} \rho_u^A(P_u) \triangleq \arg \max_{P_u} \pi_u \text{ subject to } P_u \geq \gamma P_u \\ \rho_u^B(P_u) \triangleq \arg \max_{P_u} \pi_u \text{ subject to } P_u \leq \gamma P_u \end{array}$$

Under these definitions, necessary and sufficient conditions for the existence of equilibrium with each structure are:

1. An A-equilibrium exists if and only if there exists a price pair (P_f^{eq}, P_u^{eq}) with $P_f^{eq} \leq P_u^{eq} \cdot \frac{1}{\gamma}$ such that

$$\left. \begin{array}{l} (c1) : P_f^{eq} = \rho_f^A(P_u^{eq}) \\ (c2) : \pi_f(P_f^{eq}, P_u^{eq}) \geq \pi_f(\rho_f^B(P_u^{eq}), P_u^{eq}) \end{array} \right| \begin{array}{l} (c3) : P_u^{eq} = \rho_u^A(P_f^{eq}) \\ (c4) : \pi_u(P_f^{eq}, P_u^{eq}) \geq \pi_u(P_f^{eq}, \rho_u^B(P_f^{eq})) \end{array}$$

2. A B-equilibrium exists if and only if there exists a price pair (P_f^{eq}, P_u^{eq}) with $P_f^{eq} \geq P_u^{eq} \cdot \frac{1}{\gamma}$ such that

$$\left. \begin{array}{l} (c1) : P_f^{eq} = \rho_f^B(P_u^{eq}) \\ (c2) : \pi_f(P_f^{eq}, P_u^{eq}) \geq \pi_f(\rho_f^A(P_u^{eq}), P_u^{eq}) \end{array} \right| \begin{array}{l} (c3) : P_u^{eq} = \rho_u^B(P_f^{eq}) \\ (c4) : \pi_u(P_f^{eq}, P_u^{eq}) \geq \pi_u(P_f^{eq}, \rho_u^A(P_f^{eq})) \end{array}$$

And, in both these cases (P_f^{eq}, P_u^{eq}) is the equilibrium price.

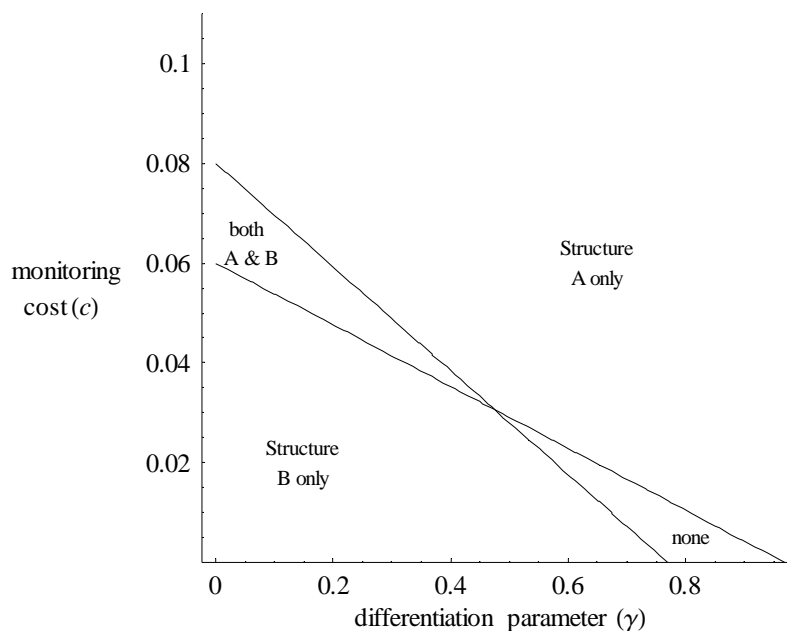


Figure 10: Equilibria as a function of monitoring cost and product differentiation

All equilibrium prices are of the form $g \cdot U_u$ or $h \cdot U_f$ where g and h are functions of c and γ but not of U_f or U_u .⁶ It then follows, because the model can be solved in terms of normalized prices $p_f \triangleq \frac{P_f}{U_f}$ and $p_u \triangleq \frac{P_u}{U_u}$, that equilibrium existence with a particular structure depends on c and γ but is not affected by the U_f and U_u values. Figure 10 illustrates.

Figure 10 displays the equilibria for all values of monitoring cost c and product differentiation γ . As observed, there is a threshold for structure A above which there exists an equilibrium with properties given by proposition 4. Similarly, there exists a threshold for structure B below which there exists an equilibrium with the structure given in proposition 4. The regions determined by the intersection of these thresholds correspond to the different results on the existence of equilibria: there exist two large regions where there is only one equilibrium, either A or B. There are also two thin regions where there are either two equilibria or no equilibria.

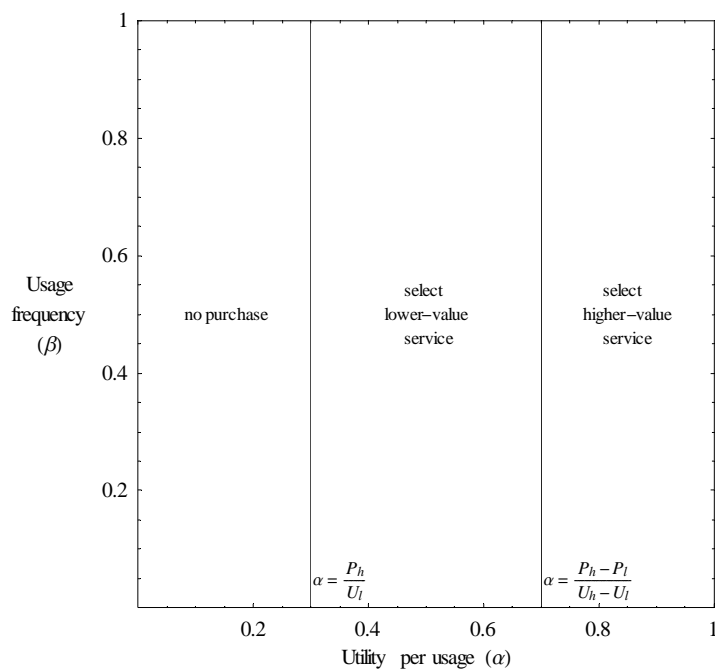


Figure 11: Market segmentation when both firms offer a metered price

4.3 Usage-fee Duopoly, differentiated products

We now model a scenario in which two firms offer products of different “quality” and both offer usage-fee pricing. Using subscripted “ h ” to indicate the firm with the higher quality product and “ l ” to indicate the firm with lower quality, the following apply:

- The firms’ prices are denoted P_h and P_l and profits are π_h and π_l .
- The products’ quality levels are U_h and U_l with $U_h > U_l$.
- The cost of metering/monitoring are c_h and c_l , and we additionally assume that $\frac{c_h}{U_h} = \frac{c_l}{U_l}$ – this assumption is primarily for ease of exposition, and it is easily relaxed.

Figure 11 illustrates the corresponding market segmentation – it consists of two nested rectangles with the higher quality firm acquiring the higher- α customers. As always under usage-fee pricing,

⁶For P_u , this can be seen by inspection of proposition 4 and it can be shown analytically for P_f .

profits are determined by the number of transactions; the profit functions are:⁷

$$\text{higher quality firm : } \pi_h = \begin{cases} \frac{1}{2} \cdot (P_h - c_h) \cdot (1 - \frac{P_h - P_l}{U_h - U_l}) & \text{if } \frac{P_l}{U_l} < \frac{P_h}{U_h} \\ \frac{1}{2} \cdot (P_h - c_h) \cdot (1 - \frac{P_h}{U_h}) & \text{otherwise} \end{cases} \quad (9)$$

and

$$\text{lower quality firm : } \pi_l = \begin{cases} \frac{1}{2} \cdot (P_l - c_l) \cdot (\frac{P_h - P_l}{U_h - U_l} - P_l) & \text{if } \frac{P_l}{U_l} < \frac{P_h}{U_h} \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

At equilibrium, both firms set prices in order to maximize their individual profits ($\max_{P_h} \{\pi_h\}$ and $\max_{P_l} \{\pi_l\}$) as given in the next proposition.

Proposition 5 *There exists a unique equilibrium in pure strategies. The equilibrium prices are:*

$$P_h = \frac{2(1 - \gamma) + (2 + \gamma)c}{4 - \gamma} \cdot U_h \text{ and } P_l = \frac{(1 - \gamma) + 3c}{4 - \gamma} \cdot U_l$$

The proposition that follows provides insight on the comparison between this duopoly situation and the previous one involving different pricing schemes, particularly at lower monitoring costs.

Proposition 6 *For the differentiated duopoly in which both firms offer usage-fees: the total duopoly profit is a decreasing function of the monitoring cost*

This proposition is in direct contrast to the dual-fee duopoly – there, the aggregate duopoly profit increases with monitoring cost despite the fact that both firms incur the cost of monitoring usage by doing so.

5 Discussion

The rise of the software-as-a-service business model has led to the rebirth of usage-metering as a pricing mechanism. However, usage alone is not an adequate measure of the willingness-to-pay for consumers of software products and services. Consumers vary in the value they derive with

⁷The constraints in (9) and (10) ensure non-negativity of prices and market segment areas.

the same amount of use. The effect of this form of consumer heterogeneity on the optimal pricing structure has not been discussed in previous literature. This paper analyzes this issue in both monopolistic and competitive settings. The basic parameters used to classify the results are the transaction costs of monitoring usage and the level of differentiation between firms.

Given a choice between fixed-fee and usage-fee schemes, a monopolist would find usage-fees to be optimal at low monitoring costs and a fixed-fee at higher monitoring costs. However, the monopolist would always find it beneficial to offer both pricing schemes. This is because the nature of market segmentation enlarges the size of the market when both pricing schemes are offered. This fact also enables firms to differentiate themselves purely on the basis of pricing mechanisms even when their service quality values are not different. However, when firms compete with different pricing schemes, lower monitoring costs lead to intense price competition. In particular, the usage-fee scheme is highly sensitive to competition, specifically when the competitor offers a fixed-fee. This effect can be relieved if both firms offer a usage-fee even though this involves incurring a corresponding monitoring cost.

Relating the results of this model to real world settings: a service provider should be cautious in offering usage-fee pricing if the competitor is a vendor offering fixed pricing. Such a strategy adversely affects both firms, particularly at low transaction costs of monitoring usage. In such cases, if the vendor chooses to offer the product in the form of a service, it might want to consider offering usage-fee pricing even though this means incurring a usage monitoring cost.

6 Appendix

Proof of lemma 1 First, it must be that g is concave-convex on $[a, b]$.⁸ That is, there is a value c such that g is strictly concave over subinterval $[a, c)$ and strictly convex over $(c, b]$. To see this: (1) g must be strictly concave at a – otherwise, $g''' > 0$ would imply that g is strictly *convex* everywhere, and this would make $g(a) = g(b) = 0$ impossible. (2) $g''' > 0$ implies that if g is convex at c then g is strictly convex over all $(c, b]$.

Next, g must have exactly one point in (a, b) at which $g' = 0$. This is seen by contradiction: assume $g'(x) = g'(y) = 0$ for some $x < y$ in (a, b) . Then, (because g is concave-convex) g must be concave at x , convex at y , and increasing between y and b . This then implies that $g(y) < g(b)$ which together with $g(b) = 0$ contradicts the premise of nonnegative g .

Finally, the strict concavity of g over (a, c) together with $g = 0$ at endpoints a and b implies that the unique inflection point must be a maximum.

Proof of proposition 1: The monopolist firm prefers usage-fee pricing to fixed pricing when $\pi_u^* \geq \pi_f^*$. Substituting for these expressions from equations (2) and (3), we have $\frac{1}{8} \cdot (1 - \frac{c_u}{U})^2 \geq 0.1$ which can be rewritten as $c \leq 0.097$ **QED**

Lemma 3 *If the price pair (P_f, P_u) is optimal and $P_f \geq P_u$, then $P_u = \frac{U}{2}(1 + c)$.*

Proof: Since $P_f \geq P_u$, the dual-fee monopolist's profits are⁹

$$\pi_d = \frac{1}{2}(P_u - c_u) \left(1 - \frac{P_u}{U}\right) \quad \text{and} \quad (11)$$

$$\frac{\partial \pi_d}{\partial P_u} = \frac{1}{2U}(c_u - 2P_u + U), \quad (12)$$

the optimality condition implies equation (12) equals 0, and this solves to

$$P_u = \frac{1}{2}(c_u + U) = \frac{U}{2}(1 + c).$$

The lemma then follows by contradiction – if $P_u \neq \frac{U}{2}(1 + c)$ then either:

⁸Although the convex portion may be empty.

⁹This is equation (4) given that the fixed-fee exceeds the usage-fee

- $\frac{\partial \pi_d}{\partial P_u} \neq 0$ – implying the prices are not actually optimal – or
- π_d does not take the form given in (11) – implying $P_f < P_u$.

Either case contradicts the lemma's premise. **QED**

Proof of proposition 2: Please note: (1) the three parts of this proposition are proven in reverse order – (iii) is first and (i) is last, and (2) the proof relies on lemma 3 that is stated with proof just above.

Recall that the firm's profit-maximization objective is $\max_{P_f, P_u} (\pi_d)$ with π_d given by (4). Because π_d is continuous in prices and is independent of P_f for $P_f > P_u$, the firm's profit maximization problem can be rewritten as the constrained optimization program

$$\max_{P_f, P_u} \left(\pi = P_f \cdot \left(1 - \frac{P_f}{P_u}\right) + \frac{P_f^2}{U} \ln \left(\frac{P_f}{P_u}\right) + \frac{1}{2} \cdot (P_u - c_u) \cdot \left(1 - \frac{P_u}{U}\right) \cdot \left(\frac{P_f}{P_u}\right)^2 \right) \quad (13a)$$

$$\text{subject to } P_f \leq P_u. \quad (13b)$$

At optimality we must have¹⁰

$$0 = \frac{\partial}{\partial P_u} [\pi - \lambda(P_f - P_u)] = \frac{\partial \pi}{\partial P_u} + \lambda. \quad (14)$$

with λ denoting the LaGrange multiplier for constraint (13b). Next is to show by contradiction that $\lambda = 0$ at optimality.

We thus assume that λ is *strictly positive* at optimal prices. $\lambda > 0$ implies that constraint (13b) is binding, so $P_f = P_u$ and: (1) π simplifies to $\frac{1}{2} \cdot (P_u - c_u) \cdot \left(1 - \frac{P_u}{U}\right)$, (2) $P_u = \frac{U}{2} (1 + c)$ (by lemma 3), and (3) $\frac{\partial \pi}{\partial P_u} = 0$ (after differentiating π and substituting P_u as just given). This, together with $\lambda > 0$ shows that equation (14) is a contradiction implying that $\lambda = 0$ at optimality.

To complete verification of (iii), $\lambda > 0$ implies $P_f < P_u$ at optimality by complementary slackness.

(ii): Solving $\frac{\partial \pi}{\partial P_u} = 0$ (with π given by (13a)) gives $P_u = \frac{(1-c) + \sqrt{(1-c)^2 + 16c}}{4}$ as the only (positive) critical point for π . The fact that this is independent of P_f together with $\frac{\partial^2 \pi}{\partial P_u^2} < 0$ at this value of P_f (easily verified by inspection of this partial), is sufficient to imply that this critical point identifies a maximum.

¹⁰The first equality is by LaGrange's method; the second is simplifying.

(i): Part (ii) tell us that there is a unique optimal P_u that is independent of P_f . Thus, it is only left to show that when P_u takes this value a unique P_f maximizes the firm's profits. To simplify the expressions that follow, we use the following definitions:

$$p_f \triangleq \frac{P_f}{U}, \quad p_u \triangleq \frac{P_u}{U}, \quad \text{and} \quad c \triangleq \frac{c_u}{U}.$$

Substituting these into the profit function π and differentiating

$$\frac{\partial \pi}{\partial p_f} = 1 - \left(\frac{2}{p_u} + \frac{2 \cdot (p_u - c) \cdot (1 - p_u)}{2 \cdot p_u^2} - 1 \right) p_f + 2p_f \ln \left(\frac{p_f}{p_u} \right)$$

and

$$\frac{\partial^2 \pi}{\partial p_f^2} = - \left[\frac{2}{p_u} + \frac{2 \cdot (p_u - c) \cdot (1 - p_u)}{2 \cdot p_u^2} \right] - 3 + 2 \ln \left(\frac{p_f}{p_u} \right)$$

The $[\cdot]$ -bracketed term is positive (after noting that $P_u > c_u$ implies $p_u > c$) and is constant in p_f , and the last term is negative (because $0 < p_f < p_u$ by part (iii) of this proposition), so $\frac{\partial^2 \pi}{\partial p_f^2}$ is unambiguously negative. π is thus concave in p_f and also in P_f . There is thus a unique profit-maximizing P_f for the given P_u . **QED**

Proof of lemma 2: (i) (by contradiction): If $P_f \geq P_u$ then: (1) For every (α, β) -customer, the utility $\alpha\beta U - P_f$ derived from the fixed-fee option is $\leq \beta(\alpha U - P_u)$, the utility derived from the usage-fee option (because $\beta \leq 1$). (2) Thus, q_f , sales by the fixed-fee firm are 0. (3) But, the continuity of π_f guarantees that the fixed-fee firm can always find a price that will supply strictly positive profits. (4) Thus, a (P_u, P_f) pair with $P_f \geq P_u$ violates this fixed firm's optimality criterion. Hence at optimal pricing for the fixed price firm, we have $P_f < P_u$

(ii) To simplify the analysis, let: $\pi'_u = \frac{\pi_u}{U}$, $p_f = \frac{P_f}{U}$, $p_u = \frac{P_u}{U}$ and $c = \frac{c_u}{U}$:

The profit function of the usage-fee firm in equation (6) becomes:

$$\pi'_u = \begin{cases} \frac{(p_u - c) \cdot (1 - p_u) \cdot p_f^2}{2 \cdot p_u^2} & \text{if } p_u > p_f \\ \frac{(p_u - c) \cdot (1 - p_u)}{2} & \text{otherwise} \end{cases}$$

Differentiating the first line of the profit function in equation (??) with respect to p_u :

$$\frac{\partial \pi'_u}{\partial p_u} = \left(-\frac{1}{p_u^2} + \frac{2c}{p_u^3} - \frac{c}{p_u^2} \right) \frac{p_f^2}{2}$$

$$\frac{\partial^2 \pi'_u}{\partial p_u^2} = \left(2 - \frac{6c}{p_u} + 2c\right) \frac{p_f^2}{2p_u^3}$$

Setting the first derivative to zero, we get $p_u^*(p_f) = \frac{2c}{1+c}$. Now $\frac{\partial^2 \pi'_u}{\partial p_u^2} \leq 0$ for $p_u \in [0, \frac{3c}{1+c}]$ and > 0 for $p_u \in (\frac{3c}{1+c}, 1]$ implying that π'_u is concave-convex in p_u . Also, at $p_u = c$, $\pi'_u = 0$ and $\frac{\partial \pi'_u}{\partial p_u} = -(1-c) \frac{p_f^2}{2} \leq 0$ at $p_u = 1$ implying that the function is concave with zero value at the lower limit of the domain and convex decreasing at the upper limit of the domain. Combining the above facts implies quasi-concavity of the objective function over the given domain. Setting the first derivative equal to zero provides a unique maximum. If $p_f < \frac{2c}{1+c}$, the best response $p_u^*(p_f) = \frac{2c}{1+c} > p_f$. If $p_f \geq \frac{2c}{1+c}$, then the quasi-concavity of the profit function dictates that $p_u^*(p_f) = p_f$

Differentiating the second line of the profit function in equation 6, we find the optimal usage-fee to be $p_u^*(p_f) = \frac{1+c}{2}$. If $p_f \leq \frac{1+c}{2}$, then $p_u^*(p_f) = \frac{1+c}{2}$ else $p_u^*(p_f) = p_f$

Combining the results above, the best response usage-fee for the entire range of fixed prices can be constructed as stated in the lemma.

Proof of proposition 3: (i) Using lemma 2, we can restrict analysis to the case where $p_f < p_u$. Similar to previous part of the proof, we set $\pi'_f = \frac{\pi_f}{U}$, $p_f = \frac{P_f}{U}$, and $p_u = \frac{P_u}{U}$. The first line of the profit function of the fixed price firm from equation (5) is simplified to give:

$$\pi'_f = p_f \cdot \left(1 - \frac{p_f}{p_u}\right) + p_f^2 \ln \left(\frac{p_f}{p_u}\right) \quad (15)$$

with:

$$\frac{\partial \pi'_f}{\partial p_f} = 1 + p_f - \frac{2p_f}{p_u} + 2p_f \ln \left(\frac{p_f}{p_u}\right)$$

and:

$$\begin{aligned} \frac{\partial^2 \pi'_f}{\partial p_f^2} &= 3 - \frac{2}{p_u} + 2 \ln \left(\frac{p_f}{p_u}\right) \\ \frac{\partial^3 \pi'_f}{\partial p_f^3} &= \frac{2}{p_f} > 0 \end{aligned}$$

Using this fact about the third derivative and that $\pi_f = 0$ at the endpoints $p_f = 0$ and $p_f = p_u$, lemma 1 provides the result that there exists a unique fixed price response to any usage-fee. From

the earlier part of this proof, we know that when $p_f < p_u$, the usage-fee firm has a unique response $p_u = \frac{2cU}{1+c}$. Since there is a unique fixed price response to this price, the resulting unique set of prices maximizes the profit of both firms given the strategy of the competitor and hence constitutes an equilibrium in pure strategies. Let this price pair be denoted by (p_f^e, p_u^e) .

We show uniqueness of this equilibrium by contradiction: suppose that (p'_f, p'_u) represents an equilibrium price pair in addition to the price pair (p_f^e, p_u^e) . By lemma 3, $p'_f < p'_u$ resulting in $p'_u = \frac{2c}{1+c}$ (again from proposition 2) which equals p_u^e . From the earlier part of this proposition, there is a unique fixed price response to this price. Hence $p'_f = p_f^e$. Hence $(p'_f, p'_u) = (p_f^e, p_u^e)$ and the equilibrium is unique.

(ii) Follows from i)

(iii) The fixed-price firm's modified profit function is given by equation (15). This profit is an increasing function of the usage-fee as shown by direct differentiation:

$$\frac{\partial \pi'_f}{\partial p_u} = \frac{p_f^2}{p_u} \left(\frac{1}{p_u} - 1 \right) > 0$$

This is true for every p_u , so it is also true for the equilibrium p_u . Also, c does not appear in the fixed-price firm's profit function. This implies that the fixed-price firm's equilibrium profit increases if and only if the equilibrium p_u increases with c . By inspection of lemma 2, we find that it does.

(iv) & (v) Let $\pi_u^*(c)$ represent the equilibrium profit for the firm offering the usage-fee at optimal prices as a function of the monitoring cost. $p_u^* < 1$ implies that customers will purchase a strictly positive number of usage-fee transactions, and $p_u^* > c$ implies that these transactions are profitable. Thus, $\pi_u^*(c) > 0$ whenever $0 < c < 1$. The stated results are then implied by the continuity of π_u^* in c (which can be verified by the implicit function theorem).

(vi) At equilibrium prices, the first derivative of the fixed price firm's profit (c.f. equation 15) as a function of price must be zero. That is:

$$\frac{\partial \pi'_f}{\partial p_f} = 1 + p_f - 2 \frac{p_f}{p_u} + 2 p_f \ln\left(\frac{p_f}{p_u}\right) = 0$$

where p_f and p_u are the equilibrium prices. Taking the limit of the above equation as $c \rightarrow 0$ gives:

$$1 + \lim_{c \rightarrow 0} p_f - 2 \cdot L + 2 \cdot \lim_{c \rightarrow 0} p_f \cdot \ln(L) = 0$$

where $L = \lim_{c \rightarrow 0} \left(\frac{p_f}{p_u} \right)$

We know that at equilibrium, $0 < p_f < p_u$. However, $\lim_{c \rightarrow 0} p_u = 0$ (using the closed form expression of p_u). Taking limits on both sides of the inequality, we have $\lim_{c \rightarrow 0} p_f = 0$. Using this in the above equation:

$$1 - 2 \cdot L = 0$$

$$\lim_{c \rightarrow 0} \left(\frac{p_f}{p_u} \right) = \frac{1}{2}$$

which is the same as:

$$\lim_{c \rightarrow 0} \left(\frac{P_f}{P_u} \right) = \frac{1}{2}$$

At $c \rightarrow 0$, we have both prices: $p_u \rightarrow 0$ and $p_f \rightarrow 0$. At zero prices, the market is fully covered. $\frac{p_f}{p_u}$ is the line of indifference between the two segments and $\frac{p_f}{p_u} \rightarrow \frac{1}{2}$ as $c \rightarrow 0$. Hence each firm gets exactly half of the potential market.

Derivation of profit functions (??) and (??): For structure A ($\frac{P_f}{U_f} < \frac{P_u}{U_u}$): we derive the number of customers who use the fixed-fee service, q_f (given by the corresponding area in figure 9) to be:

$$q_f = \left(\left(1 - \frac{P_f}{U_f} \right) - \int_{\frac{P_f}{U_f}}^1 \frac{P_f}{\alpha U_f} d\alpha \right) - \left(\int_{\frac{P_u}{U_u}}^1 \frac{P_f}{\alpha(U_f - U_u) + P_u} d\alpha - \int_{\frac{P_u}{U_u}}^1 \frac{P_f}{\alpha U_f} d\alpha \right)$$

$$= \left(1 - \frac{P_f}{U_f} \right) + \frac{P_f}{U_f} \ln \left(\frac{U_u}{U_f} \cdot \frac{P_f}{P_u} \right) - \frac{P_f}{U_f - U_u} \ln \left(\left(\frac{U_u}{P_u} \right) \cdot \left(\frac{U_f - U_u + P_u}{U_f} \right) \right)$$

and the profit function for the fixed pricing firm when $\frac{P_f}{U_f} < \frac{P_u}{U_u}$ follows.

The usage-fee firm's objective again depends on the number of transactions, now

$$\int_{\frac{P_u}{U_u}}^1 \int_0^{\frac{P_f}{\alpha(U_f - U_u) + P_u}} \beta d\beta d\alpha \text{ which equals } \frac{P_f^2 (U_u - P_u)}{2P_u U_f (P_u + U_f - U_u)},$$

and the firm's profit function follows.

For structure B ($\frac{P_f}{U_f} \geq \frac{P_u}{U_u}$): we derive the number of customers who use the fixed-fee service, q_f (given by the corresponding area in figure 9) to be:

$$\begin{aligned} q_f &= 1 - \frac{P_f - P_u}{U_f - U_u} - \int_{\frac{P_f - P_u}{U_f - U_u}}^1 \frac{P_f}{\alpha(U_f - U_u) + P_u} d\alpha \\ &= 1 - \frac{P_f - P_u}{U_f - U_u} - \frac{P_f}{U_f - U_u} \ln \left(\frac{U_f - U_u + P_u}{P_f} \right) \end{aligned}$$

and the firm's profit function when ($\frac{P_f}{U_f} \geq \frac{P_u}{U_u}$) follows.

The usage-fee firm's objective again depends on the number of transactions, now

$$\begin{aligned} &\frac{1}{2} \left(\frac{P_f - P_u}{U_f - U_u} - \frac{P_u}{U_u} \right) + \int_{\frac{P_f - P_u}{U_f - U_u}}^1 \int_0^{\frac{P_f}{\alpha(U_f - U_u) + P_u}} \beta d\beta d\alpha \text{ which equals} \\ &\frac{1}{2} \left(\frac{P_f - P_u}{U_f - U_u} - \frac{P_u}{U_u} \right) + \left(\frac{P_f}{2(U_f - U_u)} \right) \left(\frac{U_f - U_u + P_u - P_f}{U_f - U_u + P_u} \right) \end{aligned}$$

and the profit function follows.

Proof of proposition 4: Again, we use the following notation to simplify the analysis: $\pi'_f = \frac{\pi_f}{U_f}$, $\pi'_u = \frac{\pi_u}{U_u}$, $p_f = \frac{P_f}{U_f}$, and $p_u = \frac{P_u}{U_u}$

(i) Case A: ($p_u > p_f$)

For case 1, the profit function of the firm offering usage-fee pricing given by equation (8) can be simplified to:

$$\pi'_u = \frac{(p_u - c) \cdot (1 - p_u) \cdot p_f^2}{2 \cdot p_u \cdot (1 - \gamma + \gamma \cdot p_u)}$$

To find a maximum, we differentiate with respect to π_u :

$$\frac{\partial \pi'_u}{\partial p_u} = \left(\frac{p_u(1 - \gamma + \gamma p_u)(1 - 2p_u + c) - (p_u - c)(1 - p_u)(1 - \gamma + 2\gamma p_u)}{p_u^2(1 - \gamma + \gamma p_u)^2} \right) \frac{p_f^2}{2}$$

Setting $\frac{\partial \pi'_u}{\partial p_u} = 0$, we get $p_u^*(p_f) = \frac{c\gamma \pm \sqrt{c(1-\gamma+c\gamma)}}{1+c\gamma}$. For $\gamma < 1$, $\frac{c\gamma - \sqrt{c(1-\gamma+c\gamma)}}{1+c\gamma} < 0$ by inspection leaving us with $\frac{c\gamma + \sqrt{c(1-\gamma+c\gamma)}}{1+c\gamma}$ as the only root in the domain $[c, 1]$. Differentiating again with respect to p_u , we can show that $\frac{\partial^2 \pi'_u}{\partial p_u^2} < 0$ for $c \leq p_u < \frac{c\gamma}{1+c\gamma} + \frac{c^{\frac{2}{3}}(\gamma(1-\gamma+c\gamma))^{\frac{1}{3}}}{1+c\gamma} + \frac{c^{\frac{1}{3}}(1-\gamma+c\gamma)^{\frac{2}{3}}}{\gamma^{\frac{1}{3}}(1+c\gamma)}$ and

$\frac{\partial^2 \pi'_u}{\partial p_u^2} \geq 0$ for $\frac{c\gamma}{1+c\gamma} + \frac{c^{\frac{2}{3}}(\gamma(1-\gamma+c\gamma))^{\frac{1}{3}}}{1+c\gamma} + \frac{c^{\frac{1}{3}}(1-\gamma+c\gamma)^{\frac{2}{3}}}{\gamma^{\frac{1}{3}}(1+c\gamma)} \leq p_u \leq 1$. This implies that π'_u is concave-convex in p_u . Also, at $p_u = c$, $\frac{\partial \pi'_u}{\partial p_u} = \left(\frac{1-c}{c(1-\gamma+c\gamma)}\right) \left(\frac{p_f^2}{2}\right) > 0$ and at $p_u = 1$, $\frac{\partial \pi'_u}{\partial p_u} = -(1-c) \left(\frac{p_f^2}{2}\right) < 0$. So the profit function is concave increasing at the lower limit of the domain and convex decreasing at the upper limit. Hence the root $\frac{c\gamma + \sqrt{c(1-\gamma+c\gamma)}}{1+c\gamma}$ is the usage-fee firm's best response to p_f .

The profit function of the fixed price firm for case A in (c.f. equation (7)) after normalizing becomes:

$$\pi'_f = p_f \cdot (1 - p_f) + p_f^2 \ln\left(\frac{p_f}{p_u}\right) - \frac{p_f^2}{1-\gamma} \ln\left(\frac{1-\gamma+\gamma p_u}{p_u}\right)$$

Differentiating with respect to p_f :

$$\begin{aligned} \frac{\partial \pi'_f}{\partial p_f} &= 1 - p_f - \frac{2}{1-\gamma} \ln\left(\frac{1-\gamma+\gamma p_u}{p_u}\right) p_f + 2p_f \ln\left(\frac{p_f}{p_u}\right) \\ \frac{\partial^2 \pi'_f}{\partial p_f^2} &= 1 - \frac{2}{1-\gamma} \ln\left(\frac{1-\gamma+\gamma p_u}{p_u}\right) + 2 \ln\left(\frac{p_f}{p_u}\right) \end{aligned}$$

By inspection, $\left[1 - \frac{2}{1-\gamma} \ln\left(\frac{1-\gamma+\gamma p_u}{p_u}\right)\right]$ is constant in p_f and $\ln\left(\frac{p_f}{p_u}\right)$ is monotone increasing (note that $\ln(0) = -\infty$ and $\ln(1) = 0$), so $\frac{\partial^2 \pi'_f}{\partial p_f^2}$ is monotone increasing. Also at $p_f = p_u$, $\frac{\partial \pi'_f}{\partial p_f} = 1 - p_u - \frac{2p_u}{1-\gamma} \ln\left(\frac{1-\gamma+\gamma p_u}{p_u}\right) < 0$ implying that the function is decreasing at the upper limit of the domain. Hence the function is strictly quasi-concave over the domain with its maximum in the interior. Thus, there is a unique profit maximizing fixed price response for any usage-fee p_u in the region $p_f \in [c, p_u]$. The usage-fee firm has a unique best response price p_u independent of the fixed price. The fixed price firm has a unique response to this price. If an equilibrium in pure strategies exists for case A, this unique price pair characterizes the equilibrium and there is no other equilibrium.

(ii) Case B ($p_f \geq p_u$):

The profit function of the fixed price firm for structure B given by equation (7) can be rewritten as:

$$\pi'_f = p_f - p_f \left(\frac{p_f - \gamma p_u}{1-\gamma}\right) - \frac{p_f^2}{1-\gamma} \ln\left(\frac{1-\gamma+\gamma p_u}{p_f}\right)$$

To find a maximum, compute the first and second order derivatives:

$$\frac{\partial \pi'_f}{\partial p_f} = 1 + \frac{\gamma p_u}{1-\gamma} - \frac{p_f}{1-\gamma} - \frac{2p_f}{1-\gamma} \ln\left(\frac{1-\gamma+\gamma p_u}{p_f}\right) \quad (16)$$

$$\frac{\partial^2 \pi'_f}{\partial p_f^2} = \frac{1}{1-\gamma} - \frac{2}{1-\gamma} \ln \left(\frac{1-\gamma + \gamma p_u}{p_f} \right)$$

Setting the first derivative to zero to get the best response for the fixed price firm:

$$p_f^*(p_u) = \frac{1-\gamma + \gamma p_u}{3.5}$$

This price gives a local maximum for the profit function over the domain $[p_u, 1]$ for any usage-fee p_u since $\frac{\partial^2 \pi'_f}{\partial p_f^2}$ evaluated at $p_f = \frac{1-\gamma + \gamma p_u}{3.5}$ is $\frac{1-2\ln(3.5)}{1-\gamma} < 0$ implying concavity.

The profit function of the usage-fee firm for structure B given by equation (8) can be rewritten as:

$$\pi'_u = \frac{1}{2}(p_u - c) \left(\frac{p_f - \gamma p_u}{1-\gamma} - p_u + \left(\frac{p_f}{2(1-\gamma)} \right) \left(\frac{1-\gamma + \gamma p_u - p_f}{1-\gamma + \gamma p_u} \right) \right)$$

To find a maximum, compute the first and second derivatives:

$$\frac{\partial \pi'_u}{\partial p_u} = \frac{1}{4(1-\gamma)} \left(2c + 3p_f - \frac{(1-\gamma + \gamma c)p_f^2}{(1-\gamma + \gamma p_u)^2} - 4p_u \right) \quad (17)$$

$$\frac{\partial^2 \pi'_u}{\partial p_u^2} = \frac{1}{2(1-\gamma)} \left(\frac{\gamma(1-\gamma + \gamma c)p_f^2}{(1-\gamma + \gamma p_u)^3} - 2 \right) < 0 \text{ by inspection, implying concavity}$$

It is tedious but straightforward to show that the first derivative always has a unique real root in p_u . Concavity of the function implies that this price provides a local maximum over the domain $[0, p_f]$ for any given p_f . Thus, it only remains to simultaneously solve for the roots of equations (16) and 17. Doing so provides the values given in the proposition. Only one such solution exists implying the uniqueness of the equilibrium. *QED*

Proof of proposition 5: Again, we use the following notation to simplify the analysis: $\pi'_h = \frac{\pi_h}{U_h}$, $\pi'_u = \frac{\pi_u}{U_u}$, $p_f = \frac{P_f}{U_f}$, and $p_u = \frac{P_u}{U_u}$

The profit function for the firm offering the higher value service given by equation (9) can be rewritten as:

$$\pi'_h = \frac{1}{2} \cdot (p_h - c) \cdot \left(1 - \frac{p_h - \gamma p_l}{1-\gamma} \right)$$

Computing the first and second derivatives with respect to p_f :

$$\frac{\partial \pi'_h}{\partial p_h} = \frac{1}{2} \left(1 - \frac{2p_h}{1-\gamma} + \frac{\gamma p_l}{1-\gamma} + \frac{c}{1-\gamma} \right)$$

$$\frac{\partial^2 \pi'_h}{\partial p_h^2} = -\frac{1}{1-\gamma} < 0$$

The sign of the second derivative implies concavity of the profit function and hence the existence of a unique maximum. Setting the first derivative to zero, we get the best response price:

$$p_h^*(p_l) = \frac{\gamma p_l + (1-\gamma)c}{2}$$

The profit function for the firm offering the lower value service given by equation (10) can be rewritten as:

$$\pi'_l = \frac{1}{2} \cdot (p_l - c) \cdot \left(\frac{p_h - \gamma p_l}{1-\gamma} - p_l \right)$$

Computing the first and second derivatives with respect to p_l :

$$\frac{\partial \pi'_l}{\partial p_l} = \frac{1}{2} \left(\frac{p_h}{1-\gamma} - \frac{2\gamma p_l}{1-\gamma} - 2p_l + \frac{c\gamma}{1-\gamma} + c \right)$$

$$\frac{\partial^2 \pi'_l}{\partial p_l^2} = -\frac{1}{1-\gamma} < 0$$

The sign of the second derivative implies concavity of the profit function and hence the existence of a unique maximum. Setting the first derivative to zero, we get the best response price:

$$p_l^*(p_h) = \frac{p_h + c}{2}$$

Both response functions are linear giving rise to a unique equilibrium with the stated equilibrium prices.

Proof of proposition 6: Using the values derived in proposition 5, we write the total duopoly profit at optimal prices:

$$\begin{aligned} \pi' &= \pi'_h + \pi'_l \\ &= \frac{1}{2} \cdot (p_h^* - c) \cdot \left(1 - \frac{p_h^* - \gamma p_l^*}{1-\gamma} \right) + \frac{1}{2} \cdot (p_l^* - c) \cdot \left(\frac{p_h^* - \gamma p_l^*}{1-\gamma} - p_l^* \right) \end{aligned}$$

Substituting for the optimal prices from proposition 5, we differentiate the profit function with respect to the monitoring cost and make use of the envelope theorem to analyze the equilibrium profit as a function of the monitoring cost parameter:

$$\frac{\partial \pi'}{\partial c} = \frac{\partial \pi(p_h^*, p_l^*)}{\partial c} = -\frac{5(1-\gamma)(1-c)}{(4-\gamma)^2} < 0$$

implying that the total duopoly profit is decreasing with monitoring cost. **QED**

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