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Radiation trapping forces on microspheres with optical tweezers

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Axial trapping forces exerted on microspheres are predicted using a Gaussian beam electromagnetic field model and a ray-optics model, and compared with experimental measurements. Ray-optics predicts a maximum trapping efficiency $Q = -0.14$ for optically trapped polystyrene microspheres in water, compared to a measured value of -0.12 ± 0.014 for 10 μm diam microspheres. When the microspheres are composed of amorphous silica, the predicted ray-optics Q decreases to -0.11, compared to a $Q = -0.034$ predicted by the electromagnetic field model, and a measured value of -0.012 ± 0.001 for 1 μm diam microspheres. These results indicate that the two models have applicability in two different size regimes, and thus, are complementary.

Optical tweezers is the popular term for a single-beam gradient force optical trap,[1] which consists of a very strongly focused laser beam, and is usually implemented with the aid of a microscope. Optical tweezers utilize the radiation pressure of light to manipulate and hold microscopic objects, including dielectric particles and biological cells. The calculation of optical trapping forces is difficult, given the small size of the particles and the complex nature of the laser beam. Nevertheless, the ability to predict trapping forces is essential, for example, in understanding the strength of biological molecular motors,[2] as well as the elastic properties of cell membranes[3] and DNA macromolecules.[4] Previous works[1,5,6] have used a ray-optics (RO) approximation to show that optical trapping of spherical particles is possible with focused laser beams. However, this approach is independent of sphere size, and is therefore correct only for large particle diameters (>10 μm). In addition to the transverse components, a tightly focused laser beam has axial field components, which precludes its consideration as a paraxial Gaussian beam. To calculate the forces on a small sphere more accurately, an electromagnetic (EM) field model can be used to describe the interaction between a focused laser beam and a spherical particle.[7] In this letter, we compare the trapping force predictions made by the RO and EM models for convergence angles of 5.8° and 60°. We also compare the predicted trapping forces with experimental measurements on 1 and 10 μm diam spheres held by the optical trap. Finally, the trapping force is examined as a function of the sphere radius over four orders of magnitude using both models. A comparison between the two models, and with experimental data, is important for determining the accuracy and region of applicability for the models in predicting radiation forces.

Trapping forces can be calculated in terms of a nondimensional efficiency parameter Q, from the expression $F = nQP/c$, where F is the force, P is the power of the laser beam, n is the refractive index of the surrounding medium, and c is the speed of light in free space. The predictions for the RO case are determined using a model developed by Ashkin.[6] For this calculation, an incident beam is assumed to have a Gaussian intensity profile at the microscope objective aperture, with a beam diameter equal to the lens aperture radius. In the EM field model,[7] expansion coefficients are derived for an infinite series representation of the electric and magnetic fields external to a spherical particle. Once the expansion coefficients, which describe the incident and scattered laser fields, are found, the axial forces exerted on a microsphere can be derived from the Maxwell's stress tensor and can then be determined using expressions previously derived.[8] However, in order to obtain the expansion coefficients, the EM model requires an accurate expression for the radial component of the incident electric field. In this letter, fifth-order corrections[9] for the incident field components of a Gaussian laser beam are sufficiently accurate for the EM calculation, when the beam parameter $s = 1/k\omega_0$ is less than ~ 0.32 for a sphere diameter less than 2 μm and a laser spot size $\omega_0 = 0.4$ μm. In this case, the wavelength λ is taken to be 1.06 μm, where $k = 2\pi n/\lambda$ is the wave number, and the refractive index of the surrounding medium (water) is 1.33.

A comparison of the calculated axial force on a 10 μm diam polystyrene ($n = 1.57$) microsphere suspended in water ($n=1.33$), and illuminated at a wavelength of 1.06 μm with a cone angle of 5.8° ($\omega_0 = 2.5$ μm), using the RO and EM models, is shown in Fig. 1. An approximate formula for the cone angle, for small angles, is $\theta \approx \lambda/n\pi\omega_0$. While this cone angle is too small to create an optical trap, it does provide a means to directly compare the accuracy of the two force models for a large microsphere. From Fig. 1, it is seen that the RO model [curve (A)] predicts a Q in the range of 0.028–0.03, depending on the location of the sphere with respect to the focal point of the laser. In comparison, a Q in the range of 0.03–0.033 is predicted by the EM model [curve (B)]. The models are seen to be in good agreement, with a difference of less than 10%.

When the cone angle for focused laser light is increased to 60°, the RO model predicts a maximum trapping efficiency $Q = -0.14$, as shown in Fig. 1 [curve (C)]. This cone angle is representative of that produced by a 100× oil

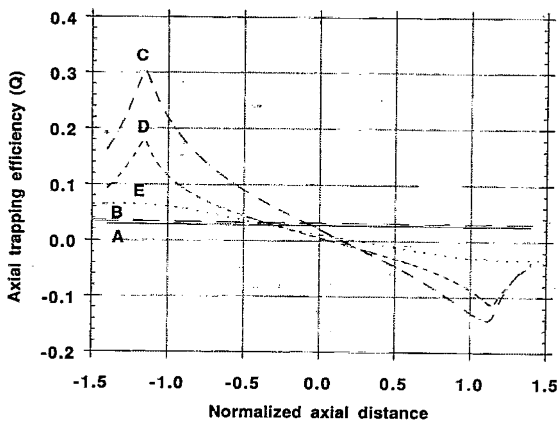

FIG. 1. Calculated axial trapping efficiency in terms of the location of the beam focus with respect to the center of the microsphere. The axial distance is normalized with respect to the sphere radius. (A) Ray optics (RO) calculation for a polystyrene ($n=1.57$) sphere within a Gaussian laser beam having a cone angle of 5.8°. (B) EM calculation for the same sphere as in (A). (C) RO calculation for a polystyrene sphere within a Gaussian beam having a cone angle of 60°. (D) Same calculation as (C), but for a sphere composed of amorphous silica ($n=1.45$). (E) EM calculation for an amorphous silica sphere having a diameter of 1 μm illuminated by a Gaussian laser beam ($\lambda = 1.06\ \mu$m) with a spot size of 0.4 μm. All of the above calculations assume a surrounding medium of water ($n=1.33$).

immersion objective, having a numerical aperture of 1.3. To measure the trapping efficiency, a 10 μm polystyrene microsphere (Duke Scientific) located in a thin, glass rectangular chamber (Vitro Dynamics) having an internal thickness of 50 μm, was trapped and moved to a position just below the upper glass surface to minimize spherical aberration. The power of the trapping beam was then decreased until the microsphere was observed to fall out of the trap. At a release power of 100 μW and an axial force of 5.6×10^{-15} N, the axial trapping efficiency (Q) is found to be equal to -0.12 ± 0.014, as shown in Table I. These data indicate that a 10 μm diam microsphere is almost within the ray-optics regime, since the RO prediction is slightly higher than the measured axial Q.

When the diameter of the microsphere is decreased to 1 μm, the EM model predicts a smaller trapping efficiency compared to the RO model, as shown in Fig. 1 [curves (E) and (D), respectively]. The laser spot size was measured to be $0.4 \pm 0.05\ \mu$m, using the method of Firester *et al.*[10] and was used in the EM model, rather than the 0.24 value predicted from $\theta \approx \lambda/n\pi\omega_0$. Note that the expression for θ was derived from a paraxial approximation to the wave equation and applies only for angles $\leqslant \sim 30°$.[11] Numerical calculations of the field amplitudes at the beam focus, using an expansion of a Gaussian beam in terms of an angular spectrum of plane waves, indicates that for a cone angle of $\sim 58°$, the spot size should be 0.44 μm,[12] compared to 0.24 μm for the paraxial prediction. The maximum backwards EM trapping Q is now seen to be -0.034, which is $\sim$ 4 times less than the value predicted by the RO model. The maximum trapping Q for $1 \pm 0.08\ \mu$m diam amorphous silica microspheres (Bangs Laboratories), suspended in water and having a refractive index of 1.45, was measured to be -0.012 ± 0.001 (Table I). Silica particles, rather than polystyrene, were utilized for the 1 μm diam measurement because their greater specific gravity (2.1 versus 1.05, respectively) make it easier to observe when they fall out of the trap. The maximum axial trapping Q predicted by the EM model decreases to -0.022 if the spot size is taken to be 0.45 μm instead of 0.4 μm. The remaining discrepancy between the predicted and measured axial Q for the 1 μm microsphere is partially attributed to spherical aberration in the 100× oil immersion lens, since an ideal lens has been assumed in the calculations. Spherical aberration of high numerical aperture objectives, for example, is a problem in the design of laser scanning confocal microscopes for 3D sectioning of thick specimens.[13] Also, the predicted axial Q using the EM model has a certain amount of error, since the incident Gaussian laser fields are not exact solutions to the field equations. The possibility that the microscope illuminator was contributing an unwanted optical or radiometric force was also checked, but no difference in the axial Q was observed when the illumination was decreased. Thus, the EM model provides a better description for optically confined particles that are in the size regime of 0.5–2.0 μm diam, typical of those particle dimensions encountered inside a biological cell.

The EM model can be used to explore the dependence of the axial Q on the size of the trapped microsphere. Calculations were performed for a polystyrene microsphere ($n=1.57$) suspended in water ($n=1.33$) and trapped at a wavelength of 1.06 μm with a spot size of 0.4 μm. Figure 2 shows that the Q has an $\sim r^3$ dependence for sphere radii between 10 and 100 nm, as expected for Rayleigh sized particles.[1] As the radius of the microsphere increases to 1 μm, a decrease in the slope of the force curve is observed. At the other extreme, for large spheres ($r > 100\ \mu$m), one is in the ray-optics regime where Q is independent of r. The calculation for the 100 μm sphere radius was performed using the RO model. Since the EM model is limited to sphere radii less than $\sim 1\ \mu$m, an interpolation curve has been fitted through the calculated data points for both models in order to predict the expected force on microspheres having radii in the range of 1 to $\sim$ 10 μm. It can be seen that the expected force changes by only a factor of ~ 2 for spheres with radii between 1 and 10 μm. Thus, the RO model can be used to predict forces within a factor of ~ 2 on particles in the size regime of greatest interest to cell biologists. Note that the force curve in Fig. 2 does not decrease in slope monotonically for sphere radii between

TABLE I. Measured axial trapping efficiency (Q) on microspheres.[a]

Sphere diameter (μm)	Power (μW)	Force (N)	(Q)
1	100	5.6×10^{-15}	-0.012 ± 0.001
10	560	3.0×10^{-13}	-0.12 ± 0.014

[a]The force exerted on the microsphere is calculated by taking the difference between the gravitational and buoyant forces. The 1 μm diam microspheres, composed of amorphous silica ($n=1.45$), were suspended in water ($n=1.33$) and had a specific gravity of 2.1. The 10 μm diam microspheres, composed of polystyrene ($n=1.57$), were also suspended in water ($n=1.33$) and had a specific gravity of 1.05.

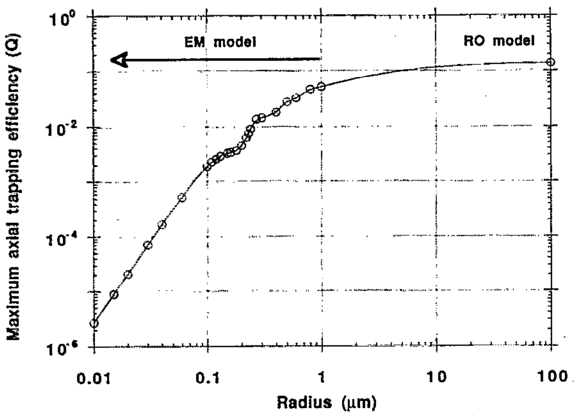

FIG. 2. Maximum axial trapping efficiency (Q) as a function of the microsphere radius. The microsphere was assumed to be polystyrene ($n = 1.57$) suspended in water ($n = 1.33$). For the EM model calculation ($r \leqslant 1$ μm), the wavelength was 1.06 μm and the spot size was 0.4 μm. The RO model calculation ($r = 100$ μm) used a cone angle of 60°. Note the r^3 dependence on (Q) for $0.01 \leqslant r < 0.1$ μm, decreasing to a r^0 dependence as r becomes very large (> 100 μm).

0.1 and 1 μm. This may be due to a Fabry–Perot resonance effect within the sphere at radii of ≈ 0.17 and 0.34 μm, which can reduce the total Q via the storage of beam energy inside the sphere.[14]

It would be desirable to have a model that provides accurate force predictions for spheres with radii between 1 and 10 μm. The primary limitation with the present EM model is the approximation used to determine the radial component of the incident electric field. Because the field expressions used to describe the Gaussian laser beam are not exact solutions to Maxwell's equations, the resulting boundary conditions are poorly fit at the surface of the sphere when the sphere size is large and the spot size is small. One alternative to using a high-order correction to the Gaussian laser beam in this size regime is to treat the focusing of the trapping laser beam as a diffraction problem. Recent calculations[15] employing the Fresnel–Kirchhoff diffraction integral, suggest that it may be possible to determine the EM fields at the trap focus for highly converging laser beams using Fourier transform techniques. This work is currently underway.

In conclusion, the EM and RO models have been shown to be complementary. Forces on a large particle ($r \geqslant 5$ μm) can be calculated using the RO model and are in good agreement with the measured Q values for a 10 μm diam sphere. Calculation of the axial forces on small particles ($r < 0.5$ μm) and large convergence angles ($\sim 60°$) can be done using the more accurate EM model. The predictions of the EM model are confirmed by measurements on 1 μm diam silica microspheres. There remains a range of particles diameters from ~ 1.0 to 10 μm where neither model provides sufficient accuracy. Nevertheless, both models provide useful information within their range of validity, and their predictions have been confirmed by experimental measurements.

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