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Real Options Models for Better Investment Decisions in Road Infrastructure under Demand Uncertainty

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Wang, Ke

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Real Options Models for Better Investment Decisions in Road Infrastructure under Demand
Uncertainty

DISSERTATION

submitted in partial satisfaction of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

in Civil Engineering

by

Ke Wang

Dissertation Committee:
Professor Jean-Daniel Saphores, Chair
Professor R. Jayakrishnan
Professor Knut Solna

2017

DEDICATION

To

my wife, Jing Gao

and

my parents, Ruiping Jia and Min Wang

in recognition of their love and support

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CURRICULUM VITAE
Ke Wang

EDUCATION

University of California, Irvine, California, USA
    PhD of Civil Engineering   2017

University of California, Irvine, California, USA
    Master of Transportation Science   2012

Harbin Institute of Technology, China
    Bachelor of Science, Road and Bridge Engineering   2008

FIELD OF STUDY

Infrastructure investment under uncertainty, network design problem and travel behavior

RESEARCH EXPERIENCES

UCTC project: Greening freight transportation*: An analysis of some social benefits from
shifting freight traffic to off-peak hours   Jan. 2011 - Jan. 2013
- Southern California traffic emission calculation based on Transmodeler simulation
- Southern California OD matrix estimation
- Southern California road network edition

IKEA & WWF: Sustainable Transportation of People (SToP) Research, Shanghai, China
                                              Jan.2009 - Aug.2009
- Policy review and trend analysis of sustainable transportation in China;
- Created an effective mathematical method forecasting CO_2 emissions generated by the trips of visitors and workers of IKEA new stores.

Measuring Traveler Dissatisfaction and Behavior Changes under Urban Traffic Congestion, Shanghai, China
                                              May.2009 - Aug.2010
- Random sampling, survey organizing and data collection

PEER-REVIEWED PUBLICATION

Wang, K. (2013). **"**Causality between built environment and travel behavior: structural equations model applied to Southern California." Transportation Research Record: Journal of the Transportation Research Board, No. 2397, 80–88.

CONFERENCE PROCEEDINGS

Bhagat, A., Wang, K., R. Jayakrishnan, Saphores, J.-D. (2015). "Schemes to expand Origin-Destination estimation results for microscopic simulation to an extended network." Transportation Research Board (TRB) 94th Annual Meeting, Washington, D.C., USA, January 11-15.

Wang, K., Saphores, J.-D. (2015). "Financing, Timing, and Capacity of a New Intercity Highway under Demand Uncertainty: The BOT Case." *Proceedings from the 2015 UCCONNECT Student Conference*, University of California, Santa Barbara, Febuary 27 - Match 1.

Wang, K., X. Shi, T. Xiao (2010). "Analysis of factors associated with shopping transport mode choice and change." *Proceedings from the 10th International Conference of Chinese Transportation Professionals*, Beijing, China, August 4-8.

TEACHING EXPERIENCE

Teaching Assistant
    2014 Spring   CEE 110: MODELING, ECONOMICS, AND MANAGEMENT
    2015 Winter   CEE 111: SYSTEMS ANALYSIS AND DECISION MAKING
    2015 Spring   CEE 110: MODELING, ECONOMICS, AND MANAGEMENT
    2016 Winter   CEE 111: SYSTEMS ANALYSIS AND DECISION MAKING
    2016 Spring   CEE 122: TRANSPORTATION SYSTEMS II: OPERATIONS & CONTROL

HONOR

2015 UCCONNECT (University of California Center on Economic Competitiveness in Transportation) Dissertation Grant

ABSTRACT OF THE DISSERTATION

Real Options Models for Better Investment Decisions in Road Infrastructure under Demand Uncertainty

By

Ke Wang

Doctor of Philosophy in Civil Engineering

University of California, Irvine, 2017

Professor Jean-Daniel Saphores, Chair

An efficient transportation system requires adequate and well-maintained infrastructure to relieve congestion, reduce accidents, and promote economic competitiveness. However, there is a growing gap between public financial commitments and the cost of maintaining, let alone expanding the U.S. road transportation infrastructure. Moreover, the tools used to evaluate transportation infrastructure investments are typically deterministic and rely on present value calculations, even though it is well-known that this approach is likely to result in sub-optimal decisions in the presence of uncertainty, which is pervasive in transportation infrastructure decisions. In this context, the purpose of this dissertation is to propose a framework based on real options and advanced numerical methods to make better road infrastructure decisions in the presence of demand uncertainty.

I first develop a real options framework to find the optimal investment timing, endogenous toll rate, and road capacity of a private inter-city highway under demand uncertainty. Traffic congestion is represented by a BPR function, competition with an existing road is captured by user equilibrium, and travel demand between the two cities

follows a geometric Brownian motion with a reflecting upper barrier. I derive semi-analytical solutions for the investment threshold, the dynamic toll rates and the optimum capacity. The result shows the importance of modeling congestion and an upper demand barrier – features that are missing from previous studies.

I then extend this real options framework to study two additional ways of funding an inter-city highway project: with public funds or via a Public-Private Partnership (PPP). Using Monte Carlo simulation, I investigate the value of a non-compete clause for both a local government and for private firms involved in the PPP.

Since road infrastructure investments are rarely made in isolation, I also extend my real options framework to the multi-period Continuous Network Design Problem (CNDP), to analyze the investment timing and capacity of multiple links under demand uncertainty. No algorithm is currently available to solve the multi-period CNDP under uncertainty in a reasonable time. I propose and test a new algorithm called "Approximate Least Square Monte Carlo simulation" that dramatically reduces the computing time to solve the CNDP while generating accurate solutions.

CHAPTER 1 INTRODUCTION

An efficient transportation system requires an adequate and well-maintained transportation infrastructure to relieve congestion, reduce accidents, improve freight productivity, and enhance economic competitiveness. However, there is a growing gap between resources invested by the public sector and the cost of maintaining, let alone expanding, the road transportation infrastructure in the United States. Indeed, the 2013 ASCE infrastructure report card gave roads nationwide a "D". While capital investments reached $91 billion annually for all levels of government, this falls dramatically short of the $170 billion that the FHWA estimates are needed annually to significantly improve road conditions and performance (ASCE, 2013). Taxes on gasoline and diesel are the primary sources of transportation funding at the state and federal levels, accounting for 90% of the Highway Trust Fund (Lowry, 2015). However, road infrastructure funding is declining due to a lack of adjustment of fuel taxes to inflation (Dumortier *et al.*, 2017) and to improvements in fuel efficiency. Indeed, between 1980 and 2012, average fleet fuel efficiency increased from 15.97 to 23.31 miles per gallon (U.S. DOT Bureau of Transportation Statistics, 2015).

Historically, the U.S. road infrastructure has been mainly funded by the public sector because of its ability to raise revenue, its cost advantage and its financial risk tolerance (Jacobson and Tarr, 1995). For example, of the $89 billion spent on road capital investments in the U.S. in 2012, only $0.6 billion came from the private sector (Werling and Horst, 2014). In contrast, private capital dominates in many other types of infrastructure investments, including rail, pipelines, telecommunications, and electrical energy. In fact, international experience demonstrates that road infrastructure could also be privately owned or managed. For example,

more than 37% of highways by length in the European Union is under concession, and 3 out of 4 km under concession are operated by private firms (Albalate *et al.*, 2009).

One popular alternative to better involves the private sector is Public-Private Partnerships (PPP). There are two main advantages of PPP for infrastructure investment (Yescombe, 2002). First, the public sector can tap private capital to finance infrastructure during difficult fiscal times, which is often the case nowadays. Second, the public sector may benefit from private sector expertise and experience to build and operate infrastructure more efficiently. In the U.S., 24 states and the District of Columbia have used public-private partnerships to help finance and build at least 96 transportation projects worth a total $54.3 billion by 2011 (Reinhart, 2011). However, PPP is more widely used outside of the U.S., including in Canada, the U.K., Spain, Australia and South Korea (Brown *et al.*, 2009).

In spite of their apparent advantages, there are concerns about Public-Private Partnerships. PPP project agreements are long-term, complicated and inflexible. As a result, PPP agreements may reduce the government's flexibility to make necessary policy adjustment since such changes may affect the profitability of a PPP project (Kashani, 2012). For example, the PPP contract for the California SR-91 toll lanes contained a non-compete clause that forbade the government from increasing highway capacity. Eventually, SR-91 toll lanes were purchased by Caltrans before the end of the concession period to remove the non-compete clause.

Another obstacle to a more widespread use of PPP and involvement from the private sector is the inadequacy of some of the tools commonly used to assess transportation infrastructure investments. Indeed, these tools are typically deterministic and rely on present value calculations, even though it is well-known that this approach is likely to result in sub-optimal decisions in the presence of uncertainty (Dixit and Pindyck, 1994), which is pervasive in

transportation infrastructure decisions. For roads, this uncertainty is driven by travel demand uncertainty, which depends on future land use developments, competing infrastructure, but also general economic conditions and fuel prices, not to mention the long time it takes to plan, design, and build new roads. In a widely cited paper, Flyvbjerg *et al*. (2006) examined the accuracy of demand predictions for 27 rail projects and 183 road projects around the world. They found that for the first year of operation, the difference between predicted and actual demand exceeded 20% in half of the road projects they examined. These discrepancies may have partly been caused by overly optimistic forecasts trying to tilt investment decisions in favor of a project, but they also highlight that predicting travel demand is far from an exact science.

In this context, the purpose of this dissertation is to propose a framework based on real options techniques and advanced numerical methods to make better road infrastructure decisions in the presence of demand uncertainty. The real options approach was selected in this dissertation because it provides powerful tools from finance for dealing with investments in non-financial (i.e., "real") assets, under uncertainty. These tools include stochastic dynamic programming, contingent claims analysis, and stochastic calculus (for a basic introduction, see Dixit and Pindyck, 1994). In particular, a real options framework can address the main weakness of the traditional NPV method, which is that it ignores the flexibility of altering investment decisions under uncertainty as new information becomes available over time.

Chapter 2 reviews the literature related to real options applications to transportation infrastructure investments and network design problems. This review provides a landscape of the literature dealing with applications of real options to transportation infrastructure investments and suggests avenues for contributing to this area of knowledge.

Chapter 3 proposes a real options framework to handle demand uncertainty for a private intercity highway project. The traffic congestion effects on the existing and proposed highways are captured by a BPR function. Travel demand between the two cities follows a geometric Brownian motion with a reflecting upper barrier (RGBM) that corresponds to total highway capacity between the two cities. This framework considers three critical decisions: the dynamic toll rate, the optimal timing of the investment, and the optimal capacity of the new highway. The proposed framework is then tested on data from the California SR-125 toll road project, which is treated as a private intercity highway.

Chapter 4 revisits the optimal investment timing problem for a public inter-city highway from the government's point-of-view. The "Rule of a Half" is used to approximate the consumer surplus of the project. The system optimal toll rate charged on the new highway is derivated to be constant regardless of demand and of highway capacities. Monte Carlo simulation is used to solve the optimal demand threshold. In addition, the investment timing problem is solved in a PPP framework (build-operate-transfer, to be specific). The government gives a concession to a private company who takes responsibility for financing, planning, designing, constructing, operating and maintaining the new highway. The value of a non-compete clause, which forbids the government from expanding the old highway during the concession is calculated using real options methods.

Chapter 5 expands the real options method to the multi-period Continuous Network Design Problem (CNDP), which aims to find the investment timing and optimal highway expansion plan for a road network. Analytical methods, finite difference method and Least Square Monte Carlo simulation (LSMC) are not applicable for solving the multi-period CNDP, because of the complexity of this problem. I, therefore, propose an algorithm that I call

"Approximate Least Square Monte Carlo simulation" (ALSMC) to solve the multi-period NDP. This algorithm applies least square regression to estimate the value of the termination payoff function without knowing the optimal capacity improvement plan. In each iteration, only a multi-period CNDP with deterministic demand needs to be solved, which dramatically (from days to minutes) reduces the computing time of each termination payoff function. The ALSMC is tested on a simple example where an exact solution is available, compared to other approaches available to deal with the single period CNDP, and illustrated on a small network for a multi-period CNDP.

Finally, Chapter 6 summarizes the main findings of this dissertation and makes some suggestions for future research.

CHAPTER 2 LITERATURE REVIEW

Chapter 2 reviews the literature related to real options applications to transportation infrastructure investments and network design problems. The application of real options papers is classified into 2 groups. One group, discussed in Section 2.1, focuses on the investment timing of transportation infrastructure. The other group of papers (Section 2.2) evaluates government guarantees in a public-private partnership. Section 2.3 reviews algorithms for deterministic Continuous Network Design Problem (CNDP) and multi-period CNDP with stochastic demand.

2.1. REAL OPTIONS APPLICATION IN TRANSPORTATION

Table 2-1 summarizes 10 published papers since 2004 that rely on Real Options (RO) to analyze transportation infrastructure investment problems. Out of these 10 papers, 6 papers deal with road projects and 4 papers are concerned with rail or transit projects. First, I review these papers according to their solution methods (simulation in 2.1.1 and analytical solutions in 2.2.2). Then, I discuss four critical assumptions involved in the modeling process of these papers: capacity choice, infrastructure competition, toll authority, demand uncertainty and the investor's objective (Section 2.1.3 to 2.1.6).

Table 2-1. Applications of real options to private-funded transportation projects

Authors (year)	Option(s)	Methodology	Toll/ transit fare	Model solution	Case study	Main findings
Garvin and Cheah (2004)	Project deferment	Binomial model with travel demand uncertainty	Pre-determined	Simulated solution	Dulles Greenway toll road project (Virginia)	The deferment option has a significant value under demand uncertainty.
Zhao *et al.* (2004)	Right of way, highway expansion, and rehabilitation	Least-square Monte Carlo (MC) simulation with uncertainty in travel demand, land price, and highway deterioration	Fixed	Simulated solution	Hypothetical example	Increasing land price could lead to highway expansion even when demand is fixed. Increases in demand encourage expansion, land reserve and rehabilitation.
Saphores and Boarnet (2006)	Project deferment	Continuous time real options model with population uncertainty	Annualized costs equally shared by residents	Analytical solution	None	Derived an explicit population threshold; for short project implement duration, ignoring uncertainty leads to investing prematurely; the reverse holds if the duration is long enough and uncertainty is high.
Chow and Regan (2011a)	Project deferment, re-design	Least-square MC simulation with travel demand uncertainty between each OD pair	No toll	Simulated solution	Hypothetical example	A real options model to find the optimal investment timing with capacity design exogenously decided from a road network design problem.
Chow and Regan (2011b)	Project deferment, re-design, re-order	Least-square MC simulation with travel demand uncertainty between each OD pair	No toll	Simulated solution	Hypothetical example	Optimal investment timing and capacity improvements on a road network. They optimize the order of investment of link improvement.
Railway						
Couto *et al.* (2012)	Project deferment	Continuous time real options with travel demand uncertainty	A power function of demand	Analytical solution	Hypothetical example	Higher high-speed rail demand uncertainty with random shocks decreases the demand threshold.
Godinho and Dias (2012)	Project deferment	MC simulation with fuel price, GDP and traffic uncertainty	No toll	Simulated solution	Douro Interior Concession in Portugal	Deferment option may add substantial value to projects. The NPV may be biased downward if calculated using only expectation of variables and ignoring uncertainty.

Authors (year)	Option(s)	Methodology	Toll/ transit fare	Model solution	Case study	Main findings
Pimentel *et al.* (2012)	Project deferment	Continuous time real options model with travel demand and project cost uncertainty	A power function of demand	Analytical solution	Hypothetical example	Optimal investment timing should be delayed under demand uncertainty
Gao and Driouchi (2013)	Project deferment	Alpha-maxmin multiple-priors expected utility with population and decision-making uncertainty	Annualized costs equally shared by residents	Analytical solution	Hypothetical example	As the ambiguity level in probability distributions increases, optimistic decision makers will decrease their investment threshold while pessimistic counterparts will do the opposite
Li *et al.* (2015)	Project deferment, technology selection	Continuous time real options model with population uncertainty	Fixed	Analytical solution	Hypothetical example	Transit investment can induce urban sprawl. Ignoring impact of investing on urban spatial equilibrium can lead to investing too late. Ignoring uncertainty can cause premature investment

2.1.1. Simulated Solutions

Five papers listed in Table 2-1 solve the investment problem by simulation. For example, Garvin and Cheah (2004) apply a binomial model to capture the demand uncertainty for a private toll road. With an exogenous toll, demand evolves according to a discrete-time process, going up or down with known probabilities at each time step. Using Monte Carlo simulation, the authors price the deferment option and demonstrate that it can have substantial value under demand uncertainty.

Godinho and Dias (2012) use Monte Carlo simulation to price the deferment option for a publicly-funded road. They estimate an autoregressive model with a normally distributed error based on historical data to forecast the GDP as a function of its value in the two preceding time steps. Moreover, fuel price is assumed to follow a geometric Brownian motion (GBM). They show that the deferment option may add substantial value to the social benefit of a project.

The other three papers rely on Least-Squares Monte Carlo simulation (Longstaff and Schwartz, 2001; Gamba, 2003), which is a powerful approach that can handle multiple forms of uncertainty and multiple options.

Zhao *et al.* (2004) build a multistage stochastic model for a toll highway with multiple embedded options: land acquisition, highway expansion, and highway rehabilitation. They assume that travel demand and land prices follow a GBM, and that highway service quality may decrease from 5 to 1 gradually at each point in time. They find the profit-maximizing solution numerically for some examples and report that a higher traffic demand requires more lanes and a larger land reserve (right-of-way acquired beyond immediate need). However, increasing land prices only drives down the land reserve, but not the number of lanes. At some demand level, higher land prices even trigger highway expansion with the same demand level.

Chow and Regan (2011a, 2011b) incorporate real options into a continuous network design problem. Traffic demand between each OD pair is assumed to follow a GBM. The travel time on each link is modeled by a BPR function. Using Least-square Monte Carlo simulation, they find the optimal investment timing, which link(s) on the road network to invest in, and the corresponding optimal capacity.

2.1.2. Analytical Solutions

The other five papers listed in Table 2-1 provide analytical results. Saphores and Boarnet (2006) consider a congestion relief project in a monocentric city where construction costs are recovered by taxing land. They derive analytically the socially optimum utility-maximizing timing of that investment, assuming that the city population follows a Geometric Brownian Motion with reflecting bounds. They conclude that a standard cost-benefit analysis (CBA) may either lead to starting the project prematurely or too late, depending on the level of uncertainty and the duration of the project construction.

Gao and Driouchi (2013) model a monocentric city that is analogous to that of Saphores and Boarnet (2006). Unlike other studies assuming that future uncertainty is characterized by a certain probability, they developed an α-maxmin multiple-priors expected utility framework to solve for the option value of a rail transit investment under Knightian uncertainty, which measures the ambiguity in population uncertainty with multiple geometric Brownian motions. They derive the population threshold as a function of the level of optimism (α) for different values of probabilistic ambiguity. They report that as the ambiguity level in probability distributions increases, optimistic decision-makers decrease their population threshold while their pessimistic counterparts do the opposite.

Li *et al.* (2015) also consider a monocentric city model with a stochastic population to study the timing and the technology of a transit project. They report that this transit investment could induce urban sprawl and that ignoring population volatility could lead to investing prematurely. Moreover, in their real options model one transit technology always dominates others regardless of population size.

Pimentel *et al.* (2012) develop a real options model to examine the optimal timing of upgrading to high-speed rail (HSR) a conventional rail line between two cities with three sources of uncertainty: travel demand, project cost, and benefit per HSR passenger. Their objective is to maximize the net benefits of travelers, which includes time and fare savings. They show analytically that increasing demand uncertainty delays the optimal investment time.

Couto *et al.* (2012) apply a similar but simplified framework that only accounts for uncertainty in demand, which they assume follows a GBM with random shocks to capture sudden changes in demand due to unanticipated future events. They report that positive demand shocks bring forward the timing of a High-Speed Rail investment and decrease the value of the option to defer the transportation investment.

2.1.3. Capacity Choice

With three exceptions (Zhao *et al.*, 2004 and Chow and Regan, 2011a, 2011b) that solved the capacity choice problem by simulation, most of the papers I reviewed assume that the capacity of the proposed project is predetermined. However, one important value of project deferment is project re-design. Building infrastructure with a lower capacity is cheaper but it may generate lower revenues and social benefits in the long run; conversely, building infrastructure with a higher capacity is more expensive upfront but it becomes more profitable and more socially

beneficial once travel demand is sufficiently high. Therefore, it makes sense to endogenize capacity, and to chose it jointly with the timing of a transportation investment.

Another reason for accounting for capacity choice is that traffic or passenger volume physically cannot exceed infrastructure capacity. Ignoring capacity limits may lead to overestimating future volumes as well as profits or social benefits. Among all papers reviewed above, only Zhao *et al.* (2004) assume that the actual traffic volume is below highway capacity.

2.1.4. Infrastructure Competition and Tolls

For infrastructure investments, the infrastructure already available in an area should intuitively impact the forecasted profits and the social benefits of the infrastructure under planning. However, most optimal investment timing studies published to date have ignored potential competition from existing infrastructure. As a result, these studies can only set toll or transit fares exogenously. For example, Garvin and Cheah (2004) assume that the private sector has no toll authority and that the toll is set by the government to slowly increase from $2 to $3 over 40 years. Zhao *et al.* (2004) and Li *et al.* (2015) set fixed toll or transit fares. Chow and Regan (2011a, 2011b) did not consider tolls, and their projects are assumed to be publically financed.

Some papers use taxes to recoup a project' costs (e.g., see Saphores and Boarnet, 2006; or Gao and Driouchi, 2013). Couto *et al.* (2012) and Pimentel *et al.* (2012) set HSR fare as a power function of demand in a framework that seeks to maximize passengers' net benefits instead of profits.

2.1.5. Geometric Brownian Motion

Since my dissertation relies on the geometric Brownian motion, it is useful to review its use in

transportation problems. The geometric Brownian motion (GBM) has been a popular stochastic process in transportation research to model uncertainty in population (Saphores and Boarnet, 2006; Gao and Driouchi, 2013; Li *et al.*, 2015), railway demand (Pimentel *et al.*, 2012), travel demand (Chow and Regan, 2011a; Chow and Regan, 2011b) and highway traffic (Zhao *et al.*, 2004; Galera and Solino, 2010). The GBM is popular because it has a number of attractive properties, apart from allowing for explicit solutions in simple investment models. Indeed, if a variable X follows a GBM, then X is lognormally distributed, and it never becomes negative.

However, the validity of the GBM hypothesis has only received limited testing in transportation. One exception is Marathe and Ryan (2005), who examined airline passenger enplanements in the U.S. from 1981 to 2001. After removing seasonal variations, their analysis of monthly data suggests that the GBM process might be appropriate to model airline passenger data. Galera and Solino (2010) run Dickey-Fuller (1979) test on the annual average daily traffic (AADT) for 11 toll highway stretches. They report that the GBM hypothesis could not be rejected for traffic volume on highways. We note, however, that their sample is relatively small (22 to 31 observations).

2.1.6. Objective Function

Out of the 10 papers reviewed in this section, 8 papers study the investment problem from the government's point-of-view. The two exceptions are Garvin and Cheah (2004) and Zhao *et al.* (2004), who model projects from the point of view of private investors. Among the 8 papers focusing on the problem of the government, three types of objective are maximized: (1) total travel time savings (Chow and Regan, 2011a, b; Couto *et al.*, 2012; Pimentel *et al.*, 2012); (2) total utility (Saphores and Boarnet, 2006; Gao and Driouchi, 2013; Li *et al.*, 2015); and (3)

consumer surplus (Godinho and Dias, 2012). Consumer surplus can measure the benefit of induced demand, while the other two objectives are only suitable for investments with inelastic demand.

2.2 GOVERNMENT GUARANTEE

Public-Private Partnerships (PPP) is a contractual agreement between the public sector and one or more private sectors that allow private sector participation in the delivery and financing of infrastructure projects. PPPs have been studied in several fields including law, finance, economics, business management, civil engineering, and planning. This literature has been growing rapidly in the last decade and includes hundreds of papers, which makes it impractical to present a review of this literature. Instead, since I found a number of recent PPP review papers, I summarize these reviews to paint a general picture of this literature and contextualize the work presented in this dissertation.

Tang *et al*. (2010) reviewed 85 empirical and non-empirical studies with interest in risk, financing, project success factors, and concession periods. Clerck *et al*. (2012) examined 125 papers published between 2004 and 2011 to generate highlights and research trends in tendering and risk management in PPP projects. Roehrish *et al*. (2014) analyzed over 1400 publications and summarized publication patterns and emerging PPP research themes. Song *et al*. (2016) conducted a scientometric review of 1036 bibliographic records. Their content analysis found that emerging trends in PPP have shifted away from concession pricing and concession periods, legislation, governance, procurement management, and critical success factors in PPP projects to focus more on risk allocation, performance evaluation, negotiation of concession contracts, real options evaluation, and contract management.

Among active PPP research themes, risk allocation, especially risk sharing mechanisms, have attracted much attention (Cheah and Liu, 2006; Huang and Chou, 2006; Chiara *et al.*, 2007; Brandao and Saraiva, 2008; Galera and Solino, 2010; Liu *et al.*, 2014). One such mechanism is the government guarantee, which is a contract where the government reduces the financial risks faced by private firms involved in a PPP. Table 2-2 lists 8 papers since 2006 that study government guarantees in PPP transportation projects using real options.

Build-Operate-Transfer (BOT) is a form of PPP that is commonly used for financing, developing and operating transportation projects (Ashuri *et al.*, 2012; Kashani, 2012; Kokkaew, 2013). In a BOT project, the private sector receives a concession from the public sector to finance, design, build, operate, and maintain a facility under the concession contract for a specific period. The concession contract allows the private sector to charge facility users and/or receive payments from the public sector to recover its investment, as well as its operating and maintenance costs.

The minimum revenue guarantee (MRG) is the most common mechanism for mitigating traffic revenue risks in BOT transportation projects (Irwin, 2007). The private sector is guaranteed to receive compensation from the government when revenues fall below a predetermined level. Cheah and Liu (2006) propose a real options model to value an MRG and a revenue cap using Monte Carlo simulation. They applied their model to the Malaysia-Singapore Second Crossing. Both the initial traffic volume after construction completion and the annual traffic growth rate are assumed to be normally distributed. The toll rate is fixed exogenously. They assume that each year, if the actual revenue is lower than a projected level, the guarantee is triggered and the government needs to pay the revenue shortfall. Conversely, if the actual traffic exceeds a certain level, a revenue cap requires the private sector to pay back the excess revenue.

Their case study shows that the value of an MRG is more sensitive to the standard deviation of initial traffic volume, and the revenue cap is more sensitive to the standard deviation of the traffic growth rate. The value of the MRG grows when traffic volatility increases and the relationship between the value of the revenue cap and the traffic volatility is not monotonic.

While most papers consider government guarantees for road projects, Huang and Chou (2006) use a real options model to value the MRG and the option to abandon for a rail BOT project. MRG is formulated as a series of European options (European options can only be exercised at their expiration date). Their analysis of the Taiwan High-speed Rail project shows that both MRG and the option to abandon add value to the project from the private-sector's point of view. However, since both the MRG and the option to abandon mitigate the risk of unexpected low revenue, they counteract each other and their values are reduced. For example, increasing the MRG level decreases the value of the option to abandon.

Carbonara *et al.* (2014) develop a real options model to optimize the MRG level that ensures the project is profitable to a private entity and economically sustainable and politically acceptable to the government. The optimal MRG level also allocates risk fairly between public and private sectors. In their model, traffic follows a geometric Brownian motion, and the MRG is seen as a series of European options. By Monte Carlo simulation, the authors find the optimal MRG for a toll road in Italy.

Another form of government guarantee is the Minimum Traffic Guarantee (MTG), which triggers the compensation from the government when traffic volume on the PPP project falls below a certain level. Both Brandao and Saraiva (2008) and Galera and Solino (2010) value the MTG using a risk-neutral procedure to determine the discount rate endogenously.

Table 2-2. Applications of real options to government guarantees in transportation projects

Authors (year)	Option(s)	Methodology	Model solution	Case study	Main findings
Road					
Cheah and Liu (2006)	Minimum Revenue Guarantee (MRG) and revenue cap	MC simulation with initial traffic volume and growth rate uncertainties	Simulated solution	Malaysia-Singapore Second Crossing project	The value of the MRG grows when traffic volatility increases; the relationship between the value of revenue cap and traffic volatility is not monotonic.
Huang and Chou (2006)	MRG and the option to abandon	Compound options pricing model with operating revenue uncertainty	Analytical solution	Taiwan High-Speed Rail BOT Project	MRG and the option to abandon counteract each other. Increasing MRG level decreases the value of the option to abandon.
Chiara *et al*. (2007)	MRG	Least-square MC simulation with traffic volume uncertainty	Simulated solution	Hypothetical example	Expand LSMC to value multiple-exercise options. An MRG that covers only a short term of the concession period can still mitigate revenue risk significantly
Brandao and Saraiva (2008)	Minimum Traffic Guarantee (MTG) and revenue cap	MC simulation with traffic volume uncertainty	Simulated solution	BR-163 roadway project in Brazil	Establish a risk-neutral procedure using risk premium from Capital Asset Pricing Model (CAPM).
Galera and Solino (2010)	MTG	Option pricing model with traffic volume uncertainty	Simulated solution	A toll highway in Spain	The value of the guarantee grows significantly when traffic volatility increases
Takashima *et al* (2010)	MRG, the ownership transfer guarantee and option to defer investment	Option pricing model with revenue uncertainty	Analytical solution	Hypothetical example	Increasing the compensation payment in the MRG leads to a decrease in the investment threshold and an increase in the transfer threshold
Carbonara *et al* (2014)	MRG and revenue cap	MC simulation with traffic volume uncertainty	Simulated solution	A toll road in Italy	A real options model to find the optimal MRG level.
Liu *et al*. (2014)	Restrictive competition guarantee valuation	MC simulation with traffic volume uncertainty	Simulated solution	A PPP highway project in China	The value of restrictive competition guarantee is positively correlated with the gap between traffic and the level of the guarantee.

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Brandao and Saraiva (2008) present an MTG real options model, where the MTG is modeled as a series of independent European options. The initial traffic follows a triangular probability distribution, and later traffic follows a geometric Brownian motion. The toll rate is constant throughout the concession period. They assume that the MTG is combined with a revenue cap. A case study of the BR-163 roadway in Brazil shows that the MTG transfers revenue risks from the private sector to the public sector and that the use of caps can help reduce the government's expose to risks.

Galera and Solino (2010) develop a real options model to value different levels of minimum traffic guarantee by simulation. Traffic is assumed to follow a geometric Brownian motion. The minimum traffic guarantee is treated as multiple independent European options. They illustrate their model by test it on a toll highway in Spain. They report that the value of the guarantee from the private sector's point of view grows significantly as traffic volatility increases.

Most studies above treat the guarantees as European options, while Chiara *et al*. (2007) develop a model to value the MRG, which is in the form of Bermudan and Australian options, and solve it using Lease-square Monte Carlo simulation. Bermudan option can be exercised one time on multiple dates, while Australian option can be exercised multiple times on multiple dates. The authors argue that European options have fixed exercise time, which is not flexible enough, and such "static contract" may cause the government to compensate too much to the private sector for low revenue. They apply their model to a hypothetical example with a concession period of 30 years. They find that an MRG covering only 15 years can already mitigate 99% of the revenue risk, while an MRG covering only 4 years can mitigate 54% of the risk.

Out of 8 papers that study government guarantees, only Liu *et al.* (2014) apply real options to evaluate the value of restricted competition for a PPP toll road project. Liu *et al.* (2014) assume that new competitor will enter the market as long as the total market demand exceeds a forecasted threshold. The government needs to compensate the project company once a new competitor emerges. As in many other studies, the toll rate is determined exogenously. Results show that the value of the restrictive competition guarantee can be very significant in a project. It is positively correlated with the level of the government guarantee and negatively correlated with actual project traffic.

Most papers reviewed in Section 2.2 set the investment timing exogenously. One exception is Takashima *et al.* (2010), who study studies the value of government guarantees and their impact on investment timing. Two guarantees are considered: MRG and ownership transfer guarantee, by which the private sector can always transfer the ownership of the project to the government for a set compensation. Revenue is assumed to follow a geometric Brownian motion. The authors find that an increase in the compensation payment induces a decrease in the investment threshold and an increase in the transfer threshold.

2.3 NETWORK DESIGN PROBLEM

Most investment timing studies using real options model isolate the proposed project from other competing infrastructures and do not consider congestion effect. However, there is another field of research, called Network Design Problem (NDP) that aims to find the optimal road capacity improvement or lane addition plan while modeling the traffic congestion and traffic equilibrium explicitly in a context of road network. But there are few papers that consider the investment timing in NDP under demand uncertainty. This dissertation bridges the gap by proposing a new

algorithm to solve the investment timing and optimal capacity improvement plan for a road network in a real options framework. Therefore, I briefly review different NDP categories and algorithms developed for single-period NDP (fixed investment timing) and multi-period NDP with stochastic demands (the problem to be solved in this dissertation).

Over the last four decades, how to optimize the design of a physical transportation network has emerged as a fundamental problem in transportation research. Related problems include road capacity design, public transit network design and facilities location layout. Dozens of studies have made remarkable contributions to both model formulations and algorithms (e.g., see Abdulaal *et al.*, 1979; Chen and Yang, 2004; Wang and Lo, 2010).

The network design problem (NDP) is bi-level by nature and can be seen as a static version of the non-cooperative, two-person game. In this static game with perfect information, each player has only one move. The leader (a transport planner) goes first, with the goal of minimizing total travel costs by improving road capacity, and anticipates all possible responses of his opponent, the follower (road users). The follower observes the leader's decision and reacts in order to achieve optimal benefits (i.e., minimizing travel time) regardless of external effects (system-wide costs).

The NDP can be classified in different ways:

(1) Discrete vs. continuous or mixed, depending on the nature of decision (investment) variables:

(a) The discrete NDP with binary decision variables only allows adding an entire lane to the road network; (b) The continuous NDP allows adding a fraction of a lane; (c) The mixed NDP includes both discrete and continuous decision variables.

(2) Based on network/demand uncertainty, the NDP is either deterministic, or at least one component of the problem (demand, road capacity, travel time, or connectivity) is stochastic. The latter aims to find robust investment decision rules under uncertainty.

(3) Based on the time horizon, the NDP can be categorized as: (a) single-period NDP, allowing a one-time investment with a given investment timing and with the objective of optimizing the performance of the road network for a single period; (b) multi-period NDP, where decisions include both selecting the size of the investments to improve capacity but also their timing, under stochastic demand and/or road capacity, with the objective of optimizing the long-term performance of the road network over a given time horizon. Multi-period NDP can be more challenging by allowing more than one investment stage on any single link. Because the decision-maker has the flexibility to expand, change, delay, and/or abandon the future investment if necessary, it is also called as flexible NDP by Ukkusuri and Patil (2009), and adaptive NDP by Chow and Regan (2011b).

(4) Based on traffic assignment method, the NDP can be called either static NDP or dynamic NDP. A static NDP uses static traffic assignment to measure traffic flows and travel times, while a dynamic NDP relies on dynamic traffic assignment (Waller and Ziliaskopoulos, 2001; Karoonsoontawong and Waller, 2007).

(5) NDP can also be characterized by the elasticity of demand: (a) An inelastic demand NDP assumes that demand, either deterministic or stochastic, is independent of travel time; (b) conversely, an elastic demand NDP introduces a demand function to model the relationship between demand and travel demand for each OD pair. Because the travelers' willingness to pay is different in an elastic NDP, usually the consumer surplus is used instead of the travel time saving in the objective function to measure the performance of the investment.

A number of studies have worked on NDP formulations and algorithms. Since the goal of Chapter 5 is to propose an algorithm for multi-period CNDP with stochastic demand, I review the literature dealing with the deterministic CNDP and the multi-period CNDP models in subsections 2.3.1 and 2.3.2.

2.3.1 Deterministic Continuous NDP (CNDP)

Abdulaal and LeBlanc (1979) were the first to propose the Hooke-Jeeves algorithm to solve a deterministic CNDP, which they tested on a medium-sized realistic network (24 nodes and 76 links). Since then, many algorithms have been proposed for solving deterministic the CNDP (Suwansirikul *et al.*, 1987; Friesz *et al.*, 1992; Yang and Yagar, 1995; Meng *et al.*, 2001; Ban *et al*., 2006). However, none of these algorithms can guarantee global optimality because of the non-convexity of the CNDP, which is due to both traffic assignment equilibrium constraints and non-linear travel time functions.

Wang and Lo (2010) formulate the CNDP as a mixed-integer linear program, which offers the important property of global optimality for the solution obtained. Following Wang and Lo, Luathep *et al*. (2011) find a link-based global optimization method for the mixed NDP by formulating it as a single-level optimization problem with a variational inequality (VI) constraint that represents the user equilibrium condition. Li *et al.* (2012) develop a global optimization method based on the concepts of gap function and penalty for the CNDP, which is transferred into a sequence of concave programs and solved by a multicutting plane method.

Since the global optimal algorithms developed so far are much slower than local optimal algorithms (Wang et al., 2015), some stochastic global search methods are more attractive in practice because of their fast convergence speed and close to global optimal solution, even

though they are usually not global optimum guaranteed. One of these methods is differential evolution, which is shown by Baskan and Ceylan (2014) that performs much better than simulated annealing and genetic algorithm.

2.3.2 Multi-period NDP

A multi-period NDP involves a time horizon and time-dependent stochastic variables. The decision-maker needs to find not only the optimal capacity improvement/addition plan, but also the optimal investment timing. There are two types of formulations for multi-period NDP: static programming (Patil and Ukkusuri, 2008; Ukkusuri and Patil, 2009) and dynamic programming (Chow and Regan, 2011b).

Patil and Ukkusuri (2008) propose a stochastic mathematical program with equilibrium constraints for a multi-period NDP (allowing multi-stage investment) with stochastic and elastic demands and solve it using the Sample Average Approximation (SAA) method. Since this method is based on Monte Carlo simulation to enumerate demand paths, its solution is stochastic and converges to the true solution as the sample size increases. In the SAA method, the sample size is increased gradually until the optimal objective function values converge. Patil and Ukkusuri (2008) assume that the potential demand follows an independent distribution at each time period, so that realized demand does not give the decision maker any new information about future demand.

Ukkusuri and Patil (2009) relax their assumption about demand independence at each time period and use a scenario tree to model demand. In this approach, during the implementation period (when the investment opportunity is valid) possible demand levels can vary at each stage, while during the post-implementation period (when the investment

opportunity is expired) demand can only go up or down by a fixed amount with known probabilities.

Chow and Regan (2011b) argue that Ukkusuri and Patil's formulation "would not explicitly treat future period investment as options as adapted processes (that depend on the realization of all the stochastic elements up to that point)." Chow and Regan (2011b) then rely on real options to formulate a multi-period stochastic network design problem, and they apply Least Square Monte Carlo (LSMC) simulation to solve two simplified cases of investment timing and optimal capacity for a CNDP. Their first case is a myopic situation where the solution only optimizes the objective at the time of the investment and does not consider possible consequences in subsequent time periods. Their second case assumes that the optimal capacity is determined at time 0 and cannot be changed in the future, even if the investment is delayed and a new capacity improvement plan should be preferred as demand evolves.

CHAPTER 3 OPTIMAL INVESTMENT TIMING AND CAPACITY CHOICE OF A PRIVATE INTERCITY HIGHWAY UNDER DEMAND UNCERTAINTY

3.1. INTRODUCTION

Given the large investments needed to plan, design, build and maintain road infrastructure, private investors expose themselves to demand uncertainty and to financial risks from lower-than-expected traffic demand on their private roads, so it is critical to model demand uncertainty properly when analyzing road infrastructure investments. Since it is well known that the traditional net present value (NPV) approach is inadequate in the presence of uncertainty (Dixit and Pindyck, 1994), I apply the Real Options (RO) approach in a continuous time framework to capture the value of the flexibility to invest under uncertainty in road infrastructure. The purpose of this chapter is to analyze the decision to build a private toll road between two cities while accounting for congestion and uncertainty in travel demand.

This chapter makes several contributions. First, I derive semi-analytical results for the optimal investment timing with an endogenous toll rate while modeling congestion with a BPR function and accounting explicitly for the competition between an existing highway and a new highway linking the same two cities. Second, I model travel demand uncertainty, and my formulation accounts for induced travel demand. Third, I expand my framework to find the optimal road capacity under demand uncertainty.

This chapter is organized as follows. Section 3.2 presents the methodology for analysis, derive some results, and contrast the real options method with the traditional NPV method. Section 3.3 applies the framework to a case study with data from the California State Route 125 South Bay Expressway project. Section 3.4 expands the methodology to find the optimal road

capacity. Section 3.5 discusses the results, summarizes limitations, and suggests ideas for future work.

3.2. METHODOLOGY

3.2.1. Project Overview

I consider two growing cities, A and B, linked by a controlled access highway whose capacity is denoted by k_o. For simplicity, this highway is the only surface transportation link between A and B. As tourism and trade grow between these two cities, this existing highway is increasingly congested, so a new road is needed. A private company is considering building and operating a new highway, which will be financed by tolls charges on the new highway.

$$\tau_o = T_o \cdot \left(1 + \omega \cdot \left(\frac{q_o}{k_o} \right)^{\delta} \right)$$

Old Highway

City A ⭕   ⭕ City B

New Highway

$$\tau_n = T_n \cdot \left(1 + \omega \cdot \left(\frac{q_n}{k_n} \right)^{\delta} \right)$$
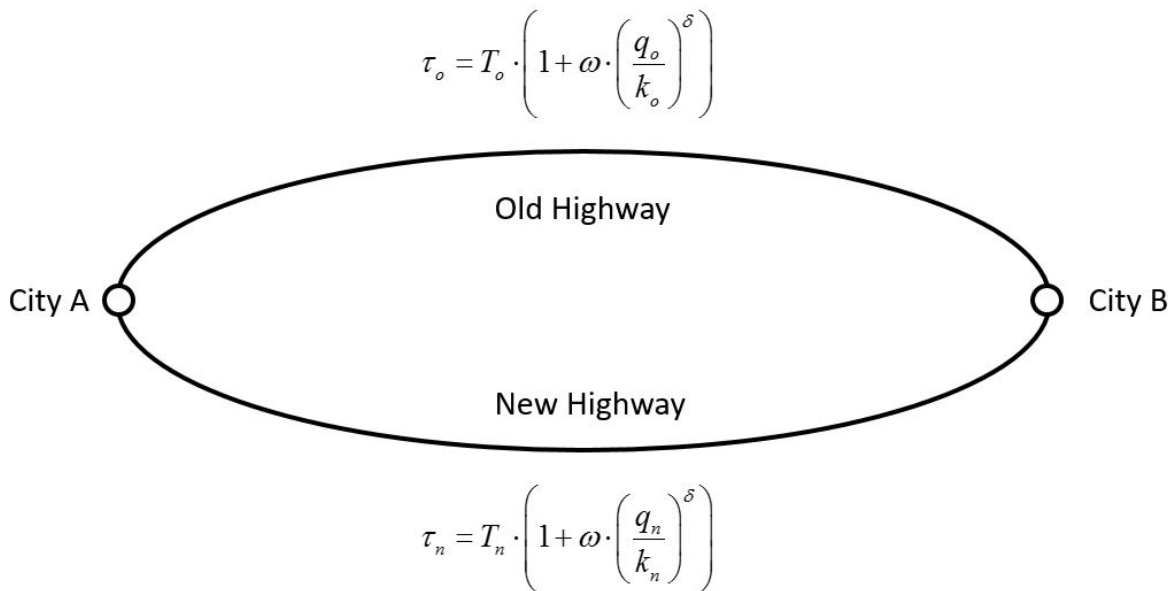
Figure 3-1. 2-link road network

Upfront costs of the new highway project include three components: land acquisition; project planning and design; and construction costs. The cost of land acquisition is given by the

present value of agriculture rents: $\int_0^\infty R_A e^{-\rho t} dt = R_A / \rho$. Second, the cost of project planning, design, and environmental approval, is denoted by C_P. Third, construction costs ar assumed to be given by $n_L \cdot C_C$, where n_L is the number of lanes of the new highway and C_C is the cost of construction per lane. The duration of the permitting, planning and construction phases is assumed to be known, and it is denoted by Δ. Finally, once the new highway starts operating, the private company needs to spend a fixed amount m per time period on maintenance and operations.

To recoup its costs and make a profit, the private company collects tolls from users of the new highway. In each time period, the flow of toll revenues at time t ($ per year) is given by:

$$R(t) = Npq_n(t), \tag{3.1}$$

where p is the toll rate ($ per vehicle), $q_n(t)$ denotes the volume of traffic (vehicles per hour) on the new highway at t, and N is a factor that converts revenues from $/hour to $/year.

The net present value (NPV) of the new highway project's total profits is the discounted flow of revenue, minus upfront costs, and minus the discounted flow of maintenance and operations costs:

$$\Omega = \int_\Delta^{+\infty} R(t) e^{-\rho t} dt - \left(\frac{R_A}{\rho} + C_P + n_L C_C \right) - \int_\Delta^{+\infty} m e^{-\rho t} dt \tag{3.2}$$

The private company would like to determine the optimal timing of investing into the new highway project as well as the capacity of this new highway and the endogenous toll rate in order to maximize the expected net present value of its profits. I make two additional assumptions. First, the capacity of the new highway is limited to a handful of values corresponding to the number of lanes in each direction. Second, I assume that the optimal toll rate is dynamic and changes based on travel demand.

To solve this problem, I first treat highway capacity as pre-determined, find the optimal investment timing and value of the project for each capacity. I then discuss how to choose between different capacities. Figure 3-2 presents an overview of my methodology.
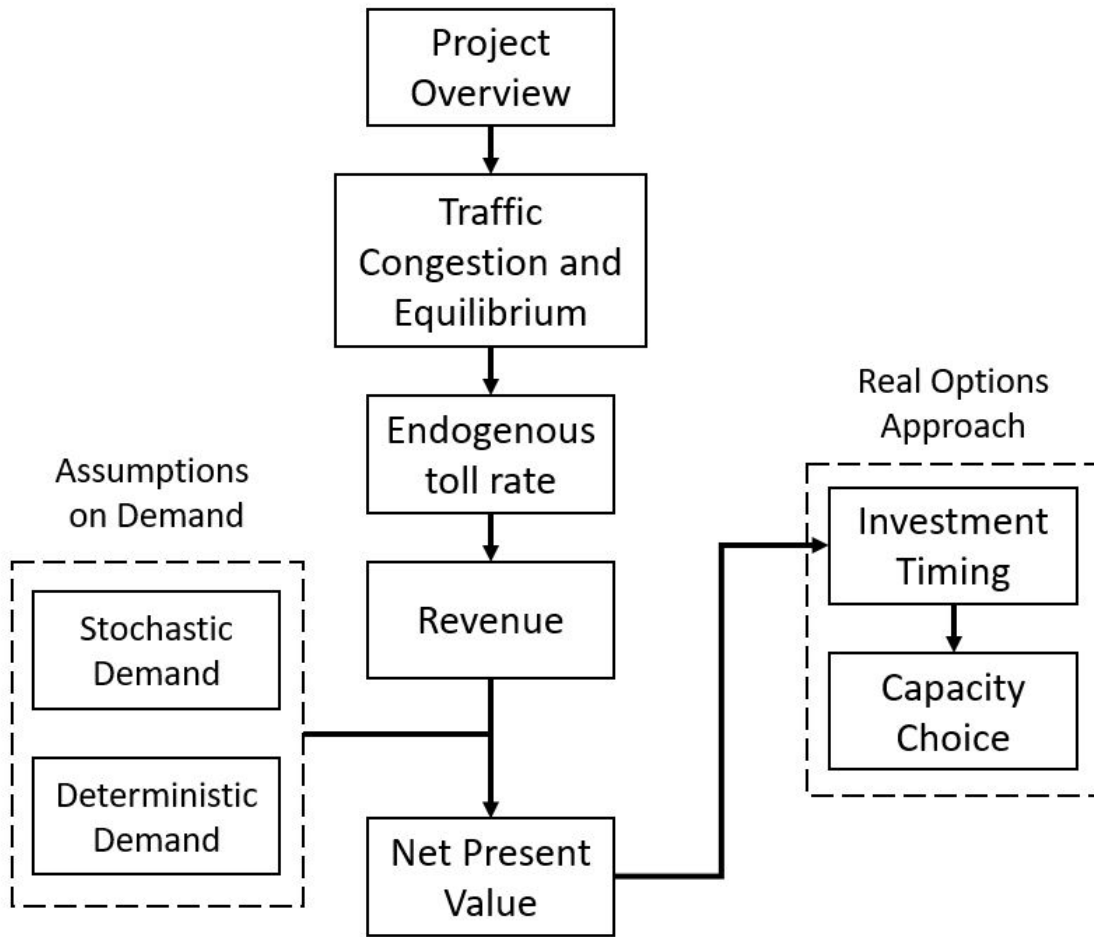
Figure 3-2. Overview of Methodology

3.2.2. Traffic Congestion and Equilibrium

As mentioned above, the flow of revenues collected depends on traffic $q_n(t)$. Given $q(t)$ $(= q_o(t) + q_n(t))$, the total traffic volume between the two cities, I derive the traffic volume on the new highway, $q_n(t)$, under user equilibrium, as a function of $q(t)$.

First, to model the travel time on each highway, I use the following Bureau of Public Roads (BPR) function:

$$\tau_i = T_i \cdot \left(1 + \omega \cdot \left(\frac{q_i}{k_i}\right)^\delta\right), \tag{3.3}$$

where (omitting the time argument from τ_i and q_i to lighten the notation):

- i is "o" for the old highway and "n" for the new highway;

- τ_i denotes the travel time on highway i when the traffic volume is q_i vehicles per hour;

- T_i is the free flow travel time;

- k_i denotes road capacity;

- ω indicates by how much travel time increases over free flow travel time at capacity; and

- δ determines how fast travel time increases with the traffic volume $q_i(t)$. Common values of ω and δ are 0.15 and 4, respectively.

The BPR function is widely used in traffic assignment partially because of its simplicity, but it is important to remember that the BPR function underestimates travel time when traffic volume is close to capacity. One way to mitigate this problem is to increase the values of ω and δ. Horowitz (1991) estimated the value of ω and δ using actual highway traffic flow data. For example, for a freeway with a design speed of 60 mph, the estimated values of ω and δ are 0.83 and 5.5 respectively, leading to a congested travel time that is 83% higher than the free-flow travel time. Another way to prevent traffic from getting too close to capacity is via access control, which is implicitly assumed here.

Under user equilibrium, travelers are indifferent between the old and the new highways because their generalized travel costs are equal. The generalized travel cost on highway i, denoted by c_i, has three components: 1) the monetary value of travel time, which is the product

of the value of time ξ and travel time τ_i; 2) variable costs, $v\tau_i$, which include fuel cost and are assumed to be proportional to travel time τ_i; 3) and an endogenous toll p for the new highway only. Hence:

$$\begin{cases} c_o = \xi \cdot \tau_o + v \cdot \tau_o \\ c_n = \xi \cdot \tau_n + v \cdot \tau_n + p \end{cases} \tag{3.4}$$

Equating c_o with c_n, substituting the expressions of τ_o and τ_n from Equation (3.4), and rearranging to obtain dimensionless quantities gives:

$$AX_n^\delta - (Q - X_n)^\delta + P + B = 0, \tag{3.5}$$

where the new dimensionless variables are

$$\begin{cases} X_n = \dfrac{q_n}{k_o}, \\[2ex] Q = \dfrac{q}{k_o}, \\[2ex] P = \dfrac{p}{\omega \cdot T_o \cdot (\xi + v)}. \end{cases} \tag{3.6}$$

and the equation parameters are given by

$$\begin{cases} A = \dfrac{T_n}{T_o} \cdot \left(\dfrac{k_o}{k_n}\right)^\delta, \\[2ex] B = \dfrac{T_n - T_o}{\omega \cdot T_o}. \end{cases} \tag{3.7}$$

A represents the ratio of the contribution of congestion to travel time for both old and new highways at the same traffic volume, and B is the ratio of the difference in free flow travel times divided by the excess travel time at capacity on the old highway.

To avoid corner solutions where all the traffic uses only one of the highways when travel demand is low, additional assumptions are needed. Since the toll rate on the new highway is

endogenous, and the private firm is assumed to maximize profit, we do not need to worry about having $c_n < c_o$ because the private firm will set the toll rate so that the last driver per unit of time who wants to drive between the two cities is indifferent between the two highways (i.e., the toll rate will be such that the free flow generalized travel costs on the old highway with no traffic $(= (\xi + v) \cdot T_o)$ will be equal to the generalized travel costs on the new highway with all the low flow traffic $(= (\xi + v) \cdot T_n \cdot (1 + \omega \cdot \left(\dfrac{q_{low}}{k_n} \right)^{\delta}) + \tau)$.

However, to prevent all the traffic from using the old highway, we need the free flow generalized travel costs on the new highway with no traffic and no toll $(= (\xi + v) \cdot T_n)$ to be smaller than or equal to the generalized travel costs on the old highway with low traffic $(= (\xi + v) \cdot T_o \cdot (1 + \omega \cdot \left(\dfrac{q_{low}}{k_o} \right)^{\delta}))$, which leads to the condition: $\left(\dfrac{q_{low}}{k_n} \right)^{\delta} \geq \dfrac{B}{A}$. From Equation (3.7), this condition is automatically satisfied if $T_n < T_o$ because then B<0 (A is always positive). In the rest of this paper, we, therefore, assume that $T_n < T_o$ to ensure user equilibrium.

3.2.3. Endogenous Toll Rate

Because costs are predetermined and not impacted by traffic volumes on the new highway, the optimal toll rate at any point in time needs to maximize the private company's instant toll revenue. Multiplying both sides of Equation (3.5) by $N^{\delta} p(q)^{\delta} k_o^{\delta}$ and rearranging leads to:

$$AR(p,q)^{\delta} - \left[R(p,q) - Np(q)q \right]^{\delta} + N^{\delta} p(q)^{\delta} k_o^{\delta} \left[\frac{p(q)}{\omega T_o (\xi + v)} + B \right] = 0, \qquad (3.8)$$

which can be seen as an implicit function of three variables: $f(R(p,q), p, q) = 0$. $R(p, q)$ can be derived from Equation (3.8) as a function of p and q. The optimal toll rate p can then be obtained as a function of q from the first-order condition $\frac{\partial R}{\partial p} = 0$.

Taking the partially derivative of Equation (3.8) and setting $\frac{\partial R}{\partial p} = 0$ leads to:

$$\frac{\partial f}{\partial p} = \delta Nq \left[R(q) - Np(q)q \right]^{\delta-1} + (\delta+1) N^\delta k_o^\delta \frac{p(q)^\delta}{\omega T_o (\zeta+v)} + \delta BN^\delta k_o^\delta p(q)^{\delta-1} = 0 \qquad (3.9)$$

Once $R(q)$ and $p(q)$ are known from Equations (3.8) and (3.9), we can get $q_n(q) = \frac{R(q)}{p(q)}$.

For the special care where $T_n = T_o$, Equation (3.8) and (3.9) can be solved to get:

$$\begin{cases} p(q) = \dfrac{\delta}{\delta+1} \beta^{\delta-1} T_o \omega(\xi+v) \dfrac{q^\delta}{k_o^\delta} \\ q_n(q) = (1-\beta)q \\ R(q) = Np(q)q_n(q) = \dfrac{\delta}{\delta+1}(1-\beta)\beta^{\delta-1} NT_o \omega(\xi+v) \dfrac{q^{\delta+1}}{k_o^\delta} \end{cases} \qquad (3.10)$$

where $\beta \in [0,1]$ is the real root of the continuous function:

$$\phi(\beta) = \frac{k_o^\delta}{k_n^\delta}(1-\beta)^\delta - \beta^\delta + \frac{\delta}{\delta+1}\beta^{\delta-1} = 0. \qquad (3.11)$$

This equation has a real root between 0 and 1 because $\phi(\beta)$ is continuous, $\phi(0) > 0$, and $\phi(1) < 0$.

When $T_n < T_o$, $R(q)$ and $p(q)$ do not have simple expressions. In that case, the private company has a competitive advantage, which allows it to charge a toll larger than the toll when $T_n = T_o$ under any given total demand. With very low travel demand, the optimal toll is 0 when $T_n = T_o$; by contrast, when $T_n < T_o$ the optimal toll can be as much as $(\xi+v)(T_o - T_n)$, in which case travel costs on the two highways are equal and all travelers use the toll road. Let us call "toll

premium" this increase in toll caused by the difference between T_o and T_n. We can solve the endogenous toll problem for the new highway project with $T_n < T_o$:

$$p(q) = \frac{\delta}{\delta+1}\beta^{\delta-1}T_o\omega(\xi+v)\frac{q^\delta}{k_o^\delta} + toll\ premium = (\xi+v)\left[\frac{\delta}{\delta+1}\beta^{\delta-1}T_o\omega\frac{q^\delta}{k_o^\delta} + (T_o - T_n)\right] \quad (3.12)$$

Equation (3.12) is valid for both $T_n < T_o$ and $T_n = T_o$, and the traffic on the new highway when $T_n < T_o$ is given by Equation (3.10). Therefore, the annual revenue of the new highway when $T_n \leq T_o$ is given by:

$$R(q) = \beta_1 q^{\delta+1} + \beta_2 q \quad (3.13)$$

where

$$\begin{cases} \beta_1 = N\dfrac{\delta T_o\omega(\xi+v)(1-\beta)\beta^{\delta-1}}{(\delta+1)k_o^\delta}, \\ \beta_2 = N(T_o - T_n)(\xi+v)(1-\beta). \end{cases} \quad (3.14)$$

Next, the expected NPV of the new highway project can be expressed as:

$$\Omega(q) = \int_\Delta^{+\infty}\left(\beta_1 E\left\{q(t)^{\delta+1}\big|q(0)=q\right\} + \beta_2 E\left\{q(t)\big|q(0)=q\right\}\right)e^{-\rho t}dt - \left(\frac{R_A}{\rho} + C_P + n_L C_C + \int_\Delta^{+\infty}me^{-\rho t}dt\right) \quad (3.15)$$

In next section, let us specify how demand evolves over time, and in Section 3.2.5, based on these assumptions, let us calculate the expected NPV of the new highway project as a function of the initial demand q.

3.2.4. Stochastic Demand

A major challenge facing the private company is the uncertainty in travel demand between the two cities. Unlike most previous studies that allow for infinite demand growth (e.g. Chow and Regan, 2011a and 2011b), I assume that travel demand has an upper reflecting barrier to reflect that the capacity of each highway is finite.

To model uncertainty in travel demand between city A and B at time t, which is denoted by $q(t)$, I assume that $q(t)$ follows a reflective geometric Brownian motion, which can be written (Harrison, 1985, pp. 80):

$$\begin{cases} dq(t) = \mu q(t) dt + \sigma q(t) dW(t) - dU(t) \\ q(t) \in (0, \overline{q}] \end{cases} \tag{3.16}$$

where $\overline{q}$ is the upper reflecting barrier; μ is the expected demand growth rates; σ is the demand volatility; $dW(t)$ is a standard Wiener process; and $U(t)$ is a non-decreasing process with $U(0) = 0$. $U(t)$ increases only when $q(t) = \overline{q}$ to keep $q(t)$ below $\overline{q}$. In particular, when $\overline{q}$ goes to infinity, $U(t)$ disappears, and the process $q(t)$ becomes a geometric Brownian motion (GBM).

The GBM is a popular stochastic process for modeling traffic demand uncertainty, partially because it allows for explicit solutions in simple investment models. Assuming that demand follows a GBM ensures that demand is never negative, which is an advantage over the standard Brownian motion. The GBM assumes that demand changes around an exponential trend, which is likely reasonable in developing countries or in fast-growing regions. In that case, an upper barrier is necessary to limit the growth of demand, and I show in Section 3.3.1.1 that the expected growth rate of the RGBM is constant when demand is far from the upper barrier, and it slows down gradually as demand gets close to the upper barrier.

$q(t)$ is the traffic flow (vehicles per hour) used in the BPR function to calculate travel times. For simplicity, I ignore intraday traffic variations, weekday-weekend pattern and traffic seasonality to only focus on annual demand trends.

Another common assumption in previous studies (e.g. Chow and Regan, 2011b; Li *et al.*, 2015) is that travel demand, traffic, and/or population size follow a single stochastic process, which is independent of transportation infrastructure investments. A more realistic setting is to

allow for demand induced by new infrastructure. In this paper, to model induced demand, I first assume that when the new highway is built, the annual growth rate of travel demand increases from μ to α (but the demand volatility is assumed unchanged for simplicity), and second, that the upper barrier on travel demand $\bar{q}$, which equals k_o before investment, rises to k_o+k_n. The new highway gives travel demand more potential to grow especially if the old highway capacity is almost fully utilized.

3.2.5. NPV of the New Highway Project

Under demand uncertainty, the expected NPV of the new highway project is given by Equation (3.15). One challenge is to calculate the expected NPV of the project, denoted by $\Omega(q)$, because to my knowledge, there is no analytical expression for $E\left\{q(t)^{\delta+1}\middle|q(0)\right\}$ when $q(t)$ follows an RGBM.

Dixit and Pindyck (1994, pp. 178-180 and 255) showed how to use contingent claim analysis to calculate $\Omega(q(0))$ without knowing explicitly $E(q(t)|q(0))$. Their model assumes the revenue itself follows an RGBM, the growth rate of revenue must be smaller than the discount rate, and there is no construction duration. I adopt their method to my framework.

First, let us consider the project with no construction duration. When the private company invests at $t = 0$ with initial demand q, the NPV is $\Omega(q)$. When delaying the investment by a small time dt, the expected change in the NPV of the new highway is:

$$E\left(d\Omega\right) = \rho\Omega\left(q\right)dt - R\left(0\right)dt + e^{-\rho(CP+dt)}E\left[R\left(CP+dt\right)\right]dt + mdt - e^{-\rho(CP+dt)}mdt \qquad (3.17)$$

The first term on the right-hand side Equation (3.17) is the capital appreciation that the private company can obtain from delaying the investment, and the second term is the present value of expected revenues right after construction completion from time 0 to dt, which will be

missed if the investment is delayed by *dt*. Instead, the company will collect the future revenue from time *CP* to *CP+dt*, which is discounted to the present (third term in Equation (3.17)). When *CP*, the concession period length, is large enough and revenue is capped due to the upper barrier on demand, the third term is approximately zero, regardless of the values of the growth rate and of the discount rate. The fourth term is the maintenance costs that would not happen from time 0 to *dt* if the investment is delayed by *dt*. Instead, the company needs to pay the maintenance costs from time *CP* to time *CP+dt*, which is discounted to the present (last term in Equation (3.17)). Again, when *CP* is large enough the last term is approximately zero.

Because *q* follows a GBM locally, based on Ito's Lemma, the left side of Equation (3.17) equals:

$$E(d\Omega) = \alpha q \Omega'(q) dt + \frac{1}{2} \sigma^2 q^2 \Omega''(q) dt \tag{3.18}$$

Combining Equations (3.17) and (3.18) give the following Cauchy–Euler equation:

$$\alpha q \Omega'(q) + \frac{1}{2} \sigma^2 q^2 \Omega''(q) - \rho \Omega(q) = -\left[\beta_1 q^{(\delta+1)} + \beta_2 q \right] + m \tag{3.19}$$

When *q* hits the upper barrier $\bar{q} = k_o + k_n$, it bounces down to stay below the barrier. So at the upper barrier, we have the boundary condition:

$$\Omega'(\bar{q}) = 0. \tag{3.20}$$

Solving Equations (3.19) and (3.20) gives the expression of $\Omega(q)$ when the construction duration is 0. To capture construction duration, I then deduct the revenue loss due to construction from $\Omega(q)$. I ignore the upper barrier $\bar{q} = k_o + k_n$ and assume that demand follows a GBM during construction, which leads to overestimating demand during construction and revenue loss.

Therefore, Equation (3.21) is a downward biased approximation of the NPV of the project with construction duration:

$$\Omega(q) = \gamma_1 q^{(\delta+1)} + \gamma_2 q - \gamma_3 + g_1 q^{\theta_1} \tag{3.21}$$

where $g_1 = -\dfrac{(\delta+1)\gamma_4 \bar{q}^{\delta} + \gamma_5}{\theta_1 \bar{q}^{\theta_1-1}}$, $\tag{3.22}$

θ_1 is the positive real root of $\sigma^2\theta^2 + (2\alpha - \sigma^2)\theta - 2\rho = 0$,

$$\gamma_1 = \begin{cases} \beta_1 \dfrac{e^{\left[(\delta+1)\left(\alpha+\frac{\delta\sigma^2}{2}\right)-\rho\right]\Delta}}{\rho-(\delta+1)\left(\alpha+\frac{\delta\sigma^2}{2}\right)} & \text{when } \rho-(\delta+1)\left(\alpha+\dfrac{\delta\sigma^2}{2}\right) \neq 0 \\[20pt] -\beta_1 \dfrac{\ln(q)}{\alpha+\left(\delta+\frac{1}{2}\right)\sigma^2} & \text{when } \rho-(\delta+1)\left(\alpha+\dfrac{\delta\sigma^2}{2}\right) = 0 \end{cases} \tag{3.23}$$

$$\gamma_2 = \begin{cases} \beta_2 \dfrac{e^{(\alpha-\rho)\Delta}}{\rho-\alpha} & \text{when } \rho-\alpha \neq 0 \\[20pt] -\beta_2 \dfrac{\ln(q)}{\alpha+\dfrac{\sigma^2}{2}} & \text{when } \rho-\alpha = 0 \end{cases} \tag{3.24}$$

$$\gamma_3 = \dfrac{me^{-\rho\Delta} + R_A + \rho(C_P + n_L C_C)}{\rho} \tag{3.25}$$

$$\gamma_4 = \begin{cases} \dfrac{\beta_1}{\rho-(\delta+1)\left(\alpha+\dfrac{\delta\sigma^2}{2}\right)} & \text{when } \rho-(\delta+1)\left(\alpha+\dfrac{\delta\sigma^2}{2}\right) \neq 0 \\[20pt] \beta_1 \Delta & \text{when } \rho-(\delta+1)\left(\alpha+\dfrac{\delta\sigma^2}{2}\right) = 0 \end{cases} \tag{3.26}$$

$$\gamma_5 = \begin{cases} \dfrac{\beta_2}{\rho-\alpha} & \text{when } \rho-\alpha \neq 0 \\[6pt] \beta_2 \Delta & \text{when } \rho-\alpha = 0 \end{cases} \tag{3.27}$$

It is noteworthy that when a GBM is unbounded, its growth rate α must be smaller than the discount rate ρ, to prevent the NPV from growing to infinity. However, with an upper barrier that limits the growth of demand, no limit is needed on the value of α. Without loss of generality and in order to keep the derivation for the rest of this paper concise, I assume that

$$\rho - (\delta + 1)\left(\alpha + \frac{\delta\sigma^2}{2}\right) \neq 0 \text{ and that } \rho - \alpha \neq 0 .$$

3.2.6. Real Options Model

In this case, the total travel demand $q(t)$ is stochastic and follows a reflective geometric Brownian motion (RGBM).

This stochastic case can be formulated as a dynamic programming problem. The private company holds an option, which gives it the right (not the obligation) to invest in a new highway, so at every point in time, it faces a binary decision: invest or continue waiting. This optimal stopping problem can be solved using Bellman's optimality principle (Dixit and Pindyck, 1994):

$$F(q) = \max\left\{\Omega(q), \ \frac{1}{1 + \rho dt} E\left[F(q + dq) | q\right]\right\} \tag{3.28}$$

where $F(q)$ is the value of the new highway project. The value of this option only depends on travel demand. $\Omega(q)$ is the termination payoff function, which is the expected NPV achieved by investing immediately. Since no immediate profit is generated by postponing the investment, the second term on the right-hand side of Equation (3.28) is simply the discounted option value in the next period.

It can be shown that there exists an optimal demand threshold, which is denoted by q^*, such that when travel demand is lower than q^*, it is optimal to postpone the investment, and it is optimal to invest immediately when $q(t)$ is higher than q^* (Dixit and Pindyck, 1994). At q^*, the

private company is indifferent between investing and waiting. Therefore, $F(q)$ satisfies the following equations:

$$F(q) = \Omega(q) > 0 \qquad \text{when } q \geq q^* \qquad (3.29)$$

$$F(q) = \frac{1}{1+\rho dt} E\big[F(q+dq)\,|\,q\big] \qquad \text{when } q \leq q^* \qquad (3.30)$$

The expression of $\Omega(q)$ was derived in Section 3.2.5. Let us now derive the expression of $F(q)$ when $q \leq q^*$. By Ito's Lemma (Dixit and Pindyck, 1994), since $q(t)$ follows a GBM locally Equation (3.30) can be expressed as:

$$F(q) = \frac{1}{1+\rho dt} E\big[F(q+dq)\,|\,q\big] = \frac{1}{1+\rho dt}\left\{F(q) + \left[\mu q \frac{\partial F}{\partial q} + \frac{1}{2}\sigma^2 q^2 \frac{\partial^2 F}{\partial q^2}\right]dt\right\} \qquad (3.31)$$

Rearranging Equation (3.31) gives:

$$\rho F(q) = \mu q \frac{\partial F(q)}{\partial q} + \frac{1}{2}\sigma^2 q^2 \frac{\partial^2 F(q)}{\partial q^2} \qquad (3.32)$$

To derive the option value function, I solve the ordinary differential equation (3.32) to find the general solution:

$$F(q) = g_2 q^{\theta_2} \qquad \text{when } q \leq q^* \qquad (3.33)$$

where g_2 is an unknown constant and θ_2 is the positive root of $\sigma^2\theta^2 + (2\mu - \sigma^2)\theta - 2\rho = 0$. Since when q gets close to 0, F(q) should also be close to 0, F(q) does not include a power function with the negative root of $\sigma^2\theta^2 + (2\mu - \sigma^2)\theta - 2\rho = 0$. To find g_2 and q*, I use the following two boundary conditions (Dixit and Pindyck, 1994):

Continuity: $\qquad F(q^*) = \Omega(q^*) \qquad (3.34)$

Smooth-Pasting: $\qquad \dfrac{\partial F(q)}{\partial q}\bigg|_{q^*} = \dfrac{\partial \Omega(q)}{\partial q}\bigg|_{q^*} \qquad (3.35)$

Proposition 1. *Under demand uncertainty assumed above, the travel demand threshold q^* is the real root of:*

$$(\theta_2 - \delta - 1)\gamma_1 q^{\delta} + (\theta_2 - 1)\gamma_2 - \theta_2\gamma_3 q^{-1} + (\theta_2 - \theta_1)g_1 q^{\theta_1 - 1} = 0 \tag{3.36}$$

To derive Equation (3.36), substitute g_1 and combine Equations (3.34) and (3.35). When $q^* < \bar{q}$, the travel demand space is divided into two regions: when current travel demand exceeds q^*, the private company should invest immediately; when current demand is smaller than q^*, waiting is optimal; and when current demand is equal to q^*, the private company is indifferent between investing and waiting.

If $q^* > \bar{q}$, q^* is unreachable. If current demand is below $\bar{q}$, it still follows a GBM locally and waiting is optimal. Once current demand is at the upper barrier, demand does not follow a GBM, and the q^* solution is not valid anymore. Moreover, the demand will decrease from the upper barrier for sure, so the private company must invest immediately or never. Therefore, the investment timing depends on the following condition:

$$\Omega(\bar{q}) > 0 \tag{3.37}$$

The idea behind Equation (3.37) is simple: if the investment generates positive expected profit at $\bar{q}$, the private company should invest immediately. Any demand below $\bar{q}$ belongs to the waiting region. The value of g_1 is determined by $F(\bar{q}) = \Omega(\bar{q})$. The smooth-pasting condition is unnecessary. If Equation (3.37) is not satisfied, the private company should never invest.

Once I get the demand threshold q^*, the coefficient g_2 of Equation (3.33) can be solved by

$$g_2 = \frac{\Omega(q^*)}{(q^*)^{\theta_2}} \tag{3.38}$$

3.2.7. Traditional NPV Method

For reference, it is useful to derive the solution of the deterministic case when there is no travel

demand volatility, and travel demand $q(t)$ grows at a constant rate μ (so $dq(t) = \mu q(t) dt$) until

it reaches the upper barrier $\overline{q} = k_o$. Once demand reaches $\overline{q}$, it stays at $\overline{q}$. After investing, travel

demand can continue to grow at rate α instead of μ. The new upper barrier $k_o + k_n$ is ignored,

since the demand level at the time of investment is under k_o, and then far away from $k_o + k_n$.

Because the change of travel demand is deterministic, I can find the total travel demand at any

given time point t before investment:

$$q(t) = \begin{cases} q(0)e^{\mu t} & 0 \leq t \leq T_{upper} \\ \overline{q} & t > T_{upper} \end{cases} \tag{3.39}$$

where

$$T_{upper} = \frac{\log \overline{q} - \log q(0)}{\mu} \tag{3.40}$$

Because the private company has no motivation to delay the project in order for the

demand to hit the upper barrier after construction completion, the investment time T should

always be less than or equal to $T_{upper} - \Delta$. Therefore, the demand at the time of investment is

$q(0)e^{\mu T}$, and demand after the investment is $q(T+t) = q(0)e^{\mu T + \alpha t}$. The NPV of the project,

which depends on initial travel demand $q(0)$ and T, is as follows:

$$\Omega(q(0),\ T) = \left\{ \int_{\Delta}^{+\infty} \left[R(q(T+t)) - m \right] e^{-\rho t} dt - \left[\frac{R_A}{\rho} + c(y_n) \right] \right\} e^{-\rho T} \tag{3.41}$$

The traditional NPV method, given an initial total travel demand $q(0)$, can be used to find

the optimal investment timing T^*, which maximizes the NPV, by solving first-order conditions.

When $T^* = 0$, the corresponding $q(0)$ is the demand threshold q^*. Once the travel demand reaches

q^*, the private company should invest in the new highway project immediately to obtain the highest NPV. To find q^*, I solve the first-order condition (a polynomial equation of q):

$$\left.\frac{\partial\Omega}{\partial T}\right|_{T=0} = \left(\rho-(\delta+1)\mu\right)\beta_1\frac{e^{((\delta+1)\alpha-\rho)\Delta}-e^{((\delta+1)\alpha-\rho)P}}{\rho-(\delta+1)\alpha}q(0)^{(\delta+1)}+\left(\rho-\mu\right)\gamma_2 q(0)-\rho\gamma_3=0 \quad (3.42)$$

The solution of Equation (3.42) gives the investment threshold under the Jorgensonian rule, which shows that investing becomes optimal when the marginal return of capital equals the user cost of capital. In the next section, I show numerically that when demand volatility is close to 0, the stochastic investment threshold from the real options approach converges to the deterministic threshold I just derived.

3.3. NUMERICAL EXAMPLE

In this section, I illustrate on a case study the framework developed in Section 3.2. Unless stated otherwise, the parameter values used in this case study are the ones listed in Table 3-1. These values are inspired by the California SR-125 toll road project that links Otay Mesa Road and the SR-54 in Spring Valley (California Department of Transportation, 2009; see Figure 3-3).

The SR-125 toll road was a public-private project. It lasted from May 2003 to November 2007 and had a price tag of $658 million (U.S. Department of Transportation, 2014). The costs of this project might have been different if it had been completely private. Moreover, the SR-125 and its alternative are not highways that directly link two cities. Because of these simplifications and of the hypothetical parameter values (e.g., R_A, m, v and ξ) used in this numerical example, results from the analysis in this section should only be seen as an illustration of my proposed framework with somewhat realistic parameter values.

Table 3-1. Definition and base value of key parameters

Name	Description	Numerical example in Section 3.3	Unit
Project-specific parameters			
C_p	Costs of planning, design and environmental approval	58 million	$
C_c	Construction cost of new highway per lane	125 million	$
R_A	Agriculture rent	7 million	$
m	Maintenance and operating costs of new highway	1 million	$
n_L	Number of lanes of the new highway	2	lane
k_L	Capacity per lane	2,000	vph
k_n	Capacity of new highway on each direction	4,000	vph
k_o	Capacity of old highway on each direction	4,000	vph
N	Ratio of traffic per year to vph	7,300	n.a.
T_n, T_o	Free-flow travel time on new and old highway	0.15, 0.25	hours
CP	Concession period length	Infinite	years
Δ	Construction duration	4.5	years
Other parameters			
c_n, c_o	Individual travel cost on new and old highway	n.a.	n.a.
$q_n(t), q_o(t)$	Traffic volume on new and old highway at time t	n.a.	vph
$q(t)$	Travel demand between two cities at time t	n.a.	vph
ω, δ	Parameters of BPR function	0.15, 4	n.a.
ρ	Annual continuously compounding discount rate	7%	n.a.
μ, α	Annual demand growth rate before and after investment	2%, 3%	n.a.
σ	Travel demand volatility	0 - 10%	n.a.
$\bar{q}$	Upper barrier of travel demand	8000	vph
ξ	Value of time	20	$ per hour
v	Variable cost rate	10	$ per hour

n.a. = not applicable.

The SR-125 toll road is 10-mile long, and it has 2 lanes in each direction. The alternative, which consists of portions of the SR-54, I-805, and SR-905, is 15 miles long and it has between 3 and 5 lanes in total. Since the SR-54, I-805 and SR-905 are partially occupied by traffic between OD pairs that are no considered in this model, for simplicity, I, therefore, set the equivalent capacity of the alternative to the SR-125 to 2 lanes in each direction (4 lanes total). I use 2,000 vehicles per hour per lane as the ultimate capacity.
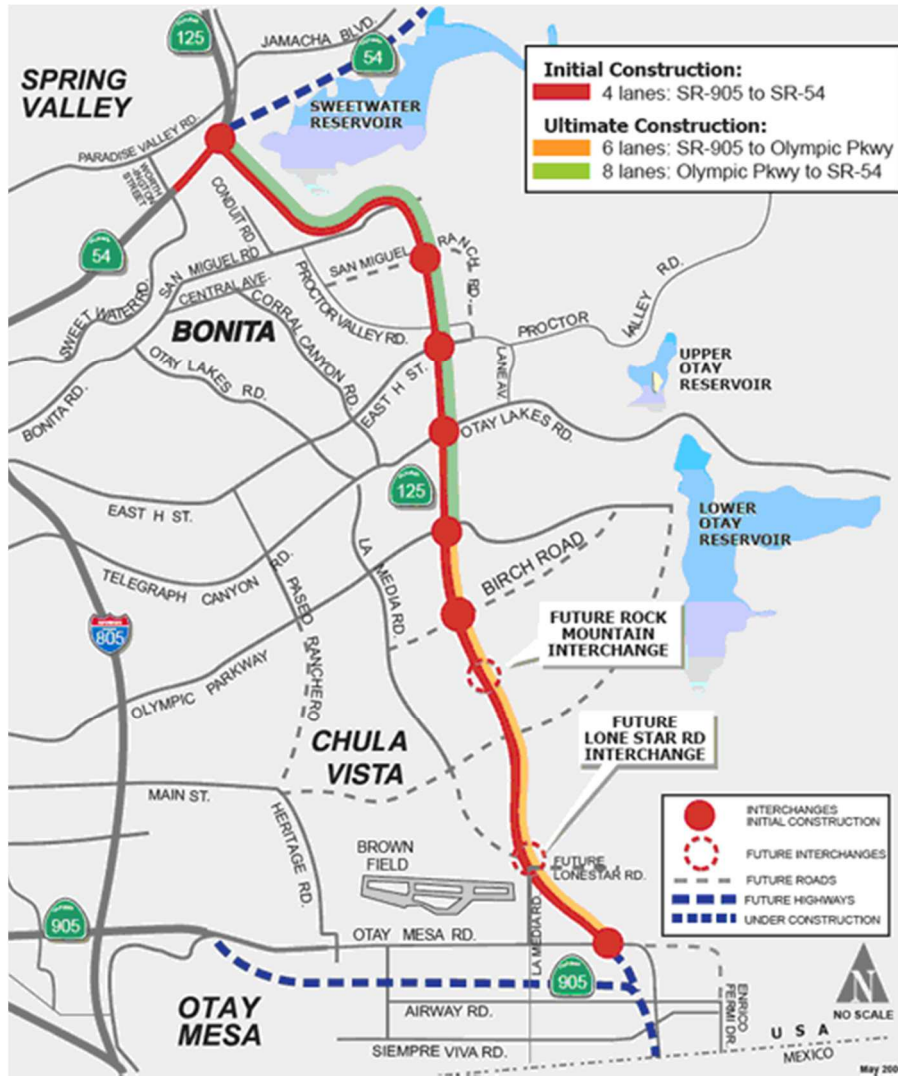
Figure 3-3. California SR-125 (from http://tollroadsnews.com/)

In the rest of this section, I first compare the simulated NPV with the analytical NPV. I then analyze the demand threshold for different parameter values and discuss their impacts on the timing of the optimal investment. For the deterministic case, I solve Equation (3.42) to get the demand threshold, and use Equation (3.36) for the stochastic case. Because there is no closed-form solution to polynomial equations of degree five or higher with arbitrary coefficients, I use the Newton-Raphson algorithm to find the solutions.

3.3.1. NPV of the SR-125

3.3.1.1. Simulated NPV

To simulate the NPV of this project, I first need to find the simulated $E\{q(t)|q(0)\}$ for each time point. Because the RGBM can be seen locally as a GBM with a restriction at $\bar{q}$, simulated $q(t)$ paths are generated with $q_0 = 1000$, $\bar{q} = 4000$, $\mu = \sigma = 3\%$ and $\Delta t = 0.001$ year as follows:

$$\text{Step 1: } q(\Delta t) = \min\left\{\bar{q}, \ q(0)\exp\left(\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma W(\Delta t)\right)\right\};$$

$$\text{Step 2: } q(\Delta t + \Delta t) = \min\left\{\bar{q}, \ q(\Delta t)\exp\left(\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma W(\Delta t)\right)\right\};$$

......

$$\text{Step } n\text{: } q(n\Delta t) = \min\left\{\bar{q}, \ q((n-1)\Delta t)\exp\left(\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma W(\Delta t)\right)\right\};$$

Steps 1 to *n* are repeated 100,000 times to generate 100,000 simulated $q(t)$ paths and then $E\{q(t)|q(0)\}$ is calculated from: $E\{q(t)|q(0)\} = \dfrac{1}{100000}\sum_{i=1}^{100000} q_i(t)$.

The expectation of $q(t)$ is shown in Figure 3-4 for 100,000 simulation paths. Because the initial demand is far from the upper barrier, it takes about 200 years for the expectation of RGBM to be stationary. Once the demand is higher than 3,000 vph, the gap between the GBM and the RGBM is substantial. Ignoring the upper barrier leads to significantly overestimating the expected travel demand and the NPV of the new highway project, especially when demand is high and the old highway is already congested.
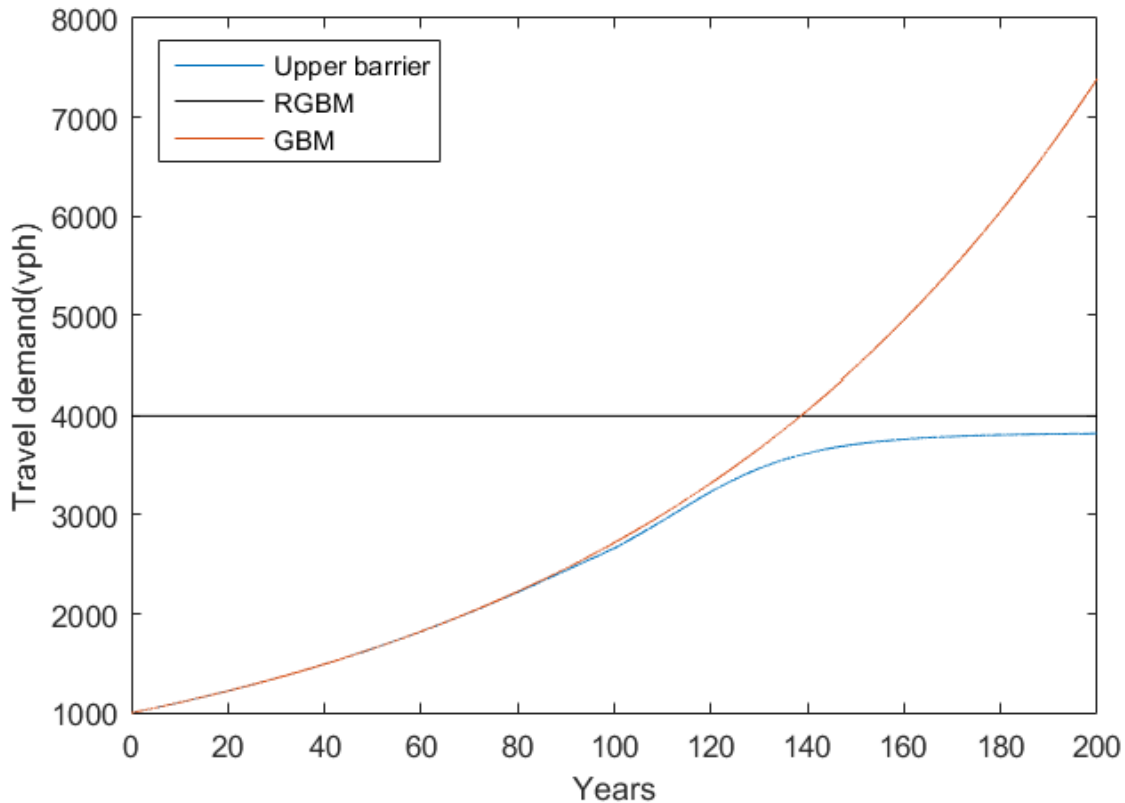
Figure 3-4. Simulated expectation of the RGBM and of the GBM

It is noteworthy that the sample size and step size Δt in the RGBM simulation are critical because theoretically the RGBM follows a GBM only when Δt→0 and a large sample size is needed to generate reliable values of the expectation of the RGBM. A large sample size is easy to satisfy. However, a finite step size, no matter how small it is, will lead to overestimating the expectation value at each time point, since the demand is less likely to be reflected back. Since the analytical NPV tends to underestimate the NPV when taking construction duration into account, the NPV based on simulated expectations of the RGBM and the analytical NPV form upper bound and lower bound of the true project NPV with a non-zero construction duration.

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3.3.1.2. Comparison of simulated NPV and analytical NPV

Figure 3-5 compares both NPVs for the SR-125 project with the parameters listed in Table 3-1 and various values of σ. The initial demand is set at 4,000 vph, which is the highest possible demand between two cities before the new highway construction. Table 3-2 shows the values of the NPVs calculated analytically and by simulation. When the annual demand volatility is under 10%, the gap between two NPVs is less than 1%. Because the true NPV is between two bounds, the error of the analytical NPV is within 1% when σ is under 10%.

The construction duration of the SR-125 is assumed to be 4.5 years, and the highest acceptable demand volatility level is 10% to keep the analytical NPV accurate. Like the results shown in Figure 3-5 and Table 3-2, analytical and simulated NPVs are calculated with various construction durations. I list the highest acceptable demand volatility level for each construction duration: 16% (0.5 year), 15% (1 year), 11% (2.5 years) and 9% (6.5 years). These volatility levels are important when discussing the demand threshold in Section 3.3.2.

Table 3-2. Analytical NPV and simulated NPV with different demand volatilities.

Demand volatility (%)	Analytical NPV ($100 million)	Simulated NPV ($100 million)	Difference between NPVs ($100 million)	Difference (% of analytical NPV)
1%	18	18	0	0.0%
2%	17.6	17.6	0	0.0%
3%	16.9	16.9	0	0.0%
4%	16.1	16.1	0	0.0%
5%	15.2	15.2	0	0.0%
6%	14.1	14.1	0	0.0%
7%	13.1	13.1	0	0.0%
8%	12.1	12.1	0	0.0%
9%	11.1	11.1	0	0.0%
10%	10.1	10.2	0.10	1.0%
11%	9.18	9.28	0.10	1.1%
12%	8.3	8.46	0.16	1.9%
13%	7.45	7.65	0.20	2.7%
14%	6.63	6.83	0.20	3.0%
15%	5.85	6.13	0.28	4.8%
16%	5.09	5.4	0.31	6.1%
17%	4.35	4.74	0.39	9.0%
18%	3.61	4.12	0.51	14.1%
19%	2.87	3.63	0.76	26.5%
20%	2.13	3.02	0.89	41.8%
21%	1.35	2.51	1.16	85.9%
22%	0.55	2.07	1.52	276.4%
23%	-0.32	1.58	1.90	n.a.
24%	-1.27	1.19	2.46	n.a.
25%	-2.32	0.77	3.09	n.a.

n.a. = not applicable.
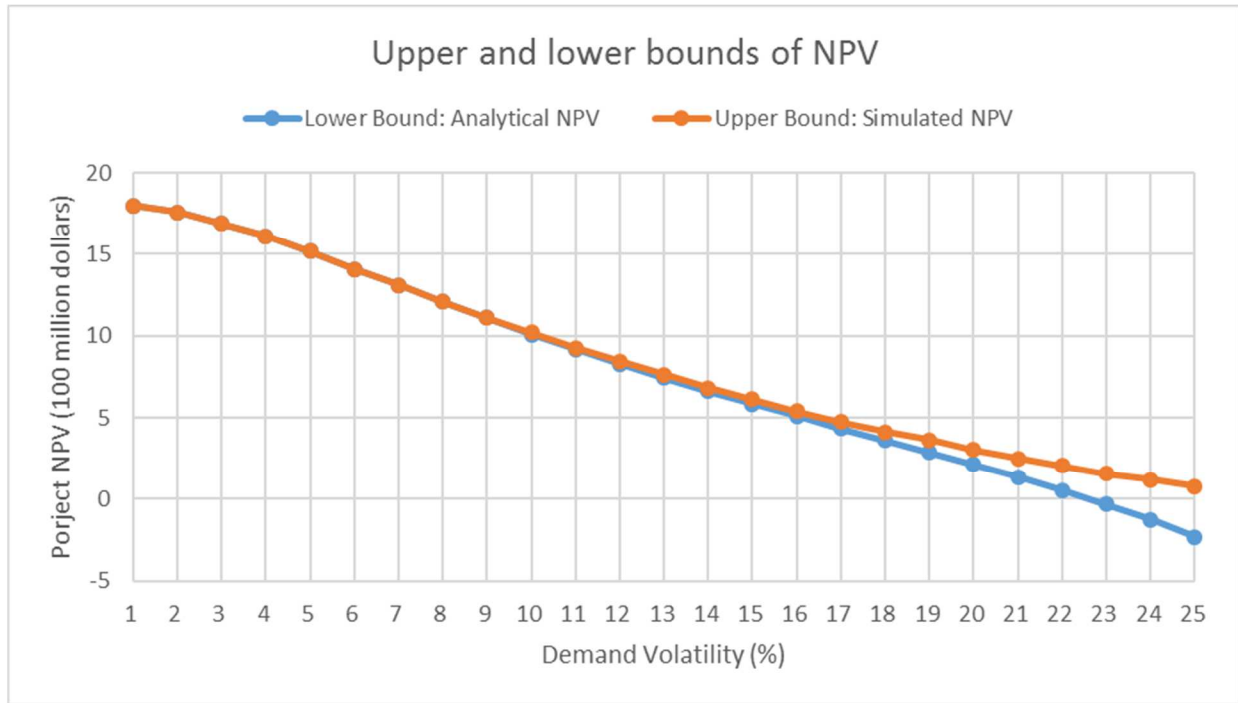
(Note: q_0 = 4000 vph, the value of all the other parameters used are listed in Table 3-1)

Figure 3-5. Analytical and simulated NPV

3.3.1.3. Impact of reflecting barrier on NPV

Figure 3-6 Panel A shows the impact of the upper barrier on the NPV of the SR-125 toll road with α = 1%, σ = 1%, ρ = 7% and an upper barrier of 8,000 vph. Compared to the NPV without upper demand barrier, the NPV with an upper barrier is lower because the upper barrier limits the potential demand increase and the corresponding toll revenues. The closer the initial demand is to the barrier, the more the NPV without the barrier overestimates the value of the project. For example, on Figure 3-6 Panel A, when the initial demand is 2,500 vph or lower, the difference between the NPVs with and without a barrier is negligible. When the demand reaches 4,000 vph, the NPV of the project is $257 million without barrier, but only $97 million (62% lower) with a reflective upper barrier at 8000 vph. It shows that the NPV can be significantly overestimated

when ignoring the upper demand barrier. It is noteworthy that when $\rho - (\delta + 1)\left(\alpha + \dfrac{\delta\sigma^2}{2}\right) < 0$,

the NPV without the upper barrier is infinite, while the NPV with barrier is still calculable as

shown in Panel B of Figure 3-6.

3.3.1.4. Impact of demand volatility and demand growth rate on NPV

The impact of demand volatility on the NPV with an upper barrier on demand is not

straightforward because demand uncertainty has two opposite impacts on NPV. First, when

ignoring the barrier, the NPV is a convex function of demand. Higher demand volatility leads to

a higher expectation of the NPV while keeping demand expectation constant. Based on Equation

(3.23), r_1, the coefficient of the first term of the expected NPV, is monotonically increasing in σ.

Therefore, when ignoring the last term of Equation (3.21), a higher demand volatility actually

raises the value of the project. Second, considering the impact of the upper barrier, a higher

demand volatility increases the probability that the demand will fall instead of staying at the

barrier. Based on Equations (3.22) and (3.26), the last term of the expected NPV is

monotonically decreasing in σ. The SR-125 project has an upper reflecting barrier at 8000 vph.

Figure 3-7 indicates that when initial demand is less than 4,000 vph, the first impact overcomes

the second and the NPV increases with higher demand volatility.
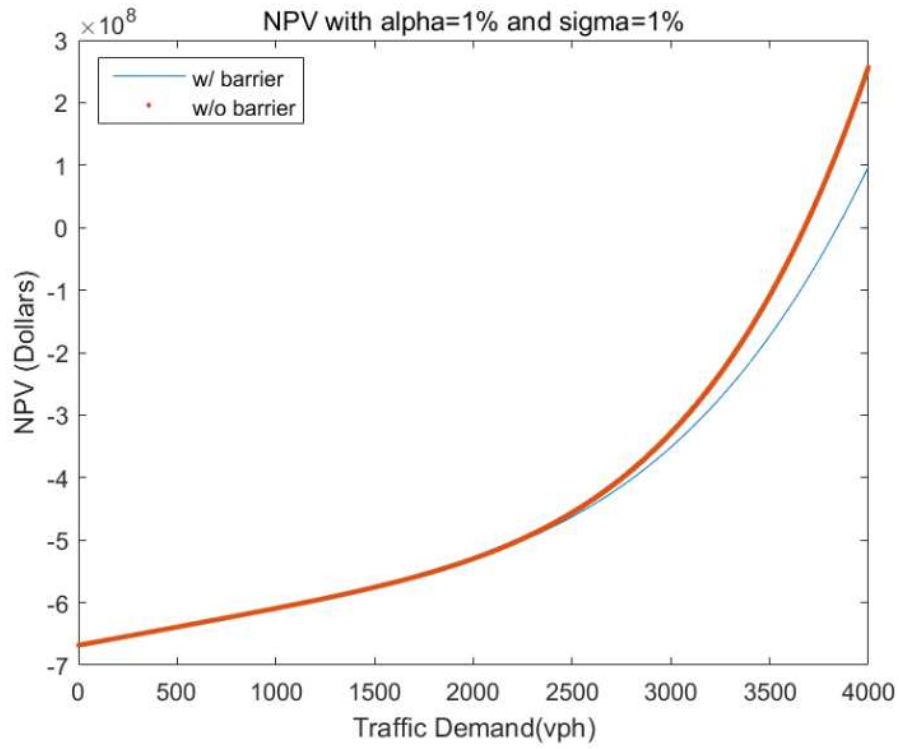
Panel A
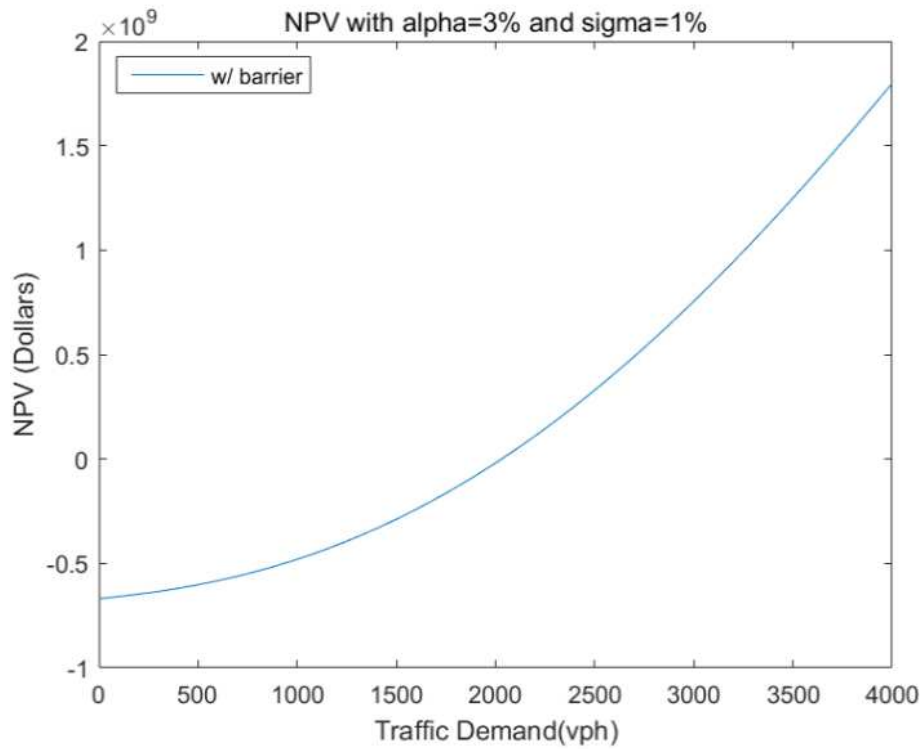
Panel B

Figure 3-6. The effect of barrier on the Net Present Value (NPV) of project
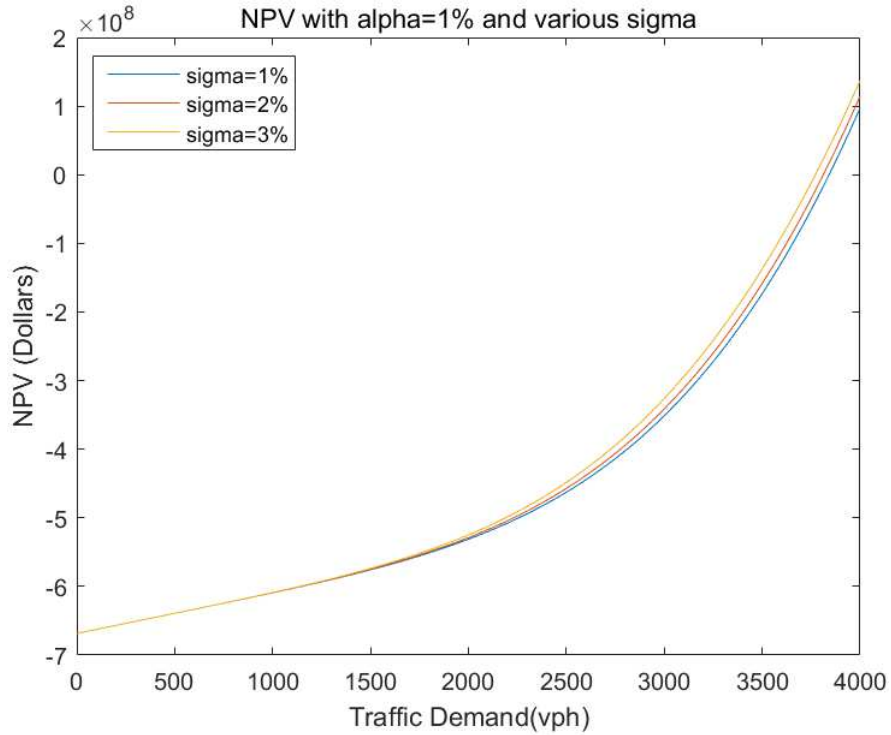
Figure 3-7. The effect of demand uncertainty on NPV with upper barrier
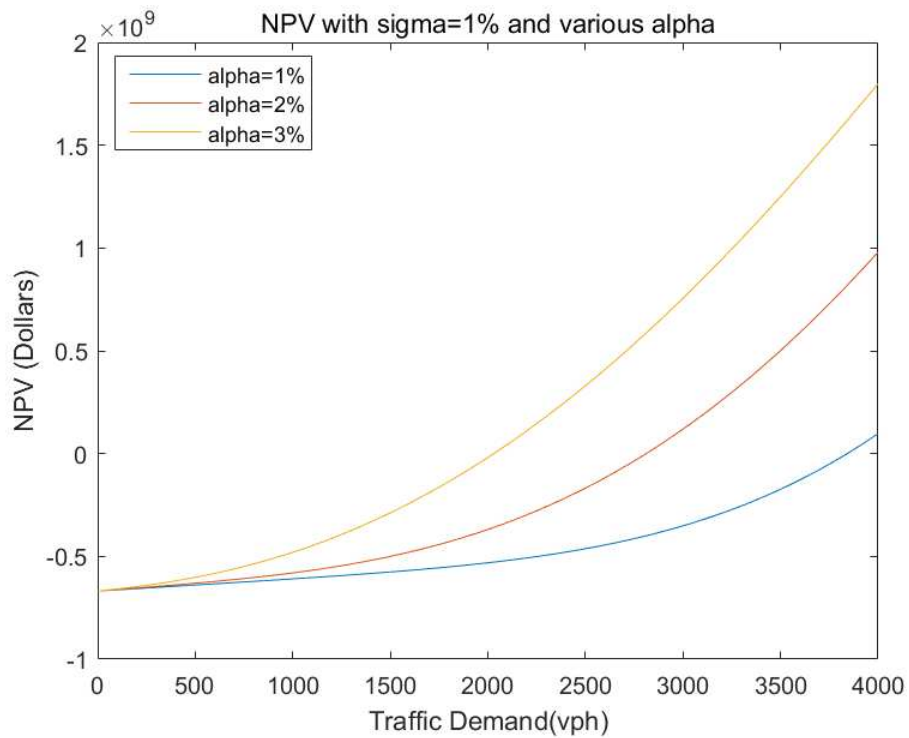
Figure 3-8. The effect of demand growth rate on NPV with upper barrier

Figure 3-8 shows that a higher demand growth rate leads to a greater NPV with an upper barrier. When initial demand is 4,000 vph, the NPV is $97 million with 1% growth rate, while the NPV is $1.8 billion with a 3% growth rate.

3.3.2. Demand Threshold

3.3.2.1. Impact of demand volatility and construction duration on demand threshold

Taking into account the upper reflecting barrier, let us examine the demand threshold for the SR-125 toll road using the methodology discussed in Section 3.2. Figure 3-9 shows the effects of demand volatility σ and construction duration Δ on demand threshold. When $\sigma=0$, the demand threshold is obtained from Equation (3.42). When $\sigma>0$, the demand threshold is solved from the stochastic results. As expected, the demand threshold obtained via real options converges to the traditional NPV result when demand volatility shrinks to 0.
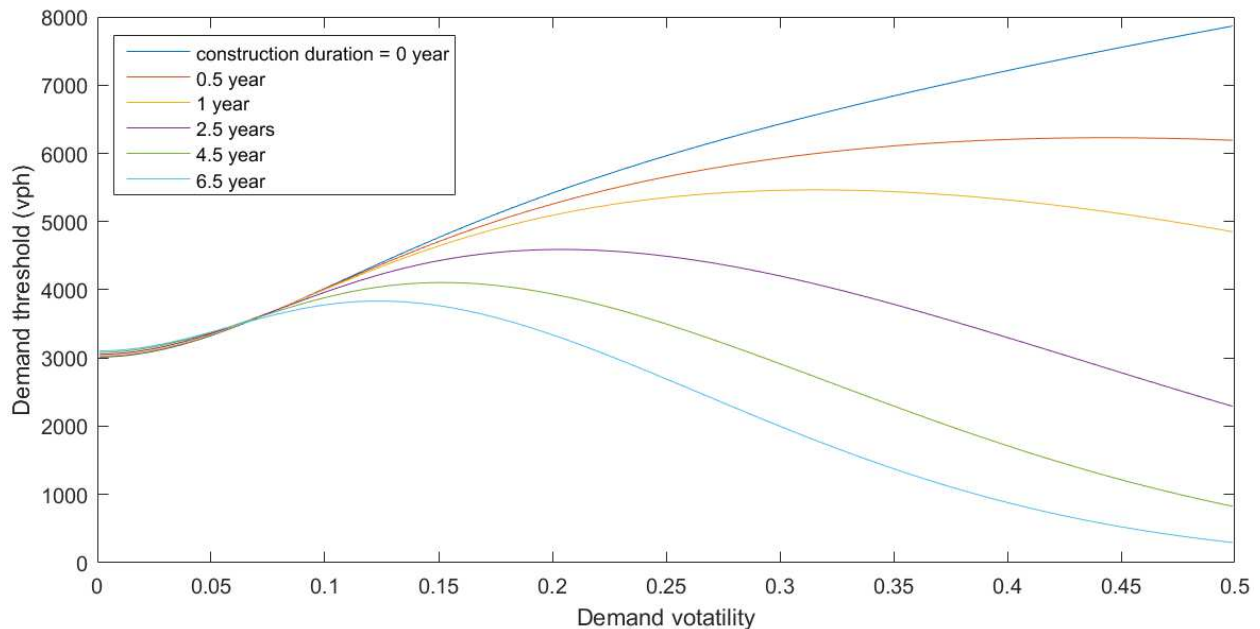
Figure 3-9. Demand thresholds of new highway with different demand volatilities

My results are similar to those of Saphores and Boarnet's study (2006), but with a caveat. When ignoring construction duration, it is optimal to invest later in congestion-relief infrastructure as uncertainty increases. By contrast, for projects that require a particularly long time-to-build, the demand threshold could rise first and then fall as uncertainty increases, but this may happen outside of realistic parameter values. Indeed, Figure 3-10 shows the demand threshold with the acceptable demand volatility levels found in Section 3.3.1.2, the demand threshold is still a monotonically increasing function of demand volatility.
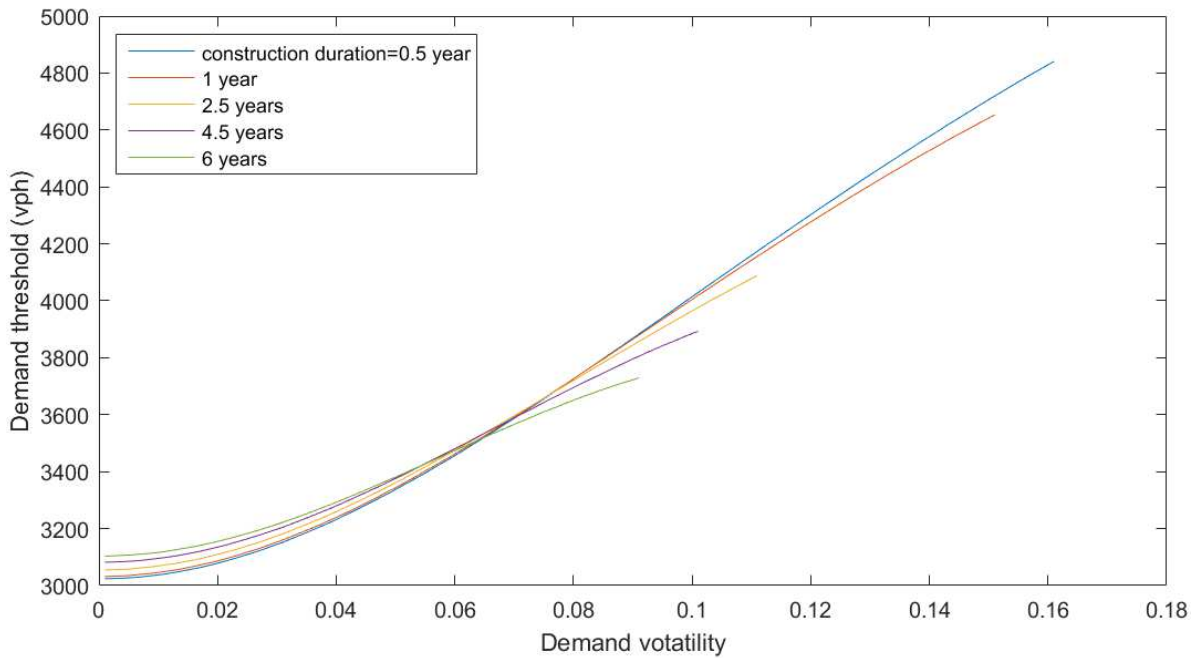
Figure 3-10. Demand thresholds within acceptable demand volatilities
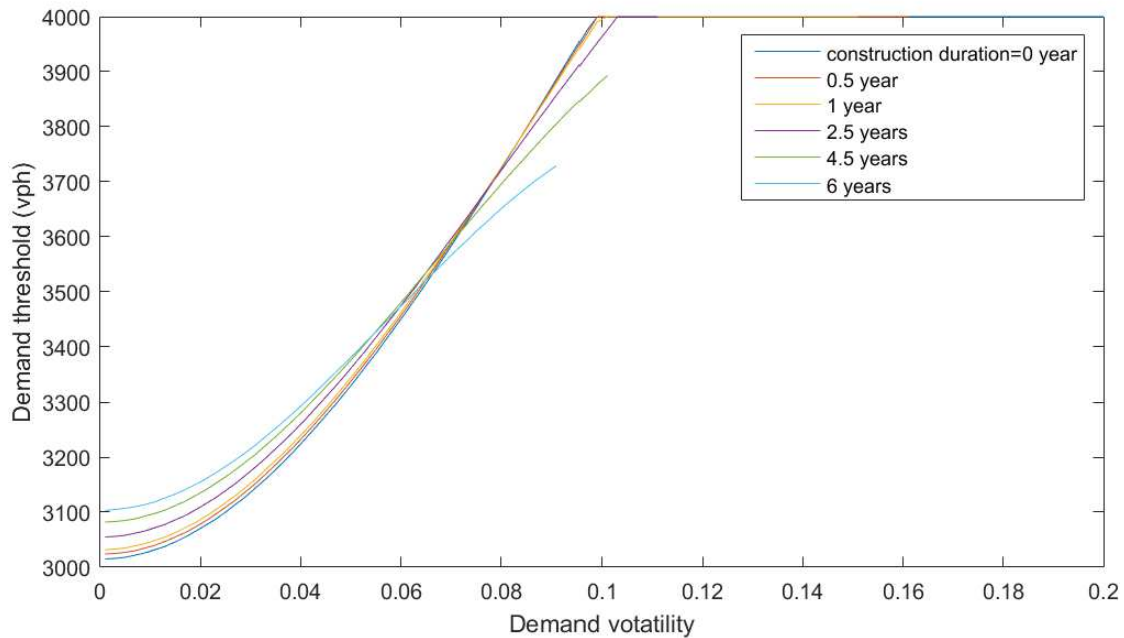
Figure 3-11. Demand thresholds within acceptable demand volatilities truncated at 4,000 vph

In this case, the demand threshold should not exceed the old highway's capacity of 4,000 vph, because when the investment decision for the new highway needs to be made, the new highway is not available to travelers. Therefore, the demand thresholds plotted in Figure 3-11 are truncated at 4,000 vph. With demand uncertainty increasing, the demand threshold stays at 4,000 vph as long as $\Omega\big(q(0)=4000\big)>0$.

3.3.2.2. Impact of other parameters on the demand threshold

All parameters tested in Figure 3-12 show that optimistic predictions of the project's profit lead to investing earlier. By contrast, any negative event, like excessive upfront costs or lower than expected demand, makes it optimal to delay the investment. The decrease in the traveler's value
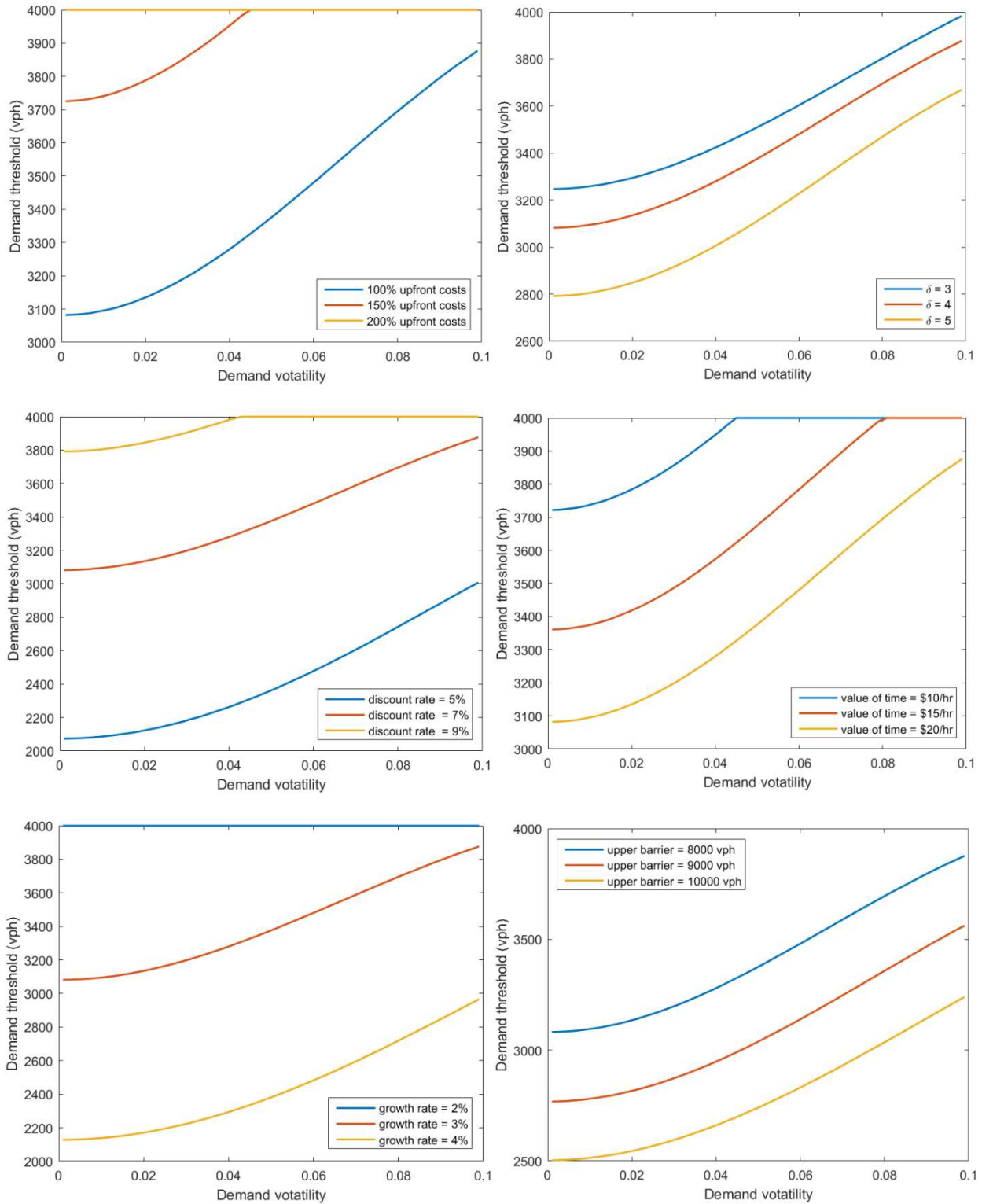
Figure 3-12. Impact of other parameters on demand threshold
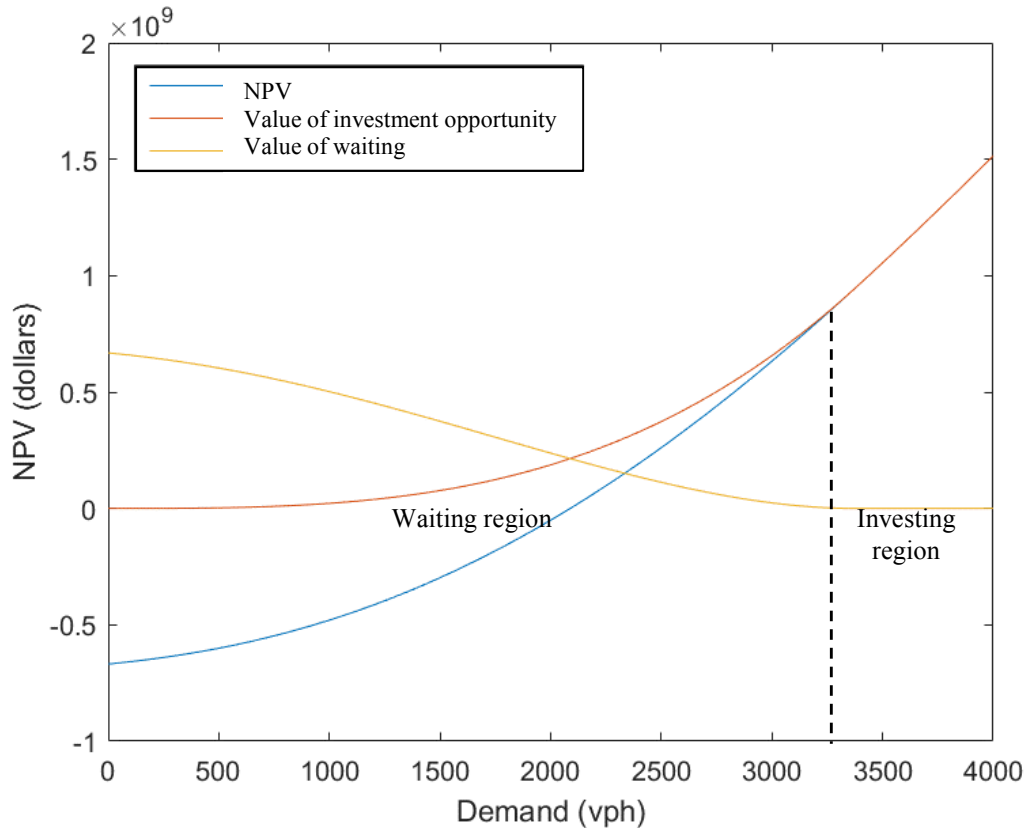
Figure 3-13. Values of NPV, investment opportunity and option to defer

of time results in low willingness to pay a toll and then entails revenue loss. Having a high discount rate discounts future revenue from the new highway and makes it more difficult to recoup upfront costs. Both lead the private company to be more conservative.

Figure 3-13 illustrates the value of the investment opportunity, the NPV and the value of the option to defer. I set the demand uncertainty level to 5% (the value of other parameters is as listed in Table 3-1). The demand threshold of 3,376 vph divides the demand space into two regions: waiting and investing. Under the demand threshold, the investment opportunity is greater than the NPV of investing immediately, so there is a positive value to defer the investment. Conversely, a higher demand justifies investing immediately and reduces the value

to the deferment option. When the demand increases beyond the threshold, the investment opportunity is equal to the NPV, the private company should invest immediately, and the value of the deferment option goes to zero.

3.4. CAPACITY CHOICE

3.4.1 Alternative Capacity

In Section 3.2 and 3.3, I consider a highway project with a predetermined capacity (2 lanes in each direction, k_n=4,000 vph). In this section, I show how to decide not only when to invest, but also what capacity to choose given an alternative capacity choice with 3 lanes in each direction and k_n=6,000 vph. With the flexibility to re-design the new highway's capacity, the decision-maker can build a 4-lane highway (2 lanes each way) when demand is low to reduce construction spending, and add 2 lanes (one in each direction) when demand is high enough to increase profits.

For the new capacity choice (6 lanes, 3 in each direction), the construction costs are assumed to increase by 50% compared to the 4-lane project. The upper barrier on demand is set at 12,000 vph, which is the total capacity of the old and new highways. I assume that the demand uncertainty level is σ = 5% for the rest of this paper. Figure 3-14 shows that the NPV of the projects is monotonically increasing with the initial travel demand irrespective of capacity. Since the 6-lane project has higher construction costs, it is not favored when demand is low, so its NPV is lower than the NPV of the 4-lane project at first. The 6-lane project becomes more attractive when demand increases, so when demand is sufficiently high, its NPV surpasses the NPV of the 4-lane project. Therefore, there exists an indifference travel demand $\tilde{q}$, at which the private company is indifferent between building 4 lanes or 6 lanes. In particular, considering two capacity choices, k_n and k_n^a ($k_n < k_n^a$), I can say $\Omega(q, k_n) \leq \Omega(q, k_n^a)$ if and only if $q \geq \tilde{q}$.
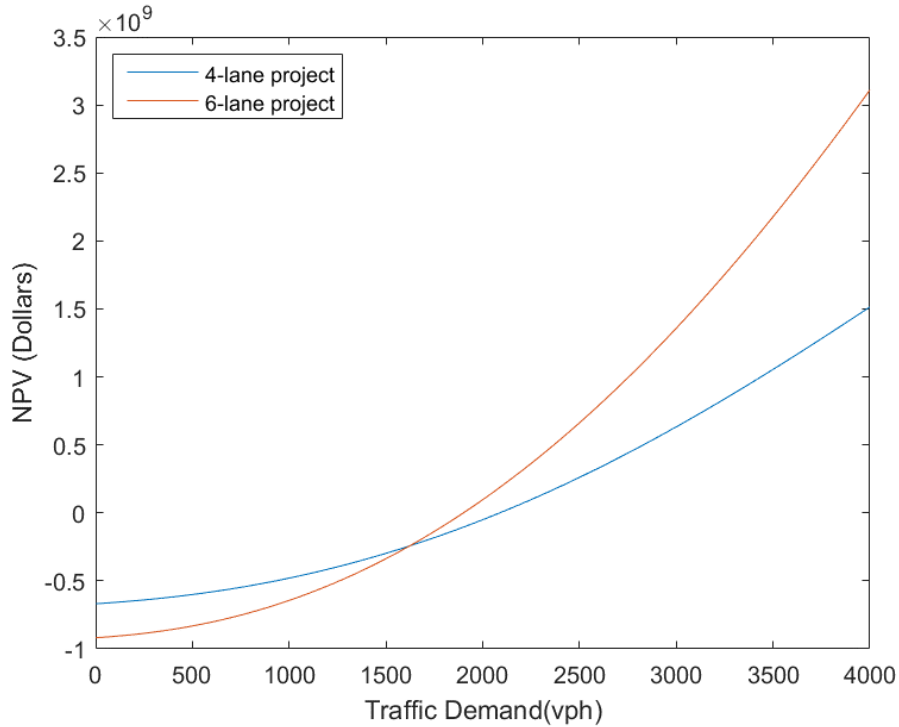
Figure 3-14. NPVs of two capacity choices

Some papers call the indifferent demand a "trigger threshold" (Li *et al.*, 2015) at which point the optimal capacity (or technology) choice shifts from one project to another. Ideally, when the demand for travel is smaller than the threshold, the smaller road capacity project is preferable because it is cheaper; conversely, when demand is higher than the threshold, the project with the larger road capacity is more profitable. However, Section 3.4.2 shows that the answer can be more complicated.

First, I solve the 6-lane project problem as a fixed capacity investment timing problem, similar to what I do in Section 3.3. Then I plot the 6-lane project's NPV $\Omega^a(q)$, the value of the investment opportunity $F^a(q)$ and the demand threshold q^{a*} along with $\Omega(q)$, $F(q)$ and q^* found in Section 3.3. The comparison is shown in Figure 3-15. Since $F^a(q) > F(q)$ over the

demand space, the 6-lane project is always a better choice regardless of demand level. However, there are circumstances when one capacity option does not dominate the other. I list all three possible cases of capacity choice problems and discuss how to solve them in Section 3.4.2.
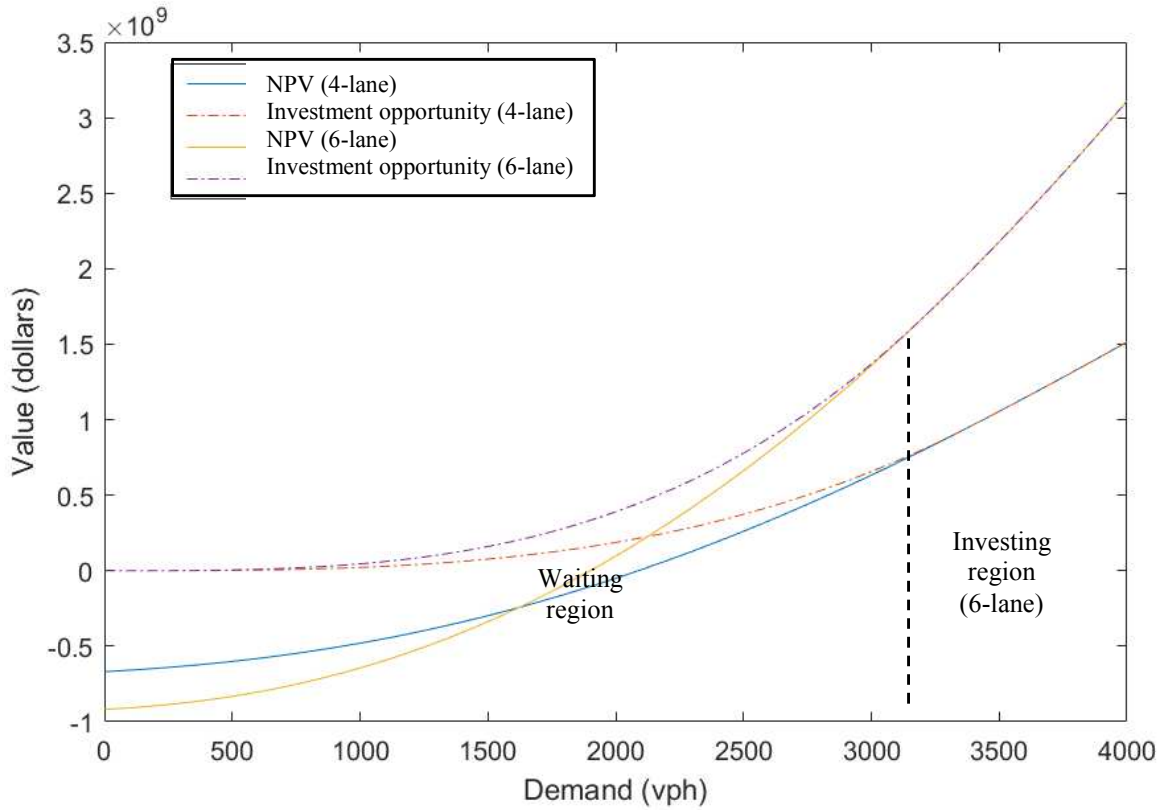
Figure 3-15. Value of investment opportunities with different capacity choices

3.4.2. Capacity Choice Cases

Solving the "value-matching" and "smooth-pasting" conditions for projects with fixed capacity k_n and k_n^a separately, I am able to get the value of four variables: g_2, g_2^a, q^* and q^{a*}. g_2 and g_2^a are coefficients in $F(q)$ and $F^a(q)$, respectively. q^* and q^{a*} are the optimal demand thresholds of the 4-lane project and 6-lane project, respectively. Based on their values and existence of $\tilde{q}$, I

can summarize the 3 cases that may arise when comparing two capacity choices k_n and k_n^a (assuming $k_n < k_n^a$):

(1)   There is no indifferent demand $\tilde{q}$ between 0 and the upper barrier $\overline{q}$.

Solution: the capacity that gives a higher NPV dominates the other, and the problem simplifies to a single capacity problem.

(2)   First, $\tilde{q}$ exists. Second, $g_2^a > g_2$. Proposition 2 proves that the high-capacity demand threshold is greater than or equal to the indifferent demand, $q^{a*} \geq \tilde{q}$. So when demand falls in $\left(0, \ q^{a*} \right]$, the high-capacity project's value of waiting $F^a(q)$ is higher than the low-capacity project's value of waiting and investing. When demand falls in $\left[q^{a*}, \overline{q} \right]$, the high-capacity project's value of investing is higher than the low-capacity project's value of waiting and investing.

Solution: the high-capacity project dominates the low-capacity project. Then the problem can be simplified to a single capacity problem.

(3)   First, $\tilde{q}$ exists. Second, $g_2 > g_2^a$. Proposition 2 proves that the low-capacity demand threshold is smaller than or equal to the indifferent demand, $q^* \leq \tilde{q}$. So when demand falls in $\left(0, \ q^* \right]$, the low-capacity project's value of waiting is greater than the high-capacity project's value of waiting and investing.

Solution: one waiting region is $\left(0, \ q^* \right]$, other regions will be discussed later.

Proposition 2. *In case (2), when $\tilde{q}$ exists and $g_2^a > g_2$, then the high-capacity demand threshold is greater than or equal to the indifferent demand, $q^{a*} \geq \tilde{q}$. In case (3), when $\tilde{q}$ exists and*

$g_2 > g_2^a$, *then low-capacity demand threshold is smaller than or equal to the indifferent demand,*

$q^* \leq \tilde{q}$.

Proof:

First, I assume that $\tilde{q}$ *exists*, $g_2^a > g_2$ and $q^{a*} < \tilde{q}$. An example is shown in Figure 3-16 Panel A.

If $q^{a*} < \tilde{q}$, then $F^a(q^{a*}) = \Omega^a(q^{a*}) < \Omega(q^{a*})$. However, if $g_2^a > g_2$, $F^a(q) > F(q) \geq \Omega(q)$ for

any given q. This is a contradiction. Therefore, if $\tilde{q}$ exists and $g_2 < g_2^a$, then q^{a*} must be greater

than or equal to $\tilde{q}$.

Similarly, I can prove that if $\tilde{q}$ exists and $g_2 > g_2^a$, then q^* must be smaller than or equal

to $\tilde{q}$. The case shown in Figure 3-16 Panel B cannot happen.
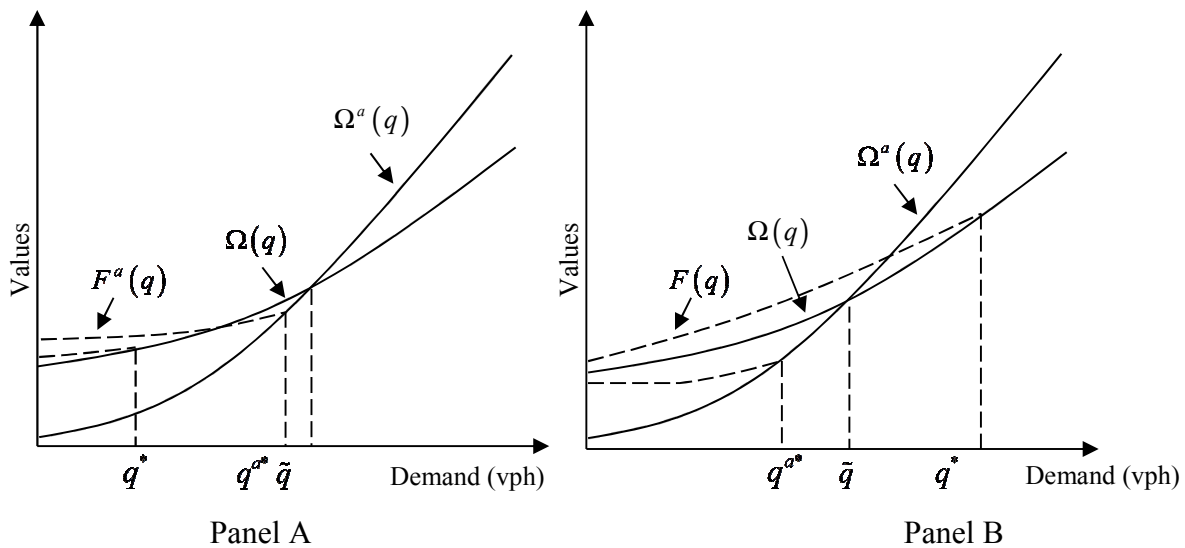
Figure 3-16. Examples of impossible cases

In the first and second cases of the capacity choice problem, one capacity can be easily

rejected, and the other one is the optimal capacity. However, in the third case, no capacity

dominates the other over the demand space. To create a scenario that satisfies the third case, I change some parameter values of the 6-lane project: first, the upper barrier of demand is set to 8,650 vph (instead of 12,000 vph) due to the limit of population size and economy activities; second, the planning cost, maintenance cost, and land cost are doubled. The construction duration is extended to 5.5 years. In that case, the solution is $\tilde{q}$ =3,734 vph, g_1^a=0.0113 and g_1 =0.0114. Since g_1^a is slightly smaller than g_1, we are in the third case. The demand threshold for the 4-lane project is q^* = 3,376 vph, which is smaller than the indifference demand $\tilde{q}$= 3,734 vph.

Dixit (1993) described a trigger strategy where the whole $[q^*, \overline{q}]$ region belongs to the investment region. However, Décamps et al. (2006) proved that $\tilde{q}$ never belongs to the investing region. Because $\dfrac{\partial \Omega(q)}{\partial q}$ is smaller than $\dfrac{\partial \Omega^a(q)}{\partial q}$ at $\tilde{q}$, according to smooth-pasting condition, by waiting for a small interval of time dt, the expected gain dominates the expected cost. Therefore, there is another waiting region $\left[q_1^*, q_2^* \right]$ around the indifference demand ($q^* < q_1^* < \tilde{q} < q_2^*$) (see Figure 3-17).

The method used to calculate q^* is discussed in Section 3.2.6. Similarly, q_1^* and q_2^* can be characterized by the following 4 boundary conditions (2 value-matching conditions and 2 smooth-pasting conditions):

$$F_w\left(q_1^*\right) = \Omega\left(q_1^*\right) \tag{3.43}$$

$$\frac{\partial F_w(q)}{\partial q}\bigg|_{q_1^*} = \frac{\partial \Omega(q)}{\partial q}\bigg|_{q_1^*} \tag{3.44}$$

$$F_w\left(q_2^*\right) = \Omega^a\left(q_2^*\right) \tag{3.45}$$

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$$\frac{\partial F_w(q)}{\partial q}\bigg|_{q_2^*} = \frac{\partial \Omega^a(q)}{\partial q}\bigg|_{q_2^*} \tag{3.46}$$

where $F_w(q)$ is the value of investment opportunity by waiting around the indifference demand.

The form of $F_w(q)$ is $F_w(q) = g_3 q^{\theta_3} + g_4 q^{\theta_4}$. θ_3 and θ_4 are the positive and negative root of

$\sigma^2\theta^2 + (2\mu - \sigma^2)\theta - 2\rho = 0$, respectively. g_3 and g_4 are the coefficients to be determined.

There are 4 equations and 4 unknown variables: g_3, g_4, q_1^* and q_2^*.
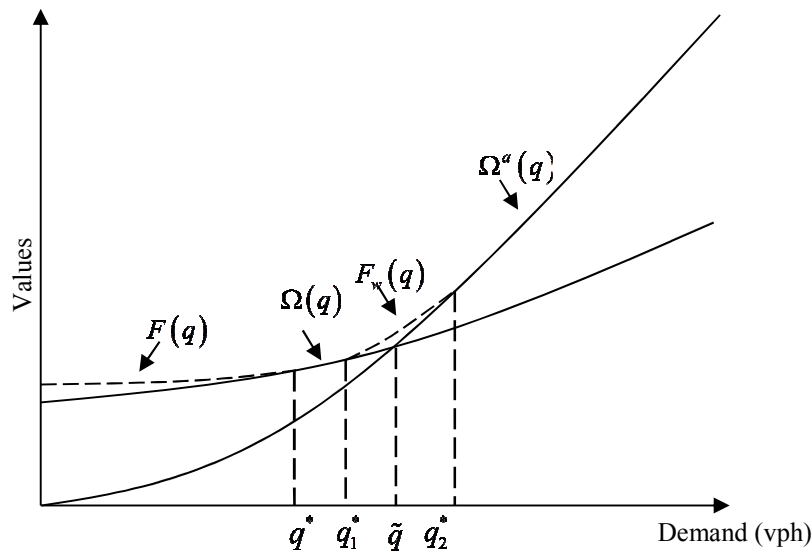
Figure 3-17. The third case of capacity choice problem

The values of q_1^* and q_2^* are 3,570 vph and 3,875 vph, respectively. So for the third case,

there are two waiting regions and two investing regions. When demand is between 3,570 vph and

3,875 vph, the deferment option can add more value than investing immediately with either

capacity choice. At the indifference demand, the value of investing is $1.27 billion, while the

value of delaying is $1.30 billion. The value of the deferment option itself is $30 million, which

increases the NPV by 2.3%. It might be insignificant to consider the relatively small waiting

region around the indifferent demand. However, for important transportation projects that have substantial impacts on revenue social welfare and for which shifting between capacity choices is costly, for example, bus transit and light rail transit, when current travel demand is close to the indifference point, the investor should wait until demand increases or decreases further into one investing region and then invest with the corresponding capacity choice.

3.5. DISCUSSION

This chapter develops an analytical framework under demand uncertainty to find the optimal timing to invest in a road project between two cities, and to estimate the endogenous toll to finance this project. Having a BPR function to model congestion makes it harder to derive a closed form solution for traffic on the new highway under user equilibrium. However, I successfully define a profit-maximizing and endogenous toll, with which the traffic on the new highway is proportional to the travel demand between two cities, and the revenue collected each time point is a power function of demand. Therefore, whether the private sector has toll authority to adjust toll rate makes a huge difference to the project's earning power. When the power of the BPR function is 4, the revenue at each time grows about 4 times faster than demand does, while with fixed toll the revenue growth rate is the same as the demand growth rate.

Demand uncertainty and the demand barrier play an important role in predicting project revenue and in determining the timing of the investment timing. With an upper barrier, the demand is limited to the sum of the capacities of both highways, and the value of the project is substantially reduced when compared to the case where demand is allowed to grow at a constant rate forever. The impact of demand uncertainty on revenue could be positive or negative. When

demand is far away from the upper barrier, ignoring uncertainty will lead to underestimating revenue, while the opposite holds when demand is close to the upper barrier.

When the project can be implemented instantly, I derive the exact solution of the Net Present Value (NPV) of the new highway project. Under increasing demand uncertainty, the demand threshold is also increasing until demand reaches the capacity of the old highway and cannot grow anymore. The option to defer adds significant value to the project and traditional NPV calculations that ignore demand volatility lead to investing prematurely. However, when construction takes time (a more realistic case), the approximate closed-form solution of the NPV is downward biased. Combined with simulation results, I find that within a reasonable range for demand uncertainty, the closed-form solution can provide an accurate estimation of true NPV and demand threshold increases with uncertainty.

By formulating the capacity choice of the new highway in terms of a number of lanes, the optimal capacity can be found by comparing every pair of capacity choices. First, the investment timing problem is solved with each capacity choice. Then, based on their NPVs and option values, one capacity choice could dominate the other, or each capacity choice could have its own demand threshold and investment region. It is noteworthy that the indifference in demand level, at which both capacity choices have the same NPV, belongs to the waiting region. My example shows that the extra value added by delaying the investment at the indifference demand level is trivial. But the waiting region between two investing regions could be more important for transit and rail technology selection, because two different technologies, such as BRT and light rail, have more substantial differences in passenger capacity and costs.

CHAPTER 4 PUBLIC ALTERNATIVES TO COMPLETE AN INTERCITY HIGHWAY UNDER DEMAND UNCERTAINTY

4.1. INTRODUCTION

In Chapter 3, I proposed a real options model to solve a series of optimal decisions for a private-funded highway project. This chapter considers the point of view to the government, which aims to build an inter-city highway to relieve growing congestion on an existing highway. The government is facing two options: one is to finance the project using public funds; the other option is to enter into a Public-Private Partnership (PPP). For the first alternative, I derive the government's optimal toll rate that minimizes the total travel time. "Rule of a Half" is used to measure the consumer surplus with induced demand. By Monte Carlo simulation, the optimal demand threshold and the value of the public-funded inter-city highway are calculated. For the second alternative, the government initiates a Build-Operate-Transfer (BOT) contract with the private sector. With toll regulation and investment timing determined by the government, the private sector's successful participation depends on the profitability of the new highway project, which would be undermined by the government's potential expansion of the existing highway in future. Using real options method, I evaluate the demand threshold and the value of the old highway expansion from the government's point of view. With the potential expansion of the existing highway under consideration, the value of the new highway project is estimated for both public and private sectors. Moreover, the value of the new highway project is re-estimated with a non-compete clause that forbids the government from expanding the existing highway during the concession period. The value of the non-compete clause is analyzed from both sectors' point of view.

This chapter is organized as follows. Section 4.2 discusses my modeling framework, including finding the optimal toll rate, demand uncertainty, the government's objective and the proposed solution method. Section 4.3 illustrates the modeling framework on a numerical example. Section 4.4 evaluates the investment opportunity for both public and private sectors in a PPP project, and the impact of a non-compete clause in a PPP contract. Section 4.5 concludes.

4.2. METHODOLOGY

4.2.1. Project Overview

Let us revisit the investment timing problem of a new intercity highway project introduced in Chapter 3, assuming this time that it is a public project. A local government is responsible for funding, designing, building and operating this new intercity highway that links city A and city B. The government faces the same land acquisition cost $\dfrac{R_A}{\rho}$, project planning, and design costs Cp, construction costs per lane Cc, construction duration Δ, and annual maintenance and operations costs m as the private firm does (see Section 3.3.1.) The government is also able to set a flexible toll rate and to collect toll revenue from users of the new highway. As in Section 2.3, I assume that the new highway capacity is predetermined.

What changes compared to the framework considered in Chapter 3 are the toll and the objective, because instead of maximizing profits the goal of the government is to maximize social benefits. The optimal toll rate and the NPV of the new highway project from a social planner's point of view are discussed below from Section 4.3.2 to 4.3.5.

4.2.2. System Optimal Toll

In Chapter 3, the investor of the new highway, a private company, can set an endogenous toll rate to maximize its revenues at any time. In this chapter, the government can also set an endogenous toll rate. However, instead of maximizing revenue, the objective of government is to minimize the total travel cost at each time point, given travel demand q. Since the toll collected on the new highway will be used to recover (at least part of) the investment costs, the toll expense is not considered a cost in the "system" and is excluded from the objective function but it enters into the traffic equilibrium condition (where users chose between both highways):

$$\underset{q_o, p}{Min} \quad (\xi + v) T_o \left[1 + w \left(\frac{q_o}{k_o} \right)^\delta \right] q_o + (\xi + v) \left[T_o \left(1 + w \left(\frac{q_o}{k_o} \right)^\delta \right) - \frac{p}{\xi + v} \right] (q - q_o) \qquad (4.1)$$

Subject to:

$$T_o \left[1 + w \left(\frac{q_o}{k_o} \right)^\delta \right] = T_n \left(1 + w \left(\frac{q - q_o}{k_n} \right)^\delta \right) + \frac{p}{\xi + v} \qquad \text{Traffic equilibrium} \qquad (4.2)$$

$$q_o \geq 0, p \geq 0 \qquad (4.3)$$

Table 4-1 defines all notations. The Lagrangian multiplier method can be used to solve this nonlinear optimization problem with an equality constraint. The new objective function with the Lagrangian multiplier is:

$$Min \; L = T_o \left[1 + w \left(\frac{q_o}{k_o} \right)^\delta \right] q - \frac{p(q - q_o)}{\xi + v} + \lambda \left[T_n \left(1 + w \left(\frac{q - q_o}{k_n} \right)^\delta \right) + \frac{p}{\xi + v} - T_o \left(1 + w \left(\frac{q_o}{k_o} \right)^\delta \right) \right] (4.4)$$

with three first-order conditions:

$$\frac{\partial L}{\partial p} = -(q - q_o) + \lambda = 0, \qquad (4.5)$$

$$\frac{\partial L}{\partial q_o} = \left(T_o - T_n\right) + \left(\delta + 1\right) w \left[T_o \frac{q_o^{\delta}}{k_o^{\delta}} - T_n \frac{\left(q - q_o\right)^{\delta}}{k_n^{\delta}} \right] = 0, \tag{4.6}$$

$$\frac{\partial L}{\partial \lambda} = T_n \left(1 + w \left(\frac{q - q_o}{k_n} \right)^{\delta} \right) + \frac{p}{\xi + v} - T_o \left(1 + w \left(\frac{q_o}{k_o} \right)^{\delta} \right) = 0. \tag{4.7}$$

Combining Equations (4.5) to (4.7), I find the system-optimal toll:

$$p = \frac{\delta}{\delta + 1} \left(\xi + v \right) \left(T_o - T_n \right) \tag{4.8}$$

The system-optimal toll is independent of travel demand and only depends on the free-flow travel time of both highways. The optimal system toll is always smaller than the private's firm optimal toll rate, which includes a toll premium $\left(\xi + v \right) \left(T_o - T_n \right)$. Therefore, under the system optimal toll rate, the generalized free-flow travel cost of the new highway is smaller than the generalized free-flow travel cost of the old highway. It is noteworthy that when travel demand

$q < k_n \left[\dfrac{T_o - T_n}{\left(\delta + 1\right) T_n w} \right]^{1/\delta}$, the demand is too low and all travelers use the new highway, so $q_o = 0$.

When $q \geq k_n \left[\dfrac{T_o - T_n}{\left(\delta + 1\right) T_n w} \right]^{1/\delta}$, the solution of q_o is the positive real root of Equation (4.9). A closed-form solution of q_o as a function of q only exists when δ equals 1, 2, 3 or 4.

$$T_n \left(\frac{q - q_o}{k_n} \right)^{\delta} - T_o \left(\frac{q_o}{k_o} \right)^{\delta} - \frac{p}{\delta w \left(\xi + v \right)} = 0 \tag{4.9}$$

Table 4-1. Definition and base value of key parameters.

Name	Description	Numerical example in Section 4.2-4.4	Unit
Project-specific parameters			
C_p	Costs of planning, design and environmental approval	58 million	$
C_c	Construction cost of new highway per lane	125 million	$
C_{ex}	Expansion cost of the old highway	250 million	$
R_A	Agriculture rent	7 million	$
m	Maintenance and operating costs of new highway	1 million	$
n_L	Number of lanes of the new highway	2	lane
k_L	Capacity per lane	2,000	vph
k_n	Capacity of new highway in each direction	4,000	vph
k_o	Capacity of old highway in each direction before expansion	4,000	vph
k_{oex}	Capacity of old highway in each direction after expansion	6,000	vph
N	Ratio of traffic per year to vph	7,300	n.a.
T_n, T_o	Free-flow travel time on new and old highway	0.15, 0.25	hours
CP	Concession period length	40	years
Δ	Construction duration	4.5	years
p	System-optimal toll rate	2.4	$/vehicle
τ	Toll rate regulated by the government	3	$/vehicle
Other parameters			
$q_{2,n}(t), q_{2,o}(t)$	Traffic volume on new and old highway at time t after new highway investment	n.a.	vph
$q_{2,oex}(t)$	Traffic volume on the old highway at time t after the old highway expansion	n.a.	vph
$q_1(t), q_2(t)$	Travel demand between two cities without and with the new highway investment, at time t	n.a.	vph
ω, δ	Parameters of BPR function	0.8, 4	n.a.
ρ	Annual continuous compounding discount rate	7%	n.a.
μ, α	Travel demand growth rate before and after investment	2%, 3%	n.a.
σ	Travel demand percentage volatility	5%	n.a.
$\bar{q}_1, \bar{q}_2$	Upper barrier of travel demand	4000, 8000	vph
ξ	Value of time	20	$ per hour
v	Variable cost rate	10	$ per hour

n.a. = not applicable.

4.2.3. Stochastic Demand

I keep the same assumptions about demand uncertainty as in Chapter 3. Without the new highway investment, the demand between the two cities, denoted by $q_1(t)$, follows a geometric Brownian motion with a growth rate μ and a volatility σ. A reflecting upper barrier $\bar{q}_1 = k_o$ caps the demand. With the investment, the demand between the two cities denoted by $q_2(t)$, has a higher growth rate α and a higher reflecting upper barrier, $\bar{q}_2 = k_o + k_n$.

4.2.4. Consumer Surplus

The amount of revenue that remains (if any) after accounting for all construction, land cost and operating costs is not the most important concern for a social planner. The main purpose of building a new highway is to relieve congestion, save travel time, reduce fuel cost and eventually increase social benefit.

Because of induced demand, to calculate the social benefit of time savings economists use the Rule of a Half (RoH). The RoH has been widely used in transportation investments (Winkler, 2015). As shown in Figure 4-1, the RoH assumes a nearly-linear demand function. Area A in Figure 4-1 represents the consumer surplus before the new highway construction. The new highway leads to a travel cost reduction between two cities and induces more travel demand. The gain in consumer surplus is a trapezoid (area B+C). The RoH states that the benefits of induced travel are worth half the per-trip saving to existing travelers. At each time t, the consumer surplus CS of the new highway is:

$$CS\big(q_1(t), q_2(t)\big) = \text{area B} + \text{area C} = TS \cdot q_1(t) + \frac{1}{2} TS \cdot \big(q_2(t) - q_1(t)\big)$$

$$= TS(t) \frac{q_1(t) + q_2(t)}{2} \tag{4.10}$$

where *TS* is travel cost saving per traveler resulting from the new highway investment:

$$TS\left(q_1(t),q_2(t)\right)=\left(\xi+v\right)T_o w\left[\left(\frac{q_1(t)}{k_o}\right)^{\delta}-\left(\frac{q_{2,o}\left(q_2(t)\right)}{k_o}\right)^{\delta}\right] \tag{4.11}$$

and $q_{2,o}\left(q_2(t)\right)$ is traffic on the old highway after the new highway construction, which is the

positive real root of Equation (4.12):

$$T_n\left(\frac{q_2(t)-q_{2,o}(t)}{k_n}\right)^{\delta}-T_o\left(\frac{q_{2,o}(t)}{k_o}\right)^{\delta}-\frac{p}{\delta w(\xi+v)}=0 \tag{4.12}$$
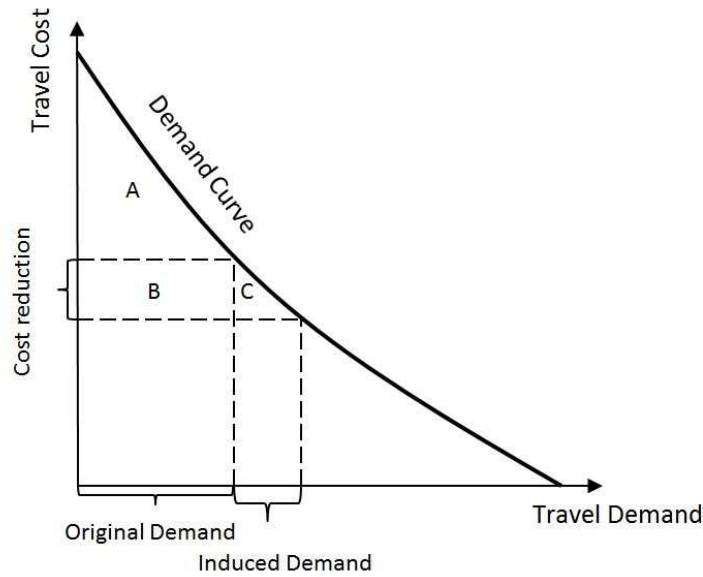
Figure 4-1. Vehicle Travel Demand Curve Illustrating the Rule of a Half

4.2.5. NPV of the New Highway Project

The objective of the government is to maximize expected social benefit, which is the expected

present value of consumer surplus and toll revenue over time, minus maintenance costs and

upfront costs. Therefore, the NPV of investing immediately with initial demand q_0 is:

$$\Omega(q_0)=E\left\{N\int_{\Delta}^{+\infty}\left[CS\left(q_1(t),q_2(t)\right)+R\left(q_2(t)\right)\right]e^{-\rho t}dt\bigg|q_0\right\}-\left[\frac{me^{-\rho\Delta}+R_A}{\rho}+C_P+n_L C_C\right] \tag{4.13}$$

where $R(q_2(t)) = pq_{2,o}(q_2(t))$ is the toll revenue at time t. See Table 4-1 for notations and parameter values. N is a factor that converts consumer surplus and revenue from \$/hour to \$/year.

Because $q_{2,o}$, which is needed in the calculation of consumer surplus and revenue, cannot be obtained analytically, and there are two RGBM demands, $q_1(t)$ and $q_2(t)$ involved, $\Omega(q_0)$ can only be calculated by numerically. For that purpose, I use Monte Carlo simulation.

4.2.6. Demand Threshold

As discussed in Section 3.2.6, the government's investment timing problem can be formulated as a dynamic programming problem. The government holds an option to invest in a new highway, so at every point in time, it faces a binary decision: invest immediately or continue waiting. The optimal stopping problem can be solved using Bellman's equation. There exist an optimal demand threshold q^*, such that when travel demand is lower than q^*, it is optimal to postpone the investment, and it is optimal to invest immediately when travel demand is higher than q^* (Dixit and Pindyck, 1994).

$$F(q) = \begin{cases} bq^{\theta_1} & \text{when } q \leq q_{ex} \\ \Omega(q) & \text{when } q > q_{ex} \end{cases} \tag{4.14}$$

where bq^{θ_1} is the value of waiting, and b is a coefficient to be determined, and θ_1 is the positive root of $\sigma^2\theta^2 + (2\mu - \sigma^2)\theta - 2\rho = 0$.

The continuity and smooth-pasting conditions need to be solved to obtain the demand threshold. The challenge is obtaining $\Omega(q)$ and its first order derivative. Since $\Omega(q)$ can only be obtained by Monte Carlo simulation, a least-square regression model of $\Omega(q)$ is used to approximate its first order derivative.

Continuity: $\quad\quad\quad\quad F\left(q^*\right) = \Omega\left(q^*\right)$ \hfill (4.15)

Smooth-Pasting: $\quad \left.\dfrac{\partial F(q)}{\partial q}\right|q^* = \left.\dfrac{\partial \Omega(q)}{\partial q}\right|q^*$ \hfill (4.16)

4.2.7. Monte Carlo Simulation

Figure 4-2 shows the procedure used for calculating $\Omega(q)$ and its first order derivative by Monte Carlo simulation and finding the optimal demand threshold.
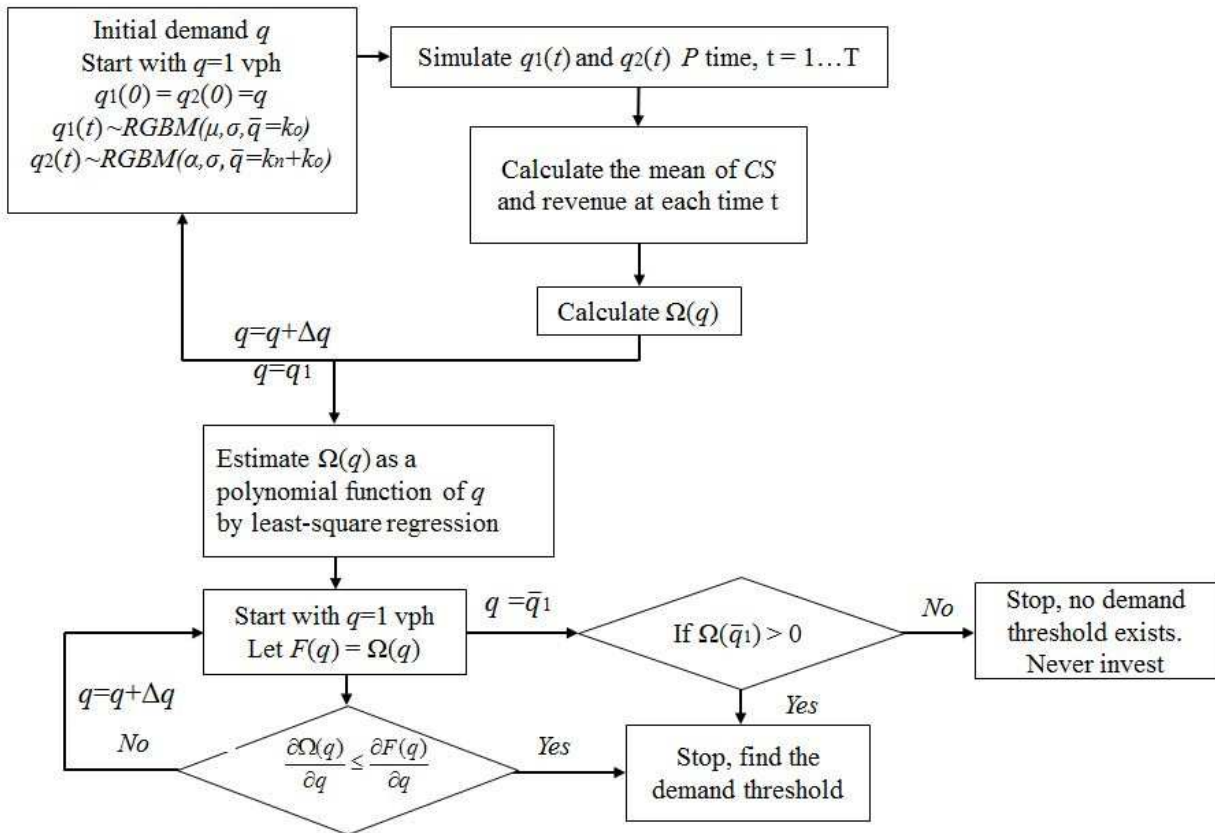
Figure 4-2. Find the optimal demand threshold by Monte Carlo simulation

4.3 A NUMERICAL EXAMPLE

In this section, I illustrate the public investment real options model developed in Section 4.2 on a case study with data inspired from the California SR-125 toll road project. Section 3.3 gives a brief introduction to the California SR-125 toll road and its competing highway. Unless specifically stated otherwise, the parameter values used in the numerical examples below are those listed in Table 4-1.
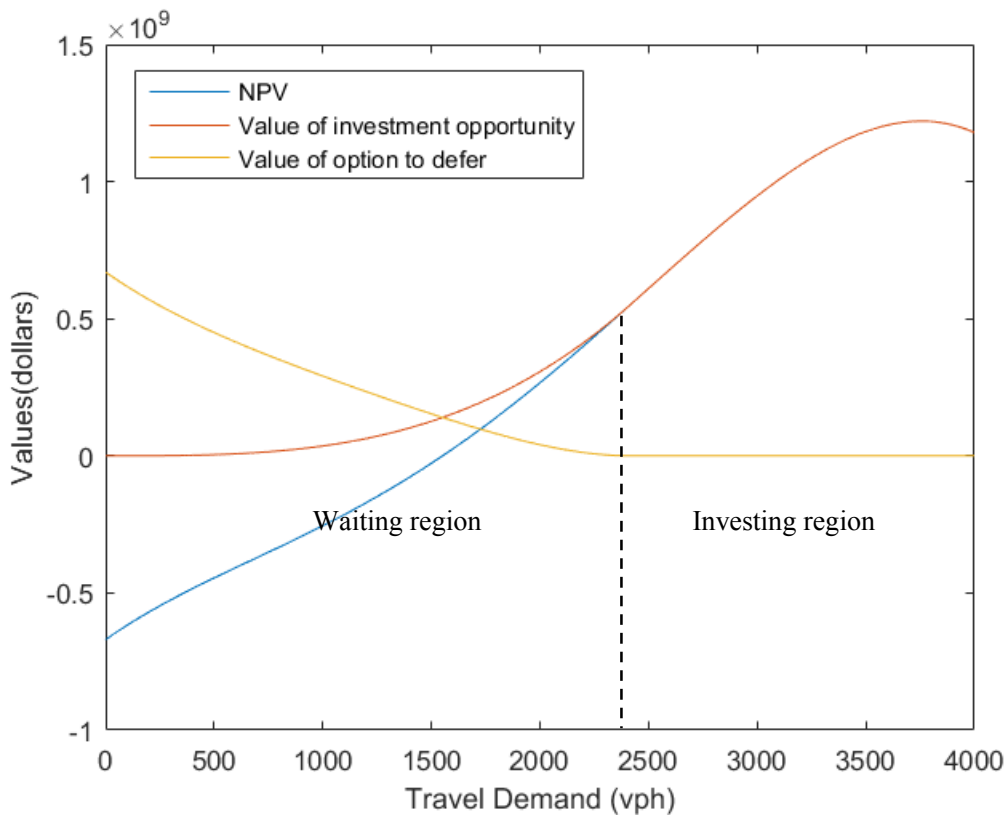
Figure 4-3. Values of NPV, investment opportunity and option to defer (the new highway project)

Figure 4-3 illustrates the value of the investment opportunity, the NPV and the value of the option to defer. The demand threshold of 2,416 vph divides the demand space into two regions: waiting and investing. Under the demand threshold, the investment opportunity is

greater than the value $\Omega(q)$ of investing immediately, so there is a positive value to defer the investment. Conversely, a higher demand justifies investing immediately and gives less value to the deferment option. When the demand increases beyond the threshold, the value of the investment opportunity is equal to $\Omega(q)$, so the government should invest immediately, and the value of waiting goes to zero.

The relationship between demand volatility and demand threshold is similar to what I found in Section 3.3. A higher volatility leads to delaying the investment. Underestimating demand uncertainty will cause investing prematurely.
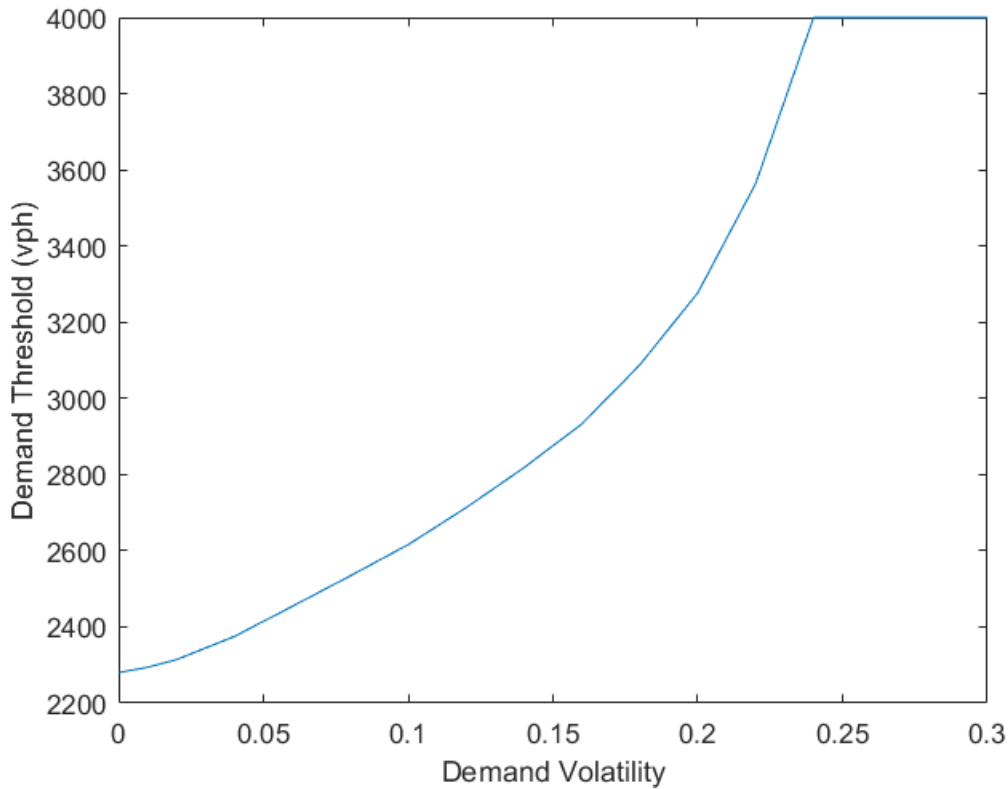
Figure 4-4. Demand thresholds of new highway with different demand volatilities

4.4 THE VALUE OF THE NON-COMPETE CLAUSE IN PPP

In this section, I discuss another option the government has to get the new highway built. Build–operate–transfer (BOT) is a form of project financing, wherein a private entity receives a concession from the public sector to finance, design, construct, and operate a new highway with a predetermined capacity k_n. After Δ years of planning, design and construction, the private sector can collect tolls from new highway's users to recoup the planning cost C_p, construction cost $n_L C_C$, and annual maintenance cost m. During the concession period, the toll is regulated by the government to be τ. At the end of the concession period CP, the operation of the new highway is transferred to the government.

The government expects significant congestion relief once the new highway is constructed. However, if future demand keeps growing, the government may find it necessary to also expand the old highway capacity to k_{oex} at the cost of C_{ex}. For simplicity, I assume that the construction duration of the old highway expansion is also Δ, and the expansion will not induce more travel demand.

If the PPP project cannot provide a competitive rate of return similar to alternative projects of comparable risk, the private sector may request a non-compete clause in the PPP contract that during the concession period the government cannot expand the old highway. This would give the private firms who entered into the PPP some reassurance that the profitability of the project will not be reduced by competing projects owned by the government, and therefore guarantee a higher rate of return.

This section is organized as follows. Section 4.4.1 finds the demand threshold and value of the old highway expansion from the government's point-of-view. Section 4.4.2 finds the demand thresholds and values of the new highway project with and without the non-compete

clause from the government's point-of-view. Section 4.4.3 finds the demand thresholds and values of the new highway project with and without the non-compete clause from the private sector's point-of-view. Section 4.4.4 evaluates the value of the non-compete clause.

4.4.1. Value of the Old Highway Expansion

The optimal investment timing of the old highway expansion can be formulated as a dynamic programming problem. The value of the expansion has the same form as Equation (4.17). There exists an optimal demand threshold q_{ex}, such that when travel demand is lower than q_{ex}, it is optimal to postpone the expansion. Otherwise, it is optimal to expand the old highway immediately. Continuity and smooth-pasting conditions need to be solved to obtain the demand threshold.

$$F_{ex}(q) = \begin{cases} b_{ex}q^{\theta_2} & \text{when } q \le q_{ex} \\ \Omega_{ex}(q) & \text{when } q > q_{ex} \end{cases} \tag{4.17}$$

where $b_{ex}q^{\theta_2}$ is the value of waiting, and b_{ex} is a coefficient to be determined, and θ_2 is the positive root of $\sigma^2\theta^2 + (2\alpha - \sigma^2)\theta - 2\rho = 0$.

$\Omega_{ex}(q)$ is the NPV of expanding the old highway immediately. Its expression is given by Equation (4.18). Since I assume that there is no induced demand after expansion, from the government's point-of-view the NPV of expansion is simply the expected present value of total travel cost savings with an infinite time horizon, minus the expansion cost C_{ex}.

$$\Omega_{ex}(q_0) = E\left\{ N \int_{\Delta}^{+\infty} q_2(t) TS_{ex}(q_2(t)) e^{-\rho t} dt \middle| q_0 \right\} - C_{ex} \tag{4.18}$$

where:

- N is a factor that converts travel cost savings from \$/hour to \$/year,

- $TS_{ex}(q_2(t))$ is travel cost savings per traveler at time t caused by the old highway expansion:

$$TS_{ex}(q_2(t)) = (\xi + v) T_o w \left(\left(\frac{q_{2,o}(t)}{k_o} \right)^{\delta} - \left(\frac{q_{2,oex}(t)}{k_{oex}} \right)^{\delta} \right),$$ (4.19)

- and $q_{2,oex}(q_2(t))$ is the traffic volume on the old highway after expansion. It is the positive real root of Equation (4.20):

$$T_n \left(\frac{q_2(t) - q_{2,oex}(t)}{k_n} \right)^{\delta} - T_o \left(\frac{q_{2,oex}(t)}{k_{oex}} \right)^{\delta} - \frac{\tau}{\delta w (\xi + v)} = 0.$$ (4.20)

Using parameter values listed in Table 4-1, $\Omega_{ex}(q_0)$ and its first order derivative can be obtained by Monte Carlo simulation. Using a similar procedure as shown in Figure 4-2, the value of q_{ex} can be found to be 4,994 vph, at which point the social benefit of the expansion option is $690 million. The coefficient b_{ex} is 4.2574. The values of expansion, waiting and investing immediately over the demand space are shown in Figure 4-5.
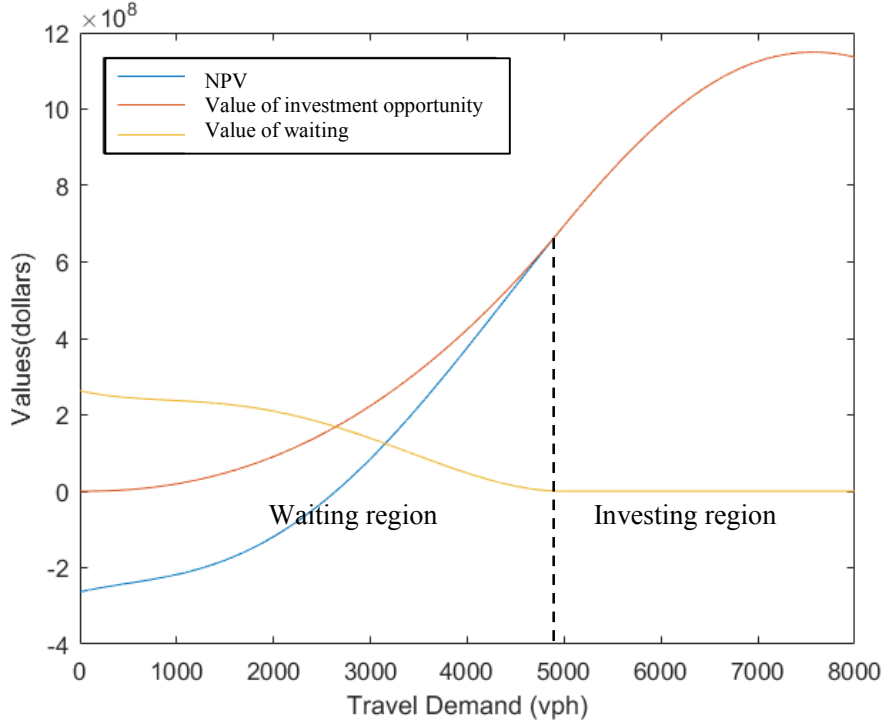
Figure 4-5. Values of NPV, investment opportunity and option to defer (old highway expansion)

4.4.2. Public Sector's Value of the New Highway Project

From the government's point-of-view, the value of the new highway project is:

$$F_g(q) = \begin{cases} b_g q^{\theta_1} & \text{when } q \leq q_g \\ \Omega_g(q) & \text{when } q > q_g \end{cases} \qquad (4.21)$$

where $b_g q^{\theta_1}$ is the value of waiting, and b_g is a coefficient to be determined,

$\Omega_g(q)$ is the value of starting the new highway project immediately.

4.4.2.1. Without the non-compete clause

From the government's point-of-view, the value of starting the new highway project immediately is given in Equation (4.22). The first term is the expected present value of consumer surplus over

time. The second term is the value of the old highway expansion. The last term is the discounted cash flow of maintenance cost after concession period plus upfront land cost.

$$\Omega_g(q_0) = E\left\{N\int_\Delta^{+\infty} CS(q_1(t), q_2(t))e^{-\rho t}dt\bigg|q_0\right\} + F_{ex}(q_0) - \left(\int_{CP}^{+\infty} me^{-\rho t}dt + \frac{R_A}{\rho}\right) \quad (4.22)$$

4.4.2.2. With non-compete clause

With the non-compete clause, the government is not allowed to expand the old highway until the end of the concession, at which point the value of expansion is $F_{ex}(q(CP))$. Therefore, the value of the project with the non-compete clause is the expected present value of $F_{ex}(q(CP))$ as the second term in Equation (4.23).

$$\Omega_g^{NC}(q_0) =$$

$$E\left\{N\int_\Delta^{+\infty} CS(q_1(t), q_2(t))e^{-\rho t}dt\bigg|q_0\right\} + E\left\{F_{ex}(q(CP))\big|q_0\right\}e^{-\rho CP} - \left(\int_{CP}^{+\infty} me^{-\rho t}dt + \frac{R_A}{\rho}\right) \quad (4.23)$$

4.4.3. Private Sector's Value of the New Highway Project

4.4.3.1. Without non-compete clause

From the private sector's point-of-view, the value of starting the new highway project immediately is given by Equation (4.24). The first term is the expected present value of revenue before the old highway expansion. The second term is the expected present value of revenue after the old highway expansion. The last term is the discounted cash flow of maintenance costs during the concession period plus upfront costs.

$$\Omega_p(q_0) =$$

$$E\left\{\int_\Delta^{T_{ex}+\Delta} R(q_2(t))e^{-\rho t}dt\bigg|q_0\right\} + E\left\{\int_{T_{ex}+\Delta}^{CP} R_{ex}(q_2(t))e^{-\rho t}dt\bigg|q_0\right\} - \left(\int_\Delta^{CP} me^{-\rho t}dt + C_P + n_L C_C\right) \quad (4.24)$$

where $R\left(q_2\left(t\right)\right) = Np\left(q_2\left(t\right) - q_{2,o}\left(t\right)\right),$ (4.25)

R_{ex} is the annual revenue that the private sector collects after the expansion of the old highway:

$$R_{ex}\left(q_2\left(t\right)\right) = Np\left(q_2\left(t\right) - q_{2,oex}\left(t\right)\right),$$ (4.26)

T_{ex} is the first time $q_2\left(t\right)$ reaches q_{ex}, which triggers then old highway expansion with Δ years of construction. The value of τ is calculated by simulation.

4.4.4.2. With the non-compete clause

With the non-compete clause, the new highway project keeps generating revenues during the whole concession period without being undermined by competition from the other highway.

$$\Omega_p^{NC}\left(q_0\right) = E\left\{\int_\Delta^{CP} R\left(q_2\left(t\right)\right)e^{-\rho t}dt \middle| q_0\right\} - \left(\int_\Delta^{CP} me^{-\rho t}dt + C_P + n_L C_C\right)$$ (4.27)

4.4.4. Value of the Non-compete Clause

The value of the NPV for both the public and private sectors with and without the non-compete clause (Equations (4.22), (4.23), (4.24) and (4.27)) can be calculated by simulation. Since the demand threshold of the new highway project is determined by the government, I calculate the corresponding demand threshold for Equations (4.22) and (4.23), and use the thresholds to calculate the NPV of the project for both the public and the private sectors. The results are plotted in Figure 4-6 and 4-7.

Without the non-compete clause, the public sector's optimal demand threshold of the new highway project is 2,039 vph, and the private sector's threshold is higher, at 2,127 vph. With the non-compete clause, the threshold increases to 2,127 vph and 2,607 vph for the public and

private sector, respectively. I assume that the starting time of the PPP after negotiation between both sectors is 2,300 vph, no matter if the non-compete clause is included. I calculate the expected profit and at 2,300 vph, which is $48.5 million without the non-compete clause and $73.8 million with the non-compete clause. The corresponding internal rates of turn for the private sector are 7.4% and 7.7%, respectively. The non-compete clause does not make the project much more attractive to the private sector. However, it causes a significant social benefit loss to the public. By signing the non-compete clause, the social benefit drops from $942 million to $795 at 2,654 vph. Figure 4-8 shows that the social benefit loss of the new highway project is much higher than the profit gain under the non-compete clause, which implies that the non-compete clause is not an efficient form of government guarantee. Even if the government subsidizes the private sector directly, the social benefit loss should be at least equal to the profit gain.
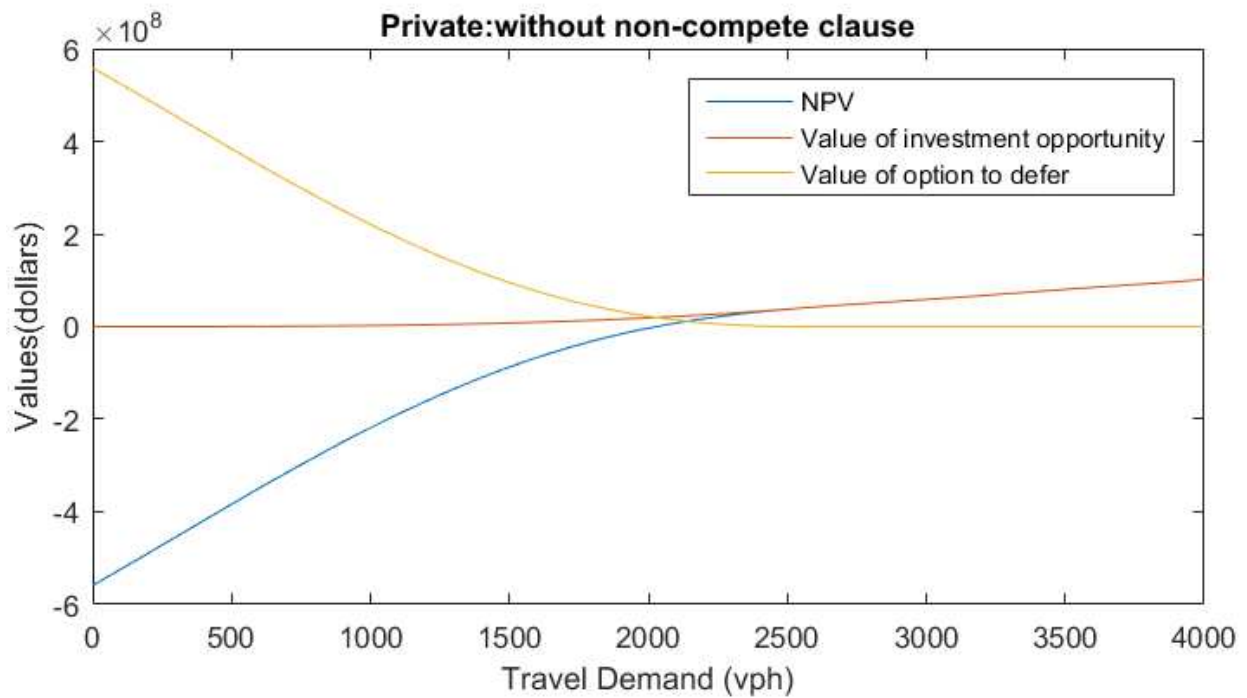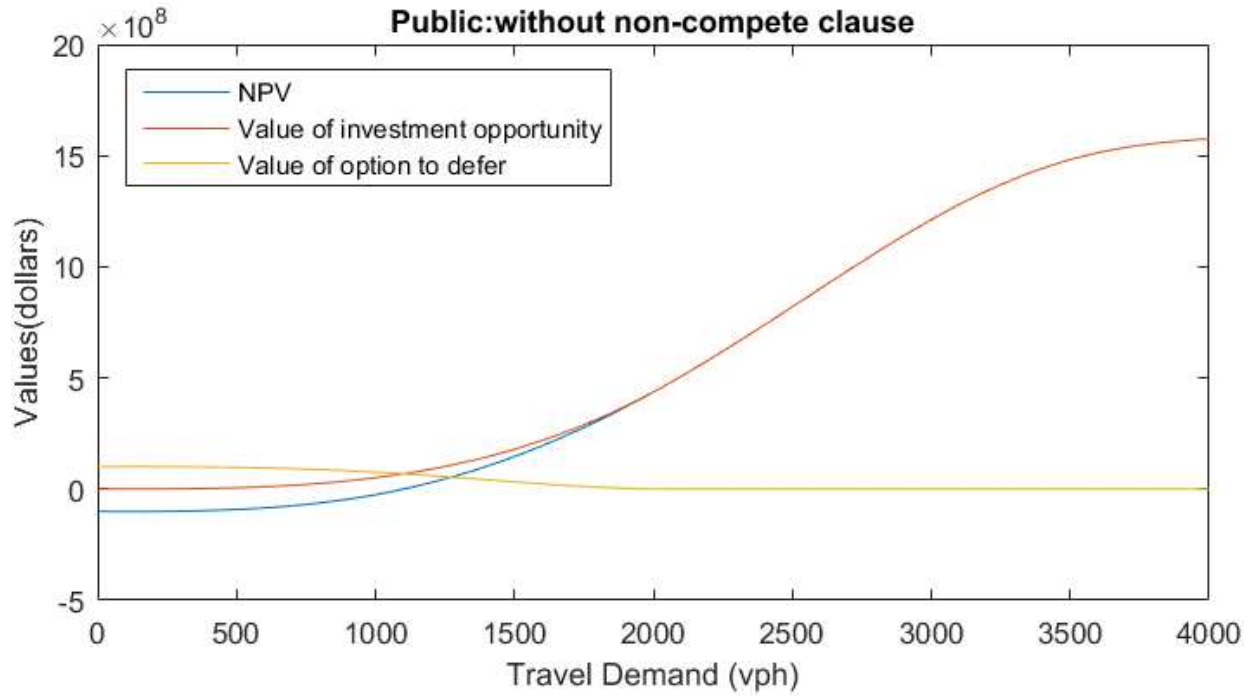
Figure 4-6. Values of NPV, investment opportunity and option to defer (without clause)
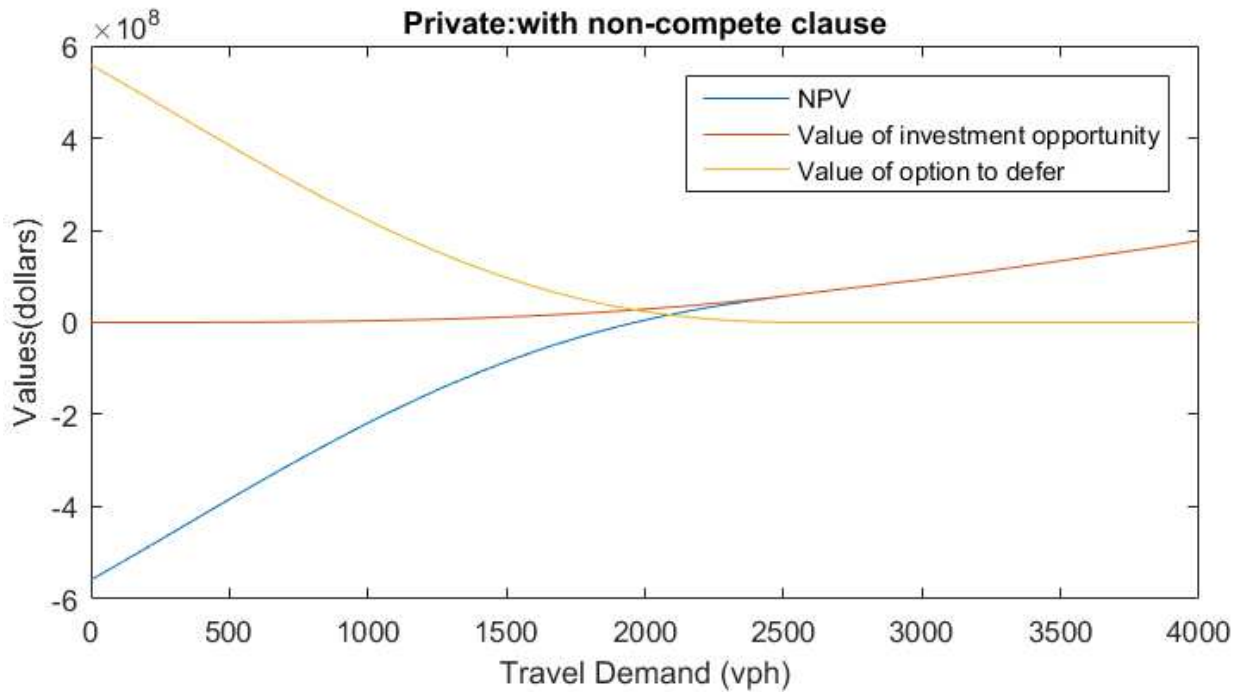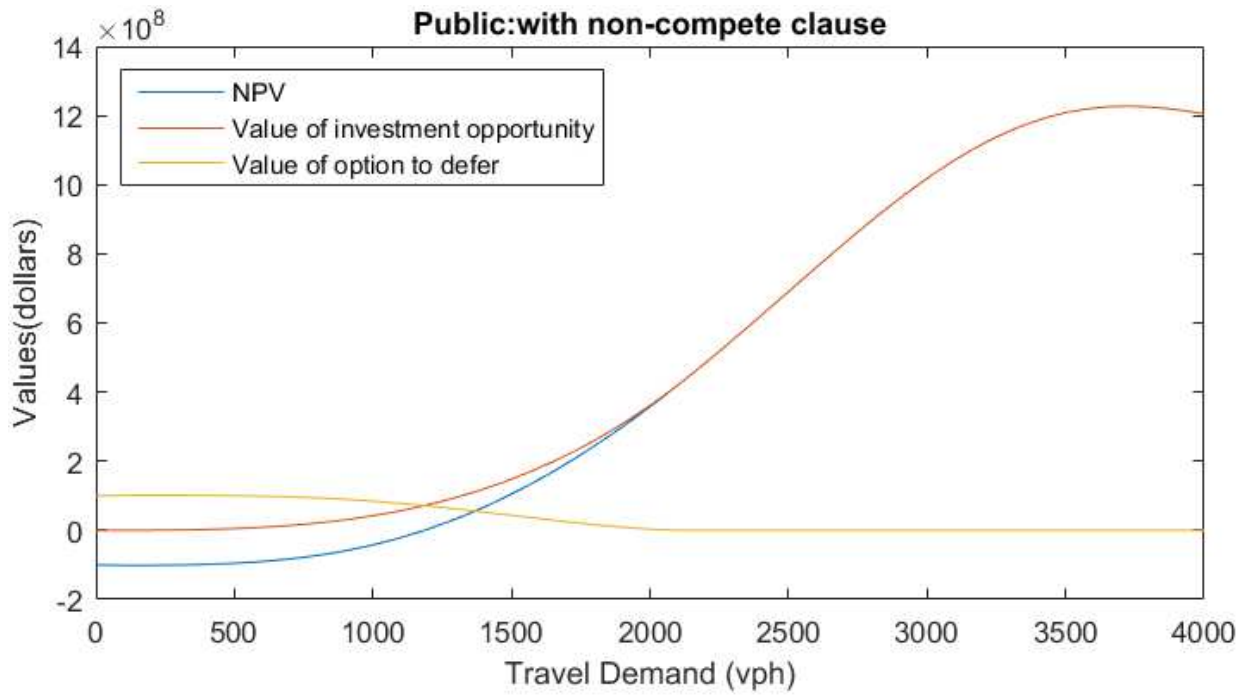
Figure 4-7. Values of NPV, investment opportunity and option to defer (with clause)
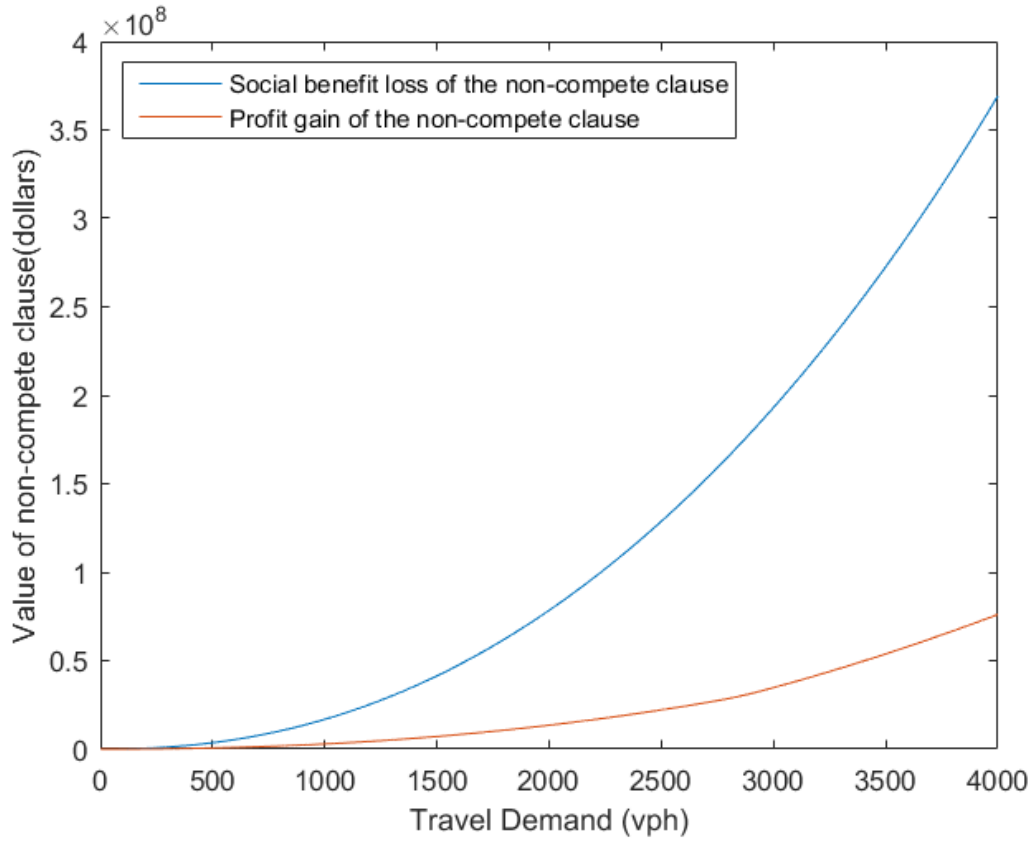
Figure 4-8. Value of the non-compete clause

4.5 DISCUSSION

This chapter revisited the optimal investment timing problem for a public inter-city highway from the government's point-of-view. The "Rule of a Half" was used to measure the consumer surplus of the project. I analytically derived the system optimal toll rate to be charged on the new highway, which was assumed to constant regardless of the demand level and highway capacities. The objective function is not analytically calculable, so Monte Carlo simulation was used to obtain the termination payoff function and its derivative and to solve the optimal demand threshold. With predetermined capacity, the demand threshold divides the demand space into a waiting region and an investing region, just as in Chapter 3. Interestingly, for the same SR-125

project, the government's demand threshold is lower than the private firm's, which implies that social-benefit-maximizing leads to investing earlier compared to a profit-maximizing company. Like the private firm, however, the government also tends to delay the investment as demand volatility increases.

In Section 4.4, I evaluated the new highway project in a build-operate-transfer framework. The government releases a concession to a private concessionaire who takes responsibility for financing, planning, designing, constructing, operating and maintaining the new highway. The private sector collects tolls, which are regulated by the government, to recover investment costs during the concession period. My focus was the analysis of the non-compete clause, which forbids the government from investing in the old highway during the public-private partnership. Real options methods were used to value the sequential options of the new highway construction and the old highway expansion under demand uncertainty. I found that the non-compete clause does increase the private sector's expected profit, but it has little impact on the internal rate of return of the project. However, the social benefit loss is much larger, which implies that in the negotiation of PPP contract, the non-compete clause may not be the best form of government guarantees to ensure the private sector's profitability.

CHAPTER 5 SOLVING THE MULTI-PERIOD NETWORK DESIGN PROBLEM WITH APPROXIMATE LEAST-SQUARE MONTE CARLO SIMULATION

5.1. INTRODUCTION

Chapter 3 and 4 analyze the timing and the capacity of a toll road project between two cities under uncertainty for a very simple road network with only two centroids (city A and B) and two links (the old highway and the new highway) where traffic from/to centroids outside the network and any additional transportation infrastructure are neglected. In this chapter, I expand the real options method to the multi-period Continuous Network Design Problem (Multi-period CNDP), which is more complex than the inter-city highway project in several ways: (1) the road network has multiple centroids and links; (2) the decision-maker is facing multi-dimensional stochastic demand between OD pairs; (3) the decision-maker needs to select a continuous capacity improvement plan for each link, so capacity is no longer limited to a small number of integer values.

By increasing the dimensions of centroids, links, stochastic variables and decision variables, the analytical and finite difference methods used in Chapter 3 and 4 are no longer applicable. Least Square Monte Carlo simulation (LSMC) proposed by Longstaff and Schwartz (2001) is a simple yet powerful approach to solve the investment timing and capacity choice problem for projects in a road network. However, the LSMC method follows an iterative procedure, and the value of the termination payoff function for the CNDP needs to be solved repeatedly given a set of simulated demands for each iteration. Solving the CNDP with stochastic demand is very time-consuming (typically measured in days on a desktop computer, even for a small network with a few centroids and links). When the objective of the CNDP covers the whole concession periods, a network investment timing and capacity choice problem

using LSMC may require calculating the values of the termination payoff function thousands of times, which would result in years of computation time (on a PC), making the problem unsolvable.

In this chapter, I propose an algorithm, which I called "Approximate Least Square Monte Carlo simulation" (ALSMC), to solve the multi-period CNDP in a reasonable amount of time on a PC. The Approximate Least Square Monte Carlo simulation applies least square regression to estimate the value of the termination payoff function without knowing the optimal capacity improvement plan. For each iteration, only a multi-period CNDP with deterministic demand needs to be solved, which dramatically reduces the computing time of each termination payoff function from days to minutes.

This chapter is organized as follows. Section 5.2 presents the original LSMC algorithm and introduces the ALSMC algorithm. The ALSMC is tested on a small road network in Section 5.3. Section 5.4 discusses the results, summarizes limitations, and suggests ideas for future work.

5.2. METHODOLOGY

In Section 5.2.1, I first introduce the original Least Square Monte Carlo (LSMC) simulation. Then, I propose an approximation of the termination payoff function in Section 5.2.2, which combined with the original LSMC becomes the Approximate LSMC simulation method. In Section 5.2.3, the Approximate LSMC is tested on a simple example, and the approximate results are compared with the exact solution.

Table 5-1. Notation

Notation	Definition
Set notation	
A	Set of links
rs	OD pair, $rs \in RS$
a	Link, $a \in A$
Variables to determine	
y_a	Link capacity improvement for link a
$y_{\max}$	The maximum link capacity improvement allowed for each link
x_a	Link flow for link a with investment
$x_{a,0}$	Link flow for link a without investment
$\mathbf{y}^*$	Vector of optimal link capacity improvement $\mathbf{y}_a^*$, $a \in A$
$\mathbf{y}^\beta$	Vector of approximate link capacity improvement $\mathbf{y}_a^\beta$, $a \in A$
$\mathbf{q}_t$	Vector of q_t^{rs}, $rs \in RS$
q_t^{rs}	Traffic demand between OD pairs rs at time t
$\mathbf{q}_t^i$	Vector of q_t^{rs} on i-th simulated demand path at time t, $rs \in RS$
Parameters given	
C_a	Parameter in link performance function for link a
B_a	Parameter in link performance function for link a
d_a	Unit cost of link capacity improvement
k_a	Capacity of link a
$\delta_{a,p}^{rs}$	Link-path incidence indicator
T	Implementation period, the expiration date of the investment opportunity
CP	Concession period
Δ	Construction duration
P	Number of demand path used in LSMC
P'	Number of demand path used in the approximation of termination payoff function

5.2.1. Least Squares Monte Carlo Simulation

Least Squares Monte Carlo simulation (LSMC) is a powerful approach for approximating the value of American options by simulation, especially when dynamic factors follow some general stochastic processes that render unsolvable the partial differential equation characterizing the value of the option, and when the value of the option depends on multiple dynamic factors that make finite difference and binomial techniques impractical. The LSMC approach can accurately

approximate the value of the option with high computational speed in a parallel environment. Let me first briefly outline the LSMC approach developed by Longstaff and Schwartz (2001) and extended by Gamba (2003) to solve multi-option valuation problem.

The investment opportunity can be treated as an American option, which is an option that can be exercised once at any time before the expiration date. Least-Square Monte Carlo simulation approximately assumes that an American option has multiple but finite N exercise times before it expires at time T, and the exercise time interval is $\Delta t = T/N$. The exercise time can be treated as continuous by increasing N. The Bellman equation corresponding to the optimal stopping problem in discrete time can be written:

$$F\left(t_n, \mathbf{q}_{t_n}\right) = \max\left\{\Omega\left(\mathbf{q}_{t_n}, \mathbf{y}^*\left(\mathbf{q}_{t_n}\right)\right), \Phi\left(t_n, \mathbf{q}_{t_n}\right) = e^{-r\Delta t} E_{t_n}\left[F\left(t_{n+1}, \mathbf{q}_{t_{n+1}}\right)\Big|\mathbf{q}_{t_n}\right]\right\} \tag{5.1}$$

where $F\left(t_n, \mathbf{q}_{t_n}\right)$ is the option value function at time t_n given state variables $\mathbf{q}_{t_n}$. $\Omega\left(\mathbf{q}_{t_n}, \mathbf{y}^*\right)$ is the termination payoff function that can be obtained by exercising the option immediately at time t_n with the optimal decision $\mathbf{y}^*$.

$$\mathbf{y}^*\left(\mathbf{q}_{t_n}\right) = \arg\max_{\mathbf{y}} \Omega\left(\mathbf{q}_{t_n}, \mathbf{y}\right) \tag{5.2}$$

The second term on the right-hand side of Equation (5.1) is the continuation value $\Phi\left(t_n, \mathbf{q}_{t_n}\right)$, which is given by the expected conditional option value $E_{t_n}\left[F\left(t_{n+1}, \mathbf{q}_{t_{n+1}}\right)\Big|\mathbf{q}_{t_n}\right]$ at time t_{n+1} discounted to time t_n. If the termination payoff value is greater than the continuation value, then it is optimal to exercise the option at time t_n. Otherwise, it is better to wait and to get a higher expected option value at time t_{n+1}.

The LSMC Algorithm

The LSMC algorithm can be summarized in the following steps:

Step 1. Given $\mathbf{q}_0$ at time t_0, the dynamics of the state variables $\mathbf{q}_{t_n} = \left\{ q_t^{rs}, \forall rs \in RS \right\}$ in the future is simulated by generating P paths for $\mathbf{q} = \left\{ \mathbf{q}_{t_n}, n = 1, 2, ..., N \right\}$. Let us denote by $\mathbf{q}_{t_n}^i$ the vector of demands between all OD pairs at time t_n along the i-th simulated path. The value of $F\left(t_n, \mathbf{q}_{t_n} \right)$ is obtained by backward dynamic programming.

Step 2. At time t_N, the continuation value is zero, because the option expires after time t_N. For each time $t_n < t_N$, $\Omega\left(\mathbf{q}_{t_n}^i, \mathbf{y}^* \left(\mathbf{q}_{t_n}^i \right) \right)$ can be calculated analytically or by simulation given $\mathbf{q}_{t_n}^i$. Therefore, at time t_N, the option value function is given by:

$$F\left(t_N, \mathbf{q}_{t_N}^i \right) = \max \left\{ \Omega\left(\mathbf{q}_{t_N}^i, \mathbf{y}^* \left(\mathbf{q}_{t_N}^i \right) \right), 0 \right\} \tag{5.3}$$

Step 3. For t_n from t_{N-1} back to t_1:

a. For paths with $\Omega\left(\mathbf{q}_{t_n}^i, \mathbf{y}^* \left(\mathbf{q}_{t_n}^i \right) \right) > 0$, which are called "in the money" paths, calculate

$$\hat{\Phi}\left(t_n, \mathbf{q}_{t_n}^i \right) = \sum_{m=0}^{M} \hat{\beta}_m f_m \left(\mathbf{q}_{t_n}^i \right). \quad f_m(X) \text{ is the basis function of } X. \text{ There are many choices of}$$

basis functions, including Hermite, Legendre, Laguerre, Chebyshev, Gegenbauer and Jacobi polynomials (Longstaff and Schwartz, 2001). One choice is the simple powers of the state variables:

$$f_0(X) = 1; f_1(X) = X; f_2(X) = X^2; \quad ... \quad ; f_M(X) = X^M \tag{5.4}$$

Use original least square to estimate the $\hat{\beta}_m$ for the M-polynomial least square regression model above. Longstaff and Schwartz (2001) states that if the stochastic state variables have state-dependent volatility, then weighted least squares or generalized least squares

might be more robust than ordinary least squares because the residuals from the regression may be heteroskedastic. However, I find that original least-square perform as well as weighted least-square in my numerical examples. Each sample observation in the regression model is the realized continuation value along one "in-the-money" path:

$$\Phi^i\left(t_n, \mathbf{q}_{t_n}^i\right) = e^{-r\Delta t} F\left(t_{n+1}, \mathbf{q}_{t_{n+1}}^i\right) \tag{5.5}$$

b. For "in the money" paths, update

$$F\left(t_n, \mathbf{q}_{t_n}^i\right) = \max\left\{\Omega\left(\mathbf{q}_{t_n}^i, \mathbf{y}^*\left(\mathbf{q}_{t_n}^i\right)\right), \hat{\Phi}\left(t_n, \mathbf{q}_{t_n}^i\right)\right\} \tag{5.6}$$

c. For other paths, update

$$F\left(t_n, \mathbf{q}_{t_n}^i\right) = \hat{\Phi}\left(t_n, \mathbf{q}_{t_n}^i\right) \tag{5.7}$$

Step 4. For t_0, the expected continuation value is the average value of all paths:

$$\Phi\left(t_0, \mathbf{q}_{t_0}\right) = \frac{e^{-r\Delta t}}{P}\sum_{i=0}^{P} F\left(t_1, \mathbf{q}_{t_1}^i\right).$$ If $\Omega\left(\mathbf{q}_{t_0}, \mathbf{y}^*\left(\mathbf{q}_{t_0}\right)\right) < \Phi\left(t_0, \mathbf{q}_{t_0}\right)$, then the best strategy is to defer the investment. If, however, $\Omega\left(\mathbf{q}_{t_0}, \mathbf{y}^*\left(\mathbf{q}_{t_0}\right)\right) \geq \Phi\left(t_0, \mathbf{q}_{t_0}\right)$, then the best strategy is to invest at time t_0.

The estimation of the continuation value has been proven to converge to the true value when the number of paths K and the number of basis functions J goes to infinity (Stentoft, 2004), and the estimation errors are asymptotically normally distributed (Clément *et al.*, 2002). One concern with the basis function is that when the number of state variables increases, it seems that the number of basis functions also grows exponentially with the dimension of the problem. However, Longstaff and Schwartz (2001) show the number of basis functions needed to obtain convergence grows much slower than exponentially.

5.2.2. Approximation of the Termination Payoff Function

The greatest challenge in solving multi-period CNDP with LSMC is how to determine the optimal capacity design $\mathbf{y}^*$ and the value of $\Omega\left(\mathbf{q}_{t_n}^i, \mathbf{y}^*\left(\mathbf{q}_{t_n}^i\right)\right)$, given $\mathbf{q}_{t_n}^i$. Since the optimal capacity depends not only on the initial demand $\mathbf{q}_{t_n}^i$ at time t_n, but also on future demands $\mathbf{q}_{t_{n+1}}$, $\mathbf{q}_{t_{n+2}}$, …, $\mathbf{q}_{t_{n+CP}}$, solving $\mathbf{y}^*$ relies on the enumeration of future demands. Therefore, calculating $\Omega\left(\mathbf{q}_{t_n}^i, \mathbf{y}^*\left(\mathbf{q}_{t_n}^i\right)\right)$ is extremely time-consuming. Taking the test network in Section 5.3 as an example, calculating a single $\Omega\left(\mathbf{q}_{t_n}^i, \mathbf{y}^*\left(\mathbf{q}_{t_n}^i\right)\right)$ with 10 concession periods using 2000 paths takes hours on a recent personal computer, even with parallel computation. Moreover, for the test network design problem with 5 implementation periods and 2000 paths, the LSMC method requires $\Omega\left(\mathbf{q}_{t_n}^i, \mathbf{y}^*\left(\mathbf{q}_{t_n}^i\right)\right)$ being solved 10000 times, which takes years to finish.

The goal of solving flexible network design problem is to know whether or not capacity improvement should be implemented at t_0 given $\mathbf{q}_{t_0}^i$, and if yes, what the optimal capacity design should be, in addition to finding the value of the investment opportunity. Because the LSMC method requires the value of $\Omega\left(\mathbf{q}_{t_n}^i, \mathbf{y}^*\left(\mathbf{q}_{t_n}^i\right)\right)$ for each time point on each path, the optimal capacity design needs to be solved for each time point on each path. However, it is unnecessary to know the optimal capacity design for any future time point. If it is better to delay the investment, the decision maker only needs to solve the flexible network design problem with the newly evolved demands at the next time point. If we could generate accurately enough estimated values of the termination payoff function $\hat{\Omega}\left(\mathbf{q}_{t_n}^i, \mathbf{y}^*\right)$ without knowing the optimal capacity design $\mathbf{y}^*$, it would be possible to substantially reduce the computation time of the flexible network design problem. This is the idea behind my approach.

I calculate $\hat{\Omega}\left(\mathbf{q}_{t_n}^i, \mathbf{y}^*\right)$ with the approximate capacity design y^β as follows:

(1) Randomly choose P' out P demand paths.

(2) For each demand $\mathbf{q}_{t_n}^j$ on path j, $j \in P'$ and $n = 1, 2, ..., N$, instead of solving $\mathbf{y}^*$ that relies on path enumeration, find an approximate $\mathbf{y}^a$ that optimizes the objective with only a single path of demands $\mathbf{q}_{t_n}^j$, $\mathbf{q}_{t_{n+1}}^j$, ..., $\mathbf{q}_{t_{n+CP}}^j$.

$$\mathbf{y}^a = \arg\max_{\mathbf{y}} \Omega\left(\mathbf{q}_{t_n}^j, \mathbf{q}_{t_{n+1}}^j, ..., \mathbf{q}_{t_{n+CP}}^j, \mathbf{y}\right) \tag{5.8}$$

(3) Based on $\mathbf{y}^a$ and the single path of demands $\mathbf{q}_{t_n}^j$, $\mathbf{q}_{t_{n+1}}^j$, ..., $\mathbf{q}_{t_{n+CP}}^j$, calculate the termination payoff value $\Omega\left(\mathbf{q}_{t_n}^j, \mathbf{q}_{t_{n+1}}^j, ..., \mathbf{q}_{t_{n+CP}}^j, \mathbf{y}^\beta\right)$

(4) Estimate $\hat{\Omega}\left(\mathbf{q}_{t_n}^j\right) = \sum_{m=0}^{M} \hat{\beta}_m f_m\left(\mathbf{q}_{t_n}^j\right)$. $f_m\left(\mathbf{q}_{t_n}^j\right)$ is the basis function of $\mathbf{q}_{t_n}^j$. Use least-square to estimate $\hat{\beta}_m$ for the M-polynomial least square regression model above. Each sample observation in the regression model is $\Omega\left(\mathbf{q}_{t_n}^j, \mathbf{q}_{t_{n+1}}^j, ..., \mathbf{q}_{t_{n+CP}}^j, \mathbf{y}^\beta\right)$ from step (3).

(5) Estimate $\hat{\Omega}\left(\mathbf{q}_{t_n}^i\right) = \sum_{m=0}^{M} \hat{\beta}_m f_m\left(\mathbf{q}_{t_n}^i\right)$ for all $i \in P$ and $i \notin P'$.

(6) Run the original LSMC using $\hat{\Omega}\left(\mathbf{q}_{t_n}^i\right)$ as the termination payoff value of each point on all demand paths. If the result shows that investing immediately is preferred, then a stochastic NDP needs to be solved to obtain the optimal capacity at time 0. If the result shows that waiting is preferred, then the investor should simply repeat the ALSMC at the next time point, until the result changes to investing immediately or the investment opportunity expires.

Finding the approximate y^β only requires a single simulated path of demands, which simplifies the stochastic NDP to a deterministic NDP and then dramatically reduces computations. By least-square regression, optimal objective values of deterministic NDP samples are used to estimate the stochastic NDP's objective value. However, because $\mathbf{y}^\beta = \arg\max \Omega\left(\mathbf{q}^j_{t_n}, \mathbf{q}^j_{t_{n+1}}, ..., \mathbf{q}^j_{t_{n+CP}}, \mathbf{y}^\beta\right)$, the deterministic NDP with y^β tends to overestimate the stochastic NDP's objective value with $\mathbf{y}^*$, $\Omega\left(\mathbf{q}^j_{t_n}, \mathbf{q}^j_{t_{n+1}}, ..., \mathbf{q}^j_{t_{n+CP}}, \mathbf{y}^\beta\right) \geq \Omega\left(\mathbf{q}^j_{t_n}, \mathbf{q}^j_{t_{n+1}}, ..., \mathbf{q}^j_{t_{n+CP}}, \mathbf{y}^*\right)$.

5.2.3. A Simple Example

Let us test this approach on a very simple investment timing problem with endogenous and continuous capacity that can also be solved analytically. Let us then compare the Approximate LSMC solution with analytical solutions obtained via real options and the tradition NPV method.

Consider a proposed highway project that is designed to generate revenues of $q_t K^{1/2}$ dollars each year. The length of the concession period CP is 10 years. q_t is the stochastic demand at year t; it is assumed to follow a geometric Brownian motion with an annual drift rate $\mu = 3\%$ and annual volatility $\sigma = 3\%$. The annual discount rate ρ is 7%. The capacity of the project is K, which is decided at the time of investment and cannot be changed after. The upfront costs include construction cost \$15*K and a start-up cost \$100. The investor can delay the investment infinitely, and the investment opportunity never expires. The construction duration Δ is 1 year, so if the investment happens at time t, the investor can collect revenues from time $t+1$ to $t+CP$.

Equation (5.9) shows the NPV of the investment at demand level q_0 with a given capacity K. With endogenous capacity K^* which maximizes the NPV (Equation (5.10)), the new NPV is updated by Equation (5.11):

$$\Omega(q_0, K) = E\left\{ \sum_{t=\Delta}^{CP} q_t e^{-\rho t} \middle| q_0 \right\} K^{1/2} - 15K - 100 \tag{5.9}$$

$$K^*(q_0) = \frac{E^2\left\{ \sum_{t=\Delta}^{CP} q_t e^{-\rho t} \middle| q_0 \right\}}{900} \tag{5.10}$$

$$\Omega(q_0, K^*(q_0)) = \frac{\left[\sum_{t=\Delta}^{CP} E(q_t|q_0) e^{-\rho t} \right]^2}{60} - 100 = \frac{\left[e^{-(\mu-\rho)CP} - e^{-(\mu-\rho)\Delta} \right]^2}{60(\mu-\rho)} q_0^2 - 100 \tag{5.11}$$

Since $E(q_t|q_0)$ has a closed-form expression, this investment problem can be solved analytically using real options. The demand threshold is $q^* = 26.98$. When $q_0 < 26.98$, it is better to delay the investment, otherwise it is better to invest immediately. For example, when $q_0 = 20$, delaying the investment has a value of \$348.59, while investing immediately is worth \$335.05.

Let us compare the analytical results when $q_0 = 20$ with the solution obtained using Approximate Least-Squares Monte Carlo simulation, assuming that $E(q_t|q_0)$ and K^* are not known. First, I simulate P demand paths with T implementation period ($T = 20$ years) and CP concession period ($CP = 10$ years). The original problem has an infinite implementation period, so the investment opportunity never expires. However, Least-Square Monte Carlo simulation must have an option expiration date, so I set a long enough implementation period T. Second, I randomly choose P' out of P paths (for now, set $P' = P$). For each time t_n and path j ($j \in P'$), with the current demand $q_0 = q_{t_n}^j$, the approximate capacity K^β is calculated as:

$$K^{\beta}\left(q_{t_n+\Delta}^{j},...,q_{t_n+CP}^{j}\right)=\frac{\left[\sum_{t=\Delta}^{CP}q_{t_n+t}^{j}e^{-\rho t}\right]^{2}}{900} \tag{5.12}$$

Obviously, $K_{t_n}^{j}$ is not the optimal capacity $K^{*}(q_{t_n}^{j})$, because the value of $K_{t_n}^{j}$ depends on the simulated demand level, instead of the expected demand level, from t_{n+1} to t_{n+10} on path j. The NPV of the project with $K_{t_n}^{j}$ and simulated demands is:

$$\Omega_{t_n}^{j}=K^{\beta}\left(q_{t_n+\Delta}^{j},...,q_{t_n+CP}^{j}\right)^{1/2}\sum_{t=1}^{10}q_{t_n+t}^{j}e^{-0.07t}-15K^{\beta}\left(q_{t_n+\Delta}^{j},...,q_{t_n+CP}^{j}\right)-100 \tag{5.13}$$

I use $\Omega_{t_n}^{j}$ as the dependent variable and $q_{t_n}^{j}$ as the independent variable ($j=1, 2, ..., P$; $t_n=1, 2, ..., T$) to estimate the least-square regression model. Therefore, the exact NPV function of Equation (5.11) can be replaced by the approximate NPV function. An example of approximated NPV function generated by simulation is Equation (5.14):

$$\hat{\Omega}(q_0)=0.0022*q_0^{3}+0.9163*q_0^{2}+4.1681*q_0-130.5669 \tag{5.14}$$

Figure 5-1 shows all the $\Omega_{t_n}^{j}$ with the analytical NPV function given by Equation (5.11).

After using Equation (5.14) to approximate $\Omega(q_0)$, the original LSMC simulation can be applied to calculate the value of the investment opportunity when the initial demand is 10. The result is compared with the analytical value. I repeat the Approximate LSMC simulation 10 times with different numbers of paths (P=500, 1000, 2500 and 5000), and then plot the errors in Figure 5-2. It shows that even with a small number of paths, the error between the analytical value and approximate value of investment opportunity is within 1%. As the number of paths increasing, the approximate value of the investment opportunity converges. With 5000 paths, the difference between the exact and the approximated solution is within 0.5%. However, as I

discussed above in Section 5.2.2, the approximated value of investment opportunity tends to be slighted higher than the exact one.
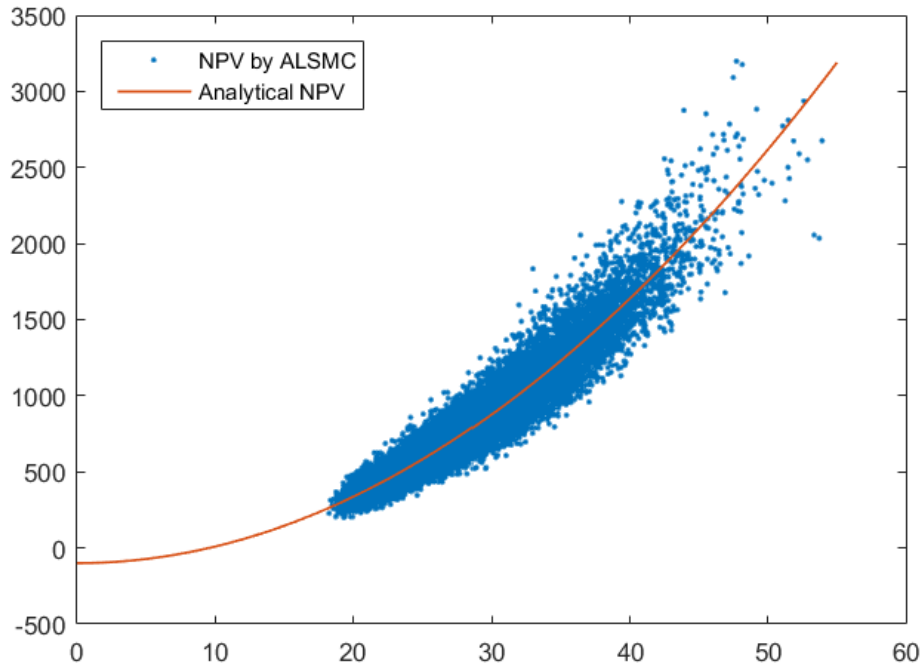
Figure 5-1. Least square estimation of Ω as a polynomial function of q

The simulated demand matrix has P paths and T implementation periods, therefore $P*T$ samples in total. The original LSMC uses P samples in least-square regression at each iteration, while the approximation of the termination payoff function uses $P'*T$ samples in least-square regression. Setting $P' = P$, as I did above, means that the sample size used in the second least-square regression model is T times as large as the sample size used in the first least-square regression model. Because the approximation of the termination payoff function is very time-consuming and the original LSMC takes almost no time, I can raise P to $P = P'*T$ to improve the accuracy of the approximate LSMC without increasing the computation time. Figure 5-3

shows the effects of increasing P on the accuracy of the approximate LSMC simulations, keeping P'=250. We see that when $P = P'*T = 5000$, the approximate LSMC simulation has the best accuracy. When P is smaller or larger than $P'*T$, the two sample sizes used in the approximation of the termination payoff function and LSMC are unbalanced, which shows that overall accuracy is impacted by the smaller sample size.
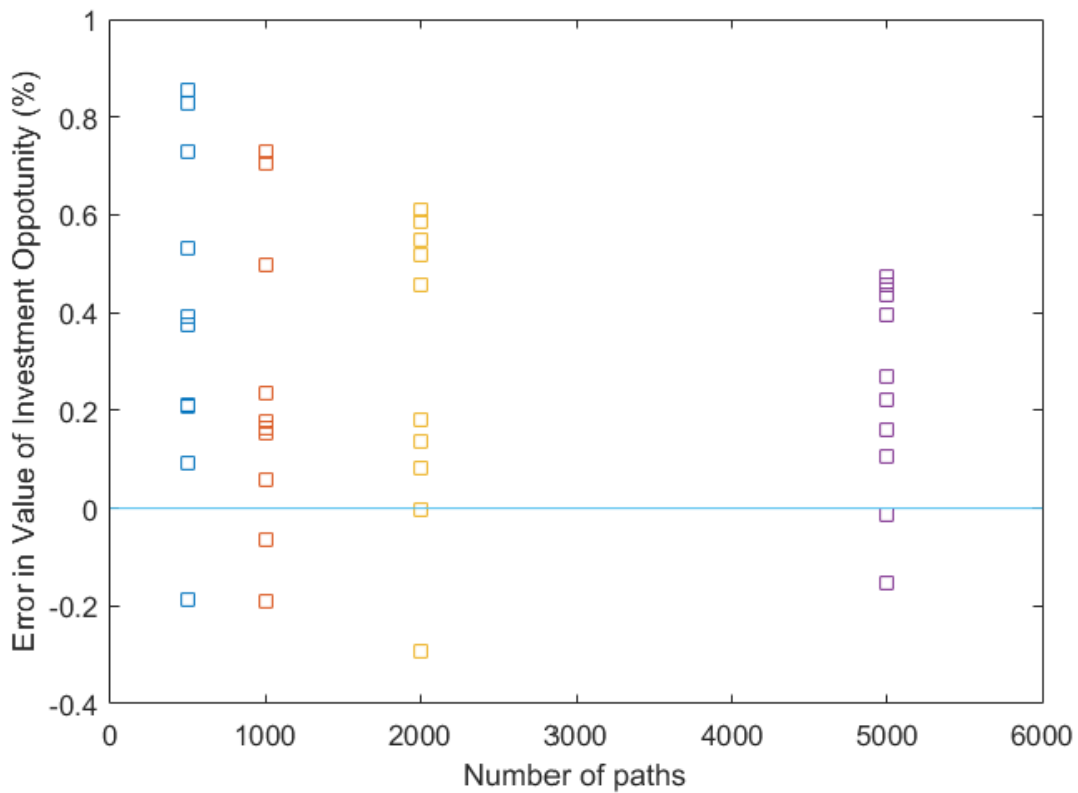
Figure 5-2. The error of approximate value of investment opportunity
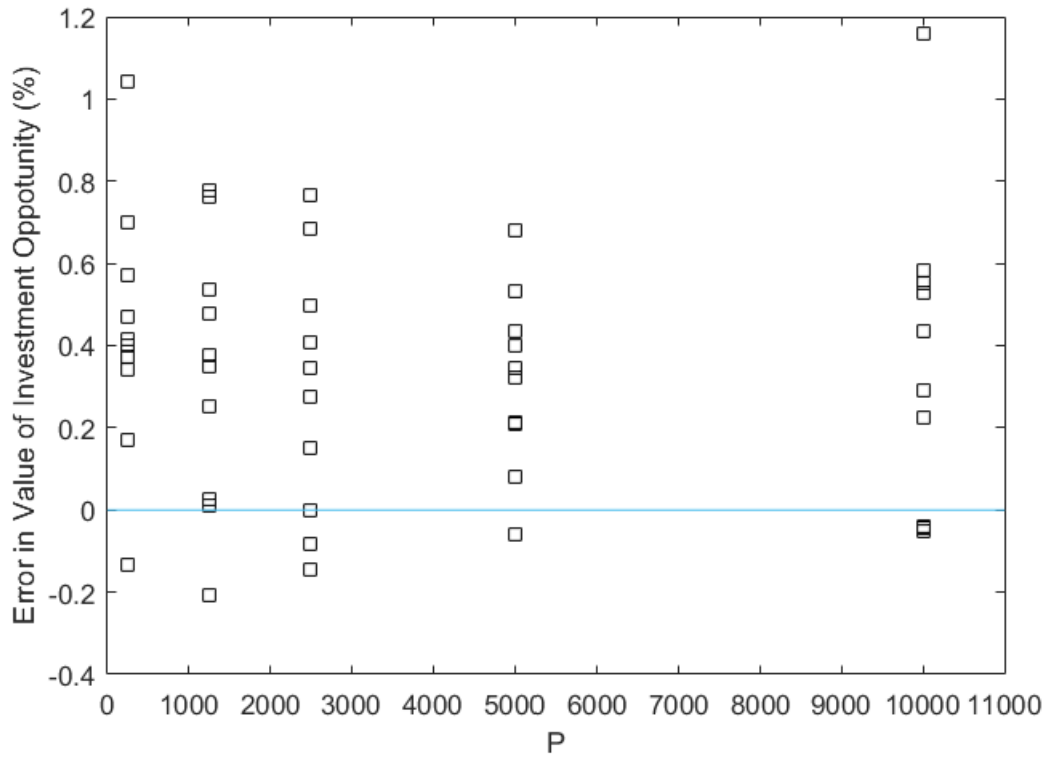
Figure 5-3. The impact of *P* on the accuracy of ALSMC (*p′* = 250)

5.3. TEST NETWORK

Let us now test the approximate LSMC method on the network shown in Figure 5-4, which has

been extensively used in the literature (Abdulaal and LeBlanc, 1979; Friesz *et al*., 1992; Wang

and Lo, 2010). This test network has 6 nodes, 16 links, and 2 OD pairs (1-6 and 6-1). The path

set for each OD pair contains 8 paths. The values of the link travel cost parameters C_a and B_a,

construction costs d_a, and link capacity k_a, which all come from Friesz *et al.* (1992) are shown in

Table 5-2.
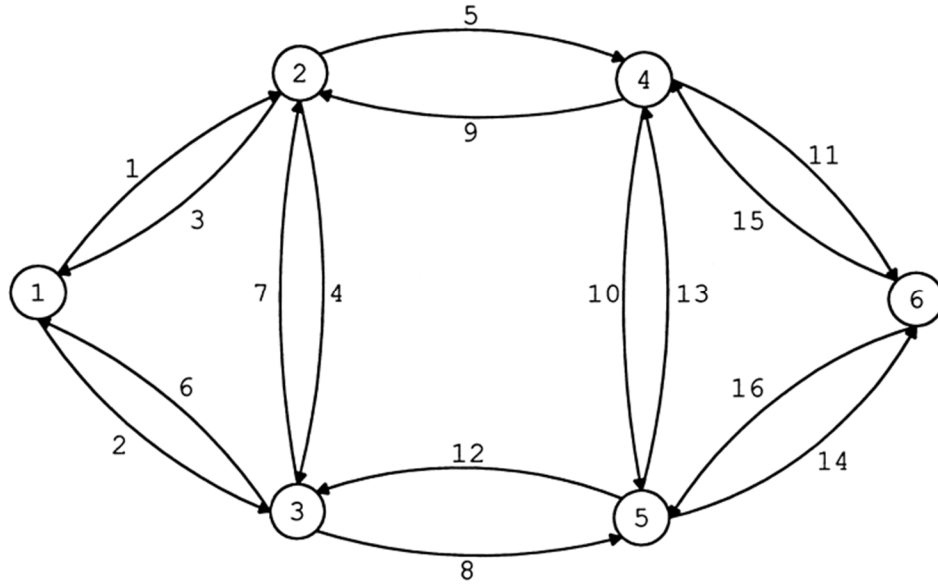
Figure 5-4. Test network

Table 5-2. Network Characteristics

Link	C_a	B_a	k_a	d_a
1	1.0	10.0	3.0	2.0
2	2.0	5.0	10.0	3.0
3	3.0	3.0	9.0	5.0
4	4.0	20.0	4.0	4.0
5	5.0	50.0	3.0	9.0
6	2.0	20.0	2.0	1.0
7	1.0	10.0	1.0	4.0
8	1.0	1.0	10.0	3.0
9	2.0	8.0	45.0	2.0
10	3.0	3.0	3.0	5.0
11	9.0	2.0	2.0	6.0
12	4.0	10.0	6.0	8.0
13	4.0	25.0	44.0	5.0
14	2.0	33.0	20.0	3.0
15	5.0	5.0	1.0	6.0
16	6.0	1.0	4.5	1.0

This test network was used as a single-period CNDP with deterministic demands (Friesz

et al., 1992; Wang and Lo, 2010; Luathep *et al*., 2011). I need to introduce a time horizon and

select additional parameters to expand the test network to a multi-period CNDP with stochastic

demands. The demand for each OD pair follows a reflective geometric Brownian motion with μ = 3% and σ = 3%; these GBM processes are assumed independent for different OD pairs. The initial demands are 5 and 10 for OD 1-6 and 6-1, respectively. The upper barriers of demand are 10 and 20 for OD 1-6 and 6-1, respectively. The annual discount rate is 7%.

The investor has a one-time opportunity to improve the capacity of any links in the test network. For each link, the maximum capacity improvement y_{max} is 20. Since Least-Square Monte Carlo applies a backward procedure and must have an expiration date, I assume that the length of the implementation period T is 5 years. So the investor can invest in year 0, 1, 2, 3, 4 and 5. The investment opportunity expires after year 5. A longer implementation can be used to approximate the case that the investment opportunity never expires. However, it does not impact the demonstration of ALSMC algorithm. The construction duration Δ is 1 year. The length of the concession period CP is 10 years. So if the investment is made in year 5, the concession period is from year 6 to year 15.

5.3.1. Net Present Value of the Project

The NPV of the capacity expansion project is the total social cost saving by investing immediately with the optimal capacity design. The total travel cost saving at each time point is given by the difference between a baseline of total travel costs without investment and total travel costs with the optimal capacity design. Total social cost saving is given by the present value of all expected total travel cost savings minus upfront investment costs:

$$\Omega(\mathbf{q}_0) =$$

$$\max_{\mathbf{y}} \left\{ \sum_{t=\Delta}^{CP} E \left\{ \sum_{a \in A} \left[C_a + B_a \left(\frac{x_{a,0}(\mathbf{q}_t)}{k_a} \right)^4 \right] x_{a,0}(\mathbf{q}_t) - \left[C_a + B_a \left(\frac{x_a(\mathbf{q}_t, \mathbf{y})}{y_a + k_a} \right)^4 \right] x_a(\mathbf{q}_t, \mathbf{y}) \middle| \mathbf{q}_0 \right\} e^{-\rho t} - d_a y_a \right\}$$

$$(5.15)$$

subject to the capacity improvement constraint:

$$0 \leq y_a \leq y_{\max} \qquad \forall a \in A \qquad (5.16)$$

In Equation (5.15), $x_{a,0}(\mathbf{q}_t)$ is the equilibrium traffic on link a at time t given demand $\mathbf{q}_t$ without capacity improvement, $x_a(\mathbf{q}_t, \mathbf{y})$ is the equilibrium traffic on link a at time t given demand $\mathbf{q}_t$ with capacity improvement plan $\mathbf{y}$. Both $x_{a,0}(\mathbf{q}_t)$ and $x_a(\mathbf{q}_t, \mathbf{y})$ can be solved using incremental traffic assignment. In ALSMC, $\Omega(\mathbf{q}_0)$ is approximated by a least-square regression model based on the total social cost saving on a single simulated demand path, with approximate capacity design y^β, which is solved using differential evolution. In Section 5.3.2, I explain how to solve CNDP with a single simulated demand path (deterministic CNDP) using differential evolution.

5.3.2. Differential Evolution

The Differential Evolution (DE) is a simple and powerful genetic algorithm which was introduced by Storn and Price (1995) to solve various optimization problems. The DE guides the initial population towards the vicinity of the global or near-global optimum solution for a given optimization problem through repeated cycles of mutation, crossover, and selection (Liu *et al*., 2010). In the DE, three control parameters are used to operationalize the optimization process. The first one is the number of populations (NP), which is the number of solution vectors used in the solution process. The second one is the mutation factor (MF), which is used to obtain mutant vector from three selected solution vectors in the population; Storn and Price (1995) recommend setting MF between 0.5 and 1. The last parameter is the crossover rate (CR), which is the probability of consideration of the mutant vector. The recommended range for the crossover rate

is [0.8, 1] by Storn and Price (1995). In this study, MF and CR are 0.5 and 0.9, respectively. The DE algorithm can be summarized as follows.

Differential Evolution Algorithm:

Step 1: Initial N (= 12) random capacity improvement plan $\mathbf{y}_i$ (i=1, 2, …, N) between 0 and y_{max}.

Step 2: For each parent $\mathbf{y}_i$, solve $x_{a,0}(\mathbf{q}_t)$ and $x_a(\mathbf{q}_t, \mathbf{y})$ using incremental traffic assignment.

   Then, calculate Ω using Equation (5.15).

Step 3: Generate child for each parent $\mathbf{y}_i$:

   With a 90% chance, do mutation and crossover: $y_{i,a} = y_{i,a} + MF*(y_{k,a} - y_{g,a})$ for all link a. k and g are two random chosen parents other than parent i. For 10% chance, the child $\mathbf{y}_i$ is the same as the parent.

   If child $y_{i,a} < 0$, set $y_{i,a} = 0$; If $y_{i,a} > y_{max}$, set $y_{i,a} = y_{max}$.

Step 4: For each child $\mathbf{y}_i$, solve $x_{a,0}(\mathbf{q}_t)$ and $x_a(\mathbf{q}_t, \mathbf{y})$ using incremental traffic assignment.

   Then, calculate Ω using Equation (5.15).

Step 5: For $i \in N$, compare the Ω value of parent $\mathbf{y}_i$ and child $\mathbf{y}_i$. If the Ω value of parent is greater than the Ω value of child, replace the parent with the child. Otherwise, keep the parent.

Step 6: Among all parents, find the best parent $\mathbf{y}_{best}$ and Ω_{best}.

Step 7: Repeat step 3-6, until Ω_{best} does not improve for 50 iterations or 500 iterations are reached.

   Many algorithms solved the test network (shown in Figure 5-4) with a single period. Differential evolution is also used to solve the test network with a single period, in order to compare their performance. The deterministic demands of two scenarios are shown in Table 5-3.

Figure 5-5 and 5-6 shows that DE converges within 500 iterations for both scenarios, which takes less than 1 minute on a 64-bit Windows 7 Intel i7-2600 4-core 3.4GHz processor with parallel computation. The solutions of two scenarios are compared with 18 algorithms proposed in previous papers (see Appendix A).

Table 5-3. Traffic demand scenarios for deterministic CNDP

Scenario	Demand from node 1 to node 6	Demand from node 6 to node 1	Maximum improvement y_{max}
I	5	10	10
II	10	20	20
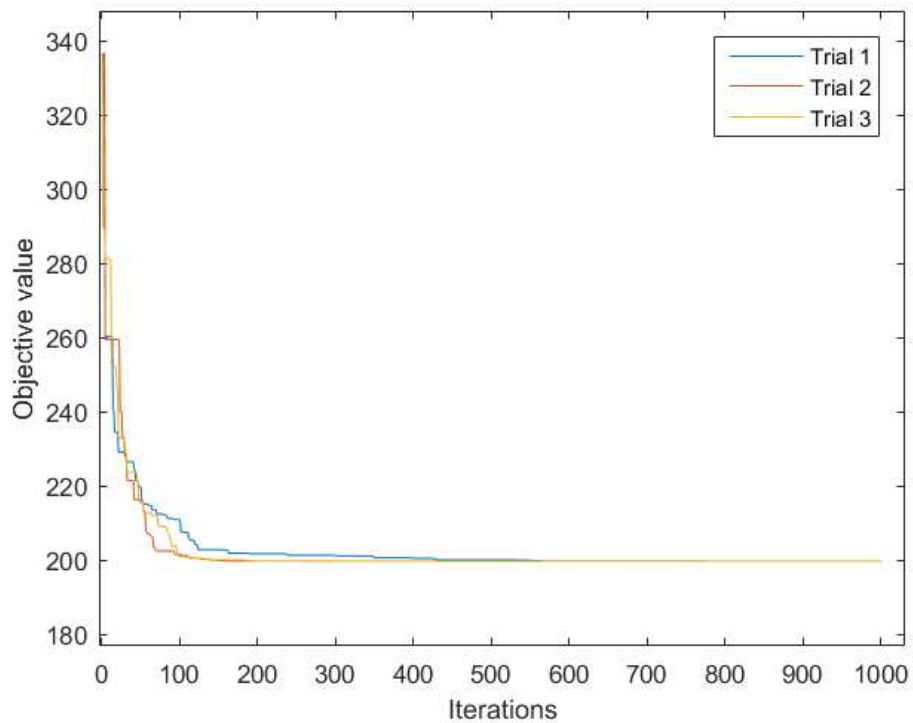
Figure 5-5. Convergence of Differential Evolution (low demand scenario)
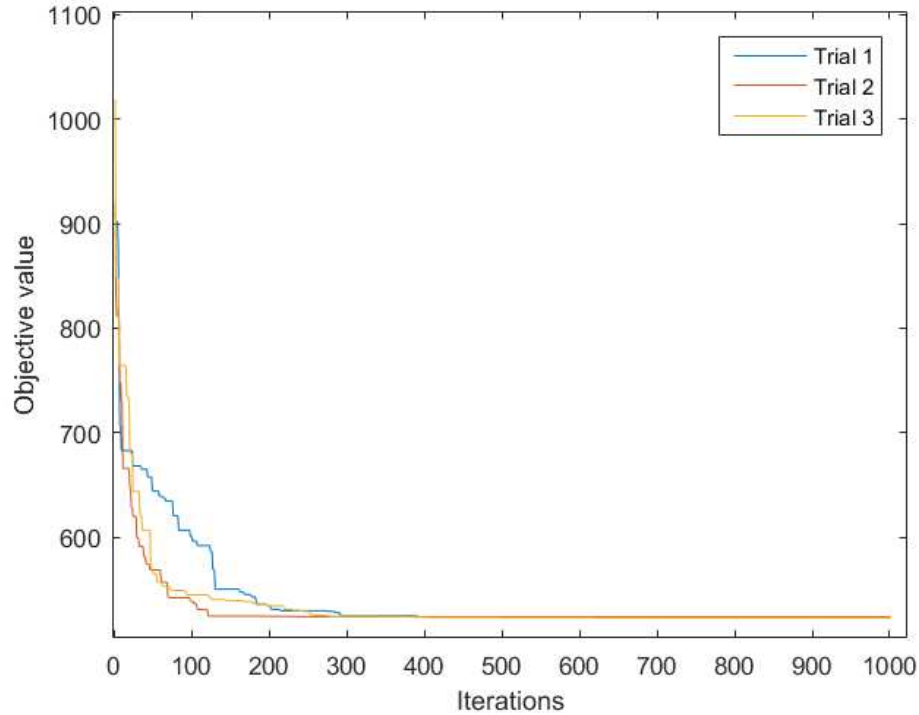
Figure 5-6. Convergence of Differential Evolution (high demand scenario)

5.3.3. Solving Multi-period CNDP with Stochastic Demand using ALSMC

ALSMC simulation is executed in Matlab 2016b with $P = 4000$ and $P' = 800$. All Matlab codes are attached in Appendix B. The total computation time is 88 hours on a 64-bit Windows 7 Intel i7-2600 4-core 3.4 GHz processor with parallel computation. The result shows that, at year 0, it is better to delay the investment, because the value of deferment is 4581, compared to the termination payoff value 2861. Allowing an option to defer the investment increases the value of the investment oppotunity by 60.1%. It also shows that even with a smaller sample size ($P = 1000$ and $P' = 200$), the values of investing immediately are already stable over different trials. But a larger sample size is necessary to get the value of waiting to converge.

Table 5-4. Value of investing immediately and waiting at t=0

Trials (P=1000, P'=200)	Value of investing at $t = 0$	Value of waiting at $t = 0$
1	2,818	4,676
2	2,885	4,358
3	3,025	4,719
4	2,888	4,573
Combination of all samples (P=4000, P'=800)	2,861	4,581

It is not practical to solve the same multi-period stochastic CNDP with the same sample size using the original LSMC simulation approach. The original LSMC method requires solving the termination payoff function repeatedly, which could be extremely time-consuming. Therefore, I am unable to compare the value of investment opportunity generated by ALSMC and LSMC. Instead, I calculate the value of investing immediately (termination payoff function) at time 0 based on 2000 simulated demand paths from time 1 to 10. Differential evolution is applied to find the optimal capacity improvement plan **y*** at time 0 and then the value of investing immediately. The total computation time is 20 hours. The value of investing immediately is 2887, which is very close to the approximated value from ALSMC, 2861. The error is within 1%. It shows that Approximate Least Square Monte Carlo simulation can provide an accurate value of the termination payoff function.

5.4. DISCUSSION

In this chapter, I propose a new algorithm - which I called "Approximate Least Square Monte Carlo simulation" (ALSMC) - that complements the original Least-square Monte Carlo simulation to solve the multi-period stochastic Continuous Network Design Problem (CNDP). The ALSMC has significant advantages over the original Least-square Monte Carlo simulation on real options problems with hard-to-solve termination payoff function. The original Least-

Square Monte Carlo (LSMC) simulation requires repeatedly solving the termination payoff function for each sample. For the multi-period CNDP with stochastic demands, solving explicitly the termination payoff function relies on demand path enumeration and finding the optimal capacity design. Usually, global search algorithms can be used to find the optimal capacity design with a large number of simulated demand paths, which is very time-consuming. By contrast, ALSMC solves each termination payoff function approximately based on a single simulated demand path, which reduces computation time dramatically. Then the termination payoff function is estimated as a polynomial function of demands using least-square regression.

In Section 5.2.3, ALSMC is tested on a simple example that can be solved analytically. Therefore, I am able to compare the exact solution with ALSMC solution. ALSMC is shown to converge with the number of demand paths increasing. Although ALSMC solution tends to overestimate the termination payoff function and the value of investment opportunity, the gap between the exact value and ALSMC's value is very smaller (within 0.5%).

In Section 5.3, ALSMC is tested on a multi-period stochastic CNDP with 16 links and 2 OD pairs. Differential evolution, which I show as a fast and accurate global search algorithm, is used to solve the termination payoff function with a single demand path. 2000 termination payoff functions are solved to estimate the least-square regression model. The ALSMC solution shows that delaying investment brings 60% more value to the project than investment immediately at time 0. The "exact" termination payoff is also calculated based on demand enumeration, which is about 1% greater than the ALSMC value. However, solving the "exact" termination payoff takes 20 hours, and the whole multi-period stochastic CNDP with 2000 number of paths and 5 implementation periods requires solving the "exact" termination payoff function 10,000 times,

which leads to a total computation time of 200,000 hours. ALSMC reduces the computation time to 88 hours and generates accurate enough solution.

CHAPTER 6 CONCLUSIONS

6.1. SUMMARY OF FINDINGS AND CONTRIBUTIONS

The contributions of this dissertation can be summarized as follows. For a private inter-city highway project (Chapter 3):

(1) It provides an analytical framework to find the optimal timing, the endogenous toll and the optimal capacity under demand uncertainty.

(2) It derives an analytical solution that enforces traffic equilibrium while using a BPR function to model congestion.

(3) The profit-maximizing toll and revenue are derived as power functions of demand between two cities. Whether the private firm has the authority to adjust toll rates makes a huge difference to the project's value. When the power of the BPR function is 4, the annual revenue grows about 5 times as fast as demand does, while with fixed toll the revenue growth rate is the same as the demand growth rate.

(4) Results show that the demand barrier has a significant influence on project revenues and therefore on the timing of the investment. With an upper barrier, the demand is limited to the sum of the capacities of both highways, and the value of the project is substantially reduced when compared to the case where demand is allowed to grow at a constant rate forever.

(5) When the project can be implemented instantly, the expression of the Net Present Value (NPV) of the new highway project is derived. Under increasing demand uncertainty, the demand threshold is also increasing until demand reaches the capacity of the old highway and cannot grow anymore. The option to defer adds substantial value to the project and traditional NPV calculations that ignore demand volatility lead to investing prematurely. However, when

construction takes time (a more realistic case), the approximate closed-form solution of the NPV is downward biased. Combined with simulation results, I find that within a reasonable range for demand uncertainty, the closed-form solution can provide an accurate estimation of true NPV and that demand threshold increases with uncertainty.

(6) By formulating the capacity choice of the new highway in terms of a number of lanes, the optimal capacity can be obtained by comparing every pair of capacity choices. First, the investment timing problem is solved with each capacity choice. Then, based on their NPVs and option values, one capacity choice could dominate the other, or each capacity choice could have its own demand threshold and investment region. It is noteworthy that the indifference demand level, at which both capacity choices have the same NPV, belongs to the waiting region. An illustration shows that the extra value added by delaying the investment at the indifference demand level can be trivial, but the waiting region between two investing regions could be more important for transit and rail technology projects.

In Chapter 4, this dissertation considers the financing of an inter-city highway when it is funded by the government or by a Public-Private Partnership (PPP). In that case,

(7) It shows how to solve the optimal investment timing problem from the government's point-of-view by Monte Carlo simulation. The "Rule of a Half" is used to approximate consumer surplus with induced demand. The system optimal constant toll rate is derived analytically, and it is always smaller than the profit-maximizing toll. A case study shows that the government's demand threshold is lower than the private firm's demand threshold. As for the private financing case, the investment should be delayed when demand volatility increases.

(8) A real options framework is developed to evaluate the non-compete clause in a public-private partnership. The non-compete clause does increase the private sector's expected profit; however,

the social benefit loss is much larger, which implies that during the negotiation of a PPP contract, the non-compete clause may not be a preferable form of government guarantee to ensure the private sector's profitability.

In Chapter 5, this dissertation extended the proposed real options framework to solve the multi-period continuous network design problem (CNDP) with stochastic demand. Contributions include:

(9) A new algorithm, called Approximate Least Square Monte Carlo simulation (ALSMC). The ALSMC can solve the multi-period stochastic CNDP significantly faster than the original LSMC by solving multiple deterministic CNDP to approximate the stochastic CNDP's termination payoff function. The ALSMC is tested on a small multi-period stochastic CNDP with 16 links and 2 OD pairs. Differential evolution is used to solve the deterministic CNDP repeatedly. The "exact" termination payoff is also calculated based on demand enumeration, which is about 1% greater than the ALSMC value. With 4000 sample size, the ALSMC simulation takes 88 hours to finish, while the original LSMC simulation would require solving the stochastic CNDP repeatedly, which could take 200,000 hours in computation time.

6.2. FUTURE RESEARCH

This dissertation assumes that demand is inelastic to transportation infrastructure investment, but it would be more realistic to model demand with a stochastic demand curve that ensures that demand is both stochastic and elastic to travel cost.

When studying the capacity choice problem in Chapter 3, future expansions of the new highway are ignored. By allowing for capacity expansion of a low-capacity project when demand grows later, allowing for future expansions would add more flexibility to the low-capacity

project and possibly make it more desirable than the high-capacity project. Another improvement would be to model demand elasticity using a stochastic demand curve. The real options method can also be used for a transit investment timing and technology selection problem, which may produce different results.

Future research on Public-Private Partnerships could look at the development of a real options model that considers the negotiation of the concession period, toll regulation, demand threshold and government guarantees simultaneously. This model would help find the optimal PPP contract that ensures a minimum rate of return for the private sector while maximizing the social benefits of the project. Moreover, the risk could be explicitly considered when looking for an optimum solution.

A case study for a large size real road network with actual demand data would better illustrate the accuracy and computational speed of the Approximate Least-Square Monte Carlo simulation. One problem that may rise when network size increases is the "curse of dimensionality" (where computations increase exponentially with network size). For an actual network with thousands of OD pairs, if we assume that the demand of each OD pair follows a different stochastic process, then the number of demand paths grows exponentially, and the problem becomes too large to solve. One solution is to classify OD pairs into groups and assume that the demand growth rates of OD pairs in the same group follow the same stochastic process. This approach would reduce the number of stochastic variables and keep the problem solvable. Another approach is to use Approximate Dynamic Programming to overcome the curse of dimensionality (Powell, 2009). All these ideas are left for future work.

APPENDIX A. Solution of the Deterministic CNDP using Differential Evolution

The benchmark network shown in Figure 5-4 has been used by many CNDP algorithms (see Table 5-4). However, instead of measuring multi-period outcomes of the capacity improvement plan, all previous papers only solved a single period of the CNDP. To compare the performance of DE with other algorithms, I solved the same single period CNDP with two demand scenarios used in previous papers. Table A-2 and Table A-3 compares DE solutions with solutions reported by papers listed in Table A-1. DE gives one of the best solutions for both scenarios within a reasonable computation time.

Table A-1. Abbreviations of algorithms for solving CNDP

Abbreviation	Name of algorithm	Source
IOA	Iterative optimization-assignment algorithm	Allsop, 1974
HJ	Hooke-Jeeves algorithm	Abdulaal and LeBlanc, 1979
EDO	Equilibrium decomposed optimization	Suwansirikul *et al.*, 1987
MINOS	Modular in-core nonlinear system	Suwansirikul *et al.*, 1987
SA	Simulated annealing algorithm	Friesz *et al.*, 1992
SAB	Sensitivity analysis-based algorithm	Yang and Yagar, 1995
AL	Augmented Lagrangian algorithm	Meng *et al.*, 2001
GP	Gradient projection method	Chiou, 2005
CG	Conjugate gradient projection method	Chiou, 2005
QNEW	Quasi-Newton projection method	Chiou, 2005
PT	PARATAN version of gradient projection method	Chiou, 2005
RELAX	Single-level nonlinear program wirh relaxation scheme	Ban *et al.*, 2006
PMILP	Path based mixed-integer linear program	Wang and Lo, 2010
LMILP	Link based mixed-integer linear program	Luathep *et al.*, 2011
NFFN	Filled function method	Hellman, 2010
DIRECT	Dividing rectangles	Hellman, 2010
EGO	Efficient global optimization	Hellman, 2010
PMC	Penalty with multicutting plane method	Li *et al.*, 2012
DE	Differential Evolution	this dissertation

Table A-2. Comparison of results for demand scenario I (low demand)

Algorithms[a]	Variables						Objective	
	y_3	y_6	y_7	y_{12}	y_{15}	y_{16}	Z^a	Gap[b]
IOA	0	6.95	0	0	5.66	1.79	214.335	7.33%
HJ	1.2	3	0	0	3	2.8	218.297	9.31%
EDO	0.13	6.26	0	0	0.13	6.26	201.197	0.75%
MINOS	0	6.58	0	0	7.01	0.22	211.25	5.78%
SA	0	3.1639	0	0	0	6.724	201.626	0.96%
SAB	0	5.8352	0	0	0.9739	6.1762	204.218	2.26%
AL	0.0062	5.2631	0.0032	0.0064	0.7171	6.7561	202.997	1.65%
GP	0	5.8302	0	0	0.87	6.109	203.78	2.04%
CG	0	6.1989	0	0	0.0849	7.5888	200.225	0.26%
QNEW	0	6.0021	0	0	0.1846	7.5438	200.619	0.46%
PT	0	5.9502	0	0	0.5798	7.1064	202.42	1.36%
RELAX	0	5.1946	0	0	0	7.5962	199.65	-0.03%
PMLP	0	5.19	0	0	0	7.5	199.628	-0.04%
LMILP	0	5.24	0	0	0	7.585	199.627	-0.04%
DE	0	5.1922	0	0	0	7.3496	199.700	0.00%

a. Refer to Table A-1 for more information of each algorithm.

b. The objective of each link capacity improvement plan is recalculated by incremental assignment with step size = 0.001 trip, which provides a more accurate value of Z than the value provided in the original paper who proposed the corresponding algorithm.

c. The percentage represent to which extent the solution provided in previous papers is inferior/superior to the DE solution.

Table A-3. Comparison of results for demand scenario II (high demand)

| Algorithms[a] | Variables | | | | | | | | | | | Objective | |
	y_1	y_2	y_3	y_6	y_7	y_8	y_9	y_{12}	y_{14}	y_{15}	y_{16}	Z^a	Gap[b]
IOA	0	4.55	10.65	6.43	0	0.59	0	0	1.32	19.36	0.78	557.842	6.69%
HJ	0	5.4	8.18	8.1	0	0.9	0	0	3.9	8.1	8.4	561.487	7.39%
EDO	0	4.88	8.59	7.48	0.26	0.85	0	0	1.54	0.26	12.52	540.198	3.32%
MINOS	0	4.61	9.86	7.71	0	0.59	0	0	1.32	19.14	0.85	557.143	6.56%
SA	0	0	10.17	5.777	0	0	0	0	0	0	17.28	533.33	2.00%
SAB	0.019	2.225	9.339	9.047	0	0.018	0	0.082	0.02	2.143	18.98	536.183	2.55%
AL	0	4.615	9.88	7.6	0.002	0.6	0.001	0.113	1.318	2.727	17.58	532.69	1.88%
GP	0.101	2.182	9.342	9.044	0	0.008	0	0.038	0.009	1.943	18.99	535.664	2.45%
CG	0.102	2.18	9.343	9.044	0	0.007	0	0.036	0.008	1.948	18.99	535.693	2.45%
QNEW	0.092	2.152	9.141	8.85	0	0.011	0	0.038	0.013	1.971	18.58	535.914	2.50%
PT	0.101	2.18	9.334	9.036	0	0.008	0	0	0.009	1.943	18.97	535.575	2.43%
RELAX	0	4.614	9.910	7.374	0	0.592	0	0	1.315	0	20	522.65	-0.04%
PMLP	0	4.41	10	7.42	0	0.54	0	0	1.18	0	19.5	522.794	-0.01%
LMILP	0	2.722	9.246	8.538	0	0	0	0	0	0	20	526.488	0.69%
NFFN	0	0.354	9.881	7.494	0	0.617	0	0	1.321	0	20	529.814	1.33%
DIRECT	0	4.623	9.872	7.412	0	0.59	0	0	1.312	0	20	522.647	-0.04%
EGO	0	4.374	7.374	17.06	0	0	0	2.893	0.432	16.15	11.2	571.619	9.33%
PMC	0	4.691	9.978	7.554	0	0.633	0	0	1.766	0	19.67	522.796	-0.01%
DE	0	4.755	10.03	7.392	0	0.717	0	0	1.590	0.018	20	522.862	0.00%

a. Refer to Table 5-4 for more information of each algorithm.

b. The objective of each link capacity improvement plan is recalculated by incremental assignment with step size = 0.001 trip, which provides a more accurate value of Z than the value provided in the original paper who proposed the corresponding algorithm.

c. The percentage represent to which extent the solution provided in previous papers is inferior/superior to the DE solution.

APPENDIX B. Matlab Code for Approximate Least-Square Monte Carlo Simulation

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%    Main    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc;clear;
% <1> Parameter setting
P = 4000; % Number of simulated demand paths
n = 800; % P'
t = 1; % option exercise interval (year)
T = 5; % Implementation periods (years)
CP = 10; % Concession period (years)
m = 1 + T + CP; % length of simulated demand paths (years)
degree = 5; % Highest degree of polynomial basis function
num_od = 2; %number of OD pairs
q_start = [5,10]; % Initial demands
qhigh = [10,20]; % Upper barrier of demands
discount = 0.07; % Discount rate
mu = 0.03; % Annual demand growth rate
sigma = 0.03; % Annual demand volatility
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% <2> Generate stochastic demand paths
q = zeros(P,m,num_od); % Allocate memory for demand paths
for od = 1: num_od
    for i = 1:m
        if i==1
            q(:,1,od) = ones(P,1)*q_start(od);
        else
            w = normrnd(0,sqrt(t),[P,1]); % Random term
            q(:,i,od)=min(qhigh(od),q(:,i-1,od).*exp( (mu-
sigma^2/2)*t*ones(P,1) + sigma*w ));
        end
    end
end
q(:,1,:)=[];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% <3.1> Estimate NPV for the first n demand paths using differential
evolution
NPV = NPV_network(q(1:n,:,:),n,T,CP,discount);
% <3.2> Least-Square regression
NPV_State=reshape(NPV(:,1:T),[T*n,1]);
X = polynomial(q(1:n,1:T,:), degree);
Alpha = regress(NPV_State,X);
% <3.3> Estimate NPV for ALL demand paths using the Least-Square model
from <3.2>
X = polynomial(q(:,1:T,:), degree);
estNPV = X*Alpha;
% <3.4> Estimate NPV of investing immediately at time 0
X = polynomial(q_start, degree);
estNPV0 = X*Alpha;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% <4> Least-Square Monte Carlo simulation
estNPV = reshape(estNPV,[P,T]);
```

```matlab
estNPV = max(0,estNPV);
value = zeros(P,T);% Value of continuation
dotime = T*ones(P,1); % Indicate when to invest
B = zeros(T-1,2*degree+1);
for i = T-1:-1:1
    idx = find(estNPV(:,i)>0); %use only in-the-money paths
    X = polynomial(q(idx,i,:), degree);
    if i == T-1
        Beta = regress(estNPV(idx,T)*exp(-discount),X);
        estDefer = X*Beta;
        B(i,:) = Beta;
        % Est. continuation is smaller than exercise
        idx_do = estDefer <= estNPV(idx,i);
        value(:,T) = estNPV(:,T);
    else
        Beta = regress(value(idx,i+1)*exp(-discount),X);
        estDefer = X*Beta;
        B(i,:) = Beta;
        % Est. continuation is smaller than exercise
        idx_do = estDefer <= estNPV(idx,i);
    end
    value(:,i) = value(:,i+1)*exp(-discount);
    value(idx(idx_do),i) = estNPV(idx(idx_do),i);
    dotime(idx(idx_do),1)= i*ones(size(idx(idx_do),1),1);

end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Expected value of waiting at time 0
ValueOfDefer = mean(value(:,1))*exp(-discount);
fprintf('Value of delaying investment is %8.2f.\n',ValueOfDefer);
fprintf('Value of investing immediately is %8.2f.\n',estNPV0);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```matlab
%%%%%%%%%%%%%%%%% Functions used in the main code %%%%%%%%%%%%%%%%%%%
function X = polynomial(q, degree)
[row,col,num_od] = size(q);
if num_od ==1 && row == 1
    X = zeros(1,col*degree+1);
    X(:,1) = 1;
    i = 2;
    for od = 1:col
        for k = 1:degree
            X(:,i) = q(1,od).^k;
            i = i+1;
        end
    end
else
    X = zeros(row*col,num_od*degree+1);
    X(:,1) = ones(row*col,1);
    i = 2;
    for od = 1:num_od
        State = reshape(q(:,:,od),[row*col,1]);
        for k = 1:degree
            X(:,i) = State.^k;
            i = i+1;
        end
    end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```matlab
%%%%%%%%%%%%%%%% Functions used in the main code %%%%%%%%%%%%%%%%%%
function NPV = NPV_network(q,n,T,CP,discount)
tic;
% <1> Calculate NPV of investing immediately for the first n demand
paths
hwait = waitbar(0,'Wait');
NPV_invest = zeros(n,T);
for i = 1:n
    str = ['Running Differential Evolution: ',num2str(100*i/n),'%
finished'];
    waitbar(i/n,hwait,str);

    for j = 1:T
        q16 = q(i,j+1:j+CP,1);
        q61 = q(i,j+1:j+CP,2);
        NPV_invest(i,j) = DE(q16,q61,CP,discount);
        toc
    end
end
close(hwait);

% <2> Use Incremental assignment to get baseline NPV without capacity
expansion
NPV_base = zeros(n,T + CP);
parfor i = 1:n
    for j = 1:T + CP
        NPV_base(i,j) = TAP(q(i,j,1),q(i,j,2),zeros(16,1));
    end
end

% <3> Calculate the difference between NPV with and without investment
NPV = zeros(n,T);
for i = 1:n
    for j = 1:T
        NPV(i,j) = NPV_base(i,j+1:j+CP)*transpose(exp(-
discount*(1:CP))) - NPV_invest(i,j);
    end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```matlab
%%%%%%%%%%%%%%%%% Functions used in the main code %%%%%%%%%%%%%%%%%%%%
function output=DE(q16,q61,CP,discount)
d=[2 3 5 4 9 1 4 3 2 5 6 8 5 3 6 1]; % Unit construction cost for each
link
%%%%%%%%%%%%%%%%%%%%%    SETTING    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% How many iterations?
iterations =  500;
%How much Population?
N = 12;
%How many parameters?
s = 16;
Capacity = zeros(s,iterations);
%Crossover Rate, between 0 and 1?
CR = 0.9;
% How much is MF, between 0 and 2?
MF = 0.5;
% Give the parameter ranges (minimum and maximum values)
Parmin = 0;
Parmax = 20;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
y = zeros(s,N);
z = y;
CF = zeros(N,CP);
Obj = zeros(iterations+1,1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%<1> Generate sample
    rd = rand(s,N);
    y = Parmin + rd.*(Parmax-Parmin);

%<2> Fitness calculation
    parfor i = 1:N
        for j = 1:CP
            CF(i,j) = TAP(q16(j),q61(j),y(:,i));
        end
    end
    F = exp(-discount*(1:CP)) * transpose(CF) + d*y;
    best = find(F==min(F),1);
    Obj(1)=F(best);

    iter = 1;
    while iter <= iterations

% <3> Mutation
    for i = 1:N
        rab = randperm(N-1);
        a = rab(1)+(rab(1)>=i)*1;
        b = rab(2)+(rab(2)>=i)*1;
        z(:,i) = y(:,i)+(rand(s,1)<=CR)*MF.*(y(:,a)-y(:,b));
    end

% <4> regularization
    z = min(Parmax,max(Parmin,z));
```

```matlab
% <5> Fitness calculation
    parfor i = 1 : N
        for j = 1:CP
            CF(i,j) = TAP(q16(j),q61(j),z(:,i));
        end
    end
    G = exp(-discount*(1:CP)) * transpose(CF) + d*z;
% <6> Crossover
    idx = find(G < F);
    for i = idx
        y(:,i) = z(:,i);
    end
    F = min(G, F);
    iter = iter+1;
    best = find(F==min(F),1);
    Obj(iter) = F(best);
end
output = Obj(iter); % Optimal NPV
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```matlab
%%%%%%%%%%%%%%%% Functions used in the main code %%%%%%%%%%%%%%%%%%%
function STCost = TAP(q16,q61,capacity) %The input y is a vector of
16-link capacity improvement
%        d        A      B      K
data = [2,       1,     10,      3;
        3,       2,      5,     10;
        5,       3,      3,      9;
        4,       4,     20,      4;
        9,       5,     50,      3;
        1,       2,     20,      2;
        4,       1,     10,      1;
        3,       1,      1,     10;
        2,       2,      8,     45;
        5,       3,      3,      3;
        6,       9,      2,      2;
        8,       4,     10,      6;
        5,       4,     25,     44;
        3,       2,     33,     20;
        6,       5,      5,      1;
        1,       6,      1,    4.5];

matrix16 = [ 1      0      0      0      1      0      0      0      0      0
1      0      0      0      0      0;
       0      1      0      0      0      0      0      1      0      0      0
0      0      1      0      0;
       0      1      0      0      1      0      1      0      0      0      1
0      0      0      0      0;
       0      1      0      0      1      0      1      0      0      1      0
0      0      1      0      0;
       1      0      0      0      1      0      0      0      0      1      0
0      0      1      0      0;
       0      1      0      0      0      0      0      1      0      0      1
0      1      0      0      0;
       1      0      0      1      0      0      0      1      0      0      1
0      1      0      0      0;
       1      0      0      1      0      0      0      1      0      0      0
0      0      1      0      0];

matrix61 = [ 0      0      1      0      0      0      0      0      1      0
0      0      0      0      1      0;
       0      0      0      0      0      1      0      0      0      0      0
1      0      0      0      1;
       0      0      0      1      0      1      0      0      1      0      0
0      0      0      1      0;
       0      0      0      1      0      1      0      0      1      0      0
0      1      0      0      1;
       0      0      1      0      0      0      0      0      1      0      0
0      1      0      0      1;
       0      0      0      0      0      1      0      0      0      1      0
1      0      0      1      0;
       0      0      1      0      0      0      1      0      0      1      0
1      0      0      1      0;
```

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```
     0     0     1     0     0     0     1     0     0     0     0
1     0     0     0     1];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
step = 0.1; % Step size of flow loading
OD61 = q61; % Demand from centroid 6 to centroid 1
OD16 = q16; % Demand from centroid 1 to centroid 6
x = zeros(16,1); % Link flow

for i = step:step:OD16 % For OD pair 1-6
    t = data(:,2) + data(:,3).*x.^4./(data(:,4)+capacity).^4;
    path16 = matrix16*t; % Calculate path travel time
    row = path16==min(path16); % Find shortest path
    x = x + transpose(matrix16(row,:))*step; %Load 0.1 flow on the
shortest path
end
for i = step:step:OD61 % For OD pair 6-1
    t = data(:,2) + data(:,3).*x.^4./(data(:,4)+capacity).^4;
    path61 = matrix61*t; % Calculate path travel time
    row = path61==min(path61); % Find shortest path
    x = x + transpose(matrix61(row,:))*step; % Load 0.1 flow on the
shortest path
end
t = data(:,2) + data(:,3).*x.^4./(data(:,4)+capacity).^4;
STCost = transpose(x)*t;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

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