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Supergravity Inspired Warped Compactifications and Effective Cosmological Constants

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Abstract

We propose a supergravity derivation of the Randall–Sundrum action as an effective description of the dynamics of a brane coupled to the bulk through gravity only. The cosmological constants in the bulk and on the brane, which appear at the classical level when solving the supergravity equations of motion, are related to physical quantities like the brane electric charge and thus inherit some of their physical properties. The most appealing property is their quantization: in $d_\perp$ extra dimensions, Λ_{brane} goes like $1/N$ and Λ_{bulk} like $N^{2/(d_\perp-2)}$. This supergravity origin also explains the apparent fine-tuning required in the Randall–Sundrum scenario. In our approach, the cosmological constants are derived parameters and cannot be chosen arbitrarily; instead they are determined by the underlying supersymmetric Lagrangian. D3-branes of type *IIB* superstring theory provide an explicit realization of our construction.

1 Introduction

The coexistence of two hierarchical scales in particle physics is probably the most challenging puzzle to solve before hoping to construct a quantum theory of gravity. When the Schwarzchild radius ($R_{\rm Sch} = 2\mathcal{G}_N m/c^2$) of a system of mass m becomes of the same order as its Compton length ($\Lambda_C = \hbar/mc$), a quantum mechanical extension of general relativity is surely needed. Therefore the natural scale of quantum gravity is the Planck mass, $\sqrt{\hbar c^5/\mathcal{G}_N} \sim 10^{19}$ GeV. Understanding how, in such a theory, the tiny electroweak scale observed in experimental particle physics can arise and be stabilized against radiative corrections constitutes the so-called 'gauge hierarchy problem'. In low energy supersymmetry [1], this vast disparity in scales can be protected from quantum destabilization. However a more fundamental explanation is certainly to be found in string theory and its latest developments. String theory relates the string scale to two other fundamental scales, namely the GUT scale connected to gauge interactions, and the Planck scale connected to the gravitational interaction. The link between these two is the geometry of extra dimensions, which can lower both scales [2] down to the TeV range [3] and thus partially answer the gauge hierarchy problem, or at least translate it into geometrical terms.

Subsequent to studies of thin shells in general relativity [4] and their revival in a M-theory context [5–7], Randall and Sundrum (RS) have recently proposed [8] a new phenomenological mechanism for solving the gauge hierarchy problem, without requiring the extra dimension to be particularly large or small–in fact it could be noncompact. An exponential hierarchy is generated by the localization of gravity near a self-gravitating brane with positive tension, obtained by solving Einstein equations. The solution is a nonfactorizable metric, *i.e.,* a metric with an exponentially decaying warp factor [9] along the single extra dimension. Restricting the Standard Model to a second parallel brane with negative tension at some distance in this transverse dimension, the electroweak scale in our world then follows from a redshifting of the Planck scale on the second brane. Since the exponential suppression by the redshift factor does not require an unnaturally large interbrane separation, the hierarchy problem can be explained without fine tuning, and without requiring any special size for the extra dimensions.

The cosmological implications of this scenario have been studied [10, 11], with emphasis on the danger of placing the Standard Model on a brane with negative tension since, for instance, the Friedmann equation governing the expansion of the universe appears with a wrong sign. A similar difficulty is also faced [12] when trying to reproduce the unification of gauge couplings. The original scenario can be modified [8, 12, 13] by maximizing the warp factor on the Standard Model brane, which can be achieved if its tension is taken positive. The two former problems are overcome but the electroweak scale seems now difficult to accommodate. More recently it has been shown that the correct cosmological expansion can be obtained if the second brane tension is negative, but not too much so [14]. Thus the RS scenario remains attractive, especially with regard to the possibility of an infinite extra dimension probed only by gravity. It is appealing that, despite a continuous Kaluza–Klein spectrum without any mass gap, Newton's law of gravity is still reproduced [8, 13, 15] within the current experimental precision. Ref. [16] also proposed explicit models where a mass gap separates the 'massless graviton' from its KK excitations while the Yukawa type deviations from the 4D Newton law remain compatible with experimental bounds.

Although the gravity localization mechanism seems to be specific to codimension one branes, several works [15, 17] have managed to extend it by considering many intersecting codimension one branes.[1] Oda and Hatanaka *et al.* [19] also obtain solutions with a more involved content of branes with a single one extra-dimension. In this context also, cosmological expansion can be reconciled with the solution to the weak scale hierarchy problem [14].

Undoubtedly, the localization of gravity by the RS mechanism has rich phenomenological and cosmological consequences [10–15, 19–22]; but at the present stage it seems lacking in generality, and it suffers from apparently *ad hoc* fine-tunings required between the cosmological constants in the bulk and on the branes, in order to obtain a solution to Einstein equations. Verlinde [23] has reexamined the RS scenario in superstring language and shown that the warp factor can be interpreted as a renormalization group scaling. In the context of the AdS/CFT correspondence, the extra dimension plays the role of the energy scale.

In this paper, we offer a derivation of the effective action used by RS, starting from a more fundamental, string-inspired origin. Recent works [5, 7, 24, 25] have studied the dynamics of a supersymmetric brane-universe; here we propose an explicit embedding of the RS model in supergravity theories and examine its physical implications, following refs. [16, 26], which have previously addressed this question at a more formal level. Our starting point will be the bosonic action of supergravity theories in ten or eleven dimensions. We emphasize that, instead of neglecting various fields specific to these actions like the dilaton and some n-differential forms, taking them into account can lead to an effective description in terms of cosmological constants. Using p-brane solutions, we construct such a description for codimension one branes, which allows us to identify the effective cosmological constants with physical quantities like the electric charge carried by the brane and its mass density on the worldvolume. Since the electric charge of a p-brane obeys a generalized Dirac quantization rule, we are led to the interesting conclusion that the cosmological constants are also quantized.

The advantage of this approach is that we derive the stress-energy tensor $T_{\hat{\mu}\hat{\nu}}$, which is needed to solve the Einstein equations, starting from an action for fundamental fields, rather than putting it in by hand. Thus our $T_{\hat{\mu}\hat{\nu}}$ is on the same footing as the Einstein tensor itself, from the point of view of fundamentality, since it follows from a symmetry principle: namely, supersymmetry fixes the form or $T_{\hat{\mu}\hat{\nu}}$ and constrains the values of the couplings appearing therein. Moreover we are able to generalize the procedure to higher codimension brane-universes (*e.g.*, 3-branes embedded in more than one extra dimension), providing some of the first such solutions. In this case the bulk energy is no longer a cosmological "constant," but depends on the distance from the brane.

[1] See also ref. [18] for a recent construction of warped compactification in two transverse dimensions.

2 Brane cosmological constant as a warp in an anti-de Sitter bulk

We begin with a review of the model studied by Randall and Sundrum [8]. This model is a particular case of the ones proposed by Chamblin and Reall [6], in which a scalar field was coupling a dynamical brane to an embedding bulk. Here we consider the restricted scenario of a static brane embedded in a spacetime curved by a bulk cosmological constant Λ_b. The physics of this model is governed by the following action:

$$S_{RS} = \int d^{p+1}x\, d^{d_\perp}y\, \sqrt{|g|} \left(\frac{\mathcal{R}}{2\kappa^2} - \Lambda_b - \Lambda \delta^{d_\perp}(\sqrt{|g_\perp|}\, y) \right) , \qquad (1)$$

where $y^I = 0$ is the location of the brane in the transverse (extra dimensional) subspace and $g_\perp$ is the determinant of the metric, assumed to be factorizable, in this subspace. The Einstein equations derived from (1) when the transverse space is flat are (Greek indices denote longitudinal coordinates, $\mu = 0 \ldots p-1$ and Latin indices are coordinates transverse to the brane, $I = 1 \ldots d_\perp$):

$$G_{\mu\nu} = -\kappa^2 \left(\Lambda_b + \Lambda \delta^{d_\perp}(\sqrt{|g_\perp|}\, y) \right) g_{\mu\nu} ; \qquad (2)$$

$$G_{IJ} = -\kappa^2 \Lambda_b\, g_{IJ} . \qquad (3)$$

Randall and Sundrum solved these equations in the case of a codimension one brane. With the ansatz

$$ds^2 = a^2(y)\, dx^\mu \otimes dx^\nu \eta_{\mu\nu} + b^2(y)\, dy \otimes dy , \qquad (4)$$

the Einstein equations reduce to

$$p\frac{a''}{a} + \frac{p(p-1)}{2} \left(\frac{a'}{a} \right)^2 - p\frac{a'}{a}\frac{b'}{b} = -\kappa^2 \left(\Lambda_b + \Lambda\, \delta(|b|y) \right) b^2 ; \qquad (5)$$

$$\frac{p(p+1)}{2} \left(\frac{a'}{a} \right)^2 = -\kappa^2 \Lambda_b\, b^2 , \qquad (6)$$

where primes denote derivatives with respect to the transverse coordinate y. For this system of equations to admit a solution that matches the singular terms, a fine-tuning between Λ_b and Λ is necessary:

$$\Lambda_b = -\frac{p+1}{8p} \kappa^2 \Lambda^2 . \qquad (7)$$

A general solution then takes the form:

$$a(y) = f(|y|) \quad \text{and} \quad b(y) = \mathcal{N} \frac{f'(|y|)}{f(|y|)} , \qquad (8)$$

where f is a regular function and the constant $\mathcal{N}$ is related to the brane cosmological constant by: $|\mathcal{N}| = -2p\epsilon/(\kappa^2\Lambda)$, ϵ being the sign of $f'(0)/f(0)$. A particular class of solution that will play an important role in our analysis corresponds to:

$$a(y) = (l + |y|/R)^{n_a} \quad \text{and} \quad b(y) = \frac{n_a \mathcal{N} R^{-1}}{l + |y|/R} , \tag{9}$$

where R and l are two positive constants. An appropriate change of coordinates brings this solution to the form proposed by Randall and Sundrum [8]: defining $X^\mu = l^{n_a} x^\mu$ and $Y = \text{sgn}(y) n_a \mathcal{N} \ln(1 + |y|/(Rl))$, the metric reads:

$$ds^2 = e^{2\,\text{sgn}(n_a)|Y/\mathcal{N}|}\, dX^2 + dY^2 . \tag{10}$$

If the brane located at the origin is identified as the "Planck brane" of Lykken–Randall [13], an electroweak scale will be generated on the "TeV brane" if and only if the power n_a is negative, which corresponds to a positive cosmological constant on the Planck brane.[2] Another motivation for requiring $n_a < 0$ comes from computing the four-dimensional effective Planck mass, $M_{Pl}^2 = M^3 \int dy\, a^2 |b|$, which is finite for $n_a < 0$ but diverges for $n_a > 0$.

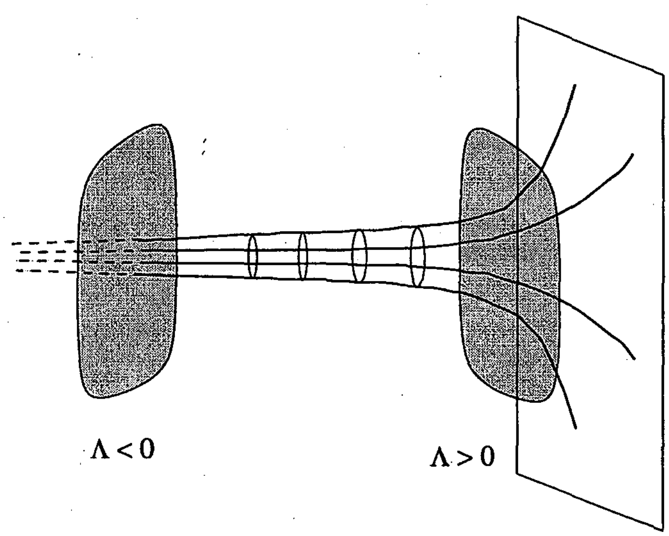

Figure 1. The boundary of an anti-de Sitter of dimension $p+2$ space is topologically $S^1 \times S^p$. In the system of coordinates x^μ and r, this boundary is located at $r = 0$ and $r = \infty$: the piece at infinity is a $p+1$-dimensional Minkowskian space, while the horizon at $r = 0$ corresponds to the union of a point and $\mathbb{R} \times S^p$. A codimension one brane embedded in this AdS space acts as a warp in the sense that it cuts a part of the bulk: a brane with a positive cosmological constant cuts the vicinity of the boundary located at the infinity, while a brane with a negative cosmological constant removes the horizon at the origin.

We can make another diffeomorphism that clarifies the geometry of the solution. Defining

[2] This connection between the signs of n_a and Λ is specific to one transverse dimension. In section 4, we will see that we can have $n_a > 0$ whereas $\Lambda > 0$. In any case, the discussion about the hierarchy problem deals with the sign of n_a only.

$r = R_0(l + |y|/R)^{n_a}$, with $R_0 = |\mathcal{N}R|$, we now obtain:

$$ds^2 = \left(\frac{r}{R_0}\right)^2 dx^2 + \left(\frac{R_0}{r}\right)^2 dr^2 \ , \tag{11}$$

where we see that the geometry of the bulk corresponds to an anti-de Sitter space of radius R_0, or at least a slice of an anti-de Sitter space, since the variable r ranges only over a part of $\mathbb{R}$. Indeed, for $n_a > 0$, the range of variation of r is restricted to $[l^{n_a}, +\infty)$, while for $n_a < 0$ this range becomes $[0, l^{n_a}]$. Although in both cases the whole AdS space is covered in the limit $l \to 0$, it is interesting to note which part is cut when $l \neq 0$. As we will argue in the appendix, the boundary of an anti-de Sitter space of dimension $p+2$ space is topologically $S^1 \times S^p$, and in the system of coordinates x^μ and r, this boundary is located at $r = 0$ and $r = \infty$: the piece at infinity is a $p+1$-dimensional Minkowskian space, while the horizon at $r = 0$ corresponds to the union of a point and $\mathbb{R} \times S^p$. So the $n_a < 0$ case, which corresponds to a positive cosmological constant Λ on the brane, removes the part at infinity, while the $n_a > 0$ case, *i.e.* $\Lambda < 0$, cuts the horizon at the origin. Note that in the AdS/CFT correspondence [27], a superconformal theory describes the dynamics of a brane near the horizon of an AdS space while this dynamics should become free near infinity [28].

As presented, the model studied by Randall and Sundrum leaves one wondering whether it can be derived from some more fundamental starting point. In particular, the *ad hoc* fine-tuning between the cosmological constants is rather mysterious and begs for a better understanding. One suggestion is that this relation might arise from the requirement that tadpole amplitudes are zero in the underlying string theory [11]. (See also ref. [29] for recent progress in this direction). Here we will see the cosmological constants as effective parameters which cannot be chosen arbitrarily, so the fine-tuning problem is ameliorated. The aim of this work is to motivate the RS model from a supersymmetry/superstring framework.

3 Effective cosmological constants from dynamics of codimension one branes

In this section, we would like to show that the theory derived from the action (1) can be seen as an effective description of a brane of codimension one, *i.e.*, of an extended object with p spatial dimensions embedded in a $(p+2)$ dimensional spacetime.

The dynamics of an object extended in p spatial directions is governed by the generalization of the Nambu–Goto action[3] [30]:

$$S_{NG} = -M_b^{p+1} \int d^{p+1}\xi \sqrt{\left| \det \left(\frac{\partial X^{\hat\mu}}{\partial \xi^a} \frac{\partial X^{\hat\nu}}{\partial \xi^b} g_{\hat\mu\hat\nu} \right) \right|} \ , \tag{12}$$

where $X^{\hat\mu}(\xi^a)$ are the coordinates in the embedding spacetime of a point on the brane characterized by its worldvolume coordinates ξ^a; M_b is the scale mass in so-called "p-brane

[3]Concerning the indices, our conventions will be the following: hatted Greek indices are spacetime indices ($\hat\mu = 0 \ldots D-1$) while Latin indices are worldvolume indices($a = 0 \ldots p$).

units" which is simply related to the Planck scale, M, in the embedding spacetime; see below eq. (16). This action is known [31] to be equivalent to:

$$\mathcal{S}_P = M_b^{p+1} \int d^{p+1}\xi \left(-\frac{1}{2}\sqrt{|\gamma|}\gamma^{ab}\partial_a X^{\hat{\mu}} \partial_b X_{\hat{\mu}} + \frac{p-1}{2}\sqrt{|\gamma|} \right) , \tag{13}$$

where γ_{ab} is an auxiliary field that gives the metric on the worldvolume.

Superbranes have been constructed [32] as classical solutions of supergravity theories in ten or eleven dimensions: they are BPS objects, since they preserve half of the supersymmetries; they have a Poincaré invariance on their worldvolume universe and also a rotational invariance in the transverse space. A p-brane is therefore coupled to the low-energy effective theory of superstrings. Below the fundamental energy scale, identified as the energy of the first massive excitations of the string, the theory can be described by supergravity theories whose bosonic spectrum contains the metric, a scalar field (the dilaton) and numerous differential forms. The bosonic effective action, in supergravity units, takes the general form ($\kappa^2 = M^{2-D}$):

$$\mathcal{S}_{eff} = \int d^D x \sqrt{|g|} \left(\frac{1}{2\kappa^2} \mathcal{R} - \frac{1}{2}\partial_{\hat{\mu}}\Phi \partial^{\hat{\mu}}\Phi - \frac{1}{(p+2)!} e^{\alpha_p \Phi} F_{\hat{\sigma}_1 \ldots \hat{\sigma}_{p+2}} F^{\hat{\sigma}_1 \ldots \hat{\sigma}_{p+2}} \right) , \tag{14}$$

where $F_{\hat{\mu}_1 \ldots \hat{\mu}_{p+2}} = (p+2)\, \partial_{[\hat{\mu}_1} A_{\hat{\mu}_2 \ldots \hat{\mu}_{p+2}]}$ is the field strength of the $(p+1)$-differential form A, whose coupling to the dilaton is measured by the coefficient α_p. The coefficient α_p is explicitly determined by a string computation: the coupling of the dilaton to differential forms from the Ramond-Ramond sector appears at one loop and thus $\alpha_p^{RR} = (3-p)/2$ in supergravity units, while the Neveu-Schwarz–Neveu-Schwarz two-form couples at tree level, so $\alpha_1^{NS} = -1$. In some cases, we can also add a Chern–Simons term ($A \wedge F \wedge F$) to the action, but it does not have any effect on the classical solutions.

The p-brane couples to a $(p+1)$-differential form, which results in the addition of a Wess–Zumino term to the free action (13):

$$\mathcal{S}_P = M^{p+1} \int d^{p+1}\xi \left(-\frac{1}{2}\sqrt{|\gamma|}\gamma^{ab}\partial_a X^{\hat{\mu}} \partial_b X^{\hat{\nu}} g_{\hat{\mu}\hat{\nu}}(X)\, e^{\beta_p \Phi} + \frac{p-1}{2}\sqrt{|\gamma|} \right.$$
$$\left. + \frac{A_{WZ}}{(p+1)!} \epsilon^{a_1 \ldots a_{p+1}}\, \partial_{a_1} X^{\hat{\mu}_1} \ldots \partial_{a_{p+1}} X^{\hat{\mu}_{p+1}} A_{\hat{\mu}_1 \ldots \hat{\mu}_{p+1}} \right) . \tag{15}$$

The functions $g_{\hat{\mu}\hat{\nu}}$ and $e^{\beta_p \Phi}$ implicitly depend on worldvolume coordinates ξ through their dependence in the embedding coordinates X. The coefficient β_p defines the "p-brane units;" it is fixed [33] by requiring the same scaling behavior for $\mathcal{S}_{eff}$ and $\mathcal{S}_P$, which leads to

$$\beta_p = -\frac{\alpha_p}{p+1} . \tag{16}$$

The relation between M_b and M then follows from the value of this coupling to the dilaton: $M_b = e^{\beta \phi_\infty/(p+1)} M$, ϕ_∞ being the vacuum expectation value of the dilaton.

To proceed, we must now relax some of the constraints imposed by supersymmetry, while still maintaining the form of the action. For example in string theories, the values of

p and D are related to one another in order to have supersymmetry on the worldvolume universe [34]. Also, as just mentioned, the coupling to the dilaton is fixed. By relaxing these constraints, we give up any claim that the following construction is a direct consequence of string theory. On the other hand it might be hoped that our results will persist in a realistic low energy limit of string theory, which includes the effects of supersymmetry breaking. In what follows, we will elucidate how the various fields, which play a crucial role for the existence of branes in supergravity, can give rise to an effective stress-energy tensor which resembles the cosmological constant terms needed for the Randall–Sundrum scenario.

The equations of motion derived from $\mathcal{S}_{eff} + \mathcal{S}_P$ are

$$\begin{aligned} G_{\hat{\mu}\hat{\nu}} &= \kappa^2\, \partial_{\hat{\mu}}\Phi\partial_{\hat{\nu}}\Phi + \frac{2\kappa^2}{(p+1)!}\, e^{\alpha_p\Phi}\, F_{\hat{\mu}\hat{\sigma}_1\ldots\hat{\sigma}_{p+1}} F_{\hat{\nu}}{}^{\hat{\sigma}_1\ldots\hat{\sigma}_{p+1}} \\ &\quad + \frac{1}{2}\left(-\kappa^2\, \partial_{\hat{\sigma}}\Phi\partial^{\sigma}\Phi - \frac{2\kappa^2}{(p+2)!}\, e^{\alpha\Phi}\, F_{\hat{\sigma}_1\ldots\hat{\sigma}_{p+2}} F^{\hat{\sigma}_1\ldots\hat{\sigma}_{p+2}}\right) g_{\hat{\mu}\hat{\nu}} + T_{\hat{\mu}\hat{\nu}}\ ; \end{aligned} \tag{17}$$

$$D_{\hat{\mu}}D^{\hat{\mu}}\Phi = \frac{\alpha_p}{(p+2)!}\, e^{\alpha_p\Phi}\, F_{\hat{\sigma}_1\ldots\hat{\sigma}_{p+2}} F^{\hat{\sigma}_1\ldots\hat{\sigma}_{p+2}} + T_\Phi\ ; \tag{18}$$

$$\partial_{\hat{\mu}_0}\left(\sqrt{|g|}\, e^{\alpha_p\Phi}\, F^{\hat{\mu}_0\ldots\hat{\mu}_{p+1}}\right) = J^{\hat{\mu}_1\ldots\hat{\mu}_{p+1}}\ ; \tag{19}$$

$$\gamma_{ab} = \partial_a X^{\hat{\mu}}\partial_b X^{\hat{\nu}} g_{\hat{\mu}\hat{\nu}}\, e^{\beta_p\Phi}\ ; \tag{20}$$

$$\begin{aligned} \partial_a\left(\sqrt{|\gamma|}\gamma^{ab}\partial_b X^{\hat{\nu}} g_{\hat{\mu}\hat{\nu}} e^{\beta_p\Phi}\right) &= \frac{1}{2}\sqrt{|\gamma|}\gamma^{ab}\partial_a X^{\hat{\sigma}_1}\partial_b X^{\hat{\sigma}_2}\partial_{\hat{\mu}}\left(g_{\hat{\sigma}_1\hat{\sigma}_2} e^{\beta_p\Phi}\right) \\ &\quad - \frac{\mathcal{A}_{WZ}}{(p+1)!}\,\epsilon^{a_1\ldots a_{p+1}}\,\partial_{a_1}X^{\hat{\sigma}_1}\ldots\partial_{a_{p+1}}X^{\hat{\sigma}_{p+1}}\, F_{\hat{\mu}\hat{\sigma}_1\ldots\hat{\sigma}_{p+1}}\ . \end{aligned} \tag{21}$$

The stress-energy tensor $T_{\hat{\mu}\hat{\nu}}$ of the brane is given by

$$T_{\hat{\mu}\hat{\nu}} = -\frac{M^{p+1}}{\kappa^2}\int d^{p+1}\xi\,\sqrt{|\gamma|}\,\gamma^{ab}\partial_a X^{\hat{\mu}'}\partial_b X^{\hat{\nu}'}\, g_{\hat{\mu}'\hat{\mu}}g_{\hat{\nu}'\hat{\nu}}\, e^{\beta_p\Phi}\,\frac{\delta^D(x-X(\xi))}{\sqrt{|g|}}\ . \tag{22}$$

The electric current created by the brane is

$$\begin{aligned} J^{\hat{\mu}_1\ldots\hat{\mu}_{p+1}} &= -\frac{\mathcal{A}_{WZ}}{2}\, M^{p+1}\int d^{p+1}\xi\,\epsilon^{a_1\ldots a_{p+1}} \\ &\quad \partial_{a_1}X^{\hat{\mu}_1}\ldots\partial_{a_{p+1}}X^{\hat{\mu}_{p+1}}\,\delta^D(x-X(\xi))\ . \end{aligned} \tag{23}$$

And the source current for the dilaton equation is

$$T_\Phi = \frac{\beta_p\, M^{p+1}}{2}\int d^{p+1}\xi\,\sqrt{|\gamma|}\,\gamma^{ab}\partial_a X^{\hat{\mu}}\partial_b X^{\hat{\nu}}\, g_{\hat{\mu}\hat{\nu}}\, e^{\beta_p\Phi}\,\frac{\delta^D(x-X(\xi))}{\sqrt{|g|}}\ . \tag{24}$$

We will solve these equations in the case of a codimension one brane and we will see in the next section how the analysis can be extended to higher codimension. First we choose a system of spacetime coordinates related to the brane:

worldvolume coordinates: $x^\mu \quad \mu = 0\ldots p$;

transverse coordinate: y ,

in the physical gauge where $X^\mu(\xi) = \xi$.

We are looking for a solution with a Poincaré invariance in $(p+1)$ dimensions, so that we can make the following ansatz for the metric:

$$ds^2 = e^{2A(y)} dx^\mu \otimes dx^\nu \eta_{\mu\nu} + e^{2B(y)} dy \otimes dy \ . \tag{25}$$

The nonvanishing components of the $(p+1)$-differential form that couples to the p-brane are

$$A_{\mu_1 \dots \mu_{p+1}} = -\epsilon_{\mu_1 \dots \mu_{p+1}} \frac{1}{\mathcal{A}_{WZ}} \, e^{C(y)} \ , \tag{26}$$

where $\epsilon_{\mu_1 \dots \mu_{p+1}}$ is the antisymmetric tensor normalized to ± 1.

It is well known that (see for instance [33] for a review), corresponding to the ansatz (25–26), the solutions of eqs (17–21) can be expressed in terms of a harmonic function $H(y)$:

$$ds^2 = H^{2n_x} \, dx^\mu \otimes dx^\nu \, \eta_{\mu\nu} + H^{2n_y} \, dy \otimes dy \ ; \tag{27}$$

$$e^\Phi = H^{n_\Phi} \, e^{\phi_\infty} \quad (\phi_\infty \text{ is the value of } \Phi \text{ at infinity}) \ ; \tag{28}$$

$$F_{y\mu_1 \dots \mu_{p+1}} = \epsilon_{\mu_1 \dots \mu_{p+1}} \frac{1}{\mathcal{A}_{WZ}} \, e^{-\alpha_p \phi_\infty / 2} \, \frac{dH^{-1}}{dy} \ ; \tag{29}$$

where the powers are given by

$$n_x = \frac{2\kappa^2}{p \, \mathcal{A}_{WZ}^2} \qquad n_y = \frac{2(p+1)\kappa^2}{p \, \mathcal{A}_{WZ}^2} \qquad n_\Phi = \frac{\alpha}{\mathcal{A}_{WZ}^2} \ . \tag{30}$$

Provided that the coefficient of the Wess–Zumino term is related to the coupling to the dilaton by

$$\mathcal{A}_{WZ}^2 = -2\kappa^2 \frac{p+1}{p} + \frac{\alpha^2}{2} \ , \tag{31}$$

the whole set of equations of motion is now equivalent to Poisson's equation,

$$\frac{d^2 H}{dy^2} = -\tfrac{1}{2} \mathcal{A}_{WZ}^2 M^{p+1} e^{-\alpha \phi_\infty / 2} \, \delta(y) \ , \tag{32}$$

the solution of which reads

$$H(y) = 1 - \tfrac{1}{4} \mathcal{A}_{WZ}^2 M^{p+1} e^{-\alpha \phi_\infty / 2} \, |y| \ . \tag{33}$$

We have normalized $H(y)$ so as to obtain a flat Minkowski space in the vicinity of the brane. At this stage, it is worth noticing that the derivation follows directly from the bosonic equations (17-21) and no supersymmetric argument has been used. The full supergravity equations also include a Killing spinor equation that can be consistently solved, provided that the coupling of the differential form to the dilaton takes its stringy value. This promotes the bosonic solution to a BPS one.

It is interesting to substitute this solution back into the Einstein equations (17) to obtain:

$$G^{\mu\nu} = -\frac{\kappa^2}{\mathcal{A}_{WZ}^2}\left(1-\frac{\alpha^2}{2\,\mathcal{A}_{WZ}^2}\right)H^{-2(n_y+1)}\,(H')^2 g^{\mu\nu} - M\,H^{-(1+n_x(p+1))}e^{-\alpha\phi_\infty/2}\,\frac{\delta(y)}{\sqrt{g_{yy}}}\,g^{\mu\nu}\ ; \tag{34}$$

$$G^{yy} = -\frac{\kappa^2}{\mathcal{A}_{WZ}^2}\left(1+\frac{\alpha^2}{2\,\mathcal{A}_{WZ}^2}\right)H^{-2(n_y+1)}\,(H')^2\,g^{yy}\ . \tag{35}$$

In the limit of decoupling between the brane and the dilaton, *i.e.*, $\alpha = 0$, which also corresponds to $n_y = (p+1)n_x = -1$ using the constraint (31), the Einstein tensor involves two constants Λ_b° and Λ°:

$$G^{\mu\nu} = -\kappa^2\left(\Lambda_b^\circ + \Lambda^\circ\frac{\delta(y)}{\sqrt{g_{yy}}}\right)g^{\mu\nu}\ ; \tag{36}$$

$$G^{yy} = -\kappa^2\Lambda_b^\circ\,g^{yy}\ . \tag{37}$$

If we keep the factors $\alpha\phi_\infty$ fixed (since ϕ_∞ could go to infinity as $\alpha\to 0$), these constants are given by

$$\Lambda_b^\circ = -\frac{p+1}{8p}M^{p+2}e^{-\alpha\phi_\infty} \quad\text{and}\quad \Lambda^\circ = M^{p+1}\,e^{-\alpha\phi_\infty/2}\ . \tag{38}$$

They can be interpreted as effective cosmological constants since the metric (27) is a solution to the Einstein equations derived from the RS action (1).

The expression of the cosmological constants in terms of supergravity quantities may give some insight into the origin of the apparently *ad hoc* fine-tuning (7) of the RS mechanism: here the cosmological constants are no longer fundamental parameters and the fine-tuning problem appears in a different way; in the present language it is a consequence of taking the limit where the dilaton decouples from the brane. Of course this represents just one point in the full parameter space. The more general solution, when the dilaton does not decouple, is a bulk energy density which depends on y, rather than a cosmological constant term. Regardless of this difference, one can still obtain an exponentially decaying warp factor, as long as n_x remains negative. The new insight, then, is that the original RS solution is only the simplest possibility within a whole class of solutions which can solve the hierarchy problem.

Furthermore, our approach links the energy densities of the brane and bulk to physical quantities like the charge associated to the electric current (23). Not only is such a charge conserved, but it also obeys Dirac's quantization rule [35]: solutions exist where the fiducial value of the electric charge is multiplied by an integer and these can be interpreted as a superposition of N parallel branes. Since in such a configuration the electric field strength is multiplied by a factor N, the RS effective cosmological constants depend on N: Λ goes like Λ°/N and Λ_b like Λ_b°/N^2. Therefore the effective cosmological constants are quantized.

A serious shortcoming with the above solution is that the dilaton decoupling regime requires a purely imaginary Wess–Zumino term (see eq. (31)), which implies an imaginary hence unphysical value for the electric charge. Therefore this solution is still just a tantalizing hint at a stringy origin for the RS proposal. To be more convincing, it is essential to overcome this problem. In the next section, we will address this issue by going to a higher number of extra dimensions, in the space transverse to the brane. It may happen that the compactification of some of these extra dimensions can be crucial, requiring a more complete analysis involving some interacting moduli fields in gauged supergravity theories[4]. The problem should also be reconsidered in a more complicated version [36] of ten dimensional *IIA* supergravity including mass terms since a codimension one supersymmetric object, the D-8 brane, has been constructed by Bergshoeff *et al.* [37]. This subject was partially addressed in the recent references [38].

When the dilaton coupling is turned on, the cosmological 'constant' in the bulk will now have a dependence on the transverse distance r. Apart from the shortcoming of not quite reproducing the RS picture, this solution does have an interesting feature regarding the cosmological constant on the brane in the physical regime where the Wess-Zumino coupling $\mathcal{A}_{WZ}$ is real: it provides an example of a negative tension brane. For $n_x > 0$, as is the case when $\mathcal{A}_{WZ}^2 > 0$, the discontinuity in the derivative of the warp factor is positive, which through the Einstein equation (5) implies that $\Lambda < 0$. This is noteworthy because negative tension branes play a prominent role in the RS solution. In the original proposal, which resembles the Hořava-Witten compactification of $d = 11$ supergravity, the TeV brane was required to have negative tension. To get correct cosmological expansion on the TeV brane in the case where the extra dimension is noncompact, it was shown [14] that negative tension branes must exist. Since this situation seems rather exotic, it is reassuring to find a model in which it arises.

In summary, our study of codimension one branes suggests that the cosmological constants introduced by Randall and Sundrum are an effective description of the dynamics of a more complicated set of fields governing the physics of a brane that couples to the bulk through gravitational interactions only. Thus those effective cosmological constants inherit some physical properties of the brane, an intriguing one being their quantization. We point out that the solution (27) belongs to the general class of solutions (9) for codimension one branes. Since the exponent n_x is negative, it follows from the general discussion of section 2 that this field configuration can solve the gauge hierarchy problem in the manner proposed by Lykken and Randall [13]. Namely, physical particle masses will be exponentially suppressed on any test-brane ("TeV brane") placed sufficiently far from the "Planck brane" featured in our solution.

4 Generalization to higher codimension brane-universe

We would now like to generalize the previous results to the case of a brane-universe of codimension greater than one. Requiring rotational invariance in the transverse space, the ansatz for the metric and for the $(p + 1)$-differential form will be a function only of the

[4]This question has been recently addressed by Behrndt and Cvetič [25]. See also ref. [5] for an earlier discussion.

distance r in the transverse space:

$$r = \sqrt{y^I y^J \delta_{IJ}} \ . \tag{39}$$

The solutions (17–21) take the same form, but the powers are now given by:

$$n_x = -\frac{2(d_\perp - 2)\kappa^2}{(p + d_\perp - 1)\,\mathcal{A}_{WZ}^2} \qquad n_y = \frac{2(p+1)\kappa^2}{(p + d_\perp - 1)\,\mathcal{A}_{WZ}^2} \qquad n_\Phi = \frac{\alpha}{\mathcal{A}_{WZ}^2}\ , \tag{40}$$

and the relation between the Wess–Zumino coupling and the dilaton coupling becomes:

$$\mathcal{A}_{WZ}^2 = 2\kappa^2 \frac{(p+1)(d_\perp - 2)}{(p + d_\perp - 1)} + \frac{\alpha^2}{2}\ . \tag{41}$$

The function H is harmonic in the transverse space:

$$\Delta_\perp H \equiv \delta^{IJ} \frac{\partial^2 H}{\partial y^I \partial y^J} = -\tfrac{1}{2} \mathcal{A}_{WZ}^2 M^{p+1} e^{-\alpha\phi_\infty/2}\, \delta^{d_\perp}(y)\ , \tag{42}$$

A particular solution is

$$H = l + \frac{\mathcal{A}_{WZ}^2 M^{p+1}}{2(d_\perp - 2)\Omega_{d_\perp - 1}}\, e^{-\alpha\phi_\infty/2}\, \frac{1}{r^{d_\perp - 2}} \tag{43}$$

where $\Omega_{d_\perp - 1}$ is the volume of $S^{d_\perp - 1}$, and l is an arbitrary constant which we will set to zero in order to obtain cosmological constants in our results. (When $d_\perp = 1$ the sphere degenerates into two points, giving $\Omega_0 = 2$.) The case of a brane of codimension two involves logarithmic behavior, and we will not specify it in the following. As before, when the dilaton decouples from the brane, the geometry can be derived from effective cosmological constants, as we will now demonstrate. The components of the Einstein tensor associated with the solution (43) are

$$\begin{aligned} G^{\mu\nu} &= -\frac{\kappa^2}{\mathcal{A}_{WZ}^2}\left(1 - \frac{\alpha^2}{2\,\mathcal{A}_{WZ}^2}\right) H^{-2(n_y+1)}\,(H')^2 g^{\mu\nu} \\ &\quad - M^{2-d_\perp} H^{-(1+n_x(p+1))} e^{-\alpha\phi_\infty/2} \frac{\delta^{d_\perp}(y)}{\sqrt{g_\perp}}\, g^{\mu\nu}\ ; \end{aligned} \tag{44}$$

$$G^{IJ} = -\frac{\kappa^2}{\mathcal{A}_{WZ}^2}\left(1 + \frac{\alpha^2}{2\,\mathcal{A}_{WZ}^2}\right)\left(2\frac{y^I y^J}{r^2} e^{-2B} - g^{IJ}\right) H^{-2(n_y+1)}\,(H')^2\ . \tag{45}$$

When the dilaton decouples, $\alpha = 0$, implying $n_x = -1/(p+1)$ and $n_y = 1/(d_\perp - 2)$. The metric can then be written as:

$$ds^2 = \left(\frac{r}{R_0}\right)^{2(d_\perp - 2)/(p+1)} dx^\mu \otimes dx^\nu\, \eta_{\mu\nu} + \left(\frac{R_0}{r}\right)^2 dy^I \otimes dy^J\, \delta_{IJ}\ . \tag{46}$$

with

$$R_0 M = \left(\frac{p+1}{(p + d_\perp - 1)\Omega_{d_\perp - 1}}\right)^{1/(d_\perp - 2)} e^{-\alpha\phi_\infty/(2d_\perp - 4)}\ . \tag{47}$$

This is the geometry of $AdS_{p+2} \times S^{d_\perp - 1}$; R_0 is the radius of the sphere and it is related to the radius of the AdS space by $R_0 = R_{AdS}(d_\perp - 2)/(p+1)$. The expression of the Einstein tensor simplifies to:

$$G^{\mu\nu} = -\kappa^2 \left(\Lambda_b^\circ + \Lambda^\circ \frac{\delta^{d_\perp}(y)}{\sqrt{g_\perp}}\right) g^{\mu\nu} ; \tag{48}$$

$$G^{IJ} = -\kappa^2 \Lambda_b^\circ \left(2\frac{y^I y^J}{R_0^2} - g^{IJ}\right) ; \tag{49}$$

where the constants Λ_b° and Λ° are given by:

$$\begin{aligned} \Lambda^\circ &= M^{p+1} e^{-\alpha\phi_\infty/2} ; \\ \Lambda_b^\circ &= \frac{d_\perp - 2}{2} \left(\frac{p + d_\perp - 1}{p+1}\right)^{d_\perp/(d_\perp - 2)} \Omega_{d_\perp - 1}^{2/(d_\perp - 2)} M^{p+d_\perp+1} e^{\alpha\phi_\infty/(d_\perp - 2)} . \end{aligned} \tag{50}$$

What allows us to interpret them as effective cosmological constants is the fact that the metric (46) is actually a solution to the Einstein equations derived from a generalized RS action:

$$\mathcal{S} = \int d^{p+1}x\, d^{d_\perp}y \sqrt{|g|} \left(\frac{\mathcal{R}}{2\kappa^2} - \Lambda_b \left(g_\perp \left(\frac{r}{R_0}\right)^{2d_\perp}\right)^{-1} \left(\frac{R}{R_0}\right)^2 - \Lambda \frac{\delta^{d_\perp}(y)}{\sqrt{g_\perp}}\right) ; \tag{51}$$

where R is defined by $R^2 = y^I y^J g_{IJ}$. It is noteworthy that when the metric in the transverse space is integrated out, this action reduces to the one introduced by RS.

In the expression (50), we notice that even if the power $n_a = (d_\perp - 2)/(p+1)$ is positive, the cosmological constant on the brane is positive. This would not be the case with only one extra dimension, but when $d_\perp > 1$ the extra transverse dimensions that live on the sphere also contribute to the singularity in the Einstein tensor and modify the singularity coming from the AdS part of the space. Nevertheless, our discussion of the hierarchy problem is unaffected by the spherical extra dimensions and thus a positive power n_a is undesirable as regards the gauge hierarchy problem, since it implies that the integral for the 4D effective Planck mass diverges. However a positive power n_a naturally generates a gauge coupling unification along the lines of the scenario proposed in [12].

Just as in the case of codimension one, the expression for the effective cosmological constants in terms of supergravity quantities leads to their quantization in multibrane configurations: the electric field-strength increases by a factor N, Λ goes to Λ°/N and Λ_b goes to $\Lambda_b^\circ N^{2/(d_\perp - 2)}$.

Not only does going to higher codimension brane-universes cure the problem of the imaginary Wess–Zumino term, but they can also be more easily embedded in a superstring framework. Indeed, a D-3 brane in type *IIB* theory does not couple to the dilaton and thus provides an explicit realization of our construction. In this context it would be interesting to incorporate in the field theoretical analysis of RS some stringy corrections to the supergravity action, like quadratic terms in curvature, for instance, since they can modify the spectrum of the Kaluza–Klein graviton's excitations.

5 Discussion

In this work we have presented solutions to the coupled equations for branes in $d_\perp$ extra dimensions and the low energy bosonic states of supergravity or superstring theories. The goal was to reproduce the effective stress-energy tensor needed for the Randall-Sundrum solution which uses gravitational trapping to solve the weak scale hierarchy problem. Let us summarize the results.

Decoupled dilaton regime

Regardless of the dimensionality of the tranvserse space, we find that the stress-energy tensor takes a simple form only in the limit that the dilaton field decouples from the brane. Then there are three cases:

$d_\perp = 1$. It is necessary to go to an unphysical value of the Wess-Zumino coupling, $\mathcal{A}_{WZ}^2 < 0$, to obtain a solution, which does however then yield exactly the bulk and brane cosmological constants needed for the RS proposal.

$d_\perp = 2$. This appears to be an uninteresting case, because $\mathcal{A}_{WZ}$ is forced to vanish, leading to trivial solutions.

$d_\perp > 2$. We now find solutions with positive Λ_b and physically acceptable values $\mathcal{A}_{WZ}^2 > 0$ for the Wess-Zumino coupling. The bulk energy term looks conventional (constant) in the brane components of $T_{\mu\nu}$, but it has a mild dependence on the bulk coordinates in the tranvserse components, T_{IJ}. The warp factor $a(Y)$ goes like $\exp(+\text{const}|Y|)$ in coordinates where Y represents the physical distance from the brane in the bulk (const > 0). Therefore the solution cannot be advocated to explain the hierarchy between the Planck and electroweak scales. This is in qualitative agreement with the $d_\perp = 2$ solution recently found in ref. [18]. It would therefore appear that the RS solution to the hierarchy problem works only in the case of a single extra dimension[5], or in the case of several intersecting branes of codimension one. On the other hand, as shown in ref. [12], despite infinitely large extra dimensions, gauge coupling unification can naturally arise as a result of the anomaly associated with the rescaling of the wave functions on the brane. Moreover the presence of the spherical extra dimensions can help to cure some phenomenological puzzles which occur when there is only one transverse dimension, such as electroweak symmetry breaking and obtaining small enough neutrino masses [12].

Coupled dilaton regime

It is interesting to also consider the solutions where the dilaton does not decouple from the brane. The bulk energy is no longer constant in these solutions, so the resulting stress-energy tensor does not have the simple form proposed by RS. Nevertheless, these solutions are equally acceptable and may have interesting physical consequences.

$d_\perp = 1$. It is now possible to have a real-valued Wess-Zumino coupling, in which case $n_x > 0$. As explained in section 3, this implies that the brane has a negative energy density,

[5]Numerical solutions which we have found in the case of $d_\perp = 2$ also support this conclusion.

which is somewhat surprising, since pure scalar field domain wall configurations always have positive tension. Since the TeV brane in the RS proposal tends to have negative tension, it may be relevant to explore the properties of such configurations.

$d_\perp = 2$. The solutions are no longer trivial, but have a logarithmic dependence on the bulk coordinate. We have not studied this special case in detail.

$d_\perp > 2$. The term in $T_{\hat{\mu}\hat{\nu}}$ which looked like a bulk cosmological constant when the dilaton coupling vanished now has nontrivial spatial dependence in the bulk. Such behavior has recently been proposed as a condition for avoiding the generic problem of the incorrect Friedmann equation for the expansion of the brane [39]. In the latter, complicated and *a priori* unmotivated expressions for the dependence of T_{55} on y were derived using the requirement of correct cosmological expansion. Although we have not yet found inflationary solutions in the present supergravity context, it would be interesting to do so in order to check whether the y dependence of T_{55} advocated in ref. [39] can be justified by the presence of nontrivial dilaton fields.

Appendix: the boundary of an anti-de Sitter space

An anti-de Sitter space of dimension $p+2$ can be seen as a hypersurface embedded in a flat space of signature $(2,p+1)$. Let $x^{\hat{\mu}}$, $\hat{\mu} = 0 \ldots p+2$, be some coordinate system in this embedding space. The anti-de Sitter space of radius R is defined by the equation:

$$x^{\hat{\mu}} x_{\hat{\mu}} \equiv -x^0 x^0 + x^1 x^1 + \ldots x^{p+1} x^{p+1} - x^{p+2} x^{p+2} = -R^2 \tag{52}$$

and the metric on AdS is the embedding metric. In a convenient system of coordinates defined by

$$X^\mu = \frac{R}{x^{p+1} + x^{p+2}} x^\mu \ , \mu = 0 \ldots p, \quad \text{and} \quad r = x^{p+1} + x^{p+2} \ , \tag{53}$$

the embedding metric factorizes:

$$ds^2 = \left(\frac{r}{R}\right)^2 \eta_{\mu\nu} \, dX^\mu \otimes dX^\nu + \left(\frac{R}{r}\right)^2 dr \otimes dr \ . \tag{54}$$

The boundary of AdS is the set of points that satisfies equation (52) at the infinity of the flat space. More precisely, we can rescale the coordinates $x^{\hat{\mu}} \to x'^{\mu} = \lambda x^{\hat{\mu}}$ and consider the limit $\lambda \to \infty$. The boundary is thus defined by the projective equations

$$-x'^0 x'^0 + x'^1 x'^1 + \ldots + x'^{p+1} x'^{p+1} - x'^{p+2} x'^{p+2} = 0 \tag{55}$$

$$x'^{\hat{\mu}} \sim \rho x'^{\hat{\mu}} \quad \text{with } \rho \in \mathbb{R} \setminus \{0\} \ , \tag{56}$$

which clearly describe $S^1 \times S^p$. In the system of coordinates (53), the set of solutions to the boundary equations has two disconnected pieces: the first one is associated with $r' \neq 0$, which is sent to $r = \infty$ by the rescaling, and it corresponds to a Minkowski space of dimension $p+1$ spanned by $x^0 \ldots x^p$; the second piece is associated with $r' = 0$, *i.e.* $r = 0$, and corresponds to the union of a point and $\mathbb{R} \times S^p$.

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References

[1] E. Witten, *Nucl. Phys.* **B188** (1981) 513.

[2] I. Antoniadis, *Phys. Lett.* **B246** (1990) 377;
E. Witten, *Nucl. Phys.* **B471** (1996) 135;
J.D. Lykken, *Phys. Rev.* **D54** (1996) 3693.

[3] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B436** (1998) 257;
K.R. Dienes, E. Dudas and T. Gherghetta, *Phys. Lett.* **B436** (1998) 55.

[4] W. Israel, *Nuovo Cim.* **44B** (1966) 1;
V.A. Rubakov and M.E. Shaposhnikov, *Phys. Lett.* **125B** (1983) 139;
A. Vilenkin, *Phys. Lett.* **133B** (1983) 177;
J. Ipser and P. Sikivie, *Phys. Rev.* **D30** (1984) 712.

[5] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *Phys. Rev.* **D59** (1999) 086001;
A. Lukas, B.A. Ovrut and D. Waldram, hep-th/9902071.

[6] H.A. Chamblin and H.S. Reall, hep-th/9903225.

[7] P. Binétruy, C. Deffayet, D. Langlois, hep-th/9905012.

[8] L. Randall and R. Sundrum, hep-ph/9905221 and hep-ph/9906064.

[9] M. Gogberashvili, hep-ph/9812296, hep-ph/9812365 and hep-ph/9904383.

[10] C. Csaki, M. Graesser, C. Kolda and J. Terning, hep-ph/9906513.

[11] J.M. Cline, C. Grojean and G. Servant, hep-ph/9906523.

[12] K.R. Dienes, E. Dudas and T. Gherghetta, hep-ph/9908530.

[13] J. Lykken and L. Randall, hep-th/9908076.

[14] J. Cline, C. Grojean and G. Servant, hep-ph/9909496.

[15] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and N. Kaloper, hep-th/9907209.

[16] A. Brandhuber and K. Sfetsos, hep-th/9908116.

[17] C. Csaki and Y. Shirman, hep-th/9908186;
A.E. Nelson, hep-th/9909001.

[18] A. Chodos and E. Poppitz, hep-th/9909199.

[19] I. Oda, hep-th/9908104 and hep-th/9909048;
H. Hatanaka, M. Sakamoto, M. Tachibana and K. Takenaga, hep-th/9909076.

[20] W.D. Goldberger and M.B. Wise, hep-ph/9907218 and hep-ph/9908447.

[21] T. Nihei, hep-ph/9905487;
N. Kaloper, hep-th/9905210;
D.J. Chung and K. Freese, hep-ph/9906542 and hep-ph/9910235;
P.J. Steinhardt, hep-th/9907080;
H.B. Kim and H.D. Kim, hep-th/9909053;
L. Mersini, hep-ph/9909494.

[22] T. Li, hep-th/9908174;
J. Lykken and S. Nandi, hep-ph/9908505;
H. Davoudiasl, J.L. Hewett and T.G. Rizzo, hep-ph/9909255;
E. Halyo, hep-th/9909127;
P. Kanti, I.I. Kogan, K.A. Olive and M. Pospelov, hep-ph/9909481.

[23] H. Verlinde, hep-th/9906182.

[24] A. Chamblin and G.W. Gibbons, hep-th/9909130;
O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, hep-th/9909134.

[25] K. Behrndt and M. Cvetic, hep-th/9909058.

[26] A. Kehagias, hep-th/9906204.

[27] J. Maldacena, *Adv. Theor. Math. Phys.* **2** (1998) 231.

[28] C. Grojean and J. Mourad, hep-th/9903164, to appear in *Nucl. Phys.* **B**.

[29] U. Ellwanger, hep-th/9909103.

[30] T. Goto, *Prog. Theor. Phys.* **46** (1971) 1560 .

[31] S. Deser and B. Zumino, *Phys. Lett.* **65B** (1976) 369;
L. Brink, P. Di Vecchia and P. Howe, *Phys. Lett.* **65B** (1976) 471;
A.M. Polyakov, *Phys. Lett.* **103B** (1981) 207.

[32] P.K. Townsend, *Phys. Lett.* **202B** (1988) 53;
A. Dabholkar, G. Gibbons, J.A. Harvey and F. Ruiz Ruiz, *Nucl. Phys.* **B340** (1990) 33;
A. Strominger, *Nucl. Phys.* **B343** (1990) 167;
M.J. Duff and K.S. Stelle, *Phys. Lett.* **B253** (1991) 113.

[33] M.J. Duff, R.R. Khuri and J.X. Lu, *Phys. Rept.* **259** (1995) 213.

[34] A. Achucarro, J.M. Evans, P.K. Townsend and D.L. Wiltshire, *Phys. Lett.* **198B** (1987) 441;
M.J. Duff and J.X. Lu, *Nucl. Phys.* **B390** (1993) 276.

[35] R.I. Nepomechie, *Phys. Rev.* **D31** (1985) 1921;
C. Teitelboim, *Phys. Lett.* **167B** (1986) 69 and *Phys. Lett.* **167B** (1986) 63.

[36] L.J. Romans, *Phys. Lett.* **169B** (1986) 374.

[37] E. Bergshoeff, M. de Roo, M.B. Green, G. Papadopoulos and P.K. Townsend, *Nucl. Phys.* **B470** (1996) 113.

[38] D. Brecher and M.J. Perry, hep-th/9908018;
A. Chamblin, M.J. Perry and H.S. Reall, *JHEP* **09** (1999) 014.

[39] P. Kanti, I.I. Kogan, K.A. Olive and M. Pospelov, hep-ph/9909481.

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