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Seismic Levee System Fragility Considering Spatial Correlation of Demands and Component Fragilities

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Seismic levee performance is most readily computed for short segments having consistent geometry, soil conditions, and seismic demands. Spatial variations of seismic demands and of segment capacities significantly influence system risk, which is critical for flood protection because any segment failure within the system can cause inundation. We present a methodology to compute the probability of seismic levee system failure conditional on individual segment fragility and spatial correlations of demands and of capacities. Seismic demands are estimated from ground motion prediction equations; their correlation is available in the literature. Capacities and their correlation are derived from levee damage observations from a levee system in Japan shaken by two earthquakes. We find seismic capacities to exhibit positive correlations over shorter distances than for demands. System fragility is computed using Monte-Carlo simulations where segment demand and capacity realizations are generated to account for spatial correlations. We find probability of system failure is lower than would be obtained under an assumption of no correlation, and damage probability increases as the number of components in the system increases.

INTRODUCTION

A levee system is comprised of earth embankments that protect a particular area from flooding. One example is the Sacramento / San Joaquin Delta, where levees protect below-sea-level "islands" and convey fresh water that is exported to more than 20 million urban and agricultural users. If a particular length of a levee is taken as a segment (e.g., 50 m in length), then a levee system is a collection of levee segments in series. Levee systems are typically

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continuous and have lengths much greater (often measured in km) than their width or height (typically measured in m).

Kwak et al. (2016) developed fragility functions for 50 m long levee segments in which probabilities of exceeding various damage thresholds are expressed as a function of peak ground velocity (*PGV*), geological conditions, and groundwater elevation. Levee segments are connected in series, so failure of one segment exposes the protected region to flood risk when the levee system is continuously 'loaded' (retaining water). Under such conditions, the levee system fragility problem involves analysis of the probability of whether at least one levee segment in the series exceeds a specific damage state. The solution of this problem depends strongly on the system length (i.e., number of segments) and correlations of capacity and demand among segments. The greater the number of segments, the higher is the opportunity for demand to exceed capacity in at least one segment.

To illustrate the importance of spatial correlation, consider two extreme cases: perfectly correlated and statistically independent. For the perfectly correlated system, the capacity and demand for each segment are random variables that are perfectly spatially correlated. Perfect correlation requires that throughout the system all realizations of demand or capacity are a uniform number of standard deviations higher or lower than the mean value, which can vary spatially. For this perfectly correlated system, the probability of system failure is equal to the maximum of the probabilities of failure of the individual segments in the system. If the system failure is denoted $P(F_S)$ and the fragility of segment i as $P(f_i)$, we have:

$$P(F_S) = \max\left[P(f_i)\right] \tag{1}$$

where arguments F_S and f_i indicate failure of system and segment i, respectively. Now consider the other extreme of statistical independence, which requires $P(F_S)$ to be computed as the complement of system survival, which in turn is the product of each individual segment surviving:

$$P(F_S) = 1 - \prod_{i=1}^{n}\left(1 - P(f_i)\right) \tag{2}$$

where n is the total number of segments. Perfect correlation and statistical independence comprise extremes known as uni-modal bounds for a series system (Ang and Tang, 2007). When segment damage states are correlated, system failure probability is between these extremes:

$$\max\left[P\left(f_i\right)\right] \le P\left(F_S\right) \le 1 - \prod_{i=1}^{n}\left[1 - P\left(f_i\right)\right] \tag{3}$$

The range of failure probabilities provided by Eq. (3) is often wide. For example, a system composed of 100 segments each with $P(f_i) = 0.05$ will have $P(F_S) = 0.05$ for perfect correlation and $P(F_S) = 0.994$ for statistical independence among segments. In general, $P(f_i)$ will vary among segments, but is selected to be constant for this simple illustration. Where the actual value of $P(F_S)$ falls between these unimodal bounds depends strongly on capacity and demand correlations among segments.

To address this problem, we present a methodology for computing $P(F_S)$ conditional on $P(f_i)$ and spatial correlations among segments. We begin in the next section by describing prior work for analysis of system fragility. We then describe the quantification of damage using Boolean variables representing damage states, define variables related to the correlation of damage states, and develop estimates of those correlations using autocorrelation analyses of levee damage data from Japan. Next we define statistical distributions and spatial correlation functions for segment-specific seismic levee capacity and demand. Demand correlations are taken from the literature (Kwak et al., 2016; Jayaram and Baker, 2009) and capacity correlations are obtained from observations of a levee system shaken by two recent earthquakes in Niigata, Japan. System-level fragility is then derived using Monte-Carlo simulations that consider the correlated segment demands and capacities. We conclude by applying the proposed approach to compute system fragility for components of a levee system protecting a Japanese city.

ANALYSIS OF SYSTEM FRAGILITY IN PRIOR WORK

Previous studies have addressed the system fragility problem for relatively simplified conditions. USACE (2008) and Wolff (2008) compute $P(F_S)$ by dividing the levee system into “reaches” with one or several characteristic lengths (typically 100 to 300 m) within which the correlation of damage is assumed perfect, whereas correlations between reaches are taken as zero. In these applications, a reach is a length of levee judged to have adequately similar geometry, soil conditions, and loading conditions that the reach can be represented by analysis of a single cross-section. For each reach, $P(f_i)$ is evaluated from geotechnical engineering models, and $P(F_S)$ is then computed using Eq. (2). For applications to flood risk in the Netherlands, Vrouwenvelder (2006) and Jongejan and Maaskant (2015) extend the ‘reach’ concept to consider non-unity correlation within reaches and potentially non-zero correlation between reaches, in both cases depending on the correlation of relevant geotechnical properties.

In an important study for the Sacramento / San Joaquin Delta region, the Delta Risk Management Strategy (URS and JRB&A, 2008) computed $P(F_S)$ by summing weighted values of $P(f_i)$, where the weights represent the probability of each reach being the "weakest link". The weight for a particular reach is proportional to $P(f_i)$, and the weights sum to unity. The resulting value of $P(F_S)$ is similar to assuming perfect correlation of damage among reaches, although $P(F_S)$ will actually be less than or equal to the maximum value of $P(f_i)$ using this approach. To account for levee system length, $P(F_S)$ is then multiplied by a correction factor ranging from 0.7 to 1.7, developed from empirical observations of flood events in the Delta.

The principal limitation of these methods is that they incorporate the spatial correlations of damage in an arbitrary manner. Defining the characteristic length of a reach is difficult without formally considering spatial correlation of resistance and demand. Characteristic lengths may be different for earthquake risk than for flood risk because soil properties that resist floods (hydraulic conductivity, erodibility) are different from those that resist earthquakes (liquefaction resistance, undrained shear strength). Moreover, high water demands posed by floods are likely more spatially correlated than ground shaking demands. A more robust solution that accounts for spatial correlations in resistance and demands is therefore needed.

A mathematical solution for system fragility can in principle be developed using an n-dimensional joint standard normal distribution function, Φ_n, in which the standard normal variate reflects the safety margin (Rackwitz and Krzykacz, 1978). The safety margin for a component i, M_i, is defined as $M_i = C_i - D_i$, where C_i and D_i are random variables representing element capacity and demand, respectively. If C_i and D_i follow normal distributions with user-defined means and standard deviations for each element, then M_i is also normally distributed. Under these conditions, the probability of system survival can be obtained by finding the space in Φ_n where the n-dimensional standard normal variates are lower than the reliability index β_i, which is defined as the mean safety margin normalized by its standard deviation, throughout all components. The equation of β_i is:

$$\beta_i = E\left(M_i\right) / \sqrt{\text{var}\left(M_i\right)} \tag{4}$$

The probability of system failure $P(F_S)$ is the complement of the probability of system survival. In practice, for systems with $n \geq 3$ components, the joint distribution Φ_n is difficult to solve for, so upper- and lower-bounded solutions based on limiting assumptions are used (Thoft-Christensen and Murotsu, 1986).

A solution for Φ_n was formulated by Dunnett and Sobel (1955) (for applications unrelated to system fragility) under the assumption that the correlation coefficient ρ between all possible element combinations is constant:

$$\Phi_{\substack{n \\ i=1-n}}\left(x_i;\rho\right)=\int_{-\infty}^{\infty}\phi\left(t\right)\prod_{i=1}^{n}\Phi\left(\frac{x_i+\sqrt{\rho}t}{\sqrt{1-\rho}}\right)dt \tag{5}$$

where x_i is a variate in the i^{th} normal distribution, i is an index for system components, ϕ and Φ are the probability density function (PDF) and cumulative density function (CDF) operators for the standard normal distribution, respectively, t is the standard normal variate and integrand, and n is the number of components (i.e., levee segments). To apply the joint distribution solution in Eq. (5) to fragility problems, as described above, it is useful to replace variate x_i with the reliability index β_i. With the substitution of β_i for x_i, Eq. (5) represents the probability of survival for a series system with equally correlated elements. The system fragility $P(F_S)$ can then be computed as the complement of system survival:

$$P\left(F_S\right)=1-\Phi_{\substack{n \\ i=1-n}}\left(\beta_i;\rho\right)=1-\int_{-\infty}^{\infty}\phi\left(t\right)\prod_{i=1}^{n}\Phi\left(\frac{\beta_i+\sqrt{\rho}t}{\sqrt{1-\rho}}\right)dt \tag{6}$$

Grigoriu and Turkstra (1979) presented a simplified version of Eq. (6) utilizing a second assumption that β_i is constant for all elements:

$$P\left(F_S\right)=1-\int_{-\infty}^{\infty}\phi\left(t\right)\left[\Phi\left(\frac{\beta_e+\sqrt{\rho}t}{\sqrt{1-\rho}}\right)\right]^n dt \tag{7}$$

where β_e is the constant reliability index for all elements. Eq. (7) is a single integration problem and as such is amenable to numerical calculation. Thoft-Christensen and Sørensen (1982) extended this solution to include non-equally correlated elements by estimating an average correlation coefficient throughout the system and applying that value in Eq. (7).

For applications to levees, there are significant limitations associated with the assumptions required to derive Eq. (7). Constant reliability index will not apply to the levee segments within a spatially distributed system – some segments will have relatively low fragility (due to low demand or high capacity) while others will be higher. Likewise, we intuitively expect the correlation of safety margin to not be constant but to vary with separation distance (closer segments well-correlated, distant segments uncorrelated).

The proposed approach, described and illustrated in subsequent sections of this paper, was developed to overcome the limitations of previous methods. The approach is fully general for series systems having non-uniform component fragilities and defined correlations among demands and capacities. Although fully general, the application emphasized here is levee flood control systems.

DAMAGE STATES AND THEIR CORRELATION

DAMAGE STATES

The damage experienced by levees during earthquakes is conveniently expressed in terms of damage states that take into consideration crack formation, crest settlement, and other factors. Kwak et al. (2016) developed fragility functions for 50 m long levee segments in which probability of exceeding a damage threshold is expressed as a function of peak ground velocity (*PGV*), surface geology, and groundwater elevation. As shown in Table 1, damage states (*DS*) range from zero for no damage to four for severe damage (e.g., levee collapse).

Table 1. Damage states assigned to levee segments (Kwak et al., 2016).

Damage state	Crack depth (cm)	Crack width (cm)	Subsidence (cm)	Description
0	0	0	0	No damage reported
1	0~100	0~10	0~10	Slight damage, small cracks
2	100~200	10~50	10~30	Moderate damage, cracks or small lateral spreading
3	200~300	50~100	30~100	Severe damage, lateral spreading
4	> 300	> 100	> 100	Levee collapse

Figure 1 shows seismic levee fragility expressed as the probability of exceeding a specific *DS* versus *PGV*, conditioned on surface geomorphology (G_N) and relative ground water elevation (D_W), defined as the elevation difference between the base of the levee and the groundwater table at the time of the earthquake. A log-normal CDF was fit to fragility data using the maximum likelihood estimation method (Kwak et al., 2016). The fragility functions are statistically lower than average for G_N category 1 (relatively firm materials in mountainous regions or gravel terrace deposits), and statistically higher than average when D_W is higher than

-1.0 m. High D_W corresponds to shallow ground water and presumably increased liquefaction risk. When discrete *DS*s are adopted to quantify damage, Boolean variables (i.e., zero or one) can be defined for cases of no damage (i.e., zero for $DS \leq ds$) and damage (one for $DS > ds$) depending on the threshold, *ds*.

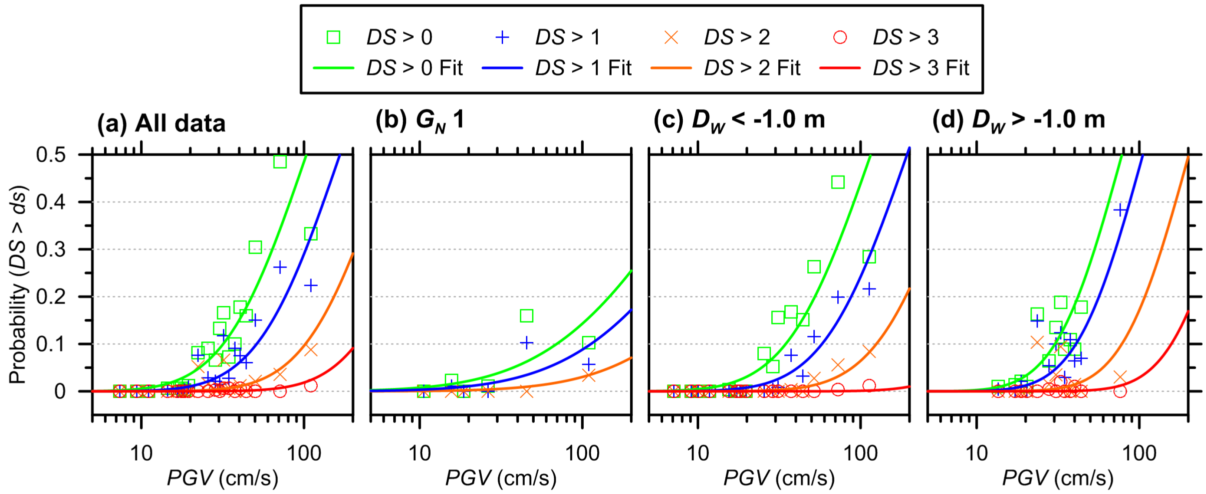

Figure 1. Probabilities of exceeding damage states (*DS*) for (a) all of the segments combined, (b) segments with geomorphic (G_N) category 1 corresponding to mountain or gravel terrace deposits, (c) groundwater depth relative to levee base (D_W) greater than 1 m (deep ground water), and (d) groundwater depth less than 1 m below levee base (shallow ground water) (Kwak et al., 2016).

CORRELATION COEFFICIENT OF DAMAGE STATES

The correlation of two random variables is often computed using residuals of the variables relative to a predictive model (residual = measured value minus predicted value) (Kutner et al., 2004; Baker and Cornell, 2006). However, the correlation of damage states is analyzed differently because damage is not expressed as continuous variables but as discrete variables as described in Table 1. If we define 'failure' (f_i) as the damage for segment *i* exceeding a particular *DS*, then, f_i can be represented as Boolean variables: $f_i = 1$ for damage and $f_i = 0$ for no damage. Kwak et al. (2015) derived the correlation coefficient of f_i (ρ_{DS}):

$$\rho_{DS} = \frac{E\left(f_i f_j\right) - \mu_{f,i}\mu_{f,j}}{\sqrt{\mu_{f,i}\left(1-\mu_{f,i}\right)\mu_{f,j}\left(1-\mu_{f,j}\right)}} \tag{8}$$

where $\mu_{f,i}$ is the mean of f_i and *E* represents the expected value operator. The correlation coefficient for survival states, which are represented as $s_i = 0$ for damage and $s_i = 1$ for no damage, is equivalent to ρ_{DS} in Eq. (8) (Kwak et al., 2015). Hence, ρ_{DS} can be expressed in terms of s_i as (Kwak et al., 2015):

$$\rho_{DS} = \frac{E\left(s_i s_j\right) - \mu_{s,i}\mu_{s,j}}{\sqrt{\mu_{s,i}\left(1-\mu_{s,i}\right)\mu_{s,j}\left(1-\mu_{s,j}\right)}} = \frac{P\left(s_i \cap s_j\right) - P\left(s_i\right)P\left(s_j\right)}{\sqrt{P\left(s_i\right)\left(1-P\left(s_i\right)\right)P\left(s_j\right)\left(1-P\left(s_j\right)\right)}} \tag{9}$$

where $P(s_i)$ is the probability of survival for a segment i and $P(s_i \cap s_j)$ is the probability of the intersection of s_i and s_j. Eqs. (8) and (9) can be used to compute ρ_{DS} from damage state data without the use of an underlying model for levee fragility. Kwak (2014) demonstrated that the correlation coefficient of residuals calculated as $R_i = f_i - P(f_i)$ is the same as that for damage states.

Direct computation of ρ_{DS} would require observations of the same levee system exposed to many earthquakes. The probability of survival for segment i [i.e., $P(s_i)$] is the mean of s_i from many samples, whose reliability is highly dependent on the number of samples, which must be from events that produce shaking that is strong enough to have the potential for causing damage. The joint distribution $P(s_i \cap s_j)$, which is the probability of survival of both segments i and j, similarly requires a large number of samples for a reliable estimate. In practice, data will seldom be available with which to compute ρ_{DS} from observed damage states. In the following section, we present an autocorrelation approach that relies on a large volume of data for a few events. This approach is investigated as a means by which to approximate ρ_{DS}.

AUTOCORRELATION COEFFICIENT AS APPROXIMATION OF CORRELATION COEFFICIENT

Autocorrelation represents the cross-correlation between a data vector and an offset, or lagged, version of the same vector in which the values in the vector appear in the same order but are shifted by the prescribed lag. The correlation is computed between the original and shifted data vectors, and the process is repeated for all possible shifts. The resulting correlation values are then plotted as a function of lag distance to develop an autocorrelation function. The autocorrelation function is equal to the damage state correlation if damage state correlation is stationary in space (i.e., if the correlation of damage states is a function only of spatial separation distance). We lack adequate observations to empirically verify whether damage state correlation is stationary, and therefore adopt a new variable ρ_{ac} to express autocorrelation.

The data set used for autocorrelation analysis consists of detailed damage observations along the Shinano River levee system in Japan following large seismic events in 2004 and 2007 in the Niigata region (details of the data set are given in Kwak et al., 2016). Figure 2 shows autocorrelations of damage state ρ_{ac} for the Boolean assignment of levee damage at $DS > 0$, 1, and 2 for three portions of the Shinano River levee systems (denoted SH1, SH2, and UO) and the two events. The values of ρ_{ac} are near unity at a separation distance near zero, and decrease approximately exponentially with separation distance. Variations of ρ_{ac} with distance are regressed as follows:

$$\rho_{ac} = \begin{cases} 1 & \text{if } x = 0 \\ c_{DS} \exp\left(-3 \times x / \alpha_{DS}\right) + \varepsilon_x & \text{if } x > 0 \end{cases} \tag{10}$$

where x is the lagged distance, c_{DS} and α_{DS} are regression coefficients, and ε_x is an error term. The regression coefficient α_{DS} is equal to the 'range' in a semi-variogram, which is the lag where ρ_{ac} becomes practically zero. Eq. (10) is divided into different equations for $x = 0$ and $x > 0$ to facilitate a more accurate fit to the data than would be afforded by forcing ρ_{ac} to be unity at $x = 0$ in the functional form for the regression (i.e., by making $c_{DS} = 1$). Accordingly, there is a step from 1.0 to c_{DS} as x becomes larger than zero. The coefficients were regressed using separation distances in the range $x \leq 1.0$ km in order to best fit the data in that critical range. Additional details on the autocorrelation analyses are given in Kwak et al. (2015).

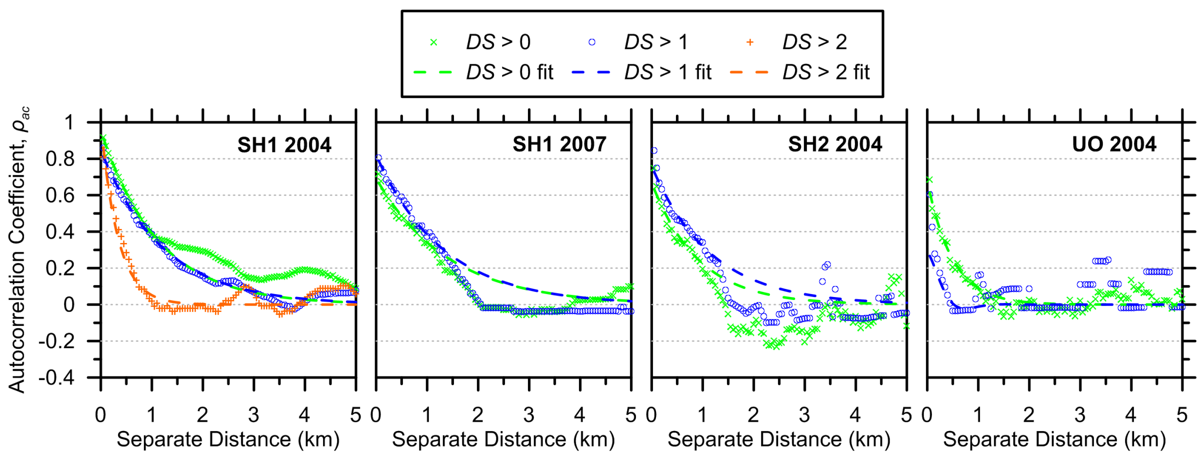

Figure 2. Autocorrelation coefficients of damage states (ρ_{ac}) for levee systems for the SH1, SH2, and UO rivers from the 2004 and 2007 Niigata earthquakes. Exponential function fits are shown. Data for $DS > 2$ are only available for SH1 in 2004 event.

As shown in Figure 2, the correlation models for $DS > 0$ and $DS > 1$ are well constrained because the results are similar for different river systems and different earthquakes. The fit curve in Figure 3 synthesizes the data from Figure 2 for $DS > 0$ and $DS > 1$, and the resulting regression coefficients are $c_{DS} = 0.77$ and $\alpha_{DS} = 3.7$ km. Regression for the $DS > 2$ case results in $c_{DS} = 0.8$ and $\alpha_{DS} = 1$ km, though this case is relatively poorly constrained.

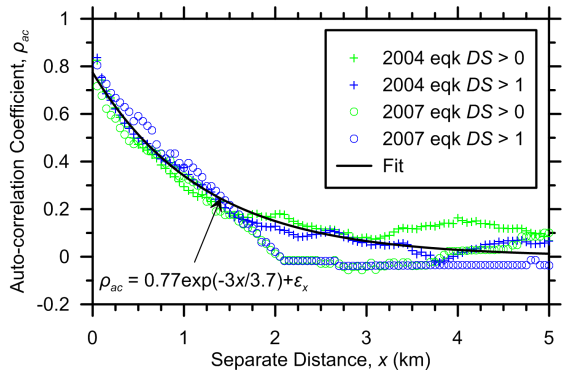

Figure 3. Auto-correlation coefficient ρ_{ac} for $DS > 0$ and 1 damage states combining all river systems for 2004 and 2007 earthquakes and a regressed fit line considering combined data set.

The functions derived here for ρ_{ac} can be taken as approximately equivalent to ρ_{DS} if damage states are stationary (in space) between segments; in other words, while the mean and standard deviation of damage states may vary in space, their correlation must depend only on separation distance and otherwise be independent of location. To check the use of ρ_{ac} for ρ_{DS} with the above assumption, Kwak et al. (2015) randomly generated Boolean variables (0 and 1) following a pre-defined function for describing ρ_{DS} as a function of separation distance, and then checked whether the ρ_{ac} calculated from the randomly generated dataset matched the values of ρ_{DS} used to generate the data. When the number of samples used in the numerical exercise was large (i.e., matching the 2762 segments available for the Shinano River system for cases with $DS > 0$ and 1), the values of ρ_{ac} compared well to ρ_{DS}. On the other hand, for the relatively sparse data set for the $DS > 2$ condition (798 segments), ρ_{ac} underestimates ρ_{DS}. Based on this analysis, we consider the available data to provide statistically meaningful estimates of ρ_{DS} for the $DS > 0$ and 1 damage thresholds and to marginally underestimate ρ_{DS} for $DS > 2$.

SYSTEM FRAGILITY UTILIZING DAMAGE DEMANDS AND CAPACITIES

We define below damage demands and capacities expressed as random variables from which segment damage probabilities can be computed. The analysis of system-level fragility requires correlation coefficients for those distributions, which are presented subsequently. System-level fragility can be defined based on variable numbers of segment failures comprising system failure. We first consider the one-segment case, in which the probability of a single levee segment exceeding a particular damage level is computed using Monte-Carlo simulation. That procedure is then extended to compute the probability of multiple damaged segments exceeding a particular damage level.

DAMAGE DEMANDS AND CAPACITIES

Damage capacity represents the "strength" or "resistance" of a levee segment against being damaged by an earthquake, and *damage demand* represents the "stress" or "load" imposed on the levee by an earthquake. In the present context, capacity is quantified as the ground motion intensity measure, which if exceeded, causes the levee segment to experience damage exceeding a damage state, and demand is quantified as the ground motion intensity measure induced by an earthquake. Based on the fragility work in Kwak et al. (2016), the selected intensity measure for representing capacity and demand is *PGV*.

For simplicity of notation, we define "failure" as occurring when demand exceeds capacity, regardless of the severity of the specific damage state. The probability of failure, P_f, for a segment is computed from capacity and demand PDFs as (adapted from Melchers, 1999):

$$P_f = P\left(C - D \leq 0\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{c\geq d} f_C\left(c\right) f_D\left(d\right) dc\,dd = \int_{-\infty}^{\infty} F_C\left(t\right) f_D\left(t\right) dt \qquad (11)$$

where f_C and F_C represent capacity PDF and CDF, respectively, f_D represents the demand PDF, and integrand t is the variate for the product of F_C and f_D. Figure 4 shows an example in which the capacity median and standard deviation are $\exp(\mu_{\ln C})$ = 104 cm/s and $\sigma_{\ln C}$ = 0.89, respectively, and the corresponding moments for demand are $\exp(\mu_{\ln D})$ = 40 cm/s and $\sigma_{\ln D}$ = 0.65 (a typical value from GMPEs). The figure illustrates the two terms in the integrand of Eq. (11) (F_C and f_D) and their product, which is a failure density function having an underlying area equivalent to P_f.

For a given capacity distribution (as in Figure 4a), if the median of demand [i.e., $\exp(\mu_{\ln D})$] is allowed to vary from zero to large values (at the upper limit of the fragility curve) with constant standard deviation, a distribution of failure probabilities P_f will be obtained that comprises a fragility function for the segment. The median of the fragility function matches the median of the capacity distribution, whereas the standard deviation of fragility function (i.e., β) is computed as:

$$\beta = \sqrt{\sigma_{\ln C}^2 + \sigma_{\ln D}^2} \qquad (12)$$

Eq. (12) supposes that demand and capacity are uncorrelated, which is generally true for seismic risk applications. In our case, we have estimates of β = 0.80-0.92 from Kwak et al. (2016) (range reflects effects of various conditioning variables) and we seek to compute $\sigma_{\ln C}$ through rearrangement of Eq. (12). Because the density of ground motion recording stations is high in the study region used in development of the fragility functions, the standard deviation of the demand estimates used by Kwak et al. (2016) is much lower (0.2-0.3) than is typical for forward predictions of ground motion (0.5-0.7). Using $\sigma_{\ln D} = 0.25$ results in estimates of $\sigma_{\ln C}$ = 0.76-0.89, which are only slightly reduced from the β values. These values of capacity dispersion are large because of many relevant factors that are not reflected in the formulation of the fragility function (e.g., site-specific soil properties related to shear strength and liquefaction resistance).

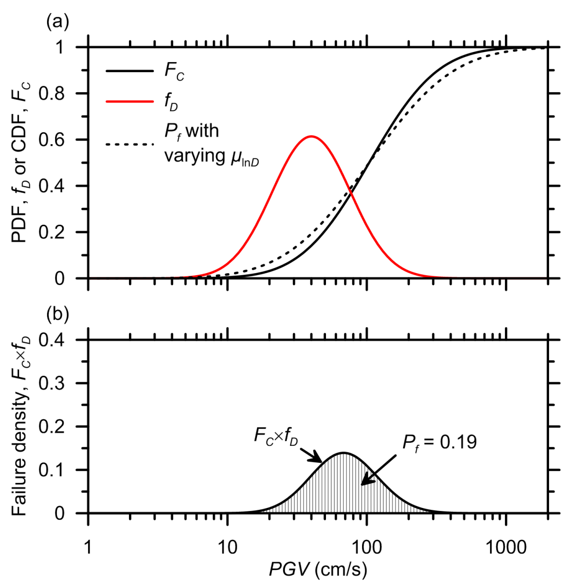

Figure 4. (a) Example CDF of capacity (F_C) and PDF of demand (f_D). (b) Failure density computed as product of F_C and f_D. The area below the failure density function is the probability of failure, P_f (after Melchers, 1999).

CORRELATION OF DAMAGE DEMANDS

The ground shaking experienced by a distributed levee system will exhibit spatially variable intensity measures (IMs). Some of those spatial variations will follow well-understood trends with respect to site-source distance (IMs tend to decrease with distance from the fault rupture) or site condition (IMs tend to increase for softer soils). However, some of those variations are, for practical purposes, random. The relatively 'deterministic' features can usually be described by a GMPE, whereas the random features can be represented by spatial variations of residuals. To facilitate analysis of the correlation of residuals, the fixed effect (approximately the mean of residuals) for earthquake k is denoted η_k and the residual between observation i and an event term-adjusted GMPE (also known as a within-event residual) is denoted ε_{ki}. We consider here the spatial correlation of ε_{ki}.

Regression analyses of semi-variance were performed using the 2004 and 2007 Niigata earthquake recordings (Kwak et al., 2012, 2016), which related a semi-variogram model to separation distance. Those relationships can be converted to correlation coefficient relationships having a similar form as follows (e.g., Goovaerts 1997):

$$\hat{\rho}_{DD,k}\left(x\right) = \exp\left(-3x / \alpha_{DD,k}\right) \tag{13}$$

where $\hat{\rho}_{DD,k}$ indicates the mean correlation coefficient for event *k*, *x* is the separation distance, and α_{DD} is equivalent to the range of a semi-variogram model (e.g., Goovaerts 1997). Values of α_{DD} were found to be 21 and 27 km for the 2004 and 2007 earthquakes, respectively.

The variation of $\hat{\rho}_{DD}$ with distance is as shown in Figure 5. Jayaram and Baker (2009) have analyzed semi-variogram data from several California earthquakes and the Chi Chi Taiwan earthquake. While they did not consider the IM of *PGV*, for the related IM of 0.5 sec PSA (Bommer and Alarcon, 2006), they find ranges of 17.1 km for widely varying geologic conditions between stations and 33.0 km for similar geologic conditions. The correlations based on this model are also shown in Figure 5, which encompass those evaluated by Kwak et al. (2012, 2016) for the Japan earthquakes.

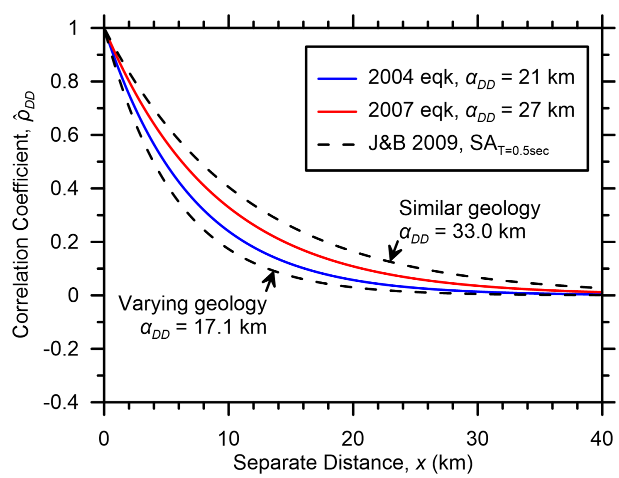

Figure 5. Correlation coefficients of damage demands (i.e., within-event residuals for *PGV*) from recordings of the 2004 and 2007 Niigata earthquakes and from Jayaram and Baker (2009).

CORRELATION OF DAMAGE CAPACITY

The correlation of damage capacity ρ_{DC} between segments cannot be calculated directly from residuals in the manner used for demand correlations because direct capacity "measurements" from observed performance are unavailable. Baker (2008) shows that the capacity distribution is the derivative of the fragility curve if demand is deterministic. When demand is not deterministic, its dispersion contributes to β (standard deviation of fragility function) per Eq. (12). In this section, we show how ρ_{DC} can be inferred from specified capacity and demand distributions and specified demand correlations. We first present equations for the probability of damage to a system consisting of two segments with correlated capacity subject to known demands. We then show how this process can be inverted to infer ρ_{DC} from the spatially correlated fragility functions and demands. We note here that we consider ρ_{DC} to be a fundamental parameter for a given levee system, whereas ρ_{DS} is an outcome of the correlated capacity and demand distributions.

Consider a levee system consisting of two segments where segment *i* has $\exp(\mu_{\ln C})$ of 108 cm/s and $\sigma_{\ln C}$ of 0.89, and segment *j* has $\exp(\mu_{\ln C})$ of 84 cm/s and $\sigma_{\ln C}$ of 0.80. Figure 6 shows randomly generated sets of capacities for both segments plotted together. In one case (Figure 6a) the capacities are uncorrelated ($\rho_{DC} = 0$), whereas in the other (Figure 6b) they are strongly correlated with $\rho_{DC} = 0.8$. If these segments are subjected to the levels of deterministic demand marked in the figure (*PGV* = 35 cm/s for levee *i* and 43 cm/s for levee *j*), segment failure is defined by the capacity realizations (dots) that are less than the demand (i.e., those in the shaded space). Furthermore, assuming these two segments constitute a series system for which failure of either segment constitutes system failure, the probability of system failure is represented by the fraction of realizations within the shaded region to the total number of realizations. The value of $P(F_S)$ depends on the correlation coefficient, being higher for $\rho_{DC} = 0$ [$P(F_S) = 0.29$] than for $\rho_{DC} = 0.8$ [$P(F_S) = 0.21$].

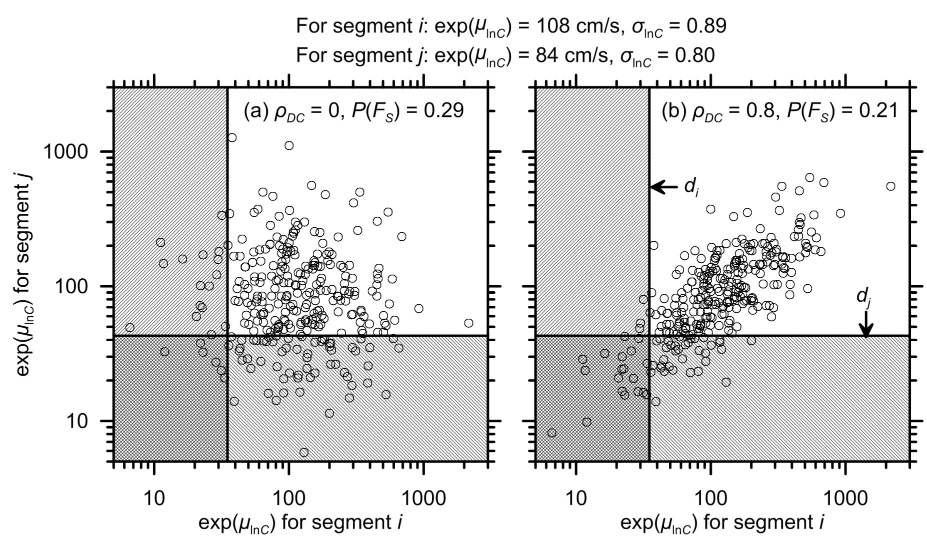

Figure 6. Randomly sampled damage capacities for two segments having different statistical moments. The capacities are (a) uncorrelated ($\rho_{DC} = 0$) and (b) correlated ($\rho_{DC} = 0.8$). Modified from Baker (2008).

The damage states for the combination of segments i and j can be represented as four sets of Boolean variables: [0 0] for failures of i and j, [0 1] for failure of i and survival of j, [1 0] for survival of i and failure of j, and [1 1] for survivals of i and j, respectively. With these damage states represented as Boolean variables, we can calculate ρ_{DS} using Eq. (9). For the case of uncorrelated capacities and deterministic demands (Figure 6a), $P(s_i) = 0.9$, $P(s_j) = 0.8$, and $P(s_i \cap s_j) = 0.72$, which provides $\rho_{DS} = 0$. For the case of correlated capacities (Figure 6b), $P(s_i)$ and $P(s_j)$ are unchanged, and $P(s_i \cap s_j) = 0.78$, which provides $\rho_{DS} = 0.5$.

The above example illustrates the calculation of ρ_{DS} for two segments having a particular correlation of capacities (ρ_{DC}), given capacity PDFs, and deterministic demands. The present problem is formulated somewhat differently, in that we seek a solution for ρ_{DC} given PDFs of capacities, a target model for the spatial correlation of damage states (ρ_{DS}), and the demand distributions for the earthquake ground motions that produced those damage states. We solve this in an iterative manner as follows:

1. Define a trial function for ρ_{DC} as follows:

$$\rho_{DC}(x) = \exp(-3x / \alpha_{DC}) \tag{14}$$

   where x is separation distance and α_{DC} is a constant for which an arbitrary trial value is selected in this initial step.

2. Using Monte-Carlo simulation, generate realizations of correlated capacities for the Shinano River system based on given PDFs of capacities and the spatial correlation structure defined in Step 1.

3. Define the appropriate correlated distribution of event-specific demands within the system and then evaluate damage states for each segment by comparing capacities with demands.

4. Calculate ρ_{DS} as a function of separation distance from the realizations in Step 3 using Eq. (9), and compare with the target ρ_{DS} in Figure 2.

5. Adjust the value of α_{DC} and repeat Steps 1 to 4 until a good match is obtained.

Figure 7 shows the values of ρ_{DC} obtained from this procedure using 50,000 Monte Carlo simulations, where the mismatch from Step 5 is less than 0.1 km. Also plotted in Figure 7 are the average values of ρ_{DS} ($\rho_{DS\text{-}avg}$) from Step 4, and the target ρ_{DS} from Figure 2. Values of coefficients α_{DC} obtained for each damage threshold are indicated in Figure 7. The value of

α_{DC} (< 10 km) in Eq. (14) is less than α_{DD} (20-30 km), indicating the spatial scale of variation of demand is larger than for capacity.

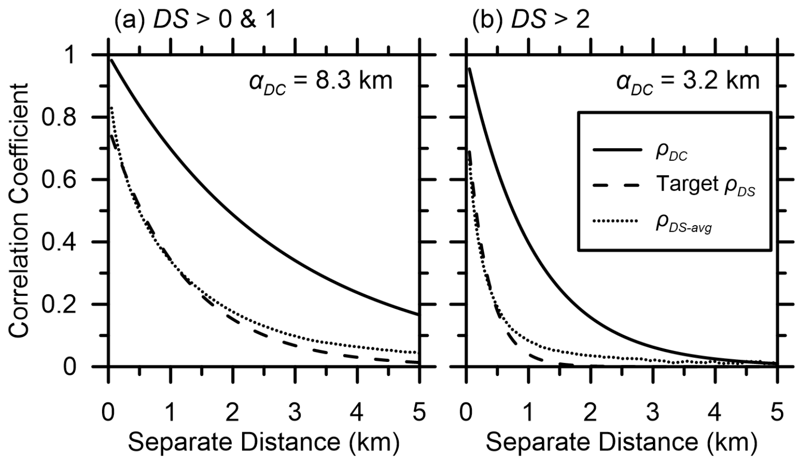

Figure 7. Correlation coefficients for damage capacities (ρ_{DC}) derived to produce correlated damage states (ρ_{DS}) from simulations that match the target equations. (a) $DS > 0$ and 1, (b) $DS > 2$.

MONTE-CARLO SIMULATION FOR EVALUATING SYSTEM FRAGILITY

With the demand and capacity correlation coefficients (ρ_{DD} and ρ_{DC}) having been defined, we now introduce a Monte-Carlo simulation approach for analysis of system fragility. In this analysis, we assume that failure occurs if at least one levee segment within the system fails because demand exceeds capacity. The steps in this approach are:

1) Generate a vector of N event terms as normally distributed random variables with zero mean and dispersion equal to the between-event standard deviation from a selected GMPE, τ_{ln}. An event-term represents approximately the average misfit of a GMPE median to the data for that event and is denoted as $\eta_{E,k}$ for event k.

2) Generate two matrices populated with normally distributed random variables with zero mean and standard deviation of unity. One matrix ($\mathbf{Z}_{DD}$) contains entries r_{ik} that are taken as modified within-event ground motion residuals that will be used to compute demands for segment i and event k (modification is described subsequently). The other matrix ($\mathbf{Z}_{DC}$) contains the random field that will be used to compute damage capacities (elements in the matrix are denoted z_{ik}). The two matrices are written as:

$$
\mathbf{Z}_{DD} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1N} \\ r_{21} & r_{22} & \cdots & r_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nN} \end{bmatrix}
$$

$$
\mathbf{Z}_{DC} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1N} \\ z_{21} & z_{22} & \cdots & z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1} & z_{n2} & \cdots & z_{nN} \end{bmatrix} \tag{15}
$$

Within the matrices, each column represents an event, and each row represents a segment. There are n segments and N events that are represented in $\mathbf{Z}_{DD}$ and $\mathbf{Z}_{DC}$.

3) Construct symmetric matrices of correlation coefficient for damage demand ($\mathbf{K}_{DD}$) and capacity ($\mathbf{K}_{DC}$), which are as follows:

$$
\mathbf{K}_{DD} = \begin{bmatrix} 1 & (\rho_{DD})_{12} & \cdots & (\rho_{DD})_{1n} \\ (\rho_{DD})_{21} & 1 & \cdots & (\rho_{DD})_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ (\rho_{DD})_{n1} & (\rho_{DD})_{n2} & \cdots & 1 \end{bmatrix}
$$

$$
\mathbf{K}_{DC} = \begin{bmatrix} 1 & (\rho_{DC})_{12} & \cdots & (\rho_{DC})_{1n} \\ (\rho_{DC})_{21} & 1 & \cdots & (\rho_{DC})_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ (\rho_{DC})_{n1} & (\rho_{DC})_{n2} & \cdots & 1 \end{bmatrix} \tag{16}
$$

where $(\rho_{DD})_{ij}$ and $(\rho_{DC})_{ij}$ represent correlation coefficients of damage demand and capacity between segments i and j, respectively. We utilize regression models for $(\rho_{DD})_{ij}$ and $(\rho_{DC})_{ij}$ [i.e., $\hat{\rho}_{DD}$ from Eq. (13) and ρ_{DC} from Eq. (14)].

4) Using the Cholesky decomposition method (Baecher and Christian, 2003), estimate matrices $\mathbf{Y}_{DD}$ and $\mathbf{Y}_{DC}$ containing correlated random variables. The entries in these matrices are denoted as r'_{ik} (used for correlated demand) and z'_{ik} (used for correlated capacity). The decomposition is performed as follows:

$$
\begin{aligned} \mathbf{Y}_{DD} &= \mathbf{S}_{DD} \times \mathbf{Z}_{DD} \\ \mathbf{Y}_{DC} &= \mathbf{S}_{DC} \times \mathbf{Z}_{DC} \end{aligned} \tag{17}
$$

where $\mathbf{S}_{DD}$ and $\mathbf{S}_{DC}$ are lower triangular matrices from Cholesky decomposition, whose multiplication with their transpose [i.e., $(\mathbf{S}_{DD})^{\mathrm{T}}$ and $(\mathbf{S}_{\mathrm{DC}})^{\mathrm{T}}$] results in the correlation matrices, $\mathbf{K}_{DD}$ and $\mathbf{K}_{DC}$. If the correlation matrix is not positive-definite, a scheme is required to make it a positive definite matrix to use Cholesky decomposition. We utilize the scheme of Qi and Sun (2006).

5) Transform r'_{ik} and z'_{ik} to damage demands (d_{ik}) and capacities (c_{ik}) in units of PGV:

$$\begin{aligned} d_{ik} &= \exp\left(r'_{ik} \times \phi_{\ln D,i} + \mu_{\ln D,i} + \eta_{E,k}\right) \\ c_{ik} &= \exp\left(z'_{ik} \times \sigma_{\ln C,i} + \mu_{\ln C,i}\right) \end{aligned} \tag{18}$$

where $\mu_{\ln D,i}$ and $\phi_{\ln D,i}$ are the natural log mean and within-event standard deviation of PGV from a GMPE, respectively, whereas $\mu_{\ln C,i}$ and $\sigma_{\ln C,i}$ are the natural log mean and standard deviation of damage capacity for segment i. It should be noted that the simulation procedure operates on many (N) realizations of a given scenario earthquake for which the mean and standard deviation of ground motions are constant between realizations; hence there is no need for subscript k in $\mu_{\ln D,i}$ and $\phi_{\ln D,i}$.

6) Find the damage state for segment i and event k (f_{ik}) as follows:

$$f_{ik} = \begin{cases} 0 & \text{if } c_{ik} > d_{ik} \\ 1 & \text{if } c_{ik} \le d_{ik} \end{cases} \tag{19}$$

7) Find the damage state for the system for event k, F_k, considering the damage states of all n segments:

$$F_k = \max\left(f_{ik}\right) \tag{20}$$

8) Estimate the probability of system failure $P(F_S)$:

$$P\left(F_S\right) = \frac{1}{N}\sum_{k=1}^{N} F_k \tag{21}$$

Following the above procedure, $P(F_S)$ can be calculated for a system given the statistical moments of damage capacity and demand distributions for each segment within the system, as well as models for the dependence of demand and capacity correlation coefficients on separation distance. We choose 50,000 realizations for Monte-Carlo simulation, from which the COV of estimated $P(F_S)$ is 0.002.

FRAGILITY ANALYSIS FOR FAILURE CRITERIA DEFINED FOR ARBITRARY NUMBER OF DAMAGED SEGMENTS

System fragilities derived in the previous section are based on the implicit assumption that a series system 'fails' if one or more segments within the system fail. This is a valid approach for analysis of flood risk associated with levees that are frequently loaded (i.e., retaining water). However, there are other applications where fragility associated with multi-segment failure is of interest. For example, if an earthquake strikes a flood control levee system while the water level is low, there is no immediate flood risk, but the number of 'failed' segments has direct impact on repair planning and costs to restore the system to full functionality. This failure criterion can be expressed as:

$$P\left(F_{S,x}\right)=P\left(X\geq x\right) \tag{22}$$

where $P(F_{S,x})$ represents the probability of having more than or equal to x segments damaged within a system. Hence, $P(F_{S,1})$ matches the system fragility computed in the previous section using Eq. (21). In this section we describe the calculation of $P(F_{S,x})$.

Recall that Step 6 in the previous section provides simulated damage states f_{ik} for n levee segments subjected to ground shaking from event k. Utilizing f_{ik}, the damage state of the levee system for event k ($F_{k,x}$) is taken as one if the number of damaged segments is greater or equal to x and is zero otherwise:

$$F_{k,x}=\begin{cases}1 & \text{if}\left(\sum_{i=1}^{n} f_{ik}\right)\geq x\\ 0 & \text{if}\left(\sum_{i=1}^{n} f_{ik}\right)< x\end{cases} \tag{23}$$

By repeating this analysis for N events, the average of $F_{k,x}$ can be computed, which is taken as $P(F_{S,x})$:

$$P\left(F_{S,x}\right)=\frac{1}{N}\sum_{k=1}^{N}F_{k,x} \tag{24}$$

EXAMPLE APPLICATION FOR PORTION OF SHINANO RIVER SYSTEM

The procedure described in the previous section is used to calculate $P(F_{S,x})$ for two scenario earthquakes applied to a 10 km interval of the Shinano River levee system providing flood protection for Nagaoka city, as shown in Figure 8. Three damage thresholds (DS > 0, 1, and 2) are considered. Demand must be computed for a scenario earthquake rather than for a uniform hazard level because a single earthquake will not mobilize uniform hazard level ground motions throughout the entire spatially-distributed system. Results are presented as probability of X segments exceeding each damage threshold, and the capacity correlation is varied to demonstrate its importance.

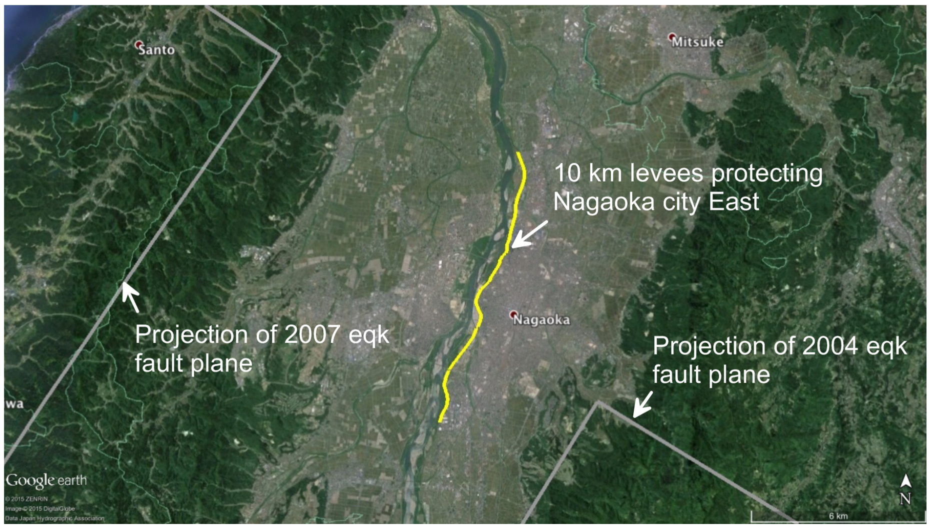

Figure 8. Aerial view of Nagaoka city region showing flood control levees along the Shinano River. Projections of fault planes for the **M** 6.6 2004 and 2007 Niigata earthquakes are shown.

The major model components required for the analysis are as follows:

1) Capacity: Capacity distributions are inferred from the fragility functions based on ground water elevation relative to levee base D_W. The median of the capacity distributions is taken as equal to the median of the fragility function, and the standard deviations are set as 0.76 for $D_W > -1$ m and 0.85 for $D_W < -1$ m per Eq. (12).

2) Demands: Demands are specified based on a scenario earthquake (location and magnitude). For the present analysis we consider two scenario events, which are repeats of the 2004 and 2007 earthquakes. In other words, we use the same fault rupture planes, which have known positions and site-to-source distances for levee segments. However, to be consistent with a forward prediction, the measured ground motions are not utilized. Rather, we consider scenario ground motions from the Boore et al. (2014) GMPE with randomized event-terms.

3) Demand correlation ρ_{DD}: We use Eq. (13) with α_{DD} = 24 km for both scenario events.

4) Capacity correlation ρ_{DC}: We use Eq. (14) with α_{DC} shown in Figure 7.

Figure 9 shows resulting system fragilities computed for damage thresholds of *DS* > 0 to 2 for each earthquake scenario (2004 and 2007 earthquakes). The 10 km levee system has the following probabilities of having at least one damaged segment:

- *DS* > 0: 76% for 2004 earthquake and 61% for 2007 earthquake
- *DS* > 1: 62% for 2004 earthquake and 41% for 2007 earthquake
- *DS* > 2: 50% for 2004 earthquake and 28% for 2007 earthquake

In the case where the capacity and demand distributions are assumed uncorrelated, the probability of damage to at least one segment is >95% for all cases except *DS* > 2 in the 2007 event (for which it is nearly 80%). In contrast, the one-segment damage probability for perfectly correlated capacity and demand is less than 30%. When capacity and demand distributions are correlated in accordance with field observations, the one-segment damage probabilities are between these unimodal bounds. This example illustrates that consideration of spatial correlation significantly influences estimation of $P(F_S)$.

Figure 9 also shows how probability of exceeding damage to more than x segments decreases as x approaches 200 (equivalent to the number of segments within the 10 km system). The uncorrelated case drops off quickly, whereas the perfectly correlated case drops off slowly. The two curves cross at a specific value of x, indicating that the probability of a large number of segments being simultaneously damaged is higher when the capacity and demand among segments are strongly correlated. Note that the uncorrelated case shown in Figure 9 represents uncorrelated demand (ρ_{DD} = 0) and uncorrelated capacity (ρ_{DC} = 0), which produces uncorrelated damage states (ρ_{DS} = 0; numerator in Eq. 9 becomes zero if $\rho_{DD} = \rho_{DC} = 0$). The perfectly correlated case represents ρ_{DD} = 1 and ρ_{DC} = 1, but not necessarily ρ_{DS} = 1. The ρ_{DS} becomes unity when segment fragilities are the same among segments and $\rho_{DD} = \rho_{DC} = 1$, but is less than unity when failure probabilities vary among segments.

Longer levee systems have more segments that could potentially be damaged, and therefore have correspondingly higher system failure probability. To evaluate the system length effect, we vary the levee system length from 0.1 to 10 km starting from the south end of the selected region using the model parameters described in the previous section, and compute system fragilities. The results in Figure 10 show that probability of damage to at least one segment increases from a lower-bound for short lengths to maxima of about 0.61, 0.41, and 0.28 for *DS* > 0, 1, and 2, respectively, for the largest considered length of 10 km. Results for uncorrelated

($\rho_{DD} = \rho_{DC} = 0$) and perfectly correlated ($\rho_{DD} = \rho_{DC} = 1$) conditions (i.e., the unimodal bounds) are also shown for reference purposes.

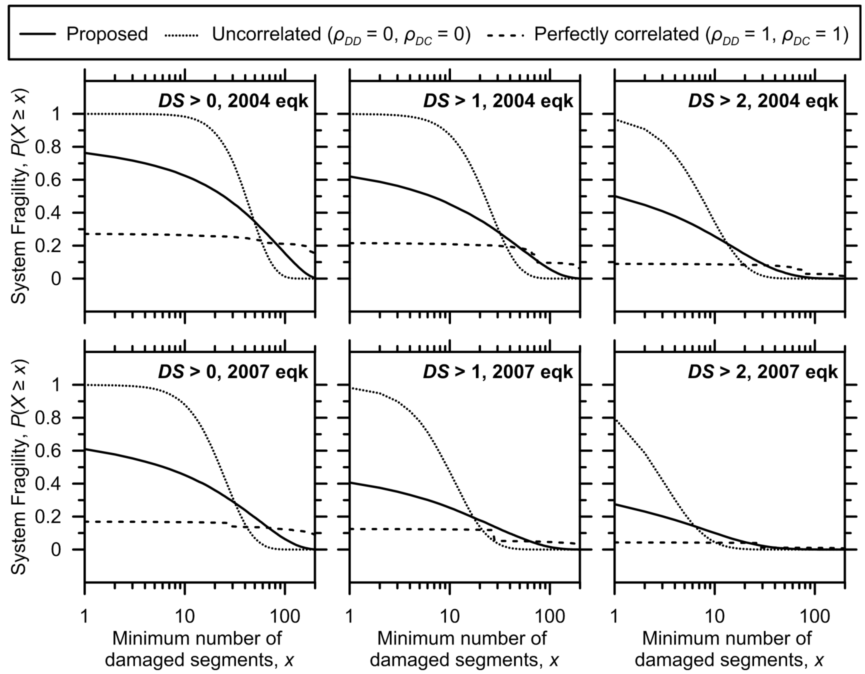

Figure 9. Fragilities for levee systems protecting Nagaoka city east for damage criteria defined on the basis of (1) three damage states and (2) variable minimum number of damaged segments, x. The simulations use the 2004 and 2007 Niigata-region earthquake scenarios having within-event and between-event variability from a GMPE (not from observation). The sudden steps shown for perfectly correlated cases are caused by changes in capacity distributions due to variable ground water conditions.

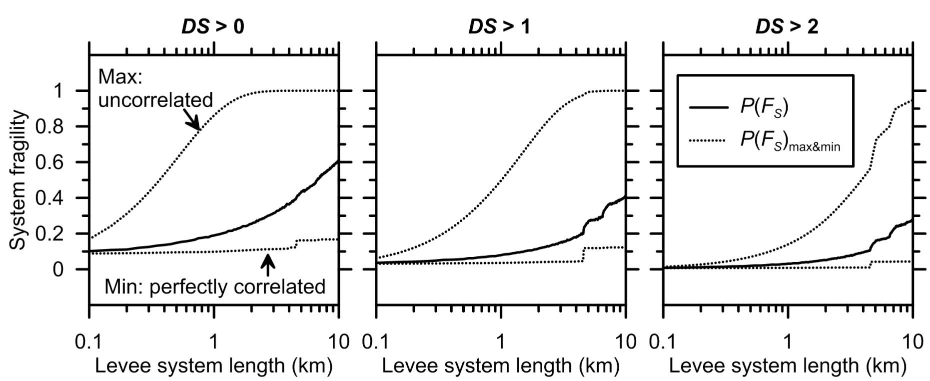

Figure 10. System fragility of levees protecting Nagaoka city east when the system length is varied from 0.1 km to its actual length of 10 km. Results apply for the 2007 Niigata earthquake scenario having GMPE-compatible within-event and between-event variability.

SUMMARY AND CONCLUSIONS

In this manuscript we extend the seismic fragility model developed for levee segments in previous work (Kwak et al., 2016) for application to a levee system by: (1) developing a methodology for evaluating correlations in levee capacities, (2) adopting models in the literature for correlation in earthquake ground shaking demands, and (3) developing a methodology for analysis of system fragility given segment fragility functions, spatially distributed demands, and appropriate correlation coefficient models for levee demand and capacity.

Critical components of the proposed procedure are demand and capacity correlation models. Demand correlations describe similarities of ground motion intensity measures in space, and are taken from the literature (Jayaram and Baker, 2009). Capacity correlations are related to spatial correlations of soil properties and groundwater levels, and were evaluated in the present work using a data set from the Shinano River in Japan. The capacity correlation structure is specific to this system, but nevertheless provides an approximate basis for assessing capacity correlations for other systems, with due consideration of associated epistemic uncertainties that can be quantified through variation of an empirical parameter (coefficient α_{DC} in Eq. 14).

The proposed methodology is applied to an example levee system with 10 km length providing flood protection for Nagaoka City in Japan. Probability of damage to at least one segment is maximum when capacity and demand are both spatially uncorrelated, whereas minimum fragilities are found when perfect correlation is applied. Our proposed method provides intermediate results that reflect the effects of the actual correlation structure.

The proposed methodology is a significant improvement over previous procedures in which spatial correlation of levee damage was treated incorrectly, and provides a means for calibrating procedures where capacity correlation is computed from correlation of geotechnical properties. The methods we propose are specific to levee systems subject to ground shaking, but could be extended to any other serial-system where demands and capacities are spatially correlated.

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