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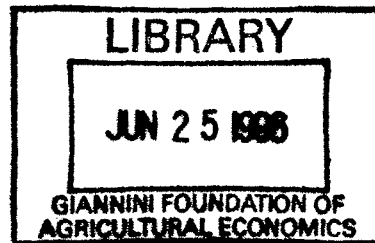
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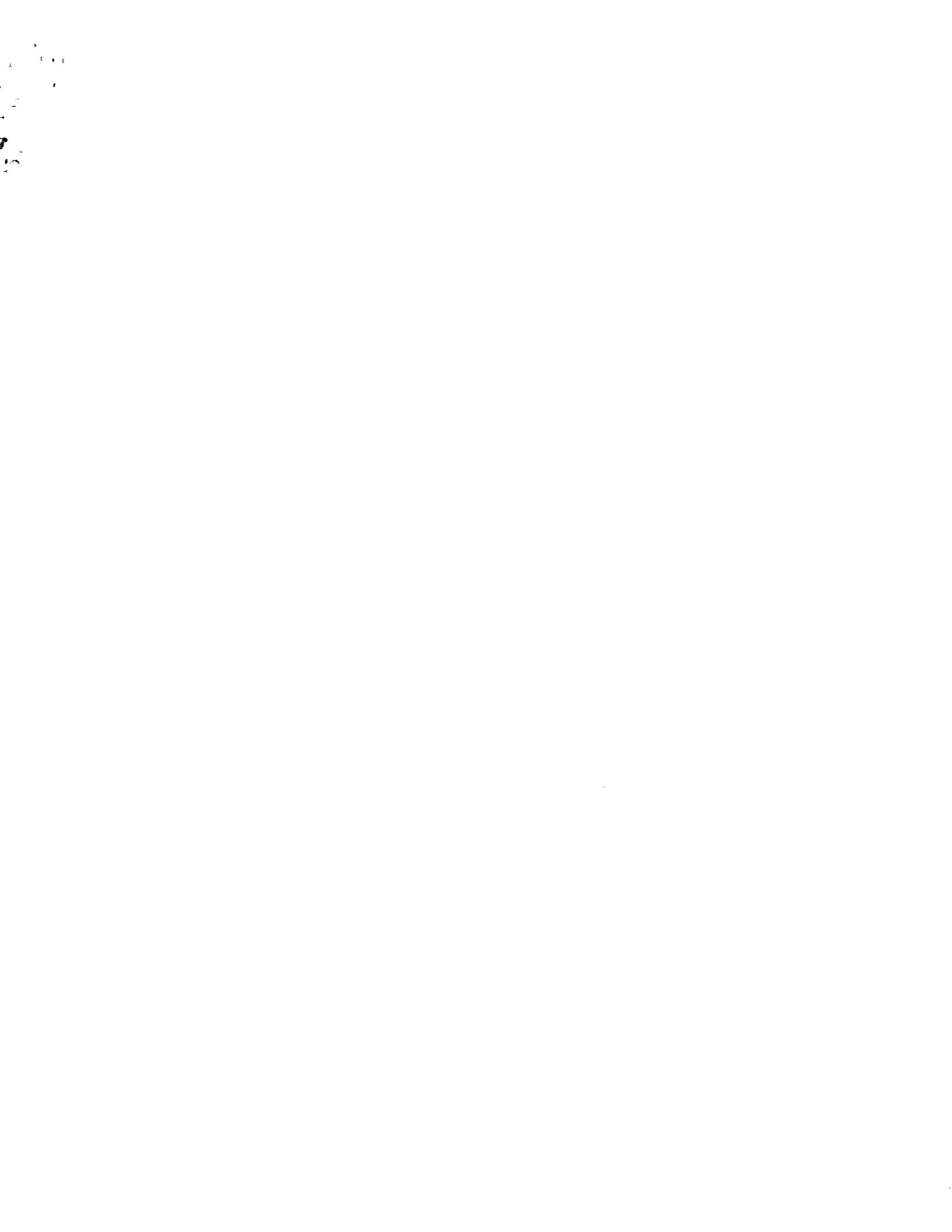
PRIVATIZATION, MARKET LIBERALIZATION AND
LEARNING IN TRANSITION ECONOMIES

by

Rachael E. Goodhue, Gordon C. Rausser and Leo K. Simon



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April 1996





**Privatization, Market Liberalization and Learning in Transition
Economies**

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ABSTRACT:

Privatization and market liberalization are widely considered to be complementary reforms in transition economies. This paper challenges this view and the closely related "big bang" approach to economic reform. Our analysis suggests that when pursued simultaneously, privatization may actually impede the transition process following market liberalization and reduce social welfare. Our result is based on an explicit model of market learning, which is a vital component of the economic transition process. Compared to a fully-functioning market in a mature market economy, a market in transition is characterized by greater uncertainty regarding market conditions, including free market equilibrium levels of prices and quantities. Market participants must learn about these conditions through their participation in the market process. When the effects of learning are incorporated into the analysis, the optimal level of privatization decreases monotonically as the level of uncertainty increases.

1. INTRODUCTION

Privatization and market liberalization are widely considered to be complementary reforms in transition economies. This paper challenges this view and the closely related “big bang” approach to economic reform. Our analysis suggests that when pursued simultaneously, privatization may actually impede the transition process following market liberalization and reduce social welfare. Our result is based on an explicit model of the market learning process. This process is an intrinsic component of any transition from a socialist economy—in which markets and market institutions are either nonexistent or highly distorted by government interventions—to a fully-functioning market economy. (For a discussion of the various facets of the transition process, see Rausser and Simon [1992].) The theoretical literature to date on the transition in Central and Eastern Europe has ignored the need for individuals to learn, through their participation in the market process, simultaneously about the features of a market in transition and the effects of government-instituted reforms. (See, for example, Grosfeld [1995], Munnell [1991] and van Brabant [1995].) We will argue in this paper that because it fails to take account of the learning process, the policy advice provided by Western experts to transition economies may be seriously flawed.

An urgent task facing policymakers in a small transition economy is to identify those subsectors of the economy in which their country will have a comparative advantage, once domestic prices have moved into line with world price levels.¹ Typically, very little information about the identity of these subsectors is provided by relative prices from the pre-transition era, since these were hugely distorted by production quotas, taxes and subsidies, and other nonmarket influences. So what economic policies will best facilitate the process of acquiring the requisite information? The standard economic advice proffered by Western economists has been to follow a “big bang” approach

¹ This concern is evidenced by the search of these governments, in particular Poland, for areas of comparative advantage, and by papers such as Hamilton and Winters [1992], and Michael, Revesz, Hare and Hughes [1993] which seek to define areas of comparative advantage based on available information.

of simultaneous, and rapid, liberalization and privatization. Proponents of this approach place tremendous faith in the efficacy of Adam Smith's "hidden hand" as a vehicle for achieving the optimal reallocation of resources: the belief is that newly privatized producers, who will be highly responsive to the newly liberalized market signals, have the best chance of identifying the optimal path of adjustment to the new market realities.

We investigate the relationship between learning and the degree of privatization in an extremely stylized model of the adjustment process. We divide the production sector into privatized and nonprivatized firms or *parastatals*. The fraction of firms in the economy that are privatized is viewed as a policy variable, and this fraction is held constant throughout the transition period.² Our privatized firms are modeled as responsive to market signals. Specifically, they base their production decisions on their private signals about market conditions and previously realized market prices. Parastatals simply select a level of production that remains fixed, regardless of market conditions. Demand and marginal cost are affine and deterministic. The sources of uncertainty in our model are all subjective. Our responsive producers know neither the intercept of the demand curve, the number of nonresponsive producers nor the amount produced by each.³ Rather than attempt to learn these parameters, our responsive producers simply attempt to predict market prices, using a naive learning rule: they form expectations about future prices by constructing weighted averages of past price realizations and their private signals.⁴

² We do not attempt to identify the optimal *rate* at which nonprivatized firms should be converted into privatized firms. While this issue is both fascinating and an important policy issue, it is also a *much* more difficult one in the context of an explicit model of learning.

³ Observations by Svejnar [1991] support our identification of the particular importance of these types of uncertainty during the transition process. He first asks how much reallocation of resources is really necessary during the early stages of the transition process and observes that the answer to this question will affect one's evaluation of potential microeconomic reforms. He then notes that the efficiency with which resources are used is another area of concern for microeconomic reforms. These two observations are reflected in our model in the uncertainty of market participants regarding the long-run comparative advantage position of the sector and the uncertainty regarding sectoral composition and responsiveness to market signals, respectively.

⁴ Since Lucas [1972], models of expectation formation such as the one we present in this paper have been widely criticized on the grounds that they postulate non-"rational" behavior by economic agents. If agents behaved in the manner we postulate, the argument runs, then arbitrage possibilities would arise and remain unexploited. While this critique is compelling when applied to models of long-run or steady-state behavior, it has much less force when applied to models of short-run—and, in particular, *transition*—behavior. Because they are operating in a transition environment, the agents in our model have not yet had enough either time or experience to "master the model," to the extent required by the rational expectations

The goal of this paper is to investigate the relationship between learning and adjustment in our model and the composition of the production sector. We construct what can be thought of as a “modified cobweb model” with time varying parameters. In general, we can distinguish three phases of the dynamic adjustment path: (i) a phase of explosive oscillations in prices and production; (ii) a phase of damped oscillations; and (iii) a phase of monotone convergence to perfect information prices. We refer to the first two phases as the *short-run* and the last phase as the *long-run*. Whether or not the price path passes through all three phases depends on parameter values. For example, if privatization is extensive and/or demand is very inelastic, the second and third phases may never be reached; if privatization is minimal and/or demand is very elastic, the first and second phases may be skipped altogether. Our first result considers the effect of increasing the fraction of privatized producers. In the short-run this change increases the volatility of prices and production, while in the long-run, prices converge more rapidly to perfect-information levels. Moreover, the length of the short-run is increasing in the number of responsive producers. In short, increasing the degree of privatization has short-run costs and long-run benefits, and the optimal resolution of this tradeoff will depend on parameter values.

The policy decision regarding privatization involves an additional tradeoff along a different dimension. Price volatility, especially in the short-run, leads to welfare losses relative to the perfect information equilibrium: our responsive producers base their production decisions on expected prices, and as volatility increases, their production levels depart further, on average, from *realized* profit maximizing levels during the short run and early in the long run. On the other hand, our nonresponsive producers are also misallocating resources. (We will assume that they are not, by

hypothesis. Indeed, as an empirical matter, it is well known that those very arbitrage opportunities which concern the critics are in fact *extremely* widespread in the early stages of transition economies. While these opportunities will no doubt be exploited eventually, if they have not already disappeared, our focus is on precisely those early stages. Moreover, given the inevitable uncertainty about market structure that characterizes all transition economies, it seems reasonable to suppose that producers may use past price observations as a forecasting tool, rather than relying upon some structural model in which they have little confidence. A related point is frequently made by econometricians in defense of their use of reduced form time-series models for short-term forecasting.

accident, already producing at perfect information levels.) A more intensive privatization process increases volatility in the short run, and hence the first kind of resource misallocation. On the other hand, since the nonresponsive sector of the economy shrinks, the second kind of resource misallocation becomes less important. Once again, the relative importance of these two effects depends on parameters such as the informativeness of privatized producers' private market signals, the elasticity of the demand curve and the rate at which producers discount past prices when they forecast future ones.

In Central and Eastern Europe and the former Soviet Union, the privatization process has proceeded at a pace that has been disappointingly low, at least from the perspective of Western donor organizations. The analysis in this paper suggests an economic rationale for this slow pace, along lines that to date has been largely unrecognized, and thus provides a normative insight for Western donor organizations and other policy advisors regarding the pace of privatization. It should be noted that we are not necessarily advocating gradualism over the big-bang approach: in some cases our model implies that a big-bang approach is optimal. Rather, the point we are making is that the quality of any transition policy advice will be compromised unless it takes due account of the relationship between privatization, liberalization and market learning.

Our approach to the gradualism versus big-bang controversy differs from the approaches that have dominated the economic literature on transitions. (See, for example, Gates, Milgrom and Roberts [1993] and Murphy, Shleifer and Vishny [1992].) Rather than modeling transition as a centrally-manipulated process, in which market participants respond perfectly to incentives set by government, we focus specifically on the functioning of transition markets when information and incentives are imperfect. We do not consider issues related to governmental credibility, political stability, and the efficacy and sustainability of reforms, although much of the policy debate has been couched in these political economic terms. (See, for example, Laban and Wolf [1993] and

Dewatripont and Roland [1992].) Unlike Dewatripont and Roland [1995], who analyze how uncertainty over outcomes may affect the choice of reform packages and ensuing political support, we treat uncertainty as an integral component of the market transition process, and consider how individuals' responses to market signals affect production, profits, prices and social welfare.

2. A MODEL OF LEARNING IN A TRANSITION ENVIRONMENT

We consider a single-market partial-equilibrium model, in which producers learn about market prices. Implicitly, our producers are learning about comparative advantage: our assumption of increasing marginal costs reflects the notion that there are alternative uses for inputs and hence increasing opportunity costs of production in our single market. This allows us to address in a simplified fashion the policy question of how a country in transition identifies those sectors within its economy in which it has a comparative advantage.

We adopt the linear-quadratic model which is the standard for learning theoretic papers (see Townsend [1978], Rausser and Hochman [1979], Bray and Savin [1986], etc.). The demand for the product is affine and deterministic, with demand curve $a - bQ$, where Q denotes the aggregate quantity produced. Both the intercept, a , and the slope, b , of the demand schedule are assumed to be positive. The total number of producers in the model, denoted by N , will be held fixed for now, but varied later in the paper, along with the intercept term a , defined as $bN + 1$. All producers have identical cost functions, but n are privatized and responsive to market signals, while the remaining $N - n$ are nonresponsive parastatals. Each parastatal produces the quantity \bar{q} , so that aggregate parastatal output is $(N - n)\bar{q}$. We also define $\alpha = \frac{n}{N}$, the share of producers that are privatized. Producers' common cost function is denoted by $C(q) = \frac{1}{2}q^2$, so that each privatized producer's expected profit maximizing level of output is identically equal to her (subjective) expectation of the market clearing price.

For each $\alpha \in [0, 1]$, we define a benchmark price $p^*(\alpha)$ with the following property: if each private producer anticipates this price and produces accordingly, then the market equilibrium price will indeed be $p^*(\alpha)$. It is defined as follows:

$$p^*(\alpha) = a - b(np^*(\alpha) + (1 - \alpha)N\bar{q}) \quad (1)$$

Henceforth, we will refer to $p^*(\alpha)$ as the *perfect information price* and suppress the reference to α except when necessary. We assume that even if all producers were parastatals, the perfect information price would still be nonnegative, i.e., that $\bar{q} \leq a/bN$. We also assume that $p^*(1) \neq \bar{q}$, i.e., that parastatals' production level differs from the level that would be Pareto optimal if all firms were responsive. Note that because $a = 1 + bN$, $p^*(1)$ is always unity.

In period zero, before any production takes place, each producer receives a signal of the perfect information price, $p_{0i} = p^* + \epsilon_i$, where the error terms (ϵ_i) are i.i.d., with mean zero and variance σ^2 . One possible interpretation is that p_{0i} is the view of market conditions that i acquires during her pre-transition experience. Producers have no other information; in particular, the magnitudes α and N are unknown. This assumption reflects the lack of knowledge regarding aggregate supply elasticity which characterizes transition economies.

In period $t = 1$, each responsive producer maximizes her profit conditional on the signal she received in period zero. That is, under the above specification she produces the quantity p_{0i} . The market clearing price in period $t = 1$ is then

$$p_1 = a - b\left(\sum_{j=1}^n p_{0j} + (N - n)\bar{q}\right). \quad (2)$$

In period $t > 1$, i 's estimate of the t 'th period price, denoted by \hat{p}_{ti} , is a convex combination of realized market prices in previous periods and her original private signal, with higher weights placed

on more recent price realizations:

$$\hat{p}_{ti} = \left(\sum_{\tau=0}^{t-1} \gamma^\tau \right)^{-1} \left[\sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1} + \gamma^{t-1} p_{0i} \right] \quad (3)$$

where $\gamma \in (0, 1)$. Here, γ is not a rate of *time* preference but rather reflects the rate at which producers discount past price information. We assume that γ is identical for all individuals. Note that

$$\begin{aligned} \hat{p}_{t+1,i} &= \left(\sum_{\tau=0}^t \gamma^\tau \right)^{-1} \left[\sum_{\tau=0}^{t-1} \gamma^\tau p_{t-\tau} + \gamma^t p_{0i} \right] \\ &= \left(\sum_{\tau=0}^t \gamma^\tau \right)^{-1} \left[p_t + \gamma \frac{\sum_{\tau=0}^{t-1} \gamma^\tau}{\sum_{\tau=0}^{t-1} \gamma^\tau} \left(\sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1} + \gamma^{t-1} p_{0i} \right) \right] \\ &= \left(\sum_{\tau=0}^t \gamma^\tau \right)^{-1} \left[p_t + \gamma \left(\sum_{\tau=0}^{t-1} \gamma^\tau \right) \hat{p}_{ti} \right] \\ &= \left(\sum_{\tau=0}^t \gamma^\tau \right)^{-1} \left[p_t + \sum_{\tau=1}^t \gamma^\tau \hat{p}_{ti} \right]. \end{aligned} \quad (4)$$

The market clearing price in period t is:

$$p_t = a - b \left(\sum_{j=1}^n \hat{p}_{tj} + (N - n)\bar{q} \right). \quad (5)$$

Note also from equations (1) and (5) that for all $t \geq 1$,

$$(p_t - p^*) = -b \sum_{j=1}^n (\hat{p}_{tj} - p^*). \quad (6)$$

That is, the path of production and prices exhibits the familiar cobweb pattern, except that the underlying parameters vary with time. It is straightforward to show that for each t and i , \hat{p}_{ti} is an unbiased⁵ estimator of the perfect information price $p^*(\alpha)$. Moreover, it can be shown that \hat{p}_{ti} is also a consistent estimator of $p^*(\alpha)$, provided that demand is not too inelastic and/or n is not too large and/or γ is not too small.⁶

⁵ That is, unbiased when viewed from the perspective of before time zero, when all uncertainty is realized. The unbiasedness property depends on our assumption that marginal costs are linear. In general, the estimator will be biased upward if producers' marginal cost curves is concave and downward if they are convex.

For a fixed parastatal share of total production, $(1 - \alpha)N$, $p^*(\alpha)$ is the price that maximizes aggregate welfare. Not surprisingly, the level of aggregate social surplus associated with $p^*(\alpha)$ increases with α , reaching its maximum when $\alpha = 1$ and all firms are responsive. Indeed, $p^*(1)$ is the (unconstrained) welfare-maximizing price, which would be realized if all producers were responsive and perfectly informed regarding market conditions.⁷

3. ANALYSIS

3.1. The Representative Price Path. We begin by fixing an arbitrary vector of private market signals and considering the dynamic path of prices and production generated by this vector. We then consider the effect on this path of increasing the number of private producers. When $t = 1$, private producers' output decisions are based exclusively on their private signals of the market price. That is, for each i , $\hat{p}_{1i} \equiv p_{0i}$. Note from equation (6) that the difference, $(p_1 - p^*)$, between the market clearing price and the perfect information price will be positive or negative depending on whether private producers have *on average* under- or over-estimated the perfect information price. In period $t = 2$, private producer i 's output is equal to her updated estimate of the market price, \hat{p}_{i2} , which is a weighted average of her original signal and the previous period's realized price, p_1 . From (3), the sum of private producers' price signals in this period is $\sum_{j=1}^n \hat{p}_{2j} = \frac{1}{1+\gamma} \left(np_1 + \gamma \sum_{j=1}^n \hat{p}_{1j} \right)$. Consider the expression $\sum_{j=1}^n (\hat{p}_{2j} - p^*)$, which is the divergence from the perfect information price of private producers' price estimates in periods two, and compare it to the corresponding expression

⁶ The condition for consistency is that $bn < \frac{1+\gamma}{1-\gamma}$. See the Appendix, Proposition 7

⁷ Indeed, $p^*(1)$ is the price resulting from privatization explicitly used by big bang advocates, who assume that producers are perfectly informed regarding market conditions.

for period one. Using equations (3) and (6), the expression is

$$\begin{aligned} \sum_{j=1}^n (\hat{p}_{2j} - p^*) &= \frac{1}{1+\gamma} \left(n(p_1 - p^*) + \gamma \sum_{j=1}^n (\hat{p}_{1j} - p^*) \right) \\ &= \frac{\gamma - bn}{1+\gamma} \sum_{j=1}^n (\hat{p}_{1j} - p^*) \end{aligned} \quad (7)$$

There are three cases to consider:

- (i) $\frac{\gamma - bn}{1+\gamma} < -1$
- (ii) $\frac{\gamma - bn}{1+\gamma} \in (-1, 0)$
- (iii) $\frac{\gamma - bn}{1+\gamma} \in (0, 1)$

Depending on which case is applicable, the dynamic path of private production and hence market price begins with either (i) explosive oscillations (ii) damped oscillations or (iii) monotone convergence. Note that the price path cannot monotonically diverge, since $\gamma - bn$ is necessarily less than $1 + \gamma$. Note also that case (i) will necessarily arise if either the number of private producers, n , is sufficiently large or if demand is sufficiently inelastic (i.e., if b is sufficiently large).

As the transition progresses, private producers sequentially revise their estimates of the market price. While earlier price observations are increasingly discounted, each new price observation has an increasingly small role in determining producers' estimates. Combining (4) and (6), we have the following relationship in period t :

$$\begin{aligned} \sum_{j=1}^n (\hat{p}_{1,t+1} - p^*) &= \frac{n}{\sum_{\tau=0}^t \gamma^\tau} (p_t - p^*) + \frac{\sum_{\tau=1}^t \gamma^\tau}{\sum_{\tau=0}^t \gamma^\tau} \sum_{j=1}^n (\hat{p}_{tj} - p^*) \\ &= \frac{\sum_{\tau=1}^t \gamma^\tau - bn}{\sum_{\tau=0}^t \gamma^\tau} \sum_{j=1}^n (\hat{p}_{tj} - p^*) \end{aligned} \quad (8)$$

(cf. the ratio of $(\gamma - bn)$ to $(1 + \gamma)$ in period 2).

In the discussion that follows, we will presume that case (i) obtains and that $\lim_{t \rightarrow \infty} \sum_{\tau=1}^t \gamma^\tau > bn$.

Under this assumption, we can distinguish three phases of the dynamic path of private producers' price expectations, according to whether t satisfies:

- (i) $(\sum_{\tau=1}^{t-1} \gamma^\tau - bn) < -\sum_{\tau=0}^{t-1} \gamma^\tau$
- (ii) $(\sum_{\tau=1}^{t-1} \gamma^\tau - bn) \in [-\sum_{\tau=0}^{t-1} \gamma^\tau, 0]$
- (iii) $\sum_{\tau=1}^{t-1} \gamma^\tau > bn$.

In phase (i), both private production and the market price, oscillate with increasing amplitude; in phase (ii), oscillations continue but they are increasingly damped; in phase (iii), private production and the market price converge monotonically to perfect information levels. We shall refer to phases (i) and (ii) as the *short run*, and to phase (iii) as the *long run*.⁸

An increase in n , the number of private producers, has three consequences. First, there is an increase in the magnitude of oscillations during the short run. Second, the duration of the short-run increases. Third, once the long run is reached, prices and production converge to perfect information levels at a faster rate. All three of these effects are illustrated in Figure 1.⁹ The graphs plot the aggregate deviation of price estimates from the perfect information price, i.e., $\sum_{j=1}^n (\hat{p}_{tj} - p^*)$, when the number of private firms is 5, 5.5 and 6, assuming that (atypically) each firm receives an identical price signal in period zero that exceeds the perfect information price.¹⁰ In the right hand box, convergence is virtually complete, so that the scale is magnified by a factor of 10^{14} . The graphs illustrate the three consequences described above. First, observe how volatility increases with n in the left-hand box. Second, observe that phase (i) (explosive oscillations) lasts for two periods when $n = 5$, for three periods when $n = 5.5$ and for four periods when $n = 6$. Third,

⁸ Obviously, the terms short and long are relative: for some parameter vectors (if $\lim_{t \rightarrow \infty} \sum_{\tau=1}^t \gamma^\tau < bn$), the short-run is forever; for others (if $\gamma > bn$), the long-run begins in period 1.

⁹ The figure depicts the price path generated by a particular vector of private signals in period zero, for an economy parameterized by the following vector: $(a = 40, b = 1, \gamma = 0.885, N = 25, \bar{q} = 0.5p^*(1))$.

¹⁰ Obviously, n should be integer-valued. Throughout the paper, we will ignore this detail, whenever it is convenient to do so.

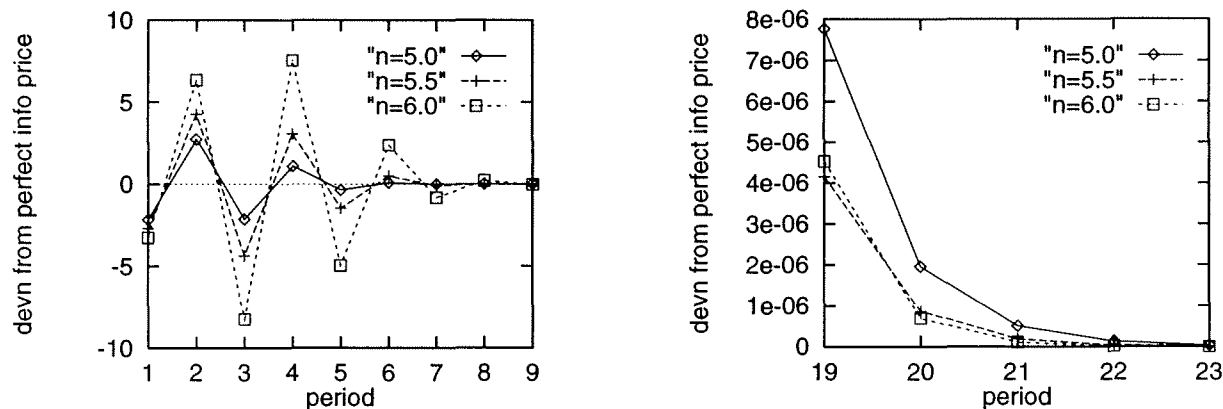


FIGURE 1. Deviations from perfect information prices.

in the right hand box, volatility decreases with n . In this particular example, clearly, the short-run effects of an increase in n overwhelmingly dominate the long-run effects. This is not necessarily true, however: as an extreme example, consider a market in transition for which the “long-run” begins in period 1 (i.e., when $\gamma > bn$).

3.2. The variance of market prices. In the preceding subsection, we considered the effect of n on the price paths generated by a given realization of uncertainty in period zero. A closely related question is the connection between n and the *variance* of price expectations. Specifically, for fixed n , this variance increases with t within phase (i), and decreases with t within phases (ii) and (iii). For fixed t , in contrast, an increase in n increases the variance of prices in the short run (phases (i) and (ii) above). Once the long run is reached, the effect of n on variance is indeterminate. These results are summarized in the following proposition.

Proposition 1. *In the short run, an increase in n increases the variance of the market price. In the long run, the effect is indeterminate.*

Proof. Recall that

$$\begin{aligned} (p_t - p^*) &= a - b \sum_{j=1}^n \hat{p}_{tj} - b(N-n)\bar{q} - p^* \\ &= -b \sum_{j=1}^n (\hat{p}_{tj} - p^*) \end{aligned} \quad (9)$$

Hence

$$\text{Var}(p_t) = b^2 \text{Var}\left(\sum_{j=1}^n \hat{p}_{tj}\right) \quad (10)$$

From (24) below (in the Appendix), the expression for $\text{Var}(\sum_{j=1}^n \hat{p}_{tj})$ is

$$\text{Var}\left(\sum_{j=1}^n \hat{p}_{tj}\right) = n \left(\prod_{s=1}^t \frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau} \right)^2 \sigma^2$$

We can now compute $\frac{d\text{Var}(p_t)}{dn}$ as follows:

$$\frac{d\text{Var}(p_t)}{dn} = \frac{b^2 \text{Var}(p_t)}{n} + 2b^2 n \sigma^2 \left\{ \prod_{s=1}^t \frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau} \right\} \left[\sum_{s=1}^t \frac{-b}{\sum_{\tau=0}^s \gamma^\tau} \left(\prod_{k \neq s} \frac{\sum_{\tau=1}^k \gamma^\tau - bn}{\sum_{\tau=0}^k \gamma^\tau} \right) \right]$$

The first term of the above expression is obviously positive for all t . Now let $t(b, \gamma, n)$ denote the first t such that $\sum_{\tau=1}^t \gamma^\tau > bn$ (i.e., $t(b, \gamma, n)$ is the beginning of the long-run). Fix $t \in [1, t(b, \gamma, n))$. The terms inside the parentheses are each the product of $t-1$ negative terms, and hence are each positive or negative, depending on whether t is odd or even. Each such term is multiplied by a negative number. Hence the summation inside the square brackets is either negative or positive, depending on whether t is odd or even. Finally, the term inside the braces is the product of t negative terms and, therefore, is also either negative or positive, depending on whether t is odd or even. Hence the entire second term is the product of either two negative or two positive numbers, and is thus positive for all t . This establishes that in the short run, i.e., for $t \in [1, t(b, \gamma, n))$, the entire expression is necessarily positive.

Now fix $t \geq t(b, \gamma, n)$. Assume that $t(b, \gamma, n)$ is an odd integer so that short-run consists of an even number of periods. In this case, the term inside the braces is positive. Also, when $s < t(b, \gamma, n)$, the product inside the parentheses is negative; otherwise it is positive. Thus, the term inside the square brackets is the sum of positive terms (for the s that are less than $t(b, \gamma, n)$) and negative terms (for the remaining s 's). Therefore, the sign of the second term in the above expression depends on the number of periods between $t(b, \gamma, n)$ and t . Even if this term is negative, however, it declines in magnitude with t , while the first term remains positive. Hence the total effect of n on the variance of price in the long-run is indeterminate. \square

One might expect the opposite result, i.e., that an increase in n would decrease the variance of prices even in the short-run. The basis for this intuition would be that as n increases, the *average* of the private market signals received by agents will be a more precise estimate of the true demand intercept. In our model, however, private producers do not know the model. In particular, they do not know n . They are not sufficiently sophisticated to infer from price observations what the average private signal must have been, and to realize that they can place greater confidence in this average signal as a predictor of actual market conditions.

3.3. Expected social surplus. In this subsection we consider the welfare implications of the comparative statics result in Proposition 1. The welfare measure we consider is *expected social surplus*, defined as the sum of expected consumer surplus and the expected producer surpluses accruing to private producers and parastatals. Expected social surplus is expressed as the sum of *perfect information social surplus*—i.e., the surplus that would arise if private producers correctly anticipated market prices—and the *social surplus gap*, defined as the shortfall of expected social surplus from the perfect information level. Both terms depend on n . This formulation allows us to highlight a tradeoff that arises each period. While the tradeoff is starkest in the short run, it

also applies to the early stages of the long run.¹¹ For the standard reasons, perfect information social surplus is positively related to n : since parastatals misallocate resources, an increase of n (or, equivalently, in $\alpha = n/N$) moves the perfect information equilibrium price $p^*(\alpha)$ closer to the Pareto optimal price $p^*(1)$. On the other hand, an increase in n reduces private producers' expected profit levels because of the variance effect described above. Since parastatal members are assumed to be risk neutral, their expected surplus is independent of the degree of price variance. Consumer surplus, on the other hand, is positively related to price variance, so that consumers actually benefit from an increase in n . Not surprisingly, however, that this benefit is more than offset by the increase in the private producer surplus gap. Thus, at least in the short run and early in the long run, an increase in n increases *both* the perfect information social surplus and the expected social surplus gap. Moreover, we will show that both measures are concave in n , so that their sum, expected social surplus, attains a unique maximum.

The effects described above depend not only on the number of private producers, n , but also on the *total* number of producers in the sector, N . Accordingly, we treat the total number of producers, N , as a variable rather than a parameter. For purposes of comparison, when we increase N we also increase demand in such a way that in the perfect-information competitive equilibrium, the price $p^*(1)$ remains constant at unity. Specifically, we set the intercept term a equal to $1 + bN$.

Now consider the expected difference between private producer profits in period t and the perfect information profit level. This difference is more negative, the greater is the variance in the sum of private producers' price expectations. The reason is that producers over-produce whenever they collectively overestimate the market price, and this results in a realized price that is below the perfect information level. Ex post, marginal cost exceeds marginal revenue. On the other hand, producers under-produce whenever they collectively underestimate the market price, and

¹¹ In the extremely long run, this tradeoff evaporates: both forces work in the same direction and expected social surplus increases with n .

this results in a realized price that is above the perfect information level. Ex post, marginal revenue exceeds marginal cost. Either way, profits are below what they would be in the absence of price variance: there are no upside gains to offset the downside losses.

The precise expression for the private producer surplus gap is as follows. Letting π_{ti} denote the i 'th producers profit level in period t and π^* denote the perfect information profit level, we have

$$\begin{aligned}
 E\left[\sum_{j=1}^n(\pi_{tj} - \pi^*)\right] &= E\sum_{j=1}^n\left(p_t\hat{p}_{tj} - \frac{1}{2}\hat{p}_{tj}^2\right) - \frac{n}{2}(p^*)^2 & (11) \\
 &= (a - b(1 - \alpha)N\bar{q})E\sum_{j=1}^n(\hat{p}_{tj} - np^*) \\
 &\quad - bE\left[\left(\sum_{j=1}^n\hat{p}_{tj}\right)^2 - (np^*)^2\right] - \frac{1}{2}\sum_{j=1}^nE(\hat{p}_{tj}^2 - p^{*2}) \\
 &= -\left\{bVar\left(\sum_{j=1}^n\hat{p}_{tj}\right) + \frac{1}{2}\sum_{j=1}^nVar(\hat{p}_{tj})\right\}
 \end{aligned}$$

The second equality is obtained by substituting into equations (1) and (5), and rearranging. The third equality follows from the second because for each j , $E\hat{p}_{tj} = p^*$. Relative to the perfect information benchmark, the first term on the right hand side of the third equality is an aggregate expected *revenue* shortfall; the second term is an aggregate expected *cost* overrun.

The other two components of the expected social surplus gap are the surplus gaps associated with nonresponsive producers (parastatals) and with consumers. The former gap is zero: parastatals' production is independent of price, and hence their expected profit levels are independent of price variance. Consumers, on the other hand, positively benefit from price variation, because of substitution possibilities: they buy less when price is high and more when it is low.¹² Letting CS_t denote expected consumer surplus in period t and CS^* the perfect information level of consumer surplus,

¹² This is a standard result from consumer theory, going back at least to Waugh (1944).

the expected consumer surplus gap in period t is

$$\begin{aligned}
E[CS_t - CS^*] &= E \left[\frac{(a - p_t)^2 - (a - p^*)^2}{2b} \right] \\
&= \frac{Var(p_t)}{2b} \\
&= \frac{bVar(\sum_{j=1}^n \hat{p}_{tj})}{2}
\end{aligned} \tag{12}$$

The last equality follows from equation (10).

Let $ESS_t(n)$ denote expected social surplus in period t and $SS^*(n, N)$ denote perfect information social surplus. (Note that both variables are conditional on n (or α) but only the former depends on t . On the other hand, only the latter depends on N .) Now define the *expected social surplus gap* in period t as the difference $SSG_t(n, N) = [ESS_t(n) - SS^*(n, N)]$. We obtain an expression for this gap by combining expressions (11) and (12):

$$\begin{aligned}
SSG_t(n, N) &= - \left\{ bVar\left(\sum_{j=1}^n \hat{p}_{tj}\right) + \frac{1}{2} \sum_{j=1}^n Var(\hat{p}_{tj}) \right\} + \frac{1}{2} bVar\left(\sum_{i=1}^n \hat{p}_{ti}\right) \\
&= -\frac{1}{2} \left\{ bVar\left(\sum_{j=1}^n \hat{p}_{tj}\right) + \sum_{j=1}^n Var(\hat{p}_{tj}) \right\} \\
&= -\frac{1}{2} \left\{ (n-1)(\gamma^{t-1})^2 + (bn+1) \left(\prod_{s=1}^t \frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau} \right)^2 \right\} \sigma^2
\end{aligned} \tag{13}$$

That this term is negative is entirely unsurprising, in spite of the fact that consumers benefit from price uncertainty. From a social perspective, uncertainty over market prices results in deadweight losses: when private producers' aggregate price expectations are overly optimistic (pessimistic), the marginal cost of the last unit of production exceeds (falls short of) its social marginal benefit ex post.

Our primary interest is in the effect of n on the social surplus gap. The gap clearly widens with n , since its magnitude increases (becomes more negative) with the variance of market prices, which in turn increases with n . Less obviously, the gap is concave (widens at an increasing rate) with n and

this fact will prove useful later, when we identify a unique maximum. The proof of the following result is relegated to the appendix (page 30).

Proposition 2. *For any $t \in (1, t(b, \gamma, n)]$,¹³ the expected social surplus gap widens with n , and at an increasing rate. (That is, both $\frac{\partial SS G_t(n, N)}{\partial n}$ and $\frac{\partial^2 SS G_t(n, N)}{\partial n^2}$ are negative.)*

3.4. Perfect information social surplus. Recall that perfect information social surplus is the level of surplus that would arise if private producers could correctly anticipate market prices. Since parastatals misallocate resources, this variable clearly increases as the share of parastatals in total production declines, at a rate that is decreasing but bounded away from zero. Once again, the proof of the following result is in the appendix (page 31).

Proposition 3. *Perfect information social surplus, $SS^*(\cdot, N)$ is concave in n , and increases with n at a rate bounded away from zero. Moreover, the rate at which $SS^*(\cdot, N)$ increases with n is increasing in N (i.e., $\frac{\partial^2 SS^*(\cdot, \cdot)}{\partial n \partial N} > 0$).*

3.5. Expected Social Surplus. Expected social surplus is the sum of perfect information social surplus and the social surplus gap. We have seen that in the short run the former increases while the latter widens with n , giving rise to a welfare tradeoff. The nature of this tradeoff is quite subtle, as Fig. 2 below illustrates. The figure plots expected social surplus for the first ten periods of a transition economy, for α values that lie between 0.23 and 0.45.¹⁴ We have selected a parameter range such that price paths begin with in phase (i), i.e., with explosive oscillations. The surface of the graph is characterized by a distinct “wrinkle” in the top left corner. Notice, however, that *for each period*, the expected surplus is concave in n . More specifically, in period 1 of the model, expected social surplus increases with n up to $n = 7$. For periods 2 through 4, however, surplus

¹³ Recall that $t(b, \gamma, n)$ is the beginning of the long-run

¹⁴ The graph was generated by the following parameter vector ($a = 30, b = 1, \gamma = 0.885, N = 15, \bar{q} = 0.8p^*(1), \sigma^2 = 0.13$).

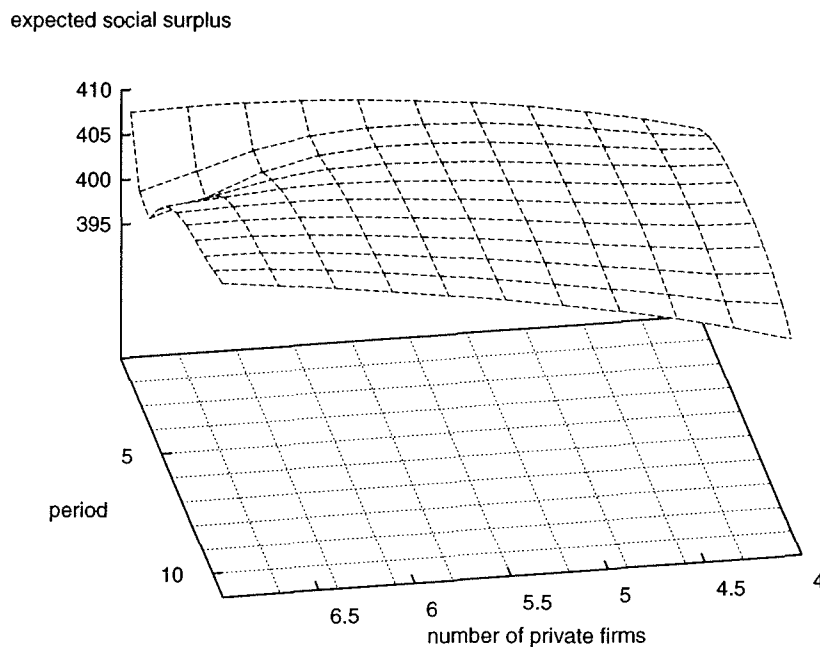


FIGURE 2. Expected Social Surplus and the Size of the Private Sector ($N=20$)

peaks at about $n = 6$ and declines rather precipitously thereafter. By the tenth period, social surplus increases quite steeply with n over the specified range.

The wrinkle may be explained as follows: At the outset (period one) the social surplus gap is rather small, and increases with n at a rate that is dominated by the increase in perfect information surplus. As phase (i) progresses and the magnitude of the price oscillations increase, the social surplus gap widens, and its rate of change with n begins to exceed the rate of change in perfect information surplus at about $n = 6$. By the time period 10 has been reached, however, the effect of the initial price uncertainty has worked its way out of the system, and the social surplus gap is more or less negligible.

3.6. Present discounted value of expected social surplus. So far, we have considered the relationship between private enterprise and expected social surplus *at a given point in time*. However,

the key policy issue our analysis addresses is: what fraction of firms should be privatized, assuming that this fraction will be fixed for the entire transition period? To address this question, we consider the decision problem facing a policymaker with discount rate δ , whose objective is to maximize the present discounted value of expected social surplus, defined as $ESS(n) = \sum_{\tau=1}^{\infty} \delta^{\tau} ESS_{\tau}(n)$, and whose only policy instrument is the level of n .

A key factor in determining the solution to this decision is the length of the short-run. As Fig. 2 suggests, the trade-off discussed in subsection 3.5 only arises in the short-run; in the long-run, by contrast, an increase in n unambiguously increases expected social surplus. It turns out, however, that an increase in n not only widens the social surplus gap in the short-run, but increases the length of the short-run itself.

Proposition 4. *The length of the short run, which is characterized by oscillating prices and quantities, is increasing in n .*

Proof. Recall that $t(b, \gamma, n)$ denotes the first t such that $\sum_{\tau=1}^t \gamma^{\tau} > bn$. (i.e., $t(b, \gamma, n)$ is the beginning of the long-run). By inspection, an increase in n raises the level that the left-hand side must reach in order to satisfy the inequality. Hence $t(b, \gamma, n)$ is increasing in n . \square

We can now address the policy issue raised above. Not surprisingly, the answer depends on the extent of subjective uncertainty about market conditions (i.e., the variance σ^2 of producers' private signals in period zero), the elasticity of the demand curve (i.e., the magnitude of b), the size of the economy (N), the rate (γ) at which producers discount past price observations and, of course, the rate (δ) at which the policymaker discounts the future. Because our determinate results so far pertain only to the short-run, yet the issue at hand necessarily involves long-run considerations, we can say something definitive about the issue only if the long-run is "sufficiently unimportant." For this reason, the result below applies only to economies that (a) are sufficiently large (i.e., large N)

to have a region of privatization levels for which “the short-run is forever,” and (b) have sufficiently small uncertainty (i.e., small σ^2) that the optimal level of privatization lies within this region.¹⁵ Specifically, we consider a two-dimensional family of economies parameterized by $(N, \sigma^2) \in \mathfrak{R}_+^2$: the economies are identical except that in the (N, σ^2) 'th economy, there are a total of N producers and the variance of private market signals is σ^2 . We let $n^*(N, \sigma^2)$ denote the level of n that maximizes $ESS(n)$. Proposition 5 below states that for economies with sufficiently large N , if uncertainty is minimal, then full privatization is optimal. As the level of uncertainty increases, the optimal level of privatization begins to decline monotonically. Eventually, for high enough levels of uncertainty, we cannot guarantee either that a unique local optimum exists or that if a unique optimum did exist, it would depend monotonically on the level of uncertainty.

Proposition 5. *For N sufficiently large, there exist scalars $0 < \underline{\sigma}^2 < \bar{\sigma}^2$ such that $n^*(N, \sigma^2) = N$, for $\sigma^2 \in [0, \underline{\sigma}^2]$, while for $\sigma^2 \in (\underline{\sigma}^2, \bar{\sigma}^2)$, $n^*(N, \cdot)$ declines monotonically as σ^2 increases.*

Proof. Define $\bar{n} = \frac{\gamma}{b(1-\gamma)}$ and note that for $n > \bar{n}$, $t(b, \gamma, n) = \infty$, i.e., “the short-run is forever” so that phase (iii) is never reached. Also define $\Delta SS^*(\bar{n}, N) = \frac{\delta}{1-\delta} (SS^*(\bar{n} + 1, N) - SS^*(\bar{n}, N))$. Since $\frac{\partial^2 SS^*(\cdot, \cdot)}{\partial n \partial N}$ is positive (Proposition 3), it follows that:

$$\Delta SS^*(\bar{n}, \cdot) \text{ is strictly increasing in } N. \quad (14)$$

Now for each (n, σ^2, N) , define $SSG(n, N, \sigma^2)$ as follows:

$$SSG(n, N, \sigma^2) = \left| \sum_{\tau=1}^{\infty} \delta^\tau SSG_\tau(n, N, \sigma^2) \right| \quad (15)$$

¹⁵ Restricting attention exclusively to the short is sufficient but not necessary for the result that follows. Specifically, the only characteristics that distinguish the short-run from the long-run are the derivative properties cited in Proposition 2. These properties also hold for t 's in the early stages of the long-run. We cannot specify in general, however, just how far into the long-run they hold.

(Note that SSG is defined to have the opposite sign of SSG_t 's.) Note from (13) that $SSG_\tau(n, N, \sigma^2)$ is linear in σ^2 and independent of N . Hence, for all n, N ,

$$SSG(n, N, \cdot) \text{ is strictly increasing in } \sigma^2 \text{ and independent of } N \quad (16)$$

Let $\bar{\zeta}^2(N)$ denote the supremum of the σ^2 's such that $SSG(\bar{n} + 1, N, \sigma^2) \leq \Delta SS^*(\bar{n}, N)$ and note from statements (14) and (16) that

$$\bar{\zeta}^2(\cdot) \text{ is strictly increasing in } N. \quad (17)$$

Now fix (N, σ^2) such that $N \geq \bar{n} + 1$ and $\sigma^2 < \bar{\zeta}^2(N)$. We now claim that:

if $ESS(\cdot, N, \sigma^2)$ attains a local maximum at $n \in [\bar{n} + 1, N]$ then
 $ESS(\cdot, N, \sigma^2)$ is globally maximized at n .

To see this, observe that $ESS(\cdot, N, \sigma^2)$ is concave on the interval $[\bar{n}, N]$, since on this region, the short-run is forever so that $ESS(\cdot, N, \sigma^2)$ is a weighted sum of concave functions. Now suppose that $ESS(\cdot, N, \sigma^2)$ attains a local maximum at $n^* \in [\bar{n} + 1, N]$. Observe that (i) the function, attains a *global maximum relative to this interval* at n^* ; (ii) $SS^*(\cdot, N)$ is strictly increasing in n on $[0, \bar{n}]$; (iii) $ESS(\cdot, N, \sigma^2)$ is everywhere less than $\frac{\delta}{1-\delta}SS^*(\cdot, N)$; (iv) for $\sigma^2 < \bar{\zeta}^2(N)$

$$\begin{aligned} \frac{\delta}{1-\delta}SS^*(\bar{n}, N) &= \frac{\delta}{1-\delta}SS^*(\bar{n} + 1, N) - \Delta SS^*(\bar{n}, N) \\ &\leq \frac{\delta}{1-\delta}SS^*(\bar{n} + 1, N) - SSG(\bar{n} + 1, N, \sigma^2) \\ &= ESS(\bar{n} + 1, N, \sigma^2) \leq ESS(n^*, N, \sigma^2) \end{aligned}$$

Observations (i)-(iv) together imply that n^* is a global maximum for $ESS(\cdot, N, \sigma^2)$ on $[0, N]$.

To complete the proof, we need to show that for N sufficiently large, $ESS(\cdot, N, \sigma^2)$ does indeed attain a local maximum at $n^* \in [\bar{n} + 1, N]$ and that this maximum depends on σ^2 as specified in the proposition. Clearly, $ESS(\cdot, N, \sigma^2)$ will attain a local maximum at N if and only if

$\frac{\delta}{1-\delta} \frac{\partial SS^*(n, N)}{\partial n} \Big|_{n=N} > \frac{\partial SSG(n, N, \sigma^2)}{\partial n} \Big|_{n=N}$. Moreover, $SSG(\cdot, \cdot, \sigma^2)$ is identically zero when $\sigma^2 = 0$, while from Proposition 3, $\frac{\partial SS^*(n, N)}{\partial n} \Big|_{n=N}$ is positive, regardless of σ^2 . On the other hand, since $SSG(n, N, \sigma^2)$ is linear in σ^2 , there exists a strictly positive function $\underline{\zeta}^2(N)$ defined by the condition:

$$\underline{\zeta}^2(N) = \inf \left\{ \sigma^2 > 0 : \frac{\delta}{1-\delta} \frac{\partial SS^*(n, N)}{\partial n} \Big|_{n=N} < \frac{\partial SSG(n, N, \sigma^2)}{\partial n} \Big|_{n=N} \right\} \quad (18)$$

Finally, since $\frac{\partial^2 SS^*(n, N)}{\partial n \partial N} \Big|_{n=N}$ is constant, while $\frac{\partial^2 SSG(\tau, n, N, \sigma^2)}{\partial n \partial N} \Big|_{n=N}$ increases with N , it follows that

$$\underline{\zeta}^2(\cdot) \text{ is strictly decreasing in } N. \quad (19)$$

Therefore, there exists $\bar{N} > 0$ such that for $N > \bar{N}$, $\bar{\zeta}^2(\cdot) > \underline{\zeta}^2(\cdot)$. It follows immediately that for $N > \bar{N}$, $n^*(N, \sigma^2) = N$, for all $\sigma^2 \in [0, \underline{\zeta}^2(N)]$, while, $n^*(N, \cdot)$ declines monotonically in σ^2 , for $\sigma^2 \in [\underline{\zeta}^2(N), \bar{\zeta}^2(N)]$. \square

The idea underlying Proposition 5 is illustrated by Fig. 3, which plots the present discounted value of expected social surplus as a function of signal variance (σ^2 ranges from zero to 0.12) and the fraction of privatized firms (α ranges from zero to unity). The policymaker's rate of time discount is 0.9 and the time horizon is 50 periods.¹⁶ For σ^2 values below 0.03, the PDV of expected social surplus increases monotonically in α , and full privatization is optimal. As σ^2 increases further, our surplus measure declines sharply at high levels of privatization, and the optimal level of privatization declines monotonically.

4. CONCLUSION

This paper is premised on the idea that there is a learning process which inevitably occurs during the transition from central planning to a market economy. This process gives rise to a welfare

¹⁶ The remaining parameter values for the figure were: ($a = 40, b = 1, \gamma = 0.885, N = 25, \bar{q} = 0.5p^*(1)$).

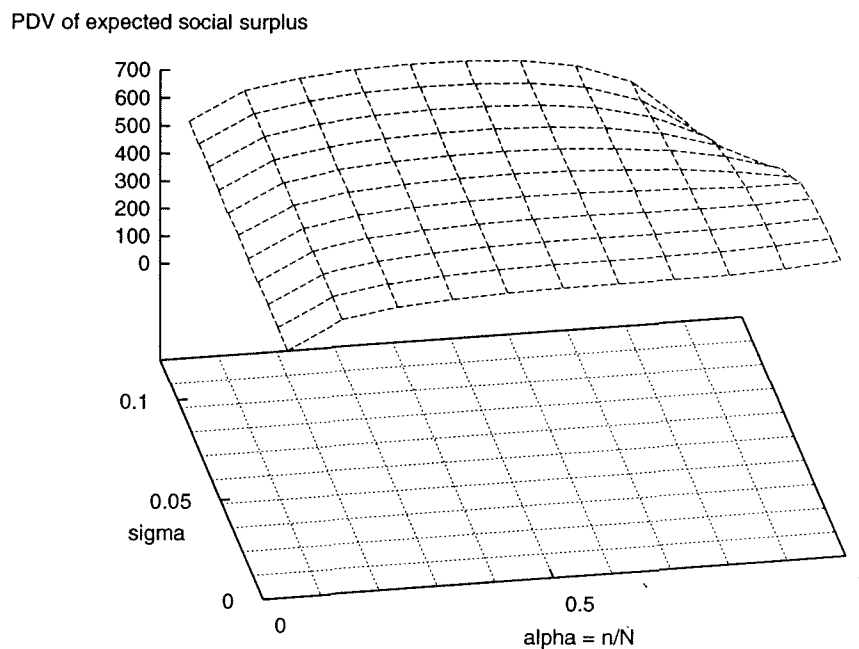


FIGURE 3. Uncertainty and the optimal degree of private enterprise

tradeoff associated with privatization policy in transition economies, when market liberalization is accompanied by uncertainty over market conditions. Our main result is that as the degree of uncertainty regarding market conditions increases, it becomes less and less likely that a policy of rapid privatization will be optimal. On the one hand, an increase in the number of responsive private producers increases the *long-run* rate of convergence to perfect information conditions, thus increasing the level of welfare that will be obtained in the very long run, once learning has been completed. These effects are welfare-enhancing. On the other hand, this increase in n also increases both short-run price and production volatility as well as the time it takes for this volatility to work its way out of the system. These effects diminish welfare.

In the vast literature on transition economies to date, proponents of the “big-bang” approach to transitions have emphasized the welfare enhancing effects of privatization, while proponents of the “gradualist” approach have largely ignored the learning issue raised in this paper. This paper

contributes to the literature by pointing out that while effective price liberalization cannot be accomplished without private enterprise, there is a possibility that too much private enterprise may diminish the effectiveness of price liberalization policies.

While policy makers in transition economies are more concerned with dynamic issues, such as the optimal *rate* at which parastatals should be privatized, than with static ones, such as the optimal *level* of privatization, our static analysis has some clear dynamic implications. Specifically, it suggests that the greater the degree of subjective uncertainty about market conditions, the more gradually should the privatization process begin. Also, government policies that support information provision and institution-building will be particularly important in the earliest stages of transitions, when their benefits are largest. In addition, information provision will be more important in industries with more privatized producers.

Rather than supporting either side of the big-bang vs. gradualism debate, our analysis adds a new dimension to the debate by emphasizing the learning process. The tradeoff we derive favors gradualism under some circumstances and big-bangs under others. Even when the learning considerations addressed in this paper would suggest a gradualist approach, gradualism may not be optimal when broader considerations, particularly political-economic ones, are taken into account. Regardless of these considerations, however, our analysis indicates that because the big-bang approach fails to acknowledge the costs of rapid privatization in a uncertain environment, its predictions will be likely to be overly optimistic except when uncertainty is minimal and learning is correspondingly unimportant.

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6. APPENDIX

6.1. Behavior of prices and expectations. In order to characterize the effect of the number of private producers on the market adjustment process, we must first characterize the behavior of \hat{p}_{ti} and p_t , and their relationship to p^* . We begin by showing that \hat{p}_{ti} is an unbiased estimator of p_t which in turn is equal in expectation to the perfect information price p^* .¹⁷

Proposition 6. \hat{p}_{ti} is an unbiased estimator of p^* .

Proof. Since $\hat{p}_{ti} = (p_{0i} + \sum_{\tau=1}^{t-1} p_\tau)/t$, and, by assumption, $E[p_{0i}] = p^*$, it suffices to show that $E[p_\tau] = p^*$, for $\tau = 1, \dots, t-1$. We begin by showing that the expectation of p_1 is p^* . Recall that private producer i sets quantity in period t equal to her estimate of the mean of p_t . In period one, this is just p_{0i} , so that:

$$\begin{aligned} E[p_1] &= E \left[a - b \left(\sum_{j=1}^n p_{j0} + (N-n)\bar{q} \right) \right] \\ &= a - b(np^* + (N-n)\bar{q}) \\ &= p^*. \end{aligned} \tag{20}$$

Now suppose that $E[p_\tau]$ equals p^* , for $\tau = 1, \dots, t-1$. We will establish that $E[p_t]$ is also equal to p^* . In this round, i sets quantity equal to $\left(\sum_{\tau=0}^{t-1} \gamma^\tau \right)^{-1} \left[\sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1} + \gamma^{t-1} p_{0i} \right]$, so that

$$E[p_t] = E \left[a - b \frac{n \sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1} + \gamma^{t-1} \sum_{j=1}^n p_{j0}}{\sum_{\tau=0}^{t-1} \gamma^\tau} - b(1-\alpha)N\bar{q} \right], \tag{21}$$

which by inspection is p^* . □

Proposition 7. If $\gamma \in (0, 1)$ satisfies $\frac{1+\gamma}{1-\gamma} > bn$, then the \hat{p}_{tj} 's are consistent estimators of p^* (i.e., for each j $\lim_{t \rightarrow \infty} \text{Var}(\hat{p}_{tj}) = 0$).

Proof. Consider the following expression, derived below, for the variance of the sum of private producers' price signals, i.e., $\text{Var}(\sum_{j=1}^n \hat{p}_{tj})$, where

$$\text{Var} \left(\sum_{j=1}^n \hat{p}_{tj} \right) = n \left(\prod_{s=1}^t \frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau} \right)^2 \sigma^2 \tag{22}$$

To establish consistency, it is sufficient to show that $\text{Var}(\sum_{j=1}^n \hat{p}_{tj})$ goes to zero with t . To establish this, it is in turn sufficient to show that there exists \bar{s} and $\epsilon > 0$ such that for all $s > \bar{s}$, the absolute

¹⁷ Note that this is true only when marginal costs are linear as in our case. In general, the estimator will be biased upward if the producer's marginal cost curve is concave and downward if it is convex.

value of the term $\frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau}$ is less than $1 - \epsilon$. To establish this, we will show that

$$1 > \lim_{s \rightarrow \infty} \left| \frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau} \right| \quad (23)$$

We have

$$\begin{aligned} & \lim_{s \rightarrow \infty} \frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau} + 1 \\ &= \frac{\frac{\gamma}{1-\gamma} - bn}{\frac{1}{1-\gamma}} + 1 \\ &= (1 + \gamma) - (1 - \gamma)bn \end{aligned}$$

But by assumption $\frac{1+\gamma}{1-\gamma} > bn$, hence condition (23) is satisfied. This establishes consistency. \square

6.2. Deriving an expression for the variance of the sum of the signals.

Proposition 8. $Var(\sum_{j=1}^n \hat{p}_{tj})$ may be expressed as a function of γ, t, b, n , and σ^2 .

Proof. Recall that

$$\begin{aligned} (p_t - p^*) &= a - b \sum_{j=1}^n \hat{p}_{tj} - b(N - n)\bar{q} - p^* \\ &= -b \sum_{j=1}^n (\hat{p}_{tj} - p^*) \end{aligned} \quad (24)$$

Hence $Var(p_t) = b^2 Var(\sum_{j=1}^n \hat{p}_{tj})$. In the display that follows, we adopt the convention that $\prod_{\tau \in \emptyset} x_\tau = 1$ (i.e., that the product of an empty set of terms is unity). Now:

$$\begin{aligned} \sum_{j=1}^n (\hat{p}_{1,t+1} - p^*) &= \frac{n}{\sum_{\tau=0}^t \gamma^\tau} (p_t - p^*) + \frac{\sum_{\tau=1}^t \gamma^\tau}{\sum_{\tau=0}^t \gamma^\tau} \sum_{j=1}^n (\hat{p}_{tj} - p^*) \\ &= \frac{\sum_{\tau=1}^t \gamma^\tau - bn}{\sum_{\tau=0}^t \gamma^\tau} \sum_{j=1}^n (\hat{p}_{tj} - p^*) \end{aligned}$$

Decomposing \hat{p}_{tj} into expressions for p_{t-1} and $\hat{p}_{t-1,j}$'s yields

$$\begin{aligned} \sum_{j=1}^n (\hat{p}_{1,t+1} - p^*) &= \left\{ \prod_{s=t-1}^t \frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau} \right\} \sum_{j=1}^n (\hat{p}_{j,t-1} - p^*) \\ &\equiv \left\{ \prod_{s=t-1}^t \frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau} \right\} \sum_{j=1}^n (\hat{p}_{j,t-1} - p^*) \end{aligned}$$

Repeating this process and iterating backward leads to

$$\sum_{j=1}^n (\hat{p}_{1,t+1} - p^*) = \left\{ \prod_{s=1}^t \frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau} \right\} \sum_{j=1}^n (\hat{p}_{j1} - p^*)$$

The \hat{p}_{j1} 's are i.i.d., with variance σ^2 . Hence:

$$\begin{aligned} \text{Var}\left(\sum_{j=1}^n \hat{p}_{tj}\right) &= \left(\prod_{s=1}^t \frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau} \right)^2 n\sigma^2 \\ &= n \left(\prod_{s=1}^t \frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau} \right)^2 \sigma^2 \end{aligned}$$

□

6.3. Deriving an expression for the sum of the variances of the signals.

Proposition 9. $\sum_{j=1}^n \text{Var}(\hat{p}_{tj})$ may be expressed as the following function of $\text{Var}(\sum_{j=1}^n \hat{p}_{tj})$:

$$\sum_{j=1}^n \text{Var}(\hat{p}_{tj}) = \frac{1}{n} \text{Var}\left(\sum_{j=1}^n \hat{p}_{tj}\right) + (n-1)(\gamma^{t-1})^2 \sigma^2. \quad (25)$$

Proof. To see this, observe that

$$\begin{aligned} \text{Var}\left(\sum_{j=1}^n \hat{p}_{tj}\right) &= \text{Var}\left(n \sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1} + \sum_{j=1}^n \gamma^{t-1} p_{j0}\right) \\ &= \text{Var}\left(n \sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1}\right) + \text{Var}\left(\gamma^{t-1} \sum_{j=1}^n p_{j0}\right) \\ &\quad + \text{Cov}\left(n \sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1}, \gamma^{t-1} \sum_{j=1}^n p_{j0}\right) \\ &= n \left\{ \text{Var}\left(\sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1}\right) + n(\gamma^{t-1})^2 \sigma^2 + \gamma^{t-1} \sum_{j=1}^n \text{Cov}\left(\sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1}, p_{j0}\right) \right\} \end{aligned} \quad (26)$$

while

$$\begin{aligned}
\sum_{j=1}^n Var(\hat{p}_{tj}) &= \sum_{j=1}^n Var\left(\sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1} + \gamma^{t-1} p_{j0}\right) \\
&= \sum_{j=1}^n \left\{ Var\left(\sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1}\right) + Var(\gamma^{t-1} p_{j0}) \right. \\
&\quad \left. + Cov\left(\sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1}, \gamma^{t-1} p_{j0}\right) \right\} \\
&= n Var\left(\sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1}\right) + (\gamma^{t-1})^2 n \sigma^2 \\
&\quad + (\gamma^{t-1}) \sum_{j=1}^n Cov\left(\sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1}, p_{j0}\right) \\
&= \frac{1}{n} Var\left(\sum_{j=1}^n \hat{p}_{tj}\right) + (n-1)(\gamma^{t-1})^2 \sigma^2.
\end{aligned} \tag{27}$$

□

6.4. Proofs of Proposition 2 and Proposition 3

Proof of Proposition 2: (For any $t \in (1, t(b, \gamma, n)]$, both $\frac{\partial SSG_t(n, N)}{\partial n}$ and $\frac{\partial^2 SSG_t(n, N)}{\partial n^2}$ are negative.)

From equation (13), the social surplus gap is:

$$SSG_t(n, N) = -\frac{1}{2} \left\{ (n-1)(\gamma^{t-1})^2 + (bn+1) \left(\prod_{s=1}^t \frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau} \right)^2 \right\} \sigma^2$$

Now, to show that this expression is concave in the short run, it is clearly sufficient to show that $\left(\prod_{s=1}^t \frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau} \right)^2 \sigma^2$ is convex. In turn, demonstrating the convexity of this expression reduces by a similar argument to demonstrating that the following expression is convex:

$$\chi(n) = \prod_{s=r+1}^t \frac{\sum_{\tau=1}^s \gamma^\tau - bn}{\sum_{\tau=0}^s \gamma^\tau}$$

To see that it is indeed convex, note that $\chi(n)$ is of the form $\left(\prod_{\tau=1}^k (\theta_\tau - \psi_\tau n) \right)^2$, where for each τ , $\theta_\tau, \psi_\tau > 0$ and $(\theta_\tau - \psi_\tau n) < 0$. The derivative of such a form is $2 \left[\prod_{\tau=1}^k (\theta_\tau - \psi_\tau n) \right] \sum_{\tau=1}^k -\psi_\tau \prod_{s \neq \tau} (\theta_s - \psi_s n)$. The expression $\prod_{\tau=1}^k (\theta_\tau - \psi_\tau n) \prod_{s \neq \tau} (\theta_s - \psi_s n)$ has $2k-1$ terms, all of which are negative.¹⁸ Since $\frac{\partial \chi(n)}{\partial n}$ is a negative linear combination of these negative terms, it must be positive.

¹⁸ If $k=1$, the set $\{s \in [1, k]; s \neq 1\}$ is empty so that by convention, $\prod_{s \neq \tau} (\theta_s - \psi_s n) = 1$.

Similarly, to establish that the second derivative is positive, it is sufficient to establish that the derivative with respect to n of $2 \left[\prod_{\tau=1}^k (\theta_{\tau} - \psi_{\tau} n) \right] \sum_{\tau=1}^k -\psi_{\tau} \prod_{s \neq \tau} (\theta_s - \psi_s n)$ is positive. We have

$$\begin{aligned} \frac{\partial^2 \chi(n)}{\partial n^2} &= \frac{\partial}{\partial n} \left\{ 2 \left[\prod_{\tau=1}^k (\theta_{\tau} - \psi_{\tau} n) \right] \sum_{\tau=1}^k -\psi_{\tau} \prod_{s \neq \tau} (\theta_s - \psi_s n) \right\} \\ &= -2 \left\{ \frac{\partial}{\partial n} \left[\prod_{\tau=1}^k (\theta_{\tau} - \psi_{\tau} n) \right] \sum_{\tau=1}^k \psi_{\tau} \prod_{s \neq \tau} (\theta_s - \psi_s n) \right. \\ &\quad \left. + \left[\prod_{\tau=1}^k (\theta_{\tau} - \psi_{\tau} n) \right] \sum_{\tau=1}^k \psi_{\tau} \frac{\partial}{\partial n} \left[\prod_{s \neq \tau} (\theta_s - \psi_s n) \right] \right\} \end{aligned} \quad (28)$$

Now $\frac{\partial}{\partial n} \left[\prod_{\tau=1}^k (\theta_{\tau} - \psi_{\tau} n) \right]$ is a negative linear combination of products of $k-1$ negative terms, and so has the same sign as $(-1)^k$. This expression is multiplied by $\sum_{\tau=1}^k \psi_{\tau} \prod_{s \neq \tau} (\theta_s - \psi_s n)$, which has the same sign as $(-1)^{k-1}$. Thus the first term inside the curly brackets has the same sign as $(-1)^{2k-1}$, which is negative. Similarly, $\frac{\partial}{\partial n} \left[\prod_{s \neq \tau} (\theta_s - \psi_s n) \right]$ is a negative linear combination of products of $k-2$ terms, and so has the same sign as $(-1)^{k-1}$. Each such term is multiplied by the product of k negative terms. Hence, the second term inside the curly brackets has the same sign as $(-1)^{2k-1}$, which is negative. Since the term inside the curly brackets is multiplied by -2 , $\frac{\partial^2 \chi(n)}{\partial n^2}$ and hence $\frac{\partial^2 \xi(n,t)}{\partial n^2}$ is positive. Therefore, the gap between perfect information and expected social surplus is concave and decreasing in n in the short run. \square

Proof of Proposition 3: (*Perfect information welfare is increasing and concave in n . Moreover, the rate at which $SS^*(\cdot, N)$ increases with n is increasing in N .*)

The proof of this proposition is based on the behavior of $p^*(\alpha)$. Recall that in equilibrium

$$p^*(\alpha) = \frac{a - b(1 - \alpha)N\bar{q}}{1 + b\alpha N}$$

Since N is held constant, α and n exhibit the same pattern of behavior. Differentiating with respect to α yields

$$\frac{dp^*}{d\alpha} = \frac{bN}{1 + b\alpha N} \left(\bar{q} - \frac{a - b(1 - \alpha)N\bar{q}}{1 + b\alpha N} \right) \quad (29)$$

$$= \frac{bN}{1 + b\alpha N} (\bar{q} - p^*(\alpha)) \quad (30)$$

Note that the sign of $\frac{dp^*}{d\alpha}$ depends on the relative magnitudes of \bar{q} and $p^*(\alpha)$, or whether the parastatals are producing more or less per person than the free market outcome where $\alpha = 1$.

This same relationship between the free market and parastatal level of production determines the sign of the second derivative, as we will show.

$$\frac{d^2 p^*}{d\alpha^2} = \frac{2(bN)^2}{(1+b\alpha N)^2} \left(\frac{a-b(1-\alpha)N\bar{q}}{(1+b\alpha N)} \right) \quad (31)$$

$$= \frac{2(bN)^2}{(1+b\alpha N)^2} (p^*(\alpha) - \bar{q}) \quad (32)$$

If $\bar{q} < p^*(\alpha)$, $\frac{dp^*}{d\alpha}$ is negative, and $\frac{d^2 p^*}{d\alpha^2}$ is positive. If $\bar{q} > p^*(\alpha)$, $\frac{dp^*}{d\alpha}$ is positive, and $\frac{d^2 p^*}{d\alpha^2}$ is negative. We will show below that these relationships determine the behavior of perfect information social surplus as a function of α .

Consider the following expression for perfect information social surplus:

$$SS^*(n, N) = (1-\alpha)(p\bar{q} - \frac{\bar{q}^2}{2}) + \frac{\alpha N(p^*(\alpha))^2}{2} + \frac{(a-p^*(\alpha))(\alpha N p^*(\alpha) + (1-\alpha)N\bar{q})}{2} \quad (33)$$

The derivative of the above expression with respect to α is

$$\begin{aligned} \frac{dSS^*(n, N)}{d\alpha} &= \frac{N}{2} ((a-\bar{q})(p^*(\alpha) - \bar{q})) + \frac{N}{2} \frac{dp^*(\alpha)}{d\alpha} (\alpha p^*(\alpha) + (1-\alpha)\bar{q} + \alpha(a-p^*(\alpha))) \\ &= \frac{N}{2} (\bar{q} - p^*(\alpha)) \left((\bar{q} - a) + \frac{bN}{1+b\alpha N} (\alpha(a-\bar{q}) + \bar{q}) \right) \end{aligned}$$

Substituting in our definition of $p^*(\alpha)$ yields:

$$\frac{dSS^*(n, N)}{d\alpha} = \frac{N}{2} (\bar{q} - p^*(\alpha))^2$$

By inspection, this expression is always nonnegative. It is zero if and only if $\bar{q} = p^*(\alpha)$. If this does not hold, then it is positive. Consequently, $\frac{dSS^*(n, N)}{d\alpha}$ is always nonnegative.

The second derivative of welfare with respect to α is

$$\begin{aligned} \frac{d^2 SS^*(n, N)}{d\alpha^2} &= N \frac{dp^*}{d\alpha} (a - \bar{q}) + \frac{N}{2} \frac{d^2 p^*(\alpha)}{d\alpha^2} (a\alpha + (1-\alpha)\bar{q}) \\ &= \frac{bN^2}{1+b\alpha N} \left((\bar{q} - p^*(\alpha))(a - \bar{q}) + (p^*(\alpha) - \bar{q}) \frac{bN}{1+b\alpha N} (a\alpha + (1-\alpha)\bar{q}) \right) \\ &= \frac{bN^2}{1+b\alpha N} \left((\bar{q} - p^*(\alpha)) \left(\frac{a}{1+bN} - \bar{q} \right) \right) \end{aligned}$$

In order to sign this derivative, consider the expression inside the parentheses. For the second derivative to be negative, the expression must be negative. Recall that $\frac{a}{1+bN}$ is equal to $p^*(1)$. This is the price that prevails when the sector is fully composed of private producers, and, more importantly, the price that prevails when parastatals produce exactly the right amount, $\bar{q} = p^*$. When parastatals produce the right amount, the expression is equal to 0. When this condition does not hold, $\frac{a}{1+bN} > p^*$, and the overall expression is negative.

Consequently, the second derivative of social surplus with respect to α is always nonpositive, and social surplus is concave in α . In fact, the second derivative is always negative except when $\bar{q} = p^*(\alpha)$.

Now we will show that the rate at which $SS^*(\cdot, N)$ increases with n is increasing in N . Here we will use n rather than α .

We will use the following derivatives of p^* that have not been previously derived:

$$\begin{aligned}\frac{dp^*}{dN} &= \frac{-b(1-\bar{q})}{1+bn} \\ \frac{d^2p^*}{dn dN} &= \frac{-b^2(1-\bar{q})}{(1+bn)^2}\end{aligned}$$

Differentiating $\frac{dSS^*}{dn}$ with respect to N results in

$$\begin{aligned}\frac{\partial^2 SS^*}{\partial n \partial N} &= -\bar{q} \frac{dp^*}{dN} + \bar{q} \frac{dp^*}{dn} + (N-n)\bar{q} \frac{\partial^2 p^*}{\partial n \partial N} + p^* \frac{dp^*}{dN} + n \frac{dp^*}{dN} \frac{dp^*}{dn} + np^* \frac{\partial^2 p^*}{\partial n \partial N} \\ &\quad + \frac{a-p^*}{2} \left(\frac{dp^*}{dN} + n \frac{\partial^2 p^*}{\partial n \partial N} \right) + \left(\frac{da}{dN} - \frac{dp^*}{dN} \right) \frac{(p^* + n \frac{dp^*}{dn} - \bar{q})}{2} \\ &\quad - \frac{\partial^2 p^*}{\partial n \partial N} \frac{(np^* + (N-n)\bar{q})}{2} - \frac{dp^*}{dn} \frac{n \frac{dp^*}{dN} + \bar{q}}{2} \\ &= \frac{dp^*}{dn} \frac{\bar{q} + bn}{2} + \frac{d^2 p^*}{dn dN} \frac{(N-n)\bar{q} + na}{2} + \frac{dp^*}{dN} \frac{a - \bar{q}}{2} + b \frac{p^* - \bar{q}}{2}\end{aligned}$$

Substituting in for the derivatives and simplifying yields

$$\frac{\partial^2 SS^*}{\partial n \partial N} = \frac{ab(1-\bar{q})^2}{(1+bn)^2}$$

Since this expression is always nonnegative, the rate at which $SS^*(\cdot, N)$ increases with n is nondecreasing in N . If $\bar{q} \neq p^*$, then $SS^*(\cdot, N)$ increases with n is increasing in N .

□