

UCLA

Recent Work

Title

Stochastic Correlation Across International Stock Markets

Permalink

<https://escholarship.org/uc/item/6vn9q79w>

Authors

Ball, Clifford A.

Torous, Walter N.

Publication Date

2000-06-07

Stochastic Correlation Across International Stock Markets

Clifford A. Ball ¹
Owen Graduate School of Management
Vanderbilt University
Nashville, TN 37203

Walter N. Torous
Anderson School of Management
UCLA
Los Angeles, CA 90095

June 7, 2000

¹We thank the Financial Markets Research Center at Vanderbilt University for financial support. Ronald Mahieu and Franz Palm provided helpful comments and suggestions on an earlier draft. We also thank the participants of the Journal of Empirical Finance Conference on Risk Management for their helpful remarks.

Abstract

This paper examines the correlation across a number of international stock market indices. As correlation is not observable, we assume it to be a latent variable whose dynamics must be estimated using data on observables. To do so, we use filtering methods to extract stochastic correlation from returns data. We find evidence that the estimated correlation structure is dynamically changing over time. We also investigate the link between stochastic correlation and volatility. In general, stochastic correlation tends to increase in response to higher volatility but the effect is by no means consistent. These results have important implications for portfolio theory as well as risk management.

1 Introduction

Modeling the dynamics of security returns and their risk characteristics remains an important task for both financial research as well as its application. For example, risk management techniques used to assess value at risk (VaR) have gained in popularity in recent years. A common approach in calculating VaR is based on the assumption that the underlying security returns are conditionally multivariate normally distributed and then uses standard portfolio theory to determine the variance of a particular portfolio to assess its risk exposure.

Advances to this approach have for the most part involved the more careful modeling of the covariance structure of the underlying security returns. In particular, most of this effort has been expended on accurately modeling the dynamics of volatility. For example, Generalized Autoregressive Conditional Heteroscedastic (GARCH) models (Engle [1982], Bollerslev [1986], and Nelson [1991]) and stochastic volatility models (Harvey, Ruiz, Shephard [1994], and Kim, Chib and Shephard [1998]) have been used to characterize the volatility of returns of common stocks and other assets.

This paper focuses on the *correlation* structure of security returns. To the extent that economic and political conditions do change over time, we would expect the correlation between international stock markets to change as well. The changing nature of this correlation is consistent with recent empirical evidence. For example, Longin and Solnik [1995] use a GARCH model to investigate the behavior of monthly international equity returns and conclude that the correlation between these returns is dynamically changing.

Ramachand and Susmel [1998] fit a switching ARCH model to weekly international stock market returns and find evidence of markedly different correlations across regimes. Using daily returns of American Depository Receipts (ADRs) to avoid non-synchronicity problems, Karolyi and Stulz [1996] find evidence of changing correlation in the daily returns of US and Japanese indices. Changing correlation also characterizes returns within domestic markets. Kroner and Ng [1998] fit a bivariate ARCH model to the weekly returns of US small and large cap portfolios and conclude that varying the restrictions placed on the evolution of variances as well as correlation can lead to markedly different model parameter estimates.

While much effort has been expended on modeling the multivariate structure of covariance, most of this research has used GARCH models. Multivariate GARCH models for conditional covariance, however, suffer from increasing parameter dimensionality and are often practical to estimate only after imposing severe restrictions, for example, assuming the correlation coefficient is constant (Bollerslev [1990]). In this paper, we model the correlation coefficient as a latent variable and use filtering methods to estimate the resultant non-linear model on the basis of observed security returns. Our approach allows for more flexibility in modeling the dynamics of correlation than the GARCH approach and provides a natural setting in which to assess whether other exogenous factors, such as stock market volatility (Solnik, Boucrelle and LeFur [1996]), statistically affect the behavior of correlation.

The plan of this paper is as follows. Section 2 details the methodology used to estimate the resultant stochastic probit model. We consider the use of both single period returns as well as longer return windows. After describing our returns data in Section 3, we present our empirical results in Section 4. The implications of stochastic correlation for

risk management are explored in Section 5. Section 6 concludes the paper.

2 Methodology

In modeling the correlation between security returns, we use two methods to assemble these returns data. The first uses *each* of the n available single period returns as an observation from which to infer the prevailing correlation. The second groups the single period returns data into m sequential non-overlapping intervals of length ts with $n = m \times ts$. Under this second approach, we assume that the correlation is constant across each interval of length ts and we model the correlation as changing across these m periods so defined. While this latter approach reduces attendant measurement errors, it does so at the expense of having fewer observations from which to estimate the parameters of the model.

2.1 Single Period Returns

We first rely on a pair of single period returns to infer the correlation prevailing between two securities. Assuming we have demeaned the return series, it follows that for a given population correlation ρ the distribution of the sample correlation coefficient, r , degenerates into a discrete distribution with either $r = +1$ or $r = -1$. Under the assumption of conditional bivariate normality, from Abramowitz and Stegun [1965] (page 937) we have

$$\begin{aligned} P[r = +1] &= \frac{1}{2} + \arcsin(\rho)/\pi, \\ P[r = -1] &= \frac{1}{2} - \arcsin(\rho)/\pi. \end{aligned}$$

To investigate the behavior of correlation, we need a model for the dynamics of ρ . For mathematical tractability, we transform the correlation onto the range of the real line and then model the transformed process as a Gaussian autoregressive process.

We consider transformations of the form

$$W = W(\rho),$$

such that $W(0) = 0$, $W(1) = +\infty$, $W(-1) = -\infty$, and $dW/d\rho > 0$. Given the form of the discrete distribution function, we also seek a transformation that permits tractable use of the *arcsin* function. To do so, let $\Phi(\cdot)$ denote the standard normal cumulative distribution function and implicitly define $W(\cdot)$ so that

$$P[r = +1] = \frac{1}{2} + \arcsin(\rho)/\pi = \Phi(W)$$

or equivalently

$$W = \Phi^{-1}\left(\frac{1}{2} + \arcsin(\rho)/\pi\right).$$

That is

$$\pi\Phi(W) - \pi/2 = \arcsin(\rho)$$

or

$$\rho = \sin(\pi\Phi(W) - \pi/2).$$

Combining, we have the following state space model:

an observation equation,

$$P[r_t = +1] = \frac{1}{2} + \arcsin(\rho_t)/\pi = \Phi(W_t), \quad t = 1, \dots, n;$$

and a transition equation,

$$W_t = \alpha + \beta W_{t-1} + \sigma \epsilon_t, \quad t = 1, \dots, n$$

which governs the dynamics of the latent variable for $\{\epsilon_t\}$ a sequence of *i.i.d.* standard normals and where $\rho_t = \sin(\pi\Phi(W_t) - \pi/2)$. The result is a *stochastic* probit model where the state variable W evolves so that the probability of the event (in this case a sample correlation of +1) varies stochastically over time.

2.1.1 Integration-Based Filtering

There are a number of ways of estimating this nonlinear model. A full-nonlinear filter may be run involving numerical integration of the latent variable. Alternatively, following Ball and Torous [1999], a single integration-based filter may be used. As we now demonstrate, in this case the nature of the filter is such that the integration may be implemented analytically thereby maintaining accuracy while reducing computational effort significantly.

Denote the sample correlation at time t by r_t and the set of sample correlations through time t by R_t . For each time point t of a bivariate return series either the two returns have the same sign or opposite sign. When the signs are the same we have $r_t = 1$. To begin with assume that the marginal distribution of W_{t-1} given observations through time $t-1$ is Gaussian:

$$f(W_{t-1} | R_{t-1}) = N(\mu_{t-1}, \sigma_{t-1}).$$

Next project to obtain the conditional distribution of W_t given R_{t-1} which will also be Gaussian

$$f(W_t | R_{t-1}) = N(\mu_{t|t-1}, \sigma_{t|t-1})$$

with mean

$$\mu_{t|t-1} = \alpha + \beta\mu_{t-1}$$

and variance

$$\sigma_{t|t-1}^2 = \beta^2\sigma_{t-1}^2 + \sigma^2.$$

Applying Bayes theorem we have

$$f(W_t, r_t | R_{t-1}) = f(r_t | W_t, R_{t-1}) \times f(W_t | R_{t-1})$$

and integrating this expression with respect to W_t gives the conditional likelihood function $f(r_t | R_{t-1})$. An alternative application of Bayes theorem generates the posterior distribution:

$$f(W_t | R_t) = f(r_t | W_t, R_{t-1}) \times f(W_t | R_{t-1}) / f(r_t | R_{t-1}).$$

Proceeding sequentially in this manner we may calculate the likelihood function:

$$\ln Like = \sum \ln f(r_t | R_{t-1}).$$

From Frühwirth-Schnatter [1994] we make the additional assumption that the posterior distribution $f(W_t | R_t)$ is also Gaussian¹ and obtain the mean and variance parameters which characterize this distribution by integration. In particular, define

$$\begin{aligned} G(\mu, \sigma) &= \int_{w=-\infty}^{w=+\infty} \Phi(w) (2\pi\sigma^2)^{-0.5} \exp(-(w - \mu)^2 / 2\sigma^2) dw \\ F(\mu, \sigma) &= \int_{w=-\infty}^{w=+\infty} (w - \mu) \Phi(w) (2\pi\sigma^2)^{-0.5} \exp(-(w - \mu)^2 / 2\sigma^2) dw \\ H(\mu, \sigma) &= \int_{w=-\infty}^{w=+\infty} (w - \mu)^2 \Phi(w) (2\pi\sigma^2)^{-0.5} \exp(-(w - \mu)^2 / 2\sigma^2) dw. \end{aligned}$$

Noting that the integrand in each case above involves the normal cumulative distribution function, a change of variable and differentiation with respect to μ allows us to obtain the partial derivative of G, F, H with respect to μ as integral expressions that now involve the normal density function rather than the normal cumulative distribution function. Maintaining Gaussian distributions under convolution implies that the resultant integrals can be expressed as Gaussian densities. Subsequent integration with respect to μ regenerates the original functions and we obtain the following results:

$$G(\mu, \sigma) = \Phi\left(\frac{\mu}{(1 + \sigma^2)^{0.5}}\right)$$

¹If the posterior is close to a normal density this approximation error is small. The results of Fahrmeir [1992] indicate that the posterior tends to normal even in cases where the observation density is extremely non-normal.

$$\begin{aligned}
F(\mu, \sigma) &= \sigma^2 (2\pi\sigma^2)^{-0.5} \exp\left(-\frac{\mu^2}{2(1+\sigma^2)}\right) \\
H(\mu, \sigma) &= \sigma^2 G(\mu, \sigma) + \mu \frac{\sigma^2}{1+\sigma^2} F(\mu, \sigma^2).
\end{aligned}$$

From these results the likelihood function can be expressed as

$$Like(r_t | R_t) = G(r_t \mu, \sigma).$$

The updating step in the filter generates the following analytic values for the mean and variance at time t given available information through time t :

$$\begin{aligned}
\mu_t &= \mu_{t|t-1} + \frac{F(r_t \mu_{t|t-1}, \sigma_{t|t-1})}{G(r_t \mu_{t|t-1}, \sigma_{t|t-1})} \\
\sigma_t^2 &= \frac{HG - F^2}{G^2} | (r_t \mu_{t|t-1}, \sigma_{t|t-1}).
\end{aligned}$$

Maximum likelihood parameter estimation requires numerical optimization of this likelihood function across the parameter space. Additionally, asymptotic standard errors are obtained from the inverse Hessian computed at the maximum likelihood estimates.²

²The model may also be estimated using Gibbs sampling. To see this define a process Z_t where

$$\begin{aligned}
Z_t &\sim N(W_t, 1) \\
r_t &= I_{Z_t > 0}
\end{aligned}$$

so that $P[r_t = 1] = \Phi(W_t)$ and $P[r_t = -1] = \Phi(-W_t)$. It will also be convenient to define $W_{\sim t}$ to represent all elements of W except W_t . The Gibbs sampler proceeds in a series of steps

1. Specify priors on parameters $\Theta = \{ \alpha, \beta, \sigma \}$. Prescribe initial values for W and Z .
2. For each t , draw from

$$f(W_t | r_t, Z_t, W_{\sim t}).$$

3. Draw from

$$f(Z_t | W_t, r_t).$$

4. Draw Θ given Z and W .
5. Go back to step 2 and sweep through the sampler.

Step 4 is simple assuming a normal-gamma conjugate prior. The more difficult computation is drawing in steps 2 and 3. Step 2 is actually straightforward also since the conditional distribution is Gaussian. Step 3 is a drawing from a truncated normal distribution which is still quite tractable to implement.

2.2 Longer Window Methods

Assuming n available single period returns, the methodology outlined in the previous section utilizes the maximum number of these observations. However, this methodology is subject to potentially significant observation error as inference about the correlation between two return series at any point in time is based solely on a pair of corresponding single period returns. This observation error can be reduced by combining several single period returns and relying on the correlation calculated using this collection or window of returns.

Define a window of length ts as a collection of ts contiguous single period returns. We now observe the sample correlation between two return series calculated using non-overlapping windows of corresponding single period returns of the two securities. As a result, given n single period returns, we have m observations where $m \times ts = n$. We assume that the true correlation, ρ , remains constant for each pair of returns in a particular window.

When $ts > 1$, the sampling distribution of the sample correlation coefficient is no longer discrete. Anderson [1984] provides a detailed analysis of this sampling distribution under the assumption of conditional bivariate normality. Simple analytic expressions are available when $ts = 2$ or $ts = 3$, but for larger values of ts either iterative formulae are

needed or truncations of hypergeometric expansions must be relied upon.³

In general, the density of the sample correlation is highly nonnormal and converges slowly to normality as ts increases. Fisher [1921], however, noted that the following transformation of the sample correlation converges to a standard normal distribution extremely quickly:

$$T(r) = 0.5 \ln \frac{1+r}{1-r} \equiv \tanh^{-1}(r).$$

Let

$$T(\rho) = 0.5 \ln \frac{1+\rho}{1-\rho} \equiv \tanh^{-1}(\rho)$$

denote the corresponding transformation of the population correlation coefficient. This transformation is monotonically increasing and has the whole real line as its range.⁴

³From Johnson, Kotz, and Balakrishnan [1995], Chapter 32, for $ts = 2$, the density of r is given by:

$$f_r^{ts}(r) = \pi^{-1}(1-r^2)^{-0.5}(1-\rho^2)(1-r^2\rho^2)^{-1}\{1+r\rho Q(r\rho)\}$$

while for $ts = 3$

$$f_r^{ts}(r) = \pi^{-1}(1-\rho^2)^{-1.5}(1-r^2\rho^2)^{-2}\{3r\rho + (1+2r^2\rho^2)Q(r\rho)\}$$

where

$$Q(r\rho) = (1-r^2\rho^2)^{-0.5} \arccos(-r\rho).$$

For larger values of ts , Johnson, Kotz, and Balakrishnan provide an iterative formula to expand the density for increasing ts .

⁴Recall that in the single returns case, $ts = 1$, we use the transformation

$$W_1(\rho) = \Phi^{-1}(0.5 + \arcsin(\rho)/\pi)$$

while for $ts > 1$ we use

$$T(\rho) \equiv W_{ts}(\rho) = \tanh^{-1}(\rho).$$

These transformations are similar to each other (verified in unreported calculations) and both share the characteristics that $W(0) = 0$, $W(1) = +\infty$, $W(-1) = -\infty$, and $dW/d\rho > 0$. The choice of a particular transformation is based on technical convenience. The transformation $W_1(\cdot)$ permits an analytic solution to the integrated filter approach while we use $W_{ts}(\cdot)$ for $ts > 1$ because this transformation applied to r converges very quickly to normality for increasing ts .

Our econometric framework may now be expressed conveniently in state-space form.

The observation equation is

$$T(r_t) = T(\rho_t) + w_t \quad t = 1, \dots, m$$

where the distribution of $\{w_t\}$ will be approximately standard normal for large ts for each t . For small values of ts , we require the exact distribution of $\{w_t\}$ which can be defined implicitly given the conditional distribution of $T(r_t) | T(\rho_t)$.⁵ The transition equation is

$$T(\rho_t) = \alpha + \beta T(\rho_{t-1}) + \sigma \epsilon_t, \quad t = 1, \dots, m$$

where the errors $\{\epsilon_t\}$ are independent standard normals.⁶

We assume that the dynamics of $T(\rho_t)$ are given by a first order autoregressive specification. The model may be easily extended to incorporate exogenous explanatory variables $\{Z_t\}$ that are hypothesized to influence correlation:

$$T(\rho_t) = \alpha + \beta T(\rho_{t-1}) + \theta Z_t + \sigma \epsilon_t, \quad t = 1, \dots, m.$$

For example, a natural choice for Z_t is a measure of return volatility. In this way we can statistically assess the effects of volatility on the behavior of stochastic correlation.

As before, we follow Ball and Torous [1999] and use a single integration-based filter to estimate the parameters of the model. Given a set of m observations $T(r_1), \dots, T(r_m)$, the likelihood function can be expressed as

$$\ln \text{Like} = \sum \ln f(T(r_t) | \mathcal{I}_{t-1})$$

⁵Observe that for any x ,

$$P[T(r_t) \leq x] = P[r_t \leq \tanh(x)].$$

The density of $T(r_t)$ is given by

$$f_{T(r)}^{ts}(x) = f_r^{ts}(\tanh(x)) \cdot d\{\tanh(x)\}/dx = f_r^{ts}(\tanh(x))(1 - \tanh(x)^2).$$

⁶In the empirical results presented later, we estimate the model with the reparameterization $\mu = \frac{\alpha}{1-\beta}$.

where $\mathcal{T}_t \equiv \{T(r_t), T(r_{t-1}), \dots, T(r_1)\}$ denotes the history of the observable through time t and $f(T(r_t) \mid \mathcal{T}_{t-1})$ denotes the conditional density of $T(r_t)$ given the history of the observable through time $t - 1$.

Approximating the prior density by a normal density with the same first and second moments as the prior, the filter is then implemented in a similar fashion to the Gaussian case. The projection, based on a normal prior, preserves normality and so can be implemented analytically. The evaluation of the conditional likelihood requires numerical integration as the measurement error is non-Gaussian, but the approximation can be made highly accurate in our case as the integration is single dimensional. Computation of the first and second moments of the updated prior each requires an additional single dimensional numerical integration and so the iterative scheme may be continued.

3 Data and Sample Statistics

We consider daily data on six major stock market indices: Canada, Germany, Hong Kong, Japan, the UK, and the US. The data are obtained from *Datastream's* FT/S&P World Stock Market files. The series are denominated in US dollars, begin on January 1st 1987 and end on May 1st 1999. To minimize the possibility of inducing spurious correlation, we eliminate common holidays across these series. We do not, however, remove an observation if it corresponds to a holiday in one series but not in another series. As a result, we have 3189 local end-of-day observations on the level of each index.

Daily returns are then computed and their summary statistics are presented in Table I. As expected, over our sample period the US market exhibited the highest average daily return while the Japanese market exhibited the lowest. Daily returns of the Hong Kong market were the most variable while the least variable market was the Canadian.

Daily returns to the US and Hong Kong markets exhibited the greatest deviations from normality as measured by their respective skewness and kurtosis.

4 Empirical Results

Since the single period returns methodology uses only information on the sign of the respective returns on each day, to obtain statistically reliable parameter estimates requires that we use as much data as possible. To minimize non-synchronicity problems, we compare daily returns to stock markets in the same time zone (Canada *vs* the US) or approximately the same (within one hour) time zone (Germany *vs* the UK, and Hong Kong *vs* Japan).

Table II provides the results of applying the single returns methodology to the daily returns of Canada *vs* the US, Germany *vs* the UK, and Hong Kong *vs* Japan. While the estimated correlation processes for the daily returns of Canada *vs* the US and Germany *vs* the UK are very close to a random walk ($\hat{\beta} \approx 0.99$), the results for the daily returns of Hong Kong *vs* Japan are consistent with a more stationary behavior in the estimated correlation process. For the Asian pair of markets we see much lower levels of average (transformed) correlation, μ , and higher levels of estimated σ than for the North American and European pairs of markets. In all cases, however, the statistical evidence points to the estimated correlation processes being stochastic.

To investigate the correlations between the world's largest equity markets - Japan, the UK and the US - requires that we compare two-day returns to accommodate the non-synchronous nature of the observations between these markets. This reduces the number of observations from 3189 one day returns to 1594 two day returns and, as a result, in this case we rely on the longer window methodology. In particular, we set $ts = 3$ so that the

correlations are calculated using approximately one trading week of corresponding returns data. The results are presented in Panel A of Table III. We see high levels of reversion in these estimated correlation processes as indicated by the estimated β parameters. In addition, in each case the mean level of correlation is significantly positive and evidence of stochastic volatility is clearly evident.

For comparison purposes, Panel B of Table III gives the results of applying the longer window methodology to the daily returns of Canada *vs* the US, Germany *vs* the UK, and Hong Kong *vs* Japan using $ts = 5$ so that one trading week of returns data is used in calculating correlations. Recall that the estimation results using the single period returns methodology may be unduly influenced by the measurement error inherent in the estimation of correlations on the basis of one day's data. From Panel B we see that consistent with the results for the Japanese, UK, and US markets, the estimated correlation processes appear to be mean reverting as well as stochastic.

Figure I plots the estimated mean level of correlation for the Germany-UK daily return series. The stochastic reverting nature of the correlation is clearly evident. The Figure also plots the corresponding exponentially smoothed correlation coefficient assuming a smoothing parameter of $\lambda = 0.97$.⁷ We can see a close correspondence between this time-varying estimate of correlation and the mean of the estimated stochastic correlation process, though, as expected, the exponentially smoothed estimate does not appear to be as variable.

Solnik, Boucrelle and Le Fur [1996] provide evidence of a positive relation between sample correlations and sample variances for monthly returns of a number of international stock indices. We now investigate whether this result continues to hold using daily returns

⁷Formally, if $\{X_t\}$ is the original series then the smoothed series, $\{SM_t\}$, is given by $SM_t = \lambda SM_{t-1} + (1 - \lambda)X_t$.

in the context of our stochastic correlation model. To do so, we add an estimate of index volatility as an explanatory variable. In particular, we use the sum of the estimated volatilities for the pair of returns used in the correlation calculations after exponentially smoothing these estimated variances with a smoothing parameter of 0.97.⁸ Table III also gives these results for the daily returns of Canada *vs* the US, Germany *vs* the UK, and Hong Kong *vs* Japan using $ts = 5$. While there is some evidence of correlation increasing in response to shocks in volatility, especially in the case of Canada *vs* the US, the effects are not particularly strong. To better understand this result, in Figure II we plot the estimated mean stochastic correlation for the German and the UK daily returns series against the exponentially smoothed measure of their market volatility. To make the comparison clear, we scale the estimated smoothed volatility to have the same mean and standard deviation as the estimated mean correlation. While it does appear at times that correlation and the volatilities move together, this is by no means a consistent phenomena. Consequently, the estimated relation between correlation and volatility is not a particularly strong one.

5 Risk Management

Value-at-Risk (VaR) is a common tool used in assessing and managing the risk of a portfolio of securities. Many of the methods used to calculate VaR are based on the assumption that security returns are conditionally normally distributed. Differences between these methods often reflect differences in the modeling of the security returns' unconditional means and, in particular, differences in the modeling of their covariance structure.

Consider a simple portfolio of two securities where the weight w_1 is placed in security

⁸Alternative smoothing parameters and alternative estimates of volatility gave similar results.

1 and the weight w_2 is placed in security 2. Assume that the security variances are σ_1^2 and σ_2^2 , respectively, and that for short return horizons the mean returns are themselves negligible. As before, we let ρ denote the correlation between the security returns.

If these parameters are known and the security returns are assumed to be bivariate normally distributed then it is straightforward to assess the percentiles of this portfolio's return. In particular, the portfolio variance is given by:

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2$$

and the VaR at the α significance level for an initial investment of I_0 is then

$$I_0\sigma_P z_\alpha$$

where z_α is that point of the standard normal distribution Z for which $P[Z > z_\alpha] = \alpha$. For a prespecified α and a known initial investment I_0 , the VaR is then the square root of a linear function of ρ . It follows then that the specification of the correlation ρ plays a key role in assessing VaR.

To better see this, consider a special case in which we invest equal amounts in two securities having equal variance σ^2 . In this case the portfolio variance is given by

$$\sigma_P^2 = \sigma^2\left(\frac{1}{2} + \frac{1}{2}\rho\right).$$

Taking the base case as $\rho = 0$, we see that the resultant VaR is

$$I_0 z_\alpha \sigma \sqrt{\frac{1}{2}},$$

while for $\rho \neq 0$, VaR is given by

$$I_0 z_\alpha \sigma_P \sqrt{\frac{1}{2} + \frac{1}{2}\rho}.$$

Hence the ratio of the VaR in the general case to the base case is given by $\sqrt{1 + \rho}$. For example, compared to $\rho = 0$, this ratio is 31.6% when $\rho = 0.1$, 50.0% when $\rho = 0.25$ and 70.7% when $\rho = 0.50$.

The behavior of this ratio for varying values of ρ allows us to assess the effects of stochastic correlation on portfolio risk. Using an estimated 95% confidence band for ρ , Figure III plots the corresponding band of upper and lower estimated VaR ratios for the Germany-UK return series through time. As can clearly be seen, sharp movements in estimated correlation are translated into sharp movements in estimated VaR.

6 Conclusions

The modeling and estimation of the stochastic covariance between security returns is a challenging problem. Much of the extant research has relied on multivariate GARCH models with severe restrictions on the parameters needed to reduce the dimensionality of the resultant parameter space.

In this paper we focus on stochastic correlation and apply our methodology to index returns corresponding to major stock markets in North America, Asia, and Europe. The inter-relation between these markets may change stochastically over time in response to shifts in government policy and other fundamental economic changes.

Rather than rely on GARCH models, we treat stochastic correlation as a latent unobservable and apply non-linear filtering methods to extract estimates of this state variable on the basis of observed returns. We provide clear empirical evidence that the correlation between the sampled index returns is indeed changing stochastically over time. While Ang and Bekaert [1998] provide evidence consistent with the covariance structure of international interest rates being subject to regime shifts, our evidence points to a diffusing

correlation structure for index returns rather than one subject to sudden large shocks. We also investigate the relation between stochastic correlation and volatility estimates. While we find this relation to be positive, in contrast to the results of Solnik et al. [1996] and others, in general, we document a statistically insignificant response in correlation to increased market volatility.

Certainly risk management techniques that ignore the stochastic component of correlation are quite likely to provide erroneous risk assessments.

References

1. Milton Abramowitz and Irene Stegun, 1965, Handbook of Mathematical Functions, Dover Publications, New York.
2. Theodore W. Anderson, 1984, An Introduction to Multivariate Statistical Analysis, (2nd edition), John Wiley and Sons, New York.
3. Andrew Ang and Geert Bekaert, 1998, "Regime Switches in Interest Rates", unpublished manuscript, Graduate School of Business, Stanford University.
4. Clifford Ball and Walter Torous, 1999, "The Stochastic Volatility of Short Term Interest Rates: Some International Evidence", Journal of Finance, 54:2339-2359.
5. Tim Bollerslev, 1986, "Generalized Autoregressive Conditional Heteroskedasticity", Journal of Econometrics, 31:307-327.
6. Tim Bollerslev, 1990, "Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Approach", Review of Economics and Statistics, 72:498-505.
7. Robert F. Engle, 1982, "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation", Econometrica, 50:987-1007.
8. Ludwig Fahrmeir, 1992, Posterior mode estimation by extended Kalman filtering for multivariate dynamic generalized linear models, Journal of the American Statistical Association 87:501-509.
9. Ronald Fisher, 1921, "On the "probable error" of a coefficient of correlation deduced from a small sample", Metron, 1:3-32.

10. Sylvia Frühwirth-Schnatter, 1994, Applied state space modelling of non-gaussian time series using integration-based Kalman filtering, Statistics and Computing 4: 259-269.
11. Andrew Harvey, Esther Ruiz and Neil Shephard, 1994, "Multivariate Stochastic Volatility Models", Review of Economic Studies, 61:247-264.
12. Norman L. Johnson, Samuel Kotz and N. Balakrishnan, 1995, Continuous Univariate Distribution, Volume 2, Second Edition, John Wiley and Sons, New York.
13. Andrew Karolyi and Rene Stulz, 1996, "Why do Markets Move Together? An Analysis of US-Japan Stock Return Comovements", Journal of Finance, 51:951-986.
14. Sangjoon Kim, Neil Shephard, and Siddhartha Chib, 1998, "Stochastic Volatility: Likelihood, Inference and Comparison with ARCH models", Review of Economic Studies, 65:361-393.
15. Ken Kroner and Victor Ng, 1998, "Modeling Asymmetric Comovements of Asset Returns", Review of Financial Studies 11:817-844.
16. Francois Longin and Bruno Solnik, 1995, "Is correlation in International Equity Returns Constant: 1960-1990?", Journal of International Money and Finance, 1995:3-26.
17. Daniel Nelson, 1991, "Conditional Heteroskedasticity in Asset Returns: A New Approach", Econometrica, 59:347-370.
18. L. Ramachand and R. Susmel, 1998, "Volatility and Cross-Correlation across Major Stock Markets", Journal of Empirical Finance, 5: 397-416.

19. Bruno Solnik, Cyril Boucrelle and Yann Le Fur, 1996, Financial Analysts Journal, 52:17-34.

Table I

Summary Statistics of Daily Returns of the Sampled International Stock Market Indices

This table provides summary statistics of the daily returns of the sampled international stock market indices. The data are obtained from *Datastream's* FT/S&P World Stock Market files. The sample period is January 1 1987 to May 1 1999 and represents 3189 local end-of-day observations on the level of each index.

Stock indices	Mean ($\times 10^{-4}$)	Standard Deviation	Skewness	Kurtosis
Canada	2.589	0.009	-1.423	28.043
Germany	3.101	0.013	-0.811	9.485
Hong Kong	4.284	0.019	-4.041	82.532
Japan	0.472	0.015	-0.107	11.184
UK	4.452	0.010	-1.154	14.055
US	5.353	0.011	-3.771	82.531

Table II

Maximum Likelihood Estimates of Stochastic Correlation Using Single Returns Methodology

This table provides maximum likelihood estimates of the dynamics of the transformed correlation W between the sampled stock indices. The dynamics of the latent variable W are assumed governed by

$$W_t = \alpha + \beta W_{t-1} + \sigma \epsilon_t.$$

We estimate the parameters β , σ , and the long-term mean of the process $\mu = \frac{\alpha}{1-\beta}$. The data are obtained from *Datastream's* FT/S&P World Stock Market files. The sample period is January 1 1987 to May 1 1999 and represents 3189 local end-of-day observations on the level of each index. Asymptotic standard errors are in parentheses and are obtained from the inverse Hessian matrix evaluated at the maximum likelihood estimates.

Stock Indices	Estimated μ	Estimated β	Estimated σ	LogLikelihood
Canada-US	0.4069 (0.059)	0.9966 (0.0028)	0.00981 (0.0047)	-1915.4626
Germany-UK	0.4340 (0.0369)	0.9879 (0.0125)	0.0175 (0.0126)	-1906.0386
Hong Kong-Japan	0.1746 (0.0268)	0.9125 (0.0534)	0.0637 (0.0345)	-2042.8821

Table III**Maximum Likelihood Estimates of Stochastic Correlation Using Longer Window Methodology**

This table provides maximum likelihood estimates of the dynamics of the transformed correlation T between the sampled stock indices. The dynamics of the latent variable T are assumed governed by

$$T(\rho_t) = \alpha + \beta T(\rho_{t-1}) + \theta Z_t + \gamma \epsilon_t.$$

We estimate the parameters β , σ , the long-term mean of the process $\mu = \frac{\alpha}{1-\beta}$, as well as the parameter θ which measures the response of correlation to changes in measured volatility, Z_t . The data are obtained from *Datastream's* FT/S&P World Stock Market files. The sample period is January 1 1987 to May 1 1999 and represents 3189 local end-of-day observations on the level of each index. Asymptotic standard errors are in parentheses and are obtained from the inverse Hessian matrix evaluated at the maximum likelihood estimates.

Panel A

Stock Indices	Estimated μ	Estimated β	Estimated σ	LogLikelihood
Japan-UK	0.296 (0.056)	0.978 (0.024)	0.025 (0.017)	-684.167
Japan-US	0.111 (0.043)	0.911 (0.161)	0.061 (0.079)	-696.498
Hong Kong-Japan	0.415 (0.078)	0.980 (0.019)	0.034 (0.017)	-704.950

Panel B

Stock indices	Estimated μ	Estimated β	Estimated σ	Estimated θ	LogLikelihood
Canada-US	0.651 (0.059)	0.946 (0.046)	0.074 (0.039)		-601.863
	0.659 (0.032)	0.752 (0.148)	0.154 (0.057)	0.033 (0.022)	-597.458
Germany-UK	0.551 (0.055)	0.954 (0.033)	0.058 (0.025)		-595.433
	0.548 (0.043)	0.926 (0.067)	0.071 (0.040)	0.005 (0.006)	-594.346
Japan-Hong Kong	0.252 (0.047)	0.963 (0.026)	0.040 (0.016)		-547.959
	0.253 (0.047)	0.962 (0.027)	0.040 (0.017)	0.001 (0.002)	-547.900

Fig 1: Germany—UK Estimated Correlation

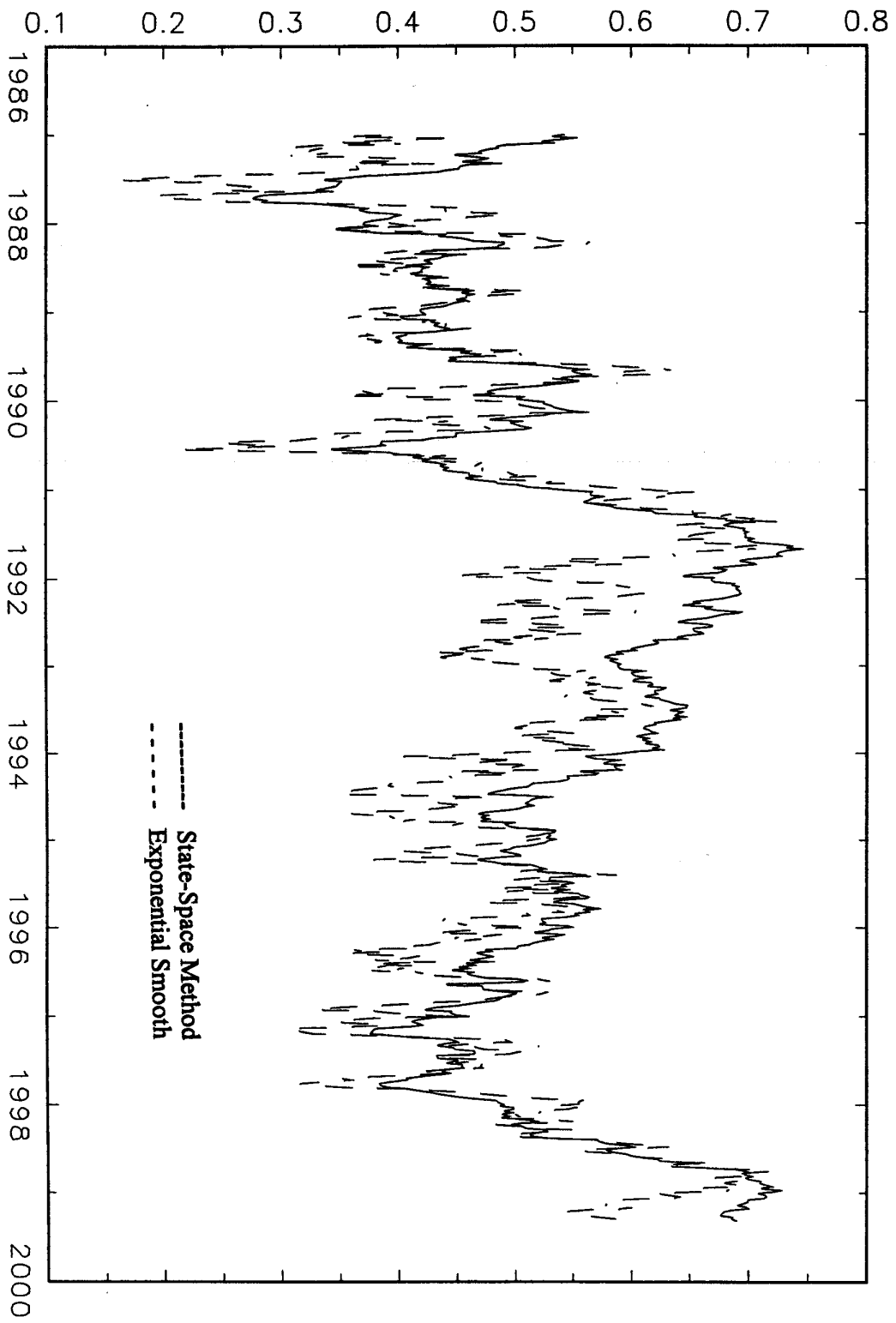
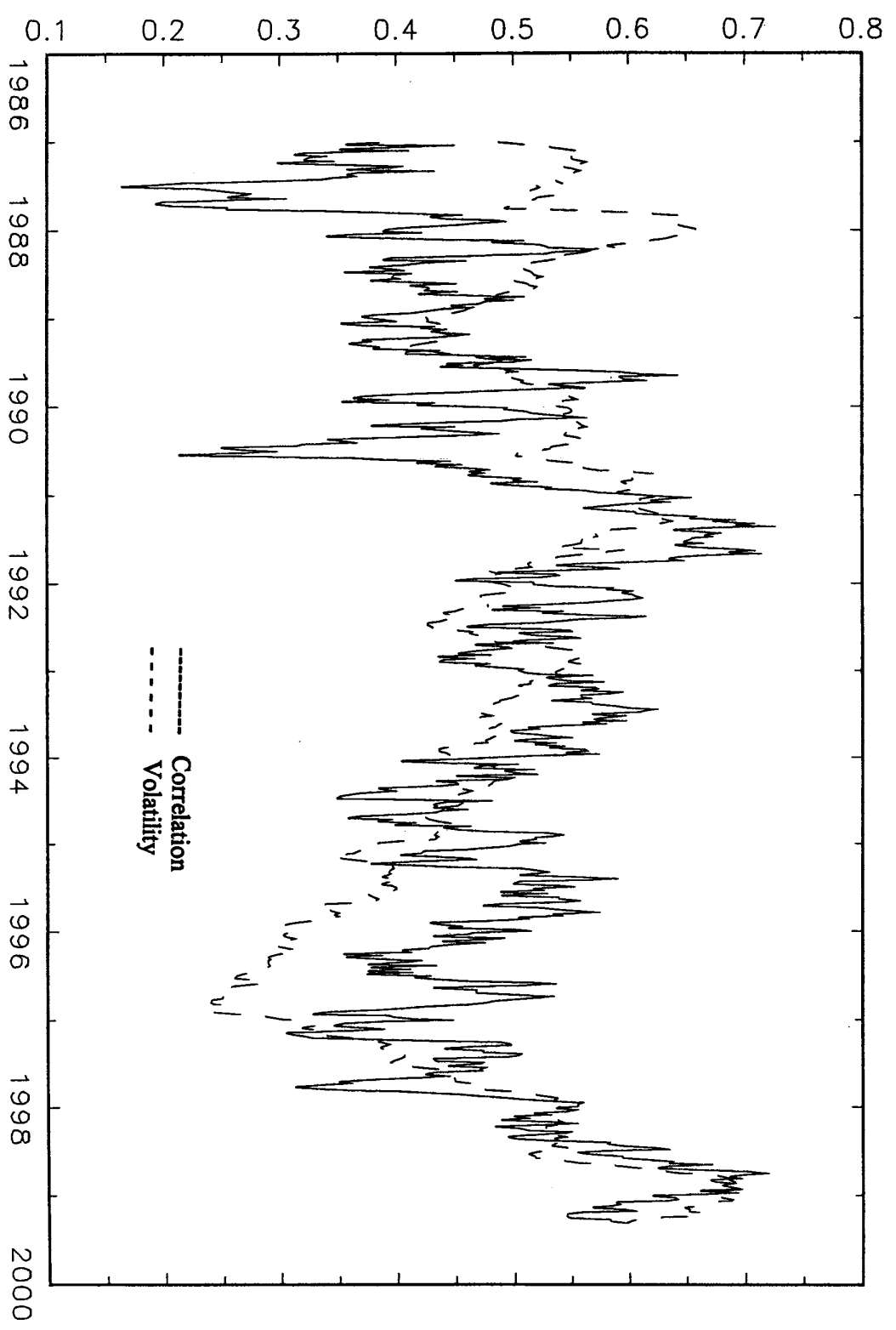


Fig II: Germany—UK Estimated Correlation — Smoothed Volatility



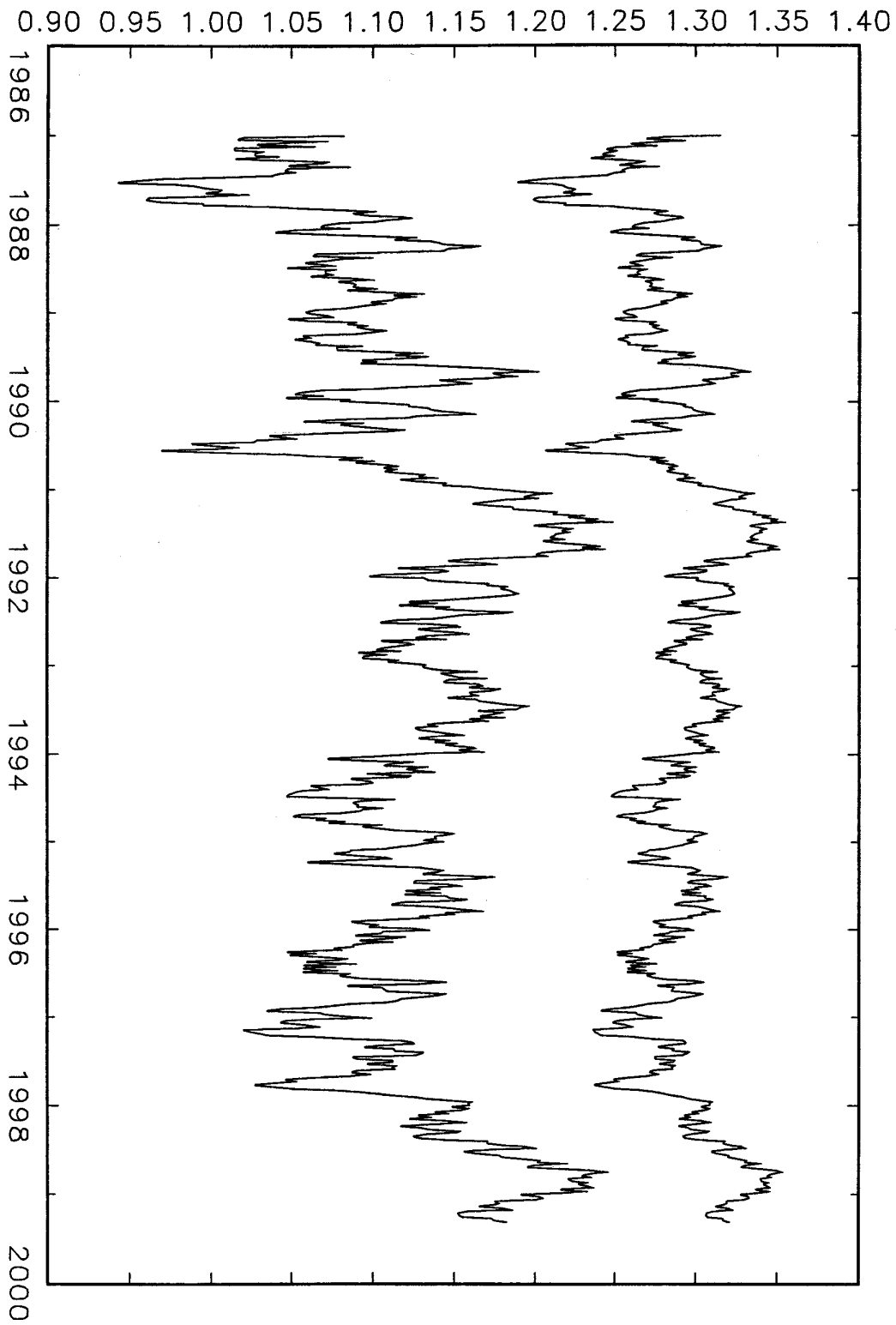


Fig III: Germany—UK 95% VaR Ratios