UCRL-11974

University of California

Ernest O. Lawrence Radiation Laboratory

FURTHER REMARKS ON THE SCALAR MESON "BOOTSTRAP"

Berkeley, California

DISCLAIMER

UCRL-11974

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

FURTHER REMARKS ON THE SCALAR MESON "BOOTSTRAP"

P. D. B. Collins

February 18, 1965

UCRL-11974

FURTHER REMARKS ON THE SCALAR MESON "BOOTSTRAP"*

P. D. B. Collins

Lawrence Radiation Laboratory
University of California
Berkeley, California

February 18, 1965

ABSTRACT

Some lacunae in our previous argument[1] that a single particle can not form a self-sustaining, or "bootstrap" system are filled. We propose a method of treating the "potential" which reconciles the Mandelstam iteration procedure with the Regge asymptotic behavior of the double spectral function, by making a subtraction of the S-wave discontinuity. This leads to a more general argument that unitarity and crossing symmetry put a very stringent limit on the magnitude of the coupling strength, and exclude the possibility of even producing the bound state corresponding to the particle.

I. INTRODUCTION

In a recent paper[1] we attempted to demonstrate that it was impossible for a scalar meson to "bootstrap" itself. We showed that there was no solution to the N/D equations for a crossing-symmetric S-matrix which had the required bound-state pole corresponding to the meson. The residue of the direct-channel pole produced in the solution, g, was much greater than the residues of the crossed-channel poles, g', which were needed to produce a bound state of the correct energy. But our result depended upon three assumptions which we now wish to examine more closely.

The first was the dominance of nearby singularities, of which we used only the crossed-channel poles and the S-wave part of the two-particle elastic unitary cut. Because of the very large coupling constant which was needed, there is some doubt as to the validity of this assumption, especially in view of results obtained in a similar non-relativistic potential problem, where comparison with the exact solution is possible. We refer to the careful analysis of the N/D method by Luming.[2] We have thus been led to try to obtain a better understanding of the limitations to our approximation to the "potential."

The second assumption was that one could neglect the fact that the input poles should be continuable in angular momentum, and could use a potential function corresponding to the exchange of an elementary particle. This would not be a good approximation if the trajectory on which the particle lay continued to high

values of the angular momentum, producing perhaps a second particle of spin 2. There was also the possibility that there might be a Pomeranchuk trajectory, with the meson lying on a secondary trajectory. This would correspond more closely to the real world, where cross sections tend to constants at high energies. Though we have not been able to use "Reggeized" potentials, we have examined the output trajectories, and find that neither of these possibilities seems to bring us nearer to a "bootstrap" solution.

We do, however, take note of the conflict between the Mandelstam iteration procedure for obtaining the elastic double spectral functions, and the requirements of Regge asymptotic behavior. We demonstrate a method for resolving the conflict in practical calculations, by explicitly subtracting the S-wave discontinuity.

Finally, in the last section we show that unitarity and crossing symmetry put a general constraint on the coupling constants, which is stringent enough to exclude the values that were necessary to produce the meson bound state. We thus have a new reason for rejecting the possibility of a "bootstrap" solution.

II. FIRST BORN APPROXIMATION

Because we wish to examine complete trajectories, we remove the threshold behavior and instead of the partial-wave amplitude $A_\ell(s)$ consider the function $B_\ell(s) = q_s^{-2\ell} A_\ell(s)$. This is necessary because the N/D method will not guarantee the correct threshold behavior for A_ℓ unless, "per impossible," we know the complete left-hand cut. In the usual way we set

$$B_\ell(s) = N_\ell(s) / D_\ell(s) , \tag{2,1}$$

where $N_\ell(s)$ has the left-hand, and $D_\ell(s)$ the right-hand, cuts of $B_\ell(s)$, and take $B_\ell(s)$ to have the same left-hand singularities as the potential function, $V_\ell(s)$, to be derived subsequently.

Thus in dispersion form we have

$$N_\ell(s) = V_\ell(s)\, D_\ell(s) - \frac{1}{\pi} \int_{s_o}^{\infty} \frac{\mathrm{Im}\left\{D_\ell(s')\, V_\ell(s')\right\}}{s' - s}\, ds' \tag{2,2}$$

and

$$D_\ell(s) = 1 + \frac{1}{\pi} \int_{s_o}^{\infty} \frac{\mathrm{Im}\left\{D_\ell(s')\right\}}{s' - s}\, ds' , \tag{2,3}$$

where $s_o = 4\,m^2$ is the elastic threshold.

The unitarity relation is

$$\mathrm{Im}\left\{D_\ell(s)\right\} = -\rho_\ell(s)\, N_\ell(s) , \tag{2,4}$$

where the phase-space factor is

$$\rho_\ell(s) = \left(\frac{s-4}{s}\right)^{\frac{1}{2}} \left(\frac{s-4}{4}\right)^{\ell}$$

with relativistic kinematics, or $\rho_\ell(s) = \left(\frac{s-4}{4}\right)^{\ell+\frac{1}{2}}$ with non-relativistic kinematics.

Combining (2,2), (2,3) and (2,4), we obtain

$$N_\ell(s) = V_\ell(s) + \frac{1}{\pi} \int_{s_o}^{\infty} ds' \, \frac{V_\ell(s') - V_\ell(s)}{s' - s} \, \rho_\ell(s') \, N_\ell(s') \,, \qquad (2,5)$$

$$D_\ell(s) = 1 - \frac{1}{\pi} \int_{s_o}^{\infty} ds' \, \frac{\rho_\ell(s') \, N_\ell(s')}{s' - s} \,. \qquad (2,6)$$

This form of the N/D equations has been preferred to that used in I. The equations have been programmed for the computer by D. C. and V. L. Teplitz,[3] and in the following calculations we have used a modified form of their program.

If we consider the force from the exchange of a spin-zero particle in both the t and u channels we have

$$V_\ell(s) = \frac{1}{2q_s^{2\ell}} \int_{-1}^{1} d(\cos\theta) P_\ell(\cos\theta) \left\{ \frac{g}{m^2 + 2q_s^2(1+\cos\theta)} + \frac{g}{m^2 + 2q_s^2(1-\cos\theta)} \right\}$$

$$= \frac{g}{q_s^{2\ell+2}} \, Q_\ell\left(1 + \frac{m^2}{2q_s^2}\right) . \qquad (2,7)$$

This is the first Born approximation to the left-hand cut, and, as in I, we find that to produce a bound state at $s = m^2$ we require $g/m^2 = 16.5$, but that then the residue of the output pole is 105, or very far from a "bootstrap" solution. In Fig. 1 we plot the position of the bound state versus the coupling constant, and in Fig. 2 we show the trajectory on which the particle lies. It will be noted that the trajectory does not rise to large values of ℓ, but has a cusp at the threshold. Because the trajectory has a branch point at threshold, the path of the second zero of the real part of the D-function (plotted as a dotted line in Fig. 2) can not be identified with the falling trajectory, but is probably reasonably close to it just above threshold. We believe that the contribution of this sort of trajectory to the potential is well represented by the $\ell = 0$ "elementary" particle form which we have used (2,7).

However, Luming[2] has shown that in the non-relativistic case the solution obtained with such large coupling constants is very far from the correct solution of the Schrödinger equation with a Yukawa potential $g\, e^{-mr}/r$. (Note that Luming uses g^2 where we use g.)

The main differences between the non-relativistic and relativistic cases are that the phase-space factor $\rho_\ell(s)$ is changed in the way explained above, and, since there is only one crossed channel, we replace g by $g/2$. The change of ρ_ℓ means that whereas relativistically one can integrate (2,5) to infinity

for $\ell < 1$ and still have a Fredholm equation, in the non-relativistic situation this is true only of $\ell < \frac{1}{2}$. Otherwise one must use a cut-off, but, as Luming shows, the results are very little dependent on the magnitude of the cut-off if it is large. We took an upper limit of 200 m^2 in all the calculations reported here, but have verified the insensitivity of the results to the value of this parameter in both the relativistic and non-relativistic cases.

In Fig. 3 we show the plot of bound-state energy versus coupling constant in the non-relativistic situation and compare it with the exact solution of the Schrödinger equation obtained by Hulthén and Laurikainer[4]. This agrees with Luming's Fig. 10. It will be observed that there is a considerable discrepancy between the two curves for $s = m^2$. However, Luming also shows that there is a great improvement if the second Born approximation to the left-hand cut is used, and we may expect this also to be true for the relativistic case.

III. THE SECOND BORN APPROXIMATION

Figure 4 shows the Mandelstam representation for the amplitude,

$$A(s,t) = \frac{g}{m^2 - t} + \frac{g}{m^2 - u} + \frac{1}{\pi}\int_{t_o}^{\infty} \frac{A_t(s,t')}{(t' - t)}\, dt' + \frac{1}{\pi}\int_{u_o}^{\infty} \frac{A_u(s,u')}{(u' - u)}\, du' , \tag{3,1}$$

or[5]

$$A(s,z_s) = A_R(s,z_s) + A_L(s,z_s) ,$$

where A_R has only the right-hand singularities, A_L the left-hand, and z_s is the scattering angle in the s channel. We define even and odd negative amplitudes by

$$A^{\pm}(s,z_s) = A_R(s,z_s) \pm A_L(s,-z_s) \tag{3,2}$$

and find, because of the symmetry in s, t, and u,

$$A^{+}(s,t) = \frac{2g}{m^2 - t} + \frac{2}{\pi^2}\iint_{}^{\infty} \rho_{st}(s',t')\left[\frac{1}{(t' - t)\,(s' - s)} + \frac{1}{(t' - t)\,(s' + t' - 4 + s)}\right] ds'\, dt' , \tag{3,3}$$

where ρ_{st} is the double spectral function.

Making the partial-wave projection of (3,3), we find

$$A_\ell^+(s) = \frac{g}{q_s^2} Q_\ell\left(1 + \frac{m^2}{2q_s^2}\right) + \frac{2}{\pi^2}\iint^\infty \rho_{st}(s',t')\left[\frac{1}{(s'-s)} + \frac{1}{(s'+t'-4+s)}\right] Q_\ell\left(1 + \frac{t'}{2q_s^2}\right) ds'\, dt' . \tag{3,4}$$

However, the second term in (3,4) has both left-hand and right-hand cuts in s, and to obtain the "potential" from this expression we must subtract the contribution of the right-hand cut ($s = s'$). Along this cut the imaginary part is

$$\frac{2}{\pi}\rho_{st}(s,t') \frac{Q_\ell\left(1 + \frac{t'}{2q_s^2}\right)}{2q_s^2} ,$$

so the contribution to A_ℓ is

$$\frac{2}{\pi^2}\iint^\infty \frac{\rho_{st}(s',\, t')}{(s'-s)} \frac{Q_\ell\left(1 + \frac{t'}{2q_{s'}^2}\right)}{2q_{s'}^2} ds'\, dt' .$$

Thus the final expression for $V_\ell(s)$ is, when we remember the threshold factor,

$$V_\ell(s) = \frac{g}{q_s^{2\ell+2}} Q_\ell\left(1 + \frac{m^2}{2q_s^2}\right) + \frac{2}{\pi^2}\iint^\infty \rho_{st}(s',\, t') \times \left\{\frac{1}{(s'-s)}\left[\frac{Q_\ell\left(1 + \frac{t'}{2q_s^2}\right)}{2q_s^{2\ell+2}} - \frac{Q_\ell\left(1 + \frac{t'}{2q_{s'}^2}\right)}{2q_{s'}^{2\ell+2}}\right] + \frac{1}{(s'+t'-4+s)} \frac{Q_\ell\left(1 + \frac{t'}{2q_s^2}\right)}{2q_s^{2\ell+2}}\right\} ds'\, dt' . \tag{3,5}$$

For the second Born approximation we require to know the elastic part of the double spectral function, which may be obtained by iterating the pole, or first Born approximation, with elastic unitarity. (See, for example, reference 6.)

The first Born approximation gives

$$A_t^{(1)}(s,t) = \pi g \, \delta(t - m^2) \quad . \tag{3,6}$$

Then $$\rho_{st}^{e\ell}(s,t) = \frac{1}{\pi q_s \sqrt{s}} \int_{s_o}^{K=o} \int_{t_o} \left[\frac{dt' \, dt'' \, A_t^{(1)}(s,t') \, A_t^{(1)}(s,t'')}{K^{\frac{1}{2}}(q_s^2, t, t', t'')} \right.$$

$$\left. + \frac{du' \, du'' \, A_u^{(1)}(s,u') \, A_u^{(1)}(s,u'')}{K^{\frac{1}{2}}(q_s^2, t, u', u'')} \right], \tag{3,7}$$

where $$K(q_s^2, t, t', t'') = \left[t^2 + t'^2 + t''^2 - 2(t\,t' + t'\,t'' + t''\,t) - \frac{t\,t'\,t''}{q_s^2} \right]. \tag{3,8}$$

Combining (3,6), (3,7), and (3,8) gives

$$\rho_{st}^{e\ell}(s,t) = \frac{2\pi g^2}{q_s \sqrt{s}} \frac{1}{\left(t^2 - 4t\,m^2 - \frac{t\,m^4}{q_s^2} \right)^{\frac{1}{2}}} \quad , \tag{3,9}$$

with a boundary at $s = \frac{4}{(t-4)} + 4.$ (3,10)

We use (3,9) substituted in (3,5) to give the "potential."

The corresponding non-relativistic expressions are

$$V_\ell(s) = \frac{g}{2q_s^{2\ell+2}} Q_\ell\left(1 + \frac{m^2}{2q_s^2}\right) + \frac{2}{\pi 2} \iint^{\infty} \rho_{st}(s', t') \frac{1}{s' - s}$$

$$\left[\frac{Q_\ell\left(1 + \frac{t'}{2q_s^2}\right)}{2q_s^{2\ell+2}} - \frac{Q_\ell\left(1 + \frac{t'}{2q_{s'}^2}\right)}{2q_{s'}^2} \right] , \qquad (3,11)$$

with[7]

$$\rho_{st}^{e\ell}(s,t) = \frac{\pi g^2}{2q_s} \frac{1}{\left(t^2 - 4t\, m^2 - \frac{tm^4}{q_s^2}\right)^{\frac{1}{2}}} \qquad (3,12)$$

reflecting the absence of the third double spectral function, and the altered unitarity condition.

Since $\rho_{st}^{e\ell}$ depends upon g^2 we can expect it to become more important as g increases. Figures 1 and 3 show the results of solving the N/D equations with these "potentials." Again in Fig. 3 we have reasonable argreement with Luming's results.

In the relativistic case a bound state is produced at m^2 with $g = 4.5\, m^2$ but $g' = 56\, m^2$, so we are no nearer to a bootstrap solution. The trajectories concerned are shown in Fig. 5. It still proves impossible to produce a secondary trajectory passing through m^2 at $\ell = 0$ however large g may be, so the chances of obtaining a bootstrap solution in this way are negligible.

However, one may object to the use of this form of the double spectral function from the point of view of continuation in angular momentum.[8] We know that the contribution of a Regge pole to the amplitude may be written

$$A(s,t) = -\frac{\pi}{2}\frac{[2\alpha(t)+1]}{\sin(\pi\alpha(t))}\,\gamma(t)\,(-q_t^2)^{\alpha(t)}\rho_{\alpha(t)}\left(1+\frac{s}{2q_t^2}\right), \qquad (3,13)$$

and our fixed $\ell = 0$ pole comes from putting

$$\alpha(t) \underset{t\to m^2}{\longrightarrow} \alpha'(m^2)\,(t-m^2)$$

and then using this for all t with

$$g = \frac{1}{2}\,\frac{\gamma(m^2)}{\alpha'(m^2)} .$$

But (3,13) shows that the use of (3,6) for $A_t^{(1)}(s,t)$ is not justified for large s, since we obtain

$$A_t^{(1)}(s,t) \underset{s\to\infty}{\propto} s^{\alpha(t)} .$$

Substituting in (3,7) would give us

$$\rho_{st}^{e\ell}(s,t) \underset{s\to\infty}{=} \iint dt'\,dt''\; s^{\alpha(t')+\alpha(t'')-1}\,\chi(\text{terms in } t,t',t'')$$

Figure 5 shows that there is a region of t from m^2 to about $10\ m^2$ for which $\mathrm{Re}\ \alpha(t) > 0$, and the integral in (3,11) is not well approximated by our use of $\alpha = 0$ in this region, and it will even diverge if there is a region where $\alpha(t) > \frac{1}{2}$. But we also know that the asymptotic behavior given by the first Mandelstam iteration is incorrect, and in fact we should have

$$\rho_{st}(s,t) \underset{s\to\infty}{\propto} s^{\alpha(t)} .$$

In other words, the elastic double spectral function does not represent the behavior of the total double spectral function for large s, and in fact (3,11) should converge providing that $\mathrm{Re}\ \alpha(t) < 0$.

In the following section we present a method of increasing the convergence of the integral (3,11) whereby only the near (small s) region of the double spectral function is important, and the asymptotic region, where the elastic double spectral function is not reliable, has little influence.

IV. A SUBTRACTION DETERMINED BY REGGE ASYMPTOTIC BEHAVIOR

In the previous work[1] we tried to represent the effect of the double spectral functions by including the force from the S-wave part of the elastic discontinuity in the crossed channels. The double spectral function, of course, gives the sum of all the partial waves, but it is still convenient to subtract the S-wave part, and then add it back in the same manner as in I. We know that the Regge asymptotic behavior of the double spectral function determines the number of subtractions needed to make integrals like (3,3) converge, and in our case, where $\alpha(t) < 1$, we need only make one subtraction of the S-wave discontinuity.

The total discontinuity in the t channel is

$$A_t(s,t) = \frac{1}{\pi}\int ds'\ \rho_{st}(s',t)\left[\frac{1}{(s'-s)}+\frac{1}{(s'+t-4+s)}\right] \qquad (4,1)$$

$$\text{and}\quad A_t^{\ell_t=0}(t) = \frac{2}{\pi}\int ds'\ \rho_{st}(s',t)\,\frac{Q_o\left(1+\frac{s'}{2q_t^2}\right)}{2q_t^2}\,ds' . \qquad (4,2)$$

Thus we may write

$$A_t(s,t) = A_t^{\ell_t=0}(t) + \frac{1}{\pi}\int \rho_{st}(s',t)\left[\frac{1}{(s'-s)}+\frac{1}{(s'+t-4+s)} - \frac{1}{q_t^2}\,Q_o\left(1+\frac{s'}{2q_t^2}\right)\right]ds' , \qquad (4,3)$$

and since

$$A_\ell(s) = \frac{g}{q_s^2} Q_\ell\left(1 + \frac{m^2}{2q_s^2}\right) + \frac{2}{\pi}\int A_t(s,t') Q_\ell\left(1 + \frac{t'}{2q_s^2}\right)\frac{dt'}{2q_s^2} , \quad (4,4)$$

we have

$$A_\ell(s) = \frac{g}{q_s^2} Q_\ell\left(1 + \frac{m^2}{2q_s^2}\right) + \frac{2}{\pi}\int A_t{}^{\ell_t = 0}(t') Q_\ell\left(1 + \frac{t'}{2q_s^2}\right)\frac{dt'}{2q_s^2}$$

$$+ \frac{2}{\pi^2}\iint \rho_{st}(s', t')\left[\frac{1}{(s' - s)} + \frac{1}{(s' + t' - 4 + s)} - \frac{1}{q_{t'}^2} Q_o\left(1 + \frac{s'}{2q_{t'}^2}\right)\right]$$

$$\times \frac{Q_\ell\left(1 + \frac{t'}{2q_s^2}\right)}{2q_s^2} \, ds' \, dt' . \quad (4,5)$$

Finally, removing the right-hand cut in analogy with Section III, we have

$$V_\ell(s) = \frac{g}{q_s^{2\ell+2}} Q_\ell\left(1 + \frac{m^2}{2q_s^2}\right) + \frac{2}{\pi}\int A_t{}^{\ell_t = 0}(t') \frac{Q_\ell\left(1 + \frac{t'}{2q_s^2}\right)}{2q_s^{2\ell+2}} dt'$$

$$+ \frac{2}{\pi^2}\iint \rho_{st}(s', t')\left\{\left[\frac{1}{(s' - s)} + \frac{1}{(s' + t' - 4 + s)} - \frac{1}{q_{t'}^2} Q_o\left(1 + \frac{s'}{2q_{t'}^2}\right)\right]\right.$$

$$\left. \times \frac{Q_\ell\left(1 + \frac{t'}{2q_s^2}\right)}{2q_s^{2\ell+2}} - \frac{1}{(s' - s)} \cdot \frac{Q_\ell\left(1 + \frac{t'}{2q_{s'}^2}\right)}{2q_{s'}^{2\ell+2}}\right\} ds' \, dt' . \quad (4,6)$$

We can immediately see that the convergence of the double integral has been improved, since

$$\frac{1}{q_{t'}^2} Q_o\left(1 + \frac{s'}{2q_{t'}^2}\right) \underset{s' \to \infty}{\to} \frac{2}{s'} ,$$

which cancels with the first term in the expansion of

$$\left[\left(\frac{1}{s' - s}\right) + \left(\frac{1}{s' + t' - 4 + s}\right)\right]$$ in powers of $\frac{1}{s'}$, leading to

$$\frac{2}{\pi^2}\iint \rho_{st}(s' , t') \chi\left(\text{terms of order } \frac{1}{s'^2}\right) ds' dt' ,$$

which will converge if $\alpha(t) < 1$. Figure 5 shows that in fact α is always less than 1, and so we have removed the difficulties described in the previous section, though at the expense of some computational complexity. The second term in (4,6) is to be evaluated by the same sort of cycling procedure we described in I, whereby we impose equality upon the discontinuities in the s and t channels.

As in the previous section we shall make the approximation of replaceing ρ_{st} by ρ_{st}^{el}. Since only the low s part is now important, this should be a good approximation.

V. THE CONFLICT WITH UNITARITY

On evaluating the double integral in (4,6) we find that its contribution to V_ℓ is negative, indicating that the S-wave (t-channel) part which we have removed is greater than the contribution of the double spectral function, if we subtract from the double spectral function's contribution that part which gives rise to the right-hand cut (in the s channel). Of course the S-wave part is smaller than the total contribution of the double spectral function, since the individual partial waves are positive, and it is only because of the removal of the right-hand cut part that the result is negative. The partial-wave series does not converge along the right-hand cut.

Since $\rho_{st}^{e\ell}$ depends on g^2, this negative contribution rapidly increases with the coupling. If we could achieve self-consistency, the second term in (4,6) would outweigh the part subtracted, so that total potential would be positive. But this term is limited by unitarity in the t channel, whereas the double spectral function is calculated with the use of unitarity in the s channel only.

When we solve the equations we find that, except for small values of g, the negative contribution of the double spectral function dominates, producing a repulsive potential. Thus except for very small g the elastic double spectral function, obtained by iterating the t-channel poles with elastic unitarity in the s channel, conflictswith the requirement of unitarity in the t channel. A unitary crossing-symmetric S-matrix can not be obtained.

The values of g which do not produce this conflict are too small for a bound state to be formed at $s = m^2$, so it is not possible to make the S-matrix crossing symmetric even with regard to the positions of the poles, as we had supposed in I.

ACKNOWLEDGMENTS

I would like to thank Professor G. F. Chew for innumerable discussions and suggestions throughout the course of this work. Thanks are also due to D. L. and V. C. Teplitz for the use of their computor program. The hospitality of the Lawrence Radiation Laboratory is gratefully acknowledged. The work was performed while I was the holder of a D. S. I. R. research fellowship.

REFERENCES

* This work was done under the auspices of the U.S. Atomic Energy Commission.

1. P. D. B. Collins, Phys. Rev. 136, B 710 (1964), subsequently referred to as I.

2. M. Luming, Phys. Rev. 136, B 1120 (1964).

3. D. C. and V. L. Teplitz, Lawrence Radiation Laboratory Report UCRL-11696, 1964 (unpublished).

4. L. Hulthén and K. V. Laurikainer, Rev. Mod. Phys. 23, 1 (1951).

5. See, for example, G. F. Chew and C. E. Jones, Phys. Rev. 135, B 208 (1964).

6. S. C. Frautschi, Regge Poles and S-Matrix Theory (W.A. Benjamin, Inc., New York, 1963) p. 59.

7. Ibid., p. 45.

8. This problem is discussed by G. F. Chew in M. Jacob and G. F. Chew, Strong-Interaction Physics (W. A. Benjamin, Inc., New York, 1964), p. 140.

FIGURE CAPTIONS

Fig. 1. Relativistic bound-state energy squared, s, vs coupling strength, g, in units of m^2.

B_1, B_2 -- second Born approximation, primary and secondary.

C_1, C_2 -- first Born approximation, primary and secondary.

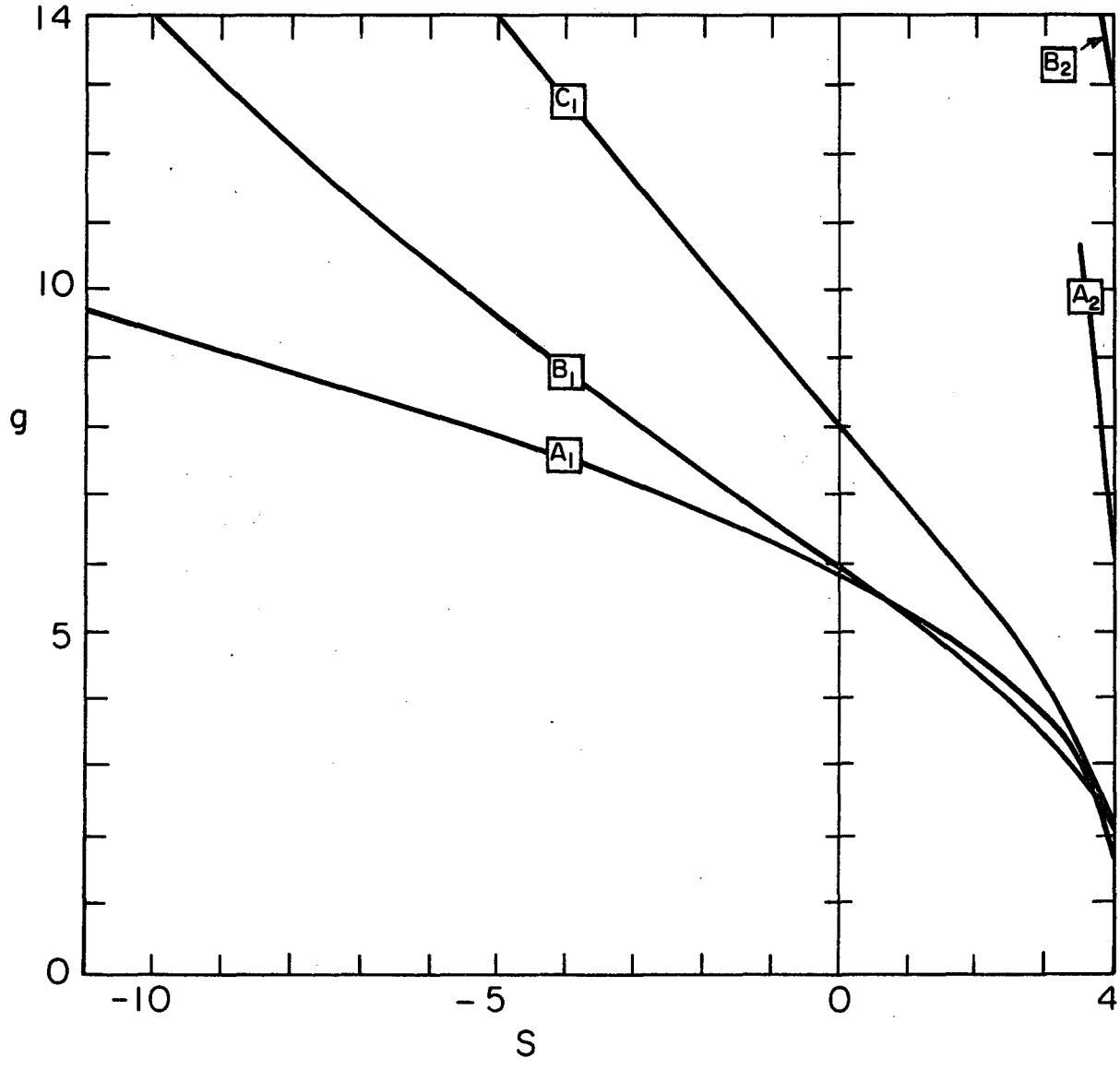
Fig. 2. Trajectory for $g = 16.5\ m^2$, relativistic first Born approximation.

Fig. 3. Nonrelativistic bound-state energy squared, s. vs coupling strength, g, in units of m^2.

A_1, A_2 -- solution of the Schrödinger equation, primary and secondary. The other labels correspond to those in Fig. 1.

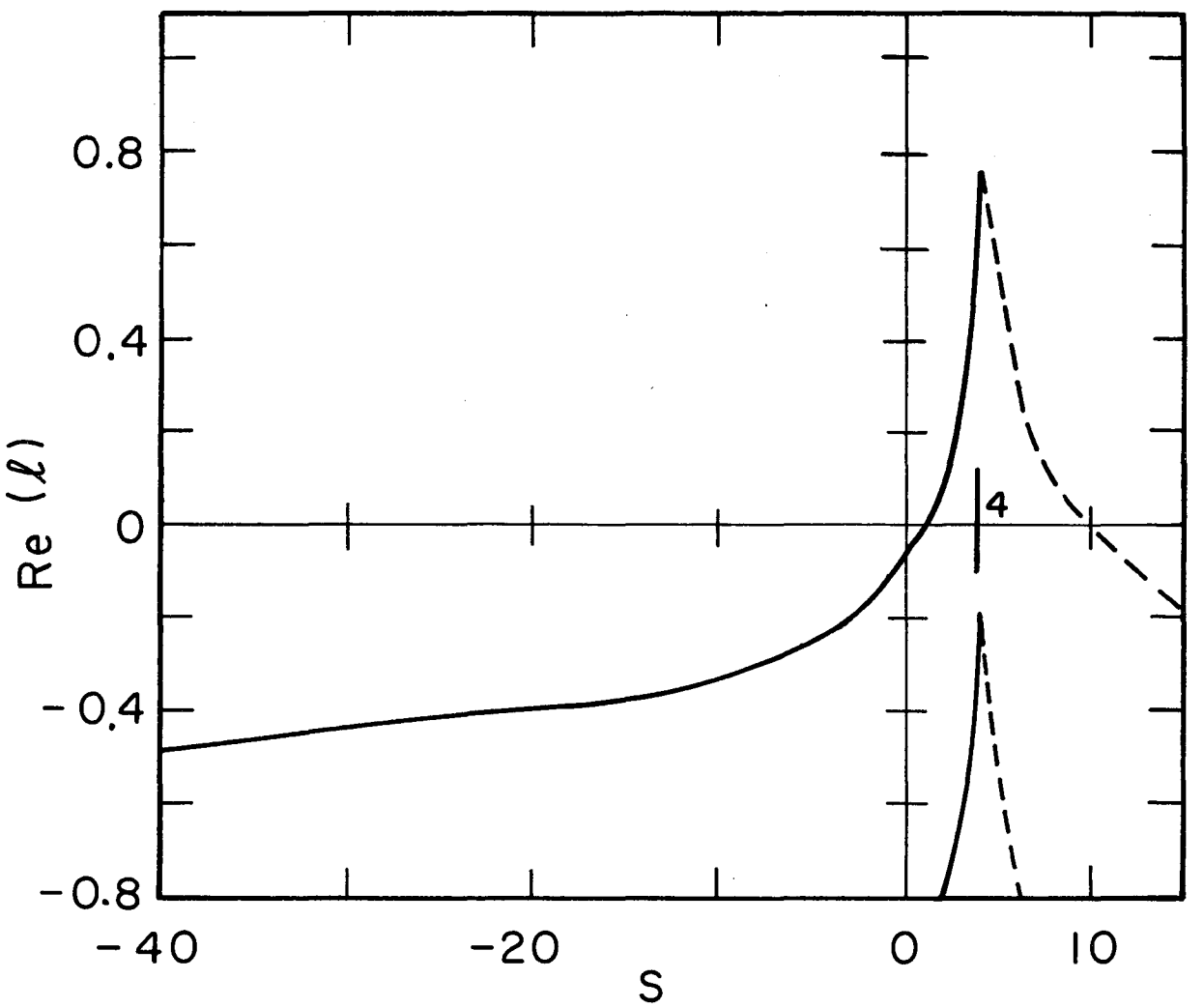
Fig. 4. The Mandelstam Representation.

Fig. 5. Trajectories for $g = 4.5\ m^2$, relativistic second Born approximation.



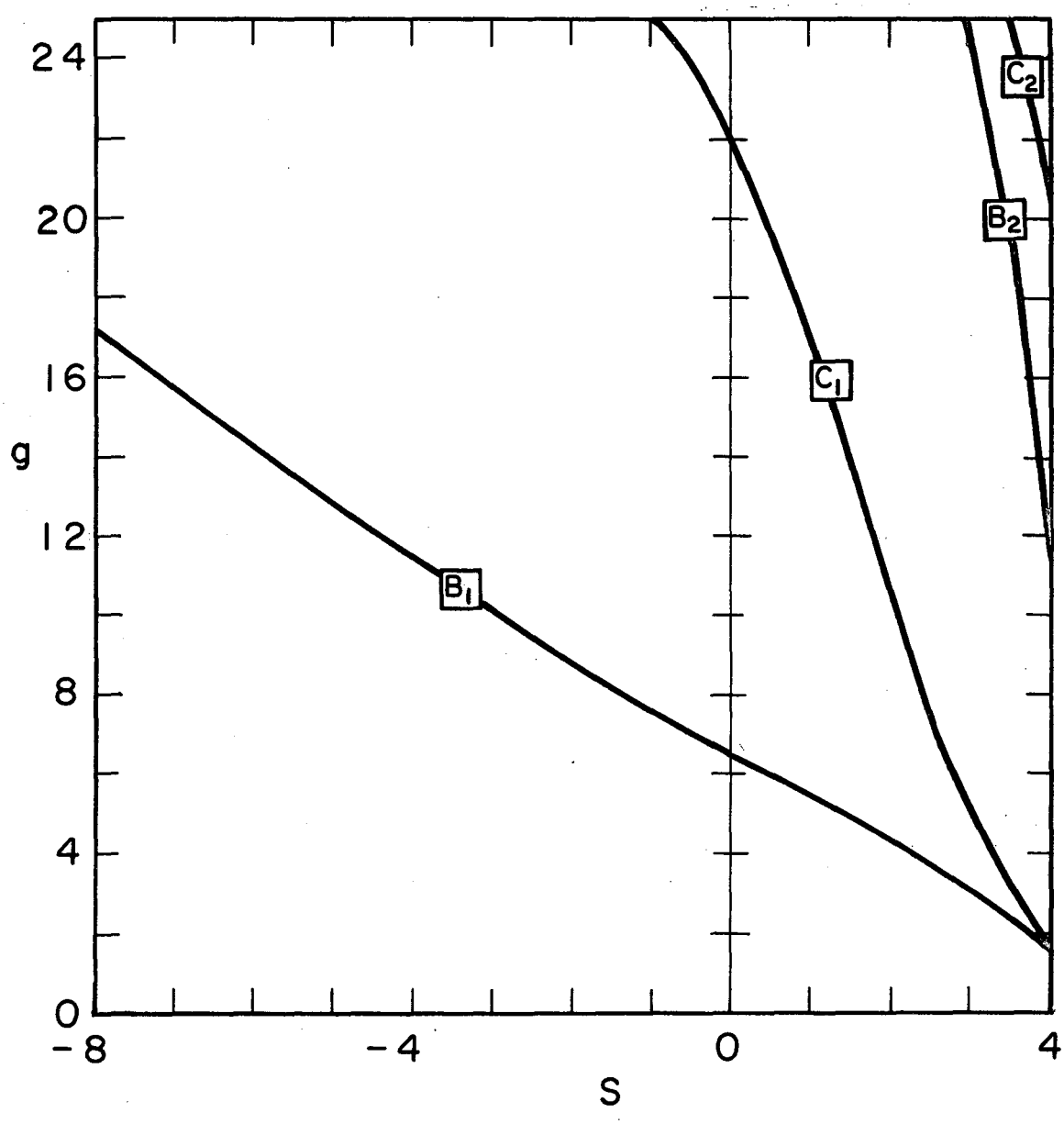
MUB-5482

Fig. 1



MUB-5483

Fig. 2



MUB-5484

Fig. 3

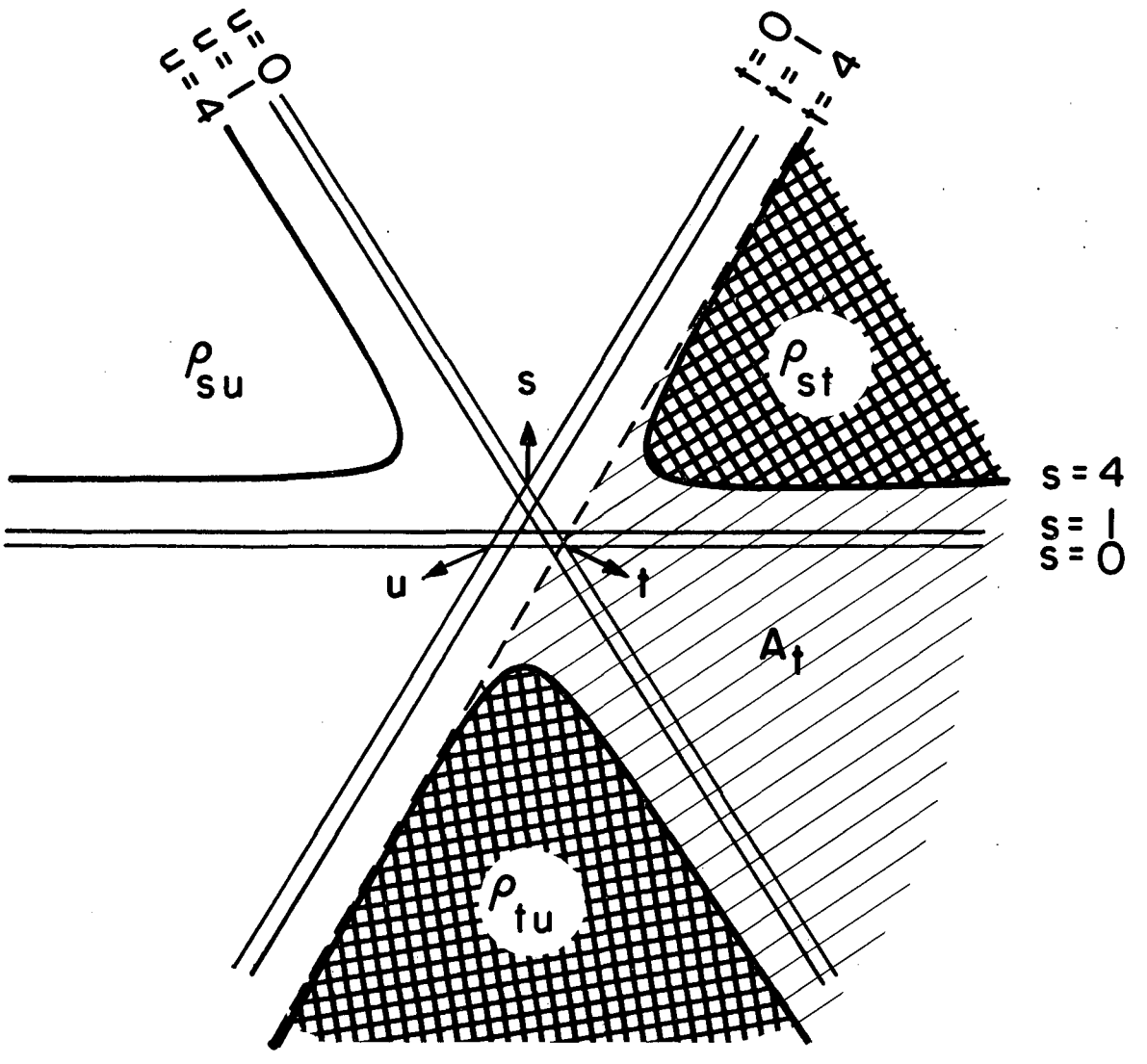
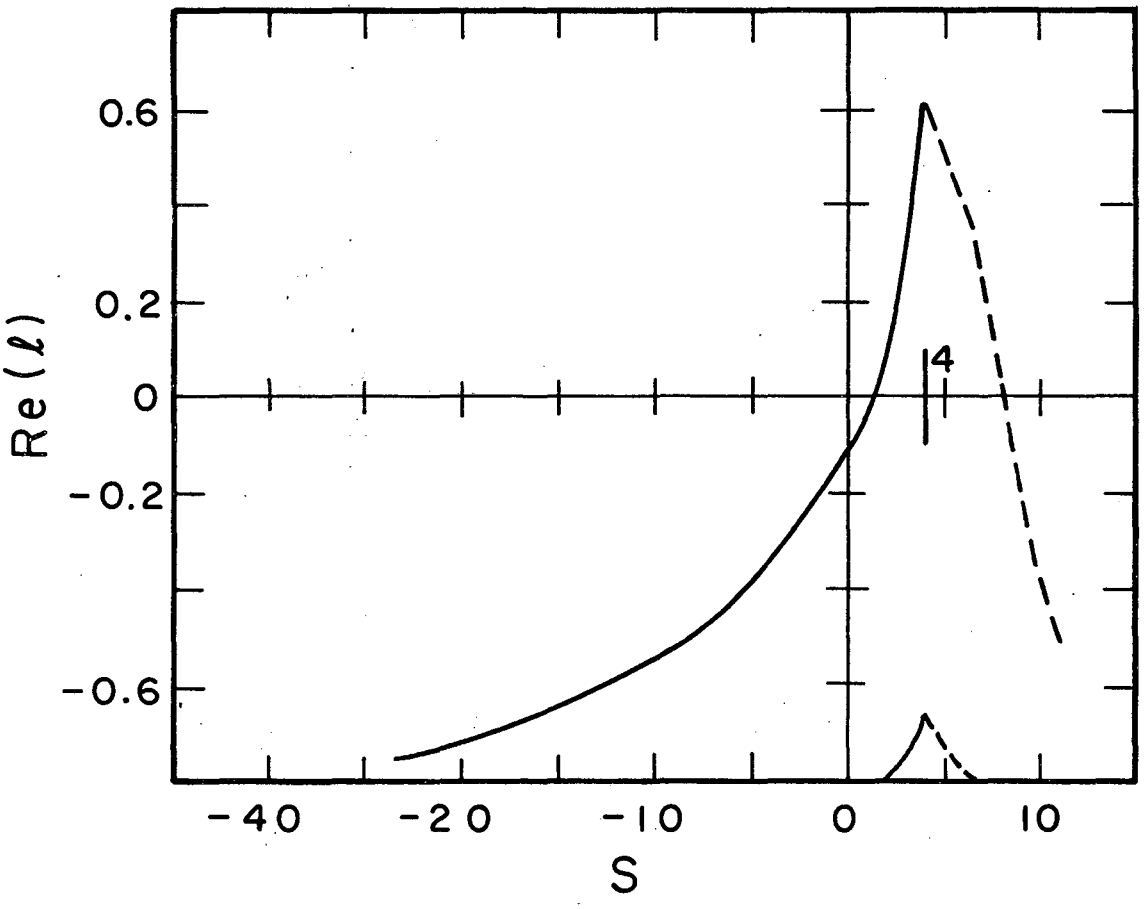


Fig. 4



MUB-5486

Fig. 5