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ON "GENERAL CONSERVATION EQUATIONS FOR MULTIPHASE SYSTEMS: 1. AVERAGING PROCEDURE" BY M. HASSANIZADEH AND W. G. GRAY

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In their paper on general conservation equations, Hassanizadeh and Gray[1] correctly point out (their criterion 1) that when an averaging operation involves integration, the integrand multiplied by the infinitesimal volume must be an additive quantity. Whether a quantity possesses the additive property or not should indeed stem from physical considerations. The additivity requirement has considerable significance in diffusion type problems. A fundamental consequence of the requirement is that intensive quantities such as pressure and temperature have first to be transformed to a quantity with additive property (physically an extensive quantity: volume, heat; mathematically a "measure") before they can be summed and averaged. This fact has been conspicuously overlooked by many previous workers[2,3,4,5,6] who defined, in general, without introducing the additivity requirement,

$$\langle\psi\rangle = \frac{1}{V}\int_V \psi dv \tag{1}$$

where ψ is any quantity. Obviously, ψdv will not be an additive quantity if ψ is fluid pressure in a porous medium or temperature. Hence (1) is, in general only a mathematical definition which may be physically reasonable in some cases (e.g., when ψ is porosity). For example, in order that fluid pressure can be averaged in a porous medium, it has to be multiplied by the storage parameter and multiplied by dv to obtain volume change. Analogously, temperature has to be multiplied by volumetric specific heat and dv to obtain heat content. Volume change and heat content are both additive quantities.

The importance of the additivity requirement was recognized by this writer[7], who formally considered the concept of additivity in analyzing the structure of the diffusion equation in the light of set theoretic concepts. Based on these, he proceeded to define the "set average" or the average of a quantity over a set of points. Depending on the nature of the set of points (one-, two- or three-dimensional) the "set average" can be a line average, an area average, or a volume average. If, instead of a set of spatial points we consider a set of mass points, the set average becomes mass average.

As pointed out by Narasimhan[7], the storage parameter or the capacity function (e.g., compressibility, specific heat) takes on a special significance in the light of the additivity requirement. The storage parameter is thus a correlation function which relates intensive and extensive quantities and is thus of fundamental importance in averaging.

The importance of additivity in regard to the physics of volume averaging has been recognized by other workers also (e.g., Marle[8]).

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