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B-L and Other U(1) Extensions of the Standard Model

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B-L and Other $U(1)$ Extensions of the Standard Model

A Dissertation submitted in partial satisfaction
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by

Nicholas Andrew Pollard

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ABSTRACT OF THE DISSERTATION

B-L and Other $U(1)$ Extensions of the Standard Model

by

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Doctor of Philosophy, Graduate Program in Physics
University of California, Riverside, December 2017
Dr. Ernest Ma, Chairperson

In Particle Physics, the Standard Model has had remarkable success at describing the experimentally observed interactions between the fundamental particles. In 2012, the LHC discovered a particle at 125 GeV which is consistent with the Higgs Boson. With this discovery the Standard Model appears to be complete. However with the experimental evidence for Dark Matter and Neutrino Mass, one is given pause to think there is physics beyond the Standard Model. By introducing extensions to the Standard Model, one is able to build upon previous triumphs while giving possible solutions to currently open and unexplained phenomena. Within this dissertation a specific class of extensions, the gauged $U(1)$, will be explored with the various consequences highlighted. Of particular interest is the Gauged $U(1)_{B-L}$, where Barionic and Leptonic number are charges. Through the careful introduction of new particles these classes of models seek to explain the outstanding Dark Matter and Neutrino masses phenomena along with their possible connections. A general look at the $U(1)$ extensions are considered as well, where the flexibility of this model class is demonstrated. This includes previously understood processes, flavor changing neutral

currents, as well as novel solutions to experimental data such as the once "observed" 750 GeV excess.

Contents

List of Figures

List of Tables

Chapter 1

Background

1.1 The Standard Model

The Standard Model (SM) of particle physics is remarkably successful at describing three of the four fundamental particle interactions of the known elementary particles under the gauge invariant symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y. \tag{1.1}$$

This symmetry describes the interactions of fermions and a single scalar, the Higgs Boson, via the force carrying gauge bosons (the photon, gluons, $W^{\pm}$, and Z bosons). The Higgs mechanism describes the spontaneous symmetry breaking of $SU(2)_L$ and explains the origins of mass for the fermions, the $W^{\pm}$ and Z bosons. With the discovery of the Higgs Boson at the LHC in 2012 [5, 6] the SM appears to be a complete, fully renormalizable quantum field theory of fundamental particle physics [7].

Despite this success there are several underlying problems for the minimal SM. For the purposes of this dissertation, neutrino mass and dark matter are of particular interest. There is a large body of experimental evidence for neutrino flavor oscillation [8], implying the existence of non-zero neutrino mass, and an overwhelming amount of astrophysical evidence for the existence of dark matter [9]. Due to both the right-handed neutrino and dark matter sterile nature, the minimal SM is insufficient to explain either phenomena.

1.2 Neutrino Mass

Experimentally neutrinos are known to oscillate between the three flavors suggesting mass differences between each,

Parameter	best-fit	3σ
$\Delta m^2_{21}\lvert 10^{-5} eV^2\rvert$	7.37	$6.93 - 7.97$
$\Delta m^2\lvert 10^{-3} eV^2\rvert$	2.50(2.46)	$2.37 - 2.63(2.33 - 2.60)$

Table 1.1: Current neutrino oscillation data. The first number refers to normal hierarchy ($m_1 < m_2 < m_3$) while the second is inverted ($m_3 < m_1 < m_2$). $\Delta m^2 = \Delta^2_{31} - \Delta^2_{21}/2$ [10]

yet within the minimal SM there is no mechanism by which neutinos obtain mass. There have been several solutions to this though, the most straight forward is to introduce some new symmetry by which three right-handed neutrinos may be introduced. These right-handed neutrinos will then be used to generate neutrino mass. Being electrically neutral, these right-handed neutrinos may be either Dirac (due to a new imposed charge) or Majorana (implying the right-handed neutrino is its own antiparticle) in nature.

A less obvious method would be to construct neutrino masses via radiative loops. In this manner new particles may be introduced with fascinating phemenological consequences. One such example was introduced in 2006 called the Scotogenic model, coming from the Greek word "scotos" meaning darkness, a Z_2 symmetry was introduced to the standard model in which the normal SM particles were even whereas three N's and a scalar doublet (η^+, η^0) are odd [11]. What is most interesting, the Scotogenic model suggests a link between dark matter and neutrino mass.

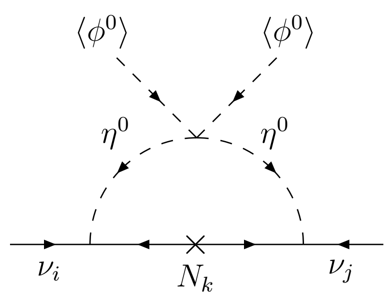

Figure 1.1: Radiative generation of neutrino mass [11]

Both of these ideas as well as their consequences will be explored in the coming chapters.

1.3 Dark Matter

The evidence for Dark Matter stems from a variety of sources, where each source illuminates several of Dark Matter's properties. For example, by looking at rotational velocity of a luminous object one would expect a Keplerian orbit, that for an object at a given radius r the rotational velocity would be given by:

$$v(r) \propto \sqrt{M(r)/r} \tag{1.2}$$

Where $M(r)$ is the amount of matter contained within the object's orbit. Once r has passed the region with the majority of the galaxies mass, it is expected that

$$v(r) \propto 1/\sqrt{r}. \tag{1.3}$$

Unexpectedly what is seen is that v becomes approximately constant for large values of r. This alludes to a massive particle which does not absorb or admit photons. While this is just one example, there are a host of experiments and phenomena such as but not including: the Cosmic Microwave Background (CMB), Gravitational Lensing, Direct Detection searches, etc.

Assuming that Dark Matter is a particle, a few important properties can be observed. First, Dark Matter must be electrically neutral. That is, Dark Matter must not be able to interact with the photon directly but does interact indirectly via it's mass. We can also infer the stability of Dark Matter, or if it is not stable that it must have a life time that is at least as long as the age of the universe. If the Dark Matter candidate in question interacts with SM particles, careful consideration must be given. If the candidate interacts directly then limits may be derived from the c_u and c_d coefficients [12, 13] or from it's cross-section [14].

1.4 Gauged $U(1)$ Extensions to the Standard Model

With both the Neutrino Mass and Dark Matter problems noted above one main avenue for progress is to introduce minor alterations to the SM such that either, or both, problem may be addressed. To do this an additional symmetry may be introduced. For example in 1975 the alteration [15, 16]:

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1) \tag{1.4}$$

known as the "Left-Right Model" was introduced. This model sought to rectify the imbalance between the left and right handed particles by both introducing a right-handed neutrino and by reforming the right-handed particles into doublets like their left-handed counterparts.

Within the context of the Left-Right Model, the electric charge is defined as:

$$Q = I_{3L} + I_{3R} + X \tag{1.5}$$

where X is the $U(1)$ charge. Using the known electric charges of the Quarks and Leptons we find the X charge to be $1/6$ for Quarks and $-1/2$ for Leptons. Thus the $U(1)$ of the Left-Right Model may be identified as $U(1)_{B-L}$. As will be commented on shortly, due to the gauged $SU(2)_R$ symmetry, three new bosons, $W_R^{\pm}$ and Z', must be included. The strongest lower mass limit on $W_R^{\pm}$ set at 4.0 TeV set by CMS [17].

Fortunately, the $U(1)$ extension is of interest for this dissertation. Thus, along with new particles introduced by the $U(1)$ extension, the properties of the new Z' boson are of concern. There are many limits set for this new boson such as, but not limited to, those set by the LHC and LUX. The benefit of this extension, underneath the new $U(1)$ charge new particles may be introduced. These particles may be right-hand neutrinos, Dark Matter candidates, or some other exotic particle, such as Lepto-Quarks.

1.4.1 Z' Mixing

After the introduction of new particles, the most obvious alteration made when introducing a new gauged symmetry is to the covariant derivative.

$$SU(2)_L \times U(1)_Y \to D_\mu = \partial_\mu - \imath g' q_W W^a_\mu \tau^a - \imath g q_B B_\mu \tag{1.6}$$

$$SU(2)_L \times U(1)_Y \times U(1)_x \to D_\mu = \partial_\mu - \imath g' q_W W^a_\mu \tau^a - \imath g q_B B_\mu - \imath g'' q_C C_\mu \tag{1.7}$$

where W^a_μ, B_μ, and C_μ are the 5 gauged bosons, τ^a are the $SU(2)_L$ generators, g, g', and g'' are the coupling constants and $q_{W,B,C}$ are the charges corresponding to the particle which D_μ is applied. For the bosons to generate mass, the Higgs Mechanism is applied, by which the modulus squared of covariant derivative applied to scalars with a vacuum expectation value (v.e.v) is observed

$$\sum_a |D_\mu \phi_a|^2. \tag{1.8}$$

While the results of eq. 1.7 are contingent on the specifics of the model being considered, it can be stated that W^1, and W^2 will mix to create the $W^{\pm}$ bosons while W^3, B, and C will mix to become the photon, Z, and Z' bosons.

1.4.2 The Triangle Anomaly

The benefit of making a gauged extension to the SM is that new fermionic particles may be introduced to explore less understood phenomena. Being a gauged theory though, special care must be taken with the charges such that the theory remains renormalizable. By looking at the current conservation, the axial vector anomaly leads to the relation:

$$\langle p, \nu, b; k, \lambda, c|\partial_\mu j^{\mu a}|0\rangle = \frac{g^2}{8\pi^2}\epsilon^{\alpha\nu\beta\lambda}p_\alpha k_\beta A^{abc} \tag{1.9}$$

where

$$A^{abc} = tr[t^a, t^b, t^c] \tag{1.10}$$

and $t^{a,b,c}$ are the generators of the symmetries used within the triangle diagram. Thus for the current $j^{\mu a}$ to remain conserved, A^{abc} must be zero. In fact $A^{abc} = 0$ is a fundamental consistency condition for chiral gauge theories [18]. Theories meeting this condition are said to be anomaly free. A detailed discussion of this derivation may be found in [19].

What is most important is how to use this information. The SM is a good first example to explore. As is well known, the particle content of the SM is listed in table 1.2,

Particle	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$(u,d)_L$	3	2	1/6
u_R	3^*	1	2/3
d_R	3^*	1	$-1/3$
$(\nu,e)_L$	1	2	$-1/2$
e_R	1	1	-1

Table 1.2: Particle content of the SM.

There are four possible anomalies, each imposes a constraint on the model. The notation used in this dissertation will be, all left-handed particles obtain a positive sign while all right-handed will obtain a negative. Note that all of the potential anomalies within the SM cancel as is seen in table 1.3. When a new $U(1)$ gauged symmetry is introduced, six new anomalies are introduced. Four of these symmetries are analogous to those demonstrated with the SM and two new restrictions, $U(1)_Y U(1)^2$ and $U(1)U(1)_Y^2$. With these six conditions, the charges for the new fermions and the alterations to the original SM must be considered. One final comment, within the SM as one changes between the families, the charge distribution is the same between analogous particles. That is, the Up, Charm and Top quarks all have the same charges. This is not necessary, as long as these conditions are met any charges may be chosen. For example within the $U(1)_{B-L}$ model, if three right-handed neutrinos are chosen then these neutrinos may have the charge distribution $(-1,-1,-1)$ [20] or $(5,-4,-4)$ [1].

Anomaly	Restriction
$SU(3)_C^2 U(1)_Y$	2*1/6 - 2/3 + 1/3 = 0
$SU(2)_L^2 U(1)_Y$	3*1/6 - 1/2 = 0
$U(1)_Y^3$	3*(2* $(1/6)^3$ -$(2/3)^3$ +$(1/3)^3$) +(2*$(-1/2)^3$ +$(1)^3$) = 0
$grav^2 U(1)_Y$	3(2*1/6 -2/3 +1/3) + (2*-1/2 +1) = 0

Table 1.3: Anomaly restrictions of the SM.

Part I

Gauged $U(1)_{B-L}$ Extensions of the Standard Model

Chapter 2

B-L with Radiative Neutrino mass and Multipartite Dark Matter

The following chapters are designed to represent the effectiveness of the gauged $B-L$ symmetry. In this chapter, a unique model is put forth. As opposed to containing just three right-handed neutrinos, seven electrically-neutral singlet fermions are introduced. The immediate difference being the former typically has tree-level neutrino mass terms while this discussion will look at radiative neutrino mass generation. Other consequences (Multipartite Dark Matter, Leptoquark Fermions) of this model will be explored. The discussion that follows is based on the work of [2].

Through a simple extension of the Standard Model of Quarks and Leptons an addition of one singlet right-handed neutrino per family is included to keep the theory anomaly-free. For sake of convenience in notation, let these three extra neutral fermion singlets N be left-handed. In this case their charges under $U(1)_{B-L}$ are (1,1,1). Their additional contributions to the axial-vector anomaly and the mixed gauge-gravitational anomaly are respectively

$$(1)^3 + (1)^3 + (1)^3 = 3, \quad (1) + (1) + (1) = 3, \tag{2.1}$$

which cancel exactly those of the SM quarks and leptons. On the other hand, it has been known for some time [21] that another set of charges are possible, i.e.

$$(-5)^3 + (4)^3 + (4)^3 = 3, \quad (-5) + (4) + (4) = 3. \tag{2.2}$$

Adding also three pairs of neutral singlet fermions with charges $(1, -1)$, naturally small seesaw Dirac masses for the known three neutrinos may be obtained [22], and a residual global $U(1)$ symmetry is maintained as lepton number. A further extension in the scalar sector allows for the unusual case of Z_3 lepton number [1] with the appearance of a scalar dark-matter candidate which is unstable but long-lived and decays to two antineutrinos. Here we consider another set of possible charges for the neutral fermion singlets, such that tree-level neutrino masses are forbidden. New scalar particles transforming under $U(1)_{B-L}$ are then added to generate one-loop Majorana neutrino masses. The breaking of $B - L$ to Z_2 results in lepton parity and thus R parity or dark parity [23] which is odd for some particles, the lightest neutral one being dark matter. A closer look at the neutral fermion singlets shows that one may be a keV sterile neutrino, and two others are heavy and stable, thus realizing the interesting scenario of multipartite dark matter.

2.1 Model

The extra left-handed neutral singlet fermions have charges $(2, 2, 2, 2, -1, -1, -3)$, so that

$$4(2)^3 + 2(-1)^3 + (-3)^3 = 3, \quad 4(2) + 2(-1) + (-3) = 3. \tag{2.3}$$

Since there is no charge $+1$ in the above, there is no connection between them and the doublet neutrinos ν with charge -1 through the one Higgs doublet Φ which has charge zero. Neutrinos are thus massless at tree level. To generate one-loop Majorana masses, the basic mechanism of Ref. [24] is adopted, using the four fermions with charge $+2$, but because of the $U(1)_{B-L}$ gauge symmetry, we need both a scalar doublet (η^+, η^0) and a scalar singlet χ^0.

Particle	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	B	L	$B-L$	copies	R parity
$Q = (u, d)$	3	2	1/6	1/3	0	1/3	3	+
u^c	3^*	1	−2/3	−1/3	0	−1/3	3	+
d^c	3^*	1	1/3	−1/3	0	−1/3	3	+
$L = (\nu, e)$	1	2	−1/2	0	1	−1	3	+
e^c	1	1	1	0	−1	1	3	+
N	1	1	0	0	−2	2	4	−
S	1	1	0	0	1	−1	2	+
S'	1	1	0	0	3	−3	1	+
$\Phi = (\phi^+, \phi^0)$	1	2	1/2	0	0	0	1	+
$\eta = (\eta^+, \eta^0)$	1	2	1/2	0	1	−1	1	−
χ^0	1	1	0	0	1	−1	1	−
ρ_2^0	1	1	0	0	2	−2	1	+
ρ_4^0	1	1	0	0	4	−4	1	+

Table 2.1: Particle content of proposed model.

The $U(1)_{B-L}$ gauge symmetry itself is broken by ρ_2^0 with charge -2 and by ρ_4^0 with charge -4. The complete particle content of this model is shown in Table 2.1.

2.2 Radiative Neutrino Mass

Using the four N's, radiative Majorana masses for the three ν's are generated as shown in Fig. 2.1. [A systematic study of this mechanism under $B-L$ (with only one fermion and three scalars) appeared [25] but does not include our case, which has four scalars.]

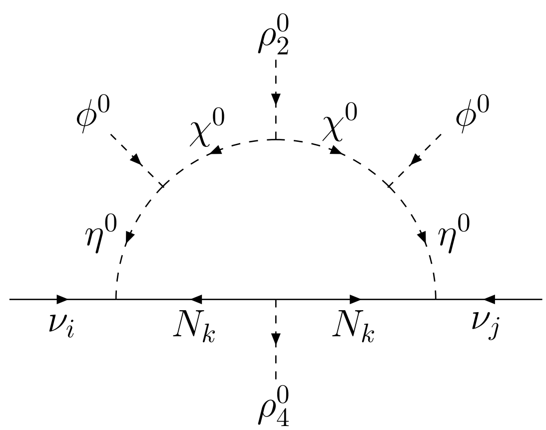

Figure 2.1: Radiative generation of neutrino mass through dark matter.

Note that N, η, χ all have odd R parity, so that the lightest neutral particle among them is a dark-matter candidate. This is the scotogenic mechanism, from the Greek 'scotos' meaning darkness. In addition to the $\eta^\dagger \Phi \chi$ trilinear coupling used in Fig. 2.1, there is also the $\eta^\dagger \Phi \chi^\dagger \rho_2$ quadrilinear coupling, which may also be used to complete the loop. There are 4 real scalar fields, spanning $\sqrt{2}Re(\eta^0)$, $\sqrt{2}Im(\eta^0)$, $\sqrt{2}Re(\chi^0)$, $\sqrt{2}Im(\chi^0)$. We denote their mass eigenstates as ζ_l^0 with mass m_l. Let the $\nu_i N_k \eta^0$ coupling be h_{ik}^ν, then the radiative neutrino mass matrix is given by [24]:

$$(\mathcal{M}_\nu)_{ij} = \sum_k \frac{h_{ik}^\nu h_{jk}^\nu M_k}{16\pi^2} \sum_l [(y_l^R)^2 F(x_{lk}) - (y_l^I)^2 F(x_{lk})], \tag{2.4}$$

where $\sqrt{2}Re(\eta^0) = \sum_l y_l^R \zeta_l^0$, $\sqrt{2}Im(\eta^0) = \sum_l y_l^I \zeta_l^0$, with $\sum_l (y_l^R)^2 = \sum_l (y_l^I)^2 = 1$, $x_{lk} = m_l^2/M_k^2$, and the function F is given by:

$$F(x) = \frac{x \ln x}{x-1}. \tag{2.5}$$

2.3 Multipartite Dark Matter

Since the only neutral particles of odd R parity are N, η^0, χ^0, there appears to be only one dark-matter candidate. However as shown below, there could be two or even four, all within the context of the existing model.

First note that $\rho_{2,4}^0$ have exactly the right $U(1)_{B-L}$ charges to make the (S, S, S') fermions massive. The corresponding 3×3 mass matrix is of the form

$$\mathcal{M}_S = \begin{pmatrix} m_{S1} & 0 & m_{13} \\ 0 & m_{S2} & m_{23} \\ m_{13} & m_{23} & 0 \end{pmatrix} \tag{2.6}$$

where m_{S1}, m_{S2} come from $\langle \rho_2^0 \rangle = u_2$ and m_{13}, m_{23} from $\langle \rho_4^0 \rangle = u_4$. If all these entries are of order 100 GeV to a few TeV, then there are three extra heavy singlet neutrinos in this model which also have even R parity. They do not mix with the light active neutrinos ν at tree level, but do so in one loop. For example, S' mixes with ν as shown in Fig. 2.2.

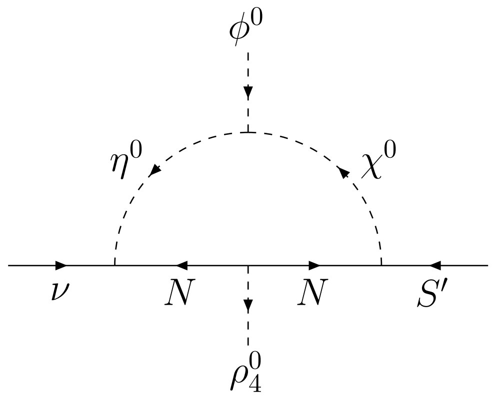

Figure 2.2: Radiative generation of $\nu - S'$ mixing.

Similarly S will also mix with ν, using the $SN\chi^0$ Yukawa coupling. However, these terms are negligible compared to the assumed large masses for (S, S, S') and may be safely ignored.

Consider now the possibility that $m_{13}, m_{23} << m_{S1}, m_{S2}$ in $\mathcal{M}_S$, then S' obtains a small seesaw mass given by

$$m_{S'} \simeq -\frac{m_{13}^2}{m_{S1}} - \frac{m_{23}^2}{m_{S2}}. \tag{2.7}$$

Let this be a few keV, then S' is a light sterile neutrino which mixes with ν only slightly through Fig. 2.2. Hence it is a candidate for warm dark matter. Whereas the usual sterile neutrino is an *ad hoc* invention, it has a natural place here in terms of its mass as well as its suppressed mixing with the active neutrinos.

We now have the interesting scenario where part of the dark matter of the Universe is cold, and the other is warm. This hybrid case was recently also obtained in a different radiative model of neutrino masses [26]. Within the present context, there is a third possibility. If we assign an extra Z_2 symmetry, under which $S_{1,2}$ are odd and all other particles even, then the only interactions involving $S_{1,2}$ come from their diagonal $U(1)_{B-L}$ gauge couplings and the diagonal Yukawa terms $f_1 S_1 S_1 (\rho_2^0)^*$ and $f_2 S_2 S_2 (\rho_2^0)^*$. This means that both S_1 and S_2 are stable and their relic abundances are determined by their annihilation cross sections to SM particles. In this scenario, dark matter has four components [27].

Since $S_{1,2}$ are now separated from S', the m_{13} and m_{23} terms in $\mathcal{M}_S$ are zero and there is no tree-level mass for S'. However, there is a one-loop mass as shown in Fig. 2.3.

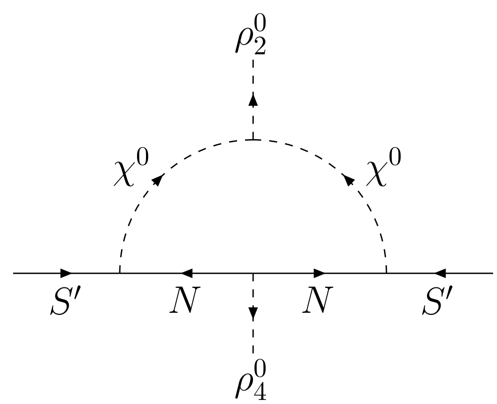

Figure 2.3: Radiative generation of S' mass.

This makes it more natural for S' to be light. A detailed study of the dark-matter phenomenology of this multipartite scenario will be given elsewhere.

2.4 Scalar Sector for Symmetry Breaking

In this model, there is only one Higgs doublet Φ which breaks the $SU(2)_L \times U(1)_Y$ electroweak symmetry, whereas there are two Higgs singlets ρ_2 and ρ_4 which break $U(1)_{B-L}$ to Z_2. The most general Higgs potential consisting of Φ, ρ_2, ρ_4 is given by

$$V = \mu_0^2\Phi^\dagger\Phi + \mu_2^2\rho_2^*\rho_2 + \mu_4^2\rho_4^*\rho_4 + \frac{1}{2}\mu_{24}[\rho_2^2\rho_4^* + H.c.] + \frac{1}{2}\lambda_0(\Phi^\dagger\Phi)^2 + \frac{1}{2}\lambda_2(\rho_2^*\rho_2)^2 + \frac{1}{2}\lambda_4(\rho_4^*\rho_4)^2 + \lambda_{02}(\Phi^\dagger\Phi)(\rho_2^*\rho_2) + \lambda_{04}(\Phi^\dagger\Phi)(\rho_4^*\rho_4) + \lambda_{24}(\rho_2^*\rho_2)(\rho_4^*\rho_4). \tag{2.8}$$

Let $\langle\phi^0\rangle = v$, $\langle\rho_2\rangle = u_2$, $\langle\rho_4\rangle = u_4$, then the minimum of V is determined by:

$$0 = \mu_0^2 + \lambda_0 v^2 + \lambda_{02}u_2^2 + \lambda_{04}u_4^2, \tag{2.9}$$

$$0 = \mu_2^2 + \lambda_{02}v^2 + \lambda_2 u_2^2 + \lambda_{24}u_4^2 + \mu_{24}u_4, \tag{2.10}$$

$$0 = u_4(\mu_4^2 + \lambda_{04}v^2 + \lambda_{24}u_2^2 + \lambda_4 u_4^2) + \frac{1}{2}\mu_{24}u_2^2. \tag{2.11}$$

The would-be Goldstone bosons are $\phi^\pm$, $\sqrt{2}Im(\phi^0)$, corresponding to the breaking of $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$, and $\sqrt{2}[u_2 Im(\rho_2) + 2u_4 Im(\rho_4)]/\sqrt{u_2^2 + 4u_4^2}$, corresponding to the breaking of $U(1)_{B-L}$ to Z_2. The linear combination orthogonal to the latter is a physical pseudoscalar A, with a mass given by:

$$m_A = \frac{-\mu_{24}(u_2^2 + 4u_4^2)}{2u_4}. \tag{2.12}$$

The 3×3 mass-squared matrix of the physical scalars $[\sqrt{2}Re(\phi^0), \sqrt{2}Re(\rho_2), \sqrt{2}Re(\rho_4)]$ is given by

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_0 v^2 & 2\lambda_{02} v u_2 & 2\lambda_{04} v u_4 \\ 2\lambda_{02} v u_2 & 2\lambda_2 u_2^2 & u_2(2\lambda_{24} u_4 + \mu_{24}) \\ 2\lambda_{04} v u_4 & u_2(2\lambda_{24} u_4 + \mu_{24}) & 2\lambda_4 u_4^2 - \mu_{24} u_2^2 / 2u_4 \end{pmatrix} \tag{2.13}$$

For $v^2 << u_{2,4}^2$, $\sqrt{2}Re(\phi^0) = h$ is approximately a mass eigenstate which is identified with the 125 GeV particle discovered at the LHC.

2.5 Gauge Sector

Since ϕ^0 does not transform under $U(1)_{B-L}$ and $\rho_{2,4}$ do not transform under $SU(2)_L \times U(1)_Y$, there is no tree-level mixing between their corresponding gauge bosons Z and Z_{B-L}. In our convention, $M_{Z_{B-L}}^2 = 8g_{B-L}^2(u_2^2 + 4u_4^2)$. The LHC bound on $M_{Z_{B-L}}$ comes from the production of Z_{B-L} from u and d quarks and its subsequent decay to e^-e^+ and $\mu^-\mu^+$. If all the particles listed in Table 2.1 are possible decay products of Z_{B-L} with negligible kinematic suppression, then its branching fraction to e^-e^+ and $\mu^-\mu^+$ is about 0.061. The $c_{u,d}$ coefficients used in the LHC analysis [12, 13] are then

$$c_u = c_d = \left[\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right] g_{B-L}^2 \times B(Z_{B-L} \to e^-e^+, \mu^-\mu^+) = 1.36 \times 10^{-2}\ g_{B-L}^2. \tag{2.14}$$

From LHC data based on the 7 and 8 TeV runs, a bound of about 2.5 TeV would correspond to $g_{B-L} < 0.24$.

2.6 Conclusion

Using gauge $U(1)_{B-L}$ symmetry, we have proposed a new anomaly-free solution with exotic fermion singlets, such that neutrino mass is forbidden at tree level. We add a number of new scalars so that neutrino masses are obtained in one loop through dark matter, i.e. the scotogenic mechanism. Because of the structure of the new singlets required for anomaly cancellation, we find a possible dark-matter scenario with four components. Three are stable cold Weakly Interaction Massive Particles (WIMPs) and one a keV singlet neutrino, i.e. warm dark matter with a very long lifetime.

Chapter 3

B-L with Residual Z_3 Symmetry

When three right-handed nuetrinos are added to the standard model under a $U(1)_{B-L}$ symmetry the most obvious charge assignment is $(-1,-1,-1)$. The discussion below is based upon [1] where the unconventional assignment $(5,-4,-4)$ is chosen. As a consequence a residual Z_3 symmetry remains after the breaking of $U(1)_{B-L}$ yielding a possible, long-lived, dark-matter candidate.

Lepton number L is a familiar concept. It is usually defined as a global U(1) symmetry, under which the leptons of the standard model (SM), i.e. e, μ, τ together with their neutrinos ν_e, ν_μ, ν_τ have $L = 1$, and all other SM particles have $L = 0$. In the case of nonzero Majorana neutrino masses, this continuous symmetry is broken to a discrete Z_2 symmetry, i.e. $(-1)^L$ or lepton parity. Consider a gauge $B - L$ extension of the SM, such that a residual Z_3 symmetry remains after the spontaneous breaking of $B - L$. This is then a realization of the unusual notion of Z_3 lepton symmetry.

3.1 Neutrino Mass

The conventional treatment of gauge $B-L$ has three right-handed singlet neutrinos $\nu_{R1}, \nu_{R2}, \nu_{R3}$ transforming as $-1, -1, -1$ under $B-L$. It is well-known that this assignment satisfies all the anomaly-free conditions for $U(1)_{B-L}$. However, another assignment [21]

$$\nu_{R1}, \nu_{R2}, \nu_{R3} \sim 5, -4, -4 \tag{3.1}$$

works as well, because

$$5 - 4 - 4 = -3, \quad (5)^3 - (4)^3 - (4)^3 = -3. \tag{3.2}$$

To obtain a realistic model with this assignment, it was recently proposed [22] that three additional neutral singlet Dirac fermions $N_{1,2,3}$ be added with $B - L = -1$, together with a singlet scalar χ_3 with $B - L = 3$. Consequently, the tree-level Yukawa couplings $\bar{\nu}_L N_R \bar{\phi}^0$ and $\bar{N}_L \nu_{R2} \chi_3$, $\bar{N}_L \nu_{R3} \chi_3$ are allowed, where $\Phi = (\phi^+, \phi^0)$ is the one Higgs doublet of the SM. Together with the invariant $\bar{N}_L N_R$ mass terms, the 6×5 neutrino mass matrix linking $(\bar{\nu}_L, \bar{N}_L)$ to (ν_R, N_R) is of the form

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & \mathcal{M}_0 \\ \mathcal{M}_3 & \mathcal{M}_N \end{pmatrix} \tag{3.3}$$

where $\mathcal{M}_0$ and $\mathcal{M}_N$ are 3×3 mass matrices and $\mathcal{M}_3$ is 3×2 because ν_{R1} has no tree-level Yukawa coupling. This means that one linear combination of ν_L is massless. Of course, if the dimension-five term $\bar{\nu}_{R1} N_L \chi_3^2$ also exists, then $\mathcal{M}_3$ is 3×3 and $\mathcal{M}_{\nu N}$ is 6×6.

The form of $\mathcal{M}_{\nu N}$ allows nonzero seesaw Dirac neutrino masses for ν [28], i.e.

$$\mathcal{M}_\nu \simeq \mathcal{M}_0 \mathcal{M}_N^{-1} \mathcal{M}_3. \tag{3.4}$$

Without the implementation of a flavor symmetry, any 3×3 $\mathcal{M}_\nu$ is possible. Although the gauge $B-L$ is broken, a residual global L symmetry remains in this model with ν, l, N all having $L = 1$. Because the pairing of any two neutral fermions of the same chirality always results in a nonzero $B-L$ charge not divisible by 3 units in this model, it is impossible to construct an operator of any dimension for a Majorana mass term which violates $B-L$. Hence the neutrinos are indeed exactly Dirac.

We now add two more scalar singlets: χ_2 with $B-L=2$ and χ_6 with $B-L=-6$. The important new terms in the Lagrangian are:

$$\bar{N}_L \nu_{R1} \chi_6, \quad \chi_2 N_L N_L, \quad \chi_2 N_R N_R, \quad \chi_2^3 \chi_6, \quad \chi_3^2 \chi_6. \tag{3.5}$$

Now $B-L$ is broken by $\langle \chi_3 \rangle = u_3$ as well as $\langle \chi_6 \rangle = u_6$, and all neutrinos become massive. If χ_2 is absent, then again a residual global L symmetry exists with $L=1$ for ν, l, N and $L=0$ for $\chi_{3,6}$. However, the existence of χ_2 shows that the residual symmetry is then Z_3, such that χ_2 and all leptons transform as $\omega = \exp(2\pi i/3)$ under Z_3 with $\chi_{3,6} \sim 1$. This is thus the first example of a lepton symmetry which is not Z_2 (for Majorana neutrinos), nor $U(1)$ or Z_4 [29, 30] (for Dirac neutrinos). Note that Z_3 is also sufficient to guarantee that all the neutrinos remain Dirac.

Although there is no stabilizing symmetry here for dark matter, χ_2 has very small couplings to two neutrinos through the Yukawa terms of Eq. (3.5) from the mixing implied by Eq. (3.3). This means that χ_2 may have a long enough lifetime to be suitable for dark matter, as shown below.

3.2 Gauge Sector

In this model, there is of course a gauge boson Z' which couples to $B-L$. Its production at the Large Hadron Collider (LHC) is due to its couplings to quarks. Once produced, it decays into quarks and leptons. In the conventional $B-L$ assignment for ν_R, its branching fractions to quarks, charged leptons, and neutrinos are 1/4, 3/8, and 3/8 respectively. These values are obtained by approximating all daughter masses as zero, then the fermionic decays will be proportional to the daughter particle's charge squared multiplied by the number of copies.

In this model, the ν_R charges are $(5,-4,-4)$, hence their resulting partial widths are very large. Assuming that Z' decays also into χ_2, the respective branching fractions into quarks, charged leptons, neutrinos, and χ_2 as dark matter are then 1/18, 1/12, 5/6, and 1/36. This means Z' has an 86% invisible width. Using the production of Z' via $u\bar{u}$ and $d\bar{d}$ initial states at the LHC and its decay into e^-e^+ or $\mu^-\mu^+$ as signature, the current bound on $m_{Z'}$ assuming $g'=g$, i.e. the $SU(2)_L$ gauge coupling of the SM, is about 3 TeV, based on recent LHC data [12, 13]. However, because the branching fraction into l^-l^+ is reduced by a factor of 2/9 in our $B-L$ model, this bound is reduced to about 2.5 TeV, again for $g' = g$. There is also a similar bound [31] from precision $e^-e^+ \to e^-e^+$ measurements at the Large Electron Positron Collider (LEP), i.e. $m_{Z'}/g' >$ a few TeV.

Particle	Contribution	B. R.
Quarks	$(1/3)^2 \times 3 \times 3 \times 2 + (1/3)^2 \times 3 \times 3 \times 2 = 4$	1/18
Charged Leptons	$(-1)^2 \times 3 + (-1)^2 \times 3 = 6$	1/12
Neutrinos	$(-1)^2 \times 3 + (5)^2 + (4)^2 + (4)^2 = 60$	5/6
χ_2	$(2)^2/2 = 2$	1/36

Table 3.1: Branching Ratio calculation for this model.

3.3 χ_2 as Dark Matter

Consider for simplicity the coupling of χ_2 to just one N, with the interaction

$$\mathcal{L}_{int} = \frac{1}{2} f_L \chi_2 N_L N_L + \frac{1}{2} f_R \chi_2 N_R N_R + H.c. \tag{3.6}$$

Let the $\nu_L - N_L$ mixing be $\zeta_0 = m_0/m_N$ and $\nu_R - N_R$ mixing be $\zeta_3 = m_3/m_N$, then the decay rate of χ_2 is

$$\Gamma(\chi_2 \to \bar{\nu}\bar{\nu}) = \frac{m_\chi}{32\pi}(f_L^2 \zeta_0^4 + f_R^2 \zeta_3^4). \tag{3.7}$$

If we set this equal to the age of the Universe (13.75×10^9 years), and assuming $m_\chi = 100$ GeV, $f_L = f_R$ and $\zeta_0 = \zeta_3$, then $f\zeta^2 = 8.75 \times 10^{-22}$. Hence

$$\sqrt{f}\zeta << 3 \times 10^{-11} \tag{3.8}$$

would guarantee the stability of χ_2 to the present day, and allow it to be a dark-matter candidate. This sets the scale of m_N at about 10^{13} GeV, which is also the usual mass scale for the heavy Majorana singlet neutrino in the canonical seesaw mechanism.

Since χ_2 interacts with nuclei through Z', there is also a significant constraint from dark-matter direct-search experiments. The cross section per nucleon is given by

$$\sigma_0 = \frac{1}{\pi}\left(\frac{m_\chi m_n}{m_\chi + A m_n}\right)^2 \left(\frac{2g'^2}{m_{Z'}^2}\right)^2, \qquad (3.9)$$

where A is the number of nucleons in the target and m_n is the nucleon mass. Consider for example $m_\chi = 100$ GeV, then $\sigma_0 < 1.25 \times 10^{-45}$ cm^2 from the LUX data [32]. This implies $m_{Z'}/g' > 16.2$ TeV, as shown in Fig. 3.1.

If $g' = g$, then $m_{Z'} > 10.6$ TeV. This limit is thus much more severe than the LHC bound of 2.5 TeV. If $g' < g$, then both the LHC and LUX bounds on $m_{Z'}$ are relaxed. However, it also means that it is unlikely that Z' would be discovered at the LHC even with the 14 TeV run.

Consider now the annihilation cross section of $\chi_2\chi_2^*$ for obtaining its thermal relic abundance. The process $\chi_2\chi_2^* \to Z' \to$ SM particles is p-wave suppressed and is unlikely to be strong enough for this purpose. We may then consider the well-studied process $\chi_2\chi_2^* \to h \to$ SM particles, where h is the SM Higgs boson. If this is assumed to account for all of the dark-matter relic abundance of the Universe, then it has recently been shown [33] that the required strength of this interaction is in conflict with LUX data except for a small region near $m_\chi = m_h/2$. On the other hand, another analysis [34] claims that a region with

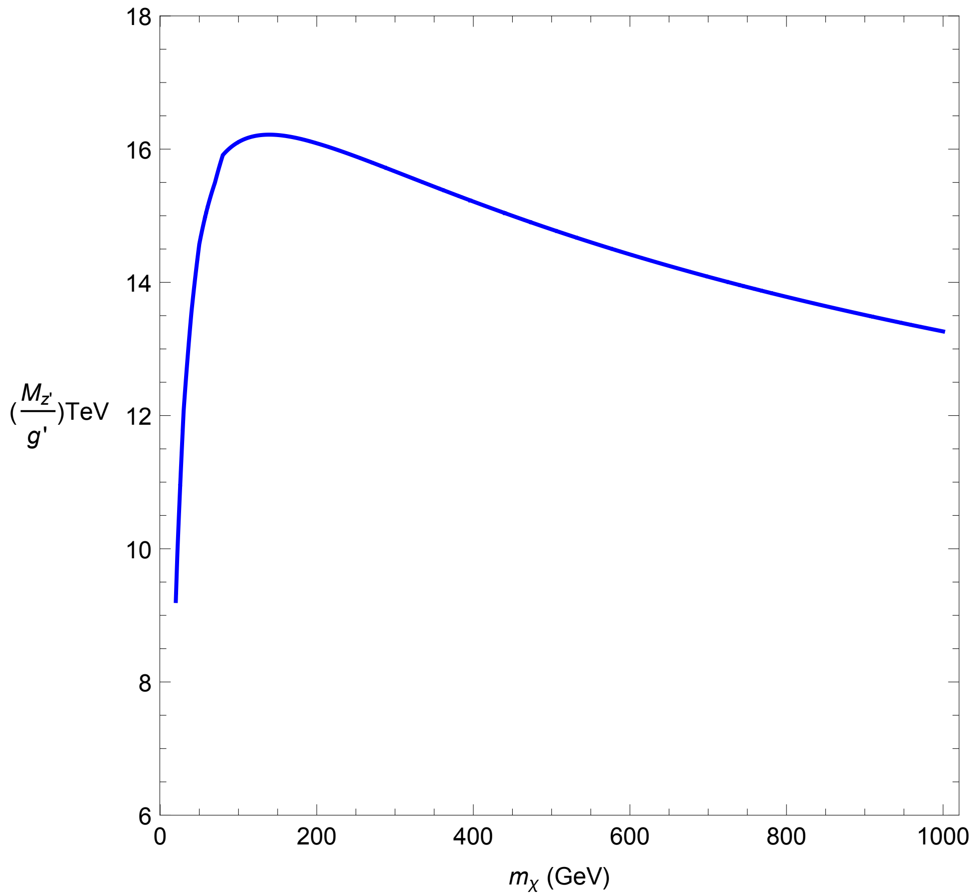

Figure 3.1: Lower bound on $m_{Z'}/g'$ versus m_χ from LUX data.

m_χ somewhat greater than m_h is still allowed.

In this chapter, we will consider the following alternative scenario. We assume that the $h\chi_2\chi_2^*$ interaction is negligible, so that neither Higgs nor Z' exchange is important for $\chi_2\chi_2^*$ annihilation. Instead we invoke the new interactions of Fig. 3.2. Since $\chi_{3,6}$ may interact freely with h, thermal equilibrium is maintained with the other SM particles. This scenario requires of course that m_χ to be greater than at least one physical mass eigenvalue in the $\chi_{3,6}$ sector.

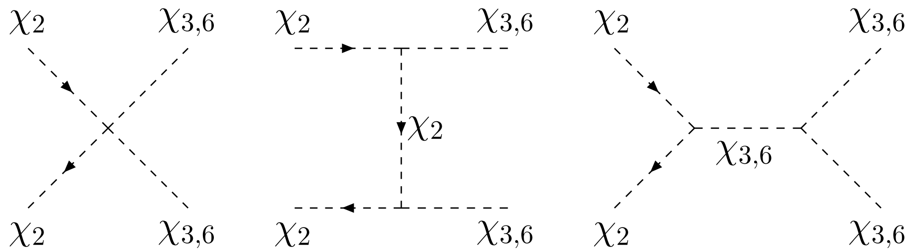

Figure 3.2: $\chi_2\chi_2^\dagger$ annihilation to $\chi_{3,6}$ final states.

To summarize, $\chi_2 \sim \omega$ under Z_3 and decays into two antineutrinos, but its lifetime is much longer than the age of the Universe. It is thus an example of Z_3 dark matter [35, 36, 37, 38, 39]. It is also different from previous Z_2 proposals [40, 41] based on Ref. [21]. It has significant elastic interactions with nuclei through Z' and Higgs exchange and may be discovered in direct-search experiments. On the other hand, its relic abundance is determined not by Z' or Higgs interactions, but by its annihilation to other scalars of this model which maintain thermal equilibrium with the SM particles through the SM Higgs boson. Note that this is also the mechanism used in a recently proposed model of vector dark matter [42].

3.4 Scalar Sector

Consider the scalar potential:

$$
\begin{aligned}
V =& -\mu_0^2(\Phi^\dagger\Phi) + m_2^2(\chi_2^*\chi_2) - \mu_3^2(\chi_3^*\chi_3) - \mu_6^2(\chi_6^*\chi_6) \\
&+ \frac{1}{2}\lambda_0(\Phi^\dagger\Phi)^2 + \frac{1}{2}\lambda_2(\chi_2^*\chi_2)^2 + \frac{1}{2}\lambda_3(\chi_3^*\chi_3)^2 + \frac{1}{2}\lambda_6(\chi_6^*\chi_6)^2 + \lambda_{02}(\chi_2^*\chi_2)(\Phi^\dagger\Phi) \\
&+ \lambda_{03}(\chi_3^*\chi_3)(\Phi^\dagger\Phi) + \lambda_{06}(\chi_6^*\chi_6)(\Phi^\dagger\Phi) + \lambda_{23}(\chi_2^*\chi_2)(\chi_3^*\chi_3) + \lambda_{26}(\chi_2^*\chi_2)(\chi_6^*\chi_6) \\
&+ \lambda_{36}(\chi_3^*\chi_3)(\chi_6^*\chi_6) + [\frac{1}{2}f_{36}(\chi_3^2\chi_6) + \text{H.c.}] + [\frac{1}{6}\lambda'_{26}(\chi_2^3\chi_6) + \text{H.c.}].
\end{aligned} \tag{3.10}
$$

Let $\langle\phi^0\rangle = v$, $\langle\chi_3\rangle = u_3$, $\langle\chi_6\rangle = u_6$, then the minimum of V is determined by:

$$
\mu_0^2 = \lambda_0 v^2 + \lambda_{03}u_3^2 + \lambda_{06}u_6^2, \tag{3.11}
$$

$$
\mu_3^2 = \lambda_3 u_3^2 + \lambda_{03}v^2 + \lambda_{36}u_6^2 + f_{36}u_6, \tag{3.12}
$$

$$
\mu_6^2 = \lambda_6 u_6^2 + \lambda_{06}v^2 + \lambda_{36}u_3^2 + \frac{f_{36}u_3^2}{2u_6}. \tag{3.13}
$$

There is one dark-matter scalar boson χ_2 with mass given by:

$$
m_\chi^2 = m_2^2 + \lambda_{02}v^2 + \lambda_{23}u_3^2 + \lambda_{26}u_6^2. \tag{3.14}
$$

There is one physical pseudoscalar boson:

$$
A = \sqrt{2}Im(2u_6\chi_3 + u_3\chi_6)/\sqrt{u_3^2 + 4u_6^2} \tag{3.15}
$$

with mass given by:

$$m_A^2 = -f_{36}(u_3^2 + 4u_6^2)/2u_6. \tag{3.16}$$

There are three physical scalar bosons spanning the basis $[h, \sqrt{2}Re(\chi_3), \sqrt{2}Re(\chi_6)]$, with 3×3 mass-squared matrix given by:

$$M^2 = \begin{pmatrix} 2\lambda_0 v^2 & 2\lambda_{03}u_3 v & 2\lambda_{06}u_6 v \\ 2\lambda_{03}u_3 v & 2\lambda_3 u_3^2 & 2\lambda_{36}u_3 u_6 + f_{36}u_3 \\ 2\lambda_{06}u_6 v & 2\lambda_{36}u_3 u_6 + f_{36}u_3 & 2\lambda_6 u_6^2 - f_{36}u_3^2/2u_6 \end{pmatrix} \tag{3.17}$$

For illustration, we consider the special case $\lambda_{03} = \lambda_{06} = 0$, so that h decouples from $\chi_{3,6}$. It then becomes identical to that of the SM, and may be identified with the 125 GeV particle discovered [5, 6] at the LHC. We now look for a solution with:

$$S = \sqrt{2}Re(-u_3\chi_3 + 2u_6\chi_6)/\sqrt{u_3^2 + 4u_6^2}, \tag{3.18}$$

$$S' = \sqrt{2}Re(2u_6\chi_3 + u_3\chi_6)/\sqrt{u_3^2 + 4u_6^2}, \tag{3.19}$$

as mass eigenstates. This is easily accomplished for example with:

$$u_3 = 2u_6, \qquad 4\lambda_3 = \lambda_6 - f_{36}/u_6. \tag{3.20}$$

In this case,

$$S = -Re\chi_3 + Re\chi_6, m_S^2 = 2\lambda_6 u_6^2 - 4\lambda_{36} u_6^2 - 4f_{36} u_6, \tag{3.21}$$

$$S' = Re\chi_3 + Re\chi_6, m_{S'}^2 = 2\lambda_6 u_6^2 + 4\lambda_{36} u_6^2, \tag{3.22}$$

$$A = Im\chi_3 + Im\chi_6, m_A^2 = -4f_{36} u_6, \tag{3.23}$$

$$m_{Z'} = 12g' u_6. \tag{3.24}$$

The couplings of $\chi_2\chi_2^*$ to S and S' are given by:

$$\chi_2\chi_2^*[u_6(\lambda_{26} - 2\lambda_{23})S + u_6(\lambda_{26} + 2\lambda_{23})S']. \tag{3.25}$$

Since S plays the same role in breaking $B-L$ as the Higgs boson h does in breaking $SU(2)_L \times U(1)_Y$, it is expected to be massive of order $\sqrt{u_3^2 + 4u_6^2} = 2\sqrt{2}u_6$. This allows $m_{S'}$ to be adjusted to be very small, then it may serve as a light scalar mediator for χ_2 as self-interacting dark matter [43]. This is not a necessary assumption of the model and requires fine tuning of scalar parameters to achieve. We merely want to demonstrate that such a possible scenario exists within our model. For $m_{S'} \simeq 0$, we need $\lambda_{36} = -\lambda_6/2$. In that case, using Eq. (3.20), we find:

$$m_S^2 = 16\lambda_3 u_6^2, \quad m_A^2 = m_S^2 - 4\lambda_6 u_6^2. \tag{3.26}$$

We assume that the relic density of χ_2 is dominated by the $\chi_2\chi_2^*$ annihilation to $S'S'$. This may have to be revised if the semi-annihilation $\chi_2\chi_2 \to \chi_2^* S'$ is sizeable. Here we simply assume that λ'_{26} is small. For illustration, we set to zero the $\chi_2\chi_2^* S'S'$ coupling, i.e. $\lambda_{23} + \lambda_{26} = 0$, as well as the $SS'S'$ coupling, i.e. $-12\lambda_3 + 6\lambda_6 + 2\lambda_{36} - f_{36}/u_6 = 0$. This implies $\lambda_3 = \lambda_6/2$ so that the $S'S'S'$ coupling is also zero and $m_A^2 = m_S^2/2$. This choice of parameters means that only the middle diagram of Fig. 3.2 contributes to the $\chi_2\chi_2^*$ annihilation cross section with:

$$\sigma \times v_{rel} = \frac{1}{64\pi m_\chi^2} \left| \frac{\lambda_{26}^2 u_6^2}{m_\chi^2} \right|^2 . \tag{3.27}$$

Equating this to the optimal value [44] of 4.4×10^{-26} cm^3 s^{-1} for the correct dark-matter relic density of the Universe, we find for $m_\chi = 100$ GeV

$$\lambda_{26} = 0.0295 \left(\frac{1 \text{ TeV}}{u_6} \right) . \tag{3.28}$$

We assume of course that $m_A > 2m_\chi$.

For S' to be in thermal equilibrium with the SM particles, we consider nonzero values of λ_{03} and λ_{06}. This is possible in our chosen parameter space if $2\lambda_{03} + \lambda_{06} \simeq 0$, so that the $S'h$ mixing is very small and yet the $S'S'h$ coupling $\lambda_{06}v/4\sqrt{2}$ and $S'S'hh$ coupling $\lambda_{06}/16$ may be significant. Note that the Sh mixing is now fixed at $(\lambda_{06}/\lambda_6)(v/2\sqrt{2}u_6)$ which may yet be suitably suppressed for h to be essentially the one Higgs boson of the SM. Even if $\lambda_{03,06}$ are negligible, the gauge interaction $S'AZ'$ may also be sufficient to maintain thermal equilibrium. This may also affect the magnitude of the self-interacting $\chi_2\chi_2^*$ cross section.

The $h \to S'S'$ decay width is given by:

$$\Gamma(h \to S'S') = \frac{\lambda_{06}^2 v^2}{256\pi m_h} = \left(\frac{\lambda_{06}}{0.04}\right)^2 0.5 \text{ MeV}. \tag{3.29}$$

It is invisible at the LHC because S' decays slowly to e^-e^+ only through its mixing with h, if $m_{S'} \sim 10$ MeV for S' as a light mediator for the self-interacting dark matter χ_2.

3.5 Conclusion

In conclusion, we have considered the unusual case of a gauge $B - L$ symmetry which is spontaneously broken to Z_3 lepton number. Neutrinos are Dirac fermions transforming as $\omega = \exp(2\pi i/3)$ under Z_3. A complex neutral scalar χ_2 exists which also transforms as ω. It is not absolutely stable, but decays to two antineutrinos with a lifetime much greater than that of the Universe. It is thus an example of Z_3 dark matter. In addition to the one Higgs boson h of the SM, there are three neutral scalars S, S', A and one heavy vector gauge boson Z'. From direct-search experiments, $m_{Z'}/g'$ is constrained to be very large, thus making it impossible to discover Z' at the LHC even with the current run. The relic abundance of χ_2 is determined by its annihilation into S' which is a candidate for the light mediator by which χ_2 obtains its long-range self-interaction.

Part II

Related Extensions and Phenomenon to $U(1)_{B-L}$

Chapter 4

Generalized Gauge $U(1)$ Family Symmetries

This chapter discusses the charge assignment of an arbitrary $U(1)$ gauged extension to the Standard Model. Restrictions placed on the particle charge assignment under the $U(1)$ extension is explored and two sample models will be proposed concluding with some effects to be observed. This chapter is based upon the previously published work in Ref. [3].

4.1 Anomaly Constraints

The Standard Model governs the interactions between the three families of quarks and leptons. Under its $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry the right-handed neutrinos ν_R do not transform. As a consequence the right-handed neutrinos are not included in the Minimal Standard Model. Since neutrinos are known to be massive [8], ν_R should be considered as additions to the Standard Model. To accomplish this, a possible new family gauge symmetry $U(1)_F$ is admitted with charges $n_{1,2,3}$ for the quarks and $n'_{1,2,3}$ for the leptons as shown in Table 4.1.

Particle	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_F$
$Q_{iL} = (u,d)_{iL}$	3	2	1/6	n_i
u_{iR}	3	1	2/3	n_i
d_{iR}	3	1	$-1/3$	n_i
$L_{iL} = (\nu,l)_{iL}$	1	2	$-1/2$	n'_i
l_{iR}	1	1	-1	n'_i
ν_{iR}	1	1	0	n'_i

Table 4.1: Fermion assignments under $U(1)_F$.

Anomaly cancellation is imposed to constrain $n_{1,2,3}$ and $n'_{1,2,3}$. The contributions of color triplets to the $[SU(3)]^2U(1)_F$ anomaly sum up to:

$$[SU(3)]^2U(1)_F \quad : \quad \frac{1}{2}\sum_{i=1}^{3}(2n_i - n_i - n_i); \tag{4.1}$$

and the contributions of $Q_{iL}, u_{iR}, d_{iR}, L_{iL}, l_{iR}$ to the $U(1)_Y[U(1)_F]^2$ anomaly sum up to:

$$U(1)_Y[U(1)_F]^2 \quad : \quad \sum_{i=1}^{3}\left[6\left(\frac{1}{6}\right) - 3\left(\frac{2}{3}\right) - 3\left(-\frac{1}{3}\right)\right]n_i^2 + \left[2\left(-\frac{1}{2}\right) - (-1)\right]n_i'^2. \tag{4.2}$$

Both are automatically zero, as well as the cubic ($[U(1)_F]^3$) and gravitational ($grav^2U(1)_F$) anomalies, because all fermions couple to $U(1)_F$ vectorially. The contributions of the $SU(2)_L$ doublets to the $[SU(2)]^2U(1)_F$ anomaly sum up to:

$$[SU(2)]^2U(1)_F \quad : \quad \frac{1}{2}\sum_{i=1}^{3}(3n_i + n'_i); \tag{4.3}$$

and the contributions to the $[U(1)_Y]^2 U(1)_F$ anomaly sum up to:

$$
\begin{aligned}
[U(1)_Y]^2 U(1)_F : &\sum_{i=1}^{3} \left[6\left(\frac{1}{6}\right)^2 - 3\left(\frac{2}{3}\right)^2 - 3\left(-\frac{1}{3}\right)^2 \right] n_i + \left[2\left(-\frac{1}{2}\right)^2 - (-1)^2 \right] n_i' \\
&= \sum_{i=1}^{3} \left(-\frac{3}{2} n_i - \frac{1}{2} n_i' \right) . \qquad (4.4)
\end{aligned}
$$

Both are zero if:

$$
\sum_{i=1}^{3} (3n_i + n_i') = 0. \qquad (4.5)
$$

n_1	n_2	n_3	n'_1	n'_2	n'_3	Model
1/3	1/3	1/3	-1	-1	-1	$B - L$ [20]
0	0	0	0	1	-1	$L_\mu - L_\tau$ [47, 48, 49, 50]
1/3	1/3	1/3	0	0	-3	$B - 3L_\tau$ [51, 52, 53, 54]
1/3	1/3	1/3	3	-3	-3	Ref. [55]
1	1	-2	1	1	-2	Ref. [56]
a	a	$-2a$	0	-1	1	Ref. [57]

Table 4.2: Examples of models satisfying Eq. (4.5).

Furthermore these results can be extended to more than three families, for example in the case of four families $n_{1,2,3} = 1/3$, $n_4 = -1$, and $n'_{1,2,3} = 1$, $n'_4 = -3$ [45, 46] as a separate gauging of B and L. There are many specific examples of models which satisfy the condition shown in Table 4.2.

We will now discuss two examples which offer some insights to the structure of mixing among the quark and lepton families. Both models have nontrivial connections

n_1	n_2	n_3	n'_1	n'_2	n'_3	Model
1	1	0	0	-2	-4	A
1	1	-1	0	-1	-2	B

Table 4.3: Two new models satisfying Eq. (4.5).

between quarks and leptons. Their structures are shown in Table 4.3.

In both cases, with only one Higgs doublet with zero charge under $U(1)_F$, quark and lepton mass matrices are diagonal except for the first two quark families. This allows for mixing among them, but not with the third family. It is a good approximation to the 3×3 quark mixing matrix, to the extent that mixing with the third family is known to be suppressed. In the leptonic sector, mixing also comes from the Majoranna mass matrix of ν_R which depends on the choice of singlets with vacuum expectation values which break $U(1)_F$. Adding a second Higgs doublet with nonzero $U(1)_F$ charge will allow mixing of the first two families of quarks with the third in both cases. As for the leptons, this will not affect Model A, but will cause mixing in the charged-lepton and Dirac neutrino mass matrices in Model B. Flavor-changing neutral currents are predicted with interesting phenomenological consequences.

4.2 Basic Structure of Model A

Consider first the structure of the 3×3 quark mass matrix $\mathcal{M}_d$ linking $(\bar{d}_L, \bar{s}_L, \bar{b}_L)$ to (d_R, s_R, b_R). Using:

$$\Phi_1 = (\phi_1^+, \phi_1^0) \sim (1, 2, 1/2; 0), \tag{4.6}$$

with $\langle \phi_1^0 \rangle = v_1$, it is clear that $\mathcal{M}_d$ is block diagonal with a 2×2 submatrix which may be rotated on the left to become:

$$\mathcal{M}_d = \begin{pmatrix} c_L & -s_L & 0 \\ s_L & c_L & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} m_d' & 0 & 0 \\ 0 & m_s' & 0 \\ 0 & 0 & m_b' \end{pmatrix} \tag{4.7}$$

where $s_L = \sin\theta_L$ and $c_L = \cos\theta_L$. We now add a second Higgs doublet:

$$\Phi_2 = (\phi_2^+, \phi_2^0) \sim (1, 2, 1/2; 1), \tag{4.8}$$

with $\langle \phi_2^0 \rangle = v_2$, so that:

$$\mathcal{M}_d = \begin{pmatrix} c_L & -s_L & 0 \\ s_L & c_L & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} m_d' & 0 & m_{db}' \\ 0 & m_s' & m_{sb}' \\ 0 & 0 & m_b' \end{pmatrix} \tag{4.9}$$

is obtained. At the same time, $\mathcal{M}_u$ is of the form:

$$\mathcal{M}_u = \begin{pmatrix} m_u' & 0 & 0 \\ 0 & m_c' & 0 \\ m_{ut}' & m_{ct}' & m_t' \end{pmatrix} \begin{pmatrix} c_R & s_R & 0 \\ -s_R & c_R & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4.10}$$

where it has been rotated on the right. Because of the physical mass hierarchy $m_u << m_c << m_t$, the diagonalization of Eq. (4.10) will have very small deviations from unity on the left. Hence the unitary matrix diagonalizing Eq. (4.9) on the left will be essentially the experimentally observed quark mixing matrix V_{CKM} which has three angles and one phase. Now $\mathcal{M}_d$ of Eq. (4.9) has exactly seven parameters, the three diagonal masses m'_d, m'_s, m'_b, the angle θ_L, the off-diagonal mass m'_{sb} which can be chosen real, and the off-diagonal mass m'_{db} which is complex. With the input of the three quark mass eigenvalues m_d, m_s, m_b and V_{CKM}, these seven parameters can be determined.

Consider the diagonalization of the real mass matrix:

$$\begin{pmatrix} a & 0 & s_1 c \\ 0 & b & s_2 c \\ 0 & 0 & c \end{pmatrix} = V_L \begin{pmatrix} a(1-s_1^2/2) & 0 & 0 \\ 0 & b(1-s_2^2/2) & 0 \\ 0 & 0 & c(1+s_1^2/2+s_2^2/2) \end{pmatrix} V_R^\dagger, \tag{4.11}$$

where $s_{1,2} << 1$ and $a << b << c$ have been assumed. To determine V_L and V_R we utilize their properties as a unitary matrix, that is:

$$V_L V_L^\dagger = V_L^\dagger V_L = V_R V_R^\dagger = V_R^\dagger V_R = 1 \tag{4.12}$$

By multiplying our matrix by it's Hermitian Conjugate on the right(left) we can eliminate $V_R^\dagger (V_L)$:

$$
\begin{pmatrix} a & 0 & s_1 c \\ 0 & b & s_2 c \\ 0 & 0 & c \end{pmatrix} \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ s_1 c & s_2 c & c \end{pmatrix} =
$$

$$
= V_L \begin{pmatrix} a(1-s_1^2/2) & 0 & 0 \\ 0 & b(1-s_2^2/2) & 0 \\ 0 & 0 & c(1+s_1^2/2+s_2^2/2) \end{pmatrix}^2 V_L^\dagger \qquad (4.13)
$$

Thus we obtain:

$$
V_L = \begin{pmatrix} 1-s_1^2/2 & -s_1 s_2 b^2/(b^2-s_1^2c^2-a^2) & s_1 \\ s_1 s_2 a^2/(b^2+s_2^2c^2-a^2) & 1-s_2^2/2 & s_2 \\ -s_1 & -s_2 & 1-s_1^2/2-s_2^2/2 \end{pmatrix}, \qquad (4.14)
$$

and:

$$
V_R^\dagger = \begin{pmatrix} 1 & s_1 s_2 ab/(b^2-a^2) & -s_1 a/c \\ -s_1 s_2 ab/(b^2-a^2) & 1 & -s_2 b/c \\ s_1 a/c & s_2 b/c & 1 \end{pmatrix}. \qquad (4.15)
$$

Hence:

$$V_{CKM} = \begin{pmatrix} c_L & -s_L & 0 \\ s_L & c_L & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_L, \tag{4.16}$$

where α is the phase transferred from m'_{db}.

Comparing the above with the known values of V_{CKM} [10], we obtain

$$s_1 = 0.00886, \quad s_2 = 0.0405, \quad s_L = -0.2253, \quad e^{i\alpha} = -0.9215 + i0.3884, \tag{4.17}$$

with $m_d = m'_d$, $m_s = m'_s$, $m_b = m'_b$ to a very good approximation.

4.3 Scalar sector of Model A

In addition to $\Phi_{1,2}$, we add a scalar singlet:

$$\sigma \sim (1, 1, 0; 1), \tag{4.18}$$

then the Higgs potential containing $\Phi_{1,2}$ and σ is given by:

$$
\begin{aligned}
V =& m_1^2\Phi_1^\dagger\Phi_1 + m_2^2\Phi_2^\dagger\Phi_2 + m_3^2\bar{\sigma}\sigma + [\mu\sigma\Phi_2^\dagger\Phi_1 + H.c.] \\
&+ \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \frac{1}{2}\lambda_3(\bar{\sigma}\sigma)^2 + \lambda_{12}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\
&+ \lambda_{12}'(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \lambda_{13}(\Phi_1^\dagger\Phi_1)(\bar{\sigma}\sigma) + \lambda_{23}(\Phi_2^\dagger\Phi_2)(\bar{\sigma}\sigma). \qquad (4.19)
\end{aligned}
$$

Let $\langle\phi_{1,2}^0\rangle = v_{1,2}$ and $\langle\sigma\rangle = u$, then the minimum of V is determined by:

$$0 = v_1(m_1^2 + \lambda_1 v_1^2 + (\lambda_{12} + \lambda_{12}')v_2^2 + \lambda_{13}u^2) + \mu v_2 u, \qquad (4.20)$$

$$0 = v_2(m_2^2 + \lambda_2 v_2^2 + (\lambda_{12} + \lambda_{12}')v_1^2 + \lambda_{23}u^2) + \mu v_1 u, \qquad (4.21)$$

$$0 = u(m_3^2 + \lambda_3 u^2 + \lambda_{13}v_1^2 + \lambda_{23}v_2^2) + \mu v_1 v_2. \qquad (4.22)$$

For m_2^2 large and positive, a solution exists with $v_2^2 << v_1^2 << u^2$, i.e.

$$u^2 \simeq \frac{-m_3^2}{\lambda_3}, \quad v_1^2 \simeq \frac{-m_1^2 - \lambda_{13}u^2}{\lambda_1}, \quad v_2 \simeq \frac{-\mu v_1 u}{m_2^2 + \lambda_{23}u^2}. \qquad (4.23)$$

Hence the scalar particle spectrum of Model A consists of a Higgs boson h very much like that of the SM with $m_h^2 \simeq 2\lambda_1 v_1^2$, a heavy Higgs boson which breaks $U(1)_F$ with $m_\sigma^2 \simeq 2\lambda_3 u^2$, and a heavy scalar doublet very much like Φ_2 with $m^2(\phi_2^+, \phi_2^0) \simeq m_2^2 + \lambda_{23}u^2$.

4.4 Gauge sector of Model A

With the scalar structure already considered, the $Z - Z_F$ mass-squared matrix is given by:

$$\mathcal{M}^2_{Z,Z_F} = \begin{pmatrix} g_Z^2(v_1^2+v_2^2)/4 & -g_Z g_F v_2^2/2 \\ -g_Z g_F v_2^2/2 & g_F^2(u^2+v_2^2) \end{pmatrix}. \tag{4.24}$$

The $Z - Z_F$ mixing is then $(g_Z/2g_F)(v_2^2/u^2)$. For $v_2 \sim 10$ GeV and $u \sim 1$ TeV, this is about 10^{-4}, well within the experimentally allowed range.

Since Z_F couples to quarks and leptons according to $n_{1,2,3}$ and $n'_{1,2,3}$, its branching fractions to e^-e^+ and $\mu^-\mu^+$ are given by $2{n'_{1,2}}^2/(12\sum n_i^2 + 3\sum {n'_i}^2)$. Since $n'_1 = 0$, we need consider only the branching fraction $Z_F \to \mu^-\mu^+$ to compare against data. For Model A, it is about 2/21. The $c_{u,d}$ coefficients used in the experimental search [12, 13] of Z_F are then:

$$c_u = c_d = 2g_F^2(2/21). \tag{4.25}$$

For $g_F = 0.13$, a lower bound of about 4.0 TeV on m_{Z_F} is obtained from the Large Hadron Collider (LHC) based on the preliminary 13 TeV data by comparison with the published data from the 7 and 8 TeV runs. Note however that if $Z_F \to e^-e^+$ is ever observed, this particular model is ruled out.

4.5 Flavor-changing interactions

Whereas the SM Z boson does not mediate any flavor-changing interactions, the heavy Z_F does because it distinguishes families. For quarks,

$$\mathcal{L}_{Z_F} = g_F Z_F^\mu(\bar{u}'\gamma_\mu u' + \bar{c}'\gamma_\mu c' + \bar{d}'\gamma_\mu d' + \bar{s}'\gamma_\mu s'). \tag{4.26}$$

Using Eqs. (4.14) and (4.15) to express the above in terms of mass eigenstates for the d sector, and keeping only the leading flavor-changing terms, we find:

$$\mathcal{L}'_{Z_F} = g_F Z_F^\mu[s_1(\bar{d}_L\gamma_\mu b_L + \bar{b}_L\gamma_\mu d_L) + s_2(\bar{s}_L\gamma_\mu b_L + \bar{b}_L\gamma_\mu s_L) - s_1 s_2(\bar{d}_L\gamma_\mu s_L + \bar{s}_L\gamma_\mu d_L)]. \tag{4.27}$$

From the experimental values of the $B^0 - \bar{B}^0$, $B_S^0 - \bar{B}_S^0$, and $K_L - K_S$ mass differences, severe constraints on $g_F^2/m_{Z_F}^2$ are obtained, coming from the operators:

$$(\bar{d}_L\gamma_\mu b_L)^2 + H.c., \quad (\bar{s}_L\gamma_\mu b_L)^2 + H.c., \quad (\bar{d}_L\gamma_\mu s_L)^2 + H.c. \tag{4.28}$$

respectively. Using typical values of quark masses and hadronic decay and bag parameters [58], we estimate the various Wilson coefficients to find their contributions as follows:

$$\Delta M_B = 4.5 \times 10^{-2} \; s_1^2 (g_F^2/m_{Z_F}^2) \text{ GeV}^3, \tag{4.29}$$

$$\Delta M_{B_s} = 6.4 \times 10^{-2} \; s_2^2 (g_F^2/m_{Z_F}^2) \text{ GeV}^3, \tag{4.30}$$

$$\Delta M_K = 1.9 \times 10^{-3} \; s_1^2 s_2^2 (g_F^2/m_{Z_F}^2) \text{ GeV}^3. \tag{4.31}$$

Using Eq. (4.17) and assuming that the above contributions are no more than 10% of their experimental values [10], we find the lower limits on m_{Z_F}/g_F to be 10.2, 9.5, 0.84 TeV respectively. This is easily satisfied for $m_{Z_F} > 4.0$ TeV with $g_F = 0.13$ from the LHC bound discussed in the previous section.

In the scalar sector, since $\Phi_{1,2}$ both contribute to $\mathcal{M}_d$, the neutral scalar field orthogonal to the SM Higgs field will also mediate flavor-changing interactions. The Yukawa interactions are:

$$\mathcal{L}_Y = \frac{h_1}{\sqrt{2}v_1}(m'_d \bar{d}'_L d'_R + m'_s \bar{s}'_L s'_R + m'_b \bar{b}'_L b'_R) + \frac{h_2}{\sqrt{2}v_2}(m'_{db} \bar{d}'_L b'_R + m'_{sb} \bar{s}'_L b'_R). \tag{4.32}$$

Extracting again the leading flavor-changing terms, we obtain:

$$\mathcal{L}'_Y = \left(\frac{h_2}{\sqrt{2}v_2} - \frac{h_1}{\sqrt{2}v_1} \right) (s_1 m_b \bar{d}_L b_R + s_2 m_b \bar{s}_L b_R - s_1 s_2 m_s \bar{d}_L s_R$$
$$- s_1 s_2 m_d \bar{s}_L d_R - s_1 s_2^2 m_d \bar{b}_L d_R - s_2^3 m_s \bar{b}_L s_R), \tag{4.33}$$

where the physical scalar $(v_1h_2 - v_2h_1)/\sqrt{v_1^2 + v_2^2} = H + iA$ is a complex field, with $m_H \simeq m_A$.

Assuming negligible mixing between H or A with the SM h (identified as the 125 GeV particle observed at the LHC), we consider the following effective operators [59]:

$$\frac{s_1^2 m_b^2}{8v_2^2}\left(\frac{1}{m_H^2} - \frac{1}{m_A^2}\right)(\bar{d}_L b_R)^2 - \frac{s_1^2 s_2^2 m_b m_d}{4v_2^2}\left(\frac{1}{m_H^2} + \frac{1}{m_A^2}\right)(\bar{d}_L b_R)(\bar{d_R} b_L) + H.c., \tag{4.34}$$

$$\frac{s_2^2 m_b^2}{8v_2^2}\left(\frac{1}{m_H^2} - \frac{1}{m_A^2}\right)(\bar{s}_L b_R)^2 - \frac{s_2^4 m_b m_s}{4v_2^2}\left(\frac{1}{m_H^2} + \frac{1}{m_A^2}\right)(\bar{s}_L b_R)(\bar{s_R} b_L) + H.c., \tag{4.35}$$

$$\frac{s_1^2 s_2^2 m_s^2}{8v_2^2}\left(\frac{1}{m_H^2} - \frac{1}{m_A^2}\right)(\bar{d}_L s_R)^2 - \frac{s_1^2 s_2^2 m_s m_d}{4v_2^2}\left(\frac{1}{m_H^2} + \frac{1}{m_A^2}\right)(\bar{d}_L s_R)(\bar{d_R} s_L) + H.c. \tag{4.36}$$

The upper bounds on $(1/v_2^2)[(1/m_H^2) - (1/m_A^2)]$ from $\Delta M_B, \Delta M_{B_s}, \Delta M_K$ are then

$$(4.5 \times 10^{-9},\ 5.3 \times 10^{-9},\ 4.5 \times 10^{-3})\ \text{GeV}^{-4}, \tag{4.37}$$

respectively, whereas those on $(1/v_2^2)[(1/m_H^2) + (1/m_A^2)]$ are

$$(1.4 \times 10^{-4},\ 1.7 \times 10^{-5},\ 8.0 \times 10^{-5})\ \text{GeV}^{-4}. \tag{4.38}$$

For $v_2 = 10$ GeV, these are easily satisfied with for example $m_H = 500$ GeV and $m_A = 520$ GeV.

4.6 Lepton sector of Model A

With the chosen $U(1)_F$ charges $(0, -2, -4)$ of Table 4.3, the charged-lepton and Dirac neutrino mass matrices ($\mathcal{M}_l$ and $\mathcal{M}_D$) are both diagonal. As for the 3×3 Majorana mass matrix $\mathcal{M}_R$ of ν_R, it depends on the choice of scalar singlets which break $U(1)_F$. We have already used $\sigma \sim 1$ [see Eq. (4.18)] to induce a small v_2 [see Eq. (4.23)]. Call that σ_1 and add $\sigma_{2,4} \sim 2, 4$, with vacuum expectation values $u_{1,2,4}$ respectively. Then

$$\mathcal{M}_R = \begin{pmatrix} M_0 & M_1 & M_2 \\ M_1 & M_3 & 0 \\ M_2 & 0 & 0 \end{pmatrix}, \tag{4.39}$$

where M_0 is an allowed invariant mas term, M_1 comes from u_2, and $M_{2,3}$ from u_4. The seesaw neutrino mass matrix is then

$$\mathcal{M}_\nu = \mathcal{M}_D \mathcal{M}_R^{-1} \mathcal{M}_D^T = \begin{pmatrix} 0 & 0 & a \\ 0 & b & c \\ a & c & d \end{pmatrix}, \tag{4.40}$$

where the two texture zeros appear because of the form of $\mathcal{M}_R$ and $\mathcal{M}_D$ being diagonal [60]. This form is known to be suitable for a best fit [61] to current neutrino-oscillation data with normal ordering of neutrino masses.

4.7 Basic structure of Model B

The quark structure of Model B is basically the same as that of Model A, with the second Higgs doublet now having two units of $U(1)_F$ charge, i.e.

$$\Phi_2 = (\phi_2^+, \phi_2^0) \sim (1, 2, 1/2; 2). \tag{4.41}$$

Hence $\sigma_2 \sim (1, 1, 0; 2)$ is needed for the $\sigma_2 \Phi_2^\dagger \Phi_1$ term in Eq. (17).

In the gauge sector, again $Z_F \to e^- e^+$ is zero, and the branching fraction $Z_F \to \mu^- \mu^+$ is now 2/51. The $c_{u,d}$ coefficients are then

$$c_u = c_d = 2g_F^2(2/51). \tag{4.42}$$

For the same choice of $g_F = 0.13$ for Model A, the present experimental lower bound from LHC data is reduced from 4.0 TeV to 3.7 TeV. For quarks,

$$\mathcal{L}_{Z_F} = g_F Z_F^\mu (\bar{u}'\gamma_\mu u' + \bar{c}'\gamma_\mu c' - \bar{t}'\gamma_\mu t' + \bar{d}'\gamma_\mu d' + \bar{s}'\gamma_\mu s' - \bar{b}'\gamma_\mu b'). \tag{4.43}$$

Using Eqs. (12) and (13) to express the above in terms of mass eigenstates for the d sector, and keeping only the leading flavor-changing terms, we find:

$$\mathcal{L}'_{Z_F} = 2g_F Z_F^\mu [-s_1(\bar{d}_L\gamma_\mu b_L + \bar{b}_L\gamma_\mu d_L) - s_2(\bar{s}_L\gamma_\mu b_L + \bar{b}_L\gamma_\mu s_L) + s_1 s_2(\bar{d}_L\gamma_\mu s_L + \bar{s}_L\gamma_\mu d_L)]. \tag{4.44}$$

This differs from Eq. (4.25) only by an overall factor of -2. As for the scalar sector, Eqs. (4.32) and (4.33) remain the same. Altogether, this means that Eqs. (4.9) to (4.17) are also valid in Model B.

4.8 Lepton sector of Model B

With the chosen $U(1)_F$ charges $(0, -1, -2)$ of Table 4.3, the charged-lepton and Dirac neutrino mass matrices are given by

$$\mathcal{M}_l = \begin{pmatrix} m'_e & 0 & m'_{e\tau} \\ 0 & m_\mu & 0 \\ 0 & 0 & m'_\tau \end{pmatrix}, \quad \mathcal{M}_D = \begin{pmatrix} m'_1 & 0 & 0 \\ 0 & m'_2 & 0 \\ m'_{31} & 0 & m'_3 \end{pmatrix}. \tag{4.45}$$

Using the scalar singlets $\sigma_1 \sim 1$ as well σ_2, the ν_R Majorana mass matrix is again given by Eq. (4.39). Now even though $\mathcal{M}_D$ is not diagonal, Eq. (4.40) is still obtained, thereby guaranteeing a best fit to current neutrino-oscillation data. The difference from Model A is the presence of $\tau - e$ transitions from the nondiagonal $\mathcal{M}_l$. However, for $m'_{e\tau}/m'_\tau < 0.1$, the branching fraction of $\tau \to e\mu^-\mu^+$ is less than 2×10^{-11}, far below the current bound of 4.1×10^{-8}.

4.9 Application to LHC anomalies

Whereas Z_F also mediates $b \to s\mu^-\mu^+$, its effect is too small in Models A and B to explain the tentative LHC observations of $B \to K^*\mu^-\mu^+$ and the ratio of $B^+ \to K^+\mu^-\mu^+$ to $B^+ \to K^+e^-e^+$ [62]. The reason is the stringent bound on m_{Z_F} from LHC data as a function of g_F through the parameters $c_{u,d}$ of Eqs. (4.25) and (4.42). Suppose we take $n_{1,2,3} = (0, 0, 1)$ and $n'_{1,2,3} = (0, -3, 0)$, then Z_F couples to only $\mu^-\mu^+$ and $b'\bar{b}'$, thus allowing for $b - s$ mixing, but $c_{u,d} = 0$. This evades the direct LHC bound, and may be used to explain the B anomalies if they persist. Of course, Eqs. (4.29) to (4.31) still hold, and a full analysis of the detailed structure of $B \to K^*\mu^-\mu^+$ will be required.

4.10 Conclusion

We have generalized the $B - L$ symmetry as a gauge $U(1)_F$ extension of the standard model, where quarks and leptons of each family may transform differently. We have considered two new examples (A and B), each with two Higgs doublets and restricted quark mass matrices consistent with data. The new Z_F gauge boson couples differently to each quark and lepton family, and is constrained by present data to be heavier than about 4 TeV if $g_F = 0.13$. Future data may reveal just such a Z_F belonging to this class of models. Flavor-changing interactions are suitably suppressed by the assignments of quarks and leptons under $U(1)_F$. In the leptonic sector, with the addition of a minimal set of Higgs singlets, a Majorana neutrino mass matrix of two texture zeros may be obtained, leading to a best fit of neutrino-oscillation data with normal ordering of neutrino masses.

Chapter 5

Phenomenology of the Utilitarian Supersymmetric Standard Model

Since the announcement [63, 64] by the ATLAS and CMS Collaborations at the Large Hadron Collider (LHC) of a diphoton excess around 750 GeV, numerous papers [65] have appeared explaining its presence or discussing its implications. In August of 2016, the 750 GeV excess was not confirmed indicating this as a statistical fluctuation [66]. In this chapter, we study the phenomenology of a model proposed in 2002 [67], which has exactly all the necessary and sufficient particles and interactions for this purpose. They were of course there for solving other issues in particle physics. However, the "observed" diphoton excess may well be a first revelation [68] of this model, including its connection to dark matter.

This 2002 model extends the supersymmetric standard model by a new $U(1)_X$ gauge symmetry. It replaces the μ term with a singlet scalar superfield which also couples to heavy color-triplet superfields which are electroweak singlets. The latter are not *ad hoc* inventions, but are necessary for the cancellation of axial-vector anomalies. It was shown in Ref. [67] how this was accomplished by the remarkable exact factorization of the sum of eleven cubic terms, resulting in two generic classes of solutions [69]. Both are able to enforce the conservation of baryon number and lepton number up to dimension-five terms. As such, the scalar singlet and the vectorlike quarks are indispensible ingredients of this 2002 model. They are thus naturally suited for explaining the observed diphoton excess. In 2010 [70], a specific version was discussed, which will be the subject of this paper as well. An important byproduct of this study is the discovery of relaxed supersymmetric constraints on the Higgs boson's mass of 125 GeV. This is independent of the diphoton excess' lack of confirmation.

5.1 Model

Consider the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ with the particle content of Ref. [67]. For $n_1 = 0$ and $n_4 = 1/3$ in Solution (A), the various superfields transform as shown in Table 1. There are three copies of $Q, u^c, d^c, L, e^c, N^c, S_1, S_2$; two copies of U, U^c, S_3; and one copy of ϕ_1, ϕ_2, D, D^c.

The only allowed terms of the superpotential are thus trilinear, i.e.

$$Qu^c\phi_2, \quad Qd^c\phi_1, \quad Le^c\phi_1, \quad LN^c\phi_2, \quad S_3\phi_1\phi_2, \quad N^cN^cS_1, \tag{5.1}$$

$$S_3UU^c, \quad S_3DD^c, \quad u^cN^cU, \quad u^ce^cD, \quad d^cN^cD, \quad QLD^c, \quad S_1S_2S_3. \tag{5.2}$$

Superfield	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$
$Q = (u, d)$	3	2	1/6	0
u^c	3^*	1	$-2/3$	1/2
d^c	3^*	1	1/3	1/2
$L = (\nu, e)$	1	2	$-1/2$	1/3
e^c	1	1	1	1/6
N^c	1	1	0	1/6
ϕ_1	1	2	$-1/2$	$-1/2$
ϕ_2	1	2	1/2	$-1/2$
S_1	1	1	0	$-1/3$
S_2	1	1	0	$-2/3$
S_3	1	1	0	1
U	3	1	2/3	$-2/3$
D	3	1	$-1/3$	$-2/3$
U^c	3^*	1	$-2/3$	$-1/3$
D^c	3^*	1	1/3	$-1/3$

Table 5.1: Particle content of proposed model.

The absence of any bilinear term means that all masses come from soft supersymmetry breaking, thus explaining why the $U(1)_X$ and electroweak symmetry breaking scales are not far from that of supersymmetry breaking. As $S_{1,2,3}$ acquire nonzero vacuum expectation values (VEVs), the exotic (U, U^c) and (D, D^c) fermions obtain Dirac masses from $\langle S_3 \rangle$, which also generates the μ term. The singlet N^c fermion gets a large Majorana mass from $\langle S_1 \rangle$, so that the neutrino ν gets a small seesaw mass in the usual way. The singlet $S_{1,2,3}$ fermions themselves get Majorana masses from their scalar counterparts $\langle S_{1,2,3} \rangle$ through the $S_1 S_2 S_3$ terms. The only massless fields left are the usual quarks and leptons. They then become massive as $\phi^0_{1,2}$ acquire VEVs, as in the minimal supersymmetric standard model (MSSM).

Because of $U(1)_X$, the structure of the superpotential conserves both B and $(-1)^L$, with $B = 1/3$ for Q, U, D, and $B = -1/3$ for u^c, d^c, U^c, D^c; $(-1)^L$ odd for $L, e^c, N^c, U, U^c, D, D^c$, and even for all others. Hence the exotic U, U^c, D, D^c scalars are leptoquarks and decay into ordinary quarks and leptons. The R parity of the MSSM is defined here in the same way, i.e. $R \equiv (-)^{2j+3B+L}$, and is conserved. Note also that the quadrilinear terms $QQQL$ and $u^c u^c d^c e^c$ (allowed in the MSSM) as well as $u^c d^c d^c N^c$ are forbidden by $U(1)_X$. Proton decay is thus strongly suppressed. It may proceed through the quintilinear term $QQQLS_1$ as the S_1 fields acquire VEVs, but this is a dimension-six term in the effective Lagrangian, which is suppressed by two powers of a very large mass, say the Planck mass, and may safely be allowed.

5.2 Gauge Sector

The new Z_X gauge boson of this model becomes massive through $\langle S_{1,2,3} \rangle = u_{1,2,3}$, whereas $\langle \phi^0_{1,2} \rangle = v_{1,2}$ contribute to both Z and Z_X. The resulting 2×2 mass-squared matrix is given by [71]

$$\mathcal{M}^2_{Z,Z_X} = \begin{pmatrix} (1/2)g_Z^2(v_1^2 + v_2^2) & (1/2)g_Z g_X(v_2^2 - v_1^2) \\ (1/2)g_Z g_X(v_2^2 - v_1^2) & 2g_X^2[(1/9)u_1^2 + (4/9)u_2^2 + u_3^2 + (1/4)(v_1^2 + v_2^2)] \end{pmatrix}. \quad (5.3)$$

Since precision electroweak measurements require $Z - Z_X$ mixing to be very small [72], $v_1 = v_2$, i.e. $\tan\beta = 1$, is preferred. With the 2012 discovery [5, 6] of the 125 GeV particle, and identified as the one Higgs boson h responsible for electroweak symmetry breaking, $\tan\beta = 1$ is not compatible with the MSSM, but is perfectly consistent here, as shown already in Ref. [70] and in more detail in the next section.

Consider the decay of Z_X to the usual quarks and leptons. Each fermionic partial width is given by

$$\Gamma(Z_X \to \bar{f}f) = \frac{g_X^2 M_{Z_X}}{24\pi}[c_L^2 + c_R^2], \tag{5.4}$$

where $c_{L,R}$ can be read off under $U(1)_X$ from Table 1. Thus

$$\frac{\Gamma(Z_X \to \bar{t}t)}{\Gamma(Z_X \to \mu^+\mu^-)} = \frac{\Gamma(Z_X \to \bar{b}b)}{\Gamma(Z_X \to \mu^+\mu^-)} = \frac{27}{5}. \tag{5.5}$$

This will serve to distinguish it from other Z' models [73].

At the LHC, limits on the mass of any Z' boson depend on its production by u and d quarks times its branching fraction to e^-e^+ and $\mu^-\mu^+$. In a general analysis of Z' couplings to u and d quarks,

$$\mathcal{L} = \frac{g'}{2} Z'_\mu \bar{f}\gamma_\mu(g_V - g_A\gamma_5)f, \tag{5.6}$$

where $f = u, d$. The c_u, c_d coefficients used in an experimental search [12, 13] of Z' are then given by

$$c_u = \frac{g'^2}{2}[(g_V^u)^2 + (g_A^u)^2]B(Z' \to l^-l^+), \quad c_d = \frac{g'^2}{2}[(g_V^d)^2 + (g_A^d)^2]B(Z' \to l^-l^+), \tag{5.7}$$

where $l = e, \mu$. In this model

$$c_u = c_d = \frac{g_X^2}{4} B(Z' \to l^- l^+). \tag{5.8}$$

To estimate $B(Z' \to l^- l^+)$, we assume Z_X decays to all SM quarks and leptons with effective zero mass, all the scalar leptons with effective mass of 500 GeV, all the scalar quarks with effective mass of 800 GeV, the exotic U, D fermions with effective mass of 400 GeV (needed to explain the diphoton excess), and one pseudo-Dirac fermion from combining $\tilde{S}_{1,2}$ (the dark matter candidate to be discussed) with mass of 200 GeV. We find $B(Z' \to l^- l^+) = 0.04$, and for $g_X = 0.53$, a lower bound of 2.85 TeV on m_{Z_X} is obtained from the LHC data based on the 7 and 8 TeV runs.

5.3 Scalar Sector

Consider the scalar potential consisting of $\phi_{1,2}$ and $S_{1,2,3}$. Whereas there are 2 copies of S_3 and 3 copies each of $S_{1,2}$, we can choose one copy each to be the one with nonzero vacuum expectation value. We then assume that the superpotential linking them is given by

$$W = f S_3 \phi_1 \phi_2 + h S_3 S_2 S_1, \tag{5.9}$$

which is of course missing some terms. We have neglected them for simplicity. Its contribution to the scalar potential is

$$V_F = f^2(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2)S_3^*S_3 + h^2(S_1^*S_1 + S_2^*S_2)S_3^*S_3 + |f\Phi_1^\dagger\Phi_2 + hS_1S_2|^2, \tag{5.10}$$

where ϕ_1 has been redefined to $\Phi_1 = (\phi_1^+, \phi_1^0)$. The gauge contribution is

$$\begin{aligned} V_D &= \frac{1}{8}g_2^2[(\Phi_1^\dagger\Phi_1)^2 + (\Phi_2^\dagger\Phi_2)^2 + 2(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - 4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)] \\ &+ \frac{1}{8}g_1^2[-(\Phi_1^\dagger\Phi_1) + (\Phi_2^\dagger\Phi_2)]^2 \\ &+ \frac{1}{2}g_X^2\left[-\frac{1}{2}\Phi_1^\dagger\Phi_1 - \frac{1}{2}\Phi_2^\dagger\Phi_2 - \frac{1}{3}S_1^*S_1 - \frac{2}{3}S_2^*S_2 + S_3^*S_3\right]^2. \end{aligned} \tag{5.11}$$

The soft supersymmetry-breaking terms are

$$\begin{aligned} V_{soft} &= \mu_1^2\Phi_1^\dagger\Phi_1 + \mu_2^2\Phi_2^\dagger\Phi_2 + m_3^2S_3^*S_3 + m_2^2S_2^*S_2 + m_1^2S_1^*S_1 \\ &+ [m_{12}S_2^*S_1^2 + A_f fS_3\Phi_1^\dagger\Phi_2 + A_h hS_3S_2S_1 + H.c.]. \end{aligned} \tag{5.12}$$

In addition, there is an important one-loop contribution from the t quark and its supersymmetric scalar partners:

$$V_t = \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2, \tag{5.13}$$

where

$$\lambda_2 = \frac{6G_F^2 m_t^4}{\pi^2} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) \tag{5.14}$$

is the well-known correction which allows the Higgs mass to exceed m_Z.

Let $\langle\phi^0_{1,2}\rangle = v_{1,2}$ and $\langle S_{1,2,3}\rangle = u_{1,2,3}$, we study the conditions for obtaining a minimum of the scalar potential $V = V_F + V_D + V_{soft} + V_t$. We look for the solution $v_1 = v_2 = v$ which implies that

$$\mu_1^2 = \mu_2^2 + \lambda_2 v^2 \tag{5.15}$$

$$0 = \mu_1^2 + A_f f u_3 + f^2(u_3^2 + v^2) + \frac{1}{2} g_X^2 \left(v^2 + \frac{1}{3}u_1^2 + \frac{2}{3}u_2^2 - u_3^2\right) + fhu_1u_2. \tag{5.16}$$

We then require that this solution does not mix the $Re(\phi_{1,2})$ and $Re(S_{1,2,3})$ sectors. The additional conditions are

$$0 = A_f f + (2f^2 - g_X^2)u_3, \tag{5.17}$$

$$0 = \frac{1}{3} g_X^2 u_1 + fhu_2, \tag{5.18}$$

$$0 = \frac{2}{3} g_X^2 u_2 + fhu_1. \tag{5.19}$$

Hence

$$u_1 = \sqrt{2}u_2, \quad fh = \frac{-\sqrt{2}g_X^2}{3}. \tag{5.20}$$

The 2×2 mass-squared matrix spanning $[\sqrt{2}Re(\phi_1^0), \sqrt{2}Re(\phi_2^0)]$ is

$$\mathcal{M}_\phi^2 = \begin{pmatrix} \kappa + g_X^2 v^2/2 & -\kappa + g_X^2 v^2/2 + 2f^2 v^2 \\ -\kappa + g_X^2 v^2/2 + 2f^2 v^2 & \kappa + g_X^2 v^2/2 + 2\lambda_2 v^2 \end{pmatrix}, \tag{5.21}$$

where

$$\kappa = (2f^2 - g_X^2)u_3^2 + \frac{2}{3}g_X^2 u_2^2 + \frac{1}{2}(g_1^2 + g_2^2)v^2. \tag{5.22}$$

For $\lambda_2 v^2 << \kappa$, the Higgs boson $h \simeq Re(\phi_1^0 + \phi_2^0)$ has a mass given by

$$m_h^2 \simeq \left(g_X^2 + 2f^2 + \lambda_2\right) v^2, \tag{5.23}$$

whereas its heavy counterpart $H \simeq Re(-\phi_1^0 + \phi_2^0)$ has a mass given by

$$m_H^2 \simeq (4f^2 - 2g_X^2)u_3^2 + \frac{4}{3}g_X^2 u_2^2 + (g_1^2 + g_2^2 - 2f^2 + \lambda_2)v^2. \tag{5.24}$$

The conditions for obtaining the minimum of V in the $S_{1,2,3}$ directions are

$$0 = m_3^2 + g_X^2 u_3^2 + \left(3h^2 - \frac{4}{3}g_X^2\right) u_2^2 + \frac{\sqrt{2}A_h h u_2^2}{u_3}, \tag{5.25}$$

$$0 = m_2^2 + 2m_{12}u_2 + \left(2h^2 + \frac{8}{9}g_X^2\right) u_2^2 + \left(h^2 - \frac{2}{3}g_X^2\right) u_3^2 + \sqrt{2}A_h h u_3, \tag{5.26}$$

$$0 = m_1^2 + 2m_{12}u_2 + \left(h^2 + \frac{4}{9}g_X^2\right) u_2^2 + \left(h^2 - \frac{1}{3}g_X^2\right) u_3^2 + \frac{1}{\sqrt{2}}A_h h u_3. \tag{5.27}$$

The 3×3 mass-squared matrix spanning $[\sqrt{2}Re(S_1), \sqrt{2}Re(S_2), \sqrt{2}Re(S_3)]$ is given by

$$m_{11}^2 = \frac{4}{9}g_X^2 u_2^2 - \frac{1}{\sqrt{2}}A_h h u_3 + \frac{1}{3}g_X^2 v^2, \quad m_{22}^2 = 2m_{11}^2 - 2m_{12}u_2, \tag{5.28}$$

$$m_{12}^2 = m_{21}^2 = 2\sqrt{2}m_{12}u_2 + A_h h u_3 + 2\sqrt{2}\left(h^2 + \frac{2}{9}g_X^2\right)u_2^2 - \frac{\sqrt{2}}{3}g_X^2 v^2, \tag{5.29}$$

$$m_{33}^2 = 2g_X^2 u_3^2 - \sqrt{2}A_h h u_2^2/u_3 + (2f^2 - g_X^2)v^2, \tag{5.30}$$

$$m_{13}^2 = m_{31}^2 = A_h h u_2 + 2\sqrt{2}\left(h^2 - \frac{1}{3}g_X^2\right)u_3 u_2, \tag{5.31}$$

$$m_{23}^2 = m_{32}^2 = \sqrt{2}A_h h u_2 + 2\left(h^2 - \frac{2}{3}g_X^2\right)u_3 u_2. \tag{5.32}$$

The 5×5 mass-squared matrix spanning $[\sqrt{2}Im(\phi_1^0), \sqrt{2}Im(\phi_2^0), \sqrt{2}Im(S_1), \sqrt{2}Im(S_2), \sqrt{2}Im(S_3)]$ has two zero eigenvalues, corresponding to the would-be Goldstone modes

$$(1,1,0,0,0) \text{ and } (v/2, -v/2, -\sqrt{2}u_2/3, -2u_2/3, u_3), \tag{5.33}$$

for the Z and Z_X gauge bosons. One exact mass eigenstate is $A_{12} = [2Im(S_1) - \sqrt{2}Im(S_2)]/\sqrt{3}$ with mass given by

$$m_{A_{12}}^2 = -6m_{12}u_2. \tag{5.34}$$

Assuming that $v^2 << u_{2,3}^2$, the other two mass eigenstates are $A \simeq -Im(\phi_1^0) + Im(\phi_2^0)$ and $A_S \simeq [u_3 Im(S_1) + \sqrt{2}u_3 Im(S_2) + \sqrt{2}u_2 Im(S_3)]/\sqrt{u_2^2 + 3u_3^2/2}$ with masses given by

$$m_A^2 \simeq (4f^2 - 2g_X^2)u_3^2 + \frac{4}{3}g_X^2 u_2^2, \tag{5.35}$$

$$m_{A_S}^2 \simeq -A_h h \left(\frac{3u_3}{\sqrt{2}} + \frac{\sqrt{2}u_2^2}{u_3} \right), \tag{5.36}$$

respectively. The charged scalar $H^\pm = (-\phi_1^\pm + \phi_2^\pm)/\sqrt{2}$ has a mass given by

$$m_{H^\pm}^2 = (4f^2 - 2g_X^2)u_3^2 + \frac{4}{3}g_X^2 u_2^2 + (g_2^2 - 2f^2)v^2. \tag{5.37}$$

5.4 Physical Scalars and Pseudoscalars

In the MSSM without radiative corrections,

$$m_{H^\pm}^2 = m_A^2 + m_W^2, \tag{5.38}$$

$$m_{h,H}^2 = \frac{1}{2}\left(m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2 \cos^2 2\beta} \right), \tag{5.39}$$

where $\tan\beta = v_2/v_1$. For $v_1 = v_2$ as in this model, m_h would be zero. There is of course the important radiative correction from Eq. (5.14), but that alone will not reach 125 GeV. Hence the MSSM requires both large $\tan\beta$ and large radiative correction, but a significant tension remains in accommodating all data. In this model, as Eq. (5.23) shows, $m_h^2 \simeq (g_X^2 + 2f^2 + \lambda_2)v^2$, where $v = 123$ GeV. This is a very interesting and important result, allowing the Higgs boson mass to be determined by the gauge $U(1)_X$ coupling g_X in addition to the Yukawa coupling f which replaces the μ parameter, i.e. $\mu = fu_3$. There is no tension between $m_h = 125$ GeV and the superparticle mass spectrum. Since $\lambda_2 \simeq 0.25$ for $\tilde{m}_t \simeq 1$ TeV, we have the important constraint

$$\sqrt{g_X^2 + 2f^2} \simeq 0.885. \tag{5.40}$$

For illustration, we have already chosen $g_X = 0.53$. Hence $f = 0.5$ and for $u_3 = 2$ TeV, $fu_3 = 1$ TeV is the value of the μ parameter of the MSSM. Let us choose $u_2 = 4$ TeV, then $m_{Z_X} = 2.87$ TeV, which is slightly above the present experimental lower bound of 2.85 TeV using $g_X = 0.53$ discussed earlier.

As for the heavy Higgs doublet, the four components $(H^\pm, H, A)$ are all degenerate in mass, i.e. $m^2 \simeq (4f^2 - 2g_X^2)u_3^2 + (4/3)g_X^2 u_2^2$ up to v^2 corrections. Each mass is then about 2.78 TeV. In more detail, as shown in Eq. (5.37), $m_{H^\pm}^2$ is corrected by $g_2^2 v^2 = m_W^2$ plus a term due to f. As shown in Eq. (5.24), m_H^2 is corrected by $(g_1^2 + g_2^2)v^2 = m_Z^2$ plus a term due to f and λ_2. These are exactly in accordance with Eqs. (5.38) and (5.39).

In the $S_{1,2,3}$ sector, the three physical scalars are mixtures of all three $Re(S_i)$ components, whereas the physical pseudoscalar A_{12} has no $Im(S_3)$ component. Since only S_3 couples to UU^c, DD^c, and $\phi_1\phi_2$, a candidate for the 750 GeV diphoton resonance must have an S_3 component. It could be one of the three scalars or the pseudoscalar A_S, or the other S_3 without VEV. In the following, we will consider the last option, specifically a pseudoscalar χ with a significant component of this other S_3. This allows the χUU^c, χDD^c and $\chi\phi_1\phi_2$ couplings to be independent of the masses of U, D, and the charged higgsino. The other scalars and pseudoscalars are assumed to be much heavier, and yet to be discovered.

5.5 Diphoton Excess

In this model, other than the addition of N^c for seesaw neutrino masses, the only new particles are U, U^c, D, D^c and $S_{1,2,3}$, which are exactly the ingredients needed to explain the diphoton excess at the LHC. The allowed S_3UU^c and S_3DD^c couplings enable the one-loop gluon production of S_3 in analogy to that of h.

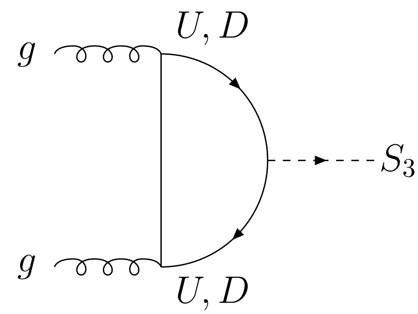

Figure 5.1: One-loop production of S_3 by gluon fusion.

The one-loop decay of S_3 to two photons comes from these couplings as well as $S_3\phi_1\phi_2$.

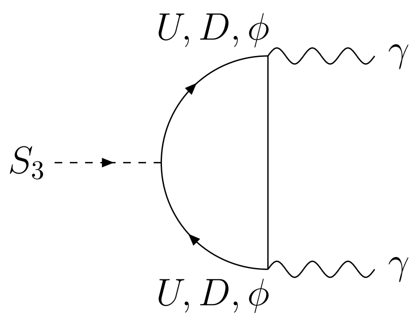

Figure 5.2: One-loop decay of S_3 to two photons.

In addition, the direct $S_1S_2S_3$ couplings enable the decay of S_3 to other final states, including those of the dark sector, which contribute to its total width. The fact that the exotic U, U^c, D, D^c scalars are leptoquarks is also very useful for understanding [74] other possible LHC flavor anomalies. In a nutshell, a desirable comprehensive picture of possible new physics beyond the standard model is encapsulated by this existing model. In the following, we assume that the pseudoscalar χ is the 750 GeV particle, and show how its production and decay are consistent with the present data.

The production cross section through gluon fusion is given by

$$\hat{\sigma}(gg \to \chi) = \frac{\pi^2}{8m_\chi}\Gamma(\chi \to gg)\delta(\hat{s} - m_\chi^2). \tag{5.41}$$

For the LHC at 13 TeV, the diphoton cross section is roughly [75]

$$\sigma(gg \to \chi \to \gamma\gamma) \simeq (100 \text{ pb}) \times (\lambda_g \text{ TeV})^2 \times B(\chi \to \gamma\gamma), \tag{5.42}$$

where λ_g is the effective coupling of χ to two gluons, normalized by

$$\Gamma(\chi \to gg) = \frac{\lambda_g^2}{8\pi} m_\chi^3. \tag{5.43}$$

Let the $\chi\bar{Q}Q$ coupling be f_Q, where Q is a leptoquark fermion, then

$$\lambda_g = \frac{\alpha_s}{\pi m_\chi} \sum_Q f_Q F(m_Q^2/m_\chi^2), \tag{5.44}$$

where [76]

$$F(x) = 2\sqrt{x}\left[\arctan\left(\frac{1}{\sqrt{4x-1}}\right)\right]^2, \tag{5.45}$$

which has the maximum value of $\pi^2/4 = 2.47$ as $x \to 1/4$. Let $f_Q^2/4\pi = 0.21$ and

$F(m_Q^2/m_\chi^2) = 2.0$ (i.e. $m_Q = 380$ GeV) for all $Q = U, U, D$, then $\lambda_g = 0.49$ TeV^{-1}. For the corresponding

$$\Gamma(\chi \to \gamma\gamma) = \frac{\lambda_\gamma^2}{64\pi} m_\chi^3, \tag{5.46}$$

the $\phi^\pm$ higgsino contributes as well as U, D. However, its mass is roughly $fu_3 = 1$ TeV, so $F(x_\phi) = 0.394$, and

$$\lambda_\gamma = \frac{2\alpha}{\pi m_\chi} \sum_\psi N_\psi Q_\psi^2 f_\psi F(x_\psi), \tag{5.47}$$

where $\psi = U, U, D, \phi^\pm$ and N_ψ is the number of copies of ψ. Using $f_\phi^2/4\pi = 0.21$ as well, $\lambda_\gamma = 0.069$ TeV^{-1} is obtained. We then have $\Gamma(\chi \to \gamma\gamma) = 10$ MeV and $\Gamma(\chi \to gg) = 4.0$ GeV.If $B(\chi \to \gamma\gamma) = 2.5 \times 10^{-4}$, then $\sigma = 6$ fb, and the total width of χ is 40 GeV, in good agreement with data [63, 64].

Note the important fact that we have considered 380 GeV for the mass of the leptoquark fermions. If they are leptoquark scalars, then their mass would be constrained by LHC data to be above 1 TeV or so. As fermions, Q has odd R parity, and must decay into the lightest supersymmetric particle, which is discussed in more detail below. We assume 200 GeV for this particle, hence there is no useful bound on m_Q at present.

As mentioned earlier, there are 2 copies of S_3 and 3 copies each of $S_{1,2}$. In addition to the ones with VEVs in their scalar components, there are 5 other superfields. One pair $\tilde{S}_{1,2}$ may form a pseudo-Dirac fermion, and be the lightest particle with odd R parity. It will couple to χ, say with strength f_S which is independent of all other couplings that we have discussed, then the tree-level decay $\chi \to \tilde{S}_1\tilde{S}_2$ dominates the total width of χ and is invisible.

$$\Gamma(\chi \to \tilde{S}_1\tilde{S}_2) = \frac{f_S^2}{8\pi}\sqrt{m_\chi^2 - 4m_S^2}. \tag{5.48}$$

For $m_\chi = 750$ GeV and $m_S = 200$ GeV, we find $\Gamma = 36$ GeV if $f_S = 1.2$. These numbers reinforce our numerical analysis to support the claim that χ is a possible candidate for the 750 GeV diphoton excess. Note also that λ_g and λ_γ have scalar contributions which we have not considered. Adding them will allow us to reduce the fermion contributions we have assumed and still get the same final results.

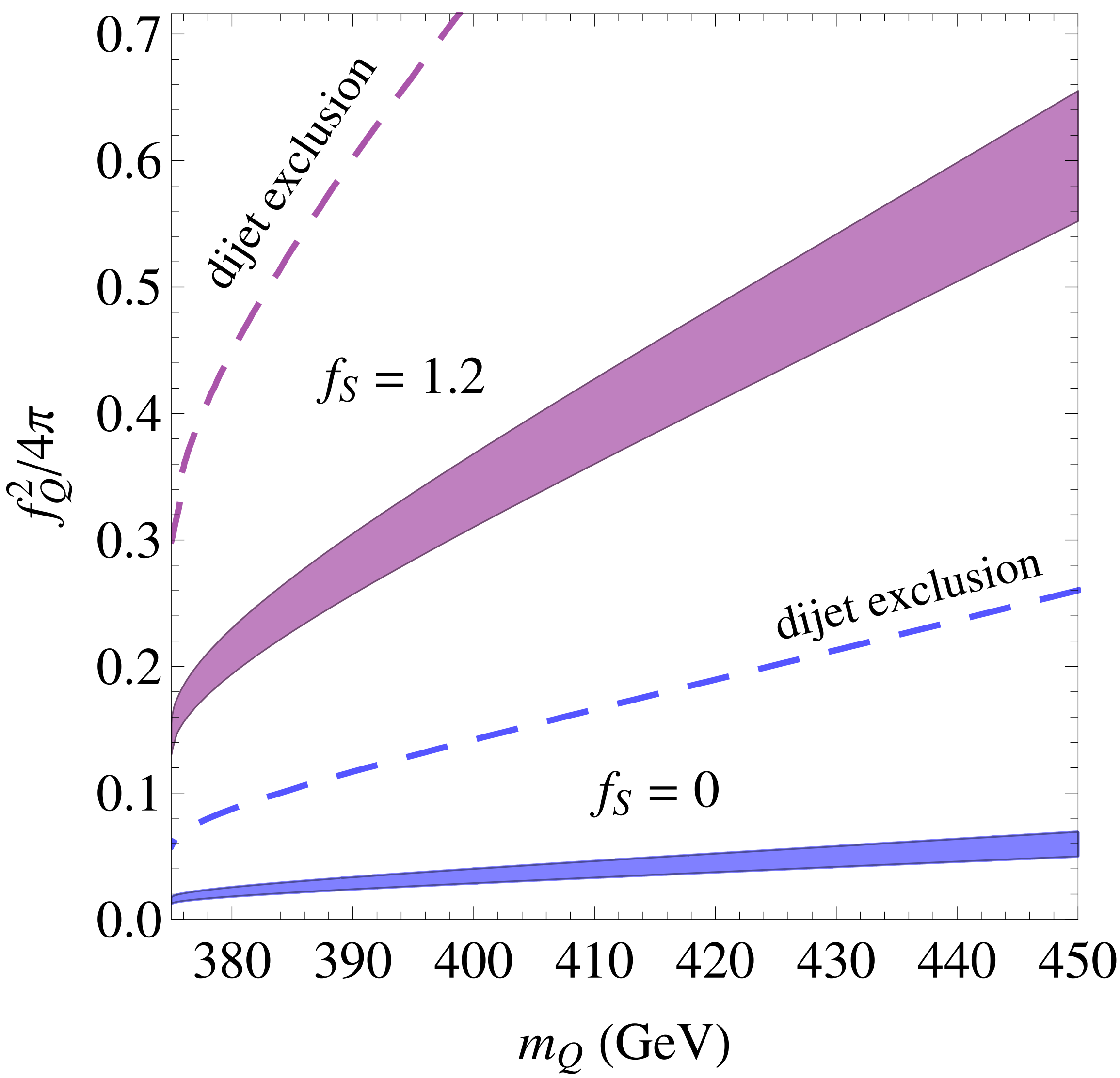

Figure 5.3: Allowed region for diphoton cross section of 6.2 ± 1 fb.

If we disregard the decay to dark matter ($f_S = 0$), then the total width of χ is dominated by $\Gamma(\chi \to gg)$, which is then less than a GeV. Assuming that the cross section for the diphoton resonance is 6.2 ± 1 fb [75], we plot the allowed values of $f_Q^2/4\pi$ versus m_Q for both $f_S = 1.2$ which gives a total width of about 40 GeV for χ, and $f_S = 0$ which requires much smaller values of $f_Q^2/4\pi$. Since χ must also decay into two gluons, we show the diject exclusion upper limits ($\sim$ 2 pb) from the 8 TeV data in each case as well. Our choice of the pseudoscalr χ to be the 750 GeV diphoton resonance is motivated by the necessity of large couplings to U, D leptoquark fermions for explaining the large width of about 40 GeV observed by ATLAS. If we take the evidence of CMS that this width is narrow, then as Fig. 3 shows, we can have much smaller couplings and much greater masses for U, D. In that case, we can use a physical scalar, with mass-squared matrix given in Eqs.(5.28) to (5.32), which is directly associated with the μ term.

5.6 Scalar Neutrino and Neutralino Sectors

In the neutrino sector, the 2×2 mass matrix spanning (ν, N^c) per family is given by the well-known seesaw structure:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix}, \tag{5.49}$$

where m_D comes from v_2 and m_N from u_1.There are two neutral complex scalars with odd R parity per family, i.e. $\tilde{\nu} = (\tilde{\nu}_R + i\tilde{\nu}_I)/\sqrt{2}$ and $\tilde{N}^c = (\tilde{N}^c_R + i\tilde{N}^c_I)/\sqrt{2}$. The 4×4 mass-squared matrix spanning $(\tilde{\nu}_R, \tilde{\nu}_I, \tilde{N}^c_R, \tilde{N}^c_I)$ is given by

$$\mathcal{M}^2_{\tilde{\nu},\tilde{N}^c} = \begin{pmatrix} m^2_{\tilde{\nu}} & 0 & A_D m_D & 0 \\ 0 & m^2_{\tilde{\nu}} & 0 & -A_D m_D \\ A_D m_D & 0 & m^2_{\tilde{N}^c} + A_N m_N & 0 \\ 0 & -A_D m_D & 0 & m^2_{\tilde{N}^c} - A_N m_N \end{pmatrix}. \tag{5.50}$$

In the MSSM, $\tilde{\nu}$ is ruled out as a dark-matter candidate because it interacts elastically with nuclei through the Z boson. Here, the A_N term allows a mass splitting between the real and imaginary parts of the scalar fields, and avoids this elastic-scattering constraint by virtue of kinematics. However, we still assume their masses to be heavier than that of $\tilde{S}_{1,2}$, discussed in the previous section.

In the neutralino sector, in addition to the 4×4 mass matrix of the MSSM spanning $(\tilde{B}, \tilde{W}_3, \tilde{\phi}_1^0, \tilde{\phi}_2^0)$ with the μ parameter replaced by fu_3, i.e.

$$\mathcal{M}_0 = \begin{pmatrix} M_1 & 0 & -g_1 v_1/\sqrt{2} & g_1 v_2/\sqrt{2} \\ 0 & M_2 & g_2 v_1/\sqrt{2} & -g_2 v_2/\sqrt{2} \\ -g_1 v_1/\sqrt{2} & g_2 v_1/\sqrt{2} & 0 & -fu_3 \\ g_1 v_2/\sqrt{2} & -g_2 v_2/\sqrt{2} & -fu_3 & 0 \end{pmatrix}, \tag{5.51}$$

there is also the 4×4 mass matrix spanning $(\tilde{X}, \tilde{S}_3, \tilde{S}_2, \tilde{S}_1)$, i.e.

$$\mathcal{M}_S = \begin{pmatrix} M_X & \sqrt{2} g_X u_3 & -2\sqrt{2} g_X u_2/3 & -\sqrt{2} g_X u_1/3 \\ \sqrt{2} g_X u_3 & 0 & hu_1 & hu_2 \\ -2\sqrt{2} g_X u_2/3 & hu_1 & 0 & hu_3 \\ -\sqrt{2} g_X u_1/3 & hu_2 & hu_3 & 0 \end{pmatrix}. \tag{5.52}$$

The two are connected through the 4×4 matrix:

$$\mathcal{M}_{0S} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -g_x v_1/\sqrt{2} & -f v_2 & 0 & 0 \\ -g_X v_2/\sqrt{2} & -f v_1 & 0 & 0 \end{pmatrix}. \tag{5.53}$$

These neutral fermions are odd under R parity and the lightest could in principle be a dark-matter candidate. To avoid the stringent bounds on dark matter with the MSSM alone, we assume again that all these particles are heavier than $\tilde{S}_{1,2}$, as the dark matter discussed in the previous section.

5.7 Dark Matter

The 5×5 mass matrix spanning the 5 singlet fermions $(\tilde{S}_1, \tilde{S}_2, \tilde{S}_1, \tilde{S}_2, \tilde{S}_3)$, corresponding to superfields with zero VEV for their scalar components, is given by:

$$\mathcal{M}_{\tilde{S}} = \begin{pmatrix} 0 & m_0 & 0 & 0 & m_{13} \\ m_0 & 0 & 0 & 0 & m_{23} \\ 0 & 0 & 0 & M_3 & M_2 \\ 0 & 0 & M_3 & 0 & M_1 \\ m_{13} & m_{23} & M_2 & M_1 & 0 \end{pmatrix}. \tag{5.54}$$

Note that the 4×4 submatrix spanning $(\tilde{S}_1, \tilde{S}_2, \tilde{S}_1, \tilde{S}_2)$ has been diagonalized to form two Dirac fermions. We can choose m_0 to be small, say 200 GeV, and $M_{1,2,3}$ to be large, of order TeV. However, because of the mixing terms m_{13}, m_{23}, the light Dirac fermion gets split into two Majorana fermions, so it should be called a pseudo-Dirac fermion.

The dark matter with odd R parity is the lighter of the two Majorana fermions, call it $\tilde{S}$, contained in the pseudo-Dirac fermion formed out of $\tilde{S}_{1,2}$ as discussed in Sec. 5.6. It couples to the Z_X gauge boson, but in the nonrelativistic limit, its elastic scattering cross section with nuclei through Z_X vanishes because it is Majorana. It also does not couple directly to the Higgs boson h, so its direct detection at underground search experiments is very much suppressed. However, it does couple to A_S which couples also to quarks through the very small mixing of A_S with A. This is further suppressed because it contributes only to the spin-dependent cross section. To obtain a spin-independent cross section at tree level, the constraint of Eqs. (5.17) to (5.19) have to be relaxed so that h mixes with $S_{1,2,3}$.

Let the coupling of h to $\tilde{S}\tilde{S}$ be ϵ, then the effective interaction for elastic scattering of $\tilde{S}$ with nuclei through h is given by

$$\mathcal{L}_{eff} = \frac{\epsilon f_q}{m_h^2} \overline{\tilde{S}} \tilde{S} \bar{q} q, \tag{5.55}$$

where $f_q = m_q/2v = m_q/(246 \text{ GeV})$. The spin-independent direct-detection cross section per nucleon is given by

$$\sigma^{SI} = \frac{4\mu_{DM}^2}{\pi A^2}[\lambda_p Z + (A - Z)\lambda_n]^2, \tag{5.56}$$

where $\mu_{DM} = m_{DM} M_A/(m_{DM} + M_A)$ is the reduced mass of the dark matter. Using [77]

$$\lambda_N = \left[\sum_{u,d,s} f_q^N + \frac{2}{27}\left(1 - \sum_{u,d,s} f_q^N\right)\right] \frac{\epsilon m_N}{(246 \text{ GeV}) m_h^2}, \tag{5.57}$$

with [78]

$$f_u^p = 0.023, \quad f_d^p = 0.032, \quad f_s^p = 0.020, \tag{5.58}$$

$$f_u^n = 0.017, \quad f_d^n = 0.041, \quad f_s^n = 0.020, \tag{5.59}$$

we find $\lambda_p \simeq 3.50 \times 10^{-8}$ GeV^{-2}, and $\lambda_n \simeq 3.57 \times 10^{-8}$ GeV^{-2}. Using $A = 131$, $Z = 54$, and $M_A = 130.9$ atomic mass units for the LUX experiment [14], and $m_{DM} = 200$ GeV, we find for the upper limit of $\sigma^{SI} < 1.5 \times 10^{-45}$ cm^2, the bound $\epsilon < 6.5 \times 10^{-4}$.

We have already invoked the $\chi \tilde{S}_1 \tilde{S}_2$ coupling to obtain a large invisible width for χ. Consider now the fermion counterpart of χ, call it $\tilde{S}'$, and the scalar counterparts of $\tilde{S}_{1,2}$, then the couplings $\tilde{S}'\tilde{S}_1 S_2$ and $\tilde{S}'\tilde{S}_2 S_1$ are also $f_S = 1.2$. Suppose one linear combination of $S_{1,2}$, call it ζ, is lighter than 200 GeV, then the thermal relic abundance of dark matter is determined by the annihilation $\tilde{S}\tilde{S} \to \zeta\zeta$, with a cross section times relative velocity given by:

$$\sigma \times v_{rel} = \frac{f_\zeta^4 m_{S'}^2 \sqrt{1 - m_\zeta^2/m_S^2}}{16\pi (m_{S'}^2 + m_S^2 - m_\zeta^2)^2}. \tag{5.60}$$

Setting this equal to the optimal value [44] of 2.2×10^{-26} cm^3/s, we find $f_\zeta \simeq 0.62$ for $m_{S'} = 1$ TeV, $m_S = 200$ GeV, and $m_\zeta = 150$ GeV. Note that ζ stays in thermal equilibrium through its interaction with h from a term in V_D. It is also very difficult to be produced at the LHC, because it is an SM singlet, so its mass of 150 GeV is allowed.

5.8 Conclusion

The utilitarian supersymmetric $U(1)_X$ gauge extension of the Standard Model of particle interactions proposed 14 years ago [67] allows for two classes of anomaly-free models which have no μ term and conserve baryon number and lepton number automatically. A simple version [70] with leptoquark superfields is especially interesting because of existing LHC flavor anomalies.

The new Z_X gauge boson of this model has specified couplings to quarks and leptons which are distinct from other gauge extensions and may be tested at the LHC. On the other hand, a hint may already be discovered with the announcements by ATLAS and CMS of a diphoton excess at around 750 GeV. It may well be the revelation of the singlet scalar (or pseudoscalar) S_3 predicted by this model which also predicts that there should be singlet leptoquarks and other particles that S_3 must couple to. Consequently, gluon fusion will produce S_3 which will then decay to two photons together with other particles, including those of the dark sector. This scenario explains the observed diphoton excess, all within the context of the original model, and not an invention after the fact.

Since S_3 couples to leptoquarks, the $S_3 \to l_i^+ l_j^-$ decay must occur at some level. As such, $S_3 \to e^+\mu^-$ would be a very distinct signature at the LHC. Its branching fraction depends on unknown Yukawa couplings which need not be very small. Similarly, the S_3 couplings to $\phi_1\phi_2$ as well as leptoquarks imply decays to ZZ and $Z\gamma$ with rates comparable to $\gamma\gamma$.

An important byproduct of this study is the discovery of relaxed supersymmetric constraints on the Higgs boson's mass of 125 GeV. It is now given by Eq. (5.23), i.e. $m_h^2 \simeq (g_X^2 + 2f^2 + \lambda_2)v^2$, which allows it to be free of the tension encountered in the MSSM. This prediction is independent of the diphoton excess.

Most importantly, since S_3 replaces the μ parameter, its association with the 750 GeV excess implies the existence of supersymmetry. If confirmed and supported by subsequent data, it may even be considered in retrospect as the first evidence for the long-sought existence of supersymmetry.

Part III

Summary

Chapter 6

Conclusion

As mentioned in chapter 1 the SM cannot accommodate the neutrino mass and dark matter experimental observations. One possible explanation is to introduce a gauged $U(1)$ extension.

In Part I, two examples of gauged $U(1)_{B-L}$ models are introduced. Chapter 2 exotic fermion singlets are introduced such that neutrino mass is generated at the one-loop level through dark matter (i.e. the scotogenic mechanism). In chapter three, three right-handed neutrinos are introduced with the unconventional charge $(5, -4, -4)$ and the $U(1)_{B-L}$ is spontaneously broken to Z_3 lepton number. These three right-handed neutrinos, along with three pairs of neutral singlets $N_{L,R}$ are connected to the SM left-handed neutrinos via two unique scalars $\chi_{3,6}$ such that the neutrinos are Dirac with see-saw mass generation. A complex neutral scalar χ_2 is also introduced under this symmetry, which while not absolutely stable, decays with a lifetime greater than that of the Universe yielding an example of Z_3 dark matter.

In Part II, three further examples of gauged $U(1)$ models are introduced. In Chapter 4 a generalized look at gauged $U(1)$ models. Under the assumption that the entire particles family contains the same charge, a restriction is derived from the triangle anomaly. Using this condition two models were put forth and how these models affected both the quark and lepton sectors was explored. In Chapter 5, a $U(1)$ supersymmetric gauged extension was made to the SM. A prediction for the 750 GeV diphoton excess was proposed as well as the LHC constraints put on the gauged Z_x boson were explored.

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