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Publication Date
2015-08-27

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HOW TO CITE THIS PAPER: Sarnecka, Barbara W. (2015). Learning to Represent Exact Numbers. *Synthese*, pp.1-18. Published online 27 August 2015.

www.escholarship.org/uc/item/6tq9j73k

Learning to Represent Exact Numbers

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This work was supported by NSF grant 0953521 to the author. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation.

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Abstract

This article focuses on how young children acquire concepts for exact, cardinal numbers (e.g., three, seven, two hundred, etc.). I believe that exact numbers are a conceptual structure that was invented by people, and that most children acquire gradually, over a period of months or years during early childhood. This article reviews studies that explore children's number knowledge at various points during this acquisition process. Most of these studies were done in my own lab, and assume the theoretical framework proposed by Carey (2009). In this framework, the counting list ('one,' 'two,' 'three,' etc.) and the counting routine (i.e., reciting the list and pointing to objects, one at a time) form a placeholder structure. Over time, the placeholder structure is gradually filled in with meaning to become a conceptual structure that allows the child to represent exact numbers (e.g., *There are 24 children in my class, so I need to bring 24 cupcakes for the party*.) A number system is a socially shared, structured set of symbols that pose a learning challenge for children. But once children have acquired a number system, it allows them to represent information (i.e., large, exact cardinal values) that they had no way of representing before.

Learning to represent exact, cardinal numbers

1. *Introduction*

This article is about how young children acquire a number system. The question is how children learn to represent exact, cardinal values such as *three* and *seven* and eventually, *2015*.

What makes this topic interesting is that a number system is a cultural product, but one that makes a big difference to individual cognition. When I recently asked my teenage son to babysit his eight-year-old brother for the evening, he replied that for $5 per hour, he would promise to stay in the house, and to call an ambulance in case of emergency. To actually play with his brother, he wanted $7.50. If he was also expected to make dinner for his brother and put him to bed, that would be $10. The eight-year-old then interrupted, offering to make his own dinner and put himself to bed for the bargain price of $7.50.

It's hard to imagine this conversation occurring without benefit of a number system, as all of us used numbers both in our thinking and in our communication with each other. This is why number learning is interesting. Whereas there are many examples of children learning new concepts (e.g., *Mammals are animals that have fur and feed their babies milk*), a number system is actually a new representational resource, enabling people who acquire it to represent a whole domain of information (i.e., information about large, exact numerosities) that would otherwise be difficult or impossible to represent.

What do I mean when I say that a child 'knows' a number? For the purposes of this article, I use the phrase 'to know a number' in the following narrow and operational way: A child 'knows' the number 'three' if, when the child is asked to give three objects to an experimenter, the child gives exactly three objects. The child must also refrain from giving three objects when asked for other numbers. Similarly, if a child is shown a picture of three objects and asked how many there are, the child should say ‘three,’ and the child should not label other set sizes as ‘three.’[1]

Of course, a child can know many things about ‘three’ and still fail to meet this narrow and operational definition. For example, the child could know that ‘three’ is a counting word, that it has something to do with how many or how much of something there is, or even that it is a number higher than one. The child could know all those things and still not know what ‘three’ means, by the operational definition used here. Similarly, a child can meet this narrow definition of ‘knowing’ three, and still not know many other things that educated adults know about it (e.g., that three is an odd number, a prime number, a rational number, the cube root of 27, etc.)

A practical reason to study children’s number learning is because children’s early numeracy seems to be related to their later educational success. A meta-analysis of six large, longitudinal datasets from the U.S., Great Britain and Canada concluded that pre-kindergarten math skills were the single best predictor not only of later math performance, but of later academic performance overall.

> . . . Early math skills have the greatest predictive power, followed by reading and then attention skills. By contrast, measures of socio-emotional behaviors, including internalizing and externalizing problems and social skills, were generally insignificant predictors of later academic performance, even among children with relatively high levels of problem behavior. Patterns of association were similar for boys and girls and for children from high and low socioeconomic backgrounds. (Duncan et al., 2007 p.1428)

[1] My thanks to an anonymous reviewer for suggesting this clarification.

These data are, of course, correlational. They show that five-year-olds who knew more about numbers than their peers grew up to be middle and high-school students who got better grades than their peers. This doesn't prove that teaching young children about numbers will make them better students later on, but neither does it rule out that possibility.

It would be especially useful if researchers could discover how to support number learning in young children from low-income backgrounds. In the United States, there is a persistent gap in mathematics achievement between students from low-income backgrounds and more privileged children—a gap that exists before children enter kindergarten and grows wider during the years that follow (e.g., Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Starkey, Klein, & Wakeley, 2004). Low-income children come to kindergarten with less number knowledge than middle-income children, and this baseline difference accounts for the gap in mathematics achievement seen at the end of third grade (Jordan, Kaplan, Locuniak, & Ramineni, 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009).

In addition to its educational importance, number learning is interesting because it may represent a case where humans acquire new representational resources. In Carey's (2009) theory, human beings have no innate cognitive system for representing exact numerosities larger than about three or four. Thus, symbols for large, exact numbers must be acquired during development. This is exciting because it represents a kind of learning that appears to be uniquely human, where socially shared symbols (in this case, numbers) get into the minds of individual people, and give those people the ability to represent information that would otherwise be difficult or impossible to represent.

This is a point on which theorists differ. Some broadly agree with Carey (2009) about the nature of the innate representations outlined below, and agree that a system for representing large, exact numbers must be acquired from one's culture (Hurford, 1987, pp. 125–127; Spelke & Tsivkin, 2001). Others propose instead that children are born with a few innate principles, which are sufficient to generate natural-number concepts. On these accounts the number system of one's culture may provide convenient labels for number concepts, but it does not provide the concepts themselves (Butterworth, 2005; Butterworth, Reeve, Reynolds, & Lloyd, 2008; Leslie, Gelman, & Gallistel, 2008). Still another view holds that children learn number concepts by constructing the axioms that define the natural numbers (Rips, Bloomfield, & Asmuth, 2008).

But these disagreements are mainly theoretical. That is, scholars do not generally dispute the findings of experiments. Instead, they argue about what those findings mean. My goal in the pages that follow is to present some findings from studies of children's number learning. I hope to give the reader a sense of the number knowledge that we actually observe and measure in young children.

2. Innate Representations of Number, or Something Like It

Humans are endowed by evolution with at least two ways of representing some number-relevant content. One is the approximate number system; the other is the ability to mentally represent small sets of, up to three or four individuals. (For detailed reviews, see Carey, 2009; Feigenson, Dehaene, & Spelke, 2004.) The important thing to note is that although each system does represent some information about numerosity, neither system has the power to represent exact, large numbers.

2.1. The Approximate Number System

Humans, like other animals, form representations of the approximate cardinal values of large sets (up to at least several hundred). The numerosity of the set (of things in the world) is represented (in the brain) by a neural magnitude that gets bigger as the represented numerosity gets bigger. For this reason, the representations are sometimes called analog magnitudes. (The whole system is sometimes called the analog-magnitude number system.) Representations of number in this system can be compared to decide which of two sets has numerically more or less. They can also be added, subtracted and multiplied (Barth et al., 2006; Dehaene, 1997, 2009; McCrink & Spelke, 2010; Nieder & Dehaene, 2009) and individuals differ in the accuracy of their number estimation. For example, almost everyone (including human infants) can tell that a cloud of 200 dots is different from a cloud of 100, and most adults can tell the difference (without counting) between 100 and 130. But only some people can see any difference between 100 and 120, and fewer still can distinguish between 100 and 110.

These representations clearly have numerical content. They refer to cardinal values, and number-relevant computations (i.e., addition, subtraction, etc.) are defined over them. Yet this system falls short of representing exact numbers: For example, it cannot represent the content 'exactly 43', as distinct from both 42 and 44. This raises the question of how we ever become able to represent large, exact numbers such as 43.

2.2. Representing Small Sets of Individuals

Humans and other animals also have the ability to represent small sets exactly. How we do this is a matter of debate. On some accounts, humans have innate concepts for small cardinalities: concepts such as *oneness*, *twoness*, *threeness*, etc. (Butterworth, 2010; Gelman & Butterworth, 2005; Gelman & Gallistel, 1978, 2004). On other accounts, we are not born with concepts for any exact, cardinal numbers—even small ones. Instead, we are born with the ability to form mental representations of individuals, and to create mental models of ongoing events in which each individual is represented by a single mental symbol (e.g., Scholl & Leslie, 1999; Simon, 1998). For example, an infant might represent a scene as having 'a big dog, another big dog, and a small dog.' Number is represented only implicitly: the mental model includes three symbols (big dog, big dog, small dog) but no symbol for 'three.' Thus, the system does not represent number explicitly. But it does support some number-relevant computations, such as numerical equality and numerical more and less, because models of small sets of individuals can be compared on the basis of 1-1 correspondence. For example, if an infant sees two objects placed behind a screen, and then the screen is raised to reveal only one object, infants look at the scene longer than at a similar scene where the screen is raised to reveal the expected two objects (e.g., Feigenson & Carey, 2003; Wynn, 1992a). Presumably, the infant has constructed a mental model in which there are two objects behind the screen. The infant compares this model to the new visual display on the basis of one-to-one correspondence and notices a discrepancy, causing him/her to look at the display longer.

Despite allowing the child to represent some number-relevant content about small sets, this system is clearly inadequate for representing large, exact numbers. First, it is not clear that the system has any symbols for cardinal values. Second, the set sizes that can be represented are limited to one, two, three and perhaps four. (In experiments where children must track small sets, young infants usually succeed with up to three items; older children and nonhuman primates usually succeed with up to four.)

How then are human beings ever able to represent numbers such as 43? On Carey's (2009) account, they learn a counting system and this counting system gives them the means to represent large, exact numbers. In this sense, when children are learning to count (or more specifically, when they are learning how counting represents number), they are actually acquiring number concepts. This learning process is explored in more detail in the studies below.

3. Acquiring a System of Representation for Exact Numbers

The first explicitly numerical activity that most children learn is counting. The young child hears adults and older children reciting a special list of words (*one, two, three*, etc.), often while pointing to objects. The child learns the list, as well as some rules about the pointing. For example, we point to each thing once and only once, and we say one word each time we point. Children start to learn these rules about counting as early as 15-18 months of age, depending on how much counting they witness (Slaughter, Itakura, Kutsuki, & Siegal, 2011). But reciting the list of counting words, even while pointing to things, is not the same as understanding how counting reveals the number of things in a set .

One task that demonstrates this gap is the Give-N task, which I will describe here because it appears in many of the studies discussed below. (The task is also sometimes called 'Give-A-Number,' 'Give-X,' or the 'verbal production task.') In this task, the child is asked to generate sets of a particular number. For example, the child might be given a bowl of 15 small plastic apples, and asked to 'Give *one* apple to the frog,' 'Give *two* apples to the frog,' 'Give *five* apples to the frog,' and so on. Children's performance on this task reveals an interesting gap between knowledge of counting (in the sense of being able to recite the list of words while pointing to objects) and knowledge of the cardinality principle (i.e., the principle that says the last word of a count tells the numerosity of the whole set.) Although many 2–4-year-olds can recite the beginning of this and point to objects just fine (e.g., when asked to count a row of apples, they say "one, two, three, four, five, six" while pointing to each apple in turn), they may be unable to 'Give *six* apples to the frog' when asked. Instead of counting (i.e., instead of reciting the number list up to *six* while adding one apple to the set with each word), they just grab a handful of apples and hand them to the frog. Even when the children are explicitly told to count (e.g. when experimenters say, 'Can you count and make sure it's six?' or, 'But the frog wanted six apples –

can you fix it so there are six?') the children point to the apples and recite the number list correctly, but do not use the result of their counting to fix the set (Le Corre, Van de Walle, Brannon, & Carey, 2006; Sarnecka & Carey, 2008).

Typically, the scene looks something like this. The child is asked to give six apples to the frog, and scoops up two handfuls of apples (which happens to be eight), dumping them on the frog's plate. The experimenter asks, "Is that six?" to which the child nods happily.

The experimenter says, "Can you count and make sure it's six?"

The child points to each apple in turn, reciting, "One, two, three, four, five, six, seven, eight."

The experimenter says, "So, is that six?"

The child nods again.

The experimenter says, "Hmm. I don't think that's six. And the frog really wanted six apples. Can you fix it so there's six?"

The child grabs *more apples* and adds them to the plate.

In other words, there is a noticeable gap between children's (early) learning of how to recite the counting list and point to objects, and their (later) conceptual understanding of how counting relates to cardinal numerosity.

Studies using the Give-N task have shown that children's performance moves through a predictable series of levels, called number-knower levels (e.g., Condry & Spelke, 2008; Le Corre & Carey, 2007; Le Corre et al., 2006; Negen & Sarnecka, 2012; Sarnecka & Gelman, 2004; Sarnecka & Lee, 2009, 2009; Slusser, Ditta, & Sarnecka, 2013; Slusser & Sarnecka, 2011; Wynn, 1990, 1992b). These number-knower levels are found in children acquiring number systems not only in English, but also in Arabic, Japanese, Mandarin Chinese, Russian, Slovenian, and even among the Tsimane', a farming and foraging society in rural Bolivia (Almoammer et al., 2013; Barner, Libenson, Cheung, & Takasaki, 2009; Li, Le Corre, Shui, Jia, & Carey, 2003; Piantadosi, Jara-Ettinger, & Gibson, 2014; Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007).

The number-knower levels are as follows. At the earliest level (called the 'pre-number-knower' level), the child makes no distinctions among the meanings of different number words. On the Give-N task, pre-number knowers might give one object for every request, or always give a handful, but the number given is unrelated to the number requested. At the next level (called the 'one-knower' level), the child knows that 'one' means one. On the Give-N task, a one-knower gives exactly one object when asked for 'one,' and gives two or more objects when asked for any other number. After this comes the 'two-knower' level. Two-knowers give one for 'one,' and two objects for 'two,' but do not reliably produce any larger sets. The two-knower level is followed by a 'three-knower' and then a 'four-knower' level.

Mathieu Le Corre and colleagues (2006) coined the term 'subset-knowers' to describe children at the one-, two-, three-, and four-knower levels, because although these children can typically recite the count list to ten or higher, they only know the exact, cardinal meanings of a subset of those words. Subset-knowers are distinct from both pre-number knowers (who do not know the exact meanings of any number words) and from cardinal-principle-knowers (who know, at least in principle, how to find the exact cardinal meaning of any number word, as high as they can count).

An obvious question to ask is, how old are children when they figure out the cardinal principle? Of course, different children reach this milestone at different times, but in a sample of 641 children (not from low-income backgrounds) children typically came to understand the cardinal principle sometime between 34 and 51 months old. That is, any time from a couple of months before their third birthday to a couple of months after their fourth birthday (Negen & Sarnecka, 2012; Sarnecka & Carey, 2008; Sarnecka & Gelman, 2004; Sarnecka et al., 2007; Sarnecka & Lee, 2009; Slusser et al., 2013; Slusser & Sarnecka, 2011). Children from lower-income backgrounds come to understand cardinality significantly later, at age 4 or older (Fluck & Henderson, 1996; Jordan & Levine, 2009).

The important thing here is that for children, counting (the list of words and the pointing-to-objects routine) forms a placeholder structure—a set of symbols that are initially defined in terms of each other (e.g., 'seven' is the word that comes after 'six' and before 'eight'), but not initially defined in terms of other knowledge the child has (e.g., the child initially does not know the cardinal meaning of 'seven'). Over time, the individual number words are filled in with meaning and become fully elaborated concepts—a process that Carey (2009) calls 'conceptual-role bootstrapping' (see also Block, 1998; Quine, 1960).

In the case of number concepts, the placeholder structure is the counting list and counting routine. The 'structure' is in the order of the list (i.e., you start counting at *one*, followed by *two*, then *three*, etc.) and in the rules about how the list is matched to objects (i.e., you say one word for each object, you point to each object once and only once, etc.) But the words and gestures are not initially defined in terms of other knowledge the child has. That is, the words don't mean anything outside the list, and nor do the gestures. The child's knowledge of counting at this point has been compared to knowledge of a nursery rhyme, or to chanting-and-gesture games like 'patty cake' or 'eenie, meenie, minie, mo.'

Under Carey's (2009) conceptual-role bootstrapping account, the long process of learning what the number words mean *is* the process of acquiring exact number concepts. If this proposal is correct, then there should be in an intermediate state of knowledge where children have acquired the symbol structure (i.e., memorized the counting list and gestures) but don't yet understand what counting words and gestures mean. The children in this intermediate state are the ones that Le Corre and colleagues (2006) termed *subset-knowers*.

What separates subset-knowers from CP-knowers is that only CP-knowers understand the *cardinal principle* (Gelman & Gallistel, 1978), which is the principle that connects counting to number-word meanings, by making the meaning of any number word a function of that word's ordinal position in the counting list. In other words, the cardinal principle guarantees that for any counting list, in any language, the sixth word in the list must mean 6, the twentieth word must mean 20, and the thousandth word must mean 1,000. To understand the cardinal principle is to understand how counting represents number.

People sometimes ask me why the subset-knower levels only go up to 'four-knower.' Why do we not see five-knowers, six-knowers, or seven-knowers? Couldn't there be a child who knows what the numbers *one* through *seven* mean, but doesn't know what *eight* means, and still doesn't understand the cardinal principle? According to the bootstrapping account, the answer is no. The reason is that without a number system, only small set sizes (up to about 4) can be represented exactly. As described above, humans have innate, nonverbal cognitive systems allowing us to represent small, exact set sizes (up to 3 or 4) as well as large, approximate set sizes (Feigenson et al., 2004). But these nonverbal systems cannot distinguish, for example, 7 from 8 items. Thus, the largest exact number that children can recognize directly seems to be 4. Numbers higher than 4 must be represented through counting.

4. Studies of Subset-Knowers and Their Partial Understanding of the Number System

If this account of where exact-number concepts come from is correct, then subset-knowers are children in the process of acquiring a new conceptual system. The meanings that these children assign to number words may be incomplete, including some but not all aspects of the exact-number concepts that adults have. The goal of the studies described below is to explore this conceptual-construction process by exploring the partial meanings that children assign to number words, as they progress toward a more complete understanding of the number system.

4.1. Understanding That Numbers Are About Quantity

One of the first things children may learn about number words is that they relate to quantity. A set of five objects can be shaken up, turned upside-down, stirred with a spoon, arranged in a line and so on, but it will still be a set of five objects. The number of objects only changes when the quantity changes—that is, when objects are added or removed.

To find out when (i.e., at what number-knower level) children understand this, Susan Gelman and I tested 54 children on a task called the Transform-Sets task (Sarnecka & Gelman, 2004). In this task, children were shown a set of five or six objects labeled with a number word (e.g. "I'm putting *six* buttons in this box."). Then some action was performed on the set (shaking the whole box, rotating the whole box, adding or removing an object), and the children were asked, "Now how many buttons? Is it five or six?" We found that even subset-knowers knew that the original number word should still apply when the box had been shaken or rotated, but the number word should change when an item had been added or removed. In other words, even before they knew exactly what the words *five* and *six* meant, children already knew that those words should only change when an item was added or removed from the set. This is an example of subset-knowers having partial knowledge of the meanings of higher number words before they understand the full, cardinal meanings.

Subset-knowers show similar intuitions when the comparison is between two sets, rather than between a set and a number word. Izard, Streri and Spelke (2014) found that subset-knowers can use one-to-one correspondence to tell when an object is missing from a set of five or six. In this study, five or six finger puppets were placed on a display stand with exactly six spaces, so that either all the spaces were filled (in trials with six puppets), or one space was left empty (in trials with five puppets). Number words were not used, but children' were encouraged to notice whether the display was full of puppets or not. The puppets were then removed from the display and placed in a box, and a few seconds later they were returned to the display. After five puppets had been retrieved from the box, the children had the opportunity to reach in to the box (they could not see into it, but they could reach in) and look for another puppet if they thought there was one left in there. Children searched longer in the trials where the display had been full (the trials with six puppets) than in the trials where a space had been empty at the start (the trials with five puppets).

In other words, even without explicitly representing that there were "five" or "six" of puppets in the set, children were able to track some information about number, and they did expect that the one-to-one correspondence between the puppets and spaces (either a puppet in every space or a space left empty) would be the same even though the puppets were taken away and then returned. Children failed to track this information, however, when the identity of the items was disrupted (e.g., when one of the puppets was removed from the box and replaced with another one), indicating that they did not clearly understand how gaining an object reverses (in terms of number) the effect of losing one.

4.2. Understanding That Numbers Are About Discontinuous Quantities

Only sets of discrete individuals have numerosity. Like the idea that numbers are about quantity, this idea is another logical prerequisite for understanding cardinality. It has been suggested that language may provide hints that numbers quantify over sets of discrete individuals because number words quantify over count nouns. For example, *five blocks* is grammatical, but **five water* is not (Bloom & Wynn, 1997). To explore this question, we tested 170 young children on what we called the Blocks and Water Task (Slusser et al., 2013). The goal was to find out whether subset knowers (who, by definition, cannot yet produce or identify a set of exactly five or exactly six objects) know that *five* and *six* refer to sets of discrete individuals, rather than masses of continuous stuff.

In the Blocks and Water task, children were presented with two cups—one containing discrete objects, the other a continuous substance. For example, one cup might contain five green blocks, while the other cup contained orange-colored water. Then the child was either asked a number question (e.g. "Which cup has five?") or a non-number question (e.g. "Which cup has orange?").

Children at all number-knower levels did very well on the non-number questions (e.g., "Which cup has orange?") and also did well on the number questions, when the cup with discrete objects contained only one or two items. (In this case, the number question was "Which cup has one?" or "Which cup has two?"). But children's performance on questions about *five* and *six* was more mixed. On these questions the CP-knowers did very well, and the three- and four-knowers also performed above chance, indicating that they had connected *five* and *six* to discontinuous quantification, even though (in the case of the three- and four-knowers) they did not yet know how to identify or generate a set of exactly five or exactly six items.

The one- and two-knowers were the most interesting. They chose the cup with discrete objects for *five* and *six* more often than chance would predict, but only if they were asked about *one* and *two* first. Children who were given the trials in the opposite order (i.e., they were first asked about sets of five and six, then about sets of one and two) chose at chance. In other words, they seemed to consider the blocks and the water both equally good examples of *five* or *six*.

This effect of trial order suggests that one- and two-knowers have at least some tacit understanding that *five* and *six* pertain to discontinuous quantities, but do not understand exactly how it works. By doing the task with low numbers first, children are primed and their tacit understanding comes to the fore, which helps them to succeed on the trials with higher numbers[2].

These results suggest that children first learn the meaning of *one* and then of *two*, including the fact that *one* picks out individuals and *two* picks out pairs. If their attention is first drawn to this information in the Blocks and Water task, and then the same question is asked about *five* and *six*, the children tentatively extend the knowledge to the higher number words—showing that they recognize *five* and *six* to be in some way the same kind of words as *one* and *two*. This supports Carey's proposal that children learn the first few numbers individually, and then infer the properties of higher numbers by generalizing what they have learned about the lower numbers.

4.3. Understanding That Later Words Denote Greater Numbers

Adults understand that to count is to repeatedly add one to a set of N, each time creating a new set of N+1. Thus, numbers occurring later in the count list represent larger cardinalities. To find out when children understand this, Susan Carey and I tested 73 children aged 24–48 months on what we called the Direction task (Sarnecka & Carey, 2008). All of these children could recite

[2] Thanks to an anonymous reviewer for suggesting this phrasing.

the count list at least to ten while pointing to one object at a time. The question was whether they understood that each word in the list denotes a larger set than the word before it.

In the Direction task, children were shown two plates, each containing six items (e.g. six red plastic stars on one plate, six yellow plastic stars on the other), and were told, “This plate has six, and that plate has six. And now I’ll move one.” Then the experimenter picked up an item from one plate and moved it to the other plate, and said, “Now there’s a plate with five, and a plate with seven. Which plate has five?” (On half the trials the question was, “Which plate has seven?”) Children were not allowed to count the items. Only four-knowers and CP-knowers succeeded on this task; one-, two-, and three-knowers performed at chance. In other words, the children who knew fewer number-word meanings did not yet seem to realize that words coming later in the list represent larger numbers.

5. *What Cardinal-Principle Knowers Know*

Several studies have identified things that only cardinal-principle-knowers (abbreviated *CP-knowers*) seem to understand about numbers. This knowledge is demonstrated on tasks where CP-knowers succeed and subset-knowers fail. These findings suggest that when children come to understand cardinality, they have not just learned a procedural rule about counting—they have actually acquired a conceptual system for representing large, exact numerosities.

5.1. CP-Knowers Have Identified Numerosity as the Relevant Dimension of Experience

Subset-knowers may know that numbers have something to do with quantity, and even that numbers quantify over sets of discrete individuals, but fail to identify numerosity (as distinct from other properties of a set) as the dimension that numbers pick out. Besides having numerosity, sets also have area, contour length, and so forth. Emily Slusser and I did a study asking when (i.e., at what knower-level) children have identified numerosity as the dimension of experience that number words pick out (Slusser & Sarnecka, 2011).

To answer this question, we tested 116 children, aged 30–48 months, on a Match-to-Sample task. In this task, the experimenter showed the child a sample picture and labeled it with the appropriate number word (e.g., “This picture has eight turtles.”) The experimenter then placed two more pictures on the table, saying, “Find another picture with eight turtles.” One picture had the same number of items as the sample picture. The other had a different number of items (either half or twice as many as the sample picture), but matched the sample picture either in total summed area or in total summed contour length of the items. Children were not allowed to count the items in the pictures. (One picture always had twice as many items as the other, so it was easy to see the difference without counting.) The experiment also included control trials, where children were asked to match pictures according to mood (happy or sad) or color. For example, “This picture has happy turtles. Find another picture with happy turtles” or, “This picture has green turtles. Find another picture with green turtles.”

We found that subset-knowers failed to extend the number words *four, five, eight,* and *ten* to another set of the same numerosity. CP-knowers, on the other hand, succeeded robustly on all types of trials. All children did just fine on the mood and color trials, showing that they understood the task. But only the CP-knowers knew that two sets of the same numerosity should be labeled by the same number word, whereas sets of visibly different numerosities should have different number words. In other words, only CP-knowers recognized numerosity as the dimension of experience that number words pick out.

Similarly, Veronique Izard and her colleagues have shown that subset-knowers seem to track the numerosity of a set only as long as its individual members do not change. When a subset-knower sees a set of (e.g.) five objects placed in a box, and then one of the objects is briefly removed and immediately returned to the box, the children know that the number of items in the box has not changed. However, when one item is removed and a *different* item is put in (even though it looks identical to the first one), subset-knowers lose track of the number of items in the box. That is, they are not sure how a change in the *identity* of an item affects the *numerosity* of the set. Interestingly, they do not have this trouble when there are only two or three items in the set, which shows that they can keep track of the 'plus-one' and 'minus-one' transformations, they simply don't know how those transformations affect set sizes above their own knower-level (Izard et al., 2014).

5.2. CP-Knowers Have Begun to Recognize That a Number +1= The Next Number

Another task where CP-knowers succeed and subset-knowers fail is the Unit task (Sarnecka & Carey, 2008). In this task, children were shown a box with five toy apples inside it, and were told, "There are five apples in this box." Then the experimenter added either one or two more apples and asked, "Now how many? Is it six or seven?" (Of course children were not allowed to count the items.) Only CP-knowers correctly judged that if we add one item to the box, the resulting number of items will be the next word in the counting list (i.e., five plus one is six), whereas if we add two items, the resulting number will be two words forward in the list (i.e., five plus two is seven). Thus it seems that CP-knowers (but not subset-knowers) have begun to connect the counting list and counting routine to the idea of a successor—that each number has a 'next' number, exactly one more, which is named by the next word in the list (but see Davidson, Eng, & Barner, 2012).

5.3. CP-Knowers Understand Numerical 'Just As Many'

Another difference between subset-knowers and CP-knowers is that CP-knowers demonstrate a much clearer understanding of ‘just as many,’ or exact equality (Izard, Pica, Spelke, & Dehaene, 2008). This is partly demonstrated by the fact that, as mentioned above, subset-knowers can use one-to-one correspondence to check that all the members of a set are present, even without a number word to represent the cardinality of the set (Izard et al., 2014).

But CP-knowers use number words to infer one-to-one correspondence between sets. For example, CP-knowers say that if you have ten flowers and ten vases, you must have exactly one flower for each vase (Muldoon, Lewis, & Freeman, 2009). In the past, I have used the word ‘equinumerosity’ to describe this knowledge, but of course children do not conceive of equinumerosity in the explicit way that philosophers do (e.g., children do not talk about bijective, surjective functions). What CP-knowers do seem to understand is that any set of, e.g., six items has just as many as any other set of six.

My colleagues and I explored this by presenting preschoolers with a scenario in which two stuffed animals (a frog and a lion) were given ‘snacks’ that were actually laminated cards with pictures of food (Sarnecka & Gelman, 2004; Sarnecka & Wright, 2013). On half of the trials, the snacks were identical (e.g., Frog and Lion each received a picture of six peaches); on the other trials, the snacks differed by one item (e.g., Frog’s picture had five muffins while Lion’s had six). The pictures showed the food evenly spaced in a line, so that sameness or difference of the two snacks was easy to see.

At the beginning of each trial, the child was asked whether the animals’ snacks were ‘just the same.’ Children almost always answered this question correctly. When they did not, the experimenter corrected the error and drew the child’s attention to the sameness or difference of the sets by saying, for example, “Well, they both have muffins, don’t they? But look! Lion has a muffin here, and Frog doesn't. Oh no, I forgot one of Frog's muffins! I'm so silly! I made a mistake!" Then the experimenter asked the child, for example, "Frog has five muffins. Do you think Lion has five or six?" (Of course, children were not allowed to count the items.)

Only the CP-knowers succeeded robustly on this task. In other words, only the CP-knowers knew that if Frog had six, and the sets were identical, then Lion must also have six. And they knew that if the sets were different, then Lion must have a different number than Frog. This is not a rule about how to recite the counting list or point to objects. It is an insight about what numbers are, and only children who understand cardinality (as measured by the Give-N task) seem to grasp it. This provides convergent evidence that when children figure out the cardinality principle, they are not only learning something about how or when to deploy the counting routine. They are actually achieving a deeper insight about what numbers are.

6. Disagreements Over How To Interpret These Data

The data reviewed above make sense within the framework of Carey's (2009) conceptual-role bootstrapping account. According to this account, concepts such as 43 are not innate. Rather, they are constructed as meanings for a set of placeholder symbols – the counting words. The studies were designed to take 'snapshots' of children's partial knowledge of the number system, such as the knowledge that number words have something to do with quantification, that number is about sets of discrete objects, and so forth. In Carey's account, children achieve a major conceptual breakthrough when they learn the cardinality principle, because that principle is the key to understanding how counting represents number. This breakthrough is sometimes called the cardinal-principle induction. The studies reviewed above link children's understanding of the cardinal principle to their understanding of the number system as a whole, including the idea of a successor (i.e., that each number is followed by a 'next' number, which is exactly one more) and the idea of 'just as many' or exact equality (e.g., that every set of six has just as many items as every other set of six.)

Of course, Carey's (2009) proposal is not the only one that exists. There are other constructivist proposals , which differ from Carey in the specifics of how number concepts might be constructed. But there are also more nativist accounts, which differ radically from Carey by saying that number concepts either cannot be learned in the way that Carey proposes, or don't have to be learned at all because they are automatically generated by the brain.

6.1 Rips's Argument Against Bootstrapping

One argument against Carey's (2009) account is made by Lance Rips and colleagues ((Rips, Asmuth, & Bloomfield, 2006; Rips et al., 2008), who point out that even after children figure out the cardinality principle, they still should not be described as representing the natural numbers, because the cardinality principle does not rule out other types of number systems, such as modular systems. Rips et al. (2006, p. B59) argue that in order to *really* have natural-number concepts, children must represent the axioms that define the natural numbers. That is, children must understand first, that there is a unique first term in the numeral sequence (e.g., "zero"); second, that if "next(k)" = "next(j)" then "k"="j"; and third, that nothing outside the list (i.e., nothing that cannot be reached from "zero" using "next") can be part of the sequence. Rips et al. argue that it is possible for a child to understand the cardinal principle and not represent these axioms. Thus, learning the cardinal principle should not be equated with constructing natural-number concepts. Instead, Rips proposes that, "children may arrive at natural numbers and arithmetic in a more top-down way, by constructing mathematical schemas." (Rips et al., 2008, p. 623)

I have a couple of responses to Rips. First, I think there is some confusion over what it means to 'represent natural numbers.' No one (certainly not Rips or Carey) suggests that four-year-olds talk explicitly about 'natural numbers,' the way that philosophers and mathematicians do. But Carey is interested in how children are able to represent large, exact cardinal values, such as 9, 43, or 2015. Sometimes Carey calls these 'positive integers,' sometimes 'natural numbers.' Rips points out that representing the exact value 2015 is not the same as representing the axioms that define the natural numbers. This is certainly true, but (to me) somewhat beside the point. I think the question of how children are able to represent 2015 is interesting in itself.

As for Rips's claim that cardinal-principle knowers do not represent the axioms that define the natural numbers, I think that is an empirical question. Certainly, children do treat 'one' as the unique first term in the sequence (e.g., Fuson, 1988). And there is evidence that even children as young as 18 months recognize the words in the counting list as separate from other kinds of words (Slaughter et al., 2011). As far as I know, no one has tested the question of whether children understand that if 'next(k)' = 'next(j)' then 'k'='j,' so that question remains open.

6.2 Leslie et al.'s Nativist Argument

A completely different account from Carey's (2009) is put forth by nativists (Butterworth, 2005; Leslie et al., 2008) who argue that the basis for all natural-number concepts is innate. According to this view, number concepts do not need to be 'constructed' at all, because they are automatically generated by the mind. When a child learns the number list of a particular language, she is merely learning a convenient way to express the number concepts that her mind automatically generates. Leslie et al. propose the following description of what this innate cognitive machinery might look like.

(i) There is an innate symbol for 1;

(ii) There is an innate representation of the successor function: $S(x)=x+1$;

(iii) Each integer symbol realized from (i) and (ii) is matched to a magnitude (i.e., to a value in the approximate number system);

(iv) The substitution operations that implement the concept of exact equality are defined over these symbols;

(v) This symbol system supports algebraic reasoning about quantity.

Under this account, natural-number concepts are not present in the mind at birth, but neither are they learned from one's culture. Instead, they are generated as needed.

> We distinguish between a generative symbol system and a realized system of symbols. A generative rule specifies the derivation of an infinite set of symbols—for example, the numbers. A realized system consists of symbols that have already been produced by running a symbol derivation and storing the result in memory. A system of arithmetic reasoning and [the concept of] ONE are mutually supportive but not because realized symbols for infinitely many numbers pre-exist in memory. What pre-exists is a procedure for generating them. (Leslie et al., 2008, p. 216)

The nativist account does not necessarily dispute the empirical findings reviewed earlier in this paper. What it disputes is the idea that when children learn a number system, they are gaining *the ability* to represent information that they could not represent before. In Carey's account, to learn a number system is to construct natural-number concepts themselves. In the nativist account, to learn a number system is simply to learn a notational system for concepts everyone already has.

One question I would pose to the nativists is, what is the evidence for this innate cognitive machinery? Studies with infants and nonhuman animals have provided a great deal of evidence for some numerical capacities (e.g., the approximate number system). But without evidence of the capacities described by (i)-(v) in the list above, the nativist proposal is pure speculation.

7. Conclusions

Decades of research on young have yielded a wealth of data about children's counting and number concepts. Children learn the counting list of their language, they learn to recite it while pointing to objects, they learn the meaning of each of the first few number words, one at a time, and eventually (according to constructivist accounts) they learn how counting represents number, which gives them a way of representing natural numbers.

Carey (2009) emphasizes the developmental discontinuity of this process. Although children are born with the ability to represent small sets exactly, and large numbers approximately, it is only when they learn a counting-list representation of the numbers that they gain the ability to represent large, exact cardinal values. Under this account, numbers are an example of a socially shared symbol system that is learned by individuals and really changes the kind of information that those individuals are able to represent. Alternatives to Carey's account argue that natural-number concepts are either generated by innate cognitive machinery (Leslie et al., 2008) or cannot be learned in the way Carey proposes (Rips et al., 2006, 2008).

Although it is useful for the sake of argument to distinguish the various positions as clearly as possible, I think they each contribute something of value to the discussion. Rips et al. (2006, 2008) raise an interesting question about when children understand the axioms that define the natural numbers, and about what it means to represent numbers.

Similarly, Leslie et al (2008) are obviously correct that humans are born with the cognitive machinery that makes it possible to learn a number system (if we weren't, then no one could ever learn a number system). But what does this innate machinery consist of? And if a person never encountered the number system of any language, would that person actually generate large, exact (but unnamed) number concepts? Is a concept such as 'the number of sheep in my flock' really the same as the concept '42'? Even if the two concepts both pick out the same numerosity, '42' seems more useful because it is part of a larger number system, which makes it easy to compare to other numbers, add it, subtract it, and so forth. Does having this powerful representational system (a number system) really change the information that we are capable of representing? Personally, I would say yes—but these are points about which reasonable people can disagree.

But I think we all agree that the question of how people get number concepts touches on classic problems of human nature. It requires us to ask what exactly is built into the mind by evolution and what is acquired from our social environment; it also requires us to ask how complex, socially shared representational systems are acquired by each new generation of learners. Certainly, the origin of number concepts is a problem that will keep us in productive disagreement for some time to come.

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