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THEORETICAL EVALUATION OF HYBRID SIMULATION FOR CLASSICAL PROBLEMS IN CONTINUUM MECHANICS

A. A. Bakhaty[1], P. L. Drazin[2], S. Govindjee[3], K. M. Mosalam[3] and M. S. Günay[4]

ABSTRACT

The primary notion of hybrid simulation is to only test part of a system physically and to simulate the rest in a computer. While this basic idea is simple to understand, there is surprisingly little theoretical work targeted towards understanding the behavior of the concept, and in particular, its theoretical limitations. In this work we present an initial investigation of the theoretical limitations of hybrid testing in the context of two canonical settings: a beam and a plate. In each case, we mathematically split the physical system into two pieces whose motion we derive in closed-form. At the splitting interface we introduce theoretical models associated with tracking and phase error of the boundary motions and forces. We are able to demonstrate that such systems are generally viable only below the first fundamental frequency of the system. Furthermore, we show there is a tendency to accumulate global errors, relative to the classical solutions, at the slightest introduction of any interface matching error but that these errors are mostly insensitive to further increase in mismatch. Finally, it is found that the different substructures of the systems are subject to excitation at their own independent natural frequencies in addition to those of the hybrid system.

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ABSTRACT

The primary notion of hybrid simulation is to only test part of a system physically and to simulate the rest in a computer. While this basic idea is simple to understand, there is surprisingly little theoretical work targeted towards understanding the behavior of the concept, and in particular, its theoretical limitations. In this work we present an initial investigation of the theoretical limitations of hybrid testing in the context of two canonical settings: a beam and a plate. In each case, we mathematically split the physical system into two pieces whose motion we derive in closed-form. At the splitting interface we introduce theoretical models associated with tracking and phase error of the boundary motions and forces. We are able to demonstrate that such systems are generally viable only below the first fundamental frequency of the system. Furthermore, we show there is a tendency to accumulate global errors, relative to the classical solutions, at the slightest introduction of any interface matching error but that these errors are mostly insensitive to further increase in mismatch. Finally, it is found that the different substructures of the systems are subject to excitation at their own independent natural frequencies in addition to those of the hybrid system.

Introduction

Hybrid testing is a class of simulation techniques that overcomes the limitations of experimentation and numerical simulation by establishing a proper communication between the two with the use of actuators and sensors. The concept is to test only part of the system and numerically simulate the rest. Of these methods, hybrid simulation [14], formerly called pseudodynamic testing [17], is the most prominent. In this method, a system is split into a computational substructure (numerical model) and a physical substructure (a specimen in the laboratory) and the governing equations of motions of the system are solved with a time-stepping algorithm. At each time step, displacements computed by the numerical algorithm are imposed via actuators on the physical substructure, whose response is measured and communicated back to update the system variables and march forward to the next time step. Naturally there exists inherent errors in the technique that arise from factors such as communication delay, experimental errors, and inaccuracies due to the numerical approximation scheme.

Originally conceived in the 1970's as an "on-line testing" method for evaluating the non-linear response of structures subject to earthquake excitation [18], the technique has seen significant development over the years [19, 10, 13, 5]. Much attention has been devoted to studying the source

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and effect of the inherent errors in hybrid testing and proposing mitigation techniques [16, 17, 15, 8, 1], but little work has been dedicated to understanding the theoretical limitations of the technique in the presence of these errors. To address this issue, we present a theoretical framework for the general assessment and evaluation of hybrid simulation (HS) — a form of hybrid testing of current interest [14].

The objective of this study is to understand how errors in a HS influence the global response of the system in an effort to characterize the limitations of the technique. In order to have a completely controlled setting for analyzing HS, we will work with a purely theoretical system in which both the traditional physical and computational parts are mathematical models. The transfer system will also be represented by simple mathematical models.

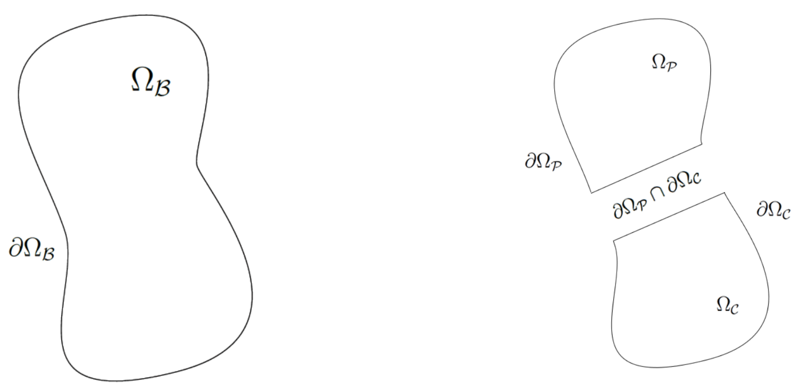
All equations will be analytically solved and thus our analysis represents the best possible case obtainable in HS. We demonstrate this concept by applying our framework to the following classical problems in continuum mechanics for which exact analytical solutions are known: a beam subjected to flexure and a plate subjected to bending.

Theoretical Framework

Consider a body $\mathcal{B}$ with domain $\Omega_{\mathcal{B}}$, depicted in Fig. 1a, whose response is determined from a governing set of equations:

$$F[\mathbf{u}(\mathbf{r},t)] = \mathbf{0}, \qquad \mathbf{r} \in \Omega_{\mathcal{B}}, \tag{1a}$$
$$\mathbf{u}(\mathbf{r},t) = \bar{\mathbf{u}}, \qquad \mathbf{r} \in \partial\Omega_{\mathcal{B}}. \tag{1b}$$

where $\mathbf{u}$ is a characteristic quantity (e.g. displacements, velocities, accelerations, etc.), $\bar{\mathbf{u}}$ is an im-



(a) Domain of system to be simulated.

(b) Hybrid domain of system to be simulated.

Figure 1. Theoretical concept of Hybrid Simulation.

posed value of that quantity on the boundary, $\mathbf{r}$ is position in space and t is time. The domain is next separated into two subdomains denoted by $\mathcal{P}$ and $\mathcal{C}$ which represent the physical and computational substructures respectively, as depicted in Fig. 1b. Without loss of generality we consider only two subdomains for simplicity, but we note that HS may have multiple physical and computational substructures [12]. Each domain's response is determined by Eq. 1 applied locally. To achieve equivalence with the full system, the following conditions are introduced:

$$\hat{\Omega}_{\mathcal{B}} = \Omega_{\mathcal{P}} \cup \Omega_{\mathcal{C}}, \tag{2a}$$
$$\hat{\mathbf{u}} = \hat{\mathbf{u}}_p \cup \hat{\mathbf{u}}_c, \tag{2b}$$
$$\partial\Omega_{\mathcal{B}} = \partial\Omega_{\mathcal{P}} \cup \partial\Omega_{\mathcal{C}} - \partial\Omega_{\mathcal{P}} \cap \partial\Omega_{\mathcal{C}}, \tag{2c}$$

where $\hat{\Omega}_{\mathcal{B}}$ is introduced as the corresponding hybrid domain of $\Omega_{\mathcal{B}}$ and $\hat{\mathbf{u}}$ is the corresponding unified response in the joint hybrid domain. From Eq. 2c and Fig. 1b, it is clear there is an interface between $\mathcal{P}$ and $\mathcal{C}$, $\partial\Omega_{\mathcal{P}} \cap \partial\Omega_{\mathcal{C}} \in \hat{\Omega}_{\mathcal{B}}$, for which additional boundary conditions on the split domain must be furnished to satisfy Eq. 1. These boundary conditions are

$$\hat{\mathbf{u}}_p(\mathbf{r}_p, t) = \mathbf{g}_p(\mathbf{r}_p, t), \qquad \mathbf{r}_p \in \partial\Omega_{\mathcal{P}} \cap \partial\Omega_{\mathcal{C}}, \tag{3a}$$
$$\hat{\mathbf{u}}_c(\mathbf{r}_c, t) = \mathbf{g}_c(\mathbf{r}_c, t), \qquad \mathbf{r}_c \in \partial\Omega_{\mathcal{P}} \cap \partial\Omega_{\mathcal{C}}. \tag{3b}$$

Here "boundary functions" denoted by $\mathbf{g}$ are introduced. These functions represent corresponding quantities in each domain at the interface, such as displacements or forces. A constraint is imposed to enforce continuity of the quantities:

$$G[\mathbf{g}_p, \mathbf{g}_c] = \mathbf{0}. \tag{4}$$

By setting corresponding boundary functions across the interface equal to each other, the solution of the full system is recovered. However by imposing a mismatch in the corresponding boundary functions we are able to model the errors characteristic of HS. One such form, representative of time delay and tracking error as encountered in HS due to finite communication time of the actuator system [9, 3], is presented as follows:

$$g_p^k = g_c^k(1 + \varepsilon_k)e^{-i\Omega d_k} \tag{5}$$

The parameters ε_k and d_k represent the magnitude and phase of the error, respectively, in the k^{th} boundary quantity, g^k. It is noted that the solution to the governing equations in each domain can be derived exactly without the use of any numerical schemes as is the case with HS and thus the only source of error in our analysis comes from the constraint presented in Eq. 5.

Application to Classical Problems

The classical problems selected, which represent a wide array of important engineering applications, are an Euler-Bernoulli beam subjected to a time harmonic concentrated bending moment (Fig. 2) and a Kirchhoff-Love plate subjected to a time harmonic edge bending moment (Fig. 3). Both problems are initially considered with linear-elastic, isotropic, homogeneous material and infinitesimal kinematics. The beam is further presented with the use of the three parameter Maxwell model for viscoelasticity [21], the so-called Standard Linear Solid. The governing equation of motion of the beam is

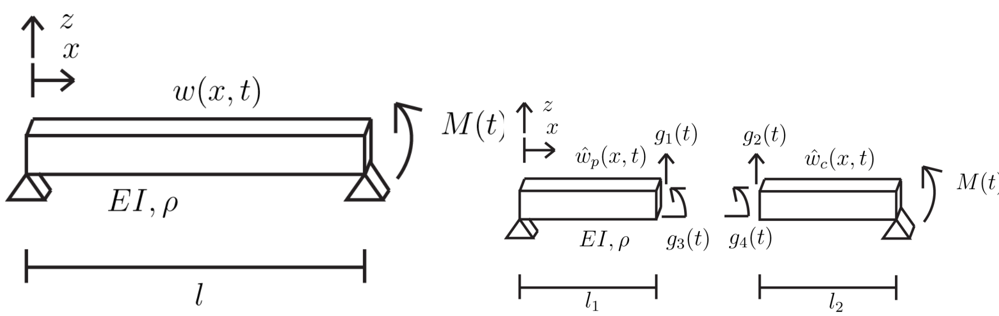
$$EI\frac{\partial^4 w}{\partial x^4} + \rho\frac{\partial^2 w}{\partial t^2} = 0, \tag{6}$$

where w is the transverse displacement, ρ is the lineal mass density, x is the longitudinal coordinate and EI is the flexural rigidity. The corresponding solution is readily available in the literature [20]. The governing equation of motion of the plate is [7]

$$D\nabla^4 w + \rho \frac{\partial^2 w}{\partial t^2} = 0, \tag{7}$$

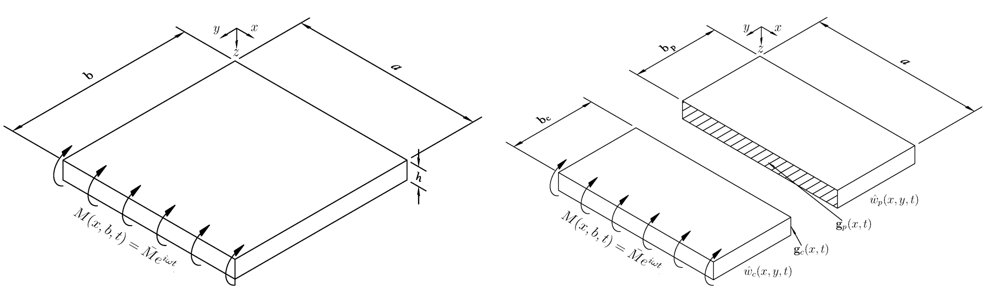
where $\nabla^4(\bullet)$ is the biharmonic operator and D is the bending stiffness. Gorman presents a Fourier series solution to the plate bending problem [6]. For brevity, the solutions to the hybrid formulations are not presented. For a comprehensive analysis of the hybrid beam and plate the reader is referred to the work of Drazin [4] and Bakhaty [2], respectively.

The boundary functions introduced in Eq. 3 are not defined explicitly but can be determined from the constraints presented in Eq. 5. For the two fourth order systems being studied, a total of four relations on each domain are needed for a unique solution. These equations are furnished by conditions on the displacement, rotation, bending moment and shear at the interface with corresponding errors of the form given by Eq. 5. This allows us to express any mismatch in the kinematic quantities and forces at the interface.



(a) True formulation. (b) Hybrid formulation.

Figure 2. Euler-Bernoulli beam subjected to harmonic end concentrated moment.



(a) True formulation. (b) Hybrid formulation.

Figure 3. Kirchhoff-Love simply supported plate subjected to a harmonic edge bending moment.

Analysis

To present a parametric error analysis of the global error in the solutions with respect to the true solution, L_2 space-time displacement norms are defined as

$$||e_{pw}||^2 = \int_\tau \int_{\Omega_\mathcal{B}} (w - \hat{w}_p)^2 \, d\Omega_\mathcal{B} \, d\tau, \qquad ||e_{cw}||^2 = \int_\tau \int_{\Omega_\mathcal{B}} (w - \hat{w}_c)^2 \, d\Omega_\mathcal{B} \, d\tau, \tag{8a}$$

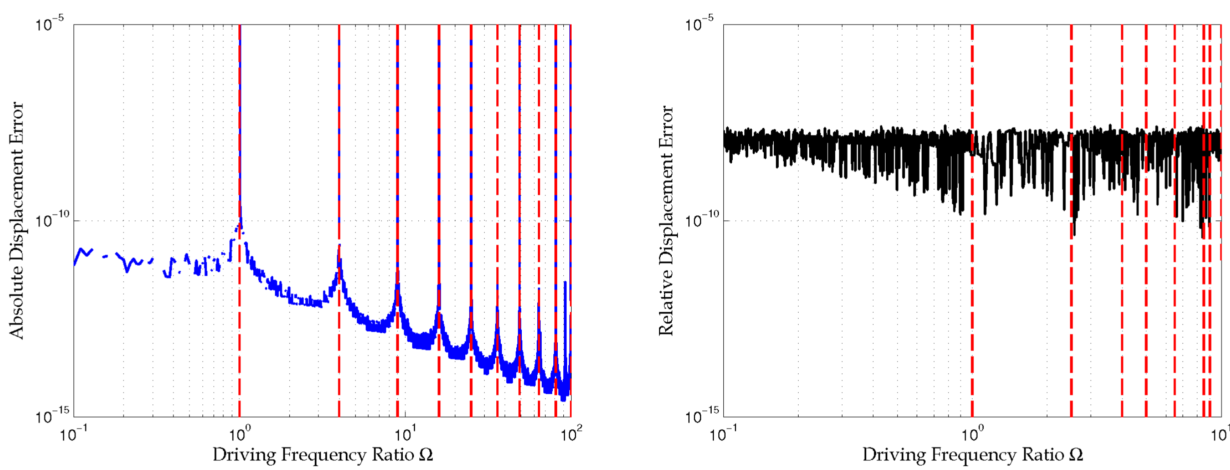
$$||e_w|| = \sqrt{||e_{pw}||^2 + ||e_{cw}||^2}. \tag{8b}$$

We also present a relative error defined as

$$||e_w||_{rel} = ||e_w||/||w||. \tag{9}$$

Validation of the Formulation

We present a validation of the proposed framework by imposing perfect continuity between the $\mathcal{P}$ and $\mathcal{C}$ domains or in other words by setting the introduced error parameters $\varepsilon_k = 0$ and $d_k = 0$ in Eq. 5. Fig. 4a presents the absolute global error given by Eq. 8b for the beam and Fig. 4b presents the relative global error given by Eq. 9 for the plate, where the parameter $\Omega = \omega/\bar{\omega}$ is the driving frequency of the harmonic excitation normalized by the fundamental frequency of the system. Due to the similarity in the results, a larger scale of driving frequencies is presented for the beam with a "close-up" of the lower range of frequencies for the plate.



(a) Beam (b) Plate

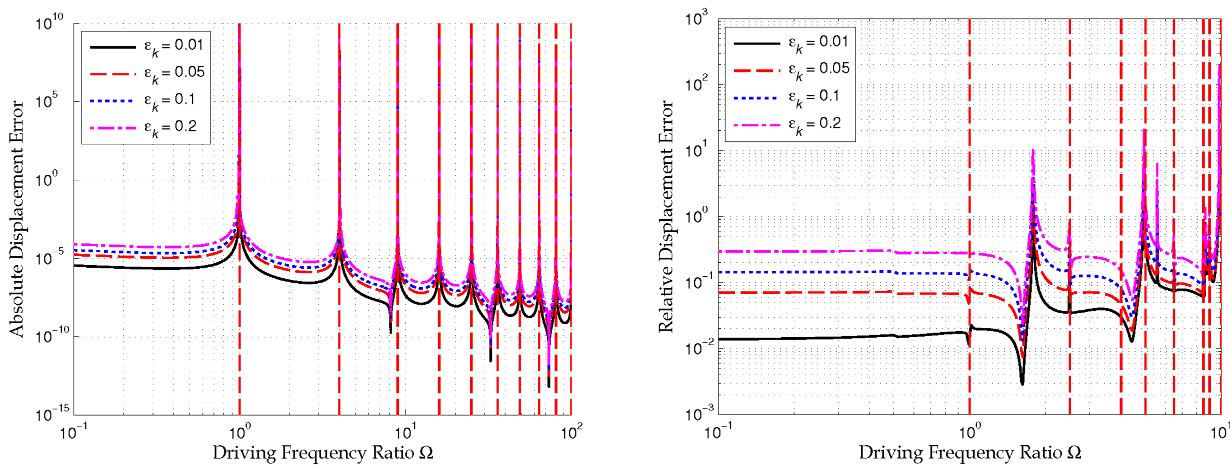
Figure 4. Frequency sweep with zero introduced errors; vertical red lines indicate natural frequencies of the original (non-hybrid) system.

It is readily observable that the hybrid formulation is consistent with the true solution in the absence of any introduced error, thus indicating a valid formulation. It is noted that the plate in Fig. 4b exhibits an error significantly greater than the machine limit for double precision. This loss of digits is the result of evaluating a large Fourier sum of hyperbolic terms in the solution of the plate [6] and not of an intrinsic error in the formulation.

Importance of the Excitation Frequency

Each curve in Fig. 5 presents a level of introduced error with fixed $d_k = 0$ in all of the boundary quantities for the beam and the plate: displacement, rotation, bending moment and shear.

Several observations are noted. 1) There is a strong tendency to accumulate errors in the vicinity of natural frequencies of the system. 2) The error becomes somewhat unpredictable above the fundamental frequency. 3) There are spikes of large errors not associated with natural frequencies. It is noted that not all of the natural frequencies of the plate are excited in Fig. 5b due to the one-sided nature of the excitation (Fig. 3). The unpredictable nature of the errors at and above the fundamental frequency indicate that hybrid tests with dominant excitation frequencies near the natural frequencies of the system may not be viable. To account for damping, the beam with viscoelastic material properties is presented later.

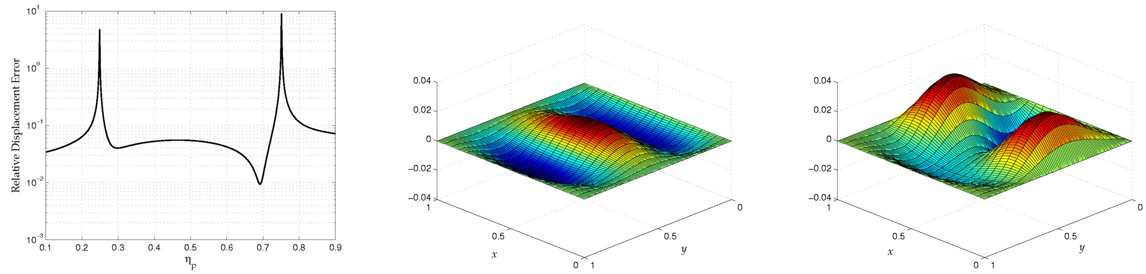


(a) Beam (b) Plate

Figure 5. Frequency sweep with ε_k errors; vertical red lines indicate natural frequencies of the original (non-hybrid) system.

Substructure Excitation of the Plate

The error spikes not associated with a natural frequency of the plate are observed to be consistent with natural frequencies of one of the individual sub-plates created by the domain split. Fig. 6a presents the relative global displacement error versus the location where the separation is made. The parameter η_p is defined as the location of separation normalized by the length of the plate orthogonal to the separation. A spike in the error is observed at one value of η_p and again at $1 - \eta_p$. This indicates each domain is excited when its "length" is η_p, which for the boundary conditions of the substructures is consistent with a natural frequency. This is confirmed in Fig. 6c which demonstrates the $\mathcal{P}$-domain being excited at one of its individual eigenmodes opposed to that of the global system. Note from Fig. 4 these errors are not present when no interface matching error is introduced and thus this effect is only realized when a mismatch is introduced at the interface.



(a) Sweep over domain separation with $\Omega = 5.58$; $\varepsilon_k = 5\%$.

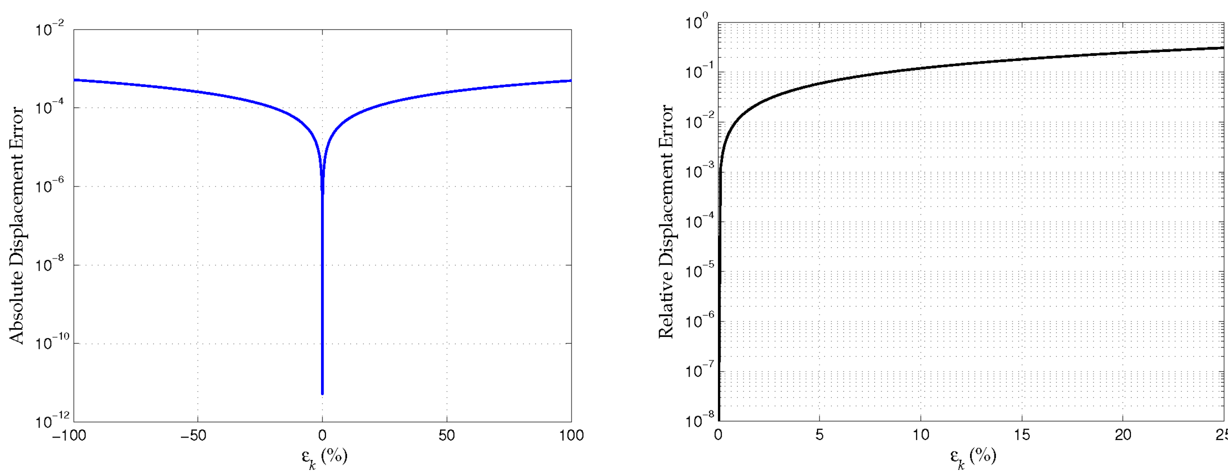
(b) True deformed shape of plate with $\Omega = 5.58$.

(c) Deformed shape of hybrid plate with $\Omega = 5.58$; $\varepsilon_k = 5\%$.

Figure 6. Excitation of the sub-domains in the presence of introduced errors.

Importance of the Introduced Error

Fig. 7 presents the global displacement error versus ε_k introduced into all four boundary quantities with $d_k = 0$ and fixed Ω. Fig. 8 presents the global displacement error versus d_k introduced into all four boundary quantities with $\varepsilon_k = 0$ and fixed Ω. For both Figs. 7 and 8 the beam is once again presented with absolute error and the plate with relative error. As with the frequency sweeps, a larger range of errors is presented for the beam and a "close-up" at smaller errors for the plate.

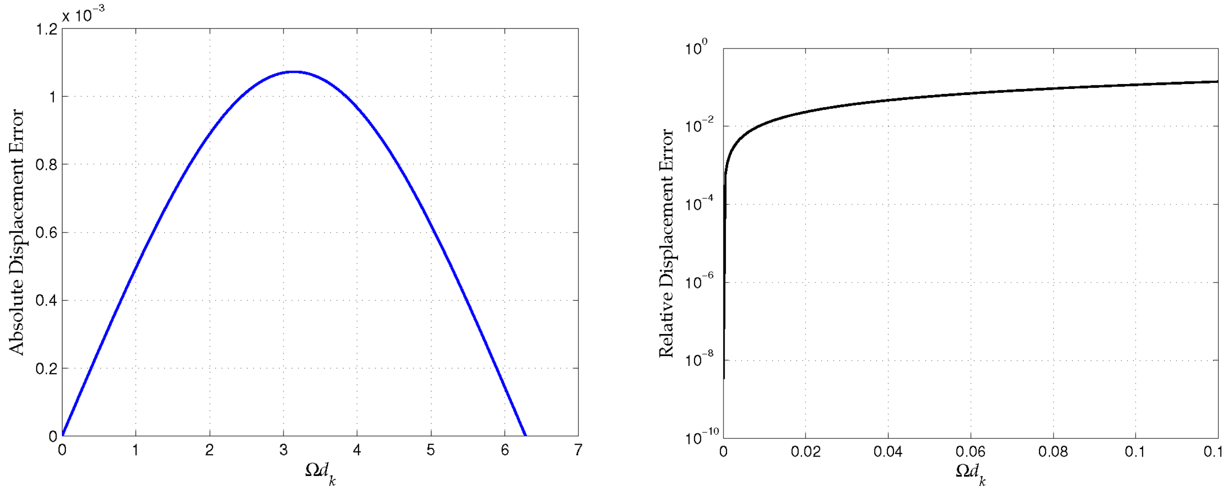


(a) Beam

(b) Plate

Figure 7. Effect of varying ε_k at $d_k = 0$.

For each system it is observed that the slightest introduction of the boundary error results in a rapid increase of the global error. The global error becomes quickly indifferent to increasing boundary errors. For both the beam and the plate the effect of the time delay, or the phase of the error d_k, has a more prominent effect on the global error than the magnitude ε_k. The error is observed to be linear with small d_k and as d_k cycles over a period of the excitation, i.e. the delay is one period out of phase, the error decreases to zero. These results indicate that significant efforts to decrease the errors in the transfer system may result in little global error reduction of the hybrid system.



(a) Beam (b) Plate

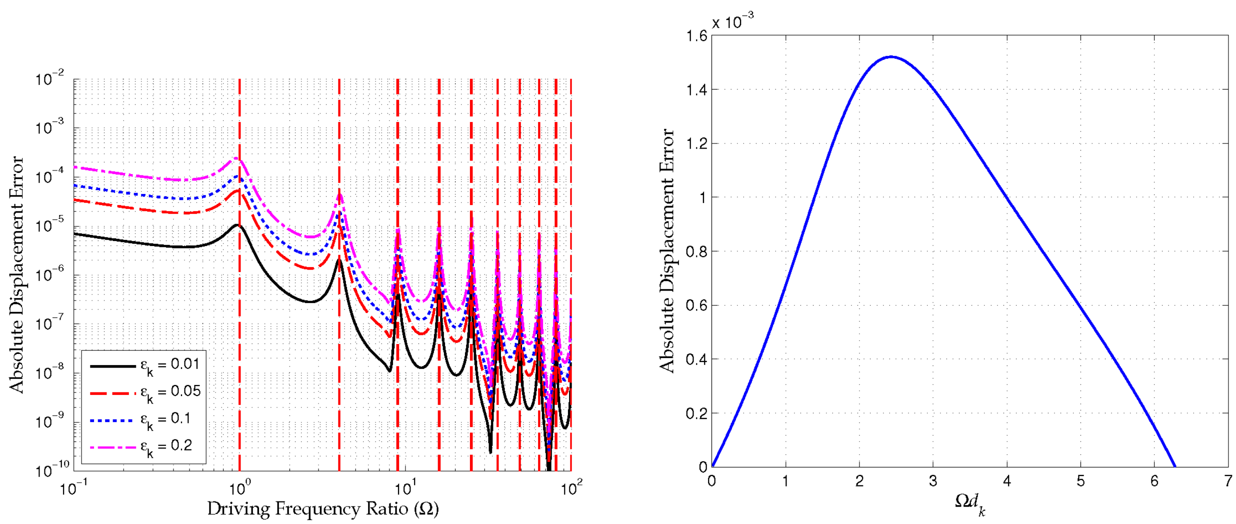
Figure 8. Effect of varying d_k at $\varepsilon_k = 0$.

Viscoelastic Material Response of the Beam

In this section we present results for the beam making use of the 3-parameter Maxwell model for viscoelasticity, which is described by the following:

$$E^* = \left[E_\infty + \frac{\omega^2 t_r^2}{1+\omega^2 t_r^2}(E_0 - E_\infty)\right] + i\left[\frac{\omega t_r}{1+\omega^2 t_r^2}(E_0 - E_\infty)\right], \tag{10}$$

where the complex modulus E^* replaces E in Eq. 7. The parameters chosen are $E_0 = E$, $E_\infty = E/2$ and $t_r = 1/\bar{\omega}$. This choice of parameters places the maximum amount of damping at the first



(a) Frequency sweep at $d_k = 0$. (b) Effect of varying d_k at $\varepsilon_k = 0$.

Figure 9. Errors for the viscoelastic beam.

resonant frequency of the beam. The curves in Fig. 9 show the absolute displacement error for the viscoelastic beam. As can be seen in Fig. 9a, the error spike due to the first resonant frequency has been drastically reduced. This is a result of the selection of t_r. Other features of the curves in Fig. 5a have become mollified near maximum damping. This indicates that damping can be used to reduce error spikes in the hybrid system, however, the average absolute displacement error over all frequencies is still on the same order as that of the elastic beam. Also, as the driving frequency increases, the curves begin to resemble those in Fig. 5a. Fig. 9b presents the global displacement error versus d_k with $\varepsilon_k = 0$ and fixed Ω. It is noted that the value of the absolute displacement error is similar to that of the elastic beam (Fig. 8a), however the curve is no longer symmetric.

Conclusion

A theoretical framework for characterizing the errors in Hybrid Simulation was presented. The method was applied to classical problems in continuum mechanics and the errors were presented with respect to analytically derived true solutions. The following is concluded from this study:

- There are unpredictable errors at and above the fundamental frequencies of the systems, with a large accumulation of errors near the resonant frequencies. This indicates that hybrid tests that excite the natural frequencies of a system may not be viable.
- The substructures of a hybrid test are subject to excitation at their individual natural frequencies in addition to the natural frequencies of the global system. These results have been corroborated by experiments [11].
- Efforts to further decrease the errors in the transfer system may result in little reduction in the global errors.
- The effect of damping reduces the errors near the frequency window of maximum damping defined by the Standard Linear Solid but overall errors outside this regime are comparable to the elastic case.

Acknowledgments

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