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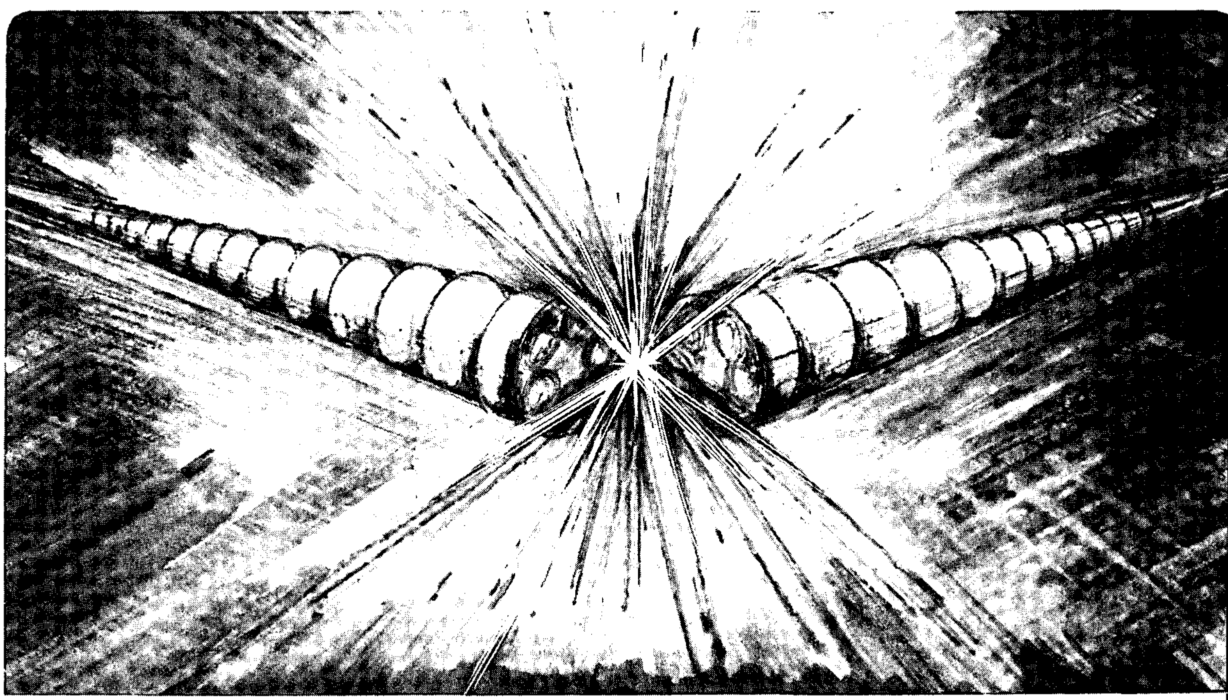

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HOLE-COUPLED RESONATORS FOR BROADLY TUNABLE INFRARED FREE ELECTRON LASERS*

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Hole-coupled resonators for broadly tunable infrared free electron lasers

ABSTRACT

We review the study of hole-coupled resonators for broadly tunable free electron laser (FEL) applications. The mode profiles inside and outside the cavity, the diffraction losses at the mirror edges and intracavity apertures, the amount of useful power coupled through the holes, and the FEL gain are calculated for several dominant azimuthal and radial modes. The FEL interaction is taken into account by constructing a propagator similar to the Fresnel integral for free space propagation. It is found that non-confocal resonators can provide efficient hole coupling over a broad wavelength range, as long as the mode beating caused by a degeneracy in the round trip loss can be avoided. The degeneracy between the azimuthally symmetric class of modes is removed by FEL interaction, and the azimuthally asymmetric modes can be suppressed by means of intracavity apertures. Therefore, in a non-confocal configuration, a hole-coupled resonator can be designed that is tunable over a broad range of wavelength by employing an adjustable intracavity aperture. On the other hand, confocal resonators are not suitable for hole coupling; Although mode beating does not occur in a confocal resonator, the hole coupling is difficult because the modes tend to avoid the hole. We provide a simple physical understanding of the difference in the performance of the confocal and non-confocal resonators. We also calculate and analyze the mode content of an empty resonator under continuous external mode injection. Such calculation is useful in interpreting experiments testing the hole coupling performance using CW lasers.

1. INTRODUCTION

An optimum use of a free electron laser requires an optical cavity that permits an efficient, broadly tunable output coupling of the coherent radiation. For an infrared (IR) FEL, a hole-coupled optical cavity with metal mirrors appears to satisfy this requirement. We have carried out an extensive study of such resonators in connection with the design of the IRFEL for the Chemical Dynamics Research Laboratory (CDRL) at Lawrence Berkeley Laboratory [1]. Hole-coupled resonator was used first in 1964 for gas lasers [2]. More recently it was used also for FELs [3,4]. Several authors have studied the scheme by either analytical or simulation approaches [5-12]. In this paper we summarize our study so far on this subject.

The criteria for the resonator design for the CDRL-FEL are as follows: The intracavity mode profile should have a good overlap with the electron beam for efficient energy extraction; A sufficient fraction of the energy should be coupled out through the hole; The out coupled mode should be of good quality for an efficient transport to the experimental station; A single cavity configuration with a fixed hole size should cover as wide a wavelength range as possible to avoid frequent change of sensitive optical components; And the mode should be insensitive to the fluctuations with the gain medium.

It is found that the confocal resonators and the non-confocal resonators (those sufficiently away from the confocal geometry) behave quite differently in their hole coupling performance; Non-confocal resonators are more effective for hole coupling than confocal resonators. This is because the amplitude of an axially symmetric mode in a non-confocal resonator is more or less peaked at the location of the hole, thus permitting an efficient outcoupling of the optical energy, while that in a confocal resonator tends to develop a null at the hole.

The limiting factor for the hole coupling performance of a non-confocal resonator is the phenomena of the mode switching, i.e., an abrupt switching of the cavity mode as the wavelength is continuously changed across a certain value [9]. The mode switching occurs when more than one dominant modes become loss-degenerate, i.e., have the same round trip loss. The mode switching must be avoided for a stable FEL operation. It is found that the FEL gain is very effective in breaking the degeneracy between the fundamental mode and azimuthally symmetric modes. Asymmetric modes could still become degenerate because they vanish on axis and therefore are not sensitive to the FEL gain. However, asymmetric modes can be suppressed by intracavity apertures. Therefore, a broadly tunable hole-coupled resonator can be designed with a fixed hole by employing an adjustable intracavity aperture. With a proper design, the coupling efficiency, which is defined by the ratio of the useful loss through the hole to the total loss, can be maintained at about 50%, and the output mode quality close to that of an Airy pattern, with 81% of out-coupled power in the fundamental free space Gaussian mode.

For confocal resonator, the mode switching does not occur, but the effective tuning range is smaller restricted by the fact that the mode tends to develop a null at the hole location. However, the confocal resonator also tends to maximize the gain by forming a mode having a better overlap with the electron beam. These properties can be useful for applications other than hole coupling [8,13].

An experimental study of the hole-coupled resonators neglecting the FEL gain may be conveniently carried out in a set-up where a resonator of properly scaled dimensions are continuously injected by a He-Ne laser. It is found that the steady state field profile of a continuously injected cavity can be made close to that of the eigenmode of the cavity by properly adjusting both the round trip phase factor and the transverse profile of the injected laser beam.

In section 2 we outline the model and approach that we used for the analysis and simulation. The calculation for the broadly tunable resonator design for the CDRL-FEL is discussed in Section 3. In Section 4, we compare the different performance of the confocal and non-confocal resonators, and offer a simple physical interpretation for the difference. The external mode injection is discussed in Section 5. Section 6 contains concluding remarks.

2. MODEL AND APPROACH

The cavity configuration under investigation is shown in Fig.(1), and consists of four elements; the right mirror with a hole, the left mirror with an adjustable aperture representing the variable mirror radius, and two intracavity apertures representing the undulator bore. The electron beam traversing the FEL interaction region provide a unidirectional gain in the region between aperture 1 and aperture 2. The important performance parameters are the fractional power loss per cavity round trip, the hole coupling efficiency defined as the ratio of the useful loss through the hole to the total loss, and the FEL gain.

The propagation through the FEL gain medium is represented by the following integral equation [9]:

$$E_m(r,z) = \int_0^b r'dr'\,[G_m(r,r',z,0) + g_0\, K_m(r,r',z,0)]\, E_m(r',0) \ , \tag{1}$$

where $E_m(r,z)$ is the azimuthal mode of order m, r and z are the radial and the axial coordinates, b is the radius of undulator bore. The function G_m is the Fresnel propagator given by

$$G_m(r,r',z,z') = \frac{(-i)^{m+1}k}{z-z'}\, J_m\!\left(\frac{krr'}{z-z'}\right) \exp\!\left[\frac{ik(r^2+r'^2)}{2(z-z')}\right] \ , \tag{2}$$

where J_m is the Bessel function of order m, $k=2\pi/\lambda$, λ is the laser wavelength. The term g_0K_m in Eq.(1) gives the correction to the free space propagation due to the FEL interaction. We have

$$K_m(r,r',z,0) = i\int_0^z dz'\int_0^{z'} dz''z''\, e^{-i\mu z''}\int_0^\infty r''dr''\ \rho(r'',z')\, G_m(r,r'',z,z')\, G_m(r'',r',z',z'') \ . \tag{3}$$

Here, $\mu=\nu/L$, ν is the FEL resonance parameter defined by $\nu=L[k_u-k(1+K^2)/2\gamma^2]$, L is the length of the undulator, γ is the relativistic energy factor, $k_u=2\pi/\lambda_u$, λ_u is undulator period, K is undulator deflection parameter, $\rho(r,z)$ is electron density profile. For propagation through free space, we set $g_0=0$. For propagation through the gain region, between aperture 1 and aperture 2, g_0 is given by (valid when the electron beam radius is constant along the undulator)

$$g_0 = \frac{8\sqrt{2}\,\pi^2\,\lambda^{3/2}\,\lambda_u^{1/2}\,N^3\,K^2}{\left(1+K^2\right)^{3/2}L^3}\,\frac{I}{I_A} \quad , \tag{4}$$

where I is the beam current, N is the number of undulator period, and I_A, the Alfvén current, is about 17030 Amperes. For simplicity, the undulator field is assumed to be helical. For a Gaussian electron beam profile Eq.(3) can be further simplified [9].

Equation (1) can be cast into a matrix form by discretizing the radial coordinate r. The total round trip matrix can be constructed by multiplying a suitable set of the free space propagation and FEL matrices for each azimuthal mode of order m. The radial modes are then determined by solving the eigenvalue problem for the round trip matrix. In general, there exists a unique set of eigenmodes for a given resonator, and each eigenmode is represented by a complex amplitude $E_{mn}(r,z)$, designated by an azimuthal mode number m and a radial mode number n. This mode will be referred to as the TEM_{mn} mode.

The FEL integral propagator, Eq.(3), was derived under the condition of small signal and low gain. The low gain assumption is generally valid for an oscillator FEL operating at saturation, assuming that the cavity loss is small. The resonator mode at saturation is determined by a simple model in which g_0 is varied until the gain balances the loss. In calculating the performance of an FEL oscillator, the electron beam size is determined by its emittance and the natural focusing of the undulator, assuming a round beam matched to the helical undulator. The undulator parameter K is determined for each wavelength by the FEL resonance condition.

3. BROADLY TUNABLE HOLE-COUPLED RESONATOR DESIGN FOR CDRL-FEL

To minimize the loss at the undulator bore, the Rayleigh range of an FEL resonator is usually chosen to be about one half of the undulator length. In the case of the CDRL-FEL, the resonator is then in the near concentric configuration, with design parameters listed in Table (1). We have carried out extensive numerical calculations for this case. The total loss for the first five low-loss radial modes in an empty cavity are shown in Fig.(2) for m=0 and m=1. The wavelength for this case is 3 µm, the hole radius is 2 mm, and the mirror radius is 32 mm for both mirrors. The transverse intensity profiles of TEM_{00}, TEM_{01} and TEM_{10} mode are show in Fig.(3a), Fig.(3b) and Fig.(3c), respectively. Notice that the dominant mode is the TEM_{10} mode in this case, which is not effective for out coupling since the intracavity amplitude vanishes at the mirror center where the hole is located. The unwanted azimuthally asymmetric modes can be suppressed by reducing effective mirror size, but this will lead to additional diffraction loss to the fundamental mode sacrificing the coupling efficiency. It turns out that the FEL interaction provides an active mode control enhancing the symmetric modes. This comes about because the symmetric modes overlap better with the electron beam relative to the asymmetric modes. Figure (4) shows the net losses (total loss minus gain) for the modes with m=0 in the same cavity as for Fig.(2), but with FEL gain. It is seen here the dominant mode is switched from the TEM_{01} mode for the empty cavity to the TEM_{00} mode by a gain medium contributing only 2.6% gain to the fundamental mode.

We have confirmed by extensive calculation that the FEL gain, even if small, can be very effective in selecting a dominant cavity mode [9,10]. Nevertheless, it is preferable to operate a laser sufficiently far from potentially degenerate configurations because fluctuations in the gain could cause a mode switching. Thus, passive mode control device such as intracavity apertures should be introduced to enhance the mode separation and guarantee the dominance of the preferred mode. As long as a mode remains dominant, its profile changes little with gain. Figure (5) illustrates this point by showing the intensity profiles at the right mirror for a single pass gain of 0%, 2.6% and 50%, respectively.

Figure (6) shows the wavelength dependence of the hole coupling performance for the CDRL-FEL, with parameters given in Table (2). The entire tuning range between 3 to 55 μm is divided into four overlapping subranges. In each subrange the hole radius is fixed while the radius of the left mirror is increased linearly with the wavelength. The dominance of the fundamental mode is maintained throughout the whole wavelength range by the combination of passive mode control due to the properly chosen radius for the left mirror and the active mode control due to the FEL gain medium. The radius of the right mirror is chosen in all cases to be large enough not to cause diffraction loss through the mirror edge. Note that the coupling efficiency and the loss remain close to 50 % and 10 % respectively. It should be emphasized that the variable mirror size is crucial for the good performance of the hole coupling in each subrange. Although not shown, the performance is not acceptable if the mirror radius were kept constant at the midrange value for each subrange; At short wavelength end, the mirror radius appears too big so that the TEM_{10} mode is the dominant one. On the other hand, the mirror radius appears too small at longer wavelength end so that the coupling efficiency is reduced. (In calculating the FEL gain for each mode, the beam current is determined by the steady state condition for the fundamental mode.)

4. COMPARISON BETWEEN CONFOCAL AND NON-CONFOCAL RESONATORS

It should be noted that all axially symmetric modes in the near concentric resonator discussed in the previous section have a peak intensity on axis even with the presence of the hole. In fact this turns out to be the case practically at any hole sizes and for nearly all resonator configurations as long as they are not too close to be confocal. The reason for this is as follows: In general the round-trip phase advances of different modes in a resonator are different. A linear superposition of different resonator modes is therefore not a resonator mode since it can not reproduce itself after one round-trip propagation. For a large mirror size without holes, these modes are the well-known Laguerre-Gaussian modes. As the mirror becomes smaller or as the hole becomes larger, these modes evolve independently. Thus, they cannot be combined to form a new mode with a different mode profile, for example, with a null at the location of the hole. This is also the reason behind the occurrence of the mode switching in non-confocal resonators.

The situation is different for a confocal resonator, for which all the radial modes with same azimuthal mode number have the same round trip phase advances. This is true at any mirror and hole sizes. A linear combination of different resonator modes may also be a resonator mode. When there is a hole on a mirror, the dominant mode corresponds to the combination that gives the lowest loss thus developing a null in the hole area [8]. Thus, the behavior of a confocal resonator is fundamentally different from that of a non-confocal one as long as the mirror size is not too small. For a given mirror size, a confocal resonator can support a set of a finite number of base modes that are not too lossy. As the mirror area is reduced, the number of modes in the base is also reduced, and less free parameters are available for the loss minimization.

Figure (7) shows the profile of the dominant axially symmetric mode of an empty confocal resonator with a hole on the right mirror. The mirror radius is 16 mm for both mirrors. Note that the mode almost completely avoids the hole. Keeping the hole radius fixed while reducing mirror radius to 10 mm, the dominant mode changes to what is shown in Fig.(8). In this case the total loss is 26% and the coupling efficiency is 44%. It can be proved that for confocal resonator the one-way losses at two mirror surfaces are exactly same at arbitrary hole and mirror sizes. This implies that the hole coupling efficiency for confocal resonator with a hole on one mirror has an exact upper limit of 50%.

Figure (9) shows the mode profile of the same resonator as for Fig.(7), except with FEL gain and a larger mirror radius of 25 mm. In the forward direction, the mode is focused around the electron beam to maximize the gain while at the same time developing a null at hole location to minimize loss. In the return pass, one would a priori expect a considerable amount of diffraction loss at the apertures; Because the mode is over-focused in the forward direction, it has to expand in the return pass. However, the loss at the apertures turns out to be quite smaller than expected, about 4%, because the mode in the confocal cavity arranges itself to minimize the net loss. Figure (10) shows the case where the aperture size is further increased from 10.5 mm to 15 mm. In this case, the mode indeed contracts more tightly around the beam in the forward pass and expands further out in the return pass. As the result, the gain is increased from 16% to 43%. Interestingly enough, the tightly focused waist around the unidirectional FEL gain medium also moves if the gain medium is displaced in the axial direction off the center of the cavity. Figure (11) shows the mode profile for the same cavity as for Fig.(10), except the undulator is moved 3 meters toward the left. The gain is further increased to 48%. The relevant parameters for the confocal resonator calculations presented in this section are given in Table (3).

5. CAVITY FIELD PROFILE UNDER CONTINUOUS EXTERNAL INJECTION

The mode calculations for an empty cavity can be benchmarked with a convenient, small scale experiment in which a He-Ne laser is continuously injected into an empty cavity through a beamsplitter. The intracavity field profile in this case can be analyzed as follows: After a steady state is reached, the following condition must be satisfied at a given intracavity transverse plane

$$E_c = E_{in} + e^{i\psi}\, \mathbf{M}\, E_c \tag{5}$$

where E_{in} and E_c denote the complex amplitudes of incident and circulating waves, **M** is the round-trip propagator matrix, $\psi=2kL_c$, and L_c is the cavity length. The incident amplitude at the reference plane can be expanded in terms of the eigenmodes of the resonator E_1, E_2, ..., as follows:

$$E_{in} = C_1E_1 + C_2E_2 + \ldots \tag{6}$$

Thus the circulating amplitude can be written in the form

$$E_c = (1-e^{i\psi}\,\mathbf{M})^{-1}E_{in} = \frac{C_1E_1}{1-e^{i(\psi+\phi_1)}|\lambda_1|} + \frac{C_2E_2}{1-e^{i(\psi+\phi_2)}|\lambda_2|} + \ldots \tag{7}$$

where $\lambda = |\lambda|\exp(i\phi)$ and λ is an eigenvalue. It is seen from Eq.(7) that the circulating amplitude is in general a combination of eigenmodes. The relative strength of eigenmodes depends on expansion coefficients $\{C_1,C_2,..\}$ and the round-trip phase factor ψ that can be experimentally controlled. Controlling the expansion coefficient $\{C_1,C_2,..\}$ is called the transverse mode matching and controlling the round-trip phase factor ψ is called the longitudinal mode matching. By a proper transverse and longitudinal mode matching, the cavity profile can be made close to any of the several dominant eigenmodes. Further details on the experiment is discussed in these proceedings [14].

6. CONCLUSION

We have shown that broadly tunable hole-coupled resonators can be designed in a non-confocal configuration by suppressing the mode switching by adjustable intracavity apertures. Confocal resonators are not suitable for the hole-coupling purpose. The difference between the confocal and non-confocal resonators can be explained by the fact that the modes of same azimuthal order are completely phase degenerate for a confocal resonator as opposed to that for a non-confocal resonator. The performance of the hole-coupled resonators can be tested in terms of a simple set-up in which the cavity is continuously injected by a CW laser.

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REFERENCES

1. K.-J. Kim, R. Byrns, S. Chattopadhyay, R. Donahue, J. Edighoffer, R. Gough, E. Hoyer, W. Leemans, J. Staples, B. Taylor and M. Xie, Proceedings of Fourteenth International Free Electron Laser Conference, Kobe, Japan, 1992, also LBL preprint PUB-5335(1992).
2. C.K.N. Patel, W.L. Faust, R.A. McFarlane and C.G.B. Garrett, Appl. Phy. Lett, 4(1964)18.
3. D.W. Feldman, H. Takeda, R.W. Warren, J.E. Sollid, W.E. Stein, W.J. Johnson, A.H. Lumpkin and R.B. Feldman, Nucl. Instr. Meth. A285(1989)11.
4. P.W. van Amersfoort, R.J. Bakker, J.B. Bekkers, R.W.B. Best, R.van Buuren, P.F.M. Delmee, B. Faatz, C.A.J. van der Geer, D.A. Jaroszynski, P. Manintveld, W.J. Mastop, B.J.H. Meddens, A.F.G. van der Meer, J.P. Nijman, D. Oepts, J. Pluygers, M.J. van der Wiel, W.A. Gillespie, P.F. Martin, M.F. Kimmit, C.R. Pidgeon, M.W. Poole and G. Saxon, Nucl. Instr. Meth., A318(1992)42.
5. D. E. McCumber, B.S.T.J., 44(1965)333 and B.S.T.J., 48(1969)1919.
6. G.T. McNice and V.E. Derr, IEEE J. Quan. Elect., 5(1969) 569.
7. M. Tsuji, H. Shigesawa and K. Takiyama, Applied Optics, 18(1979)1334.
8. R.H. Pantell. J. Feinstein and A.H. Ho, Nucl. Instr. Meth., A296(1990)638.
9. M.Xie and K.-J. Kim, Nucl. Instr. Meth. A304 (1991)792, A318(1992)877.
10. S. Krishnagopal, M. Xie, K.-J. Kim and A. Sessler, Nucl. Instr. Meth., A318(1992)661.
11. G.A. Barnett, S.V. Benson and J.M.J. Madey, "Hole Output Coupling in Mark III Free Electron Laser", These Proceedings.
12. B. Faatz, R.W.B. Best, P.W. van Amersfoort and D. Oepts, Nucl. Instr. Meth., A318(1992)665.
13. M. Ozcan and R.H. Pantell, "Free Electron Laser in a Confocal Resonators", These Proceedings.
14. W.P. Leemans, E.W. Wallace, M. Xie and K.-J. Kim, "Experimental Test of Hole-Coupled FEL Resonator Designs", These Proceedings.

TABLE 1. BASIC RESONATOR PARAMETERS FOR CDRL-IRFEL

Cavity Length [m]	24.6
Mirror Radius of Curvature [m]	12.38
Radius of Undulator Bore [mm]	10.5
Rayleigh length [m]	1
Undulator Length [m]	2
Undulator Period [cm]	5

TABLE 2. HOLE-COUPLING PARAMETERS FOR CDRL-IRFEL

Wavelength Range (μm)	Hole Radius (mm)	Left Mirror Radius (mm)	Beam Energy (MeV)	Normalized rms Emittance (mm-mrad)
3-6.84	2	19-35	55.3	7
6-13.7	2.8	30-50	39.1	8.7
12-27.4	4.4	45-65	27.7	11.1
24-54.7	6	60-100	19.6	11.1

TABLE 3. PARAMETERS FOR CONFOCAL RESONATOR CALCULATIONS

Wavelength [μm]	10
Electron Beam Energy [MeV]	30.3
Normalized rms Beam Emittance [mm-mrad]	30
Electron Beam rms Radius [mm]	0.73
Cavity Length [m]	14
Hole Radius [mm]	2
Radius of Undulator Bore [mm]	10.5 and 15

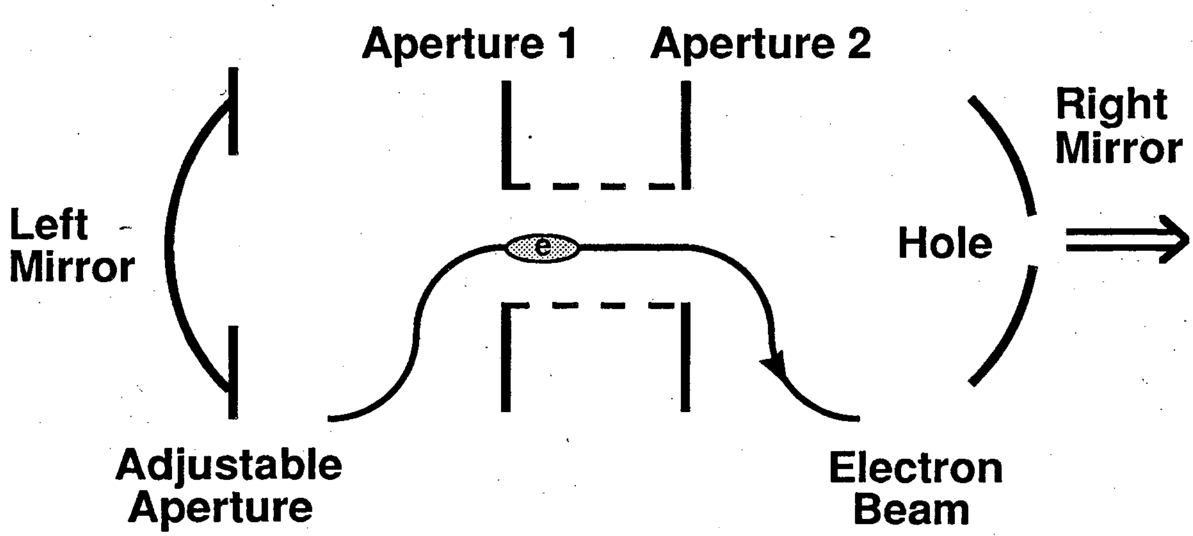

Figure 1. Side view of a cylindrical resonator consisting of two mirrors and two intracavity apertures. A circular hole is placed at the center of the right mirror. An adjustable aperture is placed in front of the left mirror for mode control. The dashed lines indicate the boundary of an undulator bore simulated by two fixed apertures. An electron beam traverses the FEL interaction region from aperture 1 to aperture 2.

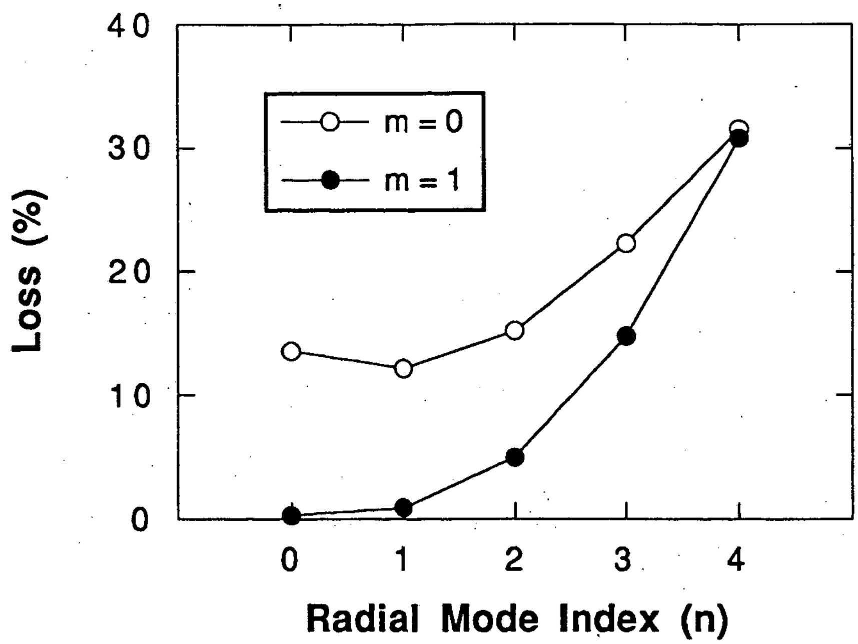

Figure 2. Total losses of five radial modes for m=0 and m=1 in an empty cavity.

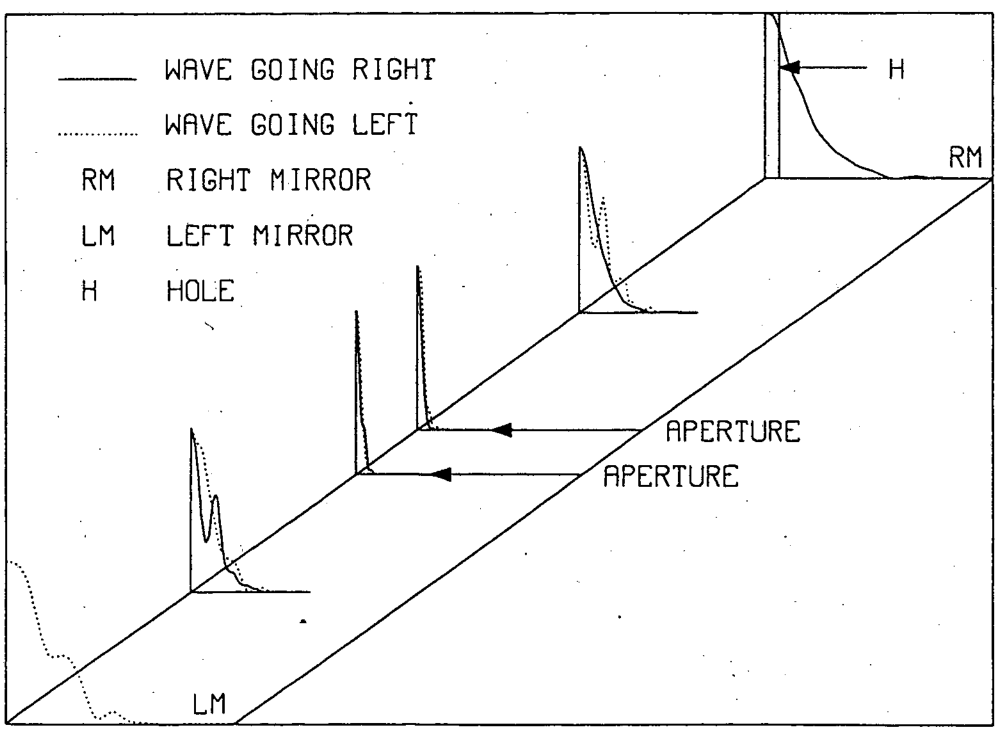

Fig.(3a)

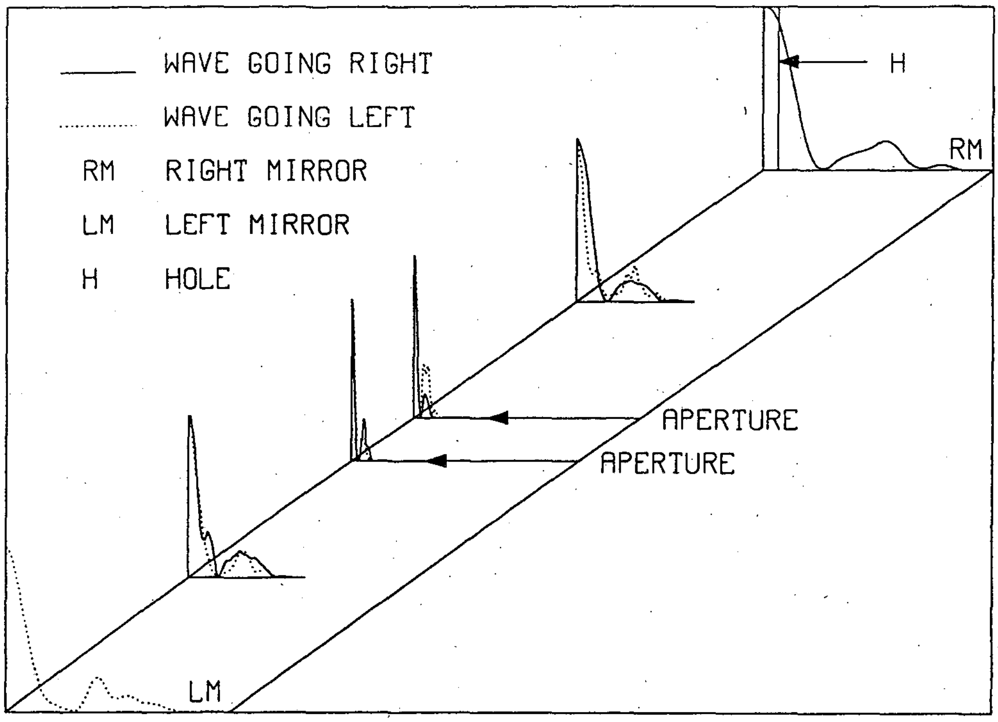

Fig.(3b)

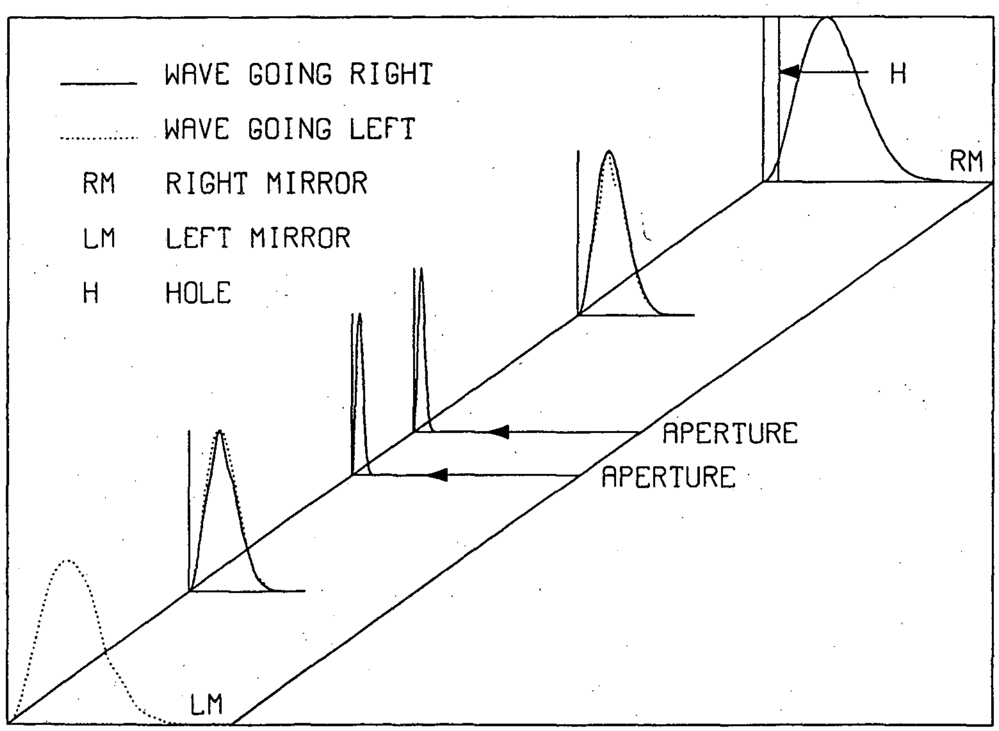

Fig.(3c)

Figure 3. Intensity profiles in the empty cavity for (a) TEM_{00} mode, (b) TEM_{01} mode and (c) TEM_{10} mode, respectively.

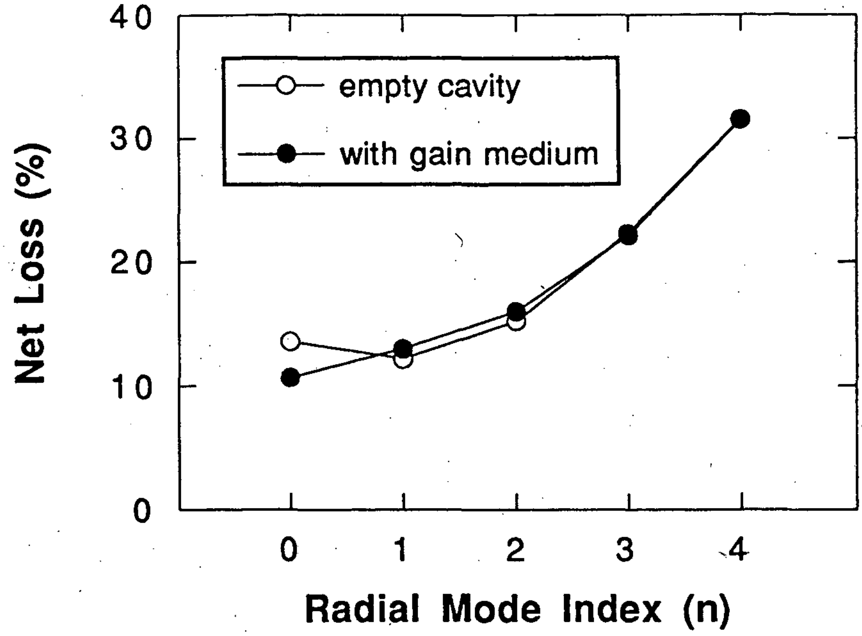

Figure 4. Net losses of five radial modes for m=0 in a cavity with and without FEL gain.

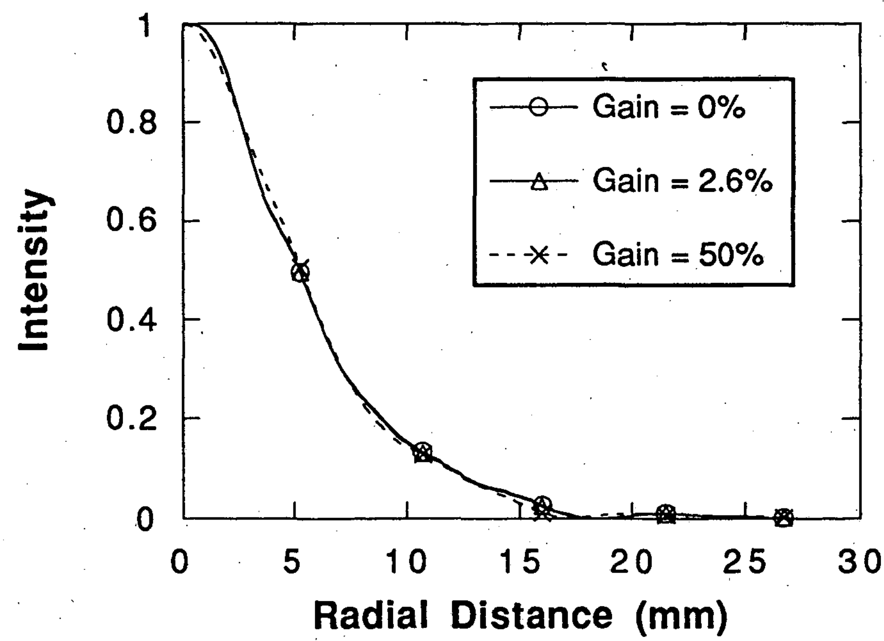

Figure 5. Intensity profiles of TEM_{00} mode at the right mirror with different FEL gains.

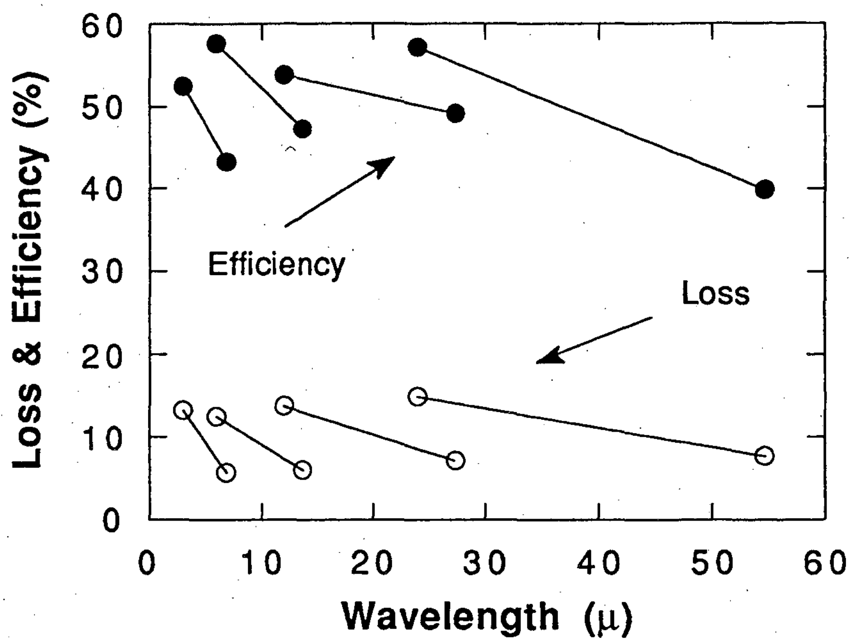

Figure 6. Hole coupling performance of the dominant TEM_{00} mode for the CDRL-IRFEL with FEL gain and an adjustable aperture, as functions of wavelength. Each line segment represents the wavelength scanning range with a fixed hole size, parameters are given in Table (2).

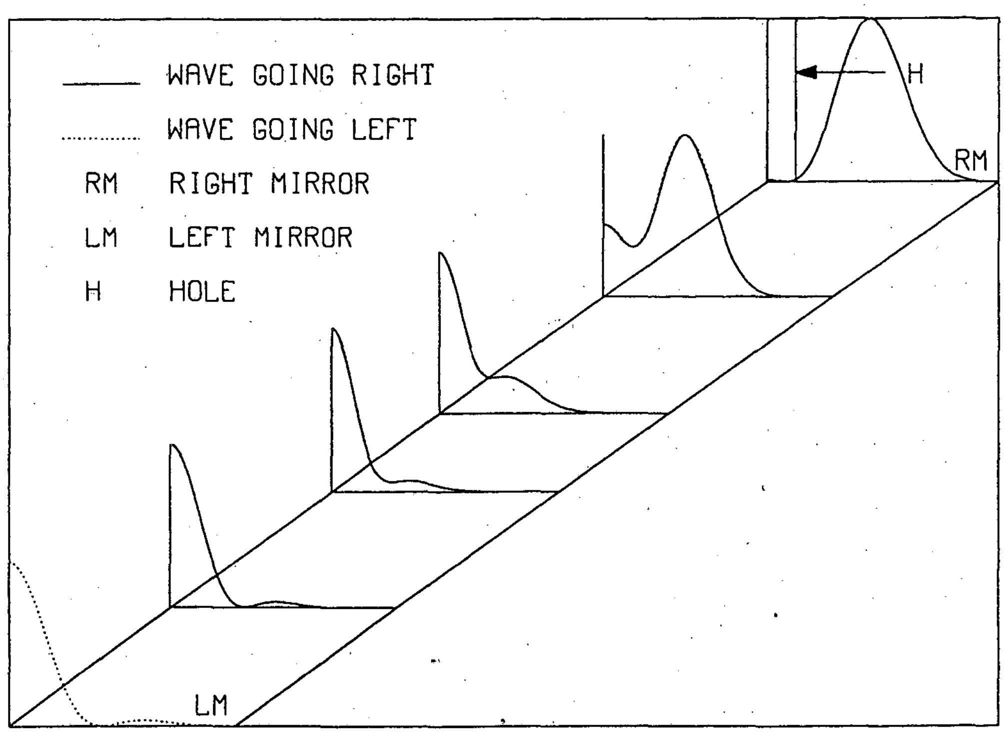

Figure 7. Intensity profile of the dominant azimuthally symmetric mode for an empty confocal resonator with 16 mm mirror radius.

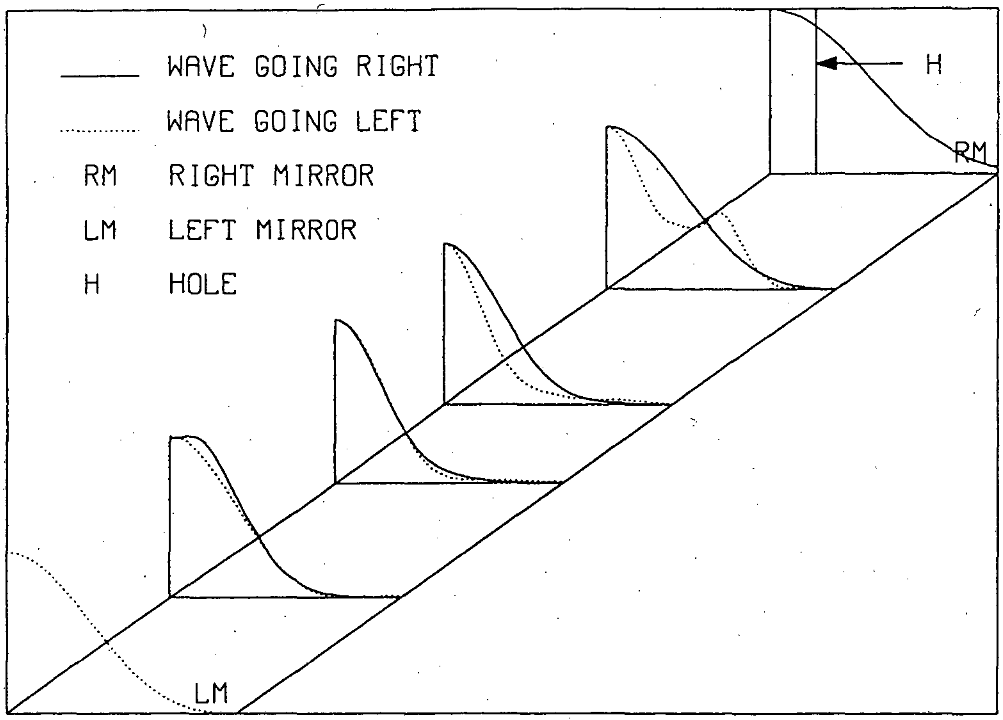

Figure 8. Same as for Fig.(7), except the mirror radii are reduced to 10 mm.

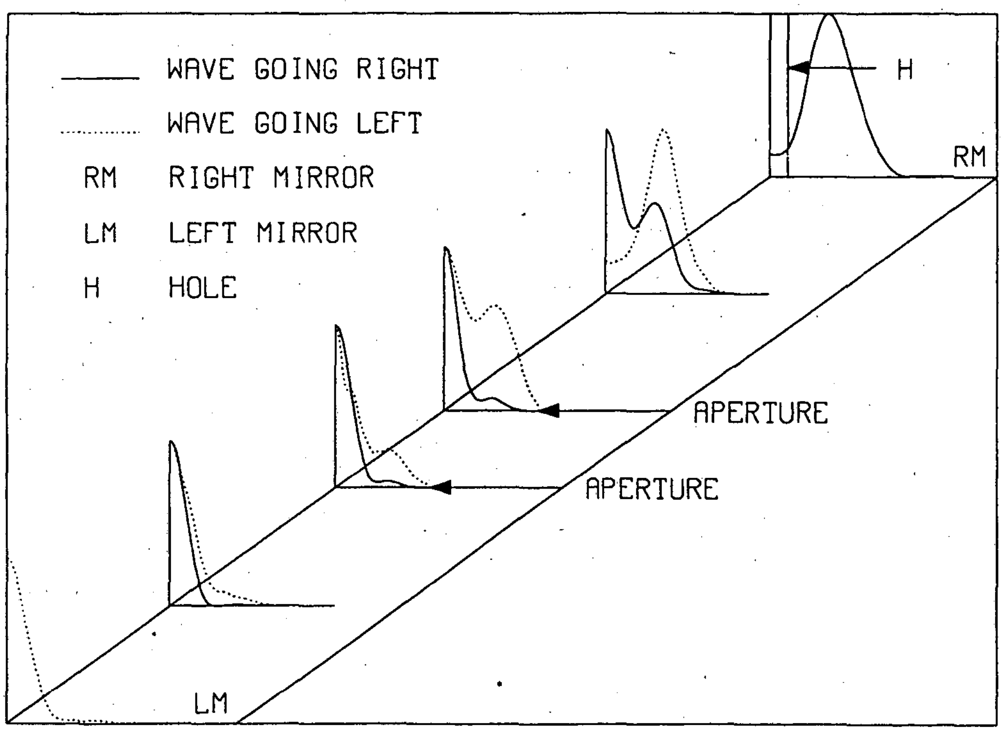

Figure 9. Intensity profile for a confocal resonator same as for Fig.(7), except with FEL gain and 25mm mirror radius, the radius of undulator bore is 10.5 mm.

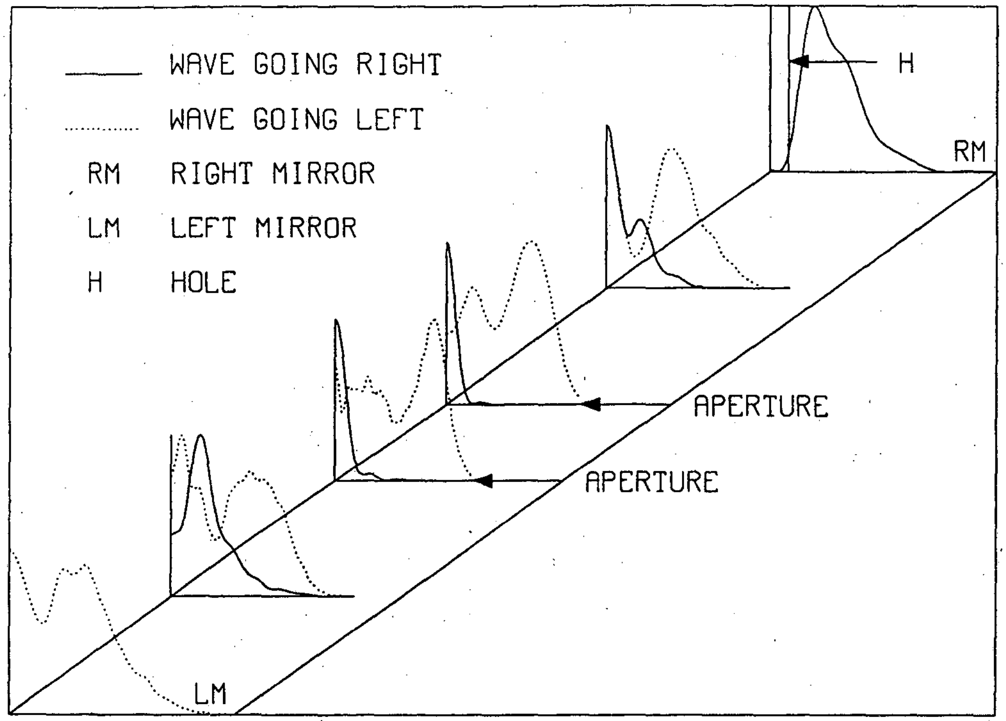

Figure 10. Same as for Fig.(9), except the radius of undulator bore is increased to 15 mm.

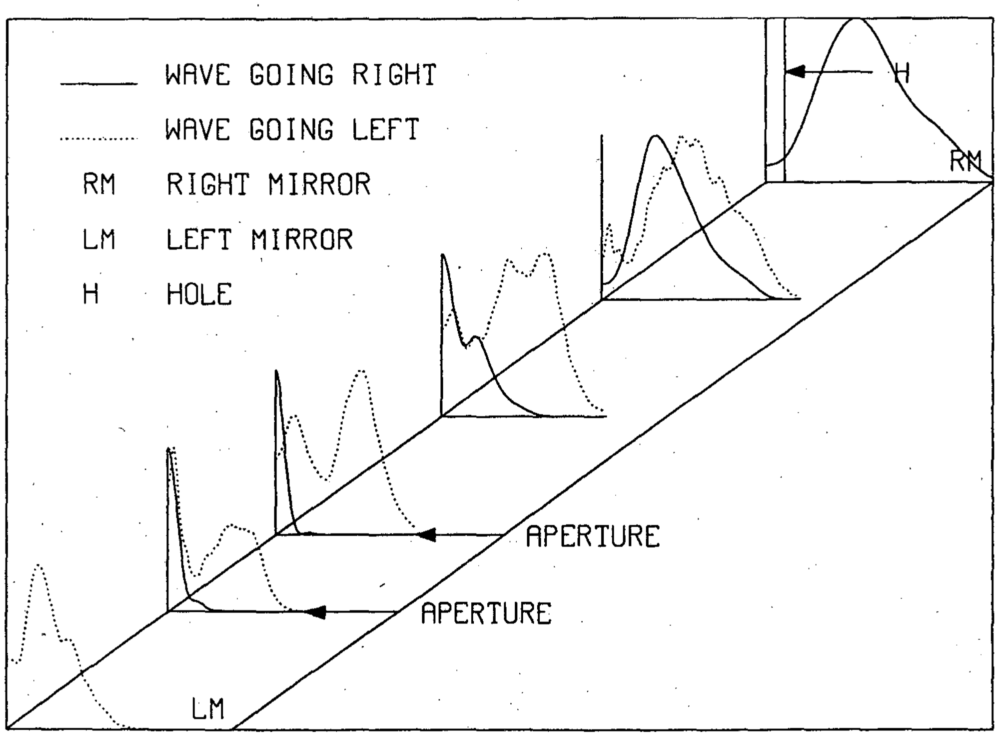

Figure 11. Same as for Fig.(10), except the undulator section is displaced 3 meters from the center of the cavity toward the left in the axial direction.

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