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HYDRODYNAMIC PRESSURES ON DAMS DURING EARTHQUAKES

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Department of Water Resources
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by

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SYNOPSIS

In this report the intensity, distribution and frequency characteristics of hydrodynamic pressures acting on dams during earthquake are discussed. First is presented a short review of previous work on this subject. Then an analysis is made of the pressures generated by the vertical as well as the horizontal components of the earthquake acceleration. In this analysis it is observed that the dam face is vertical and rigid, but the full compressibility effect of the water are considered.

The results of the study show that any significant discrepancies exist between theory and the standard design procedures for tracing hydrodynamic pressures due to earthquakes. It is evident that further consideration must be given to this factor in the design of dams in seismic regions.

HYDRODYNAMIC PRESSURES ON DAMS DURING EARTHQUAKES

Introduction

Westergaard's classical work(1) has long influenced designers in considering hydrodynamic forces on dams during earthquakes. Assumptions involved in his solution are that

(i) the dam is infinitely long and has a vertical upstream face

(ii) the reservoir extends to infinity in the upstream direction

(iii) effect of surface waves is ignored, and

(iv) the dam is rigid.

Considering the compressibility of water, the problem was solved for a harmonic ground motion in the horizontal direction perpendicular to the dam axis. The hydrodynamic pressure on the dam is shown to be opposite in phase to the ground acceleration, and thus is equivalent to the inertia forces of a "virtual mass" moving with the dam. The magnitude of this mass depends on frequency of the harmonic ground motion. Fourier spectra of past earthquakes (2) show significant amplitudes over a wide range of frequencies. Therefore the virtual mass concept would not be applicable in determining hydrodynamic responses to earthquake-type excitations.

Kotsubo(3) has shown that Westergaard's solution is valid only when the period of excitation is greater than the fundamental resonance period for water pressure.

Brahtz and Heilbron(4) demonstrated that if the reservoir is of finite length (in the upstream direction) with the upstream end immovable, the pressure increases not more than 0.5 percent if $L/H > 2$ (L is the length and H the depth of the reservoir). If the upstream end of the reservoir is assumed to vibrate with the ground, the effect of length is negligible for $L/H > 3$. The experimental results of Hoskins and Jacobsen[5] support the above conclusions. The work of Werner and Sundquist[6] also indicates that the pressure response is not sensitive to reservoir length. Recently, Bustamante et al[7] investigated the effect of length of reservoir for a range of periods of excitation wider than had been considered before. It is concluded that the influence is negligible for periods of excitation greater than the first resonance period of water pressure. For shorter periods the length plays an important role in stationary harmonic motion. The pressure distribution over the height is significantly affected by the length in this case. For a reservoir of depth H, significance of the effect of length on the hydrodynamic pressure response to an earthquake type excitation depends on the frequency characteristics of the excitation. The length will have an important influence if there are significant harmonics in the excitation with periods shorter than the first resonance period of water pressure. From values of first resonance period[7] and Fourier spectra of past earthquakes[2] it seems that reservoir length may be of some importance in case of high dams. For low dams most of the significant harmonics of ground motion have a period longer than the first resonance period and the reservoir length will have little influence on the pressures provided $\frac{L}{H} > 3$. In the present work the reservoir is taken infinetly long because in a typical practical situation reservoir behind the dam extends to a large distance.

Many authors have considered compressibility of water [1,3,5-8]. The assumption of incompressible water greatly simplifies the problem and therefore has been the subject of considerable study[7,6, 9-12]. As pointed out by Bustamante et al[7], neglecting compressibility completely changes the nature of the hydrodynamic problem. It leads to a solution which does not depend on the period of vibration for stationary harmonic motion. It is interesting to note that if the hydrodynamic pressure is independent of period of excitation then the solution for an arbitrary ground acceleration depends on its instantaneous value only. Later we shall present a proof for this. Clearly, if the pressures at any time depend only on the value of ground acceleration at that instant of time, the hydrodynamic pressures are equivalent to the inertia effects of a "Virtual mass" moving with the dam.

However, it has been shown[7] that although the assumption of incompressibility is reasonable for small values of H/T (H is depth of reservoir, T the period of harmonic ground motion), the errors become extremely large for values of T close to the resonance periods. This is particularly so for values of T close to first resonance period[7]. For an earthquake type excitation which consists of a wide range of frequencies, the major contribution to pressure response will be from those harmonic components with frequency "close" to the resonant frequences of the system. Because it is in this range of frequencies that the assumption of incompressibility leads to unacceptable errors, compressibility of water is considered in this work.

Most investigators have ignored the waves that may be generated at the free surface of water. Chen' Chzhen'-Chen[10,11] allowed for the existence of a surface wave in an incompressible fluid. Including compressibility, for harmonic excitations Bustamante et al[7] concluded that error e introduced by ignoring surface waves varies as follows

(i) $e < 0.05$ if $H/T > 4.2\sqrt{H}$

(ii) $0.05 < e < 0.20$ if $2.6\sqrt{H} < H/T < 4.2\sqrt{H}$

(iii) $e > 0.20$ if $H/T < 2.6\sqrt{H}$

(H is the reservoir depth in metres, T is the period of excitation in seconds).

Most of the significant harmonics in typical strong ground motion have period below 3 secs. Thus, from (i) for T = 3 secs., the error $e < 0.05$ if $H > 158.5$ m = 520 ft. For a reservoir depth 100 ft., $e < 0.05$ if $T < 1.32$ secs. Because the resonant periods of the system are very small (first resonance period for 100 ft. depth of water is 0.085 sec.), the contributions to the total response from harmonic components of ground motion with periods longer than 1.32 sec. will be small. This is particularly so because we are dealing with an ideal fluid. Hence, it may be concluded that errors introduced by neglecting surface waves is of the order of 0.05. Similar conclusions can be derived for other reservoir depths between 100 ft. and 520 ft. On basis of these arguments, it seems that the effect of surface waves can be ignored with little loss of accuracy.

Zangar [9] determined hydrodynamic pressure for various configurations of upstream face of the dam. Water was taken to be incompressible and therefore an electric analogy could be used. Amongst other interesting results, it was concluded that hydrodynamic pressures on a dam having an upstream face vertical for half or more of the total height will be practically same as when the upstream face is vertical for the full height. The upstream face of concrete gravity dams is almost always vertical or almost vertical for a major part of the height. Therefore, on basis of Zangar's conclusions, although they are strictly valid only when the compressibility is ignored, we shall limit our interest to the case of a vertical upstream face.

All the discussion above has been concerning responses to horizontal ground motion. Very little attention has been given to the corresponding problem for vertical ground motion. Kotsubo[13] determined the hydrodynamic response of a reservoir behind an arch dam for a harmonic ground motion. Chen' Chzhen"-Chen[11] considered the problem assuming water as incompressible.

In the present work, expressions for complex frequency responses are derived and their variation with excitation frequency studied. In case of vertical ground motion the errors associated with neglecting waves at the surface of water are discussed. Expressions for responses to arbitrary ground motions are derived. Responses to El Centro, 1940 earthquake are determined by a numerical integration scheme, for three reservoir depths. Implications of neglecting compressibility of water are discussed with reference to complex frequency responses as well as responses to earthquake type excitations. Deficiencies in the present design practice are also discussed.

Equations of Motion

Neglecting the viscosity effects of water and considering the movement as limited to small amplitudes, the motion of water is governed by the wave equation

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = \frac{W}{gk} \frac{\partial^2 \Phi}{\partial t^2} \tag{1}$$

where $\Phi(x,y,t)$ is a velocity potential such that

$$\frac{\partial u}{\partial t} = - \frac{\partial \Phi}{\partial x} \tag{2}$$

$$\frac{\partial v}{\partial t} = - \frac{\partial \Phi}{\partial y} \tag{3}$$

where u,v are respectively the x and y components of displacement of water in the coordinate system of Figure 1,

W is the unit weight of water,

g is the acceleration of gravity, and

k is the bulk modulus for water.

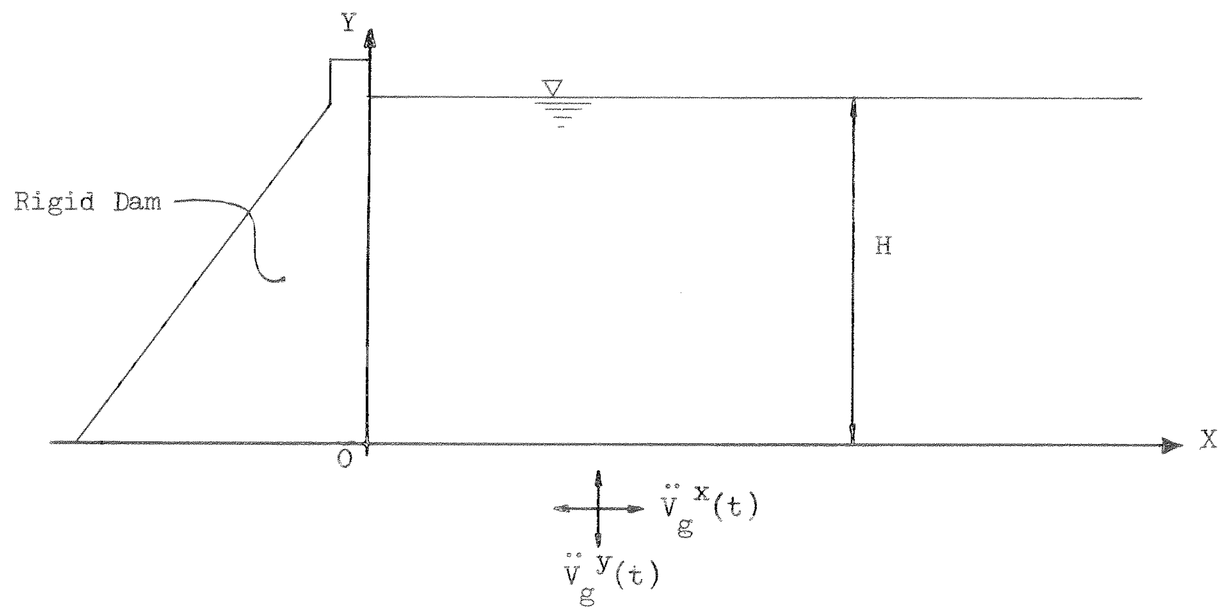


FIG. 1 COORDINATE SYSTEM

The dynamic water pressure $p = p(x,y,t)$ is related to the gradients of displacement:

$$p = -k\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \qquad (4)$$

In terms of the potential Φ,

$$p = \frac{W}{g}\ \frac{\partial \Phi}{\partial t} \qquad (5)$$

The velocity of pressure waves in water is

$$C = \left(\frac{gk}{W}\right)^{\frac{1}{2}} \qquad (6)$$

and hence the term $\frac{W}{gk}$ in Eq. 1 can be replaced by $\frac{1}{C^2}$.

Response to Horizontal Ground Motion

We shall consider an infinitely long dam with a vertical upstream face and the reservoir extending to infinity in the upstream direction. The dam is considered rigid. Compressibility of water is considered. Effect of surface waves is ignored.

Side Conditions

The boundary conditions for the problem are

(i) at the bottom, the velocity in y direction is zero.

(ii) neglecting the effect of surface waves, the water pressure at the surface is zero.

(iii) the bottom of the reservoir undergoes a prescribed horizontal motion; because the dam is considered rigid this implies the same prescribed motion of the dam.

(iv) the motions become small at large distances upstream of the dam.

These physical conditions may be expressed as

$$\frac{\partial \Phi}{\partial y}(x,o,t) = 0 \tag{7}$$

$$\frac{\partial \Phi}{\partial t}(x,H,t) = 0 \tag{8}$$

$$\frac{-\partial}{\partial t}\left(\frac{\partial \Phi}{\partial x}\right)(o,y,t) = \ddot{V}_g^{\,x}(t) \tag{9}$$

where $\ddot{V}_g^{\,x}(t)$ is the prescribed time history of ground acceleration, the superscript x denoting that the ground motion is in the direction of x-axis.

$$\Phi \rightarrow o \text{ as } x \rightarrow \infty \tag{10}$$

Considering the pressure $p = p(x,y,t)$ as the dynamic pressure which is zero at $t = 0$, the initial conditions are

$$\Phi(x,y,o) = 0 \tag{11}$$

$$\frac{\partial\Phi}{\partial t}(x,y,o) = 0 \tag{12}$$

General Remarks on Method of Solution

Procedures suitable for solving Eq. 1 for an arbitrary excitation $\ddot{V}_g^{\,x}(t)$ are: the use of a complex frequency response together with the Fourier Integral, and the use of an impulse response together with the convolution integral. These two methods are intimately related since they are essentially Fourier transforms of each other.

The latter approach is usual y advantageous for arbitrary excitations such as earthquakes. However, in this problem it is desirable to determine the complex frequency response because it provides a direct comparison with the results obtained by Westergaard[1]. It also provides a complete picture of the variation of response with excitation frequency. The unit impulse response is then obtained by determining the inverse Fourier transform of the complex frequency response.

Complex Frequency Response

It is a property of linear time invariant systems that when the excitation is steady state simple harmonic motion (without beginning or end) then the response is also steady state simple harmonic motion at the same frequency. The amplitude and phase of the response generally depend on the frequency. The frequency dependence of the amplitude and phase is described by the complex frequency response $H(\omega)$. This has the property that when the excitation is the real part of $e^{i\omega t}$ then the response is the real part of $H(\omega)\, e^{i\omega t}$

Let $\ddot{V}_g^{\,x} = 1.\ e^{i\omega t}$

Equation 9 becomes $$\frac{-\partial}{\partial t}\left(\frac{\partial \Phi}{\partial x}\right)(o,y,t) = 1.\ e^{i\omega t} \tag{13}$$

The solution for velocity potential will be of the form

$$\Phi(x,y,t) = H_{\Phi}^{x}(x,y,\omega)\, e^{i\omega t} \tag{14}$$

where $H_{\Phi}^{x}(x,y,\omega)$ is the complex frequency response for Φ. Substituting Eq. 14 in Eq. 1 and using Eq. 6 we get the Helmholtz equation in the unknown function H_{Φ}^{x}:

$$\frac{\partial^2 H_{\Phi}^{x}}{\partial x^2} + \frac{\partial^2 H_{\Phi}^{x}}{\partial y^2} + \frac{\omega^2}{C^2} H_{\Phi}^{x} = 0 \tag{15}$$

From Eq. 7 through 10 the boundary conditions on H are

$$\frac{\partial H_{\Phi}^{x}}{\partial y}(x,o,\omega) = 0 \tag{16}$$

$$H_{\Phi}^{x}(x,H,\omega) = 0 \tag{17}$$

$$\frac{\partial H_{\Phi}^{x}}{\partial x}(o,y,\omega) = \frac{-1}{i\omega} \tag{18}$$

$$H_{\Phi}^{x} \to 0 \quad \text{as } x \to \infty \tag{19}$$

Any expression of the form

$$H_{\Phi}^{x}(x,y,\omega) = e^{\alpha x}\left[A\mathrm{Cos}\sqrt{\alpha^2 + \frac{\omega^2}{C^2}}\, y + B\mathrm{Sin}\sqrt{\alpha^2 + \frac{\omega^2}{C^2}}\, y\right] \tag{20}$$

is a solution of Eq. 15, for any choice of A, B and α. Applying the boundary condition 16 gives

$$e^{\alpha x}\sqrt{\alpha^2 + \frac{\omega^2}{C^2}}\, B = 0$$

which can be satisfied for all x and ω only if

$$B = 0$$

Eq. 20 becomes

$$H_\Phi^x(x,y,\omega) = e^{\alpha x}\, A \,\mathrm{Cos}\sqrt{\alpha^2 + \frac{\omega^2}{C^2}}\, y$$

Imposing the boundary condition 17

$$e^{\alpha x}\, A \,\mathrm{Cos}\sqrt{\alpha^2 + \frac{\omega^2}{C^2}}\, H = 0$$

For non trivial solutions

$$\mathrm{Cos}\sqrt{\alpha^2 + \frac{\omega^2}{C^2}}\, H = 0$$

$$\text{i.e.} \sqrt{\alpha^2 + \frac{\omega^2}{C^2}}\, H = \frac{(2m-1)\pi}{2} \qquad m = 1,\ 2,\ 3\ \ldots$$

$$\text{Therefore, } \alpha = \pm\sqrt{\lambda_m^2 - \frac{\omega^2}{C^2}}, \qquad m = 1,\ 2,\ 3\ \ldots \tag{21}$$

$$\text{where } \lambda_m = \frac{(2m-1)\pi}{2H} \qquad m = 1,\ 2,\ 3\ \ldots \tag{22}$$

Hence $H_\Phi^x(x,y,\omega)$ may be represented as an infinite series

$$H_\Phi^x(x,y,\omega) = \sum_{m=1}^{\infty} A_m \exp\left[-x\sqrt{\lambda_m^2 - \frac{\omega^2}{C^2}}\right] \mathrm{Cos}\, \lambda_m y \tag{23}$$

the sign in Eq. 21 having been chosen to satisfy Eq. 19. The constants A_m will now be determined from the condition of Eq. 18.

From Eq. 23

$$\frac{\partial H_\Phi^x}{\partial x}(x,y,\omega) = \sum_{m=1}^{\infty} - A_m \sqrt{\lambda_m - \frac{\omega^2}{C^2}} \exp\left[-x\sqrt{\lambda_m^2 - \frac{\omega^2}{C^2}}\right] \text{Cos } \lambda_m y$$

From Eq. 18

$$\frac{\partial H_\Phi^x}{\partial x}(o,y,\omega) = \sum_{m=1}^{\infty} - A_m \sqrt{\lambda_m^2 - \frac{\omega^2}{C^2}} \text{ Cos } \lambda_m y = -\frac{1}{i\omega} \tag{24}$$

Multiplying both sides of Eq. 24 by Cos $\lambda_n y$, and integrating over (O,H) and utilizing the orthogonality property of cosine functions we get

$$A_n = \frac{4}{i\omega} \frac{(-1)^{n-1}}{\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}\ (2n-1)\pi} \tag{25}$$

Substituting Eq. 25 in Eq. 23

$$H_\Phi^x(x,y,\omega) = \frac{4}{i\omega\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}} \exp\left[-x\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}\right] \text{Cos } \lambda_n y \tag{26}$$

From Eq. 14

$$\Phi(x,y,t) = \frac{4}{i\omega\pi} e^{i\omega t} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}} \exp\left[-x\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}\right] \text{Cos } \lambda_n y \tag{27}$$

Therefore from Eq. 5 the dynamic pressure is given by

$$p^x(x,y,t) = \frac{4W}{\pi g} e^{i\omega t} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}} \exp\left[-x\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}\right] \text{Cos } \lambda_n y \tag{28}$$

The complex frequency response for dynamic water pressure is, therefore,

$$H_p^x(x,y,\omega) = \frac{4W}{\pi g}\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}} \exp\left[-x\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}\right] \operatorname{Cos} \lambda_n y \tag{29}$$

It is apparent from Eq. 29 that, in general H_p^x is a complex valued function.

Comparison with Westergaard's solution

Taking the real part of Eq. 28 and multiplying by $-\alpha g$ gives the response for $\ddot{V}_g^x = -\alpha g \cos \omega t$, which is

$$p^x(x,y,t) = \frac{-4\alpha W}{\pi}\left[\sum_{n=1}^{n_s-1} \frac{(1)^{n-1}\operatorname{Cos} \lambda_n y}{(2n-1)\sqrt{\frac{\omega^2}{C^2} - \lambda_n^2}} \operatorname{Sin}\left(\omega t - x\sqrt{\frac{\omega^2}{C^2} - \lambda_n^2}\right)\right.$$

$$\left. + \operatorname{Cos} \omega t \sum_{n=n_s}^{\infty} \frac{(-1)^{n-1} \operatorname{Cos} \lambda_n y}{(2n-1)\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}} \exp\left(-x\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}\right)\right] \tag{30}$$

where n_s = minimum value of n such that

$$\lambda_n^2 > \frac{\omega^2}{C^2}$$

The pressure on the dam is given by Eq. 30 for x = o,

$$p^x(o,y,t) = -\frac{4\alpha W}{\pi}\left[\operatorname{Sin} \omega t \sum_{n=1}^{n_s-1} \frac{(-1)^{n-1} \operatorname{Cos} \lambda_n y}{(2n-1)\sqrt{\frac{\omega^2}{C^2} - \lambda_n^2}}\right.$$

$$+ \operatorname{Cos} \omega t \sum_{n=n_s}^{\infty} \frac{(-1)^{n-1} \operatorname{Cos} \lambda_n y}{(2n-1)\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}} \tag{31}$$

Eq. 31 is directly comparable with Westergaard's solution, which using the above notation is

$$p^x(o,y,t) = -\frac{4\alpha w}{\pi} \cos \omega t \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cos \lambda_n y}{(2n-1)\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}} \tag{32}$$

It is apparent that both solutions are identical if $\lambda_1^2 > \frac{\omega^2}{C^2}$ In Eq. 32, each term of the series represents pressure either with no phase shift or a phase shift of 180° relative to the ground acceleration.

From Eq. 30 the resonant periods T_n for the dynamic water pressure are given by the condition

$$\lambda_n = \frac{\omega}{C} \quad n = 1,2,3 \ldots$$

or,

$$\frac{(2n-1)\pi}{2H} = \frac{\omega}{C}$$

or,

$$\frac{2\pi}{\omega} = \frac{4H}{(2n-1)C} \tag{33}$$

Therefore the resonant periods T_n are

$$T_n = \frac{4H}{(2n-1)C} \quad n = 1,2,3, \ldots$$

Thus it is clear that Westergaard's solution is valid only if the period of excitation is greater than the first resonant period for water pressure. This has also been established by Kotsubo[3].

Unit Impulse Response

The response to ground motion

$$\ddot{V}_g^x(t) = \delta(t)$$

where $\delta(t)$ is the Dirac delta function, is called the unit impulse response. The complex frequency response is the Fourier transform of the unit impulse response. The Fourier Integral representation of the unit impulse response $h^x(x,y,t)$ is therefore

$$h^x(x,y,t) = \frac{1}{2\pi} \int_{\infty}^{\infty} H_p^x(x,y,\omega) \quad e^{i\omega t} \, d\omega \tag{34}$$

where the superscript x denotes response to a horizontal ground motion.

From Tables of Integral Transforms(14)

$$\text{Inverse Fourier Transform of } \frac{\exp\left[-x\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}\right]}{C\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}}$$

$$= \begin{cases} 0 & , \quad t < \frac{x}{C} \\ J_o(\lambda_n\sqrt{(Ct)^2 - x^2}) & , \quad t > \frac{x}{C} \end{cases}$$

where J_o represents the Bessel Function of first kind of order zero.

The inverse Fourier transform of $H_p^x\,(x,y,\omega)$ (Eq. 29) i.e. the unit impulse response (Eq. 34) is given by

$$h^x(x,y,t) = \begin{cases} 0 & , \quad t < \frac{x}{C} \\ \frac{4WC}{\pi g} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \quad \text{Cos } \lambda_n y \; J_o(\lambda_n\sqrt{(Ct)^2 - x^2}), & \quad t > \frac{x}{C} \end{cases} \tag{35}$$

In particular the unit impulse response for the dynamic water pressure at face of the dam is

$$h^x(o,y,t) = \frac{4WC}{\pi g} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \quad \text{Cos } \lambda_n y \; J_o(\lambda_n Ct) \quad , \; t > o \tag{36}$$

Kotsubo obtained the above result by the Laplace Transform technique(3).

It is noted that although water was considered as an ideal fluid in which case there is no energy dissipating mechanism, the unit impulse response (Eq. 36) does show a decay with time. This decay of response is directly associated with the fact that the complex frequency response function has an imaginary part for certain range of frequencies. Physically it represents the loss of energy in waves moving away from the dam.

Pressure Response to Arbitrary Ground Motion

The dynamic water pressure on face of dam due to an arbitrary excitation $\ddot{V}_g^x(t)$ can be obtained from the unit impulse response (Eq. 36) by the superposition or convolution integral. Thus,

$$p^x(o,y,t) = \frac{4WC}{\pi g} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \, \text{Cos}\, \lambda_n y \int_0^t \ddot{V}_g^x(\tau)\, Jo\left[\lambda_n c(t-\tau)\right] d\tau \qquad (37)$$

Response to Vertical Ground Motion

The assumptions and limitations are as before except that the effect of surface waves is now included.

Side Conditions

Without going into details, the boundary conditions when the boundary y = o is subjected to a known excitation $\ddot{V}_g^{\,y}(t)$ (the superscript y denoting ground motion in direction of y axis) are

$$-\frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial y}\right)(x,o,t) = \ddot{V}_g^{\,y}(t) \qquad (38)$$

$$\frac{\partial^2 \Phi}{\partial t^2}(x,H,t) + g\frac{\partial \Phi}{\partial y}(x,H,t) = 0 \qquad (39)$$

$$\frac{\partial \Phi}{\partial x}(o,y,t) = 0 \qquad (40)$$

$$\Phi \text{ is bounded as } x \longrightarrow \infty \qquad (41)$$

Eq. 39 represents a wave condition at the free surface of water. If the wave motion is ignored, the condition becomes

$$\frac{\partial \Phi}{\partial t}(x,H,t) = 0 \qquad (42)$$

We shall investigate into the errors involved by ignoring the existence of surface waves.

The initial conditions are given by Eq. 11 and 12.

Complex Frequency Response

Let $\ddot{V}_g^{\,y} = 1\ e^{i\omega t}$

Eq. 38 becomes

$$-\frac{\partial}{\partial t}\left(\frac{\partial \Phi}{\partial y}\right)(x,o,t) = e^{i\omega t} \qquad (43)$$

The solution for velocity potential will be of the form

$$\Phi(x,y,t) = H_{\Phi}^{y}(x,y,\omega)\, e^{i\omega t} \tag{44}$$

The problem becomes one of solving the Helmholtz eq. (15) in the unknown function H_{Φ}^{y} subject to the boundary conditions

$$\frac{\partial H_{\Phi}^{y}}{\partial y}(x,o,\omega) = \frac{-1}{i\omega} \tag{45}$$

$$g\frac{\partial H_{\Phi}^{y}}{\partial y}(x,H,\omega) - \omega^2 H_{\Phi}^{y}(x,H,\omega) = 0 \tag{46}$$

$$\frac{\partial H_{\Phi}^{y}}{\partial x}(o,y,\omega) = 0 \tag{47}$$

$$H_{\Phi}^{y} \text{ is bounded as } x \longrightarrow \infty \tag{48}$$

Any expression of the form

$$H_{\Phi}^{y}(x,y,\omega) = e^{\alpha x}\left[A \operatorname{Cos}\sqrt{\alpha^2+\frac{\omega^2}{C^2}}\; y + B \operatorname{Sin}\sqrt{\alpha^2+\frac{\omega^2}{C^2}}\; y\right]$$

is a solution of Eq. 15.

Applying boundary condition 47 we get $\alpha = 0$.

Therefore, $$H_{\Phi}^{y}(x,y,\omega) = A \operatorname{Cos}\frac{\omega y}{C} + B \operatorname{Sin}\frac{\omega y}{C} \tag{49}$$

Applying now the boundary conditions at $y = 0$ (Eq. 45) and $y = H$ (Eq. 46) leads to two equations in the unknown constants; solving for A and B and substituting in Eq. 49 gives

$$H_{\Phi}^{y}(x,y,\omega) = -\frac{C}{i\omega^2}\;\frac{\frac{g}{\omega C}\operatorname{Cos}\frac{\omega}{C}(H-y) - \operatorname{Sin}\frac{\omega}{C}(H-y)}{\frac{g}{\omega C}\operatorname{Sin}\frac{\omega H}{C} + \operatorname{Cos}\frac{\omega H}{C}} \tag{50}$$

From Eq. 50, 44 and 5 one directly obtains the complex frequency response for water pressure,

$$H_{p}^{y}(x,y,\omega) = \frac{WC}{\omega g}\;\frac{\operatorname{Sin}\frac{\omega}{C}(H-y) - \frac{g}{\omega C}\operatorname{Cos}\frac{\omega}{C}(H-y)}{\operatorname{Cos}\frac{\omega}{C}H + \frac{g}{\omega C}\operatorname{Sin}\frac{\omega H}{C}} \tag{51}$$

It may be noted that the solution is independent of x-coordinate.

Effect of Wave Motion at the Free Surface

The solution when wave motion at the free surface is ignored is given by Eq. 50 corresponding to g = 0. Therefore

$$H_p^y(x,y,\omega) = \frac{WC}{\omega g} \quad \frac{\sin \frac{\omega}{C}(H-y)}{\cos \frac{\omega H}{C}} \tag{52}$$

Clearly the difference in the two solutions (Equations 51 and 52) depend on the magnitude of quantity $\frac{g}{\omega C}$

Fourier Spectra and Response Spectra for strong motion records of past earthquakes[2] seem to indicate that frequencies of most of the significant harmonic components lie in the range $1 < \omega < 120$ radians per second. Thus, the largest value of the above quantity is about 6.82×10^{-3} (corresponding to $\omega = 1$). However, very low frequency harmonics make very little contribution to the hydrodynamic pressure response in case of typical reservoir depths encountered in practice. This is obvious because resonant frequencies for the reservoir are rather high (e.g. fundamental resonant frequency for 300 ft. deep reservoir is 24.72 radians per second). Thus the value of the quantity $\frac{g}{\omega C}$ for harmonics of significance is considerably less than 6.82×10^{-3}.

From these arguments without going into any detailed calculations it can be concluded that the terms in Eq. 51 involving quantity $\left(\frac{g}{\omega C}\right)$ may be dropped without introducing appreciable errors. Thus, the complex frequency response given by Eq. 52 is sufficiently accurate.

Unit Impulse Response

The unit impulse response is given by the inverse Fourier Transform of the complex frequency response. From Tables of Integral Transforms [14] the Inverse Fourier transform of $\frac{C}{\omega} \quad \frac{\text{Sin}\,\frac{\omega}{C}\,(H-y)}{\text{Cos}\,\frac{\omega H}{C}}$ is given by the function

$$\frac{2C}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n-\frac{1}{2}} \quad \text{Sin}\left[(n-\tfrac{1}{2}) \quad \frac{\pi(H-y)}{H}\right] \quad \text{Sin}\left[(n-\tfrac{1}{2}) \quad \frac{\pi ct}{H}\right]$$

The unit impulse response is therefore

$$h^{y}(x,y,t) = \frac{4WC}{\pi g} \sum_{n=1}^{\infty} \frac{1}{2n-1} \quad \text{Cos}\ \lambda_n y \quad \text{Sin}\ \lambda_n ct \tag{53}$$

It is apparent from Eq. 53 that the unit impulse response function does not decay in this case; the complex frequency response (Eq. 52) is a real valued function and the system is truly undamped.

Pressure Response to Arbitrary Ground Motion

The dynamic water pressure on face of the dam due to an arbitrary known excitation $\ddot{V}_g^{\,y}(t)$ can be obtained from the unit impulse response (Eq. 53) by the convolution integral. Thus

$$p^{y}(o,y,t) = \frac{4WC}{\pi g} \sum_{n=1}^{\infty} \frac{1}{2n-1} \quad \text{Cos}\ \lambda_n y \int_{o}^{t} \ddot{V}_g^{\,y}(\tau)\ \text{Sin}\left[\lambda_n c(t-\tau)\right] d\tau \tag{54}$$

Special Case of Incompressible Fluid

Horizontal Ground Motion

When compressibility of water is considered, the complex frequency response is given by Equation 29:

$$H_p^x(0,y,\omega) = \frac{4W}{\pi g} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)\sqrt{\lambda_n^2 - \frac{\omega^2}{C^2}}} \quad \text{Cos } \lambda_n y \tag{29}$$

If water is taken as incompressible, $C = \infty$ and Eq. 29 becomes

$$H_p^x(0,y,\omega) = \frac{8WH}{\pi^2 g} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2} \quad \text{Cos } \lambda_n y \tag{55}$$

The complex frequency response for total lateral force F^x is given by

$$F^x = \int_0^H H_p{}^x(o,y,\omega)dy = \frac{16WH^2}{\pi^3 g} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \tag{56}$$

Let the total hydrostatic force on the dam be denoted by Fo, $Fo = WH^2/2$

Then

$$\frac{F^x}{Fo} = \frac{32}{\pi^3 g} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \tag{57}$$

The complex frequency responses (Eq. 55 through 57) are now independent of the excitation frequency ω and the dynamic character of the response phenomenon is completely suppressed. It may be noted that Equation 55 also corresponds to the solution for finite C and $\omega = 0$.

By the Fourier Integral Theorem, ground acceleration $\ddot{V}_g^{\,x}(t)$ may be expressed as

$$\ddot{V}_g^{\,x}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V_g^{\,x}(\omega) \; e^{i\omega t} \, d\omega \qquad (58)$$

$$\text{where } V_g^{\,x}(\omega) = \int_{0}^{s} \ddot{V}_g^{\,x}(t) \; e^{-i\omega t} \, dt \qquad (59)$$

s being the duration of the ground motion.

The hydrodynamic response to ground motion $\ddot{V}_g^{\,x}(t)$ is given by

$$p^x(o,y,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_p^{\,x}(o,y,\omega) \, V_g^{\,x}(\omega) \; e^{i\omega t} \, d\omega \qquad (60)$$

Introducing Eq. 55 into Eq. 60

$$p^x(o,y,t) = \frac{8WH}{\pi^2 g} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2} \; \text{Cos} \; \lambda_n y \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} V_g^{\,x}(\omega) \; e^{i\omega t} \, d\omega \right]$$

$$\text{or} \quad p^x(o,y,t) = \left[\frac{8WH}{\pi^2 g} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2} \; \text{Cos} \; \lambda_n y \right] \ddot{V}_g^{\,x}(t) \qquad (61)$$

It is apparent that at any time t, the hydrodynamic pressures depend only on the value of ground acceleration at that instant.

Total Lateral Force may be obtained by integrating Eq. 61

$$P^x(t) = \left[\frac{16WH^2}{\pi^3 g} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \right] \ddot{V}_g^{\,x}(t)$$

$$\text{and} \quad \frac{P^x(t)}{Fo} = \left[\frac{32}{\pi^3 g} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \right] \ddot{V}_g^{\,x}(t) = \frac{\ddot{V}_g^{\,x}(t)}{g} \, (1.0855) \qquad (62)$$

If we were only interested in total lateral force Eq. 62 could have been derived directly from Eq. 57.

Vertical Ground Motion

When compressibility of water is considered the complex frequency response for water pressure is given by Eq. 52.

$$H_p^{\,y}\,(x,y,\omega) = \frac{WC}{\omega g} \quad \frac{\operatorname{Sin}\frac{\omega}{C}(H-y)}{\operatorname{Cos}\frac{\omega H}{C}} \tag{52}$$

If water is taken as incompressible then Eq. 52 becomes

$$H_p^{\,y}(x,y,\omega) = \frac{W}{g} \lim_{\frac{\omega}{C}\to 0} \left[\frac{\operatorname{Sin}\frac{\omega}{C}(H-y)}{\frac{\omega}{C}\operatorname{Cos}\frac{\omega H}{C}} \right]$$

$$= \frac{W}{g}\,(H-y) \tag{63}$$

The complex frequency response is now independent of ω, and represents a linear variation of pressures--from zero at top to $\frac{WH}{g}$ at the base. The corresponding expression for total lateral force F^y is given by

$$F^y = \int_o^H H_p^{\,y}(o,y,\omega) = \frac{W}{G}\frac{H^2}{2}$$

and $$\frac{F^y}{Fo} = \frac{1}{g} \tag{64}$$

Response to arbitrary ground motion $\ddot{V}_g^{\,y}(t)$ can be obtained through the frequency domain using the Fourier Integral Theorem and is

$$p^y(o,y,t) = \frac{W}{g}\,(H-y) \quad \ddot{V}_g^{\,y}(t) \tag{65}$$

It is apparent that at any time t, the hydrodynamic pressures depend only on the value of ground acceleration at that instant.

The total lateral force $P^y(t)$ is given by

$$\frac{P^y(t)}{Fo} = \frac{1}{g} \quad \ddot{V}_g^{\,y}(t) \tag{66}$$

Complex Frequency Response for Total Lateral Force

Integrating Eq. 29 and 52 over the height, the complex frequency response for total lateral force on the dam may be expressed as

$$F^x(\Omega) = \frac{16WH^2}{\pi^3 g} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2\sqrt{(2n-1)^2-\Omega^2}} \qquad (67)$$

$$F^y(\Omega) = \frac{4WH^2}{\pi^2 g} \frac{1}{\Omega^2} \frac{1-\text{Cos}\,\frac{\pi\Omega}{2}}{\text{Cos}\,\frac{\pi\Omega}{2}} \qquad (68)$$

where $\Omega = \omega/\omega_1$; $\omega_1 = \pi c/2H$, being the fundamental resonant frequency for reservoir of depth H. Superscripts x and y denote response to horizontal and vertical ground motions, respectively.

Normalizing Eq. 67 and 68 with respect to the hydrostatic force $Fo = \frac{WH^2}{2}$:

$$\frac{F^x(\Omega)}{Fo} = \frac{32}{\pi^3 g} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2\sqrt{(2n-1)^2-\Omega^2}} \qquad (69)$$

$$\frac{F^y(\Omega)}{Fo} = \frac{8}{\pi^2 g} \frac{1}{\Omega^2} \frac{1-\text{Cos}\,\frac{\pi\Omega}{2}}{\text{Cos}\,\frac{\pi\Omega}{2}} \qquad (70)$$

Figure 2 shows a graphical plot of Eq. 69 and 70.

Following discussion is pertinent to the results presented in Figure 2.

(i) The amplitudes of response become large as Ω approaches 1, 3, 5, 7, 9, etc. (values corresponding to resonance). Resonance is more pronounced and "sharp" for lower values of Ω. For horizontal excitation, there is almost no increase in response for values of Ω even very close to 5, 7, 9, 11, etc. This may also be concluded directly from the structure of Equations 69 and 70.

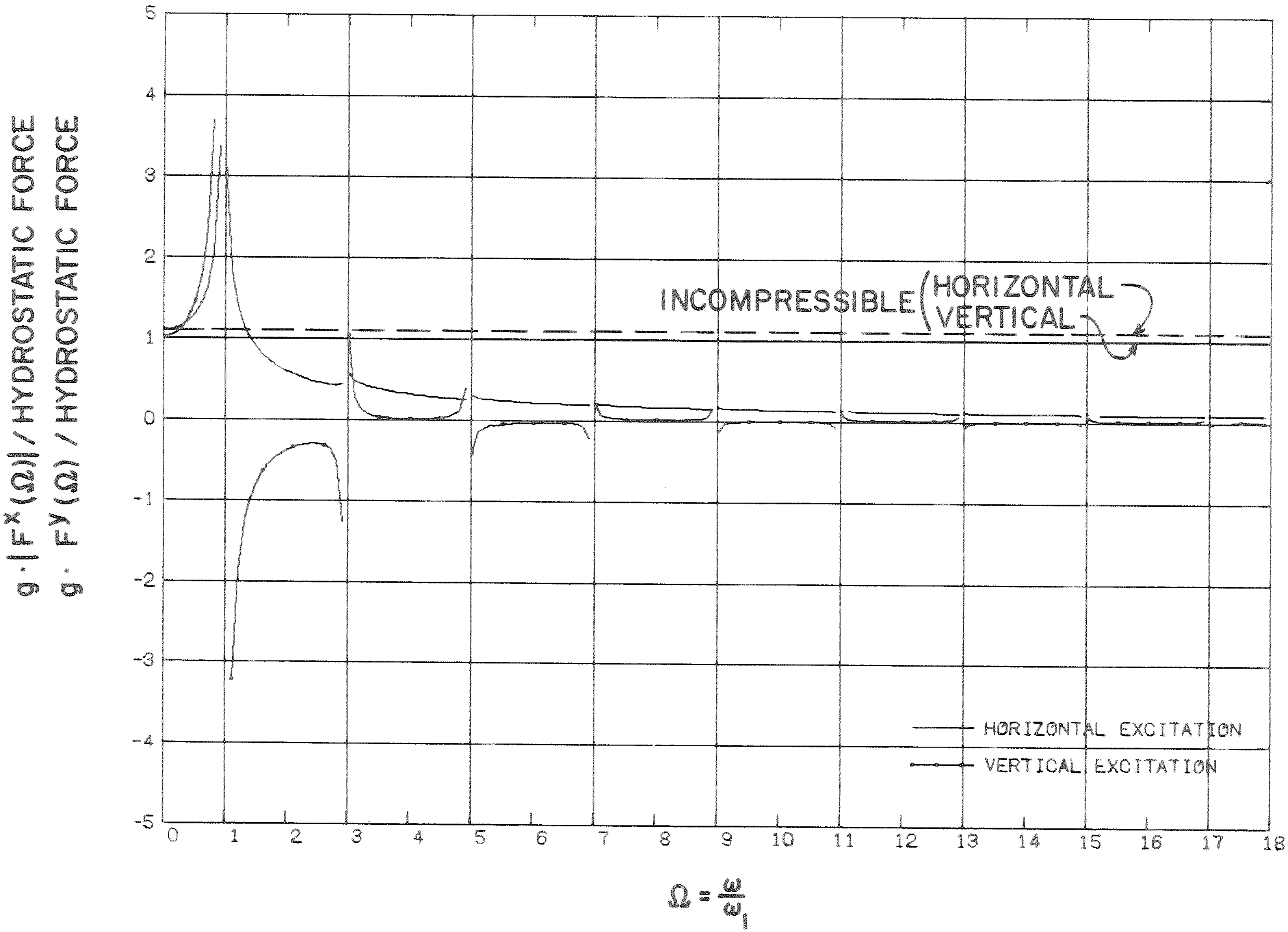


FIG. 2 -COMPLEX FREQUENCY RESPONSES FOR TOTAL HYDRODYNAMIC FORCE

(ii) For $\Omega < 3$ the total force due to vertical ground motion is about as large as that due to horizontal ground motion; for frequencies such that Ω is close to values 1 and 3 the response to vertical ground motion is larger.

(iii) Both responses diminish rapidly as Ω becomes large. However the effects of vertical ground motion for $\Omega < 3$ are not quite as small as indicated by the total pressure. This is evident from the characteristics of Equation 52--for values of $\Omega < 2$ the pressure is of same sign over the height; the pressure changes sign once if $2 < \Omega < 4$. The number of times that the sign of pressure reverses increases as Ω becomes larger, and the total force is small because the positive and negative areas partially cancel each other.

(iv) If water is taken as incompressible, the solutions are given by Eq. 57 and 64. They correspond to the limit of general (compressible) solutions as Ω approaches zero.

Although the horizontal axis in Figure 2 degenerates to a point when water is considered incompressible (because then C is infinite), the corresponding solutions are shown by horizontal lines to indicate that they are independent of excitation frequency. It is apparent that the assumption of incompressible water changes completely the response characteristics. In earthquake type excitation, frequencies are distributed over a wide range and hence this assumption may lead to large errors. This will be discussed further in connection with computed responses for El Centro 1940 earthquake.

Numerical Evaluation of Earthquake Responses

The determination of hydrodynamic pressures generated on a dam due to a prescribed earthquake involves the numerical evaluation of Equations 37 and 54. The functions $\ddot{V}_g^{\,x}(t)$ and $\ddot{V}_g^{\,y}(t)$ represent respectively the acceleration time history of a horizontal and the vertical component of the ground motion.

The form of Eq. 37 and 54 is not suitable for numerical evaluation because the upper limit of the integrals, t, appears in the integrands. If we use the expression in this form it would be necessary to start from $\tau = 0$ in evaluating the integrals for every new value of t. For Eq. 54 this difficulty can be surmounted by making use of the addition theorem for trigonometric functions

$$\text{Sin}\left\{\lambda_n C(t-\tau)\right\} = \text{Sin } \lambda_n Ct \text{ Cos } \lambda_n C\tau - \text{Cos } \lambda_n ct \text{ Sin } \lambda_n C\tau$$

Thus Eq. 54 becomes

$$p^y(o,y,t) = \frac{4WC}{\pi g} \sum_{n=1}^{\infty} \frac{1}{2n-1} \quad \text{Cos } \lambda_n y \left\{ \text{Sin } \lambda_n Ct \int_0^t \ddot{V}_g^{\,y}(\tau) \text{ Cos } \lambda_n C\tau \, d\tau - \text{Cos } \lambda_n Ct \int_0^t \ddot{V}_g^{\,y}(\tau) \text{ Sin } \lambda_n C\tau \, d\tau \right\} \qquad (71)$$

In this form the value of the integral at time $t-\Delta t$ can be utilized to obtain its value at time t.

Bessel functions do not possess addition theorems in the strict sense of the term i.e. it is not possible to express Jo(Z + z) as an algebraic function of Jo(Z) and Jo(z)[15].

The following series representation, however, exists:

$$Jo\left\{\sqrt{Z^2 + z^2 - 2Z_z \cos\Phi}\right\} = \sum_{m=0}^{\infty} E_m J_m(Z) J_m(z) \cos m\phi \qquad (72)$$

where $E_m = \begin{cases} 1 & m = o \\ 2 & m \neq o \end{cases}$

Thus, there seem to be two alternatives available to evaluate the integrals in Equation 37--(i) each time an increment is made in t, all previous values are discarded and the integration process is repeated starting from $\tau = 0$, and (ii) Eq. 72 used to express $Jo\left\{\lambda_n C(t-\tau)\right\}$ as a series in products of Bessel functions, and value of the integral for time $t-\Delta t$ used in obtaining its value at time t.

However, the second approach seems to offer no advantage over the first, which is more direct, because the evaluation of an infinite series of products of Bessel functions for each value of the argument $(t-\tau)$ is an impractically long computation process.

The first approach was therefore chosen; Simpson's rule with parabolic interpolation was used to evaluate all integrals. A computer program was developed to evaluate Eq. 37 and 71.

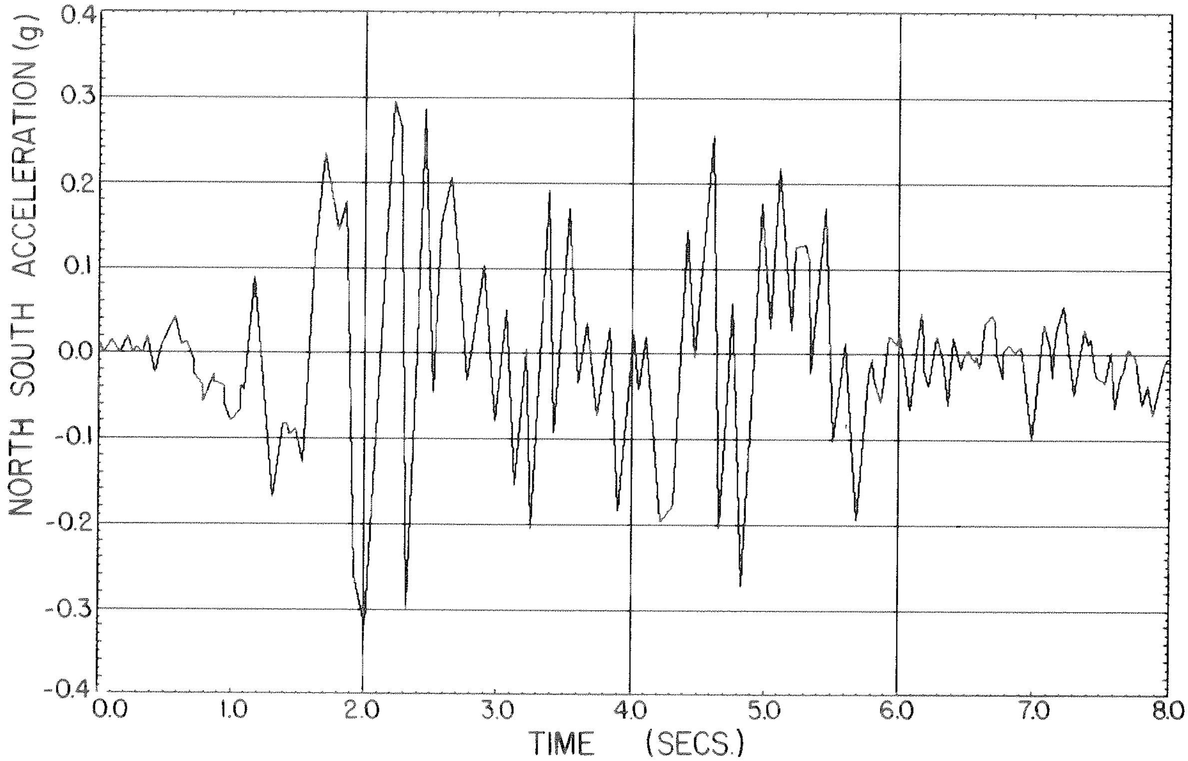
Hydrodynamic Response to El Centro, 1940 Earthquake

Figure 3 shows two components of the strong motion record obtained during the El Centro, 1940 earthquake. The peak ground acceleration in the North-South direction is 0.32g, and is 0.28g in the vertical direction (g is the acceleration due to gravity).

The physical constants chosen for water were: unit weight W = 62.5 pcf, velocity of pressure waves = 4720 fps.

Three reservoir depths were chosen--100, 300, and 600 feet. These cover the usual range of interest. The pressures were obtained by numerical integration of Eq. 37 and 51. At any instant of time the pressures may be integrated over the depth to obtain the total lateral force; the base-overturning moment can also be obtained by an integration over the depth. These quantities have been normalized with respect to the corresponding hydrostatic quantities and are plotted in Figures 4 through 6.

Figure 4 shows the time history of normalized lateral force and normalized base-overturning moment for a 100 ft. deep reservoir, the ground motion being the North-South component of El Centro, 1940 earthquake. The maximum absolute value of normalized lateral force is 0.44 and the maximum normalized overturning moment is 0.50. The variation of response with time is quite similar to the time history of ground motion itself. This is so because the frequencies of the 100 ft. deep reservoir are quite high (ω_1 = 74.1 rads per sec) and the system responds almost like a rigid body during the earthquake.



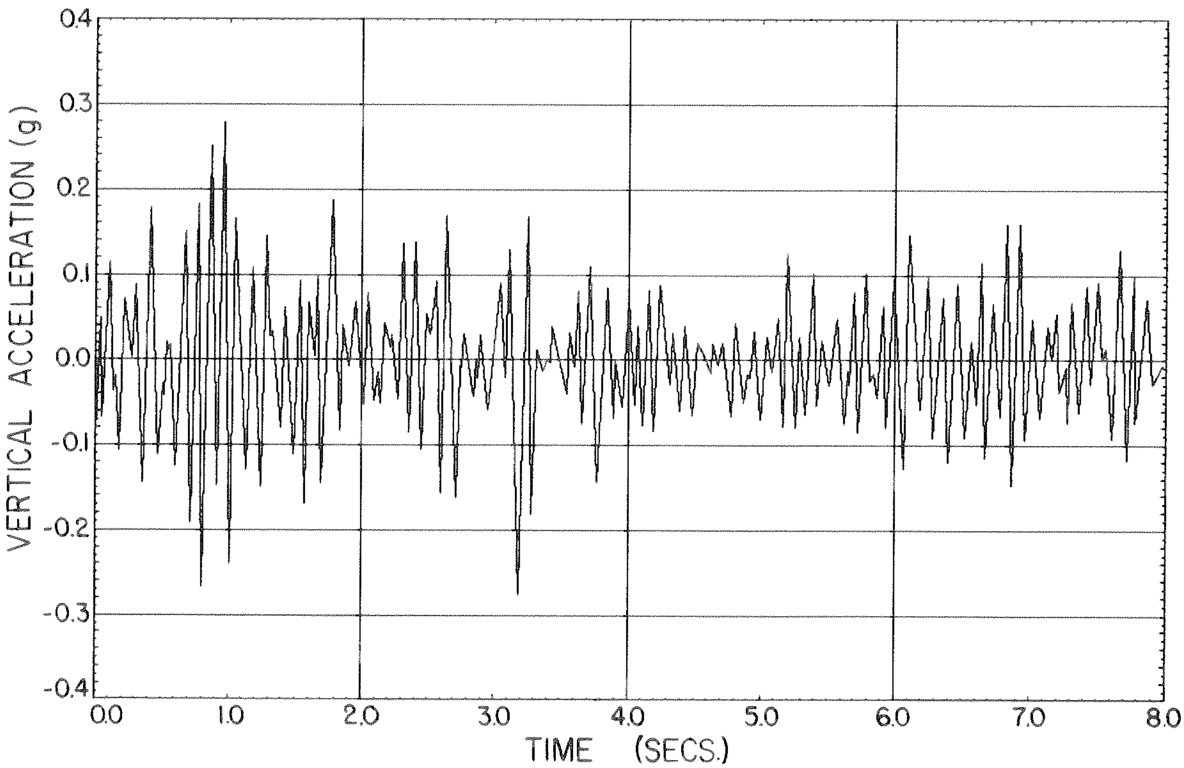
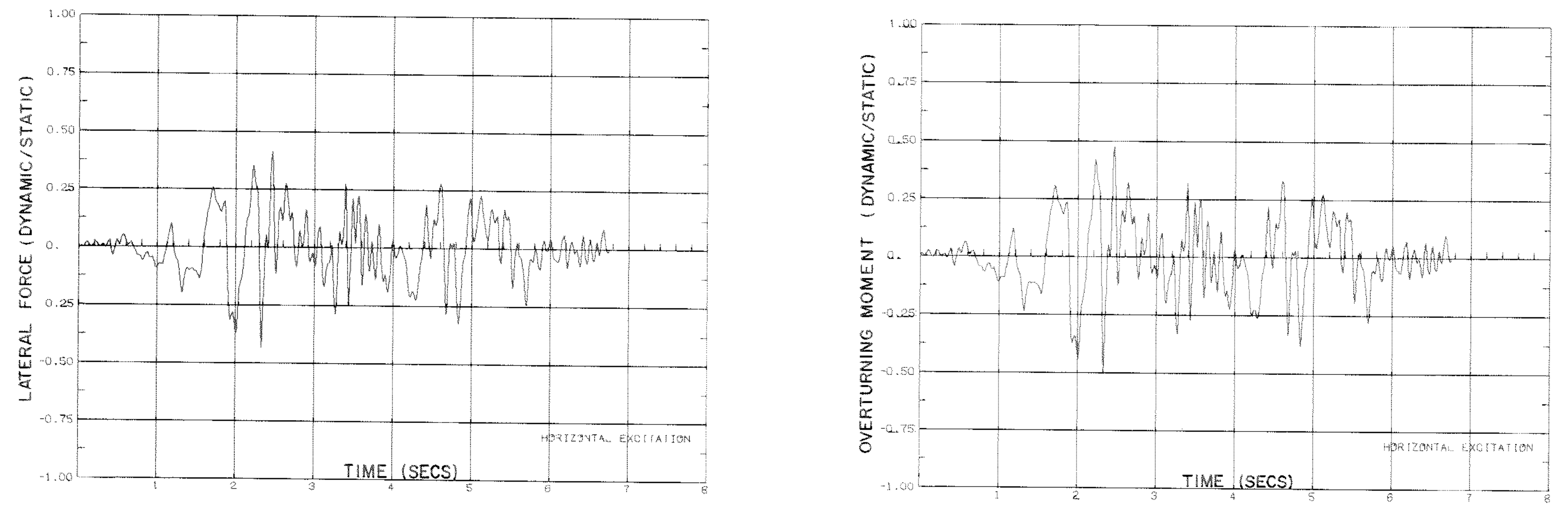
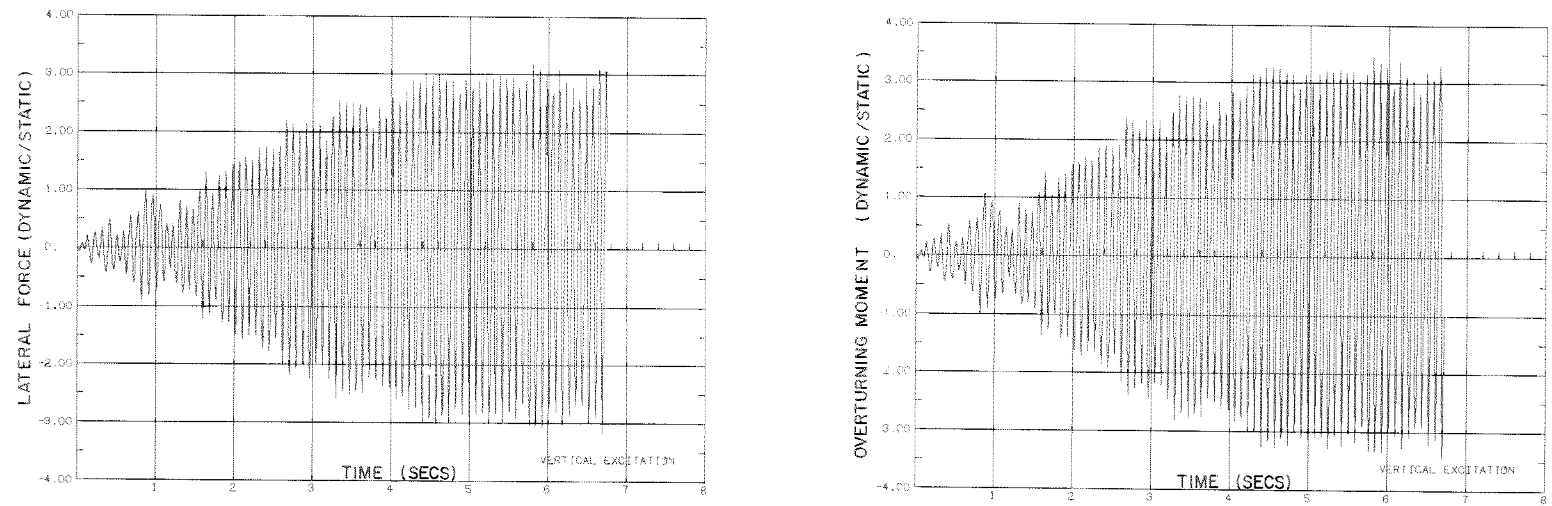


Fig. 3 - GROUND ACCELERATIONS
EL CENTRO EARTHQUAKE, MAY 18, 1940

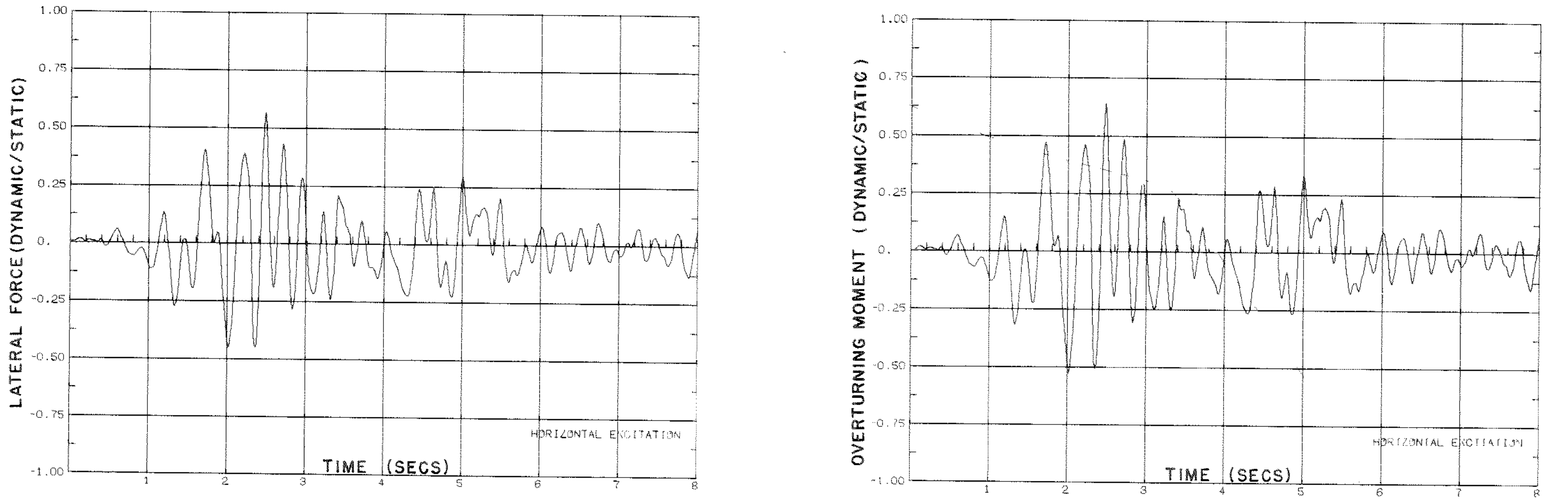


a. EXCITATION: N-S COMPONENT

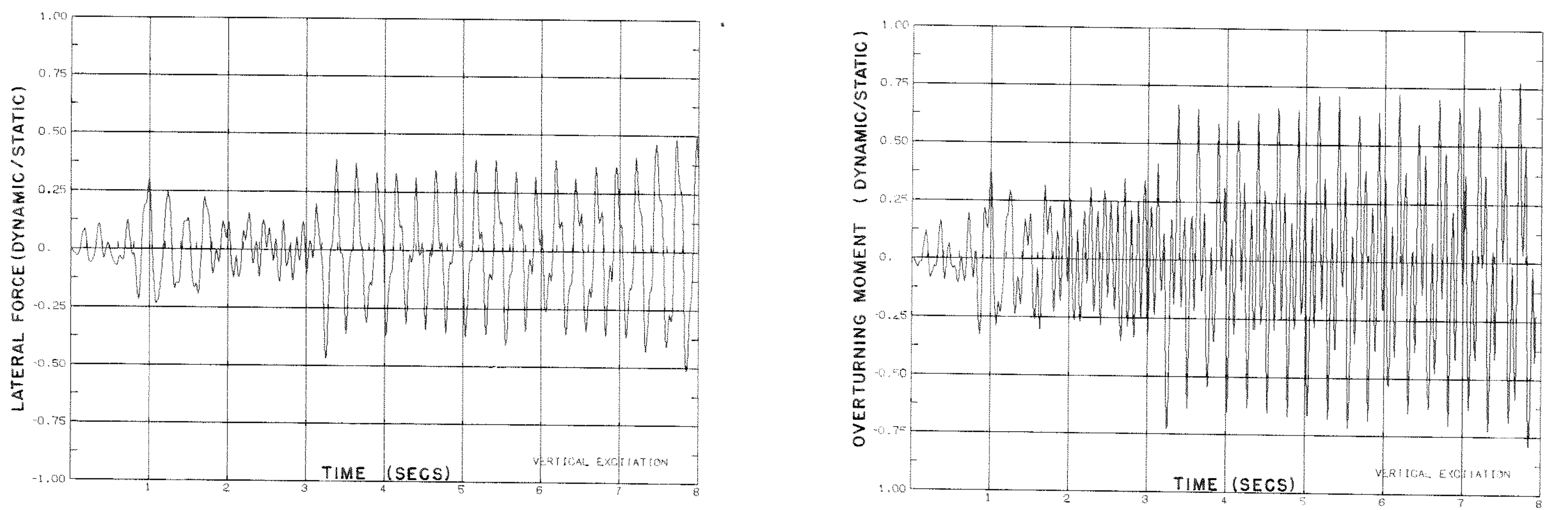


b. EXCITATION: VERTICAL COMPONENT

FIG. 4 TOTAL HYDRODYNAMIC FORCES
RESERVOIR DEPTH 100 FT
EL CENTRO, 1940 EARTHQUAKE

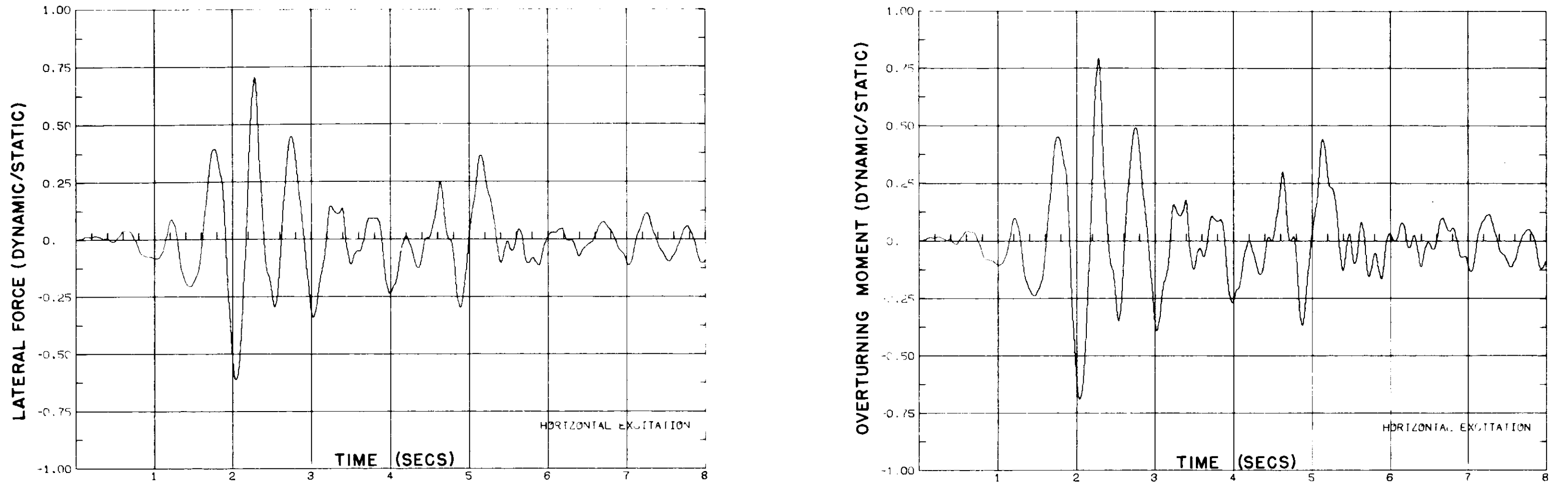


a. EXCITATION N-S COMPONENT

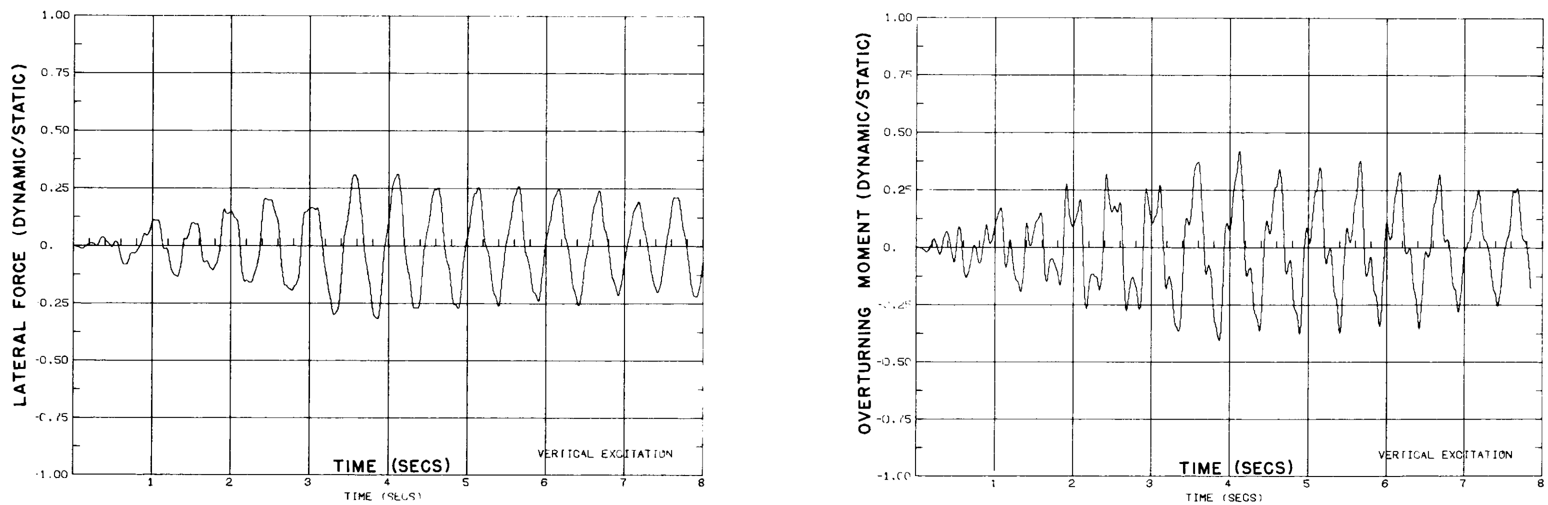


b. EXCITATION: VERTICAL COMPONENT

FIG. 5 TOTAL HYDRODYNAMIC FORCES
RESERVOIR DEPTH 300'
EL CENTRO, 1940 EARTHQUAKE



a. EXCITATION: N-S COMPONENT



b. EXCITATION: VERTICAL COMPONENT

FIG. 6 TOTAL HYDRODYNAMIC FORCES
RESERVOIR DEPTH 600'
EL CENTRO, 1940 EARTHQUAKE

The responses to vertical ground motion show a large build-up of forces oscillating at almost the first natural frequency ω_1(74.1 rads per sec or 11.8 cps). Forces as large as three times hydrostatic forces occur because (i) there is no dissipation of energy in the system and (ii) the vertical ground motion used has strong high frequency components.

Figure 5 shows time history of hydrodynamic forces on a dam storing a 300 ft. deep reservoir. The response to vertical ground motion shows predominant frequencies of 4 cps. and 12 cps. The response continues to build-up even after 8 secs. of the earthquake.

Time history of response to horizontal ground motion is now significantly different from the ground motion itself. The peak values of normalized lateral force and overturning moment are 0.57 and 0.64 respectively.

Figure 6 shows time history of hydrodynamic forces on a dam storing a 600 ft. deep reservoir. The normalized lateral force due to vertical ground motion is varying with a frequency of almost 2 cps., the amplitude being about 0.25 for a major part of the response.

The peak values of normalized lateral force and overturning moment due to horizontal ground motion are respectively 0.71 and 0.80. The time history of response is as shown in Figure 6a, and has predominant frequencies close to 2 cps.

If water were considered incompressible, the total lateral forces would vary with time in exactly the same manner as the ground accelerations themselves (Eq. 62 and 66). For horizontal ground motion, the normalized lateral force at any time would be $1.0855\alpha_g$ (Eq. 62) where α_g is the ground acceleration at that instant of time. Thus the maximum normalized lateral force due to N-S component of El Centro (1940) earthquake will be 0.35. If αg is the vertical ground acceleration at any time, the normalized lateral force on the dam at that instant of time is α (Eq. 66). Thus the maximum value of the normalized lateral force due to the vertical component of El Centro (1940) earthquake is 0.28. Comparison of results for incompressible water with those obtained by considering compressibility effects is presented in Table I, where the maximum absolute value of normalized lateral force due to El Centro (1940) earthquake (N-S component) are listed.

Table I

Comparison of Peak Responses

Reservoir Depth	Compressibility Considered	Compressibility Ignored	Percent Error
100 ft.	0.44	0.35	20
300 ft.	0.57	0.35	39
600 ft.	0.71	0.35	51

The large errors involved if compressibility of water is ignored are apparent from Table I. If peak responses are the only quantities of concern, the above comparison is indicative of the order of errors involved in an incompressible solution. If one were interested in the complete time-history of responses, discussion and estimation of errors is more complicated. For all reservoir depths, neglecting compressibility leads to response time-history identical to ground acceleration time history. This result seems reasonable for reservoir depths less than 100 ft. However, there is very little similarity between "exact" response (considering compressibility effects) and ground acceleration histories for larger reservoir depths. The frequency characteristics of hydrodynamic response, ignoring compressibility of water are completely different than the "exact" solution; their effects on a dam would therefore be quite different. Significant errors are also introduced in responses to vertical ground motion by ignoring compressibility of water.

Comparison with Present Design Practice

The present design practice[16,17] in evaluating hydrodynamic forces on dams is based on Zangar's work[9], in which water was considered as incompressible. Westergaard's results[1] may also be used to determine hydrodynamic forces[18,19]. As shown in this work, ignoring compressibility effects leads to rather large errors in hydrodynamic response and therefore it is unsatisfactory to use results based on this assumption. Westergaard's work recognizes compressibility effects but is restricted to harmonic ground motions, with period greater than the first resonant period for water pressure. As is well known, strong earthquake motions are random in character and contain a wide range of frequency components. Thus, results for harmonic ground motions cannot be applied with confidence for designing against strong earthquake motions.

Magnitudes of hydrodynamic forces typically considered in design are of the order of 10 percent of the hydrostatic forces. These are then applied on the dam as static forces for purposes of stress and stability analysis.

Results presented in this work show that much larger hydrodynamic forces may develop during strong earthquakes, e.g. the maximum value of total hydrodynamic force on a dam due to El Centro, 1940 earthquake (North-South component) for a 600 feet deep reservoir is about 71 percent of the total hydrostatic force. From the time-history of responses presented here, it is apparent that these large hydrodynamic forces act on the dam for short durations of time and that the forces reverse in direction many times during an earthquake. Considering the maximum force developed during an earthquake as a static force for purposes of stress and stability analysis would obviously lead to extremely conservative designs.

It should be noted that these computed forces are only approximations to the actual forces that may develop, because the analysis has not accounted for the flexibility of the dam. However, on the basis of these results it seems necessary to recognize the possibility of large hydrodynamic forces being developed for short durations of time.

Conclusions

Expressions have been developed for determining the complete distribution of hydrodynamic pressures over the height of the dam at any time during arbitrary vertical and horizontal ground motions. This investigation has led to the following conclusions:

1. The complex frequency response to horizontal ground motion is a complex valued function for excitation periods greater than the first resonant period for water pressure. Thus, Westergaard's solution and the concept of a "virtual mass" to represent hydrodynamic effects is not valid in this range of excitation periods.

2. Although water is considered as an ideal fluid, the unit impulse response to horizontal ground motion decays with time. This decay of response is directly associated with energy in waves moving away from the dam.

3. The response to vertical ground motion is independent of the horizontal coordinate and depends only on the depth of reservoir. The complex frequency response is real valued for all frequencies.

4. Errors that may be introduced in responses to vertical ground motion by ignoring surface waves are quite small. Complex frequency responses indicate that effects of vertical ground motion which are customarily ignored are quite significant.

5. The unit impulse response to vertical ground motion does not decay with time, thus the system is truly undamped in this case.

6. Complex frequency responses neglecting water compressibility effects are independent of excitation frequency. Significant errors are involved in this solution over the entire frequency range; the error approaches zero as the excitation frequency approaches zero.

7. The peak values as well as the frequency characteristics of hydrodynamic response to arbitrary ground motions, ignoring compressibility of water are completely different compared to the "exact" solution (considering compressibility effects); their effects on a dam would therefore be quite different.

8. Hydrodynamic forces considered in present design practice are based either on Zangar's results which ignore compressibility of water or on Westergaard's work which considers only a harmonic ground motion. There are serious deficiencies in both of these approaches.

9. Results presented indicate that forces much larger than presently considered for design purposes could develope during earthquakes. However, for economical designs it may be important to recognize that these large forces are developed usually for short durations of time and also that they may be altered by the flexibility of the dam.

The responses to vertical ground motion which have been determined here are not completely satisfactory because the mathematical model considered does not allow for any energy dissipation in the system. Even the small amounts of energy which would be dissipated in the physical system may reduce the responses considerably. In addition, if response predicted by the type of analysis presented here is large, effects of interaction between dam and reservoir and between reservoir and foundation may be significant enough to alter the response considerably.

ACKNOWLEDGMENT

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NOTATION

A. Constant

B Constant

C Velocity of pressure waves of water.

e Percent of error.

e Base of natural logarithum.

F_o Total hydrostatic force.

F^x Total lateral force in complex frequency response.

g Gravity acceleration.

H Depth of the reservoir.

H^x_ϕ Complex frequency response for ϕ.

$H(\omega)$ Complex frequency response.

h^x_p Unit impulse response.

i $= \sqrt{-1}$

J_o Bessel Function of first kind.

k Bulk modulus of water.

L Length of the reservoir.

m Summation index.

n Summation index.

n_s Minimum value of n.

P^x Total lateral force.

p Dynamic water pressure.

s Duration of the ground motion.

T Period of harmonic ground motion

T_n	Resonant period.
t	Time
u	x-component of displacement of water.
$\ddot{V}_g^y$	Horizontal ground acceleration.
$\ddot{V}_g^y$	Vertical ground acceleration.
v	y-component of displacement of water.
W	Unit weight of water.
x	Horizontal coordinate position.
y	Vertical coordinate position.
αg	Ground acceleration at any instant of time.
$\delta(t)$	Dirac Delta Function
λ_m	Constant defined by Eq. (22)
ϕ	Velocity potential.
Ω	$= \frac{\omega}{\omega_1}$
ω	Circular frequency.
ω	$= \frac{\pi C}{2H}$