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The plasma as a phase conjugate reflector

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Plasma is a nonlinear medium and two waves propagating in it interact electromagnetically with each other. If the plasma is pumped by two strong counterstreaming waves of equal frequency, and a third wave enters, the nonlinear interaction generates a fourth wave, phase conjugate to the third wave. This interaction becomes very significant if the frequency and wave vector differences between the third wave and one of the pump waves resonate with the frequency and wave vector of the ion acoustic mode of the plasma. This resonance can be predicted from a fluidlike description of the plasma, but it is shown that the Vlasov description can provide more details of behavior near the resonance. Possible applications of the emergent technology range from improved focusing of radiation in hyperthermia therapy of cancer to the formation of a microwave laser between a phase conjugate plasma reflector and a mirror, for improved radar imaging. Another application is cordless, self-guiding, power transmission.

I. INTRODUCTION

There has been much interest recently in nonlinear interaction of electromagnetic (EM) waves with matter, which lead to the phenomenon of optical phase conjugation.[1,2] In this process a material body, e.g., solid or liquid, is stimulated by electromagnetic waves in such a way that another EM wave, a "signal" wave, which is incident into the body, is reflected from the system with its wave vector reversed, and its phase conjugate. The phase conjugate "mirror" reflects the signal beam, and causes the wave to retrace its original path in a time-reversed manner.

In optics there are several methods to produce phase conjugate reflections (PCR) by using the nonlinear response of various materials to electromagnetic radiation.[3,4] In the last ten years many reports were published on both theoretical and experimental results of PCR in the optical and infrared regions of frequencies (see Ref. 1, and more recently Ref. 4). PCR can be obtained by the processes of stimulated Brillouin and Raman scattering, where a very strong signal excites an internal mode of the medium, and is reflected with its phase conjugate. The most promising process for PCR is four wave mixing (FWM), where the system is exposed to two strong pump EM waves, and a weak signal wave is PC reflected.

Nonlinear optical interactions in plasmas have attracted attention since the early 1960's (see Ref. 3, Chap. 28, and references cited therein). It is relevant to mention here the following groups of studies: (i) mixing of EM waves in the optical region,[5] (ii) parametric instabilities of EM radiation in plasmas,[6] and (iii) laser induced heating.[7]

The nature of the nonlinear interaction in a plasma is relatively predictable and calculable from first principles. If a few EM waves are incident upon a plasma, the charged particles' motion is perturbed in a nonlinear way so that collective modes are excited, and new EM waves are generated and emerge out of the system. Most of the works connected with nonlinear interaction of radiation with the plasma were concentrated on the laser heating and parametric instabilities.[8] In these cases strong EM waves are incident upon the plasma, and the interest is in the nonlinear response of the system, and not so much the outcoming waves. In the present work our main concern is the relation between the scattered EM waves and the incident waves.

The main body of studies of scattering of EM waves, due to nonlinear interaction in plasmas, concentrated on stimulated Brillouin scattering (SBS) and stimulated Raman scattering (SRS) in laser-irradiated plasmas. In these cases the threshold conditions for the parametric instabilities were investigated using nonlinear response theory.[9] The aspect of getting a phase conjugate scattered wave from SBS, which had been discussed by Zeldovich *et al*,[2] was later investigated in plasmas irradiated by strong laser sources.[10] The essential point here is that a very strong incident EM wave stirs up the plasma, and a scattered wave can be generated with its phase conjugate to the incident beam. No amplification can occur, and the reflected PC wave is usually much weaker than the signal.

If the plasma is exposed to more than one wave, a phase conjugate wave can be reflected from the system. Plasma density modulations were observed by mixing laser beams,[11] as predicted in early theoretical studies. The analysis of degenerate four wave mixing (DFWM) in plasma was reported by Steel and Lam.[12] They have calculated the third-order susceptibility of a plasma, under the influence of three EM waves with the same frequency. The dynamics of the plasma in the presence of the three waves was studied in terms of a two component (ion and electron) fluid description. If this susceptibility is used for a configuration of two counterparallel strong pump waves, and a small signal wave, a phase conjugate reflection occurs, with the reflected wave proportional to the third-order susceptibility. They conclude that the plasma is a highly viable medium, but the DFWM method is too weak. The effect of collisions on the DFWM in plasma was reported recently.[13]

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In the present paper it will be demonstrate that a plasma can be used as a PCR for radiation in the subcentimeter region if the FWM is almost degenerate. It is shown that a plasma of easily attained properties can play the role of both phase conjugate reflector and amplifier, i.e., the reflected radiation may be enhanced with respect to the incoming signal.[14] As in the DFWM configuration of PCR two counterparallel strong pump waves of the same frequency run into the plasma continuously. A small signal EM wave is incident upon the plasma with its frequency very slightly shifted with respect to the pump frequency, in such a manner that a collective mode of the plasma is excited by the mixing of the signal and one of the pumps. In this way, the nonlinear response of the plasma is enhanced because of the resonance with the collective excitation. The scattering of the second pump from the excited plasma generates a wave phase conjugate to the signal. This is a process of parametric amplification of the reflected wave. While in most optical systems the response of the nonlinear medium (say, a crystal) is quite slow, here the response is fast, being determined by the rise time of the collective excitation. It should also be pointed out that the natural frequency range of a gaseous plasma is far below the optical region, and can be as low as microwave frequencies.

In Sec. II we describe the system and indicate the basic formalism that we use. The plasma is studied in the Vlasov description, and then compared to the results of the fluid description. A general perturbation scheme is outlined for the solution of the third-order response of the plasma to the three incoming waves. In Sec. III we perform the actual calculations using some simplifying physical assumptions. After reviewing the linear response, we investigate the electron density, which is produced by the mixing of two EM waves in the plasma. The third-order current response is now generating the new EM wave, which is the phase conjugate reflection of the signal. We conclude with a discussion of the results.

II. DESCRIPTION OF THE SYSTEM

We consider a fully ionized plasma in a volume V. The average density n_0 of electrons is equal to that of the singly ionized ions. The electron temperature is T_e, and that of the ions is T_I. The plasma is assumed to be homogeneous and no external magnetic field is present. Three electromagnetic (EM) waves of similar frequencies are sent into the plasma, as depicted in Fig. 1. The EM waves are arranged so that two countermoving pump waves a and b run in the plasma continuously, and a third wave, the signal wave s, is incident to the plasma in a different direction. The angle between the signal and pumps can be arbitrary. However, if we choose 90°, the reflected signal power is doubled because of the symmetry. (At any other angle, two different gratings are generated in the plasma and two different wave fronts can be reflected.) A fourth wave c is generated by the stimulated plasma in the opposite direction from the signal wave. The purpose of the present paper is to calculate the strength of this generated wave.

We shall consider here only the case when the three

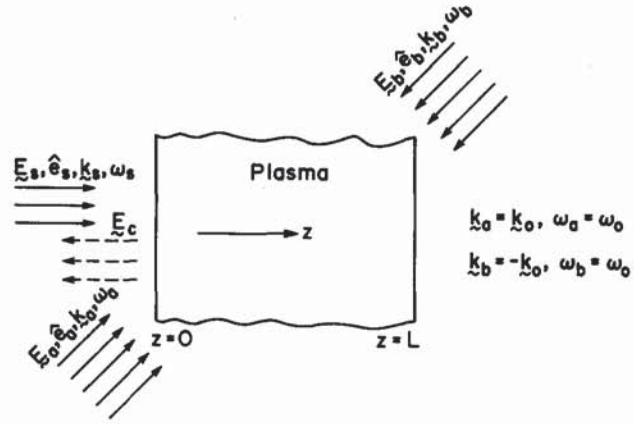

FIG. 1. The geometry of phase conjugation in the plasma. The pump waves a and b are counterpropagating, and the reflected wave is phase conjugate to the signal wave S.

incident waves are almost degenerate in their frequencies. Together with the generated fourth wave we have a system of so-called four wave mixing (FWM), with the plasma playing the role of the nonlinear medium. This method of producing a phase conjugate reflector is termed almost degenerate FWM.

A. Equations of motion

Outside the plasma we have three incoming EM waves and one outgoing EM wave. Inside the system each of the incoming waves turns into a normal mode of the plasma, that is, a linearized self-consistent solution of the plasma equations, which matches the external waves on the boundary. The outgoing wave is generated inside the plasma by these internal modes.

The plasma is described in terms of the Vlasov equation for the distribution function $f_\alpha(\mathbf{r},t;\mathbf{v})$ of the αth species ($\alpha = e$ for electrons and $\alpha = I$ for ions) in position $\mathbf{r}$, time t, and velocity $\mathbf{v}$, i.e.,

$$\frac{\partial}{\partial t} f_\alpha + \mathbf{v}\cdot\nabla f_\alpha + \frac{q_\alpha}{m_\alpha}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial}{\partial \mathbf{v}} f_\alpha = 0 . \quad (1)$$

Here q_α and m_α are the charge and mass of the αth species, c is the speed of light, and $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ are, respectively, the electric and magnetic fields in the plasma. Alternatively and also for comparison, we may also study the plasma using a fluid description. With $n_\alpha(\mathbf{r},t)$ the particle density, $\mathbf{v}_\alpha(\mathbf{r},t)$ the velocity field, and $p_\alpha(\mathbf{r},t)$ the pressure, we write the continuity equation

$$\frac{\partial}{\partial t} n_\alpha + \nabla\cdot(n_\alpha v_\alpha) = 0 , \quad (2)$$

and Newton's equation

$$\frac{d}{dt}\mathbf{v}_\alpha = -\frac{1}{m_\alpha n_\alpha}\nabla p_\alpha + \frac{q_\alpha}{m_\alpha}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right). \quad (3)$$

We will take the pressure to be $p_\alpha = T_\alpha n_\alpha$, where T_α is the temperature (in energy units) of the α species and assumed to be constant. The two descriptions are very useful when studying nonlinear behavior, and are related by

$$n_\alpha = \int d^3v\, f_\alpha, \quad n_\alpha \mathbf{v}_\alpha = \int d^3v\, \mathbf{v} f_\alpha . \tag{4}$$

The electric and magnetic fields are given by Maxwell's equations:

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{B}, \quad \nabla\cdot\mathbf{E} = 4\pi\rho ,$$
$$\nabla\times\mathbf{B} = \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E} + \frac{4\pi}{c}\mathbf{j}, \quad \nabla\cdot\mathbf{B} = 0 , \tag{5}$$

with the charge and the current densities given by

$$\rho = \sum_\alpha q_\alpha n_\alpha, \quad \mathbf{j} = \sum q_\alpha n_\alpha \mathbf{v}_\alpha . \tag{6}$$

This set of equations is now to be solved in the presence of the three external EM waves:

$$\mathbf{E}^{\rm ext}(\mathbf{r},t) = \sum_{i=a,b,s} \mathbf{E}_i^{(\rm ext)} \cos(k_i\cdot r - \omega_i t + \phi_i) . \tag{7}$$

Here $E_i^{\rm ext}$ is the amplitude of the ith wave, which is assumed to be small, i.e.,

$$\xi_i = (e/mc)E_i^{(\rm ext)}/\omega_i \ll 1 , \tag{8}$$

with e and m the electron's charge and mass. The solution will be seen to consist of an outgoing wave with amplitude proportional to the product of $E_a E_b E_s$, and thus of third order in ξ of Eq. (8).

B. Perturbation hierarchy

In principle, since we have a small expansion parameter, we can attempt to find a solution using a perturbation hierarchy of the Vlasov equation along the following lines. We first introduce Fourier transforms in space and time, i.e., for a real function $F(\mathbf{r},t)$ we write

$$F(r,t) = \frac{1}{V}\sum_{\mathbf{k}} \int \frac{d\omega}{2\pi} e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t} F(\mathbf{k},\omega) . \tag{9}$$

Using condensed notations

$$k \equiv \mathbf{k},\omega \quad \text{and} \quad \sum_k = \frac{1}{V}\sum_{\mathbf{k}} \int \frac{d\omega}{2\pi}, \tag{10}$$

we write Eq. (1) as

$$(\omega - \mathbf{k}\cdot\mathbf{v}) f_\alpha(k;\mathbf{v}) + i\frac{q_\alpha}{m_\alpha}\sum_{k'}\Big(\mathbf{E}(k-k') + \frac{1}{c}\mathbf{v}\times\mathbf{B}(k-k')\Big)\cdot\frac{\partial}{\partial v} f_\alpha(k';\mathbf{v}) = 0 , \tag{11}$$

and similarly for Maxwell's equations, which are linear. We now formally expand f in powers of ξ and iterate Eq. (11) starting from the zeroth-order distribution function $f_\alpha^0(\mathbf{v})$, and $\mathbf{E}^0,\mathbf{B}^0 = 0$. The mth-order members of the hierarchy are obtained by

$$(\omega - \mathbf{k}\cdot\mathbf{v}) f_\alpha^{(m)} + i\frac{q_\alpha}{m_\alpha}\Big(\mathbf{E}^{(m)}(k) + \frac{1}{c}\mathbf{v}\times\mathbf{B}^{(m)}(k)\Big)\cdot\frac{\partial}{\partial \mathbf{v}} f_\alpha^0(v) = I^{(m)}(k;\mathbf{v}) , \tag{12}$$

and

$$\mathbf{k}\times\mathbf{E}^{(m)}(k) = (1/c)\omega\mathbf{B}^{(m)}(k) , \quad \mathbf{k}\cdot\mathbf{E}^{(m)} = -i4\pi\rho(m) ,$$
$$\mathbf{k}\times\mathbf{B}^{(m)}(k) = -(\omega/c)\mathbf{E}^{(m)}(k) - i(4\pi/c)\mathbf{j}^{(m)}(k), \tag{13}$$
$$k\cdot\mathbf{B}^{(m)} = 0 .$$

The inhomogeneous term $I^{(m)}$ is given by the members of the expansion lower than m, i.e.,

$$I^{(m)}(k;\mathbf{v}) = -i\frac{q_\alpha}{m_\alpha}\sum_{m'=1}^{m-1}\Big(\mathbf{E}^{(m-m')}(k-k') + \frac{1}{c}\mathbf{v}\times\mathbf{B}^{(m-m')}(k-k')\Big)\cdot\frac{\partial}{\partial \mathbf{v}} f^{(m')}(k';\mathbf{v}) . \tag{14}$$

The external EM fields of Eq. (7) are to be matched with the first-order ($m=1$) electric and magnetic fields of the hierarchy. Although this perturbation procedure can be carried out up to its $m=3$ order, we will study this system using some simplified physical assumptions. This procedure is more instructive and takes more advantage of the physical parameters of the system. We will first solve for the internal EM waves using the linearized set of equations. We then argue that to second order in ξ only the electron density is of interest. Finally, we find the equation for the phase conjugate reflected EM field.

III. CALCULATION OF THE REFLECTED WAVE

A. Internal fields

The linear response of a plasma to an external transverse EM field,

$$\mathbf{E}^{\rm ext}(\mathbf{r},t) = \mathbf{E}^{(e)}\cos(\mathbf{k}_e\cdot r - \omega_e t), \quad \mathbf{k}_e\cdot\mathbf{E}^{(e)} = 0 ,$$

is treated in many textbooks. However, for later reference and for completeness we will outline it here. We start from Eq. (1) for the linear response $f^{(1)}$, $\mathbf{E}^{(1)}$, and $\mathbf{B}^{(1)}$, i.e.,

$$\frac{\partial}{\partial t} f_\alpha^{(1)} + \mathbf{v}\cdot\nabla f_\alpha^{(1)} + \frac{q_\alpha}{m_\alpha}\Big(\mathbf{E}^{(1)} + \frac{1}{c}\mathbf{v}\times\mathbf{B}^{(1)}\Big)\cdot\frac{\partial}{\partial \mathbf{v}} f_\alpha^{(0)} = 0 , \tag{15}$$

supplemented by Maxwell's equations [Eqs. (5) and (6)]. We assume a plane wave solution

$$E^{(1)}(\mathbf{r}\cdot t) = \mathbf{E}_1\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi) , \tag{16}$$

with $\omega = \omega_e\, \mathbf{k}\cdot\mathbf{E}_1 = 0$ and thus the $\mathbf{B}^{(1)}$ field is

$$\mathbf{B}^{(1)}(\mathbf{r},t) = (c/\omega)\mathbf{k}\times\mathbf{E}_1\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi) . \tag{17}$$

The linearized distribution function

$$f_\alpha^{(1)}(\mathbf{r},t;\mathbf{v}) = f_\alpha^{(1)}(\mathbf{v})\sin(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$$

is then

$$f_\alpha^{(1)}(\mathbf{v}) = \frac{q_\alpha}{m_\alpha}\frac{1}{\omega - \mathbf{k}\cdot\mathbf{v}}\Big(\mathbf{E}_1 + \frac{1}{\omega}\mathbf{v}\times\mathbf{k}\times\mathbf{E}_1\Big)\cdot\frac{\partial}{\partial \mathbf{v}} f_\alpha^0 . \tag{18}$$

If we take $f_\alpha^0(\mathbf{v})$ to be spherically symmetrical in $\mathbf{v}$, e.g., a Maxwellian distribution

$$f_\alpha^0(\mathbf{v}) = n_0(m_\alpha/2\pi T_\alpha)^{3/2}\exp[-(m_\alpha/2T_\alpha)v^2] , \tag{19}$$

we find for the density and velocity

$$n_\alpha^{(1)} = 0\,,$$

$$n_0 \mathbf{v}_\alpha^{(1)} = \frac{q_\alpha}{m_\alpha}\int d^3v \frac{1}{\omega - \mathbf{k}\cdot\mathbf{v}} \mathbf{E}_1 \cdot \frac{\partial f^0}{\partial \mathbf{v}}\,. \tag{20}$$

We consider a nonrelativistic plasma, where the thermal velocity, v_{th}, of the electrons is $v_{\text{th}} \ll c$, and thus to zeroth order in $\beta = v_{\text{th}}/c$ we find

$$\mathbf{v}_\alpha^{(1)}(\mathbf{r},t) = \mathbf{v}_\alpha^{(1)} \sin(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)\,,$$
$$\mathbf{v}_\alpha^{(1)} = (q_\alpha/m_\alpha)\mathbf{E}_1/\omega\,. \tag{21}$$

These results for $n_\alpha^{(1)}$ and $\mathbf{v}_\alpha^{(1)}$ are easily obtained from Eqs. (2) and (3) of the fluid description. Notice that, from Eqs. (17) and (21), we have

$$\nabla\times\mathbf{v}_\alpha^{(1)}(r,t) = (q_\alpha/m_\alpha c)\mathbf{B}^{(1)}(\mathbf{r},t)\,. \tag{22}$$

Since the electron to ion mass ratio m/M is very small, we find that only the electrons contribute to the current in Eq. (6),

$$\mathbf{j}^{(1)}(\mathbf{r},t) = -en_0\mathbf{v}_e^{(1)}(\mathbf{r},t)\,, \tag{23}$$

and thus Eq. (5) for the $\mathbf{E}_1$ field takes the form

$$[k^2 - \omega^2/c^2 + (\omega_e^2/c^2)\mathbf{E}_1 = 0]\,, \tag{24}$$

with the electron plasma frequency $\omega_e^2 = 4\pi e^2 n_0/m$. This leads to the plasma EM wave dispersion relation

$$\omega^2 = c^2k^2 + \omega_e^2\,, \tag{25}$$

with the internal wave vector $\mathbf{k}$ equal to $k = (1/c)\sqrt{\omega^2 - \omega_e^2}$. We shall consider only external waves with $\omega > \omega_e$, so that the EM wave can penetrate the plasma. If we further take $(\omega_e/\omega)^2 \sim 0.1$, then $k \sim \omega/c$, and $\mathbf{E}_1$ is close to the amplitude $\mathbf{E}^{(e)}$ of the external field. From now on we will not distinguish between the linearized internal fields and the external ones, and write the total linearized field inside the plasma as

$$\mathbf{E}^{(1)}(\mathbf{r},t) = \sum_{i=a,b,s} \mathbf{E}_i \cos(\mathbf{k}_i\cdot\mathbf{r} - \omega_i t + \phi_i), \quad \mathbf{E}_i\cdot\mathbf{k}_i = 0\,, \tag{26}$$

with $\mathbf{E}_i$ assumed to be given, and $\omega_i^2 = k_i^2c^2 + \omega_e^2$. We will shortly assume that because of the nonlinearities, the amplitudes of the internal fields $\mathbf{E}_a$, $\mathbf{E}_b$, and $\mathbf{E}_s$ are slowly varying functions of space on the scale of the appropriate wavelengths.

B. The density response

In anticipation of a generated EM wave $\mathbf{E}_c(\mathbf{r},t)$, which is driven by the three external EM waves, we expect the current source $\mathbf{j}$ of Eq. (5) to be of third order in ξ, i.e., to be proportional to the product of $E_aE_bE_s$. From Eq. (6) for the current we see that since $n_\alpha^{(1)} = 0$, the dominant contribution to the third-order current is the product of n_α to second order and $\mathbf{v}_\alpha$ to first order. We denote now by $\mathbf{v}_\alpha^{(a)}$ the linear velocity response to the internal field $\mathbf{E}^{(a)}$ and similarly for $\mathbf{v}_\alpha^{(b)}$ and $\mathbf{v}_\alpha^{(s)}$; we also denote by $n_\alpha^{(a,s)}$ the second-order density response to the fields (a) and (s), and so on. Furthermore, since we are interested only in a wave reflected opposite to the signal field $\mathbf{E}^{(s)}$, we conclude that the driving current in Maxwell's equations is

$$\mathbf{j}_{\text{drive}} = \mathbf{j}^{(3)}(\mathbf{r},t) = -e\big[n_e^{(a,s)}(\mathbf{r},t)\mathbf{v}_e^{(b)}(\mathbf{r},t) + n_e^{(b,s)}(\mathbf{r},t)\mathbf{v}_e^{(a)}(\mathbf{r},t)\big]\,, \tag{27}$$

where the neglected ionic current is smaller than the electronic current by the factor m/M. This calls for a simple interpretation of the driving current; namely, that the generated wave c is due to the scattering of the pump wave b from the density grating produced by the mixing in the plasma of the signal s with the pump wave a and the scattering of a from the (b,s) grating.

We turn now to the calculation of the density grating $n_\alpha^{(a,s)}(\mathbf{r},t)$ due to the interaction of the pump wave (a) and the signal wave (s) with the plasma. These calculations are similar to those of Ref. 6. Consider the two driving fields to be [see Eq. (26)]

$$\mathbf{E}^{(a)}(\mathbf{r},t) = \mathbf{E}_a \cos(\mathbf{k}_a\cdot\mathbf{r} - \omega_a t + \phi_a), \quad \mathbf{E}_a\cdot\mathbf{k}_a = 0\,,$$

and (28)

$$\mathbf{E}^{(s)}(\mathbf{r},t) = \mathbf{E}_s(\mathbf{r})\cos(\mathbf{k}_s\cdot\mathbf{r} - \omega_s t + \phi_s), \quad \mathbf{E}_s\cdot\mathbf{k}_s = 0\,.$$

We have explicitly indicated that the amplitude of the signal wave varies slowly in space, while we assumed that the possible change of the pump-wave amplitude is not considered here. In studying the density response $n_\alpha^{(a,s)}$ to the pump and signal waves special attention should be paid to the emergence of resonances at frequencies $\omega = \pm(\omega_a - \omega_s)$, and wave vectors $\mathbf{k} = \pm(\mathbf{k}_a - \mathbf{k}_s)$. In particular, we consider only the cases when the resonance frequency is much smaller than ω_a (or ω_s) but the wave vector $\mathbf{k}$ is of the order of $\mathbf{k}_a$ (or $\mathbf{k}_s$). Since this is a longitudinal response, only Poisson's equation is relevant for the mixing process.

Knowing the first-order solutions for the various quantities, we calculate the second-order distribution function $f^{(a,s)}(\mathbf{r},t;\mathbf{v}) = \tilde{f}$ using the perturbation expansion. We write Eq. (1) as

$$\frac{\partial}{\partial t}\tilde{f}_\alpha + \mathbf{v}\cdot\nabla\tilde{f}_\alpha + \frac{q_\alpha}{m_\alpha}\widetilde{\mathbf{E}}\cdot\frac{\partial}{\partial \mathbf{v}} f_\alpha^0 = \tilde{I}_\alpha\,, \tag{29}$$

where

$$\tilde{I}_\alpha = -\frac{q_\alpha}{m_\alpha}\left[\left(\mathbf{E}^{(a)}(\mathbf{r},t) + \frac{1}{c}\mathbf{v}\times\mathbf{B}^{(a)}(\mathbf{r},t)\right)\cdot\frac{\partial}{\partial\mathbf{v}} f_\alpha^{(s)}(\mathbf{r},t;\mathbf{v}) + (a \rightleftarrows s)\right] \tag{30}$$

is given in terms of Eqs. (16)–(18), and the superscript (1) is replaced by (a) or (s). In Eq.(29), the self-consistent electric field $\widetilde{\mathbf{E}}$ is longitudinal, and obeys Poisson's equation

$$\nabla\cdot\widetilde{\mathbf{E}} = 4\pi\sum_\alpha q_\alpha \tilde{n}_\alpha\,, \tag{31}$$

while the magnetic field $\widetilde{\mathbf{B}}$ is negligible. Remembering that only the terms with $\mathbf{k} = \mathbf{k}_a - \mathbf{k}_s$ and $\omega = \omega_a - \omega_s$ are of interest in Eq.(30), we write

$$\tilde{f}_\alpha(\mathbf{r},t;\mathbf{v}) = \tilde{f}_\alpha(\mathbf{v})e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}\,.$$

We solve Eq. (29) for $\tilde{f}_\alpha(\mathbf{v})$ and use

$$\tilde{n}_\alpha = \int d^3v\, f_\alpha(\mathbf{v}) \tag{32}$$

to obtain an equation for the density, i.e.,

$$\tilde{n}_\alpha + \frac{1}{4\pi q_\alpha} \chi_\alpha(\mathbf{k},\omega)(i\mathbf{k}\cdot\widetilde{\mathbf{E}}) = G_\alpha^{(a,s)}, \tag{33}$$

where

$$\chi_\alpha(\mathbf{k},\omega) = \frac{4\pi q_\alpha^2}{m_\alpha k^2} \int d^3v \frac{1}{\omega - \mathbf{k}\cdot\mathbf{v} - i\eta} \mathbf{k}\cdot\frac{\partial}{\partial \mathbf{v}} f_\alpha^0(\mathbf{v}) \tag{34}$$

is the linear susceptibility of the α species, and η is positive infinitesimal. The inhomogeneous term is given by

$$G_\alpha^{(a,s)} = \frac{1}{2}\frac{q_\alpha^2}{m_\alpha^2}\int d^3v \frac{1}{\omega - \mathbf{k}\cdot\mathbf{v}} \bigg[\left(\mathbf{E}_a + \frac{1}{c}\mathbf{v}\times\mathbf{B}_a\right) \cdot\frac{\partial}{\partial \mathbf{v}}\left(\frac{1}{\omega_b - \mathbf{k}_s\cdot\mathbf{v}}\right)\left(\mathbf{E}_s + \frac{1}{c}\mathbf{v}\times\mathbf{B}_s\right) \cdot\frac{\partial}{\partial v} f_\alpha^0 - (a \rightleftarrows s)\bigg]. \tag{35}$$

Notice that the ionic term $\widetilde{G}_I$ is smaller by the mass ratio than the electronic $\widetilde{G}_e$, and therefore is neglected; however, χ_I and χ_e are of the same order and both must be retained. With Eq. (31),

$$i\mathbf{k}\cdot\widetilde{\mathbf{E}} = 4\pi \sum_\alpha q_\alpha \tilde{n}_\alpha,$$

we solve for the electronic density response to find

$$\tilde{n}_e(\mathbf{k},\omega) = G_e^{(a,s)}[1/\epsilon(k,\omega)][1 + \chi_I(k,\omega)], \tag{36}$$

where

$$\epsilon(\mathbf{k},\omega) = 1 + \chi_I(k,\omega) + \chi_e(\mathbf{k},\omega) \tag{37}$$

is the linear dielectric function of the plasma. Equation (36) is our result for $n_e^{(a,s)}$, with $\mathbf{k} = \mathbf{k}_a - \mathbf{k}_s$ and $\omega = \omega_a - \omega_s$.

Let us make a digression to the fluid description. For comparison we outline here the calculations of $\tilde{n}_e$ in the fluid approximation. Instead of Eq. (1) we now study Eqs. (2) and (3) to second order in ξ. We rewrite Eq. (3) as

$$\frac{\partial}{\partial t}\tilde{\mathbf{v}}_\alpha + \frac{T_\alpha}{m_\alpha}\frac{1}{n_0}\nabla\tilde{n}_\alpha - \frac{q_\alpha}{m_\alpha}\widetilde{\mathbf{E}} = -(\mathbf{v}_\alpha\cdot\nabla)\mathbf{v}_\alpha - \frac{q_\alpha}{m_\alpha c}\mathbf{v}_\alpha\times\mathbf{B}, \tag{38}$$

where the nonlinear terms are placed on the right-hand side. If we use the identity

$$\nabla(\mathbf{v}_\alpha^{(a)}\cdot\mathbf{v}_\alpha^{(s)}) = (\mathbf{v}_\alpha^{(a)}\cdot\nabla)\mathbf{v}_\alpha^{(s)} + \mathbf{v}_\alpha^{(a)}\times(\nabla\times\mathbf{v}_\alpha^{(s)})$$

and Eq. (22), we have for the second-order quantities

$$\frac{\partial}{\partial t}\tilde{\mathbf{v}}_\alpha + \frac{T_\alpha}{m_\alpha}\frac{1}{n_0}\nabla\tilde{n}_\alpha - \frac{q_\alpha}{m_\alpha}\widetilde{E} = -\nabla(\mathbf{v}_\alpha^{(a)}\cdot\mathbf{v}_\alpha^{(s)}), \tag{39}$$

and the continuity equation is

$$\frac{\partial}{\partial t}\tilde{n}_\alpha + n_0\nabla\cdot\tilde{\mathbf{v}}_\alpha = 0. \tag{40}$$

Notice that the driving force of Eq. (39) is again of the form $\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \tilde{\phi})$ with $k = k_a - k_s$ and $\omega = \omega_a - \omega_s$, and that the rhs of Eq. (39) for the ions is negligible. Assume that all second-order quantities are of the form

$$\tilde{n}_\alpha(\mathbf{r},t) = \tilde{n}_\alpha e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)},$$

we find for the ions, in the small m/M limit,

$$\tilde{n}_I + (1/4\pi e)\widetilde{\chi}_i(\mathbf{k},\omega)(i\mathbf{k}\cdot\widetilde{\mathbf{E}}) = 0, \tag{41}$$

where the ionic susceptibility in the fluid description is

$$\widetilde{\chi}_I = -\omega_I^2/\omega^2, \tag{42}$$

with the ion plasma frequency $\omega_I^2 = 4\pi e^2 n/M$. For the electrons we introduce a phenomenological lifetime τ in the velocity equation and find

$$\tilde{n}_e - (1/4\pi e)\widetilde{\chi}_e(\mathbf{k},\omega)(i\mathbf{k}\cdot\mathbf{E}) = \widetilde{G}_e^{(a,s)}, \tag{43}$$

where the electron susceptibility is now

$$\widetilde{\chi}_e(\mathbf{k},\omega) = -\omega_e^2/(\omega^2 - k^2 v_{\text{th}}^2 - i\omega/\tau), \tag{44}$$

and the inhomogeneous term is

$$\widetilde{G}_e^{(a,s)} = -\frac{1}{4\pi m}\frac{\mathbf{E}_a\cdot\mathbf{E}_s}{\omega_a\omega_s}k^2\widetilde{\chi}_e(\mathbf{k},\omega). \tag{45}$$

Finally, using Poisson's equation, Eq. (31), we recover Eq. (36) for the electronic density response, with G, ϵ, and χ_I replaced by the fluid approximation quantities. This also agrees in form with the result for the density of Drake *et al.*[6]

We now return to the Vlasov description and investigate the electron density $n_e^{(a,s)}$ due to the mixing of (a) and (s), near the ion acoustic resonance of the dielectric function. If we take the thermal distribution of Eq. (19) and specify

$$(T_I/M)^{1/2} < \omega/k < (T_e/m)^{1/2},$$

Eq. (34) yields $\chi_I = -\omega_I^2/\omega^2$, and

$$\chi_e(\mathbf{k},\omega) = \omega_e^2/k^2 v_{\text{th}}^2 - i\Gamma_a, \tag{46}$$

where Γ_a is the imaginary part of the electronic susceptibility. In the fluid case we can obtain Eq. (46) from Eq. (44) if we interpret τ as being caused by Landau damping. Now we write

$$\epsilon_a(k,\omega) = 1 - \omega_I^2/\omega^2 + k_e^2/k^2 - i\Gamma_a, \tag{47}$$

where $k_e^2 = \omega_e^2/v_{\text{th}}^2$ is the inverse Debye screening length. The ion acoustic resonance occurs at $\omega = \sim c_s k$ with $c_s = \sqrt{m/M}\,v_{\text{th}}$ with width $\sqrt{(\pi/8)m/M}$ when $T_I \ll T_e$. Notice that near this resonance the second term, χ_I, in the bracket of Eq. (36) is much larger than one. Next, we calculate $G_e^{(a,s)}$ from the Vlasov description, Eq. (35), and find that near the acoustic resonance, the leading term in the $\beta + v_{\text{th}}/c$ expansion and to lowest order in $c_s/v_{\text{th}} = \sqrt{m/M}$ gives

$$G_e^{(a,s)} = -\frac{3}{8\pi}\frac{\omega_e^2}{T_e}\frac{\mathbf{E}_a\cdot\mathbf{E}_s}{\omega_a\omega_s}, \tag{48}$$

which agrees with the fluid case (for negligible Γ_a). Finally, we express the density response as

$$n_e^{(a,s)}(\boldsymbol{\kappa},\omega) = \frac{3}{8\pi}\frac{m}{M}\frac{\omega_e^4}{\omega^2}\frac{1}{T_e}\frac{\mathbf{E}_a\cdot\mathbf{E}_s}{\omega_a\omega_s}\frac{1}{\epsilon(\mathbf{k},\omega)}. \tag{49}$$

Notice that the density grating due to the mixing of the pump wave b and the signal s does not, in general, resonate when the frequencies and wave vectors are so chosen that a

and s mix strongly. Thus the driving current of Eq. (27) is dominated by the term with $n_e^{(a,s)}$ and $\mathbf{v}_e^{(b)}$.

C. Generation of the reflected wave

Now that we have the proper driving current, Eq. (27),

$$\mathbf{j}_{\text{drive}}(\mathbf{r},t) = -en_e^{(a,s)}(\mathbf{r},t)\mathbf{v}_e^{(b)}(\mathbf{r},t),$$

we return to our basic equations and solve for the reflected wave $\mathbf{E}_a^{(c)}(\mathbf{r},t)$ to third order in ξ. First, we write more explicitly the driving current. From Eq. (49) we read

$$n_e^{(a,s)}(\mathbf{r},t) = |n_e^{(a,s)}(\mathbf{k}_{as},\omega_{as})|\cos(\mathbf{k}_{as}\cdot\mathbf{r} - \omega_{as}t + \phi_n), \tag{50}$$

where $\mathbf{k}_{as} = \mathbf{k}_a - \mathbf{k}_s$ and $\omega_{as} = \omega_a - \omega_s$ are the resonant wave vector and frequency, respectively, $\phi_n = \phi_a - \phi_s + \Psi_{as}$, and $1/\epsilon = |1/\epsilon|e^{i\Psi}$. From Eq. (21) we have

$$\mathbf{v}_e^{(b)}(\mathbf{r},t) = -(e/m)(1/\omega_b)\mathbf{E}_b\sin(\mathbf{k}_b\cdot\mathbf{r} - \omega_b t + \phi_b),$$

and thus

$$\mathbf{j}_{\text{drive}}(\mathbf{r},t) = \mathbf{j}_d\cos(\mathbf{k}_c\cdot\mathbf{r} - \omega_c t + \phi_c),$$
$$\mathbf{j}_d = (e^2/2m\omega_b)\mathbf{E}_b|n_e^{(a,s)}(\mathbf{k}_{as},\omega_{as})|, \tag{51}$$

with $\mathbf{k}_c = \mathbf{k}_a - \mathbf{k}_s + \mathbf{k}_b$, $\omega_c = \omega_a - \omega_s + \omega_b$, and $\phi_c = \phi_a - \phi_s + \phi_b + \Psi_{as} - \pi/2$. The term with the frequency $\omega = \omega_a - \omega_s - \omega_b$ is discarded here. Since our phase conjugate configuration is such that the pump waves are antiparallel $\mathbf{k}_a = \mathbf{k}_0$, $\mathbf{k}_b = -\mathbf{k}_0$, and $\omega_a = \omega_b = \omega_0$, we have $\mathbf{k}_c = -\mathbf{k}_s$, and $\omega_c = \omega_0 + \omega_{as}(\omega_{as} \ll w_0)$. From charge conservation we have

$$\rho_{\text{drive}}(\mathbf{r},t) = (1/\omega_c)\mathbf{k}_c\cdot\mathbf{j}_{\text{drive}}(\mathbf{r},t). \tag{52}$$

Notice that the system is driven by a high frequency current in the direction of the pump field $\mathbf{E}_b$. We also choose $\mathbf{k}_s\cdot\mathbf{E}_b = 0$ and thus make also $\mathbf{k}_c\cdot\mathbf{E}_b = 0$ and $\rho_{\text{drive}} = 0$. Thus the plasma response (of order ξ^3) is electromagnetic, i.e., transverse in nature.

In order to find the reflected wave we use Eqs. (1) and (4)–(6) and repeat the calculations in the spirit of the section on the internal fields. However, the system is driven now by the current densities of Eq. (51). We assume again a plane wave solution [compare to Eq. (16)] for the generated wave

$$\mathbf{E}^{(c)}(\mathbf{r},t) = \hat{\epsilon}_c E_c(\mathbf{r})\cos(\mathbf{k}_c\cdot\mathbf{r} - \omega_c t + \phi_c), \tag{53}$$

where $\hat{\epsilon}_c$ is the polarization vector and $E_c(\mathbf{r})$ is the slowly varying amplitude. Solving for the self-consistent velocities and currents [compare to the calculations leading from Eq. (16) to Eq. (24)] we end up with the wave equation

$$\left(\nabla^2 + \frac{\omega_c^2}{c^2} + \frac{\omega_e^2}{c^2}\right)\mathbf{E}^c(\mathbf{r},t) = \frac{4\pi}{c^2}\frac{\partial}{\partial t}\mathbf{j}_{\text{drive}}(\mathbf{r},t). \tag{54}$$

The wave $\mathbf{E}^{(c)}(\mathbf{r},t)$ is thus polarized in the direction of the current $\mathbf{j}_{\text{drive}}$ or $\mathbf{E}_b$ [see Eq. (51)], and its amplitude varies slowly along its propagating direction $\mathbf{k}_c/k_c$, which we assume to be along the z axis, i.e.,

$$\mathbf{k}_c\cdot\nabla E_c(\mathbf{r}) \gg \nabla^2 E_c(\mathbf{r}). \tag{55}$$

Since $k_c^2 = (1/c)(\omega_c^2 + \omega_e^2/c^2)$ we find for $E_c(z)$,

$$2k_c\frac{d}{dz}E_c(z) = \frac{4\pi\omega_c}{c^2}j_{\text{drive}}. \tag{56}$$

Using Eqs. (51) and (49), and assuming that all waves have the same polarization we obtain

$$\frac{d}{dz}E_c(z) = \kappa E_s(z), \tag{57}$$

where, up to numerical factors of order unity,

$$\kappa = r_0\frac{\lambda_0}{2\pi}\frac{E_0^2}{mc^2}\left(\frac{mc^2}{T_e}\right)^2\left(\frac{\omega_e}{\omega_0}\right)^4\frac{1}{|\epsilon|}. \tag{58}$$

Here $r_0 = e^2/mc^2$ is the classical radius of the electron, and the zero subscript (0) corresponds to the pump-wave parameters. As we mentioned before, at resonance, ϵ in Eq. (47) is equal to Γ_a. If we assume Landau damping and $T_e \gg T_i$ (at least $T_e/T_i \geqslant 15$) it can be shown that

$$\epsilon = \Gamma_a = (k_e^2/k^2)\cdot(\pi m/8M)^{1/2}. \tag{59}$$

We have, now, to supplement Eq. (57) with a similar equation for the signal wave $E_s(z)$. This can be achieved by following the same procedure which led us to Eq. (57), but replacing (s) by (c) and exchanging (a) and (b). Starting with the three waves (a), (b), and (c) in the plasma (in the above phase conjugate configuration) we calculate first the density response $n_e^{(b,c)}(\mathbf{r},t)$ due to the pump wave (b) and the conjugate wave (c). This leads to the same expression for $n_e^{(b,c)}(\mathbf{k},\omega)$, where $\mathbf{k} = \mathbf{k}_b - \mathbf{k}_c (= \mathbf{k}_a - \mathbf{k}_s)$ and $\omega = \omega_c - \omega_b (= \omega_a - \omega_s)$, i.e., to the same grating in space (and time reversed). The scattering of pump wave (a) from this grating, which is represented by

$$j_{\text{drive}}(\mathbf{r},t) = -en_e^{(b,c)}(\mathbf{r},t)v_e^{(a)}(\mathbf{r},t)$$

up to third order in ξ, will drive a wave equation, similar to Eq. (54), for $E^{(s)}(r,t)$. The only difference is that now, since the vector $\mathbf{k}_s$ is parallel to z, we have, instead of Eq. (57),

$$\frac{d}{dz}E_s(z) = -\kappa E_c(z) \tag{60}$$

for the slowly varying amplitudes.

Following the standard treatment of phase conjugation by four wave mixing, almost degenerate in our case (see Ref. 1), we solve Eqs. (57) and (59). Assuming that, at the entrance to the plasma $z = 0$ (see Fig. 2), the incoming signal

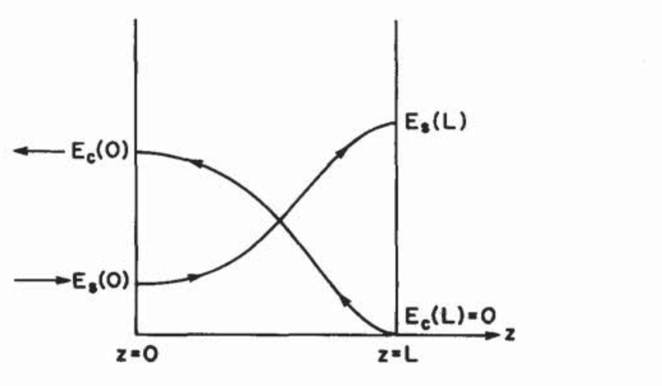

FIG. 2. Schematic generation of the amplified conjugate wave E_c by the nonlinear interaction of the signal wave E_s with the pump waves.

E_s has amplitude $E_s(0)$, and that at the back $z = L$, no conjugated wave is present, i.e., $E_c(L) = 0$, we find, for the generated conjugated wave at the entrance

$$E_c(0)/E_s(0) = \tan(\kappa L) . \quad (61)$$

A conjugated wave, with amplitude $E_c(0)$, is now reflected from the surface of the plasma. This wave is generated "backward," from $z = L$ to $z = 0$ by the nonlinear mixing of the pump waves with the signal wave, at the expense of the pump energy.

To get a feeling for the order of magnitude of κ we give a numerical estimate of κL in Eq. (60). We consider a hydrogenic plasma with $(m/M)^{1/2} = 2.4\times10^{-2}$, density $n_0 \sim 10^{12}/\text{cm}^3$, and temperature $T = 2$ eV. The power of the radiation source at $\lambda_0 = 1$ cm will be 250 W/cm^2. With $r_0 = 2.8\times10^{-13}$ cm and $\epsilon = \Gamma_a = 10^3$ we find that $\kappa \sim 0.1$ and for a 10 cm plasma $\kappa L \sim 1$. Notice that formally when $\kappa L = \pi/2$ the gain is infinite.

IV. DISCUSSION

In conclusion, we have demonstrated that a plasma with easily attainable properties can be used as an effective nonlinear medium for a phase conjugate reflector of electromagnetic waves in the subcentimeter region of wavelengths. We have exhibited that by tuning the signal frequency to be slightly shifted with respect to the pump waves, so that a collective mode of the plasma is excited, the reflected phase conjugate wave can be significantly enhanced, and amplification occurs.

Phase conjugate reflection of microwaves opens up a new technology with many applications. Since the amplification in Eq. (61) can considerably exceed unity, a cavity can be formed from a phase conjugate reflector and a simple mirror. Such a cavity is a broadband microwave amplifier, and if the amplification exceeds the mirror losses, it is a self-exciting laser. If the mirror is replaced by a set of two mirrors, forming a Fabry–Perrot étalon, a narrow band amplifier results.

Another interesting application of phase conjugate reflection from a plasma is in the field of hyperthermia, or microwave cancer therapy.[15,16] This method involves radiation heating of a portion of a patient's body by external applicators placed near the patient's skin and radiating at 0.1–2.45 GHz. The penetration into the human body is, however, poor, the extinction coefficient being of the order of 0.1–1.0 cm^{-1}. The focusing of the beam on the tumor is also hard, both because of diffraction because of the finite size of the applicators, and because of variations of the local dielectric constant inside the human body.

Use of phase conjugate microwave reflectors can contribute to this in several respects. If a small emitting dipole is inserted in the tumor—and this can be done[16]—then an outgoing signal is produced. If this signal meets, after traversing the patient's body, a phase conjugate reflector, then it can be amplified and retrace its exact path to the tumor. In this way a strong energy-bearing beam can be directed to its destination, albeit any temporal or spatial changes in the refractive index of the patient's body. Moreover, the beam will be spherically convergent on the tumor, and so will gain in strength on its way. A numerical example can demonstrate this. At a frequency of 2 GHz, the attenuation coefficient α can become, depending on the composition of the tissue, of order 1 cm^{-1}. A plane wave will propagate like $I_0 \exp(-\alpha x)$. A spherically convergent wave will converge as $(R^2/r^2) U_0 \exp[-\alpha(R - r)]$, and including the effect of a diffraction, the convergence will be of order $[(R^2 + \lambda^2)/(r^2 + \lambda^2)] I_0 \exp[-\alpha(R - r)]$. Here λ is the wavelength inside the tissue, which for 2 GHz is about 2 cm. Taking the reflector to be, e.g., at $R = 5$ cm from the tumor, the relative intensities are

r (cm)	5	4	3	2	1	0
plane propagation	1	0.37	0.14	0.05	0.02	0.007 .
spherical convergence	1	0.54	0.31	0.18	0.12	0.05

One sees that a depth of 5 cm, which is very hard to attain with 2 GHz present-day applicators (even a phased array[15]), becomes accessible with a phase conjugate reflector (the heating at $r = 5,4$, even 3 cm can be carried away by water cooling[15,16]).

As another possible application, energy can be transferred from a terrestrial power station energizing a plasma phase conjugate reflector to an orbiting satellite provided by a weak microwave transmitter. Similarly, a solar energy satellite can send energy to a terrestrial station in the form of a self-guiding, phase conjugate, amplified reflected wave. More generally, power can be transmitted from point to point in response to a weak guiding signal.

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