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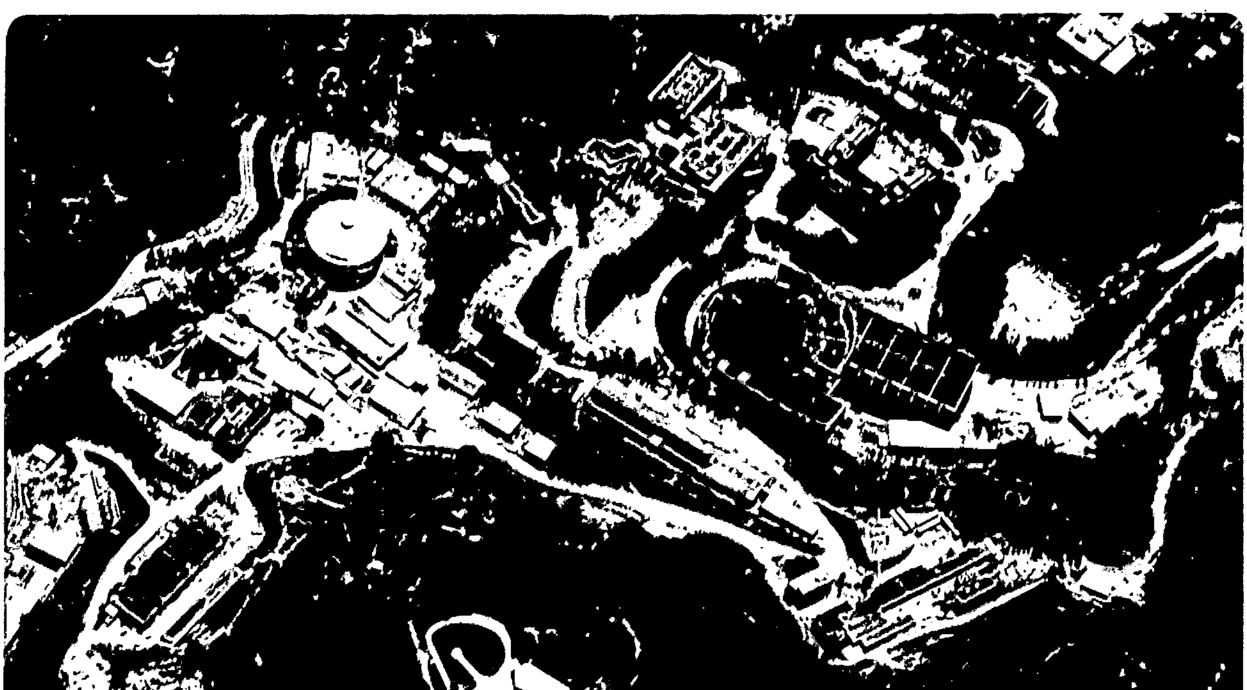

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Hadronic Flavor and CP Violating Signals of Superunification*

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Abstract

The flavor changing and CP violating phenomena predicted in supersymmetric unified theories as a consequence of the large top quark Yukawa coupling, are investigated in the quark sector and compared with related phenomena in the lepton sector, considered previously. In particular we study ε_K, $\varepsilon'_K/\varepsilon_K$, Δm_B, $b \to s\gamma$, the neutron electric dipole moment, d_n, and CP violation in neutral B meson decays, both in minimal SU(5) and SO(10) theories. The leptonic signals are generically shown to provide more significant tests of quark-lepton unification. Nevertheless, mostly in the SO(10) case, a variety of hadronic signals is also possible, with interesting correlations among them.

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1 Introduction

The most widely discussed signatures of grand unification, studied since the 1970's, are proton decay, neutrino masses, fermion mass relations and the weak mixing angle prediction. The precise measurement of the weak mixing angle at Z factories suggests that these theories should incorporate weak-scale supersymmetry, making superpartner mass relations a further signature. In recent papers we have identified new signatures for supersymmetric unification, with supersymmetry broken as in supergravity, which provide signals which are less model dependent than those of proton decay, neutrino masses and fermion mass relations. These new signatures include lepton flavor violation [1] and electric dipole moments for the electron, d_e, and for the neutron, d_n [2]. In a detailed study of the lepton signals [3], rates for $\mu \to e\gamma$ and for $\mu \to e$ conversion in atoms and values for d_e have been given over the entire range of parameter space of simple SU(5) and SO(10) models. Further searches for these signals can probe selectron mass ranges of $100 \div 200$ GeV for SU(5) and $300 \div 600$ GeV for SO(10), and are clearly very powerful.

This new class of signals arises because the top Yukawa coupling of the unified theory leads to very large radiative corrections to the masses of those superpartners which are unified with the top. In the lepton sector this leads to an important non-degeneracy of the sleptons, giving lepton flavor mixing matrices at neutral gaugino vertices. It is clear that this phenomena is not limited to the lepton sector, and the purpose of this paper is to study the flavor changing and CP violating phenomena induced by this mechanism in the quark sector. In particular we study ε_K, $\varepsilon'_K/\varepsilon_K$, Δm_B, $b \to s\gamma$, d_n and CP violation in neutral B meson decay. We address the following questions:

(A) How strong a limit is placed on the parameter space of unified models by present measurements of hadronic flavor and CP violation?

(B) Can future measurements of hadronic flavor and CP violation provide a test of supersymmetric unification?

(C) If so, how does the power of these probes compare with the lepton signals?

The answers to these questions are crucial in determining the optimal experimental strategy for using this new class of signatures to probe unified theories. For example, it is crucial to know whether new gluino-mediated contributions to ε_K are so large that the resulting constraints on the parameter space preclude values of $\Gamma(\mu \to e\gamma)$ and d_c which are accessible to future experiments.

If gluino-mediated flavor changing effects are found to be very large, what are the best experimental signatures? Three possibilities are:

i) A pattern of CP violation in neutral B meson decays which conflicts with the prediction of the SM.

ii) Predictions for ε_K and Δm_B which deviate from SM predictions for measured values of m_t and V_{ub}.

iii) A prediction for B_s meson mixing (x_s/x_d) which differs from the prediction of the SM.

In Section 2 we define the minimal SU(5) and SO(10) models. The superpartner spectrum for these models is discussed in Section 3. In Section 4 both analytic and numerical results are given for the hadronic processes of interest in the minimal SO(10) model. We illustrate why in the SU(5) case the hadronic signals are less relevant. A study of these results, and a comparison with the predictions for the lepton signals, allows us to answer questions (A), (B) and (C) above. We aim at an overall view rather than at a detailed analysis of the various effects. In Section 5 we mention aspects of the assumptions which underlie our signatures. Our results are summarized in Section 6, where we also show that our conclusions are not specific to the minimal models, but are more generally true.

2 The Minimal Models

In this paper we give results for flavor-changing and CP violating processes in two minimal superunified models, one based on SU(5) and the other on SO(10). The flavor structure of the models is constructed to be particularly simple, and the corresponding flavor mixing matrices of the low energy supersymmetric theory possess a very simple form, which directly reflects the unified group. Nature is likely to be more complicated. In the conclusions we discuss the extent to which our results are expected to hold in more general models. The predictions of the minimal models provide a useful reference point. They provide a clean estimate of the size of the effects to be expected from the top Yukawa coupling in theories where the top quark is unified with other particles of the the third generation. There are many additional flavor and CP violating effects which could be generated from other interactions of the unified theory and could be much larger then those considered here. While cancellations between different contributions can never be excluded, the contribution given here provides a fair representation of the minimal amount to be expected. Circumstances which could lead to a significant reduction of the signals are discussed in Section 5.

A crucial assumption, discussed in detail in Section 5, is that the supersymmetry breaking is communicated to the fields of the Minimal Supersymmetric Standard Model (MSSM) at a scale above the unification mass, M_G. For the analysis of this paper we assume the communication occurs at the reduced Planck scale, M_{Pl}, as in supergravity theories [4], and furthermore we assume that at this scale the supersymmetry breaking is universal. This means that all scalars acquire a common supersymmetry break mass, m_0^2 and all trilinear superpotential interactions generate a supersymmetry breaking trilinear scalar interaction with common strength given by the parameter A_0. Similarly, there is a common gaugino mass M_0. This boundary condition is not crucial to our effect; it is the simplest which involves no flavor violation, so we can be sure that the signals we calculate originate only from radiative effects of the top quark Yukawa coupling.

Before introducing the two minimal unified models, we review the flavor and CP violating signals induced by the top quark Yukawa coupling of the MSSM [5, 6, 7]. The universal boundary condition on the supersymmetry breaking interactions leads to the conservation of individual lepton numbers in the MSSM, so we discuss only the quark sector, where the superpotential can be written as:

$$W_{\text{MSSM}} = Q\overline{\boldsymbol{\lambda}}_U U^c H_2 + Q\boldsymbol{\lambda}_D D^c H_1 \tag{1}$$

where $\boldsymbol{\lambda}_D = \boldsymbol{V}^*\overline{\boldsymbol{\lambda}}_D, \boldsymbol{V}$ is the Kobayashi-Maskawa (KM) matrix, and $\overline{\boldsymbol{\lambda}}_U$ and $\overline{\boldsymbol{\lambda}}_D$ are real and diagonal Yukawa coupling matrices. Throughout this paper we assume that the largest eigenvalue of $\boldsymbol{\lambda}_D$, λ_b, is sufficiently small that the only Yukawa coupling which need be kept in the renormalization group (RG) scaling of the theory is that of the top quark, λ_t. In the large $\tan\beta$ region there will be additional effects. The one loop RGE of the MSSM, including λ_t effects, is well known [5, 6]. For our purposes the most important effect is the reduction of the scalar masses of Q_3 and U_3^c beneath that of the other squarks. This lightness of the $\tilde{t}_L$, $\tilde{b}_L$ and $\tilde{t}_R$ squarks is very well-known; it is a feature which appears in the radiative breaking of SU(2) $\otimes$ U(1) which occurs in this theory. When the left-handed down quarks are rotated by the matrix $\boldsymbol{V}$ to diagonalize the quark mass matrix, the non-degeneracy of $\tilde{b}_L$ with $\tilde{s}_L$ and $\tilde{d}_L$ implies that if this rotation is also performed on the squarks they will acquire an off diagonal mass matrix. In this paper we work in a mass basis for the squarks, so that the rotation $\boldsymbol{V}$ is done only on the d_L fermions not on the $\tilde{d}_L$ scalars. This results in the appearance of the KM matrix at the neutralino gauge vertices. In particular, for the gluino $\tilde{g}$ we find

$$\mathcal{L}_{\text{MSSM}} \supset \sqrt{2}g_3(\tilde{d}_L^* T^a \boldsymbol{V} d_L)\tilde{g}^a. \tag{2}$$

The phenomenological effects of this flavor mixing at the gaugino vertex are known to be slight. There are gluino mediated box diagram contributions to $K^0\bar{K}^0$ and $B^0\bar{B}^0$ mixing. The contribution to Δm_K is negligible, while that to ε_K and Δm_B is less than 10% of the SM contribution [5, 6]. Such precise statements are possible because the mixing matrix appearing in (2) is the KM matrix, and because we know that the gluino and squark masses are larger than 150 GeV in the MSSM. Because the mixing matrix introduces no new phases, the extra contribution to $B^0\bar{B}^0$ mixing does not effect CP violation in B meson decays [7, 8]. The asymmetries for $B_d \to \pi^+\pi^-$, $B_d \to \psi K_s$ and $B_s \to \rho K_s$ are proportional to $\sin 2\hat{\alpha}$, $\sin 2\hat{\beta}$ and $\sin 2\hat{\gamma}$ where, as in the SM, $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ are the angles of the unitarity triangle which closes: $\hat{\alpha} + \hat{\beta} + \hat{\gamma} = \pi$.

The superpenguin contribution to $\varepsilon'_K/\varepsilon_K$ is less than about 5×10^{-4} [9] and, given the theoretical uncertainties, is unlikely to be distinguished from the SM penguin contribution. In the MSSM a significant flavor changing effect is in the process $b \to s\gamma$ [6, 7, 10]. The recent experimental results from CLEO show that B.R.$(b \to s\gamma)$ is in the range $(1 \div 4) \cdot 10^{-4}$, at 95% confidence level. For $m_t = 175 \pm 15$ GeV the SM prediction is B.R.$(b \to s\gamma) = (2.9 \pm 1.0) \cdot 10^{-4}$. These results provide a considerable limit to the MSSM. However since the MSSM also involves a charged Higgs loop contribution, the limit does not apply directly to the gluino loop contribution, which involves the vertex of equation (2).

The Yukawa interactions for the minimal supersymmetric SU(5) theory are given by

$$W_{\text{SU(5)}} = T\overline{\boldsymbol{\lambda}}_U T H + T\boldsymbol{\lambda}_D \bar{F}\bar{H} \tag{3}$$

where T and $\bar{F}$ are 10 and $\bar{5}$ representations of matter, H and $\bar{H}$ are 5 and $\bar{5}$ Higgs supermultiplets, and the down Yukawa matrix can be taken to have the form $\boldsymbol{\lambda}_D = \boldsymbol{P}\boldsymbol{V}^*\overline{\boldsymbol{\lambda}}_D$. $\boldsymbol{V}$ is the KM matrix, $\boldsymbol{P}$ is a diagonal phase matrix with two physical phases and $\overline{\boldsymbol{\lambda}}_{U,D}$ are real and diagonal. Beneath M_G phase rotations can be performed so that $\boldsymbol{P}$ does not appear in the low energy interactions. The Yukawa interactions become those of the MSSM of equation (1) for the quarks, as well as $E^c\boldsymbol{\lambda}_E LH$, for the leptons, with $\boldsymbol{\lambda}_D = \boldsymbol{V}^*\overline{\boldsymbol{\lambda}}_D$ and $\boldsymbol{\lambda}_E = \boldsymbol{V}_G^*\overline{\boldsymbol{\lambda}}_E$, where $\boldsymbol{V}$ is the running KM matrix and $\boldsymbol{V}_G$ its value at M_G. For a given λ_t the scalar non-degeneracy for $\tilde{t}_L, \tilde{b}_L$ and $\tilde{t}_R$ are larger than in the MSSM. This is due to the modified numerical coefficients in the RGE above M_G. More importantly, since τ_R is unified with the top quark, the $\tilde{\tau}_R$ has a mass which is lowered compared to that of $\tilde{e}_R$ and $\tilde{\mu}_R$. This means that, in the mass basis for both fermions and scalars, in addition to neutral gaugino flavor mixing for d_L (as in equation (2)), there is also gaugino flavor mixing for e_R. Schematically representing the MSSM flavor mixing in the gauge couplings by

$$(\bar{u}\boldsymbol{V}d), \qquad (\tilde{d}^*\boldsymbol{V}d) \tag{4}$$

that for the minimal SU(5) theory can be written

$$(\bar{u}\boldsymbol{V}d), \qquad (\tilde{d}^*\boldsymbol{V}d), \qquad (\tilde{e}^{c*}\boldsymbol{V}_G e^c) \tag{5}$$

where all fermion fields are left-handed.

In SO(10) theories an entire generation is represented by a single spinor: 16. The Yukawa interaction 16 $\boldsymbol{\lambda}$16Φ, where Φ is a 10 dimensional Higgs multiplet, gives mass to the all the fermions, but does not allow generation mixing. We consider a minimal SO(10) model [2] with Yukawa interactions which can be put in the form

$$W_{\mathrm{SO(10)}} = 16\bar{\boldsymbol{\lambda}}_U 16\Phi_U + 16\boldsymbol{\lambda}_D 16\Phi_D. \tag{6}$$

All scalars of the third generation are split in mass from those of lighter generations, so that flavor mixing matrices appear at all neutral gaugino vertices, except those of the up sector. Beneath M_G the Yukawa interactions have the form

$$W'_{\mathrm{SO(10)}} = Q\bar{\boldsymbol{\lambda}}_U U^c H_2 + Q\boldsymbol{V}^*\bar{\boldsymbol{\lambda}}_D \boldsymbol{P}^{*2}\boldsymbol{V}^\dagger D^c H_1 + E^c \boldsymbol{V}_G^* \bar{\boldsymbol{\lambda}}_E \boldsymbol{P}^{*2}\boldsymbol{V}_G^\dagger L H_1 \tag{7}$$

where an asymmetric basis between left and right has been chosen such that $\boldsymbol{V}$ is the usual KM matrix, and $\boldsymbol{P}$ is a diagonal phase matrix with two independent phases, which we choose as

$$\boldsymbol{P}^2 = \begin{pmatrix} e^{i\hat{\varphi}_d} & 0 & 0 \\ 0 & e^{i\hat{\varphi}_s} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{8}$$

Using the schematic notation of equations (4) and (5), the flavor mixing of the minimal SO(10) theory has the structure

$$(\bar{u}_L \boldsymbol{V} d_L), \quad (\tilde{d}^* \boldsymbol{V} d), \quad (\tilde{d}^{c*} \boldsymbol{V}\boldsymbol{P}^2 d^c), \quad (\tilde{e}^{c*} \boldsymbol{V}_G e^c), \quad (\tilde{L}^* \boldsymbol{V}_G \boldsymbol{P}^2 L). \tag{9}$$

The flavor mixing structure of the minimal models is summarized by equations (4), (5) and (9), and the phenomenological consequences of these forms are the subject of Section 4 of this paper. The effects can be classified into two types:

(A) $(\tilde{d}^* \boldsymbol{V} d)$ effects. Although the mixing matrix is identical for MSSM and the minimal SU(5) and SO(10) models, the effects in the unified models are amplified because the modified coefficients in the unified RGE lead to larger non-degeneracies between $\tilde{b}_L$ and $\tilde{d}_L/\tilde{s}_L$. This is, however, not the dominant effect.

(B) Mixing in the d_R, e_R and e_L sectors. We have explored the consequences of lepton flavor violation in previous papers [1, 3] and found the signals for $\mu \to e\gamma$ and $\mu \to e$ conversion to be of great interest, especially in SO(10) where the mixing in both helicities implies that amplitudes for the processes can be proportional to m_τ rather than to m_μ. Also in the SO(10) case there are important contributions to the electron and neutron electric dipole moments, which, in a standard basis and notation for the KM matrix, are proportional to $\sin(\hat{\varphi}_d - 2\hat{\beta})$ [2, 3]. In this paper we compare these signals to the hadronic flavor violating ones.

3 The Scalar Spectrum

The masses of the scalars of the third generation receive important radiative corrections from the large λ_t coupling in SU(5) and SO(10) theories. The resulting spectrum provides an important signature of unification, which we present in this section.

In the minimal models there are 6 parameters which play a fundamental role in determining the spectrum, flavor and CP violating signals discussed in this paper. In more general models other parameters may enter, and we discuss this in Section 6. The 6 parameters are λ_t (the top quark coupling), m_0 (the common scalar mass at M_{Pl}), M_0 (the common gaugino mass at M_{Pl}), A_0 (the common coefficient of the supersymmetry breaking tri-scalar interactions at M_{Pl}), B (the coefficient of the Higgs boson coupling $h_1 h_2$ at low energies) and μ (the supersymmetric Higgsino mass parameter). The solutions of the RGE for the MSSM, minimal SU(5) and minimal SO(10) models has been given previously, including all one loop λ_t effects [3]. We do not repeat that analysis here, but rather recall the strategy which we take to deal with this large parameter space

λ_{tG} for our purposes it is most useful to parameterize the top Yukawa coupling by its value at the unification scale $\lambda_{tG} = \lambda_t(M_G)$. This is because the large radiative effects which generate our signals are induced by the top quark coupling in the unified theory. Now that the top quark has been found, it may be argued that λ_{tG} should be given in terms of other parameter $\lambda_{tG} = \lambda_{tG}(m_t, \tan\beta, \alpha_3)$, where $\tan\beta = v_2/v_1$ is the ratio of Higgs vacuum expectation values. In fact, for low values of $\tan\beta$, λ_{tG} has a strong dependence on α_3, and hence we prefer to keep λ_{tG} as the independent parameter. For larger values of $\tan\beta$, for example $\tan\beta \approx 10$, and with $m_t = 175 \pm 15\,\mathrm{GeV}$, λ_{tG} cannot be larger than unity. However, the prediction for m_b/m_τ requires a larger value of λ_{tG}, and hence we will not consider these larger values of $\tan\beta$ in this paper. Much larger values of $\tan\beta$, comparable to m_t/m_b, do allow large λ_{tG}, but in this case there will be many extra important renormalizations induced by the large coupling λ_b, which we have not included. Hence this paper does not consider the $\tan\beta \approx m_t/m_b$ case.

m_0 is traded for the mass of the right hand scalar electron $m_{\tilde{e}_R}$, since this is of more physical interest.

M_0 is traded for the low energy SU(2) gaugino mass parameter M_2. Note that while M_0/m_0 may be taken arbitrarily large, this is not true for $M_2/m_{\tilde{e}_R}$, which is restricted to be less than about unity. This is because a large value of M_0 generates large scalar masses through renormalization, especially in the unified theory where Casimirs are large [3] (we are insisting on $m_0^2 > 0$).

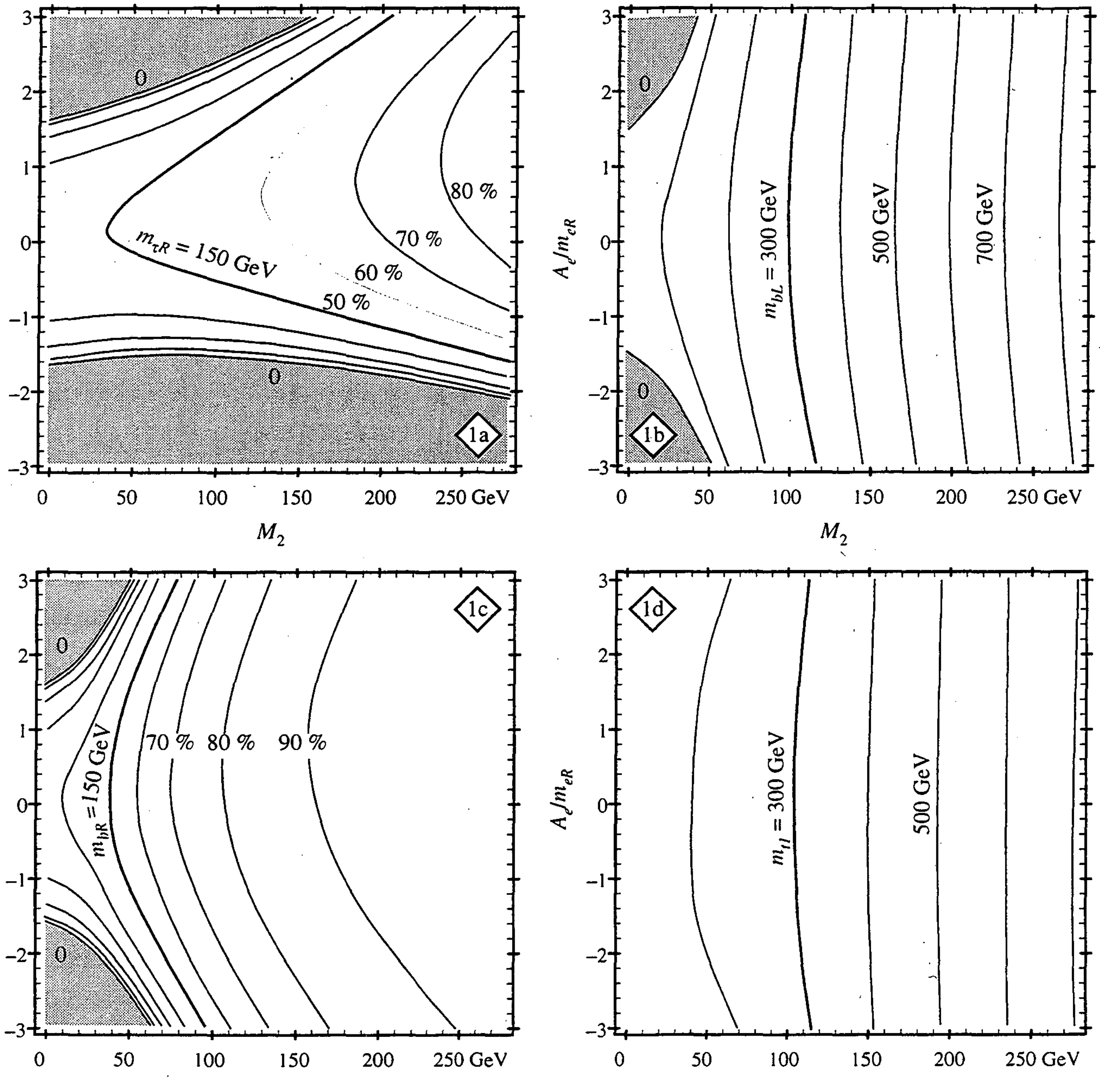

Figure 1: Contour plots of the masses of the third generation scalars in minimal SO(10) for $m_{\tilde{e}_R} = 300\,\mathrm{GeV}$ and $\lambda_{tG} = 1.25$: (a) $m_{\tilde{\tau}_R}/m_{\tilde{e}_R}$; (b) $m_{\tilde{b}_L}$; (c) $m_{\tilde{b}_R}/m_{\tilde{d}_R}$ and (d) the lightest stop for $\mu < 0$.

A_0 is traded for A_e, where the selectron trilinear scalar coupling is $A_e \lambda_e \tilde{L}_e \tilde{e}^c h_1$. The dimensionless parameter $A_e/m_{\tilde{e}_R}$ is restricted to be in the range -3 to $+3$ for reasons of vacuum stability.

B appears in the Higgs potential. On minimizing this potential, B is traded for $\tan\beta$.

μ appears in the Higgs potential. When this potential is minimized, μ^2 is determined by M_Z^2.

Hence the relevant parameter space is $\{\lambda_{tG}, m_{\tilde{e}_R}, M_2, A_e, \tan\beta, \text{sign}(\mu)\}$. All our signals are displayed in the $\{M_2, A_e/m_{\tilde{e}_R}\}$ plane, where M_2 and $A_e/m_{\tilde{e}_R}$ are allowed to run over their entire range. These planes are shown for representative choices of $\{\lambda_{tG}, m_{\tilde{e}_R}, \tan\beta\}$ and negative μ. Our conclusions do not depend on the sign of μ.

How large are the non-degeneracies amongst the scalars induced by the coupling λ_{tG} in the unified theory? A simple guess would be that the fractional breaking of degeneracies would be $\approx \lambda_{tG}^2/16\pi^2 \ln(M_{\text{Pl}}/M_{\text{G}})$, which is a few percent for $\lambda_{tG}^2 = 2$. In fact, the unified theory leads to a large Casimir, and also λ_{tG} may get larger above M_{G}, resulting in non-degeneracies which are an order of magnitude larger than this simple guess.

Numerical results are shown in Figure 1 for the case of $m_{\tilde{e}_R} = 300\,\text{GeV}$ in the minimal SO(10) theory. The results are insensitive to $\tan\beta$ and to the sign of μ. There is a large sensitivity to λ_{tG}. We take $\lambda_{tG} = 1.25$, which is below the fixed point value implied by the running of the Yukawa coupling from M_{G} to M_{Pl} [3]. Figures 1a and 1b, with relatively minor modifications, apply also to the minimal SU(5) case with $\lambda_{tG} = 1.4$. Over roughly half of the A_e/M_2 plane, the fractional non-degeneracies are above 30%. The fractional non-degeneracy is larger for the sleptons that for the squarks. This is because a radiative correction to all squark masses proportional to the gluino mass tends to restore the squark degeneracy. We call this the *"gluino-focussing"* effect; it is especially prominent for large gaugino masses. In SO(10) the non-degeneracies of the left-handed and right-handed squarks are very similar. The same is true for left and right-handed sleptons. This is the most important difference between the minimal SU(5) and SO(10) models: in the SU(5) case the left-handed sleptons are essentially degenerate, as are the right-handed down squarks.

The distinctive, large scalar non-degeneracies of Figure 1 will provide an important indication of unification. A precise measurement of these non-degeneracies will provide an essential component of the elucidation of the flavor structure of the unified theory.

4 Signals of minimal SO(10)

The minimal SO(10) model has flavor mixing angles at all neutral gaugino vertices, except those involving the up quark. Furthermore, the weak scale theory involves two additional phases, $\hat{\varphi}_s$ and $\hat{\varphi}_d$, beyond those of the MSSM, as can be seen from equations (8) and (9). The presence of flavor mixing at neutral gaugino vertices for both helicities of e and d, together with these extra phases, gives a much richer flavor structure to the minimal SO(10) model compared to that of the MSSM or minimal SU(5) theory. In fact, for this general reason, the hadronic signals in minimal SU(5) are not especially interesting. An explicit numerical calculation shows that, although somewhat larger than the corresponding effects in the MSSM, the gluino exchange contributions to the hadronic observables, in SU(5), do not compete with the leptonic flavor violating signals and are not considered anymore hereafter.

The strong signals in the lepton sector have been stressed before [1, 2, 3], and are briefly recalled here. The process $\mu \to e\gamma$ is induced by a chirality breaking operator which involves the dipole moment structure $(\sigma^{\mu\nu}F_{\mu\nu})$. In many theories, for example the minimal SU(5) theory, this chirality breaking implies that the amplitude is proportional to m_μ. However, flavor mixing in supersymmetric theories breaks chirality, if it occurs in both e_L and e_R sectors, and hence in the minimal SO(10) theory terms in the amplitude for $\mu \to e\gamma$ appear which are proportional to m_τ. This gives a large rate for $\mu \to e\gamma$, as illustrated in Figure 2a, for $\tan\beta = 2$, $\lambda_{tG} = 1.25$, $m_{\tilde{e}_R} = 300\,\text{GeV}$ and $\mu < 0$. Figures 3a and 4a show the $\mu \to e\gamma$ rate with all the same parameters as in Fig. 2a except for $\lambda_{tG} = 0.85$ (Fig. 3a) or for a scale $M = 2.0 \cdot 10^{17}\,\text{GeV}$ for the universal initial condition on all scalars and gaugino masses (Fig. 4a). A similar set of diagrams proportional to m_τ dominates d_e, which is related to the $\mu \to e\gamma$ branching ratio by a simple formula, valid over all regions of parameter space

$$\frac{d_e}{10^{-27}\,e \cdot \text{cm}} = 1.3 \sin(\hat{\varphi}_d - 2\hat{\beta}) \sqrt{\frac{\text{B.R.}(\mu \to e\gamma)}{10^{-12}}}. \tag{10}$$

where the KM matrix elements are taken to be approximately real, except for $V_{td} = |V_{td}|\, e^{-i\hat{\beta}}$ and $V_{ub} = |V_{ub}|\, e^{-i\hat{\gamma}}$. With this relation, Figures 2a, 3a, 4a can also be used to predict $d_e/\sin(\hat{\varphi}_d - 2\hat{\beta})$. We know of no reason why $\hat{\varphi}_d$ should cancel $2\hat{\beta}$, which comes from the KM matrix, so that we do no expect $\sin(\hat{\varphi}_d - 2\hat{\beta})$ to be much less unity. The process of $\mu \to e$ conversion in atoms is induced by two operators: one is the chirality breaking dipole operator involving $(\sigma^{\mu\nu}F_{\mu\nu})$, with an amplitude proportional to m_τ, while the other is the chirality conserving operator involving $(\gamma^\mu \partial^\nu F_{\mu\nu})$. The derivative in this operator has a scale of the momentum transfer, which is set by m_μ, so that these contributions are subdominant. The dominance of the $(\sigma^{\mu\nu}F_{\mu\nu})$ operator implies that in titanium the ratio $\Gamma(\mu \to e)/\Gamma(\mu \text{ capture})$ is 200 times smaller than $\text{B.R.}(\mu \to e\gamma)$. This result applies over all regions of parameter space of the minimal SO(10) model. In any event, it is simply a reflection of the dominance of the $\sigma^{\mu\nu}F_{\mu\nu}$

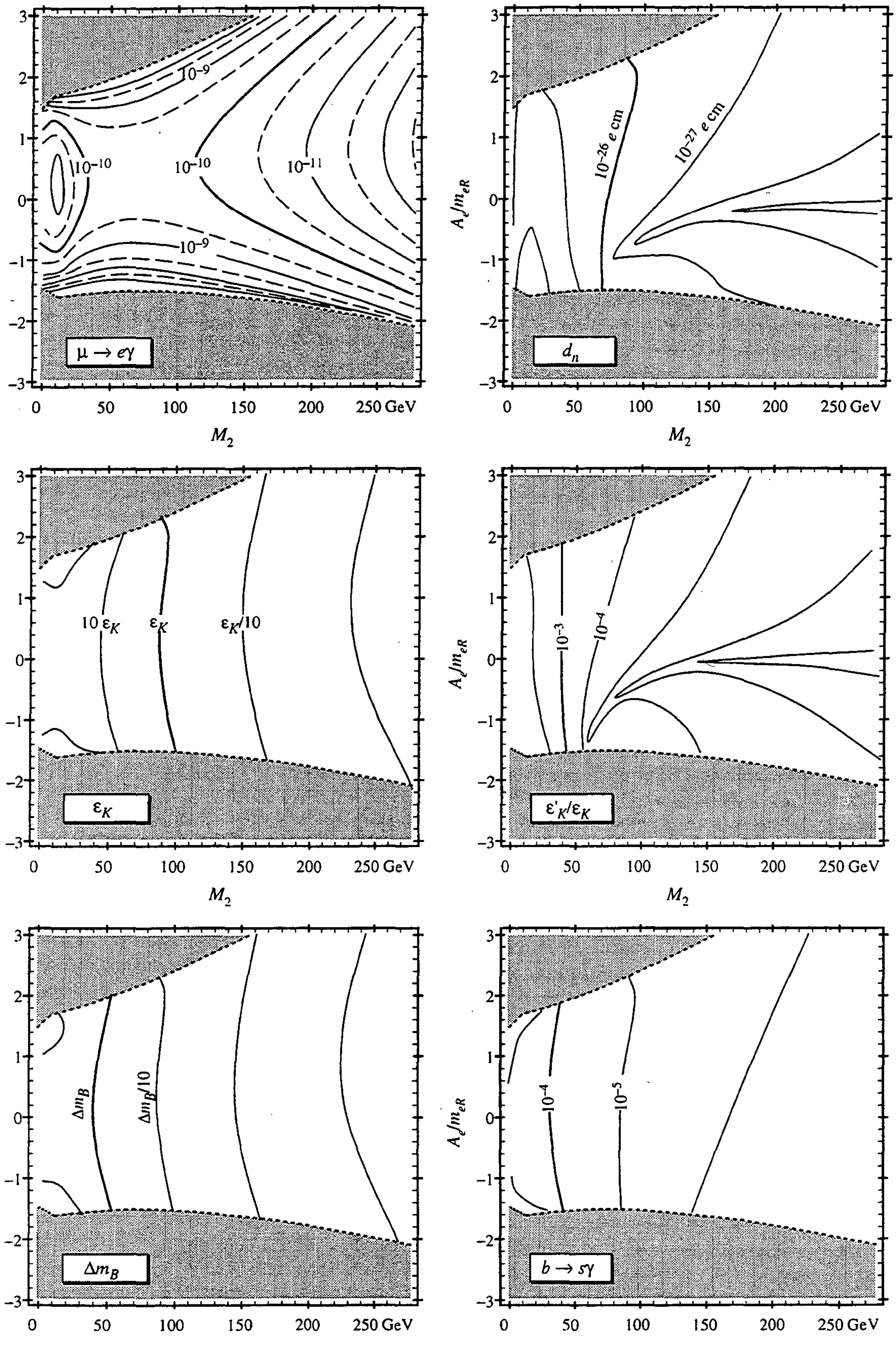

Figure 2: Contour plots in minimal SO(10) for $m_{\tilde{e}_R} = 300\,\mathrm{GeV}$, $\lambda_{tG} = 1.25$, $\mu < 0$, $\tan\beta = 2$, and maximal CP violating phases (see text) for (a) B.R.$(\mu \to e\gamma)$; (b) d_n; (c) ε_K; (d) $\varepsilon'_K/\varepsilon_K$; (e) Δm_B; (f) B.R.$(b \to s\gamma)$. In the hadronic observables only the gluino exchange contribution is included.

operator, and hence cannot be construed as a unique signature of SO(10). However, the processes $\mu \to e\gamma$, $\mu \to e$ conversion and d_e are very incisive probes of SO(10) superunification, and in the rest of this section we compare them with probes in the hadronic sector.

ε_K and d_n

The dominant gluino-mediated diagram contributing to the $\Delta S = 2$ effective Lagrangian involves the exchange of one $\tilde{d}_L$ type squark and one $\tilde{d}_R$ type squark. In the limit of keeping only the $\tilde{b}$ contribution, and setting $m_{\tilde{b}_L} = m_{\tilde{b}_R} = M_3$, this diagram gives:

$$\mathcal{L}_{\text{eff}}^{\Delta S=2} = \frac{\alpha_3^2(M_3)}{12M_3^2} |V_{ts}V_{td}|^2 e^{i(\hat{\varphi}_d - \hat{\varphi}_s)} y^2 [2(\bar{d}_R^a s_L^b)(\bar{d}_L^b s_R^a) - 6(\bar{d}_R^a s_L^a)(\bar{d}_L^b s_R^b)] \tag{11}$$

where color indices a, b are shown explicitly. The parameter $y \approx 0.77$ appears because two of the flavor mixing matrices are right-handed, and

$$(V_G)_{ti} = yV_{ti} \tag{12}$$

where $i = d, s$. This LR contribution is larger than the LL and RR contributions by about an order of magnitude, due to the $(m_K/m_s)^2$ enhancement of the hadronic matrix element. Such an effect is characteristic of SO(10), since it is not there in the MSSM or in minimal SU(5). We use the vacuum insertion approximation:

$$\langle K^0|(\bar{d}_R^a s_L^a)(\bar{d}_L^b s_R^b)|\bar{K}^0\rangle = 3\langle K^0|(\bar{d}_R^a s_L^b)(\bar{d}_L^b s_R^a)|\bar{K}^0\rangle = \frac{1}{2}\left(\frac{m_K^2 f_K}{m_s + m_d}\right)^2,$$

as seen in lattice calculations [12]. f_K is normalized in such a way that $f_K \simeq 120\,\text{MeV}$. Note that here and elsewhere we do *not* include the QCD corrections, unless otherwise stated.

The $\Delta S = 2$ gluino-mediated amplitude is important for ε_K rather than for Δm_K, and it gives:

$$\begin{aligned} |\varepsilon_K|^{\tilde{g}}_{\text{SO(10)}} &= \frac{\alpha_3^2(M_3)}{9\sqrt{2}M_3^2} \frac{f_K^2 m_K^3}{(m_s + m_d)^2 \Delta m_K} y^2 |V_{ts}V_{td}|^2 \sin(\hat{\varphi}_d - \hat{\varphi}_s) = \\ &\simeq 2.2 \cdot 10^{-2} \sin(\hat{\varphi}_d - \hat{\varphi}_s) \left(\frac{300\,\text{GeV}}{M_3}\right)^2 \left|\frac{V_{ts}V_{td}}{4 \cdot 10^{-4}}\right|^2 \left(\frac{180\,\text{MeV}}{m_s + m_d}\right)^2. \end{aligned} \tag{13}$$

At first sight equation (13) would appear to exclude colored superpartners less than about 1 TeV; however our simple analytic estimates are considerable overestimates as they neglect the compensating effects of $\tilde{d}_L$, $\tilde{s}_L$ exchange, and they do not give the full dependence on the superpartner parameter space. Nevertheless, the importance of $|\varepsilon_K|^{\tilde{g}}_{\text{SO(10)}}$ is borne out by the numerical results, which we discuss shortly.

The two most powerful hadronic probes of the minimal SO(10) model are ε_K and d_n, hence we now give our analytic results for d_n which we take to be $\frac{4}{3}d_d$, where $\mathcal{L}_{\text{eff}} = \frac{1}{2}d_d \cdot \bar{d}\sigma^{\mu\nu}F_{\mu\nu}i\gamma_5 d$ and

$$d_d = e\frac{\alpha_3(M_3)}{54\pi M_3^2} m_b(M_3) y |V_{td}|^2 \frac{A_b + \mu\tan\beta}{M_3} \sin(\hat{\varphi}_d - 2\hat{\beta}) \tag{14}$$

where y is given in equation (12) and we use, as before, the analytic approximation of keeping only the gluino diagram with internal $\tilde{b}$ squark, and set $m_{\tilde{b}_L} = m_{\tilde{b}_R} = M_3$. The parameter $m_b(M_3)$ is the running b quark mass renormalized at M_3. This gives

$$d_n = 4.2 \times 10^{-26}\, e \cdot \text{cm} \times \frac{m_b(M_3)}{2.7\,\text{GeV}} \left|\frac{V_{td}}{0.01}\right|^2 \frac{y}{0.77} \left(\frac{300\,\text{GeV}}{M_3}\right)^2 \frac{A_b + \mu\tan\beta}{M_3} \sin(\hat{\varphi}_d - 2\hat{\beta}). \tag{15}$$

In Figure 2b we show the numerical contour plot for $|d_n/\sin(\hat{\varphi}_d - 2\hat{\beta})|$, and in Figure 2c a contour plot of $|\varepsilon_K|^{\tilde{g}}_{\text{SO(10)}}/\sin(\hat{\varphi}_d - \hat{\varphi}_s)|$, where $\varepsilon_K|^{\tilde{g}}_{\text{SO(10)}}$ is the contribution to ε_K from the gluino box diagram only. The roughly vertical contours, at least in ε_K, reflect the structure imposed on the scalar non-degeneracy by gluino focussing, shown in Figures 1b and 1c. This is in marked contrast to the lepton signals of $\mu \to e\gamma$ and d_e shown in Figure 2a, which reflect the slepton non-degeneracy of Figure 1a. Figures 2a, 2b, 2c clearly show that a large λ_{tG}, as suggested by $b-\tau$ unification, with the running of the RGE in the full range from M_{Pl} to M_{G}, leads to $\mu \to e\gamma$ as the dominant probe of the minimal SO(10) model. Already the present bound of $5 \cdot 10^{-11}$ on the rate excludes a large portion of the parameter space. Outside this range, both the gluino exchange contribution to d_n and ε_K are, anyhow, negligibly small. The situation does change, however, if one looks at Fig.s 3 and 4. As noticed in the previous section, the gluino focussing effect makes non-degeneracy in the squark sector less prominent than in the slepton sector and, as such, also less sensitive to a reduction in λ_{tG} and/or in the scale for the initial condition of the RGE. In turn, although $\mu \to e\gamma$ remains as a very sensitive probe, it is now possible that gluino mediated contributions to d_n and ε_K become relevant, with gluinos in the $(200 \div 300)$ GeV mass range.

For superpartner parameters such that $|\varepsilon_K|^{\tilde{g}}_{\rm SO(10)} = \varepsilon_K = 2\cdot 10^{-3}$, and for equal phases: $\hat{\varphi}_d - \hat{\varphi}_s = \hat{\varphi}_d - 2\hat{\beta}$, d_n is predicted to be very close to its present experimental limit. Hence ε_K and d_n provide roughly comparable probes of this new physics. However, the new physics in ε_K must be disentangled from the SM background.

A crucial point emerges from Figures 3c, 4c. For a given $\hat{\varphi}_d - \hat{\varphi}_s$ it is only over a relatively small region of the plane that $\varepsilon_K|^{\tilde{g}}_{\rm SO(10)}$ will make a contribution to ε_K that we can disentangle from the SM contribution. The same statement applies to the planes drawn for different values of $\{\lambda_{tG}, m_{\tilde{e}_R}, \tan\beta, {\rm sign}\mu\}$. This is partly due to the gluino focussing effect on the scalar masses, but is also because the SM involves B_K, V_{td}, m_t in such a way that it will be very hard to identify contributions which are at the level of $\varepsilon_K/5$ or less. Contrast this to the situation with d_n, where each factor of 10 improvement in the experimental limit rules out large areas of parameter space. For this reason we view d_n as an excellent probe of the SO(10) model. It has a dependence on the superpartner parameters which is somewhat orthogonal to that of d_e, as can be seen by comparing Figures 3a and 4a with 3b and 4b.

The neutron electric dipole induced by the KM phase in the MSSM has been recently studied in ref. [13] and is found to be below $10^{-27} e\cdot$cm. In the approximation of neglecting all Yukawa couplings except the top one in the RGEs, as done here, there is no one loop contribution to d_n in the MSSM as in minimal SU(5).

$\varepsilon'_K/\varepsilon_K$

Much present experimental effort is aimed at determining the size of CP violation in the direct decays of neutral K mesons: $\varepsilon'_K/\varepsilon_K$. How large are the gluino-mediated penguin contributions to this? The SM contribution is dominated by W exchange generation of the penguin operator $\bar{d}\gamma^\mu s\, \partial^\nu G_{\mu\nu}$, where $G_{\mu\nu}$ is the gluon field strength, with coefficient $\propto {\rm Im} V_{ts}V^*_{td}/M^2_W$. In either the MSSM or minimal SU(5) or SO(10) models, the gluino-mediated penguin contribution does not compete because M_W is replaced with a larger superpartner mass $m_{\tilde{q}}$ or M_3.

However, an interesting new possibility emerges in the minimal SO(10) model: a contribution to $\varepsilon'_K/\varepsilon_K$ from a gluino-mediated chromoelectric dipole moment operator proportional to m_b. The relevant $\Delta S = 1$ effective Lagrangian is, with our usual analytic assumptions:

$$\begin{aligned}\mathcal{L}^{\Delta S=1}_{\rm eff} &= g_3(\Lambda_{\rm QCD})\frac{\alpha_3(M_3)}{36\pi M_3^2}\frac{A_b+\mu\tan\beta}{M_3} m_b(M_3) y\, |V_{ts}V_{td}| \times \\ &\times \left\{ e^{i(\hat{\varphi}_d-\hat{\beta})}\bar{d}_R\sigma^{\mu\nu}\frac{\lambda^a}{2}s_L G^a_{\mu\nu} + e^{i(\hat{\beta}-\hat{\varphi}_s)}\bar{d}_L\sigma^{\mu\nu}\frac{\lambda^a}{2}s_R G^a_{\mu\nu}\right\}. \end{aligned} \qquad (16)$$

No exact proportionality relation holds between d_n and $\varepsilon'_K/\varepsilon_K$ since the photon is attached only to the internal squark line, whereas the gluon, in the chromoelectric dipole moment, may also be attached to the gluino line.

To evaluate (16) we use matrix elements [14]

$$\begin{aligned}\langle\pi\pi, I=0|g_s\bar{d}_R\sigma^{\mu\nu}\frac{\lambda^a}{2}s_L G^a_{\mu\nu}|K\rangle &= -\langle\pi\pi, I=0|g_s\bar{d}_L\sigma^{\mu\nu}\frac{\lambda^a}{2}s_R G^a_{\mu\nu}|K\rangle = \\ &= \sqrt{3}\frac{11}{8}\frac{f_K^2}{f_\pi^3}\frac{m_K^2}{m_s}m_\pi^2 D \approx 0.37\,{\rm GeV}^2,\end{aligned}$$

where $D = m_K^2/\Lambda^2_{\rm QCD} \approx 0.3$, giving

$$\frac{|\varepsilon'_K|^{\tilde{g}}_{\rm SO(10)}}{|\varepsilon_K|} = \frac{w\,|{\rm Im}\langle\mathcal{L}^{\Delta S=1}_{\rm eff}\rangle|}{\sqrt{2}\,|\varepsilon_K|\,{\rm Re}A_0} = 3.1\times 10^{-4}\left(\frac{300\,{\rm GeV}}{M_3}\right)^2\frac{A_b+\mu\tan\beta}{M_3}\frac{\sin(\hat{\varphi}_d-\hat{\beta})+\sin(\hat{\varphi}_s-\hat{\beta})}{2}. \qquad (17)$$

We have used $w = 1/22$, ${\rm Re}A_0 = 3.3\cdot 10^{-7}\,$GeV, $|\varepsilon_K| = 2.3\times 10^{-3}$ and $m_b(M_3) = 2.7\,$GeV. This is to be compared with the expectation from the SM for $m_t = (175\pm 15)\,$GeV: $\varepsilon'_K/\varepsilon_K = (3\div 10)\cdot 10^{-4}$ [14].

The numerical results for $\varepsilon'_K/\varepsilon_K$ are shown in Figures 2d, 3d, 4d for $[\sin(\hat{\varphi}_d-\hat{\beta})+\sin(\hat{\varphi}_s-\hat{\beta})] = 2$. Comparing Figures b for d_n and d for $\varepsilon'_K/\varepsilon_K$ one finds that, in the region where these predictions could be of experimental interest, there is an approximate numerical relation

$$\left|\frac{\varepsilon'_K}{\varepsilon_K}\right|^{\tilde{g}}_{\rm SO(10)} \simeq 10^{-4}\left[\frac{\sin(\hat{\varphi}_d-\hat{\beta})+\sin(\hat{\varphi}_s-\hat{\beta})}{2\sin(\hat{\varphi}_d-2\hat{\beta})}\right]\times\frac{d_n}{10^{-26}e\cdot{\rm cm}}. \qquad (18)$$

Hence we see that, for the phase ratio in square brackets of unity, the gluino-mediated contribution to $\varepsilon'_K/\varepsilon_K$ is already constrained to be not greater than the SM contribution. Given the theoretical uncertainties in both the penguin and the chromoelectric dipole matrix elements, we find it unlikely that the gluino-mediated contribution to $\varepsilon'_K/\varepsilon_K$ could be identified in this case.

Δm_{B_d}

The rest of this section is devoted to a discussion of B meson signatures of the minimal SO(10) model. The gluino-mediated box diagrams for neutral B meson mixing induce an effective Lagrangian

$$\mathcal{L}_{\text{eff}}^{\Delta B=2} = \frac{\alpha_3^2(M_3)}{12M_3^2}\,|V_{td}|^2 \Big\{ e^{2i\hat\beta}(\bar d_L\gamma^\mu b_L)^2 + y^2 e^{2i(\hat\varphi_d-\hat\beta)}(\bar d_R\gamma^\mu b_R)^2 + y e^{i\hat\varphi_d}[2(\bar d_R^i b_L^j)(\bar d_L^j b_R^i) - 6(\bar d_R^i b_L^i)(\bar d_L^j b_R^j)] \Big\} + (d\to s, \hat\beta\to 0). \quad (19)$$

Using the vacuum insertion approximation, this leads to a contribution to the mass difference for the neutral B_d mesons of

$$\Delta m_{B_d}|^{\tilde g}_{\mathrm{SO(10)}} = \frac{2\alpha_3^2(M_3)}{9M_3^2}\,|V_{td}|^2 f_{B_d}^2 m_B \left|\frac{1}{4}e^{2i\hat\beta} + \frac{y^2}{4}e^{2i(\hat\varphi_d-\hat\beta)} + y e^{i\hat\varphi_d}\right| \quad (20)$$

where the three terms correspond to LL, RR and LR contributions respectively. For $K^0\bar K^0$ mixing the LR terms dominate because of a factor of m_K^2/m_s^2 enhancement of the matrix element. No such factor occurs in the B system, but the vacuum insertion approximation suggests that the LR term still dominates, giving

$$\Delta m_{B_d}|^{\tilde g}_{\mathrm{SO(10)}} \simeq 2.7\cdot 10^{-10}\,\mathrm{MeV}\left(\frac{300\,\mathrm{GeV}}{M_3}\right)^2\left(\frac{f_B}{140\,\mathrm{MeV}}\right)^2, \quad (21)$$

with f_B normalized in the same way as f_K. In the limit that the LR operator contributions dominate both $\varepsilon_K|^{\tilde g}_{\mathrm{SO(10)}}$ and $\Delta m_{B_d}|^{\tilde g}_{\mathrm{SO(10)}}$, we can write a relation

$$\frac{|\varepsilon_K|^{\tilde g}_{\mathrm{SO(10)}}}{\Delta m_{B_d}|^{\tilde g}_{\mathrm{SO(10)}}} \simeq \frac{1}{2\sqrt2}\frac{f_K^2}{f_B^2}\frac{m_K^3}{\Delta m_K(m_s+m_d)^2 m_B}\,y\,|V_{ts}|^2\sin(\hat\varphi_d-\hat\varphi_s) \quad (22)$$

which is approximately independent of the superpartner spectrum and of $|V_{td}|$. Inserting numbers:

$$\frac{\Delta m_{B_d}|^{\tilde g}_{\mathrm{SO(10)}}}{3.5\cdot 10^{-10}\,\mathrm{MeV}} \simeq \frac{0.1}{\sin(\hat\varphi_d-\hat\varphi_s)}\frac{|\varepsilon_K|^{\tilde g}_{\mathrm{SO(10)}}}{2.3\times 10^{-3}} \times \left(\frac{f_B}{140\,\mathrm{MeV}}\right)^2\left(\frac{m_s+m_d}{0.18\,\mathrm{GeV}}\right)^2\left|\frac{0.04}{V_{ts}}\right|^2 \quad (23)$$

demonstrating that $\Delta m_{B_d}|^{\tilde g}_{\mathrm{SO(10)}}$ can only be a large fraction of the observed Δm_{B_d} if $\sin(\hat\varphi_d-\hat\varphi_s)$ is small, unless the vacuum insertion approximation for the LR operator is an overstimate.

The numerical results for $\Delta m_{B_d}|^{\tilde g}_{\mathrm{SO(10)}}$, assuming dominance of the LR contribution, are shown as a contour plot in Figures 2e, 3e, 4e.

The contours of Figures 2e, 3e, 4e are normalized to the observed value $\Delta m_{B_d} = 3.5\cdot 10^{-10}\,\mathrm{MeV}$. As in the comparison of d_n and ε_K with the leptonic signal, for values of the parameters as in Fig. 2, $\Delta m_{B_d}|^{\tilde g}$ is constrained to be too small to be of interest. We therefore consider only the cases of Fig.s 3, 4. A useful parameter in our discussion of the phenomenology is

$$r = \frac{\Delta m_{B_d}|^{\tilde g}_{\mathrm{SO(10)}}}{\Delta m_{B_d}|_{\mathrm{SM}}}, \quad (24)$$

with the top mass in the SM contribution set to 175 GeV. In particular, it is convenient to consider three regions of the supersymmetric parameter space: A, B and C:

A $r \ll 0.1$. In this region we find that all gluino-mediated contributions to the hadronic observables provide only very small deviations from the SM predictions. The only exception to this is d_n. From Figures 3e and 4e we see that this is a very large region.

B $r\approx 0.1$. A point with $r=0.1$ is provided by: $\lambda_{tG}=0.85$, $\tan\beta=2$, $m_{\tilde e_R}=300\,\mathrm{GeV}$, $A_e/m_{\tilde e_R}=2$ and $M_2=80\,\mathrm{GeV}$. At this point, $M_3=250\,\mathrm{GeV}$, $m_{\tilde b}=200\,\mathrm{GeV}$, $m_{\tilde q}=400\,\mathrm{GeV}$ and $m_{\tilde t_1}=100\,\mathrm{GeV}$. This illustrates that region B can be reached without taking superpartner masses too close to their present lower limits.

C $r\approx 0.5$. An example of a point in this region is provided by: $\lambda_{tG}=0.85$, $\tan\beta=2$, $m_{\tilde e_R}=300\,\mathrm{GeV}$, $A_e/m_{\tilde e_R}=1$ and $M_2=50\,\mathrm{GeV}$. At this point, other masses are approximately: $M_3=150\,\mathrm{GeV}$, $m_{\tilde b}=150\,\mathrm{GeV}$, $m_{\tilde q}=300\,\mathrm{GeV}$ and $m_{\tilde t}=100\,\mathrm{GeV}$. The gluino mass is now below 200 GeV, so we expect that this region will be probed at the Fermilab collider. It is clear that values of r larger than about 1 are excluded by present limits on the gluino mass.

The majority of our discussion will concern $0.05 < r < 1$ (which includes regions B and C) as this is the region where the hadronic signatures are important. However, it is important to realize that much of the parameter space has $r \ll 0.1$, and hence can only be probed by the lepton signals.

To discuss the phenomenology of these parameter regions, it is important to consider the theoretical predictions for $|\varepsilon_K|$ and for Δm_{B_d}, which include both SM and gluino mediated contributions (neglecting other supersymmetric contributions). We find a useful approximation to be:

$$|\varepsilon_K| \approx 2.26 \times 10^{-3} \frac{\eta B_K}{0.5} \left| \frac{V_{td}}{0.01} \right|^2 [1.8 \sin 2\hat{\beta} + 11.5 r \sin(\hat{\varphi}_d - \hat{\varphi}_s)] \tag{25}$$

and

$$\Delta m_{B_d} \simeq 2.1 \times 10^{-10}\,\mathrm{MeV} \frac{\eta B_B}{0.5} \left(\frac{f_B}{140\,\mathrm{MeV}} \right)^2 \left| \frac{V_{td}}{0.01} \right|^2 \left| e^{2i\hat{\beta}} + r e^{i\hat{\varphi}_d} \right| \tag{26}$$

where in each equation the first term, involving $\hat{\beta}$, is the SM result while the second term, involving r, is the supersymmetric contribution. Note that we have set $m_t = 175\,\mathrm{GeV}$. We have also introduced a QCD correction η times a fudge factor B_K (B_B) for the matrix elements of the appropriate operators.

A natural expectation is that all phases, $\hat{\beta}$, $\hat{\gamma}$, $\hat{\varphi}_d$ and $\hat{\varphi}_s$, and their differences, are of order unity. This would exclude region C as $|\varepsilon_K|$ is predicted to be too large. We will discuss regions A and B when the phases are large. In region A there is little to say, the supersymmetric contributions provide small corrections, especially for Δm_{B_d}. In region B supersymmetric contributions to $|\varepsilon_K|$ are as important as the SM contribution, however the corrections to Δm_{B_d} are small. Fits to the data will therefore yield the usual value for $|V_{td}|$, but $\sin 2\hat{\beta}$ will be replaced by $[\sin 2\hat{\beta} + 7r \sin(\hat{\varphi}_d - \hat{\varphi}_s)]$ and will change by a large amount.

In the small region C, $\sin(\hat{\varphi}_d - \hat{\varphi}_s) \lesssim 0.1$. The supersymmetric corrections to Δm_{B_d} can be significant, so that $|V_{td}|$ may change by as much as 50%. Fits to data are now more complicated as they involve $\hat{\beta}$, $\hat{\varphi}_d$ and $\hat{\varphi}_s$. Since all phases have the same origin, it is plausible that in region C they are all small, of order 0.1. In this case the CP violation which has been observed in nature is produced dominantly by sources other than the KM matrix. Although we do not find it likely, the KM matrix could be real in regions B and C.

Figures 3,4b and 3,4d show the behavior of d_n and $\varepsilon'_K|^{\tilde{g}}_{SO(10)}/\varepsilon_K$ in these regions. Region C is clearly excluded by d_n unless $\hat{\varphi}_d - 2\hat{\beta}$ is a small phase, which again suggests that all phases should be small in this region. In regions B and C, d_n is close to discovery. A search to the level of $10^{-27} e \cdot \mathrm{cm}$ will probe a substantial fraction of region A. In regions B and C, the supersymmetric contribution to $\varepsilon'_K/\varepsilon_K$ is expected to be at the 10^{-4} level. Whether it can be distinguished from the SM contribution is very dependent on the sizes of the phases which appear, $\hat{\varphi}_d - \hat{\beta}$ and $\hat{\varphi}_s - \hat{\beta}$, compared to the phase $\hat{\varphi}_d - \hat{\varphi}_s$ that occurs in ε_K.

Δm_{B_s}

The expression for the gluino-mediated contribution to B_s mixing is obtained from equation (20) by the replacements: $V_{td} \to V_{ts}$, $\hat{\beta} \to 0$, $\hat{\varphi}_d \to \hat{\varphi}_s$ and $f_{B_d} \to f_{B_s}$, giving

$$\Delta m_{B_s}|^{\tilde{g}}_{SO(10)} = \frac{2\alpha_3^2(M_3)}{9M_3^2} |V_{ts}|^2 f_{B_s}^2 m_B \left| \frac{1}{4} + \frac{y^2}{4} e^{2i\hat{\varphi}_s} + y e^{i\hat{\varphi}_s} \right|. \tag{27}$$

If the LR contributions dominate $\Delta m_B|^{\tilde{g}}_{SO(10)}$, we find

$$\frac{x_s}{x_d} \simeq \left| \frac{V_{ts}}{V_{td}} \right|^2 \frac{f_{B_s}^2}{f_{B_d}^2} \left| \frac{1 + r e^{i\hat{\varphi}_s}}{e^{2i\hat{\beta}} + r e^{i\hat{\varphi}_d}} \right| \tag{28}$$

valid for any value of r. Deviations from the SM prediction are $\ll 10\%$, $\simeq 10\%$, $\approx 100\%$ for regions A, B and C.

CP violations in B decays

When a tagged neutral B meson decays to CP eigenstate a, there is an oscillatory term in the decay rate proportional to $\sin(\phi_M + \phi_a)\, \sin(\Delta m_B\, t)$ which is of opposite sign for B^0 and $\bar{B}^0$ decay and therefore violates CP. The phase ϕ_M is the phase of the appropriate B meson mixing amplitude M_{12}, while the phase ϕ_a is the CP violating phase of the decay amplitude for $B^0 \to a$. The values of $\sin(\phi_M + \phi_a)$ for various a in the SM are shown in the first column of Table 1.

In supersymmetric theories ϕ_a is the same as in the SM: diagrams involving superpartners provide only very small corrections to b quark decay amplitudes. Hence the possible signals of new physics are via the mixing amplitude phase ϕ_M. In the MSSM and minimal SU(5) models the supersymmetric contributions to the B mixing amplitude have the same phase as the SM contribution. Hence ϕ_M is unaltered, and the first column of Table 1 applies also to the MSSM and minimal SU(5) theories. However, as can be seen from equations (20) and (28), in

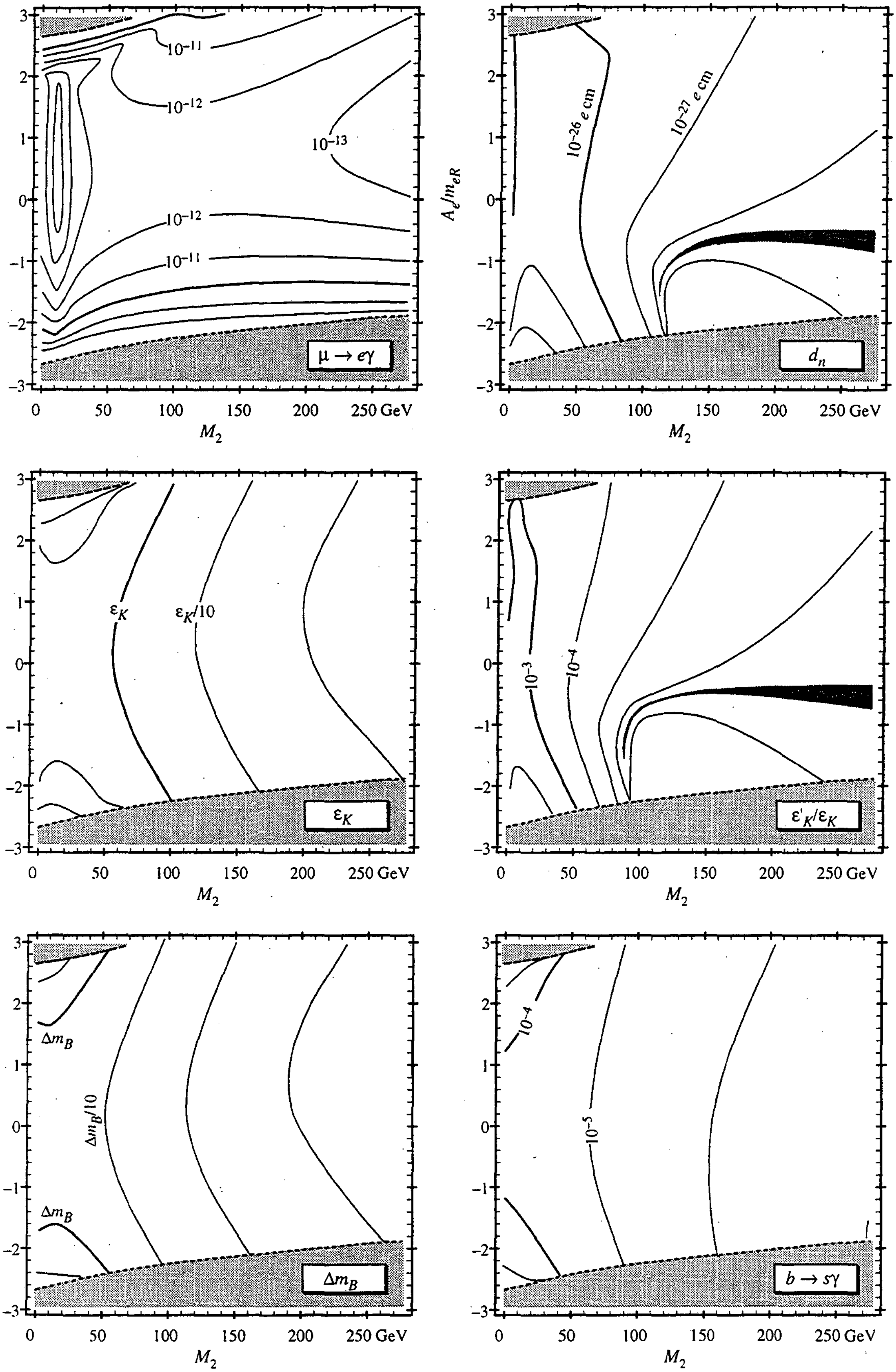

Figure 3: Same as in fig. 2 except for $\lambda_{tG} = 0.85$.

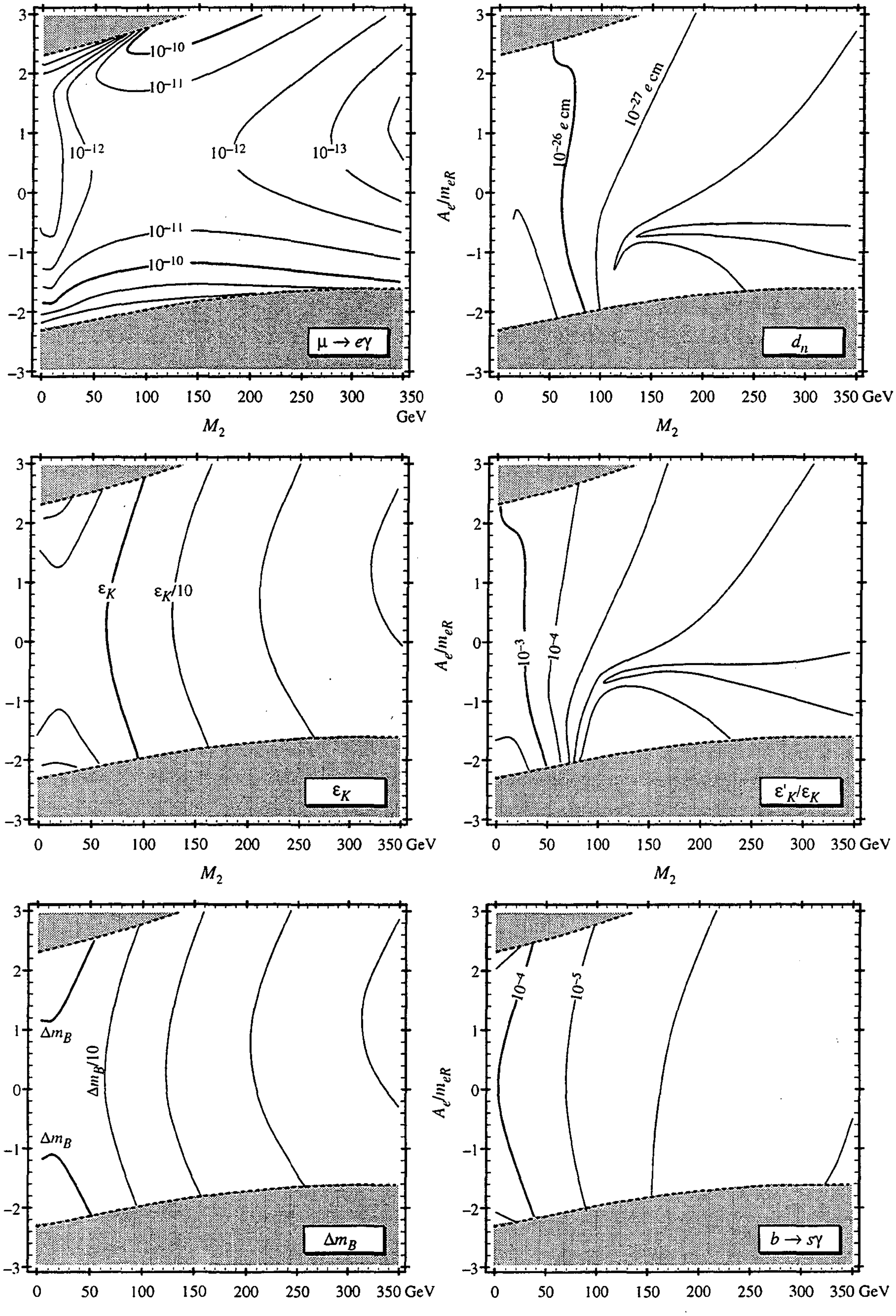

Figure 4: Same as in fig. 2 except for the initial conditions on the RGEs taken at $2.0 \cdot 10^{17}$ GeV.

	Standard Model and Minimal SU(5)	Minimal SO(10)
d_e, d_n	—	$\sin(\hat{\varphi}_d - 2\hat{\beta})$
ε_K	$\sin 2\hat{\beta}$	$\sin(\hat{\varphi}_d - \hat{\varphi}_s)$
$\varepsilon'_K/\varepsilon_K$	$\sin\hat{\beta}$	$\sin(\hat{\varphi}_d - \hat{\beta}) + \sin(\hat{\varphi}_s - \hat{\beta})$
$B_d \to \pi^+\pi^-$	$\sin(2\hat{\beta} + 2\hat{\gamma})$	$\sin(\hat{\varphi}_d + 2\hat{\gamma})$
$B_d \to \psi K_s$	$\sin 2\hat{\beta}$	$\sin\hat{\varphi}_d$
$B_d \to D^+D^-$	$\sin 2\hat{\beta}$	$\sin\hat{\varphi}_d$
$B_s \to \rho K_s$	$\sin 2\hat{\gamma}$	$\sin(\hat{\varphi}_s + 2\hat{\gamma})$
$B_s \to \psi K_s$	—	$\sin\hat{\varphi}_s$

Table 1: $\hat{\beta}$ and $\hat{\gamma}$ are defined by: $V_{td} = |V_{td}|\, e^{-i\hat{\beta}}$, $V_{ub} = |V_{ub}|\, e^{-i\hat{\gamma}}$. $\hat{\varphi}_{d,s}$ are defined by equations (8) and (9). "—" indicates signal is too small to be of experimental interest. For B meson decays: in the Standard Model and minimal SU(5) theory the entry gives the CP violating coefficient of the $\sin \Delta m_B t$ oscillatory term. For the minimal SO(10) model the entry gives the contribution to this coefficient from the gluino exchange contribution to M_{12}. This must be combined with the SM contribution, as shown in equations (25) and (26).

the minimal SO(10) model the supersymmetric contributions to $B_{d,s}$ mixing have phases $\simeq \hat{\varphi}_{d,s}$. In the case that these supersymmetric contributions to B meson mixing dominate the SM contribution, the quantity $\sin(\phi_M + \phi_a)$, for various final states a, is shown in the 2nd column of Table 1. This situation or $r \approx 1$ can occur, but over most of parameter space $r < 1$. Since

$$M_{12}^d \simeq \left|M_{12}^d\right|_{\rm SM} (e^{2i\hat{\beta}} + r\, e^{i\hat{\varphi}_d}) \tag{29a}$$

$$M_{12}^s \simeq |M_{12}^s|_{\rm SM} (1 + r\, e^{i\hat{\varphi}_s}) \tag{29b}$$

and the relevant mixing phase ϕ_{M_i} is the phase of M_{12}^i, we find that in regions A and B

$$\phi_M^d \simeq 2\hat{\beta} + r\sin(\hat{\varphi}_d - 2\hat{\beta}) \tag{30a}$$

$$\phi_M^s \simeq r\sin\hat{\varphi}_s \tag{30b}$$

Hence when r is small the deviations from the SM pattern of CP violation in neutral B meson decays is proportional to r, and is also small.

In region C the phases ϕ_M^i deviate considerably from the SM form. For example for $r = 1$, $\{\phi_M^d, \phi_M^s\} = \{\frac{\hat{\varphi}_d}{2} + \hat{\beta}, \frac{\hat{\varphi}_s}{2}\}$, which differs greatly from $\{2\hat{\beta}, 0\}$ of the SM. In this region we have argued that it is likely that all phases are small, in which case the mixing phases are $\{2\hat{\beta} + r\hat{\varphi}_d/(1+r), r\hat{\varphi}_s/(1+r)\}$. The most notable feature is that, unlike the SM, all asymmetries should be small. We stress again that region C only corresponds to a very small portion of the parameter space.

$b \to s\gamma$

Finally we consider the process $b \to s\gamma$. The effective Lagrangian for $b \to s\gamma$ can be written in the general form:

$$\mathcal{L}_{\rm eff}^{b\to s\gamma} = \frac{e}{2} m_b(m_b)[A_L \bar{s}_R \sigma^{\mu\nu} b_L F_{\mu\nu} + A_R \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}] \tag{31}$$

in which case the branching ration for $b \to s\gamma$ is given in terms of the semi-lepton branching ratio via

$$\begin{aligned} \text{B.R.}(b \to s\gamma) &= \text{B.R.}(b \to ce\bar{\nu}) \frac{48\pi^3\alpha}{G_F^2} \frac{|A_L|^2 + |A_R|^2}{|V_{cb}|^2 I(1 - \frac{2}{3\pi}\alpha_3(m_b) f)} \\ &= 1.3 \times 10^{13}\,\text{GeV}^4(|A_L|^2 + |A_R|^2) \end{aligned} \tag{32}$$

where $I \simeq 0.5$ is a phase-space factor and $f \simeq 2.4$ is a QCD correction factor, both occurring in B.R.$(b \to ce\bar{\nu})$.

In our usual analytic approximation we have

$$A_R^{\tilde{g}} = \frac{8}{27} \frac{\alpha_3(M_3)}{12\pi M_3^2} |V_{ts}| \left(-7 + \eta_b \frac{A_b + \mu\tan\beta}{M_3}\right) \tag{33a}$$

$$A_L^{\tilde{g}} = ye^{i\varphi_s} A_R^{\tilde{g}} \tag{33b}$$

where $\eta_b = m_b(M_3)/m_b(m_b)$. We therefore obtain

$$\text{B.R.}(b \to s\gamma)|_{\text{SO}(10)}^{\tilde{g}} = 1.1 \cdot 10^{-4} \left(\frac{300\,\text{GeV}}{M_3}\right)^4 \left(1 - \eta_b \frac{A_b + \mu \tan\beta}{7M_3}\right)^2 . \tag{34}$$

Note that this branching ratio is obtained by simply squaring the gluino amplitude, and it ignores the SM and charged Higgs contributions, chargino contributions and their interferences.

The numerical result for the gluino contribution to $b \to s\gamma$ are shown in Fig. 2f, 3f, 4f. In view of the uncertainties on the SM contribution to this process, they can hardly play a significant role in any situation. The rate for $b \to s\gamma$ is on the other hand known to place a constraint on the parameter space of the MSSM mostly determined from charged Higgs and chargino exchanges [10]. We notice that in the parameter space displayed in all plots of Fig.s 1÷4 the charged Higgs mass ranges from 300 GeV to 1000 GeV. Correspondingly only a very small region of the SO(10) parameter space is excluded by $b \to s\gamma$, where the $\mu \to e\gamma$ and d_e signatures can be seen.

In the minimal SO(10) model the best signatures are the lepton flavor violating processes and the electric dipole moments of the electron and neutron. These signatures can be probed by future experiments over a wide range of parameter space. Over some of this parameter space gluino-mediated contributions to ε_K are significant. Over a restricted region of parameter space gluino-mediated contributions to $\varepsilon'_K/\varepsilon_K$ and to Δm_B could be identified. The latter could lead to deviations from the pattern of CP violations in neutral B meson decays expected in the SM. In certain small regions of parameter space the deviations from the SM could be very large. However, over most of parameter space, the relative merits of the various signals are as summarized in Table 2, shown in the conclusions.

5 The Assumptions.

The flavor and CP violating signals which we compute are induced by the top Yukawa coupling of the unified theory. Although the calculations of this paper are done in specific simple models, the signals occur in any theory which satisfies three criteria (barring some kind of flavor symmetry restoration at the unification scale):

i. At least one helicity of the τ lepton is unified in the same representation as the top quark.

ii. Supersymmetry is effectively unbroken down to the weak scale.

iii. The supersymmetry breaking parameters are hard (have no power-law momentum dependence) at the scale M_G of the unified interactions.

It is certainly possible to construct theories without each of these assumptions. However, the predominant paradigm of supersymmetric unification does satisfy all three criteria. In this section we give arguments in favor of each of these assumptions.

In unified theories with three generations only, it is inevitable that the first assumption is justified. In SU(5) or SO(10) there must be some lepton in the same irreducible representation as the top quark. This could not be dominantly the e or μ, otherwise the signals that we are discussing, such as $\mu \to e\gamma$, would be much larger than the present experimental limit. Hence, to very high accuracy, the top quark is unified with the τ lepton in this case.

In unified models with $N+3$ generations and N mirror generations, there is no fundamental reason why the top quark and τ need be in the same representation [16]. The lepton unified with the top quark could be superheavy. The states of the light generations will be determined by the structure of the superheavy masses which marry the N mirror generations to N of the generations. These mass matrices may break the unified group so that the light states do not fill out complete representations of the unified group. Although such rearrangement of generations is possible, it would typically lead to a Kobayashi-Maskawa matrix with order unity intergenerational mixing, and hence appears to us not to be preferred.

The second assumption, of weak-scale supersymmetry, is motivated by the successful prediction of the weak mixing angle, at the percent level, in superunified models. Furthermore, the dynamical breaking of the electroweak symmetry induced by the large top Yukawa coupling connects the scale of supersymmetry breaking to the Z boson mass.

We believe the third assumption is that which is most open to question. There is no compelling physical mechanism for supersymmetry breaking. If the flavor and CP violating signals are shown to be absent to a high degree, then it may be a sign that the supersymmetry breaking is soft at scale M_G, and is not convincing evidence that quark-lepton unification is false. If the breaking of supersymmetry is communicated to the particles of the MSSM at energy scales much less that M_G, then the supersymmetry breaking interactions will not reflect any information about the unification at higher energy, and our signals disappear. Our signals are present in theories where supersymmetry breaking occurs in a hidden sector (with fields Z_i) such as can occur in supergravity [4]. This sector is called "hidden" because beneath some scale M there are no renormalizable interactions which couple the hidden fields to those of the MSSM (denoted Φ_a). Thus beneath M the communication between these sectors is solely via non-renormalizable operators such as $M^{-1}[Z_i \Phi_a \Phi_b \Phi_c]_F$, $M^{-2}[Z_i^\dagger Z_j \Phi_a^\dagger \Phi_b]_D$. An important assumption is that the physics at scale M, which generates these operators is flavor-blind, treating all generations equally. Considering the D operator for simplicity, its coefficient at the scale M can therefore be written as $\lambda_{ij}\delta_{ab}$. On renormalizing this operator to lower energies it will receive radiative corrections from the interactions of both observable and hidden sectors. However the hidden sector interactions are flavor-blind, so these renormalizations maintain the form $\lambda_{ij}\delta_{ab}$ and simply renormalize λ_{ij}. When supersymmetry breaks in the hidden sector we insert F_i vacuum expectation values into the operator to generate a supersymmetry breaking mass for the observable scalar fields $m^2_{ab} = (\lambda_{ij}F^*_i F_j/M^2)\delta_{ab}$. In the absence of observable sector renormalizations this is a universal mass. However, the factor δ_{ab} appeared because of the flavor independence of the physics at scale M which generated these non-renormalizable operators. Beneath M, the observable interactions, which do depend on flavor, renormalize the coefficient away from proportionality to δ_{ab}. Furthermore, as far as the observable interactions are concerned, it is simply a question of renormalizing the mass operator $\Phi^\dagger_a\Phi_b$ from M down to low energies.

This framework is not ideal for two reasons. Firstly we do not understand why the physics at M which generates these operators should be flavor independent. If it grossly violated flavor symmetry between the lightest two generations, it would lead to $m_{\tilde{e}}$ and $m_{\tilde{\mu}}$ being very different, giving B.R.$(\mu \to e\gamma) \approx 10^{-4}$. Hence we simply impose this initial flavor independence as an experimental necessity. Secondly, supersymmetry breaking occurs at an intermediate scale, $F_i^{1/2} \approx (M_W M_{\rm Pl})^{1/2}$, the origin of which is not understood.

Nevertheless, this framework can occur in the context of $N = 1$ supergravity theories, in which case M is the reduced Planck mass, $M_{\rm Pl}$. So far it has appeared preferable to alternative schemes with softer supersymmetry breaking, at least because gravity provides the desired non-renormalizable interactions.

6 Conclusions

In this paper we have studied hadronic flavor and CP violating phenomena generated by the large top quark coupling in supersymmetric grand unified theories. We have computed the gluino-mediated contributions to ε_K, $\varepsilon'_K/\varepsilon_K$, Δm_B, $b \to s\gamma$, d_n and CP violation in neutral B meson decays in two simple models. The physics at the unified scale, M_G, is reflected at low energies in the scalar superpartner spectrum and in flavor mixing matrices at neutral gaugino vertices, which have characteristic forms for the minimal SU(5) and SO(10) models. In the minimal SU(5) model the flavor mixing matrices occur at all neutral gaugino vertices for the d_L and e_R sectors, while in the minimal SO(10) model mixing occurs also in the d_R and e_L sectors.

An important, universal, feature of the hadronic signals is that they have a much larger dependence on the gaugino mass than the leptonic signals. A large gluino mass contributes a large flavor-independent radiative correction to the squark masses, thus reducing the non-degeneracies produced in the unified theory. This gluino focussing effect can be seen in Figures 1b,c,d where the squark mass shows a strong dependence on the gaugino mass. In the lepton sector the gaugino focussing is much less important, as can be seen from a comparison of Figures 1a and 1b,c,d.

The hadronic flavor-changing and CP violating effects of the minimal SU(5) theory are very similar in nature to those of the MSSM, although numerically somewhat larger. The most important limit on the parameter space is therefore provided by $b \to s\gamma$, and it is unlikely that the gluino mediated contribution be dominant [11]. However, there remain large regions of parameter space where the rare μ processes, such as $\mu \to e\gamma$, are large and provide the only probe of this new flavor physics.

The additional flavor mixing matrices of the minimal SO(10) model make the hadronic flavor and CP violating signals larger and richer than in the SU(5) model, as was also the case for the leptonic channels. A study of the contour plots of Figures 2,3,4 shows that a critical role is played by the value of λ_{tG} and/or of the scale M for the initial conditions on the RGEs. The hadronic flavor and CP violating signals can be significant, relative to the leptonic ones, only for relatively low values of λ_{tG} and/or M. This is an indirect consequence of the gluino focussing effect. In such a case, even for a not too light gluino, the discovery of d_n may be possible.

As the gluino mass is lowered, with all phases of order unity, the first process which acquires an important gluino-mediated contribution is ε_K. Most striking is the possibility that, even with colored scalars heavier than 300 GeV, ε_K may receive non-KM supersymmetric contributions as large as the SM contribution. This could be identified by a failure of the SM to accommodate the observed values of ε_K, Δm_B and $|V_{ub}|$. At present such fits are limited by the f_B^2 uncertainty in Δm_B, which amounts to a 50% effect. In this region, where the supersymmetric contribution to ε_K is comparable to the SM one, and where all phases are of order unity, Δm_B receives a correction from gluino-mediated diagrams at most of $(10 \div 20)$%. This leads to deviations from the SM pattern of CP violation in neutral B meson decay at most of $(10 \div 20)$% level.

For still lighter values of the gluino mass, in the region of 200 GeV, the gluino mediated contribution to ε_K is so large that a combination of phases must be made small. This suggests that in this region all the CP violating phases are small. Nevertheless the gluino-mediated contribution to Δm_B can be comparable to that of the SM, meaning that although the CP asymmetries in B meson decay are small they show very large deviations from those predicted by the SM. The most salient features of our results are summarized in Table 2.

We have chosen to study the minimal SU(5) and SO(10) models because the origin of the flavor violating effects

	Minimal	
	SU(5)	SO(10)
$\mu \rightarrow e\gamma, \mu \rightarrow e$	$\surd\surd$	$\surd\surd$
d_e, d_n	—	$\surd\surd$
CP violation in B^0 decays	—	$\surd$
ε_K	—	**
$\varepsilon'_K/\varepsilon_K$	—	*
Δm_B	—	*
$b \rightarrow s\gamma$	*	*

Table 2: summary of flavor and CP violating signals:

- $\surd\surd$ very important searches
- $\surd$ significant searches
- — not relevant
- * constraint on parameters
- ** dominant constraint

are dominated by the top quark coupling of the unified theory, and because the flavor mixing matrices are simply related to the KM matrix. In more general models one expects that

- The flavor mixing matrices at the gaugino vertices have the same hierarchical pattern of mixing as the KM matrix, but have entries which differ numerically from those of the KM matrix.
- The squark and slepton masses may receive important radiative corrections to their mass matrices from couplings in the unified theory other than λ_t.

How will our conclusions be modified for these theories? The differing flavor mixing matrices increase the uncertainties in the amplitudes. Hence, the relative importance of ε_K, $b \rightarrow s\gamma$, Δm_B and $\mu \rightarrow e\gamma$ may change, causing the contours of Figures 2, 3, 4 to shift by, say, factors of 3. This could mean that the modifications to CP violation in B decays are larger (or smaller) than for the minimal models. The additional radiative corrections to the scalar mass matrices will similarly increase uncertainties. Those radiative corrections which produce further non-degeneracies will enhance our effects, while radiative corrections which produce flavor-changing scalar masses could add or subtract to our effects, depending on the signs. Barring some sort of flavor symmetry restoration at M_G, precise cancellations are unlikely, and certainly would not be expected to occur in more than one process. Hence we believe that, to within factors of 2 or 3 in amplitude, the results of this paper can be interpreted as the minimum expected signatures of all models which satisfy the assumptions discussed in the previous section.

The gluino-focussing effect will be present in all theories. It is unaffected by changes in the flavor mixing angles, and its effects are enhanced if the unified theory produces larger squark non-degenerecies than discussed here. Hence we can state very generally that:

(A) Hadronic flavor and CP violating processes exclude only very small regions of parameter space, those with low gluino mass;

(B) For slightly higher values of the gluino mass, there are very interesting contributions, especially to ε_K but also to Δm_B, which could be discovered by the failure of SM fits to these quantities and by future measurements of CP violation in B decays.

(C) Lepton flavor violation, such as $\mu \rightarrow e\gamma$, and electric dipole moments, d_e and d_n, provide the most powerful probe of this flavor physics of unified theories. This is because, unlike the hadronic probes, the signals could be observed over a very wide region of parameter space.

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